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Superradiant instability of Kerr-de Sitter black holes in scalar-tensor theory

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ABSTRACT: We investigate in detail the mechanism of superradiance to render the instability of Kerr-de Sitter black holes in scalar-tensor gravity. Our results provide more clues to examine the scalar-tensor gravity in the astrophysical black holes in the universe with cosmological constant. We also discuss the spontaneous scalarization in the de Sitter background and find that this instability can also happen in the spherical de Sitter configuration in a special style.

KEYWORDS: Black Holes, Classical Theories of Gravity

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1 Introduction

Scalar-tensor (ST) theory is an alternative theory that describes gravity beyond Einstein's general relativity (GR). This theory was originally conceived by Jordan and later generalized by Brans and Dicke [1]. The primary motivation comes from cosmology to search for a theory incorporating Mach's principle which is not explicitly embodied in GR. In the ST theory, the gravitational coupling is determined by all matter in the universe. The cosmological distribution of matter will affect local gravitational experiments [2, 3]. Currently, interests in ST theory come from many aspects. First, a fundamental scalar field coupled to gravity is an unavoidable feature of unified theory such as supergravity, string theory [4], etc. Second, the inflationary scenarios of universe and dark energy may be described by ST theory [2, 3].

Experimentally, the differences between ST theory and GR are too small to be detected in the weak field regime, i.e, in the solar system [5, 6]. Particular interest of the phenomenology of the ST theory is in the strong gravity regime. It is expected that the strong gravity test can be a possible way to distinguish the ST theory from GR.

For compact stars in ST theory, unexpected phenomenon has been discovered compared with GR [7–11]. For the black hole (BH) background in ST theory, when the scalar field settles to a constant, we can get the same BH solution as that in GR [12, 13]. However the perturbations will behave differently as the two theories have different dynamics [14, 15]. It has been shown that compared with the Kerr BH background in GR, the existence of

a scalar mode in the spectrum of perturbations around a Kerr BH in the ST theory leads to remarkable effects [16, 17]. When the matter configuration surrounding the Kerr black hole is dense enough, the BH will be forced to develop scalar hair. On the other hand, when the BH rotates sufficiently fast, superradiant instability can occur. Neither of these instabilities has been observed in the Kerr BH solution background in GR. This gives the hope that BHs can be used as probes in order to distinguish ST theory from GR.

Considering that our universe has a positive cosmological constant [18], we will extend the discussions in the asymptotically flat Kerr BH backgrounds [16, 17] to Kerr-de Sitter configurations. We will examine in detail the mechanism that can render the instability of Kerr-de Sitter BHs driven by superradiance in the ST gravity. To trigger superradiant instability, two necessary conditions need to be satisfied [19], namely that the BH must have superradiance and the existence of a potential well outside the BH to trap the scattered wave. The superradiant instability in GR has been discussed thoroughly, see review [20] and references therein. In this work, we will examine how the instability depends on the matter profile, the rotation of the BH and the cosmological constant. Furthermore we will investigate the spontaneous scalarization in the Kerr-de Sitter BH background. Comparing with the stability analysis in GR [21] where no instability was found for Kerr-de Sitter BH configurations, our results will provide more possible potential observational clues to distinguish the ST gravity from GR in the BH backgrounds in the real universe.

The organization of the paper is as follows. In section II, we will briefly present the framework of ST theory and describe the Kerr-de Sitter BH. In section III, we will derive the perturbation equation of the scalar field. In section IV, we will calculate the frequencies of the perturbation numerically and list our results. In section V, we will discuss the spontaneous scalarization for spherical de Sitter BHs. The last section is devoted to discussion and summary.

2 Kerr-de Sitter black hole in scalar-tensor theory

The general action of the ST theory in the Jordan frame is [2, 3]

$$S = \frac{1}{16\pi} \int d^4x \sqrt{-g} (F(\phi)R^J - Z(\phi)g_{\mu\nu}\partial^\mu\phi\partial^\nu\phi - U(\phi)) + S_m(\Psi_m; g_{\mu\nu}), \tag{2.1}$$

where R^J is the Ricci scalar of metric $g_{\mu\nu}$, ϕ is a scalar field and $U(\phi)$ is the scalar field potential. $S_m(\Psi_m; g_{\mu\nu})$ is the action describing matter Ψ_m which is minimally coupled to $g_{\mu\nu}$ and ϕ . Functions F , Z and U represent specific theories within the class, up to a degeneracy due to the freedom of redefining the scalar [22]. When $F = \phi$, $Z = \omega_0/\phi$ and $U = 0$, one gets the Brans-Dicke theory [1]. The ST theory can be viewed as a low-energy limit of a bosonic string theory when $F = \phi$, $Z = -\phi^{-1}$.

Performing a conformal transformation and the field redefinition

$$g_{\mu\nu}^E = F(\phi)g_{\mu\nu},$$

$$\Phi(\phi) = \frac{1}{\sqrt{4\pi}} \int d\phi \left(\frac{3}{4} \frac{F'(\phi)^2}{F(\phi)^2} + \frac{1}{2} \frac{Z(\phi)}{F(\phi)} \right)^{1/2}, \tag{2.2}$$

$$A(\Phi) = F^{-1/2}(\phi),$$

$$V(\Phi) = \frac{U(\phi)}{F^2(\phi)},$$

we get the action in the Einstein frame (For more details, see [2, 3])

$$S = \frac{1}{16\pi} \int d^4x \sqrt{-g^E} (R^E - 8\pi g_{\mu\nu}^E \partial^\mu \Phi \partial^\nu \Phi - V(\Phi)) + S_m(\Psi_m; A(\Phi)^2 g_{\mu\nu}^E). \quad (2.3)$$

In Einstein frame, the scalar field is minimally coupled to gravity, but the matter field is coupled to the metric $A(\Phi)^2 g_{\mu\nu}^E$. The field equations in the Einstein frame can be derived by varying action (2.3).

$$G_{\mu\nu}^E = 8\pi \left(T_{\mu\nu}^E + \partial_\mu \Phi \partial_\nu \Phi - \frac{g_{\mu\nu}^E}{2} (\partial\Phi)^2 \right) - \frac{g_{\mu\nu}^E}{2} V(\Phi), \quad (2.4)$$

$$\nabla_\mu \nabla^\mu \Phi = -\frac{A'(\Phi)}{A(\Phi)} T^E + \frac{V'(\Phi)}{16\pi}. \quad (2.5)$$

Here $T_{\mu\nu}^E \equiv -\frac{2}{\sqrt{-g^E}} \frac{\delta S_m}{\delta g_{\mu\nu}^E}$. We assume that $\Phi = \Phi_0 = const$ is a solution to the above equations and around Φ_0 we have the following analytical expansions

$$V(\Phi) = \sum_{n=0} V_n (\Phi - \Phi_0)^n, \quad (2.6)$$

$$A(\Phi) = \sum_{n=0} A_n (\Phi - \Phi_0)^n. \quad (2.7)$$

For abbreviations, we denote $\varphi \equiv \Phi - \Phi_0$ and $\alpha_n \equiv A_n/A_0$. Then to the order $\mathcal{O}(\varphi)$, the equations of motion become

$$G_{\mu\nu}^E + \frac{V_0}{2} g_{\mu\nu}^E = 8\pi T_{\mu\nu}^E - \frac{g_{\mu\nu}^E}{2} V_1 \varphi, \quad (2.8)$$

$$\nabla_\mu \nabla^\mu \varphi = -\alpha_1 T^m + \frac{V_1}{16\pi} + (\alpha_1^2 - 2\alpha_2) T^m \varphi + \frac{V_2}{8\pi} \varphi. \quad (2.9)$$

The term V_0 is related to the cosmological constant $\Lambda \equiv \frac{V_0}{2}$. V_1 is a linear potential which will be dropped off since linear potential is seldom met in nature. V_2 is related to a standard mass term. α_1 describes the effective coupling between the scalar and matter. To have a constant scalar solution $\Phi = \Phi_0 = const$ as we mentioned above, from (2.9) we should set $\alpha_1 = 0$. In addition, observations, such as weak gravity constraints and tests for violation of the strong equivalence principle, seem to require α_1 to be negligibly small [23, 24]. The configuration with constant scalar and $\alpha_1 \simeq 0$ was argued most likely as an approximate solution in most viable ST theories [16, 17]. Finally, the equations of motion can be reduced to

$$G_{\mu\nu}^E + \Lambda g_{\mu\nu}^E = 8\pi T_{\mu\nu}^E, \quad (2.10)$$

$$\nabla_\mu \nabla^\mu \varphi = \left(\frac{V_2}{8\pi} - 2\alpha_2 T^m \right) \varphi. \quad (2.11)$$

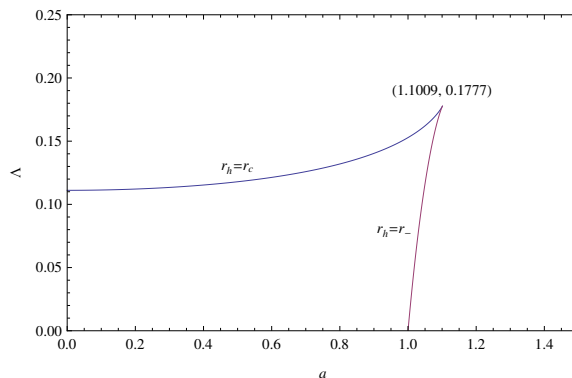


Figure 1. Parameter space for Kerr-de Sitter BH in $a - \Lambda$ plane with fixed $M = 1$. Kerr-de Sitter BH solutions are allowed for the parameters (a, Λ) in the region between the two curves.

Here $\mu_s^2 \equiv \frac{V_2}{8\pi} - 2\alpha_2 T^m$ plays the role of effective mass square of the scalar field. When the backreaction of the matter field on the spacetime geometry is negligible (“the probe limit”), from (2.10) we can have the Kerr-de Sitter BH as a solution. In Boyer-Lindquist coordinates, the metric reads

$$\begin{aligned}
 ds^2 = & -\frac{\Delta_r}{(1+\alpha)^2 \rho^2} (dt - a \sin^2 \theta d\phi)^2 + \frac{\Delta_\theta \sin^2 \theta}{(1+\alpha)^2 \rho^2} (adt - (a^2 + r^2)d\phi)^2 \\
 & + \frac{\rho^2}{\Delta_r} dr^2 + \frac{\rho^2}{\Delta_\theta} d\theta^2,
 \end{aligned} \tag{2.12}$$

in which

$$\begin{aligned}
 \alpha &= \frac{\Lambda a^2}{3}, \Delta_\theta = 1 + \alpha \cos^2 \theta, \rho^2 = r^2 + a^2 \cos^2 \theta, \\
 \Delta_r &= (a^2 + r^2) \left(1 - \frac{\alpha}{a^2} r^2\right) - 2Mr.
 \end{aligned} \tag{2.13}$$

For some ranges of parameters, $\Delta_r = 0$ exists four real roots. Three positive real roots correspond to three horizons: the inner BH Cauchy horizon r_- , the outer BH event horizon r_h and the cosmological horizon r_c . The remaining negative root has no physical meaning. We focus only on cases in which $\Delta_r = 0$ has four real roots, so that we have Kerr-de Sitter BH. In figure 1, we plot the allowed parameter space. We can see that values of (a, Λ) should lie in the region between two curves where we fix $M = 1$. The upper nearly horizontal curve corresponds to the degeneracy of the cosmological horizon and the BH event horizon $r_h = r_c$, while the lower nearly vertical curve represents the degeneracy between the BH event horizon and the Cauchy horizon $r_h = r_-$. The two curves intersect at the point $(a, \Lambda) \sim (1.1009, 0.1777)$, at which $r_- = r_h = r_c$ occurs.

The surface gravities κ and angular velocities Ω on horizons are [21]

$$\kappa_i = \frac{1}{2(1+\alpha)(r_i^2 + a^2)} \frac{d\Delta_r(r_i)}{dr}, \quad \Omega_i = \frac{a}{r_i^2 + a^2}, \tag{2.14}$$

where $r_i = \{r_h, r_c\}$.

In this paper, we will mainly focus on this Kerr-de Sitter BH configuration in the probe limit and study its superradiant stability under scalar perturbation. Different from the stability analysis in [21], we will look for separable solutions of the Klein-Gordon equations with effective mass μ_s , not just the canonical mass of a massive scalar field. The effective mass reflects the influence of the ST theory in the presence of matter surrounding the BH.

3 Perturbation equations and superradiance

3.1 Perturbation equations

The dynamics of the scalar field in Kerr-de Sitter background is described by (2.11). To make comparison with the result of Kerr BH in the asymptotically flat spacetime reported in [16, 17], we take the following form for the effective mass

$$\mu_s^2(r) = \frac{2\Lambda}{3} + \frac{G(r)}{\rho^2}, \quad G(r) = \beta\Theta(r - r_0)(r - r_0) \left(\frac{1}{r^n} - \frac{1}{r_c^n} \right), \quad (3.1)$$

which can reduce to the form in Kerr case [16, 17] when the cosmological constant $\Lambda \rightarrow 0$. Here $\Theta(r - r_0)$ is a heaviside function, r_0 is the place where the matter distribution starts from and β indicates the strength of coupling between scalar and matter field. Note that $\beta \propto \alpha_2$ in (2.11) and large positive values of α_2 are not constrained by observations, so we can take arbitrary large β . In fact, we will find that only large enough β can trigger the instability in the next section. The term $\frac{2\Lambda}{3}$ plays the role of the canonical mass term of a massive scalar. For Kerr-de Sitter background, it equals to $\frac{1}{6}R_g$ where R_g is the Ricci scalar of spacetime. So eq. (2.11) is equivalent to the equation of motion of a massive conformally coupled scalar.

The Klein-Gordon equation (2.11) is separable when effective mass (3.1) is adopted. By assuming a separation $\varphi = R(r)S(\theta)e^{-i\omega t + im\phi}$, the angular part of (2.11) turns out to be

$$\frac{\partial_\theta(\sin\theta\Delta_\theta\partial_\theta S)}{\sin\theta S} - (1 + \alpha)^2 \frac{B^2}{\Delta_\theta} - 2\alpha \cos^2\theta = -\lambda, \quad (3.2)$$

with λ the separation constant and $B \equiv a\omega \sin\theta - \frac{m}{\sin\theta}$. For Schwarzschild BH, $a = \alpha = 0$, the separation constant $\lambda = l(l - 1)$ in which l is the angular momentum number. For more general BHs, there is no explicit analytical expression of λ . For Kerr spacetime, $\Delta_\theta = 1$ and the above equation becomes a spheroidal equation which has been studied in detail in [25–32]. And λ can be expanded as a series of $a\omega$ for small $a\omega$, whose explicit form can be found in [26, 27, 32]. For Kerr-de Sitter BH, eq. (3.2) is a generalized spheroidal equation and the explicit expansion of separation constant λ in small $a\omega$ is [33, 34]

$$\begin{aligned} \lambda = & l(l + 1) + \alpha(-l(l + 1) + 2m^2 - l^2H(l) + (l + 1)^2H(l + 1)) \\ & + [-2m - 2m\alpha(1 - H(l) + H(l + 1))]a\omega \\ & + \left[H(l + 1) - H(l) + \alpha \left(H(l + 1) - H(l) + 2((l + 1)^2H(l + 1) - l^2H(l)) \right. \right. \\ & \left. \left. - lH^2(l) + (l + 1)H^2(l + 1) - \frac{H(l)H(l + 1)}{l(l + 1)} \right) \right] (a\omega)^2 + \mathcal{O}((a\omega)^3), \end{aligned} \quad (3.3)$$

with $H(l) \equiv \frac{2(l^2-m^2)l}{(2l-1)(2l+1)}$ and l being the angular momentum which takes an integer or half integer number satisfying $l \geq m$.

The radial part of (2.11) is

$$\Delta_r \frac{d}{dr} \left(\Delta_r \frac{d}{dr} R \right) + \left[(1 + \alpha)^2 K^2 - \Delta_r \left(\frac{2\alpha}{a^2} r^2 + G(r) + \lambda \right) \right] R = 0, \quad (3.4)$$

where $K = \omega(a^2 + r^2) - am$.

We look for the complex frequencies of the perturbation $\omega = \omega_R + i\omega_I$. We will concentrate on looking for unstable modes with $\omega_I > 0$, which signal the instability with the growing behavior of the perturbations. To calculate the perturbation frequencies, we need to impose suitable boundary conditions,

$$R \rightarrow \begin{cases} (r - r_h)^{-i(\omega - m\Omega_h)/2\kappa_h} & r \rightarrow r_h, \\ (r - r_c)^{i(\omega - m\Omega_c)/2\kappa_c} & r \rightarrow r_c. \end{cases} \quad (3.5)$$

This boundary condition is physically acceptable, which corresponds to a purely ingoing wave at the BH event horizon and an outgoing wave at the cosmological horizon. In asymptotically flat spacetime, for example the Kerr BH, there is no cosmological horizon and the outgoing wave condition is put at the infinity. It was argued in [35, 36] that when the conditions at infinity in asymptotically flat BH are altered in de Sitter black hole, the usual power-law scenario in the late time tail of the perturbation in asymptotically flat BH does not necessarily survive in the de Sitter background. Thus it is of interest to examine the superradiant stability in the Kerr-de Sitter configuration and compare with the result disclosed in the Kerr BH [16, 17].

In general, it is difficult to solve the perturbation equations analytically. So special numerical techniques have been developed to calculate the perturbations of BHs [37, 38], such as Pöschl-Teller potential method [39–41], finite difference method [42–44], WKB approximation [45–47], continued fraction method [28–30], direct integration method (also called shooting method) [48, 49], etc. In this paper, we will use the direct integration method to calculate the frequencies of the perturbations. The algorithm of this method can be described briefly as follows: with the boundary conditions (3.5), we can perform two integrations: (1) solve the radial equation from the BH event horizon by integration to a matching point $r = r_m$; (2) solve the radial equation from cosmological horizon to the matching point. Then by matching the two integration results, we can derive the frequencies of the perturbations. The numerical results are given in the next section.

3.2 Superradiance condition

Before doing numerical calculations, let us briefly derive the superradiant condition for Kerr-de Sitter BH. Taking the following variables

$$dr_* = \frac{r^2 + a^2}{\Delta_r} dr, \quad \Psi = \sqrt{r^2 + a^2} R, \quad (3.6)$$

the radial equation (3.4) can be written in the form of Schrödinger equation

$$\frac{d^2 \Psi}{dr_*^2} + (\omega^2 - V) \Psi = 0, \quad (3.7)$$

in which the effective potential reads

$$\begin{aligned}
 V &= \omega^2 - \frac{V_0}{(r^2 + a^2)^2} + \frac{\Delta_r}{(r^2 + a^2)^3} \left(r \frac{d\Delta_r}{dr} + \frac{a^2 - 2r^2}{r^2 + a^2} \Delta_r \right), \\
 V_0 &= (1 + \alpha)^2 K^2 - \Delta_r \left(\frac{2\alpha}{a^2} r^2 + G(r) + \lambda \right).
 \end{aligned}
 \tag{3.8}$$

In a scattering experiment, solution of (3.7) has the following asymptotic behavior

$$\Psi \sim \begin{cases} \mathcal{T} e^{-i(1+\alpha)(\omega - m\Omega_h)r_*} & \text{as } r_* \rightarrow -\infty (r \rightarrow r_h), \\ e^{-i(1+\alpha)(\omega - m\Omega_c)r_*} + \mathcal{R} e^{i(1+\alpha)(\omega - m\Omega_c)r_*} & \text{as } r_* \rightarrow \infty (r \rightarrow r_c). \end{cases}
 \tag{3.9}$$

The above boundary conditions correspond to an incident wave of unit amplitude, $e^{-i(1+\alpha)(\omega - m\Omega_c)r_*}$, plunges in from the cosmological horizon and gives rise to a reflected wave of amplitude \mathcal{R} going back to the cosmological horizon and a transmitted wave of amplitude \mathcal{T} at the BH event horizon. Considering that the effective potential is real, so Ψ^* (complex conjugate of Ψ) is also a solution of (3.7). The Wronskian of the two linearly-independent solutions, $W(\Psi, \Psi^*) \equiv \Psi \frac{d}{dr_*} \Psi^* - \Psi^* \frac{d}{dr_*} \Psi$, is a constant independent of r_* . Equating values of the Wronskian at the BH event horizon and at the cosmological horizon, we get

$$1 - |\mathcal{R}|^2 = \frac{\omega - m\Omega_h}{\omega - m\Omega_c} |\mathcal{T}|^2.
 \tag{3.10}$$

Then we can see that if $(\omega - m\Omega_h)(\omega - m\Omega_c) < 0$, we have $|\mathcal{R}|^2 > 1$. This means that the amplitude of the reflected wave is larger than that of the incident wave, and superradiance phenomenon occurs. So we get the condition to trigger the superradiance [50, 51]

$$m\Omega_c < \omega < m\Omega_h.
 \tag{3.11}$$

When there is no cosmological constant, from (24) and (25), we see that the superradiant condition goes back to that for the Kerr BH.

4 Numerical results of the superradiant instability

We list our numerical results in this section. We fix the BH mass parameter $M = 1$, $n = 3$ in the matter profile and the angular indexes $l = m = 1$. There are four free parameters left to affect the numerical results, namely the cosmological constant Λ , the angular momentum per unit mass a , the location of the matter shell r_0 and the coupling between matter and the scalar field β . To see the effect of rotation on the instability, in the following we will consider three cases with $a = 0.99, 0.7, 0.3$, respectively. And in each case, we will choose some values of Λ to see the influence of the cosmological constant on the instability.

4.1 $a = 0.99$

In this subsection, we fix the angular momentum per unit mass of the BH and examine the influences of the other parameters on the superradiant stability.

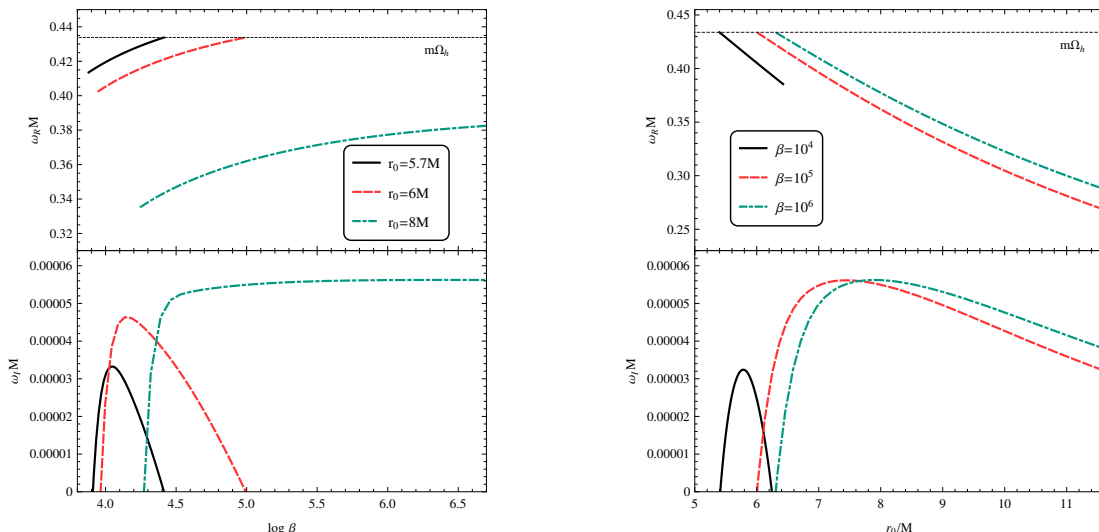


Figure 2. Superradiant instability details for Kerr BH. The outer BH event horizon $r_h = 1.14107$ and the angular velocity on the outer horizon $\Omega_h = 0.4338$.

In figure 2, we repeat the result of the superradiant instability in the Kerr BH background and we find that our numerical results are in good agreement with that reported in [16, 17]. This gives us confidence in generalizing our numerical calculation to examine the stability in the Kerr-de Sitter backgrounds.

From figures 3–5, we list the superradiant instability details for the Kerr-de Sitter black hole backgrounds with the cosmological constant $\Lambda = 0.0003, 0.03, 0.1$ respectively. The results are similar to that in the Kerr black hole case. When the real parts of the perturbation frequencies satisfy the superradiant condition (3.11), the imaginary parts of the frequencies have positive values, indicating the existence of the superradiant instability. In the figures, since Ω_c is too small at the cosmological horizon, it is not shown.

In the Kerr-de Sitter BH backgrounds, we see that with the increase of the cosmological constant, the BH event horizon r_h increases, which leads the angular velocity on the BH event horizon to decrease for the BH with the same a [21, 52–54]. The decrease of the upper bound in the superradiant condition (25) results in the decrease of the real part of the perturbation frequencies. This can be seen clearly from figures 3–5. In addition, we can read from figures 3–5 that the imaginary part of the frequencies also decreases with the increase of the cosmological constant in the Kerr-de Sitter background. Thus the superradiant instability is quenched with the increase of the cosmological constant. Physically this can be understood by looking at the effective potential in figure 6. Fixing parameters a, r_0, β , we see that with the increase of the cosmological constant in the Kerr-de Sitter BH background, the potential well becomes shallower and the barrier to reflect radiation back becomes lower. This leads to fewer outgoing waves to be reflected back and stored in the potential well, which explains why the superradiant instability is quenched for large cosmological constant.

Comparing figures 3–5, we find that to compensate the influence brought by increasing the cosmological constant, we need larger β or smaller r_0 to trigger the superradiant in-

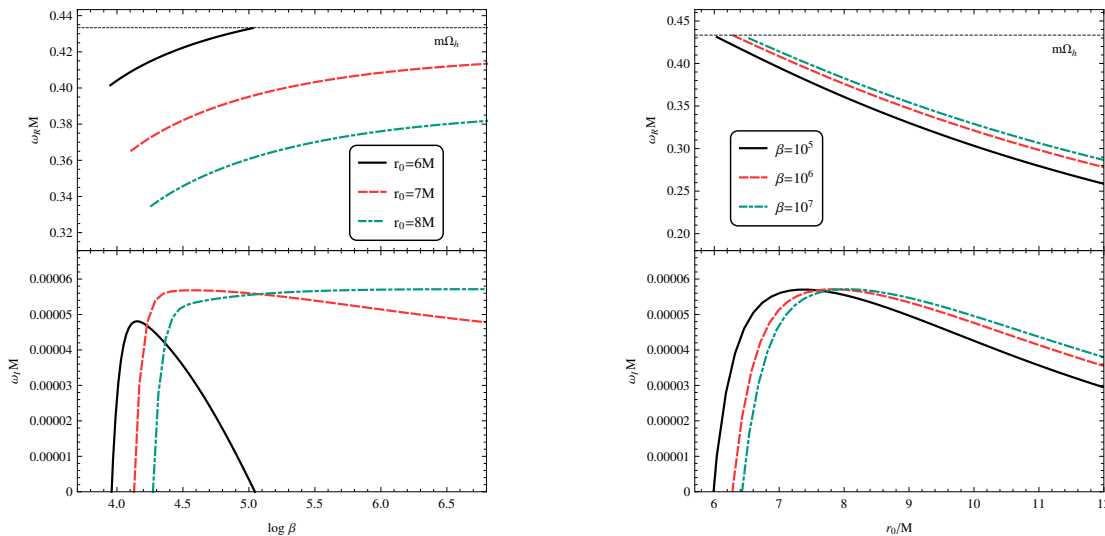


Figure 3. Superradiant instability details for Kerr-de Sitter BH with $\Lambda = 0.0003$. The BH event horizon and cosmological horizon locate at $r_h = 1.1421$, $r_c = 98.9447$. The angular velocity on the BH horizon $\Omega_h = 0.4333$.

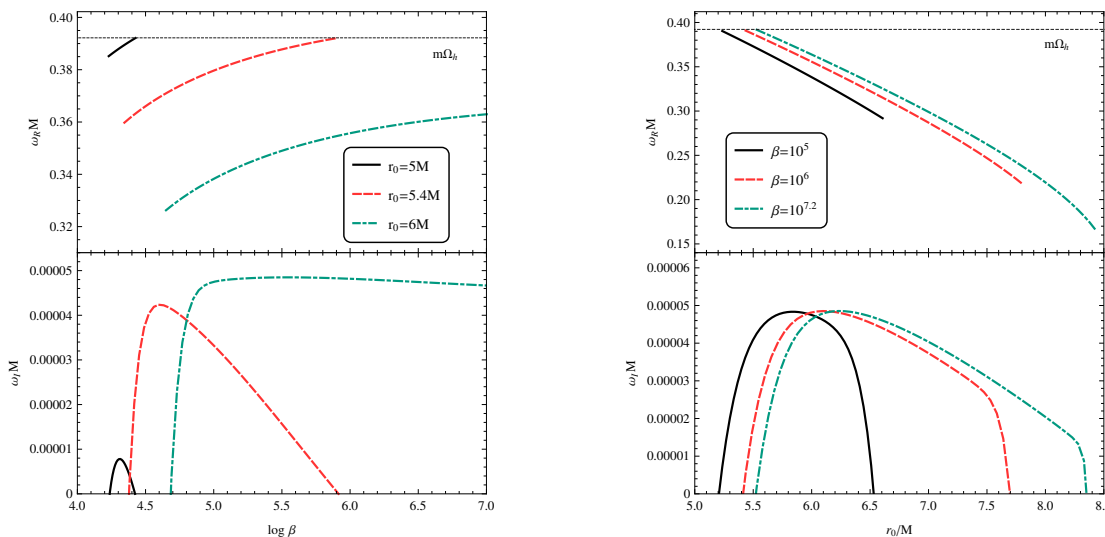


Figure 4. Superradiant instability details for Kerr-de Sitter BH with $\Lambda = 0.03$. The BH event horizon and cosmological horizon locate at $r_h = 1.2426$, $r_c = 8.8077$. The angular velocity on the BH horizon $\Omega_h = 0.3922$.

stability in the Kerr-de Sitter background. From (3.1), we can see that larger β or smaller r_0 leads to bigger effective mass and so that higher effective potential barrier. Objective pictures of the effective potentials due to the influences of β and r_0 are shown in figure 7. It is clear that for fixed r_0 , the increase of β increases the potential barrier height which is able to reflect more radiations back. Fixing β , the decrease of r_0 has the same effect. Besides if we look closely at the potentials in figure 7 near the BH event horizon, the first potential barrier is higher so that the developed potential well is deeper for bigger β as well

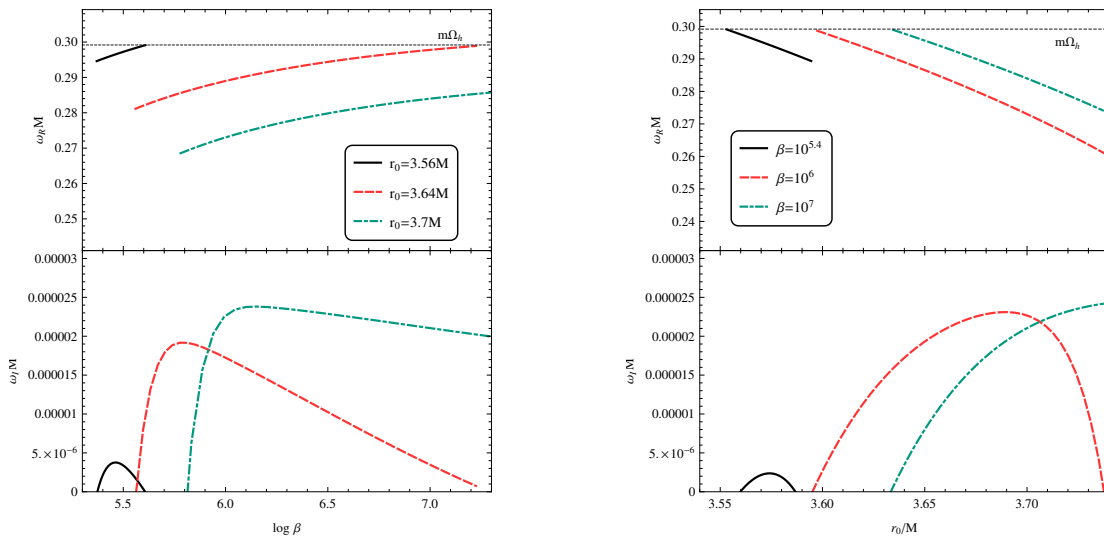


Figure 5. Superradiant instability details for Kerr-de Sitter BH with $\Lambda = 0.1$. The BH event horizon and cosmological horizon locate at $r_h = 1.5261$, $r_c = 3.9722$. The angular velocity on the BH horizon $\Omega_h = 0.2992$.

as for smaller r_0 . Thus more outgoing wave can be reflected back by the higher potential wall and accumulated in the deeper potential well. This accounts for the phenomenon that with the increase of the cosmological constant we need to count on bigger β and smaller r_0 to spark off the superradiant instability.

From figures 3–5 we also observe some similar properties to those of the Kerr BH reported in [16, 17]. When the matter shell is too close to the Kerr-de Sitter BH with very small r_0 , there is no superradiant instability, since the real part of the frequencies scales as $1/r_0$ and too small r_0 will make the frequency violate the superradiant condition. If r_0 is not small enough, the superradiant instability can appear, but this instability will be quenched as soon as the superradiant condition is saturated. The value of r_0 where the instability starts to appear decreases with the increase of the cosmological constant adopted. When r_0 is big, the positive imaginary part of the frequencies can last for sufficiently large β . This is because that once β is big enough to provide high enough potential barrier to bounce back the outgoing wave, further increasing β to raise the height of the potential wall will not contribute more to the instability. We find that the above mentioned properties keep for all values of the cosmological constant in the Kerr-de Sitter BH background.

4.2 $a = 0.7$ and $a = 0.3$

In this subsection we will fix $\Lambda = 0.03$ and show the results of superradiant instability for $a = 0.7$ and $a = 0.3$. Combining figures 8,9 and figure 4, we can show the influence of the angular momentum per unit mass on the superradiant instability for the Kerr-de Sitter BH background.

From figures 8,9 and combining with figure 4 for $a = 0.99$, we can see that when a decreases, the upper limit for the superradiant condition decreases which forces the upper value of the real part of the frequency to decrease. Considering that the real part of

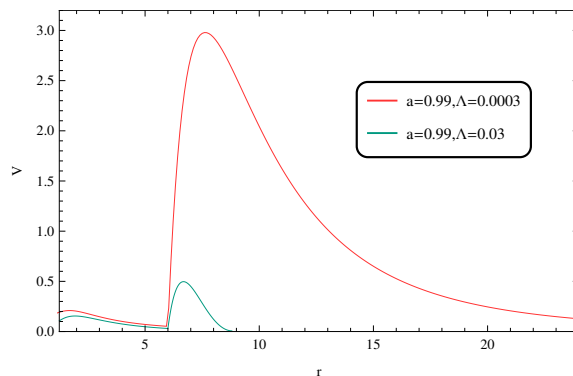


Figure 6. Effective potential behavior for the change of the cosmological constant. We fixed $a = 0.09, \beta = 10^{4.81}, r_0 = 6M$.

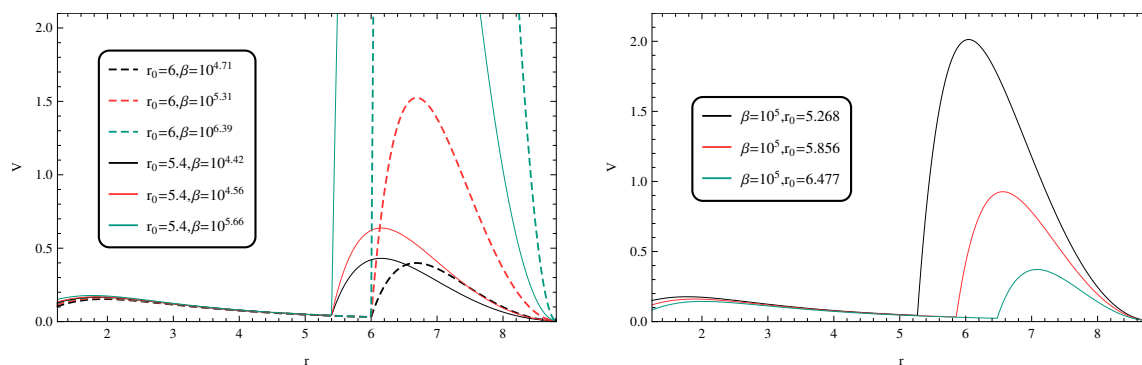


Figure 7. (Left) Effective potential behavior for the change of β . We fixed $a = 0.09, \Lambda = 0.03$. The solid lines are for $r_0 = 5.4$ and dashed lines are for $r_0 = 6$. (Right) Effective potential behavior for the change of r_0 . We fixed $a = 0.99, \Lambda = 0.03, \beta = 10^5$.

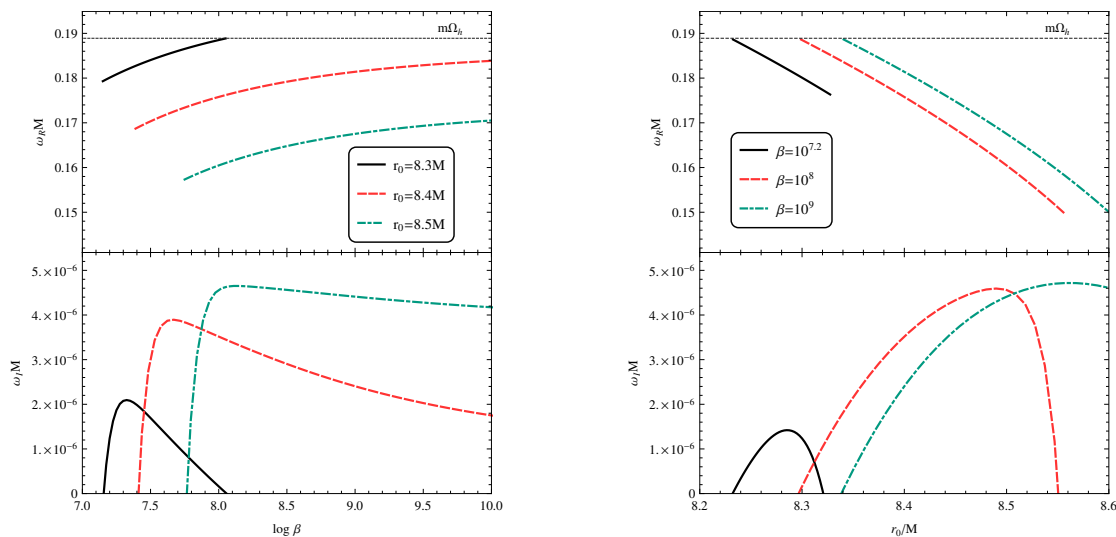


Figure 8. Superradiant instability details for Kerr-de Sitter BH with $a = 0.7, \Lambda = 0.03$. The BH event horizon and cosmological horizon locate at $r_h = 1.7932, r_c = 8.7984$. The angular velocity on the black hole horizon $\Omega_h = 0.1889$.

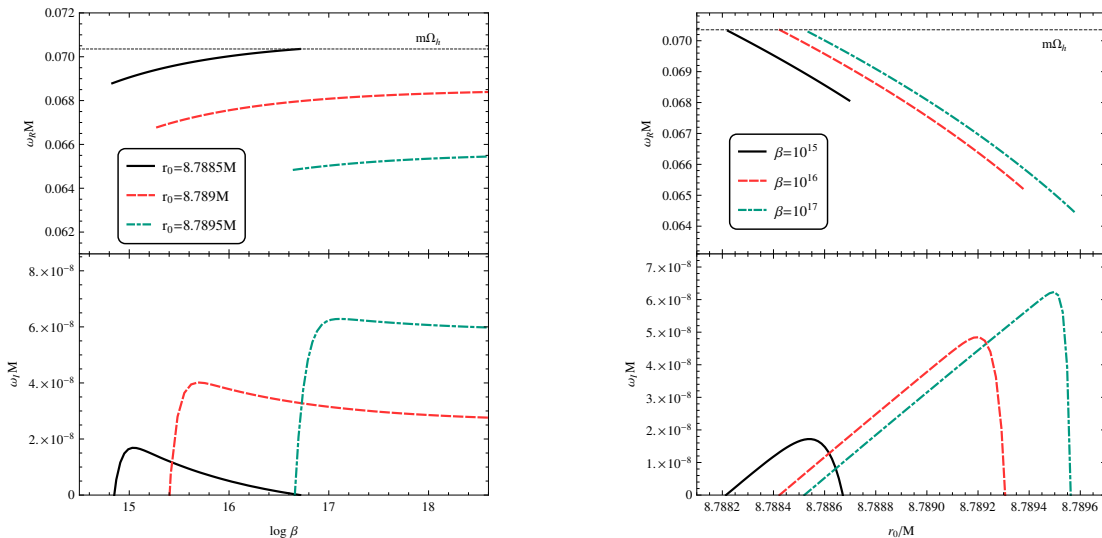


Figure 9. Superradiant instability details for Kerr-de Sitter BH with $a = 0.3$, $\Lambda = 0.03$. The BH event horizon and cosmological horizon locate at $r_h = 2.0431$, $r_c = 8.7906$. The angular velocity on the black hole horizon $\Omega_h = 0.0704$.

the frequency scales with $1/r_0$, we learn that for the Kerr-de Sitter BH with the same cosmological constant but slower rotation, the superradiant instability can occur when the matter shell is put a bit further away with bigger r_0 . Furthermore we observe that with the decrease of a , the imaginary part of the frequency falls closer to zero, which indicates that the instability becomes milder. This is understandable because when the BH rotates slower, it will be harder to extract the rotational energy through superradiance. Fixing Λ, r_0, β , we show the potential behavior with the change of a in figure 10. It is clearly shown that with the decrease of a , the potential barrier to reflect the outgoing wave becomes lower and the potential well becomes shallower. This can be used to account for the observation that lower a makes the superradiant instability milder. We further exhibit the property of the potential when the imaginary part of the frequency reaches the peak for small β in figures 4,8,9 in the left panel of figure 12. For the limiting (r_0, β) to trigger the strongest superradiant instability, we see that for the lower a we have shallower potential barrier to store the reflected perturbation. This again explains the reason that the superradiant instability is weaker when a is smaller.

4.3 The minimum a to accommodate the superradiant instability

In the last subsection, we observe that lowering a will make superradiant instability milder. We can see this from figure 9 where $a = 0.3$ and $\Lambda = 0.03$. In that case, we need large enough β to trigger instability. The physical reason is that it is harder to extract the rotational energy from the Kerr-de Sitter BH background when a is small.

Now the question comes, whether there is the minimum value of a to allow the superradiant instability? We have calculated the result, for $\Lambda = 0.1$, $a_{\min} \sim 0.54$. We calculate the corresponding unstable modes for this critical case, and the results are shown

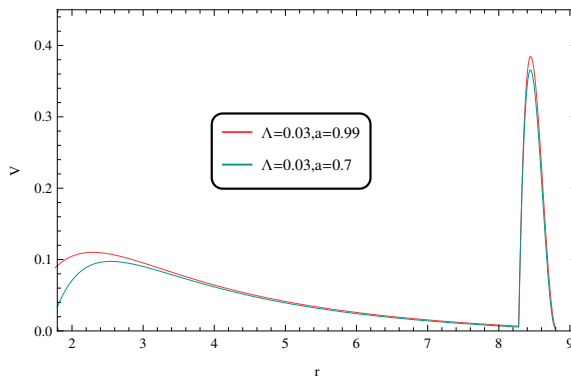


Figure 10. Effective potential behavior with the change of a , where we fix $\Lambda = 0.03, \beta = 10^{7.2}, r_0 = 8.28$.

in figure 12. Comparing to figure 5 with $a = 0.99$, we can see that β is needed to be very large to spark off superradiant instability and the instability is very mild with very small ω_I . Below this a_{\min} , no matter how big is the coupling between the matter and the scalar, the superradiant instability will not happen. For the Kerr BH background, we have $a_{\min} \sim 0.18$. We see that a_{\min} increases with the increase of the cosmological constant Λ . This is reasonable because the increase of Λ makes the superradiant instability harder to occur as we discussed above. From the right panel of figure 11, we illustrate the potential behavior of the minimum a for $\Lambda = 0.1$ to spark off superradiant instability. We see that at the minimum a , the potential is very flattened and the potential well to store the reflected outgoing perturbation is very shallow. Although the potential wall is very high to bounce back the outgoing perturbation, the perturbation will be harder to accumulate near the BH to cause the superradiant instability. This accounts for the very mild instability at this critical case. Below this a_{\min} , all the reflected outgoing perturbation will fall inside the BH without any obstacle.

5 Spontaneous Scalarization

In [16, 17], the authors uncovered a new instability caused by the distribution of matter around BHs. When the matter configuration is dense enough, the BH is forced to develop scalar hair. This is called the spontaneous scalarization. In their argument, they considered that the matter distribution is spherically symmetric and its backreaction on the geometry is negligible. In this probe limit the background metric is a Schwarzschild BH. They have constructed nonlinear, hairy solutions of ST theories with a Schwarzschild BH at the center.

In this section, we will generalize their argument to the spherically symmetric de Sitter BHs. We focus on the four-dimensional Schwarzschild-de Sitter BH at the center sounded by spherical shell of scalar field. Adopting the decomposition of the scalar field $\varphi(t, r, \theta, \phi) = \sum_{lm} e^{-i\omega t} Y_{lm}(\theta, \phi) \frac{\Psi_{lm}(r)}{r}$, the radial wave equation of the scalar field obeys

$$\frac{d^2 \Psi_{lm}(r)}{dr_*^2} + [\omega^2 - V(r)] \Psi_{lm}(r) = 0, \tag{5.1}$$

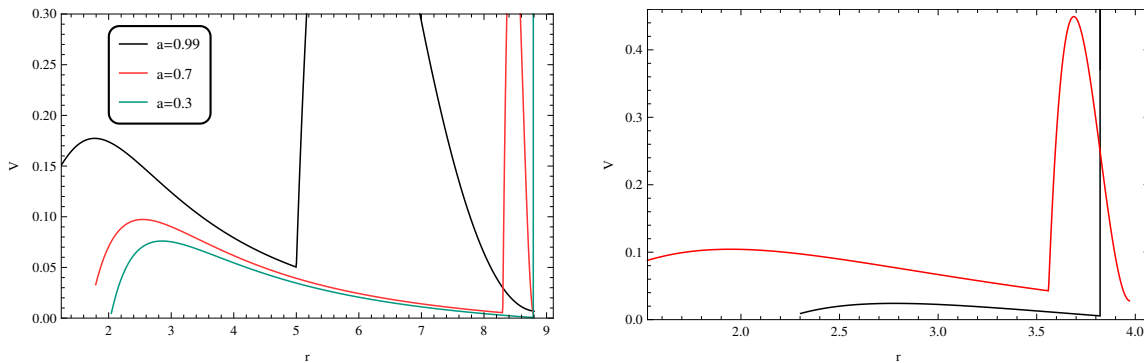


Figure 11. (Left) Effective potential behaviors with the change of a . We fix $\Lambda = 0.03$ and choose r_0, β values to have the smallest peak of ω_I in figures 4,8,9. For $a = 0.99$, we choose $(r_0, \beta) = (5, 10^{4.31})$; for $a = 0.7$, we select $(r_0, \beta) = (8.3, 10^{7.325})$; and for $a = 0.3$, we adopt $(r_0, \beta) = (8.7885, 10^{15.03})$. (Right) Effective potential behavior for the minimum a to accommodate superradiant instability when $\Lambda = 0.1$ (the black one). For comparison we also plot the line when $a = 0.99$ and $\Lambda = 0.1$ (the red one). We choose $(r_0, \beta) = (3.56, 10^{5.46})$ for $a = 0.99$ and $(r_0, \beta) = (3.82156, 10^{14.3879})$ for $a = 0.54$, respectively.

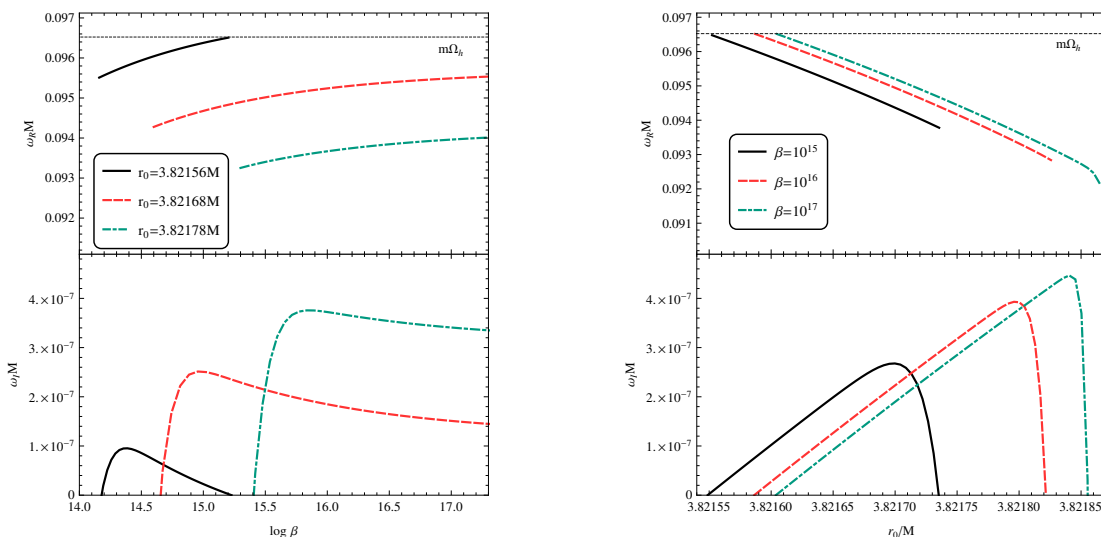


Figure 12. Superradiant instability details for Kerr-de Sitter BH with $a = 0.54, \Lambda = 0.1$. The BH event horizon and cosmological horizon locate at $r_h = 2.30281, r_c = 3.82226$. The angular velocity on the BH horizon $\Omega_h = 0.0965227$.

in which

$$V(r) = f \left(\frac{l(l+1)}{r^2} + \frac{1}{r} \frac{df}{dr} + \mu_s^2(r) \right), \tag{5.2}$$

$$f = 1 - \frac{2M}{r} - \frac{\Lambda}{3} r^2.$$

Here we have taken the tortoise coordinate defined as $dr/dr_* = f$. $\Psi_{lm}(r)$ is required to satisfy boundary conditions, namely the outgoing wave at the cosmological horizon and

ingoing wave at the BH event horizon. The frequency ω is complex, $\omega = \omega_R + i\omega_I$. The unstable modes correspond to $\omega_I > 0$.

According to the well known result in quantum mechanics [55], the sufficient condition for this potential to lead to an instability is $\int_{-\infty}^{\infty} V(r) dr_* = \int_{r_h}^{r_c} \frac{V(r)}{f} dr < 0$. Combining with (5.2), this condition becomes

$$\int_{r_h}^{r_c} \mu_s^2(r) dr < -l(l+1) \left(\frac{1}{r_c} - \frac{1}{r_h} \right) - M \left(\frac{1}{r_c^2} - \frac{1}{r_h^2} \right) - \frac{2\Lambda}{3}(r_c - r_h). \quad (5.3)$$

It is obvious that this sufficient condition can be satisfied if the effective mass square is sufficiently negative. It is straightforward to prove that for higher dimensional and charged de Sitter BHs, the instability condition can also be fulfilled if the effective mass square is negative enough.

To understand the instability due to the spontaneous scalarization in the presence of matter, we focus on the final state of spherically symmetric configurations described by

$$ds^2 = -h(r)dt^2 + \frac{1}{f(r)}dr^2 + r^2d\Omega^2. \quad (5.4)$$

The scalar field Φ can be integrated from the Klein-Gordon equation, obeying $\partial_r \Phi = \frac{Q}{r^2 \sqrt{fh}}$, where Q is the scalar charge (the integral constant). The BH event horizon ($f(r_h) = 0$) is surrounded by the matter shell, regularity at the BH event horizon requires $Q = 0$ and $\Phi = \Phi_- = const$ inside the matter shell. For the same reason, at the cosmological horizon out of the matter shell, $f(r_c) = 0$, which also demands $\Phi = \Phi_+ = const$ there. So the scalar field can only be constants both inside and outside the matter shell. However, these two constants, Φ_- and Φ_+ , can be different due to the existence of the matter shell. That is, there is a finite jump of the scalar field over the matter shell, which leads to a nontrivial configuration. This configuration is very different from that discussed in [16, 17] without cosmological horizon but only BH event horizon in the final spherical configuration, which does not have regularity condition out of the matter shell so that the scalar field Φ is not limited to be a constant out of the shell and can develop a coordinate-dependent profile [16, 17].

In our case, due to the constant profile of the scalar field inside and outside the matter shell, the final metric on both sides will take Schwarzschild-de Sitter forms but with different parameters (M, Λ). The cosmological constants Λ_{\pm} on both sides are related to the scalar field by the relation $\Lambda_{\pm} = V(\Phi_{\pm})$. If the matter shell is made of perfect fluid, the surface stress-energy tensor takes the form

$$S_{ab}^E = \sigma u_a u_b + P(\gamma_{ab} + u_a u_b), \quad (5.5)$$

where σ, P denote density and pressure respectively. u_a is the on-shell four velocity and γ_{ab} is the induced metric on the shell. According to the Israel-Darmois junction condition, the jump of the metric over the matter shell can be expressed in terms of the shell composition.

In particular, for a static shell at $r = R$, we have [16, 17, 57]

$$\sigma = -\frac{1}{4\pi R} \left(\sqrt{f_+} - \sqrt{f_-} \right) \tag{5.6}$$

$$P = \frac{1}{8\pi R} \left(-4\pi R\sigma + \sqrt{f_+} \frac{Rf'_+}{2f_+} - \sqrt{f_-} \frac{Rf'_-}{2f_-} \right). \tag{5.7}$$

In summary, in spherical de Sitter BH background, spontaneous scalarization can also happen but in a special style. We should note that the jump of the scalar field over the matter shell discussed above is due to the simplification of the thin shell. In a more realistic configuration, matter will have non-vanishing support as treated in [7–11].

6 Summary and discussion

In this paper, we have studied the stability of Kerr-de Sitter BH surrounded by matter shell in ST theories. We have found that similar to the asymptotically flat Kerr BH [16, 17], there exists superradiant instability in the Kerr-de Sitter BH configurations.

In the Kerr-de Sitter BH background, we have observed that with the increase of the cosmological constant, the superradiant instability becomes harder to be triggered. It needs to put the matter shell closer to the hole and increase the coupling between the matter and scalar field to spark off the instability. We have got the physical understanding of the phenomenon we observed by examining the potential behavior.

Further, we have examined the influence of the angular momentum per unit mass on the superradiant instability. We have disclosed that slower rotation will make the superradiant instability harder to happen. There is a minimum value of the angular momentum per unit mass to allow the appearance of the superradiant instability. This minimum a increases when the cosmological constant becomes bigger. The physical reasons behind this phenomenon have also been explained.

On the superradiant instability study, the difference between asymptotically flat Kerr black hole and Kerr-de Sitter BH is very little. The main reason is that the superradiant instability is mainly influenced by the potential barrier to reflect back the outgoing perturbation and the potential well between the BH horizon and the potential barrier to store the reflected perturbation. It has little to do with the boundary condition at the infinity for the asymptotically flat spacetime or at the cosmological horizon for the de Sitter spacetime.

We have also examined the spontaneous scalarization in the de Sitter BH background. Different from the superradiant instability, now the boundary at the cosmological horizon becomes important in the investigation. Different from the asymptotically flat BH case [16, 17], the regularity condition at the cosmological horizon enforces the scalarization to take a special style, that is the scalar field takes a constant profile outside the shell as well as inside the shell in the spherical de Sitter BH background. Since we only focused on the spherical de Sitter configurations, one question we may ask is that whether a more general scalarization can happen once the de Sitter BH background starts to rotate. Further investigations on this topic are called for.

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