Spin-dependent constraints on blind spots for thermal singlino-higgsino dark matter with(out) light singlets

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ABSTRACT: The LUX experiment has recently set very strong constraints on spin-independent interactions of WIMP with nuclei. These null results can be accommodated in NMSSM provided that the effective spin-independent coupling of the LSP to nucleons is suppressed. We investigate thermal relic abundance of singlino-higgsino LSP in these so-called spin-independent blind spots and derive current constraints and prospects for direct detection of spin-dependent interactions of the LSP with nuclei providing strong constraints on parameter space. We show that if the Higgs boson is the only light scalar the new LUX constraints set a lower bound on the LSP mass of about 300 GeV except for a small range around the half of Z^0 boson masses where resonant annihilation via Z^0 exchange dominates. XENON1T will probe entire range of LSP masses except for a tiny Z^0 -resonant region that may be tested by the LZ experiment. These conclusions apply to general singlet-doublet dark matter annihilating dominantly to $t\bar{t}$. Presence of light singlet (pseudo)scalars generically relaxes the constraints because new LSP (resonant and non-resonant) annihilation channels become important. Even away from resonant regions, the lower limit on the LSP mass from LUX is relaxed to about 250 GeV while XENON1T may not be sensitive to the LSP masses above about 400 GeV.

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1 Introduction

Weakly Interacting Massive Particle (WIMP) has been considered as one of the most attractive candidates for dark matter (DM). WIMPs have been intensively searched for in direct detection experiments and the limits on the DM scattering cross-section on nuclei improved by several orders of magnitude over the last decade. Currently the most constraining limits come from the LUX [1] experiment which will be soon improved by the XENON1T [2] experiment which is expected to be superseded by the LZ [3] experiment in near future. In many theories, WIMP interactions with nuclei are mediated by the Higgs and Z^0 bosons. The electroweak-strength couplings of WIMP to the Higgs and Z^0 bosons were excluded already few years ago by XENON100 [4] as noted in ref. [5]. The recent LUX constraints pushed models to regions of parameter spaces where these couplings are strongly suppressed i.e. to the vicinity of so-called blind spots in direct detection.

Supersymmetric (SUSY) extensions of the Standard Model (SM) generically provide a WIMP candidate in the form of the lightest neutralino which is often the lightest sparticle

(LSP). Some earlier studies of neutralino DM include refs. [6]–[25]. Blind spots for neutralino DM has been identified e.g. in refs. [5, 26–28] for Minimal Supersymmetric Standard Model (MSSM) and in refs. [29, 30] for Next-to-Minimal Supersymmetric Standard Model (NMSSM). Several recent papers emphasized a big impact of new LUX constraints on the spin-independent (SI) scattering cross-section on the parameter space of MSSM [31, 32] and NMSSM [33–35]. A universal conclusion of these papers is that viable points in the parameter space still exist but they reside very close to blind spots for SI scattering cross-section.

In the present article we study implications of the assumption that the SI scattering of the LSP is so small (below the neutrino background) that probably it will never be detected in direct detection of its SI interactions with nuclei. We also demand that the LSP has thermal relic abundance in agreement with the Planck measurement $\Omega h^2 \approx 0.12$ [36]. These two assumptions lead to interesting predictions for the model parameter space. We focus on NMSSM with singlino-higgsino LSP but many of our conclusions are valid also for more general singlet-doublet DM models (studied e.g. in refs. [37]–[40]). We investigate how the resulting parameter space can be constrained by direct detection experiments focusing on the spin-dependent (SD) LSP interactions with nuclei. We assess the impact of new LUX results presented at the Moriond 2017 conference [41] on the parameter space, as well as sensitivity of future XENON1T and LZ experiments. Rather than doing huge numerical scans of the NMSSM parameter space we study separately several classes of SI blind spots identified in ref. [29]. This way it becomes possible to understand which effects have the biggest impact on the constraints. In particular, we emphasize the role of light singlets in relaxing the constraints.

The rest of the article is organized as follows. In section 2 we introduce the model and conventions used. In the next two sections we focus on general NMSSM. In section 3 we discuss the case in which singlet-like states are heavy and SI cross-section is solely determined by the exchange of the Higgs boson. In section 4 we discuss how both the relic density and blind spot condition is affected by the presence of light singlets. In section 5 we analyze the $\mathbb{Z}_3$ invariant NMSSM. We reserve section 6 for our conclusions.

2 Model and conventions

In this section we collect formulae useful for the analysis performed in the rest of this work. We adopt conventions as in the previous paper [29] where more details may be found. We start with the most general NMSSM with the superpotential and the soft terms given by

$$W = W^{\text{MSSM}} + \lambda S H_u H_d + f(S), \quad (2.1)$$

$$-\mathcal{L}_{\text{soft}} = -\mathcal{L}_{\text{soft}}^{\text{MSSM}} + m_{H_u}^2 |H_u|^2 + m_{H_d}^2 |H_d|^2 + m_S^2 |S|^2 \\ + \left(A_\lambda \lambda H_u H_d S + \frac{1}{3} A_\kappa \kappa S^3 + m_3^2 H_u H_d + \frac{1}{2} m_S'^2 S^2 + \xi_S S + \text{h.c.} \right), \quad (2.2)$$

where S is a chiral SM-singlet superfield. In the simplest version, known as the scale-invariant or $\mathbb{Z}_3$ -symmetric NMSSM, $m_3^2 = m_S'^2 = \xi_S = 0$ while $f(S) \equiv \kappa S^3/3$. In more general models $f(S) \equiv \xi_F S + \mu' S^2/2 + \kappa S^3/3$.

The mass squared matrix for the neutral CP-even scalar fields, in the basis $(\hat{h}, \hat{H}, \hat{s})$ related to the interaction basis by a rotation by angle β (see [29] for details), reads:

$$M_s^2 = \begin{pmatrix} M_{\hat{h}\hat{h}}^2 & M_{\hat{h}\hat{H}}^2 & M_{\hat{h}\hat{s}}^2 \\ M_{\hat{H}\hat{h}}^2 & M_{\hat{H}\hat{H}}^2 & M_{\hat{H}\hat{s}}^2 \\ M_{\hat{s}\hat{h}}^2 & M_{\hat{s}\hat{H}}^2 & M_{\hat{s}\hat{s}}^2 \end{pmatrix}, \quad (2.3)$$

where

$$M_{\hat{h}\hat{h}}^2 = M_Z^2 \cos^2(2\beta) + \lambda^2 v_h^2 \sin^2(2\beta) + \Delta_{\hat{h}\hat{h}}, \quad (2.4)$$

$$M_{\hat{H}\hat{H}}^2 = (M_Z^2 - \lambda^2 v_h^2) \sin^2(2\beta) + \frac{2}{\sin(2\beta)} \left(\mu A_\lambda + \frac{\mu \langle \partial_S f \rangle}{v_s} + m_3^2 \right) + \Delta_{\hat{H}\hat{H}}, \quad (2.5)$$

$$M_{\hat{s}\hat{s}}^2 = \frac{1}{2} \lambda v_h^2 \sin 2\beta \left(\frac{A_\lambda + \langle \partial_S^2 f \rangle}{v_s} - \langle \partial_S^3 f \rangle \right) + \langle (\partial_S^2 f)^2 + \partial_S f \partial_S^3 f \rangle - \frac{\langle \partial_S f \partial_S^2 f \rangle}{v_s} \\ + A_\kappa \kappa v_s - \frac{\xi_S}{v_s} + \Delta_{\hat{s}\hat{s}}, \quad (2.6)$$

$$M_{\hat{h}\hat{H}}^2 = \frac{1}{2} (M_Z^2 - \lambda^2 v_h^2) \sin 4\beta + \Delta_{\hat{h}\hat{H}}, \quad (2.7)$$

$$M_{\hat{h}\hat{s}}^2 = \lambda v_h (2\mu - (A_\lambda + \langle \partial_S^2 f \rangle) \sin 2\beta) + \Delta_{\hat{h}\hat{s}}, \quad (2.8)$$

$$M_{\hat{H}\hat{s}}^2 = \lambda v_h (A_\lambda + \langle \partial_S^2 f \rangle) \cos 2\beta + \Delta_{\hat{H}\hat{s}}, \quad (2.9)$$

$\Delta_{\hat{h}_i \hat{h}_j}$ are radiative corrections, v_s , $v_h \sin \beta$ and $v_h \cos \beta$ are VEVs of the singlet and the two doublets, respectively. The mass eigenstates of M_s^2 are denoted by h_i with $h_i = h, H, s$ (h is the 125 GeV scalar discovered by the LHC experiments). These mass eigenstates are expressed in terms of the hatted fields with the help of the diagonalization matrix $\tilde{S}$:

$$h_i = \tilde{S}_{h_i \hat{h}} \hat{h} + \tilde{S}_{h_i \hat{H}} \hat{H} + \tilde{S}_{h_i \hat{s}} \hat{s}. \quad (2.10)$$

The mass squared matrix for the neutral pseudoscalars, after rotating away the Goldstone boson, has the form

$$M_p^2 = \begin{pmatrix} M_{\hat{A}\hat{A}}^2 & M_{\hat{A}\hat{a}}^2 \\ M_{\hat{a}\hat{A}}^2 & M_{\hat{a}\hat{a}}^2 \end{pmatrix}, \quad (2.11)$$

where

$$M_{\hat{A}\hat{A}}^2 = \frac{2}{\sin(2\beta)} \left(\mu A_\lambda + \frac{\mu \langle \partial_S f \rangle}{v_s} + m_3^2 \right) + \Delta_{\hat{A}\hat{A}}, \quad (2.12)$$

$$M_{\hat{A}\hat{a}}^2 = \lambda v_h (A_\lambda - \langle \partial_S^2 f \rangle) + \Delta_{\hat{A}\hat{a}}, \quad (2.13)$$

$$M_{\hat{a}\hat{a}}^2 = \frac{1}{2} \lambda v_h^2 \sin 2\beta \left(\frac{A_\lambda + \langle \partial_S^2 f \rangle}{v_s} + \langle \partial_S^3 f \rangle \right) + \langle (\partial_S^2 f)^2 - \partial_S f \partial_S^3 f \rangle - \frac{\langle \partial_S f \partial_S^2 f \rangle}{v_s} \\ - 3A_\kappa \kappa v_s - 2m_S'^2 - \frac{\xi_S}{v_s} + \Delta_{\hat{a}\hat{a}}, \quad (2.14)$$

Diagonalizing M_p^2 with the matrix $\tilde{P}$ one gets the mass eigenvalues $a_j = a, A$:

$$a_j = \tilde{P}_{a_j \hat{a}} \hat{a} + \tilde{P}_{a_j \hat{A}} \hat{A}. \quad (2.15)$$

After decoupling of the gauginos (assumed in this work) the neutralino mass sub-matrix describing the three lightest states takes the form:

$$M_{\chi^0} = \begin{pmatrix} 0 & -\mu & -\lambda v_h \sin \beta \\ -\mu & 0 & -\lambda v_h \cos \beta \\ -\lambda v_h \sin \beta & -\lambda v_h \cos \beta & \langle \partial_S^2 f \rangle \end{pmatrix}. \quad (2.16)$$

Trading the model dependent term $\langle \partial_S^2 f \rangle$ for one of the eigenvalues, m_{χ_j} , of the above neutralino mass matrix we find the following (exact at the tree level) relations for the neutralino diagonalization matrix elements:

$$\frac{N_{j3}}{N_{j5}} = \frac{\lambda v_h}{\mu} \frac{(m_{\chi_j}/\mu) \sin \beta - \cos \beta}{1 - (m_{\chi_j}/\mu)^2}, \quad (2.17)$$

$$\frac{N_{j4}}{N_{j5}} = \frac{\lambda v_h}{\mu} \frac{(m_{\chi_j}/\mu) \cos \beta - \sin \beta}{1 - (m_{\chi_j}/\mu)^2}, \quad (2.18)$$

where N_{j3} , N_{j4} and N_{j5} denote, respectively, the two higgsino and the singlino components of the j -th neutralino mass eigenstate while $j = 1, 2, 3$ and $|m_{\chi_1}| \leq |m_{\chi_2}| \leq |m_{\chi_3}|$. Using the last two equations and neglecting contributions from decoupled gauginos one can express the composition of the three lighter neutralinos in terms of: λv_h , m_χ/μ and $\tan \beta$. Later we will be interested mainly in the LSP corresponding to $j = 1$, so to simplify the notation we will use $m_\chi \equiv m_{\chi_1}$. The physical (positive) LSP mass is given by $m_{\text{LSP}} \equiv |m_\chi|$.

In the present work we are interested mainly in two properties of the LSP particles: their cross-sections on nucleons and their relic abundance. The spin-dependent LSP-nucleon scattering cross section is dominated by the Z^0 boson exchange and equals

$$\sigma_{\text{SD}}^{(N)} = C^{(N)} \cdot 10^{-38} \text{ cm}^2 (N_{13}^2 - N_{14}^2)^2, \quad (2.19)$$

where $C^{(p)} \approx 4$, $C^{(n)} \approx 3.1$ [42, 43]. The spin-independent cross-section for the LSP interacting with the nucleus with the atomic number Z^0 and the mass number A is given by

$$\sigma_{\text{SI}} = \frac{4\mu_{\text{red}}^2}{\pi} \frac{[Zf^{(p)} + (A-Z)f^{(n)}]^2}{A^2}, \quad (2.20)$$

where μ_{red} is the reduced mass of the nucleus and the LSP. When the squarks are heavy, as assumed in the present work, the effective couplings $f^{(N)}$ ($N = p, n$) are dominated by the t-channel exchange of the CP-even scalars [44]:

$$f^{(N)} \approx \sum_{i=1}^3 f_{h_i}^{(N)} \equiv \sum_{i=1}^3 \frac{\alpha_{h_i \chi \chi} \alpha_{h_i N N}}{2m_{h_i}^2}. \quad (2.21)$$

Further details may be found in appendix A. Formulae for the LSP annihilation cross section and its relic density are much more complicated (some of them are collected in appendices B and C).

The main goal of the present work is to identify regions of the NMSSM parameter space for which the singlino-higgsino LSP particles fulfill three conditions: 1) have very small, below the neutrino background, SI cross-section on nuclei (SI blind spots); 2) have small SD cross-sections to be consistent with present experimental bounds; 3) have relic density close to the experimentally favored value $\Omega h^2 \approx 0.12$ so the LSP can play the role of a dominant component of DM.¹ Of course, such points in the parameter space must be consistent with other experimental constraints e.g. those derived from the LHC [48] and LEP data [49, 50]. In the next sections we discuss solutions fulfilling all the above mentioned conditions, starting with the simplest case of blind spots without interference effects and then for blind spots for which such effects are crucial. We investigate also modifications present in $\mathbb{Z}_3$ invariant NMSSM.

3 Blind spots without interference effects and relic density

In this section we consider situation when the SI blind spot (BS) takes place without interference effects i.e. when all three contributions to the effective coupling $f^{(N)}$ in eq. (2.21) are very small. Two of them are small because the corresponding scalars, s and H , are very heavy while f_h is suppressed due to smallness of $\alpha_{h\chi\chi}$. In the next subsection we discuss the simplest case in which the mixing among the scalars may be neglected. Then the effects of such mixing will be taken into account.

3.1 Without scalar mixing

When the interference effects and scalar mixing may be neglected the SI blind spot condition has the following simple form:

$$\frac{m_\chi}{\mu} - \sin 2\beta = 0, \quad (3.1)$$

which corresponds to vanishing Higgs-LSP-LSP coupling for $m_s, m_H \rightarrow \infty$. In figure 1 the dependence of Ωh^2 on m_{LSP} and $\tan \beta$ for such blind spots is shown for some specific values of λ and κ (the latter parameter does not influence the situation as long as the resonance with the lightest pseudoscalar is not considered). Some parts of the $(m_{\text{LSP}}, \tan \beta)$ plane are excluded by the upper bounds on σ_{SD} from LUX [41] and IceCube [51] experiments (and also by the LEP data). Particularly important are the new LUX constraints which exclude large part of the parameter space with $\Omega h^2 = 0.12 \pm 0.02$. We note that in the allowed part of the parameter space presented in figure 1 correct thermal relic abundance is obtained for singlino-dominated LSP. It is not difficult to understand the results shown in figure 1 using (approximate) analytic formulae.

For given values of m_χ and $\tan \beta$, the blind spot condition (3.1) together with eqs. (2.17) and (2.18) may be used to obtain the LSP composition. For example, the combination

¹In order to take into account theoretical uncertainties in the relic abundance calculations we consider parameters leading to $\Omega h^2 = 0.12 \pm 0.02$ when calculated with the help of `MicrOMEGAs` 4.3.1 [45] and `NMSSMTools` 5.0.2 [46, 47] which we use to calculate the NMSSM spectrum.

Figure 1. Contour lines of Ωh^2 (obtained with MicrOMEGAs 4.3.1 [45] and NMSSMTools 5.0.2 [46, 47]) as functions of m_{LSP} and $\tan\beta$ for blind spots with decoupled all Higgs particles except the SM-like one. Dashed and dotted blue lines correspond to eq. (3.6) and (3.7), respectively, after substituting (3.2), for $\Omega h^2 = 0.12$. Red region depicts the points with $m_{\chi_1^\pm} < 103 \text{ GeV}$ which are ruled out by LEP [49]. Yellow area is forbidden because of eq. (3.9), whereas green/cyan one due to LUX/IceCube (IC) limits on spin-dependent LSP interaction with nucleons. Vertical red lines correspond to (from left to right) $m_{\text{LSP}} = m_{Z^0}/2$, $m_h/2$, m_W , m_t .

which determines the LSP coupling to the Z^0 boson is given by (see appendix A)²

$$|N_{13}^2 - N_{14}^2| = \left(\frac{\lambda v_h}{m_\chi} \right)^2 \frac{\sin^2 2\beta}{|\cos 2\beta|} \left[1 + \left(\frac{\lambda v_h}{m_\chi} \right)^2 \tan^2 2\beta \right]^{-1}. \quad (3.2)$$

All three components (gauginos are decoupled) may be expressed in terms of the above combination using

$$N_{13}^2 = \cos^2 \beta \frac{|N_{13}^2 - N_{14}^2|}{\cos 2\beta}, \quad (3.3)$$

$$N_{14}^2 = \sin^2 \beta \frac{|N_{13}^2 - N_{14}^2|}{\cos 2\beta}, \quad (3.4)$$

$$N_{15}^2 = 1 - \frac{|N_{13}^2 - N_{14}^2|}{\cos 2\beta}. \quad (3.5)$$

The above expressions are valid as long as blind spot condition (3.1) is satisfied. For the LSP masses for which the annihilation cross section is dominated by the s-channel Z^0 exchange this is enough to calculate the LSP relic density to a good accuracy. The approximate formulae are (see the appendix B for the details):

$$\Omega h^2 \approx 0.1 \left(\frac{0.3}{N_{13}^2 - N_{14}^2} \right)^2 \frac{m_{Z^0}^2}{4m_\chi^2} \left[\left(\frac{4m_\chi^2}{m_{Z^0}^2} - 1 + \frac{\bar{v}^2}{4} \right)^2 + \frac{\Gamma_Z^2}{m_{Z^0}^2} \right], \quad (3.6)$$

²There are some corrections caused by not totally decoupled particles. In our numerical scan we took $M_1, M_2 \approx 4 \text{ TeV}$, $m_H, m_s \approx 4 \text{ TeV}$, $m_{\tilde{q}} \approx 3 \text{ TeV}$.

Figure 2. Contour lines of $\Omega h^2 = 0.10, 0.12, 0.14$ as functions of m_{LSP} and $|N_{13}^2 - N_{14}^2|$ for two LSP mass regimes (blue and red dashed lines in the left plot were obtained from eq. (3.6) whereas dotted lines in the right plot from eq. (3.7)). Yellow area is forbidden because of eq. (3.9). Grey color in the left plot denotes the regions in which h resonance and W^+W^-/Z^0Z^0 channels may be important and affect the results. Green/cyan areas correspond to LUX/IceCube limits on spin-dependent LSP interaction with nucleons [41], [51]. Dashed (continuous) green lines in both plots correspond to the precision of the future XENON1T (LZ) experiment of SD direct interaction of the LSP with neutrons [52].

for m_{LSP} of order $m_{Z^0}/2$ (and below the W^+W^- threshold) and

$$\Omega h^2 \approx 0.1 \left(\frac{0.05}{N_{13}^2 - N_{14}^2} \right)^2 \left[\sqrt{1 - \frac{m_t^2}{m_\chi^2}} + \frac{3}{4} \frac{1}{x_f} \left(1 - \frac{m_t^2}{2m_\chi^2} \right) \frac{1}{\sqrt{1 - \frac{m_t^2}{m_\chi^2}}} \right]^{-1}, \quad (3.7)$$

above the $t\bar{t}$ threshold. They reproduce very well the relic density calculated numerically using **MicrOMEGAs**, as may be seen in figure 1.

The composition of the LSP is crucial not only for its relic density but also for some of the experimental constraints. One gets the following upper bounds on the combination (3.2):

$$(N_{13}^2 - N_{14}^2)^2 \lesssim 1.8 \cdot 10^{-3} \left(1 - \frac{4m_\chi^2}{m_{Z^0}^2} \right)^{-3/2} \quad (3.8)$$

from the LEP bound for invisible decays of Z^0 [18, 50] and

$$(N_{13}^2 - N_{14}^2)^2 \lesssim \frac{(\sigma_{\text{SD}}^{(N)})^{\text{exp}}}{C^{(N)} \cdot 10^{-38} \text{cm}^2} \quad (3.9)$$

from experiments sensitive to the SD interactions of the LSP with protons or neutrons for which $C^{(p)} \approx 4$, $C^{(n)} \approx 3.1$, respectively [42, 43]. These upper bounds on $(N_{13}^2 - N_{14}^2)^2$ as functions of the LSP mass are shown in figure 2.

One can find the allowed range of m_{LSP} in the vicinity of $m_{Z^0}/2$. The points in the left plot in figure 2 at which the blue and red dashed lines enter the green region (excluded by LUX constraints on the SD cross-section) determine the limiting values of m_{LSP} for which

Figure 3. Contour lines of $\Omega h^2 = 0.12$ (obtained with **MicrOMEGAs**) as functions of m_{LSP} and $\tan \beta$ for a few values of λ (dashed lines in the left plot was obtained from eq. (3.6) after substituting (3.2) whereas dotted lines in the right plot from eq. (3.7) after substituting (3.2)) for blind spots with decoupled all Higgs particles except the SM-like one. Left plot: green areas and thick red lines denote the points excluded by LUX and the LEP chargino searches, respectively; thick blue points correspond to eq. (3.10) and (3.11). Right plot: green region and thick orange line depict the points excluded by LUX and points for which the stop masses above 5 TeV are necessary to obtain the correct Higgs mass (even when the contribution from the stop mixing is maximized), respectively. The red lines denoted by LP depict regions with a Landau pole below the GUT scale.

the Z^0 resonance may give the correct relic density of the LSP. To be more accurate we used the results for Ωh^2 obtained from **MicrOMEGAs** and found the limiting LSP masses to be approximately 41 and 46.5 GeV.³ Substituting the corresponding values of $|N_{13}^2 - N_{14}^2|$ into eq. (3.2) we find the following linear dependence:

$$\tan \beta \approx 42 \times \lambda \quad \text{for } m_\chi \approx 41 \text{ GeV}, \quad (3.10)$$

$$\tan \beta \approx 44 \times \lambda \quad \text{for } m_\chi \approx 46.5 \text{ GeV}. \quad (3.11)$$

The values of $\tan \beta$ necessary to obtain good relic abundance of the LSP become larger when moving to values of m_{LSP} closer to the peak of the resonance (which is slightly below $m_{Z^0}/2$). The situation is illustrated in the left plot of figure 3. One can see that in the region allowed by LUX large $\tan \beta$ is required unless λ is small (~ 0.1 in this case). Of course the above limits may become stronger (i.e. for a given λ larger $\tan \beta$ might be required) when SD direct detection experiments (especially based on the interactions with neutrons) gain better precision. For instance, the LZ experiment will be able to explore the entire region considered here — see the left plot in figure 2.

In the LSP mass range between the W^+W^- and $t\bar{t}$ thresholds the annihilation cross section is dominated by gauge boson (W^+W^-/Z^0Z^0) final states with the chargino/neutralino exchanged in the t channel. The related couplings are proportional to the higgsino components of the LSP (the gauginos are decoupled) which for the blind spot (3.1) are related to the LSP- Z^0 coupling by eqs. (3.3) and (3.4). The values of N_{14} (N_{13} is smaller by factor

³These results were obtained for the case of $\Omega h^2 = 0.12$ but they do not change much when the uncertainty in the calculation of the relic abundance is taken into account.

$1/\tan\beta$) necessary to get $\Omega h^2 \sim 0.12$ lead to too large σ_{SD} and are excluded by both LUX and IceCube data (see figure 1). Thus, the LSP masses in the range $m_{W^+} \lesssim m_{\text{LSP}} \lesssim m_t$ are excluded. The only way to have correct relic abundance consistent with all experimental constraints is to go to very small values of λ in order to suppress SI cross-section below the LUX constraint also away from the blind spot (3.1) and increase $\tan\beta$ such that the Higgs mass constraint is fulfilled.⁴ More flexibility in the parameter space may appear if some additional particles are exchanged and/or appear in the final state of the LSP annihilation (such situations will be discussed in the next sections).

For $m_{\text{LSP}} \gtrsim 160 \text{ GeV}$ annihilation into $t\bar{t}$ (via s-channel Z^0 exchange) starts to be kinematically accessible so smaller higgsino component suffices to have large enough annihilation cross-section to fit $\Omega h^2 \approx 0.12$. In consequence, $\Omega h^2 \approx 0.12$ is obtained for somewhat larger $\tan\beta$ than between the W^+W^- and $t\bar{t}$ thresholds and smaller SD cross-section is predicted. As a result, IceCube [51] constraints are satisfied for $m_{\text{LSP}} \gtrsim 175 \text{ GeV}$. However, the new LUX constraints exclude m_{LSP} up to about 300 GeV for $\Omega h^2 = 0.12$. This lower bound on m_{LSP} may change by about 50 GeV when the uncertainties in the calculation of the relic abundance are taken into account. It becomes stronger (weaker) for smaller (bigger) values of Ωh^2 . We should also emphasize that the lower bound on the LSP mass from SD constraints are the same for the whole class of singlet-doublet fermion DM as long as it annihilates dominantly to $t\bar{t}$. In particular, similar lower bound of 300 GeV on the LSP mass was recently set by LUX on the well-tempered neutralino in MSSM [31]. One can also see in figure 2 that the correct relic abundance requires $|N_{13}^2 - N_{14}^2| \sim 0.05$ which, depending on $\tan\beta$, translates to $N_{15}^2 \sim 0.9 - 0.95$ (see eq. (3.5)) and such values may be explored by XENON1T. The right panel of figure 3 shows values of $\tan\beta$ necessary to get $\Omega h^2 = 0.12$ as a function of m_{LSP} and λ . Contrary to the Z^0 resonance case small values of $\tan\beta$ are preferred and hence moderate or large λ (in order to have big enough Higgs mass at the tree level). However, too small values of $\tan\beta$ lead, for a given big value of λ , to a Landau pole below the GUT scale. Thus, the assumption of perturbativity up to the GUT scale and the requirement $\Omega h^2 = 0.12$ give constraints which result in a λ -dependent upper bound on the mass of the LSP. For example, $m_{\text{LSP}} \lesssim 700 \text{ GeV}$ for $\lambda = 0.7$ (see figure 3) and $m_{\text{LSP}} \lesssim 800 \text{ GeV}$ for $\lambda = 0.6$. Let us also note that for large LSP masses coannihilation becomes non-negligible. This effect relaxes the upper bound on $\tan\beta$ and is increasingly important as λ decreases, as can be seen in figure 3 from comparison of full result by MicrOMEGAs and the approximated one with only $t\bar{t}$ included.

Let us comment on two features of the $\Omega h^2 \approx 0.12$ curves in figure 1. First: there are no signs of a resonant annihilation with the h boson exchange in the s-channel. This is simply the consequence of the blind spot condition leading to vanishing (or at least very small) LSP-Higgs coupling. This is characteristic of all blind spots without interference effects. Second: $\tan\beta$ decreases with m_χ for all $m_\chi > m_{Z^0}/2$ with the exception of the vicinity of the $t\bar{t}$ threshold. This is related to the fact that the annihilation cross section is directly (Z^0 in the s channel) or indirectly (the VV final states) connected to

⁴Note that a well-tempered bino-higgsino LSP in MSSM with the mass between the W and t masses cannot accommodate all constraints since in that case the mixing in the neutralino sector is controlled by the gauge coupling constant which is fixed by experiment, see e.g. refs. [9, 31].

the value of $(N_{13}^2 - N_{14}^2)$ given by eq. (3.2). The r.h.s. of (3.2) is a decreasing function of m_χ and decreasing function of $\tan\beta$ (for $\tan\beta > 1 + \lambda v_h/m_\chi$). Thus, in order to keep it approximately unchanged the increase of m_χ must be compensated by the decrease of $\tan\beta$ (other parameters determining the annihilation cross section may change this simple relation only close to the $t\bar{t}$ threshold and below the Z^0 resonance).

Another comment refers to constraints obtained from the indirect detection experiments. The IceCube upper bounds on σ_{SD} change by orders of magnitude depending on what channels dominate the LSP annihilation. This can be already seen in the simple case discussed in this subsection. The lower bound on $\tan\beta$ obtained from the IC data (as a function of m_{LSP}) visible in figure 1 drops substantially above the $t\bar{t}$ threshold because the $t\bar{t}$ pairs give softer neutrinos as compared to the W^+W^- pairs [51]. Moreover, the latest LUX results on σ_{SD} lead to stronger bounds in almost all cases. Only for quite heavy LSP the IC limits are marginally stronger, as may be seen in the right panel of figure 2.

To sum up, in this section we identified two crucial mechanisms (Z^0 resonance and annihilation into $t\bar{t}$) which may give correct relic density and are allowed by the experiments. However, both of them rely on the Z^0 boson exchange in the s channel and therefore are proportional to the LSP- Z^0 coupling, which controls also the SD cross section of the LSP scattering on nucleons. Therefore, the future bounds on such interaction will be crucial in order to constrain the parameter space. In fact, XENON1T is expected to entirely probe regions of the parameter space in which annihilation into $t\bar{t}$ dominates while LZ will be able to explore the entire region of Z^0 resonance. It is also worth noting that the situation presented in figure 1 may change if we consider light pseudoscalar a with mass $m_a \sim 2m_\chi$. Such resonance for singlino-dominated LSP (we require $\kappa \neq 0$) is controlled mainly by the mixing in (pseudo)scalar sector and hence may not be so strongly limited by the SD direct detection experiments. For instance, we checked with MicrOMEGAS that for m_a in a few hundred GeV range we can easily obtain points in parameter space with correct relic density and σ_{SD} below the future precision of the LZ experiment. In principle, the effect of light pseudoscalar may be also important for $2m_\chi \gtrsim m_h + m_a$ when the LSP starts to annihilate into ha state which, depending on κ , may suppress the $t\bar{t}$ channel and may weaken the IceCube limits. However, in the case considered in this subsection, the contribution from the ha channel may be important only for large mixing in the pseudoscalar sector. This requires quite large values of A_λ which leads to unacceptably small values of the Higgs mass. We will come to these points in the next sections where the annihilation channels involving the singlet-dominated pseudoscalar may play a more important role.

3.2 With scalar mixing

Next we consider the case when the mixing among scalars is not negligible and affects the blind spot condition (3.1), which is now of the form:

$$\frac{\tilde{S}_{h\hat{s}}}{\tilde{S}_{h\hat{h}}} \equiv \gamma \approx -\eta, \quad (3.12)$$

where η , defined by

$$\eta \equiv \frac{N_{15}(N_{13} \sin\beta + N_{14} \cos\beta)}{N_{13}N_{14} - \frac{\kappa}{\lambda}N_{15}^2}, \quad (3.13)$$

depends on the LSP composition (some formulae expressing η in terms of the model parameters may be found in appendix A). In equation (3.12) we introduced also parameter γ describing the mixing of the SM-like Higgs with the singlet scalar. This mixing can be expressed (for $m_s \gg m_h$ assumed in this section) in terms of the NMSSM parameters as:

$$\frac{\tilde{S}_{h\hat{s}}}{\tilde{S}_{h\hat{h}}} \approx \lambda v \frac{(A_\lambda + \langle \partial_S^2 f \rangle) \sin 2\beta - 2\mu}{m_s^2} \approx \text{sgn}((A_\lambda + \langle \partial_S^2 f \rangle) \sin 2\beta - 2\mu) \frac{\sqrt{2|\Delta_{\text{mix}}|m_h}}{m_s}, \quad (3.14)$$

where $\Delta_{\text{mix}} \equiv m_h - \hat{M}_{hh}$ is the shift of the SM-like Higgs mass due to the mixing [53]. For $m_s > m_h$ this shift is negative so we prefer it has rather small absolute value.

The scenario of higgsino-dominated LSP with $\Omega h^2 \approx 0.12$ is very similar to the analogous case in the MSSM model and requires $|\mu| \approx 1.2 \text{ TeV}$. Even the present results from the direct and indirect detection experiments constrain possible singlino admixture in the higgsino-dominated LSP to be at most of order 0.1. So small singlino component leads to negligible changes of μ necessary to get the observed relic density of DM particles.⁵

Thus, similarly as before, we focus on SI blind spots with $\Omega h^2 \approx 0.12$ for singlino-dominated LSP. In this case and for non-negligible κ (more precisely, when $|N_{13}N_{14}| \ll \frac{|\kappa|}{\lambda} N_{15}^2$) the blind spot condition may be approximated by:

$$\frac{m_\chi}{\mu} - \sin 2\beta \approx \gamma \frac{\kappa}{\lambda} \frac{\mu}{\lambda v_h} \left(1 - \left(\frac{m_\chi}{\mu} \right)^2 \right). \quad (3.15)$$

This condition is a quadratic equation for μ and has solutions only when

$$\cos^2 \beta > \frac{1}{2} - \left| \frac{1}{2} + \gamma \frac{\kappa}{\lambda} \frac{m_\chi}{\lambda v_h} \right|. \quad (3.16)$$

One can see from (3.15) that for $\gamma\kappa\mu > 0$ we have always $m_\chi\mu > 0$ and the LSP has more higgsino fraction than when condition (3.1) holds. In the opposite case i.e. $\gamma\kappa\mu < 0$ we can have either $m_\chi\mu > 0$ with slightly smaller higgsino fraction or strongly singlino-dominated LSP with $m_\chi\mu < 0$. However, for values of $|\gamma|$ small enough not to induce large negative Δ_{mix} the higgsino component of the LSP with $m_\chi\mu < 0$ is too small to obtain $\Omega h^2 \approx 0.12$. Therefore, from now on we focus on the case $m_\chi\mu > 0$. Solving eq. (3.15) for the ratio m_χ/μ and substituting the solution into eq. (3.2) one can find the difference of two higgsino components of the LSP. For small $|\gamma|$ it can be approximated as (3.2) with a small correction:

$$\begin{aligned} |N_{13}^2 - N_{14}^2| \approx & \left(\frac{\lambda v_h}{m_\chi} \right)^2 \frac{\sin^2 2\beta}{|\cos 2\beta|} \left[1 + \left(\frac{\lambda v_h}{m_\chi} \right)^2 \tan^2 2\beta \right]^{-1} \\ & + \gamma \frac{\kappa}{\lambda} \frac{\lambda v_h}{m_\chi} \frac{2}{|\cos 2\beta|^3} \left[1 - \left(\frac{\lambda v_h}{m_\chi} \right)^2 \frac{\sin^4 2\beta}{\cos^2 2\beta} \right] \left[1 + \left(\frac{\lambda v_h}{m_\chi} \right)^2 \tan^2 2\beta \right]^{-2}. \end{aligned} \quad (3.17)$$

⁵In fact, bigger changes of μ come from not totally decoupled gauginos, e.g. for M_1, M_2 of order 5 TeV.

Figure 4. The same as in figure 3 but now the regions with non-zero s - h mixing are depicted by colored areas: orange (yellow) corresponds to the results from MicrOMEGAs (from eqs. (3.6) and (3.15)). We took $m_\chi > 0$, $\mu > 0$, $m_s = 600$ GeV and $|\gamma| \leq 0.025$ (corresponding to $|\Delta_{\text{mix}}| \lesssim 1$ GeV), where $\gamma < 0$ (> 0) refers to the lower (upper) limit of a given area. The shift of the orange region with respect to the yellow one for LSP masses above about m_s is due to the annihilation channels containing s in the final state and co-annihilation effects which become more important for larger LSP masses.

As we already mentioned discussing eq. (3.2), the first term in the r.h.s. of the last equation is a decreasing function of $\tan \beta$ (unless $\tan \beta$ is very close to 1). So, in order to keep the value of $|N_{13}^2 - N_{14}^2|$ necessary to get $\Omega h^2 \approx 0.12$, the contribution from the second line of (3.17) may be compensated by increasing (decreasing) the value of $\tan \beta$ when $\gamma \kappa \mu > 0$ (< 0).⁶ This effect from the s - h mixing is illustrated in figure 4. One can see that indeed, depending on the sign of $\gamma \kappa \mu$, we can have smaller or larger (in comparison to eq. (3.1)) values of $\tan \beta$ for a given LSP mass, while keeping $\Omega h^2 \approx 0.12$. In particular, non-negligible Higgs-singlet mixing may relax the upper bound on the LSP mass arising from the perturbativity up to the GUT scale. It is also important to emphasize that the resonant annihilation with s scalar exchanged in the s channel is quite generic for this kind of blind spot (see the right plot in figure 4, where we have chosen $m_s = 600$ GeV) because it is easier to have substantial s - h mixing when the singlet-dominated scalar is not very heavy. Moreover, the presence of resonant annihilation via s exchange can relax the lower limit on the LSP mass from LUX constraints on the SD cross-section, as seen in the right panel of figure 4, because in such a case smaller higgsino component is required to obtain correct relic density.

4 Blind spots and relic density with light singlets

Now we move to the case when the singlet-dominated scalar is lighter than the SM Higgs. Neglecting the effect from the heavy doublet H exchange for the SI cross section (i.e. setting f_H to zero) the blind spot condition may be written in the following form [29]:

$$\frac{\gamma + \mathcal{A}_s}{1 - \gamma \mathcal{A}_s} = -\eta, \quad (4.1)$$

⁶The sign of the second term in the r.h.s. of (3.17) is determined by the sign of product $\gamma \kappa m_\chi$ but for considered here case of $m_\chi \mu > 0$ it is the same as that of $\gamma \kappa \mu$.

where

$$\mathcal{A}_s \approx -\gamma \frac{1+c_s}{1+c_h} \left(\frac{m_h}{m_s} \right)^2 \quad (4.2)$$

and parameters

$$c_{h_i} \equiv 1 + \frac{\tilde{S}_{h_i \hat{H}}}{\tilde{S}_{h_i \hat{h}}} \left(\tan \beta - \frac{1}{\tan \beta} \right) \quad (4.3)$$

measure (in the large $\tan \beta$ limit) the ratio of the couplings, normalized to the SM values, of the h_i ($= s, h$) scalar to the b quarks and to the Z^0 bosons.

In the rest of this section we will consider singlino-like LSP, because the case of higgsino-dominated LSP does not differ much from the one described in section 3.2. The blind spot condition (4.1), analogously to a simpler case (3.12), may be approximated by a quadratic equation for μ

$$\frac{m_\chi}{\mu} - \sin 2\beta \approx \frac{\gamma + \mathcal{A}_s}{1 - \gamma \mathcal{A}_s} \frac{\kappa}{\lambda} \frac{\mu}{\lambda v_h} \left(1 - \left(\frac{m_\chi}{\mu} \right)^2 \right) \quad (4.4)$$

which has solutions only if

$$\cos^2 \beta > \frac{1}{2} - \left| \frac{1}{2} + \gamma \frac{\kappa}{\lambda} \frac{m_\chi}{\lambda v_h} \frac{1 - \frac{1+c_s}{1+c_h} \left(\frac{m_h}{m_s} \right)^2}{1 + \gamma^2 \frac{1+c_s}{1+c_h} \left(\frac{m_h}{m_s} \right)^2} \right|. \quad (4.5)$$

The last condition may be interpreted as the upper bound on $\tan \beta$ (lower bound on $\cos \beta$). It is nontrivial when its r.h.s. is positive i.e. when the second term under the absolute value is negative but bigger than -1 . The bound is strongest when that term equals $-\frac{1}{2}$. However, usually the absolute value of that term is smaller than $\frac{1}{2}$ because the h - s mixing measured by γ is rather small. So, typically the bound on $\tan \beta$ becomes stronger with increasing LSP mass or increasing $|\kappa|$ (with other parameters fixed). We focus again on the more interesting case $m_\chi \mu > 0$ because for $m_\chi \mu < 0$ the blind spot condition may be satisfied only for the LSP strongly dominated by singlino which typically leads to too large relic density.⁷ Since in this section we consider $m_s < m_h$, $|\mathcal{A}_s|$ is typically larger than $|\gamma|$ (unless c_s and/or c_h deviate much from 1, which under some conditions may happen which we discuss in more detail later in this section) – in such a case the condition (4.5) is always fulfilled for $\gamma \kappa \mu < 0$ (see figure 5), whereas for $\gamma \kappa \mu > 0$ (see comment in footnote 6) there is an upper bound on $\tan \beta$ which gets stronger for larger values of $|\gamma \kappa|$.

In the following discussion we focus on big values of $|\gamma|$ because they lead to a relatively big positive contribution to the Higgs mass from the Higgs-singlet mixing [53]. In our numerical analysis we take $|\gamma| = 0.4$ which corresponds to $\Delta_{\text{mix}} \sim 4 \text{ GeV}$. In order to emphasize new features related to the modification of the BS condition we also consider rather large values of $|\kappa| \sim \mathcal{O}(0.1)$. For such choices of the parameters there are no viable blind spots for $\gamma \kappa \mu > 0$ in accord with the discussion above so we focus on $\gamma \kappa \mu < 0$.

⁷For $m_\chi \mu < 0$, $\Omega h^2 \approx 0.12$ may be obtained only when resonant annihilation via singlet (pseudo)scalar exchange is dominant (we describe this phenomenon for $m_\chi \mu > 0$) or $\lambda \ll \kappa$ which is less interesting from the point of view of the Higgs mass.

Figure 5. The same as in figure 1 but for $m_s < m_h$. We took $m_\chi > 0$, $\mu > 0$, $m_H = 4 \text{ TeV}$, $m_s = 95 \text{ GeV}$ and $|\gamma| = 0.4$, which corresponds to $\Delta_{\text{mix}} \sim 4 \text{ GeV}$. In brown areas NMSSMTools reports unphysical global minimum (UM) while in grey ones the LHC constraints on the Higgs production and decay are violated (LHC). Black regions in the upper right corners denote the points where the condition (4.5) does not hold.

The fact that our blind spot condition now comes from destructive interference between f_h and f_s amplitudes (rather than vanishing Higgs-LSP coupling) influences strongly the relation between the DM relic density, especially in small LSP mass regime, and other experimental constraints. Now the Higgs-LSP coupling is not negligible so the LSP mass below $m_h/2$ is forbidden, or at least very strongly disfavored, by the existing bounds on the invisible Higgs decays [54]. Thus, resonant annihilation with Z^0 or s (in this section we consider s lighter than h) exchanged in the s channel can not be used to obtain small enough singlino-like LSP relic abundance. As concerning the h resonance: it may be used but only the “half” of it with $m_{\text{LSP}} \gtrsim m_h/2$ (this effect is visible in all panels of figure 5). However, we found that other experimental constraints, such as the ones from the LHC and/or LUX exclude even this “half” of the h resonance when the mixing parameter $|\gamma|$ is large.

In general NMSSM the masses of singlet-like scalar s and pseudoscalar a are independent from each other so let us first consider the situation when a is heavy. The case with $m_a = 1 \text{ TeV}$ is presented in the upper left panel of figure 5. The contours of $\Omega h^2 = 0.10, 0.12, 0.14$ above the $t\bar{t}$ threshold are quite similar to the case with heavy singlet. The only difference is that now somewhat larger values of $\tan \beta$ are preferred but even in this case they cannot exceed about 5.

Let us now check what happens when the lighter pseudoscalar is also singlet-dominated (i.e. $a_1 = a$) and relatively light. The existence of such light pseudoscalar is very important

for both the relic abundance of the LSP and the constraints from the IceCube experiment. Let us now discuss these effects in turn.

The DM relic density is influenced by a pseudoscalar in two ways. First is given by possible resonant annihilation with a exchanged in the s channel. This possibility is interesting only for $m_a \gtrsim m_h$ because for lighter a one still has problems with non-standard Higgs decays constrained by the LHC data (see above).⁸ However, as for any narrow resonance, the DM relic density may be in agreement with observations only for a quite small range of the DM mass (for a given a mass). One can see this in three panels (except the upper left one) of figure 5. Second effect is related to new annihilation final states including the singlet-dominated pseudoscalar, namely sa , ha , aa (in addition to similar channels involving only scalars: ss , sh , hh). It is best illustrated in the upper right panel of figure 5 for $m_a = 260$ GeV. In this case the sa threshold roughly coincides with the $t\bar{t}$ threshold. Near this threshold the curves of constant $\Omega h^2 \approx 0.12$ go up towards bigger values of $\tan\beta$ and leave the region excluded by the LUX data for smaller LSP mass than in the case with heavy singlets. The reason is quite simple. With increasing contribution from the annihilation channels mediated by (non-resonant) pseudoscalar exchange smaller contribution from the channels mediated by Z^0 exchange is enough to get the desired value $\Omega h^2 \approx 0.12$. Moreover, smaller LSP- Z^0 coupling is obtained for bigger values of $\tan\beta$ so larger values of $\tan\beta$ are preferred than in the case with heavy a . As a result the lower possible LSP mass consistent with the LUX SD limits is almost 100 GeV smaller when a is relatively light. The precise values depend on the relic density and for $\Omega h^2 = 0.12$ is around 200 GeV instead of around 300 GeV as in the case with heavy singlets.

The behavior of the $\Omega h^2 = 0.12 \pm 0.02$ curves close to and slightly above the $t\bar{t}$ threshold depends on the parameters. Particularly important is the sign of γ . We see from the right panels of figure 5 that even for the same mass of the pseudoscalar, $m_a = 260$ GeV in this case, the plots are very different for different signs of γ . Most differences originate from the fact that $\gamma > 0$ implies $c_h > 1$ and $c_s < 1$ while for $\gamma < 0$ the inequalities are reversed. There are two important implications of these correlations which we describe in the following.

Firstly, the LHC constraints from the Higgs coupling measurements are stronger for $\gamma > 0$ because in such a case the Higgs coupling (normalized to the SM) to bottom quarks is larger than the one to gauge bosons. In consequence, the Higgs branching ratios to gauge bosons is suppressed as compared to the SM. Moreover, non-zero γ results in suppressed Higgs production cross-section so if $|\gamma|$ is large enough the Higgs signal strengths in gauge boson decay channels is too small to accommodate the LHC Higgs data which agree quite well with the SM prediction. Moreover, a global fit to the current Higgs data shows some suppression of the Higgs coupling to bottom quarks [48]⁹ which disfavors $c_h > 1$, hence also large $\gamma > 0$. It can be seen from the upper right panel of figure 5 that for $\gamma = 0.4$ the

⁸The situation may be different if the blind spot condition is of the standard form (3.1) — we will come to this point at the end of this section.

⁹The current fit also indicates an enhancement of the top Yukawa coupling. In NMSSM suppression of the bottom Yukawa coupling is correlated with enhancement of the top Yukawa coupling which has been recently studied in refs. [55, 56].

LHC excludes some of the interesting part of the parameter space which is allowed by LUX due to the LSP annihilations into sa final state. The LHC constraints can be satisfied for small values of γ but this comes at a price of smaller Δ_{mix} , hence somewhat heavier stops.

Secondly, $|\mathcal{A}_s|$ is larger for $\gamma < 0$ than in the opposite case (see eq. (4.2)). Moreover, since deviations of c_s and c_h from 1 grow with $\tan\beta$, $|\mathcal{A}_s|$ increases (decreases) with $\tan\beta$ for negative (positive) γ . For $\gamma > 0$, this implies that for large enough $\tan\beta$ the r.h.s. of the blind spot condition (4.4) changes sign and $\gamma\kappa\mu < 0$ is no longer preferred. Equivalently, the upper bound on $\tan\beta$ in eq. (4.5) gets stronger as $\tan\beta$ grows so it is clear that at some point condition (4.5) is violated. The appearance of the violation of the blind spot condition is clearly visible at large $\tan\beta$ in the upper panels of figure 5 (the black regions). For $\gamma < 0$ instead the blind spot condition may be always fulfilled for $\gamma\kappa\mu < 0$ (by taking e.g. appropriate value of μ).¹⁰ Moreover, there is interesting phenomenon that may happen for $\gamma < 0$ above the $t\bar{t}$ threshold which is well visible in the lower right panel of figure 5. The values of $\tan\beta$ corresponding to $\Omega h^2 = 0.12$ grow rapidly just above the $t\bar{t}$ threshold and there is a gap in the LSP masses for which there are no solutions with SI BS and observed value of Ω . Such solutions appear again for substantially bigger m_{LSP} (above 300 GeV in this case). The reason why $\Omega h^2 = 0.12$ curve is almost vertical near $m_{\text{LSP}} \sim m_t$ and the gap appears is related to the fact that m_χ/μ varies very slowly with $\tan\beta$, which results in fairly constant $|N_{13}^2 - N_{14}^2|$ which determines the $t\bar{t}$ annihilation cross-section, hence also Ωh^2 . The weak dependence of m_χ/μ on $\tan\beta$ originates from the fact that for increasing $\tan\beta$ both sides of the blind spot condition (4.4) grow. The l.h.s. grows because of decreasing $\sin(2\beta)$ while the r.h.s. due to increasing c_s . Of course, the fact that these two effects approximately compensate each other relies on specific choice of parameters and does not necessarily hold e.g. for different values of κ .

The presence of light a influences also the IceCube constraints in a way depending on the LSP mass. For $m_\chi \gtrsim (m_a + m_s)/2$ (assuming $m_a > m_s$) the IceCube constraints are very much relaxed and become practically unimportant for the cases discussed in this section. This is so because the additional annihilation channels (into sa , ha , ss , sh) at $v = 0$ lead to softer neutrinos as compared to otherwise dominant VV channels (or $t\bar{t}$ channel for even heavier pseudoscalar). The situation is different (and more complicated) for LSP masses between the W^+W^- and as thresholds. In this region one can have destructive/constructive interference between Z^0 and a -mediated amplitudes¹¹ for $b\bar{b}$ annihilation at $v = 0$ which strengthens/reduces the constraints (IceCube limit on SD cross section are two orders of magnitude stronger for VV than for $b\bar{b}$). In our case the effect depends on the sign of κ : for $\kappa < 0$ the IceCube limits are strengthen (relaxed) for $m_\chi \lesssim m_a$ ($\gtrsim m_a$) and vice versa for $\kappa > 0$ — see the lower panels of figure 5.

In the examples considered in this section and shown in figure 5 we have chosen the sign of μ to be positive (then m_χ is also positive because, as discussed earlier, there are no interesting solutions with $m_\chi\mu < 0$). The corresponding solutions with negative μ , and also changed signs of other parameters like κ , γ and A_κ , are qualitatively quite similar.

¹⁰The only exception is when $c_h < -1$ but this may happen only for very large $\tan\beta$ and/or very light H .

¹¹Note that these are the only non-negligible amplitudes which have non-zero a term in σv expansion — see (B.1).

Of course there are some quantitative changes. Contours in the corresponding plots are slightly shifted towards smaller or bigger (depending on the signs of other parameters) values of $\tan \beta$.

As explained in detail in ref. [29], one can also get vanishing spin-independent cross section when the standard BS condition (3.1) is fulfilled. Then, the scalar sector has no effect on the blind spot condition. In such a case, we can have as large Δ_{mix} as is allowed solely by the LEP and LHC constraints, irrespective of the DM sector. Interestingly, the standard BS condition appears also, in a non-trivial way, when $\frac{|\kappa|}{\lambda}$ is relatively small (we still consider singlino-dominated LSP) and both terms in the denominator of (3.13) are comparable and approximately cancel each other. The blind spot condition (3.12) requires η to be fixed and not very large. From eq. (A.4) we see that in such a case a small denominator, for a singlino-dominated LSP, may be compensated by small factor $(m_\chi/\mu - \sin 2\beta)$ which means that the simplest BS condition (3.1) is approximately fulfilled. In both cases, the analysis performed in subsection 3.1 holds. However, it should be noted that small $|\kappa|$ weakens the singlet self-interaction $\sim \kappa S^3$ and hence may decrease the above-mentioned effects from the a exchange.

5 $\mathbb{Z}_3$ -symmetric NMSSM

All the analysis presented till this point apply to a general NMSSM. In this section we focus on the most widely studied version of NMSSM with $\mathbb{Z}_3$ symmetry. In this model there is no dimensionful parameters in the superpotential:

$$W_{\text{NMSSM}} = \lambda S H_u H_d + \kappa S^3/3, \quad (5.1)$$

while the soft SUSY breaking Lagrangian is given by (2.2) with $m_3^2 = m_S'^2 = \xi_S = 0$. This model has five free parameters less than general NMSSM which implies that some physical parameters important for dark matter sector are correlated. The main features of $\mathbb{Z}_3$ -symmetric NMSSM relevant for phenomenology of neutralino dark matter are summarized below:

- $\text{sgn}(m_\chi \mu) = \text{sgn}(\kappa)$
- LSP dominated by singlino implies

$$|\kappa| < \frac{1}{2}\lambda. \quad (5.2)$$

- Neither singlet-like scalar nor singlet-like pseudoscalar can be decoupled due to the following tree-level relation (for singlino LSP after taking into account the leading contributions from the mixing with both scalars coming from the weak doublets, $\hat{h}$ and $\hat{H}$):

$$m_s^2 + \frac{1}{3}m_a^2 \approx m_{\text{LSP}}^2 + \gamma^2(m_s^2 - m_h^2). \quad (5.3)$$

Masses of both singlet-dominated scalar and pseudoscalar are at most of order m_{LSP} .

Figure 6. Contours of $\Omega h^2 = 0.12$ in the plane $m_{\text{LSP}}\text{-}\tan\beta$ assuming SI blind spot with $m_\chi > 0$ and $\mu > 0$ in the $\mathbb{Z}_3$ -invariant NMSSM for several values of λ with $m_s = 200$ (left panel) and 500 GeV (right panel). The color code is the same as in the right panel of figure 3. The lines denoted by UM depict regions in which NMSSMTools reports unphysical global minimum.

- Phenomenologically viable (small) Higgs-singlet mixing leads to the following tree-level relation:

$$M_A \equiv |M_{\hat{A}\hat{A}}| \approx \frac{2|\mu|}{\sin(2\beta)} \sqrt{1 - \frac{\kappa \sin(2\beta)}{2\lambda}} \approx \frac{2|\mu|}{\sin(2\beta)}, \quad (5.4)$$

where the last approximation is applicable for large $\tan\beta$ and/or singlino-like LSP and forbids resonant LSP annihilation via heavy Higgs exchange. Such resonance may be present only for $\lambda \ll 1$ since only in such a case significant deviation from relation (5.4) is possible. Important constraints on dark matter sector of $\mathbb{Z}_3$ -symmetric NMSSM follow from relation (5.3) which we discuss in more detail in the following subsections.

5.1 Heavy singlet scalar

Let us first discuss the case of heavy singlet scalar in which only the Higgs exchange is relevant in the SI scattering amplitude and the SI blind spot has the standard form (3.1). In this case $|\gamma|$ must be close to zero to avoid large negative correction to the Higgs mass and eq. (5.3) implies $m_{\text{LSP}} > m_s$. This is demonstrated in figure 6 where it is clearly seen that for $m_{\text{LSP}} \lesssim m_s$ there are no solutions (due to a tachyonic pseudoscalar).¹²

We also note that eq. (5.3) implies that resonant LSP annihilation via singlet-like scalar or pseudoscalar is typically not possible in this case.¹³ On the other hand, eq. (5.3) implies that the LSP annihilation channel into sa via a exchange is almost always open (for small mixing γ and $m_s > m_h$ this channel is kinematically forbidden only in a small region of the parameter space for which $m_a \approx 3m_s$). This allows for smaller annihilation rate into

¹²In figure 6 vanishing $\hat{h}\text{-}\hat{s}$ mixing is assumed but for small non-zero mixing γ (preferred by the Higgs mass) the results are similar.

¹³Due to loop corrections to eq. (5.3) one may find some small regions of resonant annihilation mediated by a singlet for m_s not far above m_h and large λ close to the perturbativity bound. We discuss this effect in more detail in subsection 5.2 because it is more generic for $m_s < m_h$.

$t\bar{t}$, hence also for smaller higgsino component of the LSP and larger $\tan\beta$. In consequence, larger LSP masses consistent with $\Omega h^2 = 0.12$ and perturbativity up to the GUT scale are possible than in the case with both singlets decoupled (compare figure 6 to figure 3). For the same reason large enough LSP masses are beyond the reach of XENON1T, as seen from figure 6.

5.2 Light singlet scalar

The situation significantly changes when singlet-like scalar is light, especially if the Higgs-singlet mixing is not small (which enhances the Higgs mass if $m_s < m_h$). This is because the blind spot condition changes to eq. (4.4). Moreover, for light singlet the loop corrections to condition (5.3) can no longer be neglected which under some circumstances allows for resonant LSP annihilation via the s-channel exchange of a .

In the $\mathbb{Z}_3$ -symmetric NMSSM the singlet-dominated pseudoscalar a plays quite important role for the relic density of the singlino-dominated LSP. First we check if and when the s-channel exchange of a may dominate the LSP annihilation cross section and lead to the observed relic density. Of course, this may happen if we are quite close to the resonance, i.e. when $m_a \approx 2m_{\text{LSP}}$. It occurs that it is not so easy to fulfill this requirement in the $\mathbb{Z}_3$ -symmetric model. This is related to the condition (5.3) which, for $m_a \approx 2m_{\text{LSP}}$ and after taking into account the loop corrections in eqs. (2.6) and (2.14), may be rewritten in the form

$$m_s^2 + \frac{1}{3}m_{\text{LSP}}^2 + \gamma^2(m_h^2 - m_s^2) \approx \Delta_{\hat{s}\hat{s}} + \frac{1}{3}\Delta_{\hat{a}\hat{a}}. \quad (5.5)$$

The l.h.s. of the above expression is positive so this condition can not be fulfilled without the loop contributions. The last equation may be treated as a condition for the size of the loop corrections necessary to have resonant annihilation of the LSP mediated by the pseudoscalar a . In order to understand qualitatively the impact of condition (5.5) on our analysis it is enough to consider the following simple situation: we assume that the scalar mixing γ is negligible and the BS is approximated by (3.1). On the r.h.s. of eq. (5.5) we take into account only the first term of the loop correction $\Delta_{\hat{s}\hat{s}}$ [57]

$$\Delta_{\hat{s}\hat{s}} \approx \frac{1}{2\pi^2} \lambda^2 \mu^2 \ln\left(\frac{M_{\text{SUSY}}^2}{\mu^2}\right) + \frac{1}{2\pi^2} \kappa^2 m_{\text{LSP}}^2 \ln\left(\frac{M_{\text{SUSY}}^2}{m_{\text{LSP}}^2}\right) \quad (5.6)$$

(the second term is subdominant because $m_{\text{LSP}}^2 \ll \mu^2$ for a singlino-dominated LSP and $\kappa^2 < \frac{1}{4}\lambda^2$ due to condition (5.2)). In such approximation and for $\tan\beta \gg 1$ condition (5.5) simplifies to

$$m_s^2 \approx m_{\text{LSP}}^2 \left[\left(\frac{\lambda \tan\beta}{2\pi} \right)^2 \ln\left(\frac{2M_{\text{SUSY}}}{m_{\text{LSP}} \tan\beta} \right) - \frac{1}{3} \right]. \quad (5.7)$$

For given values of λ and m_s any change of m_{LSP} must be compensated by appropriate change of $\tan\beta$. The expression in the square bracket has a maximum as a function of $\tan\beta$ approximately at $1.2M_{\text{SUSY}}/m_{\text{LSP}}$. Thus, to keep the r.h.s. constant in order to stay close to the resonance one has to decrease $\tan\beta$ for small m_{LSP} and increase for large m_{LSP} . In our numerical examples presented in figure 7 we fix $M_{\text{SUSY}} = 4 \text{ TeV}$ so the maximum of the square bracket corresponds to $\tan\beta$ about 30 (10) for the LSP masses of 150 (500)

Figure 7. The same as in figure 5 but in the $\mathbb{Z}_3$ -invariant NMSSM and $\gamma = -0.1, 0, 0.1$.

GeV. For small $|\gamma|$ the a resonance occurs at the blind spot for $\tan \beta$ of order 10. That is why $\tan \beta$ typically decreases with m_{LSP} , as can be seen from figure 7. Local minimum for $\tan \beta$ is present only in the lower panel of figure 7 because more negative values of γ lead in general to larger $\tan \beta$ (see figure 8 and discussion at the end of this section). Nevertheless, in every case the $\Omega h^2 = 0.12$ curves corresponding to the a resonance have horizontal-like behavior: do not change very much with the LSP mass (and have values of $\tan \beta$ of order 10 for $M_{\text{SUSY}} = 4$ TeV that we use in our numerical examples). This should be compared to the general case when such curves are almost vertical (narrow ranges of the LSP mass but wide ranges of $\tan \beta$) — see figure 5. This difference comes from the fact that in the general model there are more parameters and eq. (5.3) is not fulfilled.

Figure 7 shows that there are two situations for which BS and correct value of DM relic density are still compatible with the latest bound on DM SD cross-section. One is the above discussed case of resonant annihilation with the light pseudoscalar exchanged in the s channel. The second one occurs for smaller $\tan \beta$ but bigger m_{LSP} and corresponds to annihilation via non-resonant exchange of particles in the s channel. Usually the main contribution to the annihilation cross-section in such a case comes from the exchange of Z^0 boson decaying into $t\bar{t}$ final state. This process allows to avoid the LUX bounds on σ_{SD} for m_{LSP} above about 300 GeV but is not sufficient to push σ_{SD} below sensitivity of XENON1T, as discussed in section 3. The situation changes when new final state channels, especially as , open. Then not only the present bounds on σ_{SD} may be easily fulfilled but some parts of the parameter space are beyond the XENON1T reach. We see from figure 7 that for light singlets the lower limit on the LSP mass from LUX may be relaxed to about 250 GeV. The effect of annihilation into light singlets is even more important for heavier LSP so XENON1T may not be sensitive to LSP masses above about 400 GeV.

Figure 8. Contour lines of $\Omega h^2 = 0.10, 0.12, 0.14$ as functions of $\tan \beta$ and γ in the $\mathbb{Z}_3$ -invariant NMSSM. Left panel: resonant annihilation via a exchange for $m_{\text{LSP}} = 150$ GeV. Right panel: non-resonant annihilation for $m_{\text{LSP}} = 300$ and 450 GeV. The green parts of the contours are excluded by LUX. The parabola-like curves show dependence of Δ_{mix} (on the right horizontal axes) on γ .

In both cases discussed above the allowed values of $\tan \beta$ are correlated with the LSP mass. The exact form of such correlation depends on the $\hat{s}\text{--}\hat{h}$ mixing parameter γ . Quite generally values of $\tan \beta$ decrease with γ . This is illustrated in figure 8 where the bands of allowed $\tan \beta$ as functions of γ are shown for a few values of the LSP mass. This correlation between $\tan \beta$ and γ can be easily understood from eqs. (4.2)–(4.4). The first factor on the r.h.s of (4.4) grows in the first approximation like $-\gamma$. This can not be compensated by decreasing κ because in the $\mathbb{Z}_3$ -symmetric NMSSM κ is fixed by the LSP mass. The BS condition (4.4) with increasing r.h.s. may be fulfilled by decreasing the absolute value of the negative contribution to its l.h.s. i.e. by increasing $\tan \beta$.

6 Conclusions

Motivated by the recent strong LUX constraints we investigated consequences of the assumption that the spin-independent cross-section of singlino-higgsino LSP scattering off nuclei is below the irreducible neutrino background. We determined constraints on the NMSSM parameter space assuming that the LSP is a thermal relic with the abundance consistent with Planck observations and studied how present and future constraints on spin-dependent scattering cross-section may probe blind spots in spin-independent direct detection.

In the case when all scalars except for the 125 GeV Higgs boson are heavy the new LUX constraints exclude the singlino-higgsino masses below about 300 GeV unless the LSP mass is very close to the half of the Z^0 boson mass (between about 41 and 46 GeV). In the allowed region LSP dominantly annihilates to $t\bar{t}$ and $\tan \beta$ must be below about 3.5 (assuming perturbative values of λ up to the GUT scale) with the upper bound being stronger for smaller λ and heavier LSP. There is also an upper bound of about 700 GeV assuming perturbativity up to the GUT scale. We found that XENON1T has sensitivity to exclude the entire region of dark matter annihilating dominantly to $t\bar{t}$. This conclusion

apply to general models of singlet-doublet dark matter. On the other hand, the LSP resonantly annihilating via Z^0 boson exchange is possible only for large $\tan\beta$ unless λ is very small e.g. for $\lambda > 0.5$, $\tan\beta \gtrsim 20$. Only small range of LSP masses around the resonance of about 2 GeV is beyond the XENON1T reach while LZ is expected to probe Z^0 resonance completely. In all of the above cases the LSP is dominated by singlino. Current and future constraints can be avoided also for very pure higgsino with mass in the vicinity of 1.1 TeV.

The situation significantly changes when singlet-like (pseudo)scalars are light. Firstly, the presence of light CP-even singlet scalar modifies the condition for spin-independent blind spot when its mixing with other Higgs bosons is non-negligible. Depending on the sign of the mixing angle between the singlet and the 125 GeV Higgs preferred values of $\tan\beta$ may be either increased or decreased, as compared to the case with heavy singlet. Interestingly, $\tan\beta$ is increased when the Higgs coupling to bottom quarks is smaller than that to gauge bosons which is somewhat favored by the LHC Higgs coupling measurements.

Secondly, the presence of light singlets opens new annihilation channels for the LSP. As a result, correct relic abundance requires smaller higgsino component of the LSP which relaxes spin-dependent constraints. We found that resonant annihilation via exchange of singlet pseudoscalar is possible even in the $\mathbb{Z}_3$ -invariant NMSSM. Interestingly, even far away from the resonant region the lower limit on the mass of LSP annihilating mainly to $t\bar{t}$ may be relaxed to 250 GeV. For larger LSP masses sa may become dominant annihilation channel and the LSP masses above 400 GeV may be beyond the reach of XENON1T.

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A LSP-nucleon cross sections

In this appendix we collect several expressions useful in discussing the SI and SD cross-sections of LSP on nuclei.

The couplings of the i -th scalar to the LSP and to the nucleon, appearing in the formula (2.19) for the SI cross-section, after decoupling the gauginos are approximated,

respectively, by

$$\alpha_{h_i\chi\chi} \approx \sqrt{2}\lambda \left[\tilde{S}_{h_i\hat{h}} N_{15} (N_{13} \sin \beta + N_{14} \cos \beta) + \tilde{S}_{h_i\hat{H}} N_{15} (N_{14} \sin \beta - N_{13} \cos \beta) + \tilde{S}_{h_i\hat{s}} \left(N_{13} N_{14} - \frac{\kappa}{\lambda} N_{15}^2 \right) \right], \quad (\text{A.1})$$

$$\alpha_{h_i NN} \approx \frac{m_N}{\sqrt{2}v} \left[\tilde{S}_{h_i\hat{h}} \left(F_d^{(N)} + F_u^{(N)} \right) + \tilde{S}_{h_i\hat{H}} \left(\tan \beta F_d^{(N)} - \frac{1}{\tan \beta} F_u^{(N)} \right) \right]. \quad (\text{A.2})$$

The LSP couplings to pseudoscalars, important for the relic abundance calculation, are approximated by ref. [57]

$$\alpha_{a_i\chi\chi} \approx i\sqrt{2}\lambda \left[N_{15} \tilde{P}_{i1} (\sin \beta N_{14} + \cos \beta N_{13}) + \tilde{P}_{i2} \left(N_{13} N_{14} - \frac{\kappa}{\lambda} N_{15}^2 \right) \right], \quad (\text{A.3})$$

where $\tilde{P}_{ij}$ are elements of the matrix diagonalizing the pseudoscalar mass matrix defined in eq. (2.15).

Parameter η defined by eq. (3.13) and convenient for the discussion of SI blind, using eqs. (2.17) and (2.18), may be written in the form

$$\eta = \frac{\frac{\lambda v}{\mu} \left(1 - \left(\frac{m_\chi}{\mu} \right)^2 \right) \left(\frac{m_\chi}{\mu} - \sin 2\beta \right)}{\left(\frac{\lambda v}{\mu} \right)^2 \left[\left(1 + \left(\frac{m_\chi}{\mu} \right)^2 \right) \frac{\sin 2\beta}{2} - \frac{m_\chi}{\mu} \right] - \frac{\kappa}{\lambda} \left(1 - \left(\frac{m_\chi}{\mu} \right)^2 \right)^2}. \quad (\text{A.4})$$

With the help of eqs. (2.17) and (2.18), the combination of the LSP components crucial for σ_{SD} , $(N_{13}^2 - N_{14}^2)$, may be written as:

$$N_{13}^2 - N_{14}^2 = \frac{\left[1 - (m_\chi/\mu)^2 \right] (1 - N_{15}^2) \cos 2\beta}{1 + (m_\chi/\mu)^2 - 2(m_\chi/\mu) \sin 2\beta}. \quad (\text{A.5})$$

We can see immediately that the cross-section disappears in the limit of $\tan \beta = 1$ or a pure singlino/higgsino LSP. The ratio of the higgsino to the singlino components of the LSP may be calculating from eqs. (2.17) and (2.18):

$$\frac{1 - N_{15}^2}{N_{15}^2} = \left(\frac{\lambda v_h}{\mu} \right)^2 \frac{1 + (m_\chi/\mu)^2 - 2(m_\chi/\mu) \sin 2\beta}{\left[1 - (m_\chi/\mu)^2 \right]^2}. \quad (\text{A.6})$$

Using this relation we may rewrite formula (A.5) in the form:

$$N_{13}^2 - N_{14}^2 = \frac{\left[1 - (m_\chi/\mu)^2 \right] \cos 2\beta}{1 + (m_\chi/\mu)^2 - 2(m_\chi/\mu) \sin 2\beta + \left[1 - (m_\chi/\mu)^2 \right]^2 (\mu/\lambda v)^2}. \quad (\text{A.7})$$

B The LSP (co)annihilation channels

In this appendix we will use the following expansion of σv around $v = 0$:

$$\sigma v = a + bv^2 + \mathcal{O}(v^4). \quad (\text{B.1})$$

Then, the relic density may be written as [58]:

$$\Omega h^2 \approx \frac{9.4 \times 10^{-12} \text{ GeV}^{-2} x_f}{a + 3b/x_f}, \quad (\text{B.2})$$

where $x_f \approx 25$.

B.1 Resonance with the Z^0 boson (unitary gauge)

Let us consider the LSP annihilation into the SM fermions (except the t quark¹⁴) via Z^0 exchange in s channel. The expansion coefficient a and b in eq. (B.1) are equal to:

$$a = \frac{g^4}{32\pi} \frac{(N_{13}^2 - N_{14}^2)^2}{(4m_\chi^2 - m_{Z^0}^2)^2 + m_{Z^0}^2 \Gamma_{Z^0}^2} \left(1 - \frac{4m_\chi^2}{m_{Z^0}^2}\right)^2 \times \sum_F c_F m_F^2 \sqrt{1 - \frac{m_F^2}{m_\chi^2}}, \quad (\text{B.3})$$

$$b \approx b_0 = \frac{g^4}{32\pi} \frac{(N_{13}^2 - N_{14}^2)^2}{(4m_\chi^2 - m_{Z^0}^2)^2 + m_{Z^0}^2 \Gamma_{Z^0}^2} \times \frac{2m_\chi^2}{3} \sum_F c_F (2\beta_F^2 - 2\beta_F + 1), \quad (\text{B.4})$$

where $g \equiv \sqrt{(g_1^2 + g_2^2)}/2$, $c_F = 1$ for leptons and 3 for quarks, whereas $\beta_F = 2|q_F| \sin \theta_W^2$. The 0 index in b_0 parameter means that we put fermion masses to 0 (which is a very good approximation for $m_\chi \sim m_{Z^0}/2$; of course $a_0 = 0$). The sum over the SM fermions (except the t quark) in (B.4) equal ~ 14.6 . It is worth noting that $b_0 \sim m_\chi^2$ and $a \sim m_F^2(1 - 4m_\chi^2/m_{Z^0}^2)$ which means that $b \gg a \sim 0$ (in contrary to naive expectation). Moreover, the terms proportional to higher powers of v^2 in σv (for $m_\chi \gg m_F$) are suppressed with respect to bv^2 term in geometric way by $v^2/4$. Therefore we can approximate $\sigma v \approx b_0 v^2$ and hence expressed the relic density in the form of eq. (B.2). We will however improve slightly this approach (see appendix C) and write our formula in the following form:

$$\Omega h^2 \approx 0.1 \left(\frac{0.3}{N_{13}^2 - N_{14}^2} \right)^2 \frac{m_{Z^0}^2}{4m_\chi^2} \left[\left(\frac{4m_\chi^2}{m_{Z^0}^2} - 1 + \frac{\bar{v}^2}{4} \right)^2 + \frac{\Gamma_{Z^0}^2}{m_{Z^0}^2} \right]. \quad (\text{B.5})$$

where the term proportional to $\bar{v}^2 \approx 0.5^2$ stems from the fact that the dark matter particles posses some thermal energy during the freeze-out. Eq. (B.5) reproduces very well the results obtained from MicrOMEGAs far from the resonance (see e.g. figure 1), however very close to the resonance, especially for $m_\chi \lesssim m_{Z^0}/2$, the difference may be sizable (figure 3).

B.2 Annihilation into $t\bar{t}$ via Z^0

In this case the dominant contribution also comes from Z^0 exchange in s channel but in contrary to the previous paragraph $m_\chi \sim m_F (= m_t)$. Therefore the statement that $b \gg a$ is now longer true. It becomes clear when we write down the expression for a and b terms

¹⁴The effect from the t quark appears for $m_\chi \sim m_t$ which is quite far from the resonance — we will discuss this case separately in the next paragraph.

in the limit $m_\chi \gg m_{Z^0}$:

$$a \approx \frac{3g^4}{32\pi} (N_{13}^2 - N_{14}^2)^2 \frac{m_t^2}{m_{Z^0}^4} \sqrt{1 - \frac{m_t^2}{m_\chi^2}}, \quad (\text{B.6})$$

$$b \approx \frac{3g^4}{32\pi} (N_{13}^2 - N_{14}^2)^2 \frac{m_t^2}{m_{Z^0}^4} \frac{1}{4} \left(1 - \frac{m_t^2}{2m_\chi^2}\right) \frac{1}{\sqrt{1 - \frac{m_t^2}{m_\chi^2}}}. \quad (\text{B.7})$$

One can see that for $m_\chi \sim m_t$ both terms are comparable whereas for larger m_χ we have $a/b \approx 4$ and eq. (B.6) suffices (as we would expect, the terms proportional to higher powers of v^2 are suppressed for $m_\chi \gg m_t, m_{Z^0}$ as $v^2/4$). Similarly to eq. (B.5) we can find the expression for Ωh^2 . Combining (B.2) with (B.6) and (B.7) we get:

$$\Omega h^2 \approx 0.1 \left(\frac{0.05}{N_{13}^2 - N_{14}^2} \right)^2 \left[\sqrt{1 - \frac{m_t^2}{m_\chi^2}} + \frac{3}{4} \frac{1}{x_f} \left(1 - \frac{m_t^2}{2m_\chi^2}\right) \frac{1}{\sqrt{1 - \frac{m_t^2}{m_\chi^2}}} \right]^{-1}. \quad (\text{B.8})$$

The above equation works well for $m_\chi \gtrsim 175$ GeV (see figure 1), however for $m_\chi \approx m_t$ we have to be more careful because the expansion in v^2 breaks down. One can see that for $m_\chi \gg m_t, m_{Z^0}$ the square bracket in (B.8) equals roughly 1 and Ωh^2 depends on $|N_{13}^2 - N_{14}^2|$ only. Similarly to the case of the resonance with Z^0 , the crucial experimental bounds comes from SD direct detection (see right plot in figure 2).

It is worth pointing out that both a and b coefficients in (B.6) and (B.7) come purely from $-p^\mu p^\nu / m_{Z^0}^2$ term in Z^0 propagator. It was noticed long time ago [59, 60] that taking into account this term is also crucial for DM annihilation in galactic halos ($v^2 \approx 10^{-6}$) for $m_\chi \sim m_{Z^0}/2$. This is because the a coefficient in (B.3) vanishes which causes large dip in the annihilation cross section.

C Improved formula for Ωh^2 near a resonance

The method described below may be found e.g. in [58, 61]. Let us consider a general expression for σv for scalar dark matter (with mass m) annihilating via s channel exchange of a particle with mass M and total decay width Γ :

$$\sigma v = \frac{\alpha}{(s - M^2)^2 + \Gamma^2 M^2}. \quad (\text{C.1})$$

For simplicity we assume we assume $\alpha = \text{const}$ which is generally not the case, however we are mainly focused on the effect on Ωh^2 coming from the denominator in (C.1). Using dimensionless quantities $\delta \equiv 4m^2/M^2 - 1$, $\gamma \equiv \Gamma/M$ and considering non-relativistic approximation $s = 4m^2/(1 - v^2/4) \approx 4m^2(1 + v^2/4)$ we get:

$$\sigma v = \frac{\alpha/M^4}{(\delta + v^2/4)^2 + \gamma^2}. \quad (\text{C.2})$$

Let us now define $Y(x) \equiv \frac{n}{s}$, where $x = T/m$, and write

$$\frac{1}{Y(\infty)} - \frac{1}{Y(x_d)} = m M_{\text{Pl}} \frac{g_s}{\sqrt{g}} \sqrt{\frac{\pi}{45}} \int_{x_d}^{\infty} \frac{\langle \sigma v \rangle}{x^2} dx. \quad (\text{C.3})$$

Parameter x_d is defined as a moment in thermal evolution of DM when the term $1/Y(x_d)$ starts to be small and can be safely neglected. Dark matter relic abundance can be then calculated by double integration over v and x :

$$\Omega h^2 = \frac{2.82 \cdot 10^8}{\text{GeV}} m Y(\infty) \quad (\text{C.4})$$

$$\approx \frac{2.82 \cdot 10^8}{\text{GeV}} \frac{1}{M_{\text{Pl}}} \frac{\sqrt{g}}{g_s} \sqrt{\frac{45}{\pi}} \left[\frac{1}{2\sqrt{\pi}} \int_0^{\infty} dv (\sigma v) v^2 \int_{x_d}^{\infty} dx \frac{e^{-v^2 x/4}}{\sqrt{x}} \right]^{-1} \quad (\text{C.5})$$

Note that we changed the usual order of integration. We will now perform the simpler integral over x , obtaining:

$$\frac{1}{2\sqrt{\pi}} \int_0^{\infty} dv (\sigma v) v^2 \int_{x_d}^{\infty} dx \frac{e^{-v^2 x/4}}{\sqrt{x}} = \int_0^{\infty} (\sigma v) v \operatorname{erfc}(v/2\sqrt{x_d}) dv. \quad (\text{C.6})$$

Substituting here eq. (C.2) and $\operatorname{erfc}(v/2\sqrt{x_d}) \approx 1 - \sqrt{x_d/\pi} v + \dots$ we can easily find simple expressions for Ωh^2 for some hierarchical values of δ and γ e.g. $\delta \ll \gamma$ etc.

In the case of fermionic dark matter our expression (C.1) generalizes to:

$$\sigma v = \frac{\alpha f(s)}{(s - M^2)^2 + \Gamma^2 M^2}. \quad (\text{C.7})$$

Now we have to perform the following integral

$$\dots \frac{\alpha}{M^4} \int_0^{\infty} \frac{f(v) v \operatorname{erfc}(v/2\sqrt{x_d})}{(\delta + v^2/4)^2 + \gamma^2} dv. \quad (\text{C.8})$$

In order to proceed further we have to specify the formula for $f(s)$. In the case of LSP annihilation into fermions via Z^0 exchange the dominant contribution is $f(v) \sim v^2$ – see appendix B.1. Analytical form of the above integral is very complicated even for such simple expression for $f(v)$. The numerator in eq. (C.8) has a maximum for some specific value of v . Therefore we will take the denominator in front of the integral, substituting $v \rightarrow \bar{v}$, where $\bar{v}$ is defined as a mean value of the numerator. Then we have:

$$\dots \frac{\alpha}{M^4} \frac{1}{(\delta + \bar{v}^2/4)^2 + \gamma^2} \int_0^{\infty} v^3 \operatorname{erfc}(v/2\sqrt{x_d}) dv = \dots \frac{\alpha}{M^4} \frac{3/x_d^2}{(\delta + \bar{v}^2/4)^2 + \gamma^2}, \quad (\text{C.9})$$

$$\bar{v} \equiv \frac{\int_0^{\infty} v v^3 \operatorname{erfc}(v/2\sqrt{x_d}) dv}{\int_0^{\infty} v^3 \operatorname{erfc}(v/2\sqrt{x_d}) dv} = \frac{64}{15\sqrt{\pi}} x_d^{-1/2}. \quad (\text{C.10})$$

For $x_d = 25$ we have $\bar{v} \approx 0.5$. The above method effectively includes the fact that the dark matter particles possess some thermal energy during their freeze-out. Other cases of $f(v)$ can be also easily analyzed and compared with numerical results.

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References

- [1] LUX collaboration, D.S. Akerib et al., *Results from a search for dark matter in the complete LUX exposure*, *Phys. Rev. Lett.* **118** (2017) 021303 [[arXiv:1608.07648](https://arxiv.org/abs/1608.07648)] [[INSPIRE](#)].
- [2] XENON collaboration, E. Aprile et al., *Physics reach of the XENON1T dark matter experiment*, *JCAP* **04** (2016) 027 [[arXiv:1512.07501](https://arxiv.org/abs/1512.07501)] [[INSPIRE](#)].
- [3] D.C. Malling et al., *After LUX: the LZ program*, [arXiv:1110.0103](https://arxiv.org/abs/1110.0103) [[INSPIRE](#)].
- [4] XENON100 collaboration, E. Aprile et al., *Dark matter results from 225 live days of XENON100 data*, *Phys. Rev. Lett.* **109** (2012) 181301 [[arXiv:1207.5988](https://arxiv.org/abs/1207.5988)] [[INSPIRE](#)].
- [5] C. Cheung, L.J. Hall, D. Pinner and J.T. Ruderman, *Prospects and blind spots for neutralino dark matter*, *JHEP* **05** (2013) 100 [[arXiv:1211.4873](https://arxiv.org/abs/1211.4873)] [[INSPIRE](#)].
- [6] U. Ellwanger and C. Hugonie, *The semi-constrained NMSSM satisfying bounds from the LHC, LUX and Planck*, *JHEP* **08** (2014) 046 [[arXiv:1405.6647](https://arxiv.org/abs/1405.6647)] [[INSPIRE](#)].
- [7] A. Butter et al., *Invisible Higgs decays to hooperons in the NMSSM*, *Phys. Rev. D* **93** (2016) 015011 [[arXiv:1507.02288](https://arxiv.org/abs/1507.02288)] [[INSPIRE](#)].
- [8] J. Bramante, N. Desai, P. Fox, A. Martin, B. Ostdiek and T. Plehn, *Towards the final word on neutralino dark matter*, *Phys. Rev. D* **93** (2016) 063525 [[arXiv:1510.03460](https://arxiv.org/abs/1510.03460)] [[INSPIRE](#)].
- [9] M. Badziak, A. Delgado, M. Olechowski, S. Pokorski and K. Sakurai, *Detecting underabundant neutralinos*, *JHEP* **11** (2015) 053 [[arXiv:1506.07177](https://arxiv.org/abs/1506.07177)] [[INSPIRE](#)].
- [10] D. Barducci, A. Belyaev, A.K.M. Bharucha, W. Porod and V. Sanz, *Uncovering natural supersymmetry via the interplay between the LHC and direct dark matter detection*, *JHEP* **07** (2015) 066 [[arXiv:1504.02472](https://arxiv.org/abs/1504.02472)] [[INSPIRE](#)].
- [11] J. Bramante et al., *Relic neutralino surface at a 100 TeV collider*, *Phys. Rev. D* **91** (2015) 054015 [[arXiv:1412.4789](https://arxiv.org/abs/1412.4789)] [[INSPIRE](#)].
- [12] L. Roszkowski, E.M. Sessolo and A.J. Williams, *Prospects for dark matter searches in the pMSSM*, *JHEP* **02** (2015) 014 [[arXiv:1411.5214](https://arxiv.org/abs/1411.5214)] [[INSPIRE](#)].
- [13] T. Cohen, M. Lisanti, A. Pierce and T.R. Slatyer, *Wino dark matter under siege*, *JCAP* **10** (2013) 061 [[arXiv:1307.4082](https://arxiv.org/abs/1307.4082)] [[INSPIRE](#)].
- [14] M. Low and L.-T. Wang, *Neutralino dark matter at 14 TeV and 100 TeV*, *JHEP* **08** (2014) 161 [[arXiv:1404.0682](https://arxiv.org/abs/1404.0682)] [[INSPIRE](#)].
- [15] N. Nagata and S. Shirai, *Higgsino dark matter in high-scale supersymmetry*, *JHEP* **01** (2015) 029 [[arXiv:1410.4549](https://arxiv.org/abs/1410.4549)] [[INSPIRE](#)].
- [16] J. Bramante, A. Delgado, F. Elahi, A. Martin and B. Ostdiek, *Catching sparks from well-forged neutralinos*, *Phys. Rev. D* **90** (2014) 095008 [[arXiv:1408.6530](https://arxiv.org/abs/1408.6530)] [[INSPIRE](#)].
- [17] G. Grilli di Cortona, *Hunting electroweakinos at future hadron colliders and direct detection experiments*, *JHEP* **05** (2015) 035 [[arXiv:1412.5952](https://arxiv.org/abs/1412.5952)] [[INSPIRE](#)].
- [18] L. Calibbi, J.M. Lindert, T. Ota and Y. Takanishi, *LHC tests of light neutralino dark matter without light sfermions*, *JHEP* **11** (2014) 106 [[arXiv:1410.5730](https://arxiv.org/abs/1410.5730)] [[INSPIRE](#)].

- [19] L. Calibbi, J.M. Lindert, T. Ota and Y. Takanishi, *Cornering light neutralino dark matter at the LHC*, *JHEP* **10** (2013) 132 [[arXiv:1307.4119](#)] [[INSPIRE](#)].
- [20] H. Baer, V. Barger and D. Mickelson, *Direct and indirect detection of higgsino-like WIMPs: concluding the story of electroweak naturalness*, *Phys. Lett. B* **726** (2013) 330 [[arXiv:1303.3816](#)] [[INSPIRE](#)].
- [21] H. Baer, V. Barger and H. Serce, *SUSY under siege from direct and indirect WIMP detection experiments*, *Phys. Rev. D* **94** (2016) 115019 [[arXiv:1609.06735](#)] [[INSPIRE](#)].
- [22] M. Beneke et al., *Relic density of wino-like dark matter in the MSSM*, *JHEP* **03** (2016) 119 [[arXiv:1601.04718](#)] [[INSPIRE](#)].
- [23] M. Beneke, A. Bharucha, A. Hryczuk, S. Recksiegel and P. Ruiz-Femenia, *The last refuge of mixed wino-Higgsino dark matter*, *JHEP* **01** (2017) 002 [[arXiv:1611.00804](#)] [[INSPIRE](#)].
- [24] J. Cao, Y. He, L. Shang, W. Su and Y. Zhang, *Natural NMSSM after LHC Run I and the Higgsino dominated dark matter scenario*, *JHEP* **08** (2016) 037 [[arXiv:1606.04416](#)] [[INSPIRE](#)].
- [25] N. Arkani-Hamed, A. Delgado and G.F. Giudice, *The well-tempered neutralino*, *Nucl. Phys. B* **741** (2006) 108 [[hep-ph/0601041](#)] [[INSPIRE](#)].
- [26] P. Huang and C.E.M. Wagner, *Blind spots for neutralino dark matter in the MSSM with an intermediate m_A* , *Phys. Rev. D* **90** (2014) 015018 [[arXiv:1404.0392](#)] [[INSPIRE](#)].
- [27] A. Crivellin, M. Hoferichter, M. Procura and L.C. Tunstall, *Light stops, blind spots and isospin violation in the MSSM*, *JHEP* **07** (2015) 129 [[arXiv:1503.03478](#)] [[INSPIRE](#)].
- [28] T. Han, F. Kling, S. Su and Y. Wu, *Unblinding the dark matter blind spots*, *JHEP* **02** (2017) 057 [[arXiv:1612.02387](#)] [[INSPIRE](#)].
- [29] M. Badziak, M. Olechowski and P. Szczerbiak, *Blind spots for neutralino dark matter in the NMSSM*, *JHEP* **03** (2016) 179 [[arXiv:1512.02472](#)] [[INSPIRE](#)].
- [30] C. Cheung, M. Papucci, D. Sanford, N.R. Shah and K.M. Zurek, *NMSSM interpretation of the Galactic Center excess*, *Phys. Rev. D* **90** (2014) 075011 [[arXiv:1406.6372](#)] [[INSPIRE](#)].
- [31] M. Badziak, M. Olechowski and P. Szczerbiak, *Is well-tempered neutralino in MSSM still alive after 2016 LUX results?*, *Phys. Lett. B* **770** (2017) 226 [[arXiv:1701.05869](#)] [[INSPIRE](#)].
- [32] P. Huang, R.A. Roglans, D.D. Spiegel, Y. Sun and C.E.M. Wagner, *Constraints on supersymmetric dark matter for heavy scalar superpartners*, *Phys. Rev. D* **95** (2017) 095021 [[arXiv:1701.02737](#)] [[INSPIRE](#)].
- [33] J. Cao, Y. He, L. Shang, W. Su, P. Wu and Y. Zhang, *Strong constraints of LUX-2016 results on the natural NMSSM*, *JHEP* **10** (2016) 136 [[arXiv:1609.00204](#)] [[INSPIRE](#)].
- [34] U. Ellwanger, *Present status and future tests of the higgsino-singlino sector in the NMSSM*, *JHEP* **02** (2017) 051 [[arXiv:1612.06574](#)] [[INSPIRE](#)].
- [35] C. Beskidt, W. de Boer, D.I. Kazakov and S. Wayand, *Perspectives of direct detection of supersymmetric Dark Matter in the NMSSM*, *Phys. Lett. B* **771** (2017) 611 [[arXiv:1703.01255](#)] [[INSPIRE](#)].
- [36] PLANCK collaboration, P.A.R. Ade et al., *Planck 2015 results. XIII. Cosmological parameters*, *Astron. Astrophys.* **594** (2016) A13 [[arXiv:1502.01589](#)] [[INSPIRE](#)].
- [37] T. Cohen, J. Kearney, A. Pierce and D. Tucker-Smith, *Singlet-doublet dark matter*, *Phys. Rev. D* **85** (2012) 075003 [[arXiv:1109.2604](#)] [[INSPIRE](#)].

- [38] C. Cheung and D. Sanford, *Simplified models of mixed dark matter*, *JCAP* **02** (2014) 011 [[arXiv:1311.5896](#)] [[INSPIRE](#)].
- [39] L. Calibbi, A. Mariotti and P. Tziveloglou, *Singlet-doublet model: dark matter searches and LHC constraints*, *JHEP* **10** (2015) 116 [[arXiv:1505.03867](#)] [[INSPIRE](#)].
- [40] S. Banerjee, S. Matsumoto, K. Mukaida and Y.-L.S. Tsai, *WIMP dark matter in a well-tempered regime: a case study on singlet-doublets fermionic WIMP*, *JHEP* **11** (2016) 070 [[arXiv:1603.07387](#)] [[INSPIRE](#)].
- [41] P. da Silva, *Direct dark matter searches with LUX*, talk given at the http://moriond.in2p3.fr/VHEPU/2017/transparencies/2_monday/1_morning/1_pascoal_da_silva.pdf Rencontres de Moriond conference on Very High Energy Phenomena in the Universe, March 18–25, La Thuile, Italy (2017).
- [42] G. Chalons, M.J. Dolan and C. McCabe, *Neutralino dark matter and the Fermi gamma-ray lines*, *JCAP* **02** (2013) 016 [[arXiv:1211.5154](#)] [[INSPIRE](#)].
- [43] G. Belanger, F. Boudjema, A. Pukhov and A. Semenov, *Dark matter direct detection rate in a generic model with micrOMEGAs 2.2*, *Comput. Phys. Commun.* **180** (2009) 747 [[arXiv:0803.2360](#)].
- [44] G. Jungman, M. Kamionkowski and K. Griest, *Supersymmetric dark matter*, *Phys. Rept.* **267** (1996) 195 [[hep-ph/9506380](#)] [[INSPIRE](#)].
- [45] G. Bélanger, F. Boudjema, A. Pukhov and A. Semenov, *MicrOMEGAs_3: a program for calculating dark matter observables*, *Comput. Phys. Commun.* **185** (2014) 960 [[arXiv:1305.0237](#)] [[INSPIRE](#)].
- [46] U. Ellwanger, J.F. Gunion and C. Hugonie, *NMHDECAY: a Fortran code for the Higgs masses, couplings and decay widths in the NMSSM*, *JHEP* **02** (2005) 066 [[hep-ph/0406215](#)] [[INSPIRE](#)].
- [47] U. Ellwanger and C. Hugonie, *NMHDECAY 2.0: an updated program for sparticle masses, Higgs masses, couplings and decay widths in the NMSSM*, *Comput. Phys. Commun.* **175** (2006) 290 [[hep-ph/0508022](#)] [[INSPIRE](#)].
- [48] ATLAS and CMS collaborations, *Measurements of the Higgs boson production and decay rates and constraints on its couplings from a combined ATLAS and CMS analysis of the LHC pp collision data at $\sqrt{s} = 7$ and 8 TeV*, *JHEP* **08** (2016) 045 [[arXiv:1606.02266](#)] [[INSPIRE](#)].
- [49] ALEPH collaboration, A. Heister et al., *Search for charginos nearly mass degenerate with the lightest neutralino in e^+e^- collisions at center-of-mass energies up to 209-GeV*, *Phys. Lett. B* **533** (2002) 223 [[hep-ex/0203020](#)] [[INSPIRE](#)].
- [50] SLD ELECTROWEAK GROUP, DELPHI, ALEPH, SLD, SLD HEAVY FLAVOUR GROUP, OPAL, LEP ELECTROWEAK WORKING GROUP, L3 collaboration, *Precision electroweak measurements on the Z resonance*, *Phys. Rept.* **427** (2006) 257 [[hep-ex/0509008](#)] [[INSPIRE](#)].
- [51] ICECUBE collaboration, M.G. Aartsen et al., *Improved limits on dark matter annihilation in the Sun with the 79-string IceCube detector and implications for supersymmetry*, *JCAP* **04** (2016) 022 [[arXiv:1601.00653](#)] [[INSPIRE](#)].
- [52] P. Cushman et al., *Working group report: WIMP dark matter direct detection*, [arXiv:1310.8327](#) [[INSPIRE](#)].
- [53] M. Badziak, M. Olechowski and S. Pokorski, *New regions in the NMSSM with a 125 GeV Higgs*, *JHEP* **06** (2013) 043 [[arXiv:1304.5437](#)] [[INSPIRE](#)].

- [54] CMS collaboration, *Searches for invisible decays of the Higgs boson in pp collisions at $\sqrt{s} = 7, 8$ and 13 TeV*, *JHEP* **02** (2017) 135 [[arXiv:1610.09218](#)] [[INSPIRE](#)].
- [55] M. Badziak and C.E.M. Wagner, *Enhancing the Higgs associated production with a top quark pair*, *JHEP* **05** (2016) 123 [[arXiv:1602.06198](#)] [[INSPIRE](#)].
- [56] M. Badziak and C.E.M. Wagner, *Enhanced Higgs associated production with a top quark pair in the NMSSM with light singlets*, *JHEP* **02** (2017) 050 [[arXiv:1611.02353](#)] [[INSPIRE](#)].
- [57] U. Ellwanger, C. Hugonie and A.M. Teixeira, *The next-to-minimal supersymmetric standard model*, *Phys. Rept.* **496** (2010) 1 [[arXiv:0910.1785](#)] [[INSPIRE](#)].
- [58] K. Griest and D. Seckel, *Three exceptions in the calculation of relic abundances*, *Phys. Rev. D* **43** (1991) 3191.
- [59] K. Griest, M. Kamionkowski and M.S. Turner, *Supersymmetric dark matter above the W mass*, *Phys. Rev. D* **41** (1990) 3565.
- [60] K. Griest, *Cross-sections, relic abundance and detection rates for neutralino dark matter*, *Phys. Rev. D* **38** (1988) 2357 [*Erratum ibid.* **D 39** (1989) 3802].
- [61] M. Duch and B. Grzadkowski, *Resonance enhancement of dark matter interactions: the case for early kinetic decoupling and velocity dependent resonance width*, [arXiv:1705.10777](#) [[INSPIRE](#)].