Published for SISSA by Springer

Received: *March 4, 2016*
Accepted: *July 12, 2016*
Published: *July 15, 2016*

On a residual freedom of the next-to-leading BFKL eigenvalue in color adjoint representation in planar $\mathcal{N} = 4$ SYM

Sergey Bondarenko and Alex Prygarin

Physics Department, Ariel University,
Ariel 40700, territories administered by Israel

E-mail: sergeyb@ariel.ac.il, alexanderp@ariel.ac.il

Abstract: We discuss a residual freedom of the next-to-leading BFKL eigenvalue that originates from ambiguity in redistributing the next-to-leading (NLO) corrections between the adjoint BFKL eigenvalue and eigenfunctions in planar $\mathcal{N} = 4$ super-Yang-Mills (SYM) Theory. In terms of the remainder function of the Bern-Dixon-Smirnov (BDS) amplitude this freedom is translated to reshuffling correction between the eigenvalue and the impact factors in the multi-Regge kinematics (MRK) in the next-to-leading logarithm approximation (NLA). We show that the modified NLO BFKL eigenvalue suggested by the authors in ref. [1] can be introduced in the MRK expression for the remainder function by shifting the anomalous dimension in the impact factor in such a way that the two and three loop remainder function is left unchanged to the NLA accuracy.

Keywords: 1/N Expansion, Supersymmetric gauge theory

ArXiv ePrint: 1603.01093

doi:10.1007/JHEP07(2016)081

Contents

1 Introduction

The kernel of the BFKL (Balitsky-Fadin-Kuraev-Lipatov) [2–6] equation contains real and virtual gluon emissions. The virtual gluon emissions are included in the infrared divergent gluon Regge trajectory. At present, the BFKL kernel is known to the next-to-leading (NLO) [5–20] order for arbitrary color group representation in both QCD and its supersymmetric extensions. The infrared (IR) divergences cancel between the real and the virtual part of the kernel than projected on the singlet color state. This cancellation does not happen for the BFKL kernel in the adjoint representation, but despite being infrared divergent it can be useful in some applications, which also determine the way one treats the IR terms. For example, in the Bartels-Kwiecinski-Praszalowicz (BKP) [21, 22] approach of interacting reggeized gluons one can remove "halves" of two gluon trajectories, while in calculations of the corrections to the Bern-Dixon-Smirnov (BDS) [23] amplitude one removes trajectory of single reggeized gluon in order to obtain the IR finite expression for the remainder function. The eigenvalue of reduced IR finite BFKL kernel obtained in the "BDS-like" way is commonly known as the adjoint BFKL eigenvalue. The adjoint BFKL eigenvalue at the leading order was calculated by Bartels, Lipatov and Sabio Vera [24] and its NLO expression was found by Fadin and Lipatov [25]. Then its higher loop corrections were calculated order by order [26] and in all orders from near-collinear limit using integrability techniques as well as a non-trivial analytic continuation from the collinear to Regge kinematics [27, 28].

The BFKL approach to the helicity amplitudes in the Regge kinematics was extensively studied over the last years [29–50] and was very useful in understanding higher order corrections of the BFKL eigenvalue and the impact factor. However, already at the next-to-next-to-leading level the adjoint BFKL eigenvalue was shown to have an alerting feature of having a non-vanishing limit as $\nu \to 0$ after setting the conformal spin $n = 0$ [34], which in not compatible with existence of a constant BFKL eigenfunction. It was shown [36] that one way to solve this problem is to account for corrections to the cusp anomalous dimension in the impact factor. In the previous paper the authors [1] suggested that a more natural way would be to redefine the notion of the adjoint BFKL eigenvalue exploiting some ambiguity in its definition. This ambiguity is related to the way one removes IR terms as well as how one redistributes the NLO corrections between the eigenvalue and the impact factor. Moreover, the energy scale in the leading logarithm approximation is not fixed at the leading order and becomes to be important at the NLO level. The authors claimed that there is enough freedom to modify the adjoint BFKL eigenvalue in such a way that the corresponding BDS remainder function is left intact with the next-to-leading logarithm accuracy. In the present paper we show in details how the freedom of redistributing NLO corrections between the BFKL eigenvalue and eigenfunctions is realized in the remainder function as interchanging the corresponding corrections between the eigenvalue and the impact factor. The paper is organized as follows. In the next section we discuss the origin of the freedom of redistributing the NLO corrections between the eigenvalue and eigenfunctions of the reduced BFKL kernel. Then we show how this freedom is realized in the BDS remainder function modifying the eigenvalue and the impact factors, and explain why this procedure does not affect the final expression of the remainder function to the NLA accuracy.

2 Residual freedom of BFKL eigenvalue and eigenfunctions

In the multi-Regge kinematics (MRK) the effective summation parameter is $a \ln \frac{s}{s_0}$, where s is the center of mass energy squared and s_0 is some energy scale. The leading contribution is of the order of $(a \ln s/s_0)^{L-1}$, where L is a loop order in the perturbative expansion, while the next-to-leading contribution is suppressed by one power of log of s, namely $a\,(a \ln s/s_0)^{L-2}$ and commonly is referred to as the next-to-leading logarithm approximation (NLA). In the present paper we consider only two and three loop BDS remainder function $R_6|_{MRK,2\to 4}$ to the NLA accuracy in MRK for the $2 \to 4$ gluon scattering. To this accuracy the remainder function is fully determined by the energy scale s_0, the leading-order (LO) and the next-to-leading order (NLO)[1] impact factor as well as the LO and the NLO adjoint BFKL eigenvalue in the planar limit. However, there is a residual freedom related to redistribution of the NLO correction between the BFKL eigenvalue, the impact factor and the energy scale. This freedom is equivalent to a freedom of redistributing the NLO corrections between the BFKL eigenvalue and the eigenfunction in such a way that the BFKL kernel is not affected. To illustrate this statement we schematically write the

[1]For some historical reasons one says that the scattering amplitude is calculated in the next-to-leading logarithm approximation (NLA), but the corresponding BFKL eigenvalue is the next-to-leading order (NLO) eigenvalue. Same for impact factors.

reduced infrared finite BFKL kernel $\mathcal{K}$ in the following form

$$\omega \otimes \phi \otimes \phi^* = \mathcal{K}, \tag{2.1}$$

where ω and ϕ are the eigenvalue and the eigenfunction of the BFKL equation. The reduced infrared finite BFKL kernel is obtained by removing the Regge gluon trajectory from the full infrared divergent BFKL kernel in the adjoint representation of the color gauge group. One uses the reduced adjoint BFKL kernel in calculations of the BDS remainder function because the gluon Regge trajectory is built in the BDS amplitude by construction.

The eigenfunctions of the LO kernel are also eigenfunctions of the NLO kernel (cf. [51]) for the singlet case. In the color adjoint BFKL for the reduced kernel the situation is the same, which allowed Fadin and Lipatov to calculate the NLO eigenvalue [25]. We denote it as follows

$$\omega = \omega_{\mathrm{LO}} + a\,\omega_{\mathrm{NLO}} \tag{2.2}$$

and thus according to ref. [25] it reads

$$(\omega_{\mathrm{LO}} + a\,\omega_{\mathrm{NLO}}) \otimes \phi_{\mathrm{LO}} \otimes \phi^*_{\mathrm{LO}} = \mathcal{K}_{\mathrm{LO}} + a\,\mathcal{K}_{\mathrm{NLO}} \tag{2.3}$$

for the coupling constant $a = g^2 N_c/(8\pi^2)$. In their previous paper [1] the authors argued that the NLO eigenvalue ω_{NLO} can be modified

$$\omega_{\mathrm{NLO}} \to \tilde{\omega}_{\mathrm{NLO}} = \omega_{\mathrm{NLO}} + \Delta\omega_{\mathrm{NLO}} \tag{2.4}$$

to comply with the Hermitian separability properties without affecting the remainder function to this order. This can be done by pushing some of the NLO corrections to the eigenfunction as follows[2]

$$(\omega_{\mathrm{LO}} + a\omega_{\mathrm{NLO}} + a\Delta\omega_{\mathrm{NLO}}) \otimes (\phi_{\mathrm{LO}} + a\phi_{\mathrm{NLO}}) \otimes (\phi_{\mathrm{LO}} + a\phi_{\mathrm{NLO}})^* = \mathcal{K}_{\mathrm{LO}} + a\mathcal{K}_{\mathrm{NLO}}, \tag{2.5}$$

which leaves the kernel $\mathcal{K}_{\mathrm{LO}} + a\,\mathcal{K}_{\mathrm{NLO}}$ unchanged. The possibility of the suggested modification of the NLO eigenvalue $\omega_{\mathrm{NLO}} \to \omega_{\mathrm{NLO}} + \Delta\omega_{\mathrm{NLO}}$ was criticised by Fadin and Fiore [52] based on their calculations using only the LO eigenfunction that is naturally inconsistent with BFKL kernel because

$$(\omega_{\mathrm{LO}} + a\,\omega_{\mathrm{NLO}} + a\,\Delta\omega_{\mathrm{NLO}}) \otimes \phi_{\mathrm{LO}} \otimes \phi^*_{\mathrm{LO}} \neq \mathcal{K}_{\mathrm{LO}} + a\,\mathcal{K}_{\mathrm{NLO}}. \tag{2.6}$$

The authors absolutely agree with the conclusion made by Fadin and Fiore that the expression

$$\Delta\omega_{\mathrm{NLO}} \otimes \phi_{\mathrm{LO}} \otimes \phi^*_{\mathrm{LO}} \tag{2.7}$$

on its own does not make much sense, but our statement is equivalent to compensating this term with the NLO corrections to the eigenfunction in such a way that the kernel is left unchanged. In calculus this corresponds to passing to another expansion basis for some function, which does not affect the function itself rather modifies the coefficients of its expansion, the BFKL eigenvalue in our case. To the best of our knowledge, the uniqueness

[2] A similar procedure for NLO eigenfunctions of the singlet BFKL kernel was considered in refs. [53, 54].

JHEP07(2016)081

of the eigenfunction of the NLO BFKL kernel was never discussed before. We leave the analysis of the uniqueness and possible forms of the NLO BFKL eigenfunction for our further publications and only want to show how this residual freedom is realized for the BDS remainder function.

3 BDS remainder function in multi-Regge kinematics

The planar BDS amplitude in $\mathcal{N}=4$ SYM possesses correction starting at two loop order of the perturbative expansion. One of the reasons for this is the fact that the analytic properties of the BDS amplitude are not compatible with cut singularities in the complex angular momentum plane, called Mandelstam or Regge cuts. The Mandelstam cut contributions cannot be obtained exponentiating the one loop result in momentum space, in the way the BDS amplitude is constructed. For the $2\to 4$ amplitude with two produced particles k_1 and k_2 the correction to the BDS amplitude, the so-called remainder function is a function of three conformal cross ratios $u_i (i=1,2,3)$ in dual momentum space expressed in terms of the Mandelstam invariants as follows

$$u_1=\frac{ss_2}{s_{012}s_{123}},\quad u_2=\frac{s_1t_3}{s_{012}t_2},\quad u_3=\frac{s_3t_1}{s_{123}t_2}, \tag{3.1}$$

where s, s_i and s_{ij} are related to the center of mass energies, while t_i stand for the momentum transfer. In the multi-Regge kinematics (MRK) $s\gg s_{012},s_{123}\gg s_1,s_2,s_3\gg t_1,t_2,t_3$ the cross ratios greatly simplify

$$1-u_1\to 0,\ \tilde{u}_2=\frac{u_2}{1-u_1}\propto 1,\ \tilde{u}_3=\frac{u_3}{1-u_1}\propto 1 \tag{3.2}$$

and u_1 possesses a phase in the Mandelstam region $u_1=|u_1|e^{-i2\pi}$. The analytic form of the correction to the BDS amplitude in the multi-Regge kinematics was introduced by Bartels, Lipatov and Sabio Vera [24] and its more general form [25, 45] reads

$$\begin{aligned}\exp\left[R_6+i\pi\delta_{\rm MRK}\right]|_{{\rm MRK},2\to 4}=&\cos\pi\omega_{ab} \\ &+i\frac{a}{2}\sum_{n=-\infty}^{+\infty}(-1)^n\left(\frac{w}{w^*}\right)^{\frac{n}{2}}\int_{-\infty}^{+\infty}\frac{d\nu}{\nu^2+\frac{n^2}{4}}|w|^{2i\nu}\,\Phi_{\rm Reg}(\nu,n)\left(-\frac{1}{1-u_1}\frac{|1+w|^2}{|w|}\right)^{\omega^{\rm adj}(\nu,n)}.\end{aligned} \tag{3.3}$$

Here ω_{ab} and $\delta_{\rm MRK}$ are given by

$$\omega_{ab}=\frac{1}{8}\gamma_K(a)\ln|w|^2,\quad \delta_{\rm MRK}=\frac{1}{8}\gamma_K(a)\ln\frac{|w|^2}{|1+w|^4} \tag{3.4}$$

and $\gamma_K(a)\simeq 4a-4a^2\zeta_2+\ldots$ is the cusp anomalous dimension known to any order in the perturbative expansion [55]. The complex variable w is related to the transverse momenta of the produced particles k_1, k_2 and the momentum transfers q_1, q_2 and q_3 as follows

$$w=\frac{q_3k_1}{k_2q_1}=|w|e^{i\phi_{23}},\quad |w|^2=\frac{u_2}{u_3},\quad \cos\phi_{23}=\frac{1-u_1-u_2-u_3}{2\sqrt{u_2u_3}}. \tag{3.5}$$

The energy dependence in (3.3) is encoded in (3.3) by

$$\frac{1}{1-u_1}\frac{|1+w|^2}{|w|}=\frac{s}{s_0} \tag{3.6}$$

as well as the function $\omega^{\mathrm{adj}}(\nu,n)$, which is the eigenvalue of the reduced infrared finite color adjoint BFKL kernel in the planar $\mathcal{N}=4$ SYM. The propagation of the BFKL state is then convolved with a product of two impact factors given by

$$\frac{(-1)^n}{\nu^2+\frac{n^2}{4}}\,\Phi_{\mathrm{Reg}}(\nu,n). \tag{3.7}$$

To keep the connection with previous publications we refer to $\Phi_{\mathrm{Reg}}(\nu,n)$ as the impact factor in the (ν,n) space.

We are interested only in the NLO corrections and write

$$\Phi_{\mathrm{Reg}}(\nu,n)=1+a\,\Phi^{(1)}_{\nu,n}+\dots \tag{3.8}$$

as well as

$$\omega^{\mathrm{adj}}=-a\left(E^{(0)}_{\nu,n}+aE^{(1)}_{\nu,n}+\dots\right), \tag{3.9}$$

where $E^{(0)}_{\nu,n}$ and $E^{(1)}_{\nu,n}$ are the LO and NLO adjoint BFKL eigenvalues and $\Phi^{(1)}_{\nu,n}$ is the NLO impact factor.

The leading order adjoint BFKL eigenvalue [24] reads

$$E^{(0)}_{\nu,n}=-\frac{1}{2}\frac{|n|}{\nu^2+\frac{n^2}{4}}+\psi\left(1+i\nu+\frac{|n|}{2}\right)+\psi\left(1-i\nu+\frac{|n|}{2}\right)-2\psi(1) \tag{3.10}$$

and also can be written as follows (see eq. (85) of ref. [24])

$$E^{(0)}_{\nu,n}=\frac{1}{2}\left(\psi\left(i\nu+\frac{n}{2}\right)+\psi\left(-i\nu+\frac{n}{2}\right)+\psi\left(+i\nu-\frac{n}{2}\right)+\psi\left(-i\nu-\frac{n}{2}\right)\right)-2\psi(1). \tag{3.11}$$

At first sight those two representation of the LO eigenvalue are not the same numerically and even have different analytic structure, for example setting $\nu=0$ and then taking limit $n\to 0$. However, one should remember that they are always considered under the integral over ν and the sum over integer n, and thus they are equivalent, which can be shown using the reflection identity of the digamma function.

The adjoint NLO BFKL eigenvalue $E^{(1)}_{\nu,n}$ was calculated by Fadin and Lipatov [25] using LO eigenfunctions from the reduced infrared safe BFKL kernel in the color adjoint representation in planar $\mathcal{N}=4$ SYM. In the most compact form it can be written as follows (cf. [45])

$$E^{(1)}_{\nu,n}=-\frac{1}{4}D^2_\nu E_{\nu,n}+\frac{1}{2}VD_\nu E_{\nu,n}-\zeta_2E_{\nu,n}-3\zeta_3 \tag{3.12}$$

in terms of

$$V\equiv-\frac{1}{2}\left[\frac{1}{i\nu+\frac{|n|}{2}}-\frac{1}{-i\nu+\frac{|n|}{2}}\right]=\frac{i\nu}{\nu^2+\frac{n^2}{4}} \tag{3.13}$$

as well as derivative defined by $D_\nu=-i\partial_\nu$.

The NLO impact factor

$$\Phi^{(1)}_{\nu,n} = -\frac{1}{2}\left(E^{(0)}_{\nu,n}\right)^2 - \frac{3}{8}N^2 - \zeta_2, \tag{3.14}$$

where

$$N \equiv \mathrm{sgn}(n)\left[\frac{1}{i\nu+\frac{|n|}{2}} + \frac{1}{-i\nu+\frac{|n|}{2}}\right] = \frac{n}{\nu^2+\frac{n^2}{4}}, \tag{3.15}$$

was calculated by Lipatov and one of the authors [31] analytically continuing the exact two loop remainder function found by Goncharov, Spradlin, Volovich and Vergu [56] to the Mandelstam region, and then was redefined by Fadin and Lipatov [25] to fit the energy scale in agreement with the Regge factorization property. In the next section we review properties of the adjoint NLO BFKL eigenvalue and discuss a residual freedom of redistributing NLO corrections between the eigenvalue and the corresponding impact factor.

4 Hermitian separability and transition from singlet to adjoint BFKL eigenvalue

In ref. [1] the authors argued that it is possible to modify the NLO eigenvalue

$$E^{(1)}_{\nu,n} \to E^{(1)}_{\nu,n} + \Delta E^{(1)}_{\nu,n} \tag{4.1}$$

accompanied by a change of the impact factor and the energy scale in such way that the remainder function in (3.3) remains intact to the next-to-leading logarithm (NLA) accuracy in MRK. This modification of the NLO eigenvalue of the BFKL kernel in the color adjoint representation was needed to have the Hermitian separability and to establish a non-trivial connection with the corresponding NLO eigenvalue in the color singlet state. The authors also suggested the exact form of $\Delta E^{(1)}_{\nu,n}$, namely

$$\begin{aligned}\Delta E^{(1)}_{\nu,n} = \frac{1}{2}&\left[\psi\left(1+i\nu+\frac{|n|}{2}\right) - \psi\left(1-i\nu+\frac{|n|}{2}\right)\right] \\ &\times\left[-\frac{i\nu\,|n|}{\left(\nu^2+\frac{n^2}{4}\right)^2} + \psi'\left(1+i\nu+\frac{|n|}{2}\right) - \psi'\left(1-i\nu+\frac{|n|}{2}\right)\right].\end{aligned} \tag{4.2}$$

Below we review the main results of ref. [1] and discuss the motivation for the modification of the adjoint NLO eigenvalue given by (4.1) and (4.2). The BFKL equation describes a bound state of two reggeized gluons in an arbitrary color state. The BFKL equation is a Schrödinger type equation, with an eigenvalue being a function of the anomalous dimension ν,[3] and the conformal spin n. By the BFKL eigenvalue one typically means the eigenvalue of the BFKL equation projected on colorless state, called singlet state. It is well known that if BFKL is projected on color adjoint state (color state of one gluon) it reduces to one

[3]The anomalous dimension of the twist-2 operators is usually denoted by $\gamma = 1/2 + i\nu$ in the BFKL approach. In our analysis, we deal with the integration variable ν, which we call for simplicity "the anomalous dimension".

reggeized gluon if both reggeized gluons in the bound state attached to the same object, e.g. quark line. This does not happen in the six-particle helicity amplitudes, where the two reggeized gluons in the bound state are attached to different vertices resulting in color adjoint BFKL equation (for more details see ref. [24]).

Both the singlet and the adjoint leading order BFKL equations are solved exploiting conformal invariance, in the coordinate space for the singlet BFKL and in the dual momentum space for the adjoint BFKL. The conformal groups are defined in quite different spaces, but the LO eigenfunctions and eigenvalues are very similar. Roughly speaking the LO eigenfunctions of the singlet BFKL equation are $\rho^{i\nu+n/2}\bar{\rho}^{i\nu+n/2}$, while for the adjoint BFKL the eigenfunctions are $k^{i\nu+n/2}\bar{k}^{i\nu+n/2}$, where ρ and k are the complex coordinate and momentum correspondingly.[4]

The singlet BFKL eigenvalue $\chi(\nu,n)$ and adjoint BFKL eigenvalue $E_{\nu,n}$ are also very similar in the leading order. They are both built of digamma function of an argument shifted by 1/2. Namely, the singlet BFKL leading order reads

$$\chi(n,\gamma)=-\frac{1}{2}\left(\psi\left(\frac{1}{2}+i\nu+\frac{n}{2}\right)+\psi\left(\frac{1}{2}-i\nu+\frac{n}{2}\right)+\psi\left(\frac{1}{2}+i\nu-\frac{n}{2}\right)+\psi\left(\frac{1}{2}-i\nu-\frac{n}{2}\right)\right)+2\psi(1)$$

while the adjoint BFKL eigenvalue is given by

$$E^{(0)}_{\nu,n}=\frac{1}{2}\left(\psi\left(i\nu+\frac{n}{2}\right)+\psi\left(-i\nu+\frac{n}{2}\right)+\psi\left(+i\nu-\frac{n}{2}\right)+\psi\left(-i\nu-\frac{n}{2}\right)\right)-2\psi(1).$$

The next-to-leading eigenvalues calculated using LO eigenfunctions are quite different. The adjoint NLO eigenvalue is constructed solely of polygamma functions and their derivatives, while the singlet NLO eigenvalue is built of a new type of function, Lerch transcendent and its generalizations. Another difference between the two is that the NLO singlet eigenvalue can be written in the form of the Bethe-Salpeter equation separating holomorphic $i\nu+n/2$ and antiholomorphic $i\nu-n/2$ coordinates as was shown by Kotikov and Lipatov [15], while the adjoint eigenvalue in the original form calculated by Fadin and Lipatov [25] does not possess this property called Hermitian separability as it was shown in ref. [1]. To make the adjoint NLO eigenvalue to comply with the Hermitian separability property the authors suggested in ref. [1] to modify the adjoint NLO eigenvalue adding a term given by (4.2). In the present paper we show that this modification can be introduced without affecting the resulting remainder function to the NLO accuracy.

The authors also argued in ref. [1] that their proposal of modifying the adjoint NLO is supported by an observation that the modified adjoint NLO BFKL eigenvalue can be reproduced by *ad hoc* procedure of replacing sign alternating sums in the singlet NLO expression by sums of constant sign accompanied by a shift of 1/2 in the argument. This transition from singlet to the adjoint BFKL eigenvalue is of yet unknown nature and is still to be checked for validity at higher orders. However, many higher order corrections to the adjoint eigenvalue available as well as the recently calculated NNLO singlet eigenvalue for $n=0$ show the same structure, namely singlet eigenvalue is built of sign alternating sums

[4]The actual form of the eigenfunctions is a bit more involved, it accounts for momentum transfer and includes a proper normalization.

JHEP07(2016)081

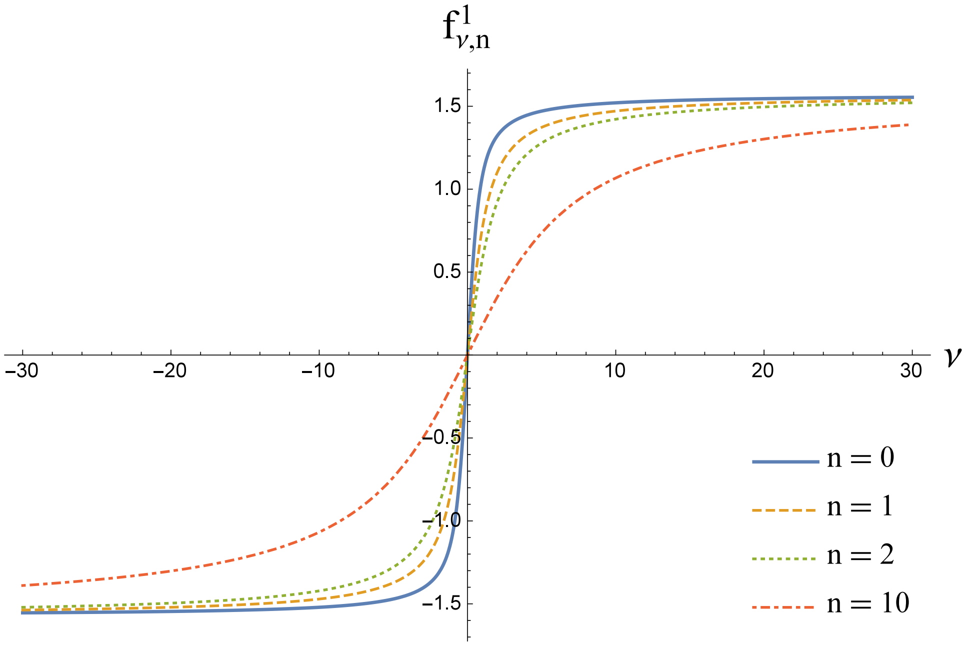

Figure 1. The plot of $f^{(1)}_{\nu,n}$ as a function of ν for different values of the conformal spin n.

with only one negative index, while the adjoint eigenvalue is constructed of polygamma functions and their derivatives. This hints a possibility that this rather simple prescription for the transition from the singlet to the adjoint eigenvalue is valid at higher orders as well.

In the next section we show how the suggested modification of the adjoint NLO eigenvalue can be introduced without changing the remainder functions to the required NLA accuracy.

5 Rescaling anomalous dimension

Below we show that the required modification of the NLO eigenvalue can be made by changing the anomalous dimension as follows

$$\nu \to \nu + a\, f^{(1)}_{\nu,n} \tag{5.1}$$

where

$$f^{(1)}_{\nu,n} = \frac{1}{2i}\left[\psi\left(1+i\nu+\frac{|n|}{2}\right)-\psi\left(1-i\nu+\frac{|n|}{2}\right)\right] = \mathrm{Im}\left[\psi\left(1+i\nu+\frac{|n|}{2}\right)\right]. \tag{5.2}$$

The change of the integration variable ν does not change the integral in (3.3) thus satisfying the condition that one modifies the NLO eigenvalue $E^{(1)}_{\nu,n}$ leaving the remainder function R_6 intact to the NLA order in MRK.

One can see from figure 1 that the function $f^{(1)}_{\nu,n}$ is well behaved in the region of the integration over ν and is limited by $\pm\pi/2$, which makes it to be compatible with the perturbative expansion in a and negligible for large values of ν in change of variable

$\nu \to \nu + a f^{(1)}_{\nu,n}$. In particular, for $n=0$ its asymptotic behaviour is given by

$$f^{(1)}_{\nu,n}|_{n=0} \underset{\nu\to\pm\infty}{\simeq} \pm\frac{\pi}{2} - \frac{1}{2\nu} + \mathcal{O}\left(\frac{1}{\nu^2}\right). \tag{5.3}$$

Other useful features of $f^{(1)}_{\nu,n}$ are that it is real and antisymmetric in ν as well as the fact that

$$\nu + a f^{(1)}_{\nu,n}|_{n=0} \underset{\nu\to 0}{\simeq} \nu(1 + a\,\zeta_2) + \mathcal{O}(\nu^2). \tag{5.4}$$

The expansion in (5.4) seems to compensate the NLO correction to the cusp anomalous dimension

$$\frac{\gamma_K}{4a}(\nu + a f^{(1)}_{\nu,n}|_{n=0}) \underset{\nu\to 0}{\simeq} \nu + \mathcal{O}(\nu^2), \tag{5.5}$$

and is likely related to a way one removes the infrared divergent Regge gluon trajectory from the color adjoint BFKL kernel.

In the appendix we demonstrate how the residual freedom of redistributing NLO corrections is realized for the remainder function in multi-Regge kinematics. We show that a simple change of the integration variable given by (5.1) can be used to redistribute the NLO corrections between that eigenvalue and the impact factor in such a way that the resulting expression naturally incorporates the modified NLO BFKL eigenvalue in (4.2) while leaving the whole expression for the remainder function at two and three loops intact to the required NLA accuracy.

As it is shown in the appendix, introducing the change of variable given by (5.1) we can write the expression for the remainder function in (3.3) in the following form

$$\begin{aligned} &\exp\left[R_6 + i\pi\delta_{\mathrm{MRK}}\right]|_{\mathrm{MRK},2\to 4} = \cos\pi\omega_{ab} \\ &+ i\frac{a}{2}\sum_{n=-\infty}^{+\infty}(-1)^n\left(\frac{w}{w^*}\right)^{\frac{n}{2}}\int_{-\infty}^{+\infty} d\nu \frac{|w|^{2i\left(\nu + a f^{(1)}_{\nu,n}\right)}}{\left(\nu + a f^{(1)}_{\nu,n}\right)^2 + \frac{n^2}{4}}\tilde{\Phi}_{\mathrm{Reg}}(\nu,n)\left(-\frac{1}{1-u_1}\frac{|1+w|^2}{|w|}\right)^{\tilde{\omega}^{\mathrm{adj}}(\nu,n)}. \end{aligned} \tag{5.6}$$

The modified NLO BFKL eigenvalue reads

$$\tilde{\omega}^{\mathrm{adj}} = -a\left(E^{(0)}_{\nu,n} + a E^{(1)}_{\nu,n} + a\Delta E^{(1)}_{\nu,n} + \dots\right), \tag{5.7}$$

where $\Delta E^{(1)}_{\nu,n}$ is given by (4.2), and the modified NLO impact factor is expressed through

$$\tilde{\Phi}_{\mathrm{Reg}}(\nu,n) = 1 + a\,\Phi^{(1)}_{\nu,n} + a\Delta\,\Phi^{(1)}_{\nu,n}\dots \tag{5.8}$$

where $\Delta\,\Phi^{(1)}_{\nu,n}$ is defined by

$$\Delta\,\Phi^{(1)}_{\nu,n} = \partial_\nu f^{(1)}_{\nu,n} = \frac{1}{2}\left[\psi'\left(1 + i\nu + \frac{|n|}{2}\right) + \psi'\left(1 - i\nu + \frac{|n|}{2}\right)\right] = \mathrm{Re}\left[\psi'\left(1 + i\nu + \frac{|n|}{2}\right)\right]. \tag{5.9}$$

The new integral representation of the MRK remainder function in (5.6), which naturally incorporates the modified NLO BFKL eigenvalue presents the main result of this manuscript. It is worth emphasizing that the remainder function at two and three loops remains the same with the NLA accuracy and not affected by redistribution of the NLO corrections between the eigenvalue and the impact factor.

It is clear that the modification of the eigenvalue given in (5.7) does not change the whole expression for the remainder function to the required NLA accuracy, because it merely follows from the change of the integration variable. On the other hand the meaning of the variable ν has been changed and it is related to the original ν through $\nu \to \nu + a f^{(1)}_{\nu,n}$. Those two coincide at the leading order and slightly differ at the next-to-leading order. The function $f^{(1)}_{\nu,n}$ is limited by $\pm\frac{\pi}{2}$ for any value of the original ν and n and thus is significant only in a small region of the integration over ν even for reasonably large values of the coupling constant in the perturbative expansion. The original meaning of n as a conformal spin remains the same and is not affected by the change of ν. The dependence of the anomalous dimension on the conformal spin through $f^{(1)}_{\nu,n}$ can be interpreted as a breaking of the axial symmetry in the complex (w, w^*) plane by causing the dilation to depend on the angle.

6 Conclusions and outlook

In this paper we discuss a residual freedom of redistributing next-to-leading order corrections between the eigenvalue and impact factors for color adjoint BFKL in planar $\mathcal{N} = 4$ SYM. This freedom originates from an arbitrariness in solving the BFKL equation by either the eigenfunctions of the LO BFKL kernel or some other eigenfunctions, that can possess NLO corrections. The full solution is then expanded in the basis of the new eigenfunctions and therefore the eigenvalue, being a coefficient of this expansion, is modified under a change of the expansion basis.

We showed that this residual freedom can be exploited to rewrite the two and three loop expression for the remainder function at the NLA accuracy in MRK in such a way that it naturally incorporates the modified NLO BFKL eigenvalue suggested by the authors in ref. [1] and is given by (4.1) together with (3.12) and (4.2). This modification requires some redefinition of the NLO impact factor in the (ν, n) space. The impact factor in the (ν, n) space is a convolution of the impact factor in the momentum space and the BFKL eigenfunction and it is natural to expect that a change of the eigenfunction is translated into a change of this convolution even for same the impact factor in the momentum space. The modifications to the eigenvalue and the impact factor are of the same origin and therefore they cancel each other leaving the expression for the remainder function unchanged.

The main motivation for modifying the NLO BFKL eigenvalue in the color adjoint state was to restore the Hermitian separability property present in the singlet case. It also helped to establish a non-trivial connection between the adjoint and the singlet NLO eigenvalues effectively replacing the alternating sums in singlet eigenvalue by sums of the constant sign. This connects between the cylindrical topology of the singlet BFKL and the plane topology of the adjoint BFKL.

The empirical recipe of the transition from the singlet to the adjoint eigenvalue may serve a powerful tool for higher loop calculations provided it holds beyond NLO order. It is very encouraging that the known corrections to the adjoint BFKL eigenvalue can be expressed in terms of the constant sign sums while the only known singlet NNLO BFKL eigenvalue (for zero value of the conformal spin) calculated by N. Gromov, F. Levkovich-Maslyuk and G. Sizov [57][5] is written in terms of alternating harmonic sums with only one negative index.

The residual freedom of redistributing higher order corrections is also present beyond NLA accuracy and is not fixed by the choice of the NLO eigenvalue and the impact factors presented in this paper. This freedom can be exploited to establish a connection between the eigenvalues of the BFKL kernel in color singlet and adjoint states to all orders in the perturbative expansion.

Acknowledgments

The authors are thankful to Jochen Bartels, Victor Fadin and Lev Lipatov for enlightening discussions on the BFKL Physics. A.P. is especially indebted to Jochen Bartels for his hospitality at Hamburg, where this project was initiated.

A Modifying next-to-leading eigenvalue and impact factor

In this section we demonstrate how the residual freedom of redistributing NLO corrections is realized for the remainder function in multi-Regge kinematics. In the following we show that a simple change of the integration variable given by (5.1) can be used to redistribute the NLO corrections between that eigenvalue and the impact factor in such a way that the resulting expression naturally incorporates the modified NLO BFKL eigenvalue in (4.2) while leaving the whole expression for the remainder function at two and three loops intact to the required NLA accuracy.

We write the relevant integral in (3.3) as follows

$$\begin{aligned}\int_{-\infty}^{+\infty}\frac{d\nu}{\nu^2+\frac{n^2}{4}}|w|^{2i\nu}\,\Phi_{\rm Reg}(\nu,n)\,\left(-\frac{1}{1-u_1}\frac{|1+w|^2}{|w|}\right)^{\omega^{\rm adj}(\nu,n)}\\ =\int_{-\infty}^{+\infty}\frac{d\nu}{\nu^2+\frac{n^2}{4}}|w|^{2i\nu}\,\Phi_{\rm Reg}(\nu,n)\exp\left[-\omega^{\rm adj}(\nu,n)\left(\ln(1-u_1)+i\pi+F(w)\right)\right],\end{aligned} \tag{A.1}$$

where

$$F_w=\frac{1}{2}\ln\frac{|w|^2}{|1+w|^4}. \tag{A.2}$$

Expanding the integrand of (A.1) to the NLA order we get

$$\begin{aligned}a\,\Phi_{\rm Reg}(\nu,n)\exp\left[-\omega^a(\ln(1-u_1)+F_w+i\pi)\right]\simeq a+a^2\ln(1-u_1)E^{(0)}_{\nu,n}\\ +a^2\left(E^{(0)}_{\nu,n}F_w+\Phi^{(1)}_{\nu,n}+i\pi E^{(0)}_{\nu,n}\right)+\frac{a^3}{2}\left(\ln(1-u_1)E^{(0)}_{\nu,n}\right)^2\\ +a^3\ln(1-u_1)\left(E^{(1)}_{\nu,n}+i\pi\left(E^{(0)}_{\nu,n}\right)^2+E^{(0)}_{\nu,n}\left(E^{(0)}_{\nu,n}F_w+\Phi^{(1)}_{\nu,n}\right)\right),\end{aligned} \tag{A.3}$$

[5] See also a parallel calculation by Velizhanin [58].

where $F_w = \frac{1}{2}\ln\frac{|w|^2}{|1+w|^4}$ is a function of transverse momentum, while $E^{(0)}_{\nu,n}$, $E^{(1)}_{\nu,n}$ and $\Phi^{(1)}_{\nu,n}$ are functions of ν and n.

Next we substitute the integration variable in (A.1) as follows

$$\nu \to \nu + a f^{(1)}_{\nu,n}. \tag{A.4}$$

It is convenient to consider the effect of this substitution term by term. First we note that

$$E^{(0)}_{\nu,n}\Big|_{\nu\to\nu+af^{(1)}_{\nu,n}} \simeq E^{(0)}_{\nu,n} + iaf^{(1)}_{\nu,n} D_\nu E^{(0)}_{\nu,n}, \tag{A.5}$$

where we define $D_\nu = -i\partial_\nu$. Plugging this into (A.3) we get

$$\begin{aligned} a\,\Phi_{\text{Reg}}(\nu,n)\exp\left[-\omega^a(\ln(1-u_1)+F_w+i\pi)\right]|_{\nu\to\nu+af^{(1)}_{\nu,n}} &\simeq a + a^2\ln(1-u_1)E^{(0)}_{\nu,n} \\ + a^2\left(E^{(0)}_{\nu,n}F_w + \Phi^{(1)}_{\nu,n} + i\pi E^{(0)}_{\nu,n}\right) + \frac{a^3}{2}\left(\ln(1-u_1)E^{(0)}_{\nu,n}\right)^2 \\ + a^3\ln(1-u_1)\left(E^{(1)}_{\nu,n} + \Delta E^{(1)}_{\nu,n} + i\pi\left(E^{(0)}_{\nu,n}\right)^2 + E^{(0)}_{\nu,n}\left(E^{(0)}_{\nu,n}F_w + \Phi^{(1)}_{\nu,n}\right)\right), \end{aligned} \tag{A.6}$$

where

$$\begin{aligned} \Delta E^{(1)}_{\nu,n} = if^{(1)}_{\nu,n}D_\nu E^{(0)}_{\nu,n} &= \frac{1}{2}\left[\psi\left(1+i\nu+\frac{|n|}{2}\right) - \psi\left(1-i\nu+\frac{|n|}{2}\right)\right] D_\nu E^{(0)}_{\nu,n} \\ &= \frac{1}{2}\left[\psi\left(1+i\nu+\frac{|n|}{2}\right) - \psi\left(1-i\nu+\frac{|n|}{2}\right)\right] \\ &\quad\times\left[-\frac{i\nu\,|n|}{\left(\nu^2+\frac{n^2}{4}\right)^2} + \psi'\left(1+i\nu+\frac{|n|}{2}\right) - \psi'\left(1-i\nu+\frac{|n|}{2}\right)\right], \end{aligned} \tag{A.7}$$

which gives exactly the modification of the NLO eigenvalue suggested by the authors (cf. [1]), namely

$$E^{(1)}_{\nu,n} \to E^{(1)}_{\nu,n} + \Delta E^{(1)}_{\nu,n}. \tag{A.8}$$

Other terms in the integrand of (A.1) are also affected by the substitution (A.4). In particular, we write

$$\frac{|w|^{i2\nu}}{\nu^2+\frac{n^2}{4}}\Bigg|_{\nu\to\nu+af^{(1)}_{\nu,n}} = \frac{|w|^{i2\left(\nu+af^{(1)}_{\nu,n}\right)}}{\left(\nu+af^{(1)}_{\nu,n}\right)^2+\frac{n^2}{4}}. \tag{A.9}$$

For clarity of presentation we choose not to expand the expression in (A.9).

The Jacobian gives

$$d\nu \to \left(1 + a\partial_\nu f^{(1)}_{\nu,n}\right)d\nu \tag{A.10}$$

and its easy to see that we can absorb the second term in the brackets of (A.10) in the redefinition of the NLO impact factor[6] as follows

$$\tilde{\Phi}_{\text{Reg}}(\nu,n) = 1 + a\,\Phi^{(1)}_{\nu,n} + a\Delta\,\Phi^{(1)}_{\nu,n}\dots \tag{A.11}$$

[6]In fact, what we call the NLO impact factor in the (ν,n) space is a product of $\Phi^{(1)}_{\nu,n}$ and the expression in (A.9) times $(-1)^n$.

where

$$\Delta\Phi^{(1)}_{\nu,n} = \partial_\nu f^{(1)}_{\nu,n} = \frac{1}{2}\left[\psi'\left(1+i\nu+\frac{|n|}{2}\right)+\psi'\left(1-i\nu+\frac{|n|}{2}\right)\right] = \mathrm{Re}\left[\psi'\left(1+i\nu+\frac{|n|}{2}\right)\right]. \tag{A.12}$$

Finally, combining the expanded terms we write the expression for the remainder function

$$\exp\left[R_6 + i\pi\delta_{\mathrm{MRK}}\right]|_{\mathrm{MRK},2\to4} = \cos\pi\omega_{ab} \tag{A.13}$$
$$+\,i\frac{a}{2}\sum_{n=-\infty}^{+\infty}(-1)^n\left(\frac{w}{w^*}\right)^{\frac{n}{2}}\int_{-\infty}^{+\infty}d\nu\frac{|w|^{2i\left(\nu+af^{(1)}_{\nu,n}\right)}}{\left(\nu+af^{(1)}_{\nu,n}\right)^2+\frac{n^2}{4}}\tilde{\Phi}_{\mathrm{Reg}}(\nu,n)\left(-\frac{1}{1-u_1}\frac{|1+w|^2}{|w|}\right)^{\tilde{\omega}^{\mathrm{adj}}(\nu,n)}$$

for modified NLO BFKL eigenvalue

$$\tilde{\omega}^{\mathrm{adj}} = -a\left(E^{(0)}_{\nu,n} + aE^{(1)}_{\nu,n} + a\Delta E^{(1)}_{\nu,n} + \ldots\right), \tag{A.14}$$

where $\Delta E^{(1)}_{\nu,n}$ is given by (A.7), and for the modified NLO impact factor

$$\tilde{\Phi}_{\mathrm{Reg}}(\nu,n) = 1 + a\ \Phi^{(1)}_{\nu,n} + a\Delta\ \Phi^{(1)}_{\nu,n}\ldots \tag{A.15}$$

where $\Delta\ \Phi^{(1)}_{\nu,n}$ is defined in (A.12).

Strictly speaking, expanding the integral in (A.1) in powers of the coupling constant a after the substitution $\nu \to \nu + af^{(1)}_{\nu,n}$ we have also to expand the upper and lower limits of the integral, which are now some functions of the coupling constant a. However, as it was already mentioned, both the function $f^{(1)}_{\nu,n}$ and its derivative are limited as $\nu \to \pm\infty$ and thus the terms coming from the expansion of the upper and lower limits vanishes for any finite value of a. Another fine point is the case of $n = 0$, which has to be considered with a special care because of the infrared divergence and should be understood as the principal value of the integral over ν as it was shown for one loop in the appendix of ref. [24].

The new integral representation of the MRK remainder function in (A.13), which naturally incorporates the modified NLO BFKL eigenvalue presents the main result of this manuscript. It is worth emphasizing that the remainder function at two and three loops remains the same with the NLA accuracy and not affected by redistribution of the NLO corrections between the eigenvalue and the impact factor.

One can consider other ways of writing (A.13), for example expanding some terms in (A.9). This would redefine the energy scale function F_w and further change the impact factor, but bring no new insight into the problem under discussion making the result less transparent.

References

[1] S. Bondarenko and A. Prygarin, *Hermitian separability and transition from singlet to adjoint BFKL equations in $\mathcal{N}=4$ super Yang-Mills Theory*, arXiv:1510.00589 [INSPIRE].

[2] L.N. Lipatov, *Reggeization of the Vector Meson and the Vacuum Singularity in Nonabelian Gauge Theories*, *Sov. J. Nucl. Phys.* **23** (1976) 338 [INSPIRE].

[3] V.S. Fadin, E.A. Kuraev and L.N. Lipatov, *On the Pomeranchuk Singularity in Asymptotically Free Theories*, *Phys. Lett.* **B 60** (1975) 50 [INSPIRE].

[4] E.A. Kuraev, L.N. Lipatov and V.S. Fadin, *Multi-Reggeon Processes in the Yang-Mills Theory*, *Sov. Phys. JETP* **44** (1976) 443 [INSPIRE].

[5] E.A. Kuraev, L.N. Lipatov and V.S. Fadin, *The Pomeranchuk Singularity in Nonabelian Gauge Theories*, *Sov. Phys. JETP* **45** (1977) 199 [INSPIRE].

[6] I.I. Balitsky and L.N. Lipatov, *The Pomeranchuk Singularity in Quantum Chromodynamics*, *Sov. J. Nucl. Phys.* **28** (1978) 822 [INSPIRE].

[7] V.S. Fadin, *Regge trajectory of a gluon in the two loop approximation*, *JETP Lett.* **61** (1995) 346 [INSPIRE].

[8] V.S. Fadin, M.I. Kotsky and R. Fiore, *Gluon Reggeization in QCD in the next-to-leading order*, *Phys. Lett.* **B 359** (1995) 181 [INSPIRE].

[9] V.S. Fadin, R. Fiore and A. Quartarolo, *Reggeization of quark quark scattering amplitude in QCD*, *Phys. Rev.* **D 53** (1996) 2729 [hep-ph/9506432] [INSPIRE].

[10] M.I. Kotsky and V.S. Fadin, *Reggeization of the amplitude of gluon-gluon scattering*, *Phys. Atom. Nucl.* **59** (1996) 1035 [INSPIRE].

[11] V.S. Fadin, R. Fiore and M.I. Kotsky, *Gluon Regge trajectory in the two loop approximation*, *Phys. Lett.* **B 387** (1996) 593 [hep-ph/9605357] [INSPIRE].

[12] J. Blumlein, V. Ravindran and W.L. van Neerven, *On the gluon Regge trajectory in $O(\alpha_S^2)$*, *Phys. Rev.* **D 58** (1998) 091502 [hep-ph/9806357] [INSPIRE].

[13] V. Del Duca and E.W.N. Glover, *The high-energy limit of QCD at two loops*, *JHEP* **10** (2001) 035 [hep-ph/0109028] [INSPIRE].

[14] A.V. Kotikov and L.N. Lipatov, *NLO corrections to the BFKL equation in QCD and in supersymmetric gauge theories*, *Nucl. Phys.* **B 582** (2000) 19 [hep-ph/0004008] [INSPIRE].

[15] A.V. Kotikov and L.N. Lipatov, *DGLAP and BFKL equations in the $N=4$ supersymmetric gauge theory*, *Nucl. Phys.* **B 661** (2003) 19 [*Erratum ibid.* **B 685** (2004) 405] [hep-ph/0208220] [INSPIRE].

[16] V.S. Fadin and R. Fiore, *The dipole form of the BFKL kernel in supersymmetric Yang-Mills theories*, *Phys. Lett.* **B 661** (2008) 139 [arXiv:0712.3901] [INSPIRE].

[17] V.S. Fadin and D.A. Gorbachev, *Nonforward color octet BFKL kernel*, *JETP Lett.* **71** (2000) 222 [INSPIRE].

[18] V.S. Fadin and D.A. Gorbachev, *Nonforward color-octet kernel of the Balitsky-Fadin-Kuraev-Lipatov equation*, *Phys. Atom. Nucl.* **63** (2000) 2157 [INSPIRE].

[19] V.S. Fadin, R. Fiore and A. Papa, *The quark part of the nonforward BFKL kernel and the 'bootstrap' for the gluon Reggeization*, *Phys. Rev.* **D 60** (1999) 074025 [hep-ph/9812456] [INSPIRE].

[20] R.E. Gerasimov and V.S. Fadin, *Scalar contribution to the BFKL kernel*, *Phys. Atom. Nucl.* **73** (2010) 1214 [INSPIRE].

JHEP07(2016)081

[21] J. Bartels, *High-Energy Behavior in a Nonabelian Gauge Theory. 2. First Corrections to $T_{n\to m}$ Beyond the Leading LNS Approximation*, Nucl. Phys. **B 175** (1980) 365 [INSPIRE].

[22] J. Kwiecinski and M. Praszalowicz, *Three Gluon Integral Equation and Odd c Singlet Regge Singularities in QCD*, Phys. Lett. **B 94** (1980) 413 [INSPIRE].

[23] Z. Bern, L.J. Dixon and V.A. Smirnov, *Iteration of planar amplitudes in maximally supersymmetric Yang-Mills theory at three loops and beyond*, Phys. Rev. **D 72** (2005) 085001 [hep-th/0505205] [INSPIRE].

[24] J. Bartels, L.N. Lipatov and A. Sabio Vera, *$N = 4$ supersymmetric Yang-Mills scattering amplitudes at high energies: The Regge cut contribution*, Eur. Phys. J. **C 65** (2010) 587 [arXiv:0807.0894] [INSPIRE].

[25] V.S. Fadin and L.N. Lipatov, *BFKL equation for the adjoint representation of the gauge group in the next-to-leading approximation at $N = 4$ SUSY*, Phys. Lett. **B 706** (2012) 470 [arXiv:1111.0782] [INSPIRE].

[26] L.J. Dixon and M. von Hippel, *Bootstrapping an NMHV amplitude through three loops*, JHEP **10** (2014) 065 [arXiv:1408.1505] [INSPIRE].

[27] B. Basso, S. Caron-Huot and A. Sever, *Adjoint BFKL at finite coupling: a short-cut from the collinear limit*, JHEP **01** (2015) 027 [arXiv:1407.3766] [INSPIRE].

[28] J.M. Drummond and G. Papathanasiou, *Hexagon OPE Resummation and Multi-Regge Kinematics*, JHEP **02** (2016) 185 [arXiv:1507.08982] [INSPIRE].

[29] J. Bartels, L.N. Lipatov and A. Sabio Vera, *BFKL Pomeron, Reggeized gluons and Bern-Dixon-Smirnov amplitudes*, Phys. Rev. **D 80** (2009) 045002 [arXiv:0802.2065] [INSPIRE].

[30] L.N. Lipatov and A. Prygarin, *Mandelstam cuts and light-like Wilson loops in $\mathcal{N} = 4$ SUSY*, Phys. Rev. **D 83** (2011) 045020 [arXiv:1008.1016] [INSPIRE].

[31] L.N. Lipatov and A. Prygarin, *BFKL approach and six-particle MHV amplitude in $\mathcal{N} = 4$ super Yang-Mills*, Phys. Rev. **D 83** (2011) 125001 [arXiv:1011.2673] [INSPIRE].

[32] J. Bartels, L.N. Lipatov and A. Prygarin, *MHV amplitude for $3 \to 3$ gluon scattering in Regge limit*, Phys. Lett. **B 705** (2011) 507 [arXiv:1012.3178] [INSPIRE].

[33] J. Bartels, L.N. Lipatov and A. Prygarin, *Collinear and Regge behavior of $2 \to 4$ MHV amplitude in $N = 4$ super Yang-Mills theory*, arXiv:1104.4709 [INSPIRE].

[34] L.J. Dixon, C. Duhr and J. Pennington, *Single-valued harmonic polylogarithms and the multi-Regge limit*, JHEP **10** (2012) 074 [arXiv:1207.0186] [INSPIRE].

[35] J. Pennington, *The six-point remainder function to all loop orders in the multi-Regge limit*, JHEP **01** (2013) 059 [arXiv:1209.5357] [INSPIRE].

[36] S. Caron-Huot, *When does the gluon reggeize?*, JHEP **05** (2015) 093 [arXiv:1309.6521] [INSPIRE].

[37] Y. Hatsuda, *Wilson loop OPE, analytic continuation and multi-Regge limit*, JHEP **10** (2014) 38 [arXiv:1404.6506] [INSPIRE].

[38] L. Lipatov, A. Prygarin and H.J. Schnitzer, *The Multi-Regge limit of NMHV Amplitudes in $\mathcal{N} = 4$ SYM Theory*, JHEP **01** (2013) 068 [arXiv:1205.0186] [INSPIRE].

[39] J. Bartels, J. Kotanski, V. Schomerus and M. Sprenger, *The Excited Hexagon Reloaded*, arXiv:1311.1512 [INSPIRE].

JHEP07(2016)081

[40] J. Bartels, A. Kormilitzin and L. Lipatov, *Analytic structure of the $n = 7$ scattering amplitude in $\mathcal{N} = 4$ SYM theory in the multi-Regge kinematics: Conformal Regge pole contribution*, Phys. Rev. **D 89** (2014) 065002 [arXiv:1311.2061] [INSPIRE].

[41] J. Bartels, V. Schomerus and M. Sprenger, *Heptagon Amplitude in the Multi-Regge Regime*, JHEP **10** (2014) 67 [arXiv:1405.3658] [INSPIRE].

[42] J. Bartels, A. Kormilitzin and L.N. Lipatov, *Analytic structure of the $n = 7$ scattering amplitude in $\mathcal{N} = 4$ theory in multi-Regge kinematics: Conformal Regge cut contribution*, Phys. Rev. **D 91** (2015) 045005 [arXiv:1411.2294] [INSPIRE].

[43] J. Bartels, V. Schomerus and M. Sprenger, *The Bethe roots of Regge cuts in strongly coupled $\mathcal{N} = 4$ SYM theory*, JHEP **07** (2015) 098 [arXiv:1411.2594] [INSPIRE].

[44] V.S. Fadin, R. Fiore, L.N. Lipatov and A. Papa, *Moebius invariant BFKL equation for the adjoint representation in $N = 4$ SUSY*, Nucl. Phys. **B 874** (2013) 230 [arXiv:1305.3395] [INSPIRE].

[45] L.J. Dixon, J.M. Drummond, C. Duhr and J. Pennington, *The four-loop remainder function and multi-Regge behavior at NNLLA in planar $N = 4$ super-Yang-Mills theory*, JHEP **06** (2014) 116 [arXiv:1402.3300] [INSPIRE].

[46] J. Broedel and M. Sprenger, *Six-point remainder function in multi-Regge-kinematics: an efficient approach in momentum space*, JHEP **05** (2016) 055 [arXiv:1512.04963] [INSPIRE].

[47] V.S. Fadin and R. Fiore, *Discontinuites of BFKL amplitudes and the BDS ansatz*, Nucl. Phys. **B 901** (2015) 115 [arXiv:1507.06560] [INSPIRE].

[48] T. Bargheer, G. Papathanasiou and V. Schomerus, *The Two-Loop Symbol of all Multi-Regge Regions*, JHEP **05** (2016) 012 [arXiv:1512.07620] [INSPIRE].

[49] L.J. Dixon, M. von Hippel and A.J. McLeod, *The four-loop six-gluon NMHV ratio function*, JHEP **01** (2016) 053 [arXiv:1509.08127] [INSPIRE].

[50] L.J. Dixon and I. Esterlis, *All orders results for self-crossing Wilson loops mimicking double parton scattering*, arXiv:1602.02107 [INSPIRE].

[51] V.S. Fadin and L.N. Lipatov, *BFKL Pomeron in the next-to-leading approximation*, Phys. Lett. **B 429** (1998) 127 [hep-ph/9802290] [INSPIRE].

[52] V.S. Fadin and R. Fiore, *On Hermitian separability of the next-to-leading order BFKL kernel for the adjoint representation of the gauge group in the planar $N = 4$ SYM*, Eur. Phys. J. **C 76** (2016) 230 [arXiv:1512.08471] [INSPIRE].

[53] S. Bondarenko, *Conformal intercept of BFKL Pomeron with NLO running coupling constant corrections*, arXiv:0808.3175 [INSPIRE].

[54] G.A. Chirilli and Y.V. Kovchegov, *Solution of the NLO BFKL Equation and a Strategy for Solving the All-Order BFKL Equation*, JHEP **06** (2013) 055 [arXiv:1305.1924] [INSPIRE].

[55] N. Beisert, B. Eden and M. Staudacher, *Transcendentality and Crossing*, J. Stat. Mech. **0701** (2007) P01021 [hep-th/0610251] [INSPIRE].

[56] A.B. Goncharov, M. Spradlin, C. Vergu and A. Volovich, *Classical Polylogarithms for Amplitudes and Wilson Loops*, Phys. Rev. Lett. **105** (2010) 151605 [arXiv:1006.5703] [INSPIRE].

[57] N. Gromov, F. Levkovich-Maslyuk and G. Sizov, *Pomeron Eigenvalue at Three Loops in $\mathcal{N} = 4$ Supersymmetric Yang-Mills Theory*, Phys. Rev. Lett. **115** (2015) 251601 [arXiv:1507.04010] [INSPIRE].

[58] V.N. Velizhanin, *BFKL Pomeron in the next-to-next-to-leading approximation in the planar $N = 4$ SYM theory*, arXiv:1508.02857 [INSPIRE].