Erratum: Nonlocal probes of thermalization in holographic quenches with spectral methods

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In the published version of this paper, the final equation in (A.26) on page 49 should appear as

$$g_c(\tau, 0) = 6a_{2,2}. \quad (\text{A.26})$$

The incorrect version of this equation was implemented in the numerical solution of the system, and propagated to several places throughout the paper. The interested reader may find a revised version of the manuscript at [1].

Note that the corrected boundary condition in equation (A.26) leads to a monotonic apparent horizon area, as opposed to the nonmonotonic behaviour in the published version of figures 1 and 2 (see the appendix for the new versions). In fact, it can be shown from constraint equation (3.18) evaluated at the horizon $\rho = \frac{1}{\alpha}$, that the change in the apparent horizon area is given by

$$\frac{d}{d\tau} \delta V_a = \left[\frac{1}{4\alpha} \left(\partial_\tau \hat{\phi} \right)^2 \right]_{\rho=\frac{1}{\alpha}} \implies \delta V_a = \int_{-\infty}^{\tau} dt \left[\frac{1}{4\alpha} \left(\partial_t \hat{\phi} \right)^2 \right]_{\rho=\frac{1}{\alpha}}, \quad (1)$$

implying that the area of the apparent horizon *monotonically increases* with time, in agreement with the area theorems of [2] (see also [3]).¹

Text

Here we list the required changes to the text of the paper.

- Throughout the body of this paper (including the abstract) the claim is made that the thermalization times of the two-point function and entanglement entropy (EE) exceeds the thermalization time of the one-point function for wide enough separations of $2y_m$. However, as can be seen in the new version of figure 15 in the appendix, the new thermalization times of the two-point function and EE do not have longer thermalization times than the one-point function for the largest values of y_m considered. This weakens these claims in the paper. However, as the behaviour for both observables' thermalization times is linear and monotonic, we expect that the thermalization times of the two-point function and EE will still exceed that of the one-point function for wide enough regions.
- As discussed above, statements about the nonmonotonicity of the area density of the apparent horizon are no longer correct, and should be disregarded.
- Because of the new table (see the next section), the discussion on page 16 should be changed from “In the table, we see that the equilibration times of the horizon become approximately constant for small α ” to “In the table, we see that the equilibration times of the horizon become approximately constant for small α , (although we see some variation for the event horizon).”

¹We would like to thank Mukund Rangamani and Moshe Rozali for valuable discussions regarding the area theorems.

- Because of the changes in figure 17 (see the appendix to this erratum), the discussion in the parentheses on page 40 should be changed from “note that in figure 17, the equilibration curve for $\alpha = 1$ is not significantly later deep in the bulk than near the boundary, while the effect becomes more pronounced for the smaller values of α ,” to “Note that in figure 17, the equilibration curve for $\alpha = 1$ is an exception, equilibrating earlier at most points in the bulk than at the boundary (even the deepest part). The effect of the integrand equilibrating later in the bulk than near the boundary is visible for the smaller values of α , which correspond more to the universal behaviour related to instantaneous quenches (see appendix A.4)”.

Table

Table 1 on page 17 showed equilibration times of the event and apparent horizon that changed after the numerical code was corrected. The corrected version of this table is the following:

α	1	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{8}$	$\frac{1}{16}$
$\tau_{eq}[\delta V_e]$	.166	1.14	1.48	1.40	0.99
$\tau_{eq}[\delta V_a]$	.385	1.69	2.37	2.85	3.11
$\tau_{th}[\langle \mathcal{O}_3 \rangle]$	3.41	6.16	15.25	30.46	61.14
$\alpha \tau_{th}[\langle \mathcal{O}_3 \rangle]$	3.41	3.08	3.81	3.81	3.82

Table 1. The equilibration times of the area densities of the event and apparent horizons and the thermalization time for the one-point correlator, which thermalizes as p_2 , (as defined by the 2% threshold of equation (4.20)), for different values of the quenching parameter α . We also give $\alpha \tau_{th}[\langle \mathcal{O}_3 \rangle]$ which corresponds to the physical time, as discussed in the main text.

Figures

Figures and captions that need to be replaced due to the corrected numerics are listed below. For readability, the new figures are contained in the appendix to this erratum.

- figure 1 and caption on page 14.
- figure 2 on and caption on page 14.
- figure 3 and caption on page 15.
- figure 4 and caption on page 22.
- figure 5 and caption on page 23.
- figure 7 and caption on page 24.
- figure 8 and caption on page 25.

- figure 9 and caption on page 33.
- figure 10 and caption on page 34.
- figure 11 and caption on page 35.
- figure 12 and caption on page 35.
- figure 15 and caption on page 38.
- figure 17 and caption on page 39.

Acknowledgments

We are indebted to Mukund Rangamani and Moshe Rozali for discussions about apparent horizon that allowed us to track a mistake in the earlier version of the code: namely, the incorrect boundary condition for $g_c(\tau, 0)$ in equation (A.26).

Appendix: new figures

Figure 1. Plots of the evolution of the perturbation of the area of the event horizon (blue) and the apparent horizon (purple) for various quenching times α . The plots are (from left to right, top to bottom) for $\alpha = 1, \frac{1}{2}, \frac{1}{4}$ and $\frac{1}{8}$, respectively. Note that the areas of both the apparent horizon and the event horizon necessarily increase monotonically with time.

Figure 2. Evolution of the perturbation of the area of the apparent horizon δV_a for quenching time $\alpha = 1$ computed using (4.9) (blue, dashed) and (1) (magenta, solid). The agreement provides a highly nontrivial test of the constraint equation (3.18).

Figure 3. Here we compare the thermalization measure (4.20) for the apparent and event horizons (δV_a in purple and δV_e in blue) with the thermalization measure for the normalizable mode p_2 of the scalar field (orange), as well as its non-normalizable mode p_0 in red. The plots are (from left to right, top to bottom) for $\alpha = 1, \frac{1}{2}, \frac{1}{4}$ and $\frac{1}{8}$, respectively. In all cases, the horizon thermalizes before the one-point function, and this becomes more noticeable for smaller α .

Figure 4. The thermalization measure as defined in (4.20) of the perturbation of the two-point functions for different-sized geodesics. The evolution is a function of the rescaled boundary time τ_* . The plots are, from left to right, top to bottom, for $\rho_m = 0.1\rho_h$, $0.5\rho_h$, $0.9\rho_h$, $0.99\rho_h$ and $0.999\rho_h$. In each plot the thermalization measure is shown for quenching parameters $\alpha = 1$ (blue), $\alpha = \frac{1}{2}$ (purple), $\alpha = \frac{1}{4}$ (brown) and $\alpha = \frac{1}{8}$ (green). Note that the smaller α is, the longer equilibration takes, in this rescaled boundary time.

Figure 5. An alternative view of figure 4. The same plots are shown, but with the thermalization measures being functions of the un-rescaled boundary time $\alpha\tau_*$. In this case one can see that the smaller α is, the shorter equilibration tends to take, from an absolute point of view.

Figure 7. We show the equilibration times of $\mathcal{L}_2$ for various values of ρ_m as a function of the inverse of the quenching parameter α , for $\alpha = 1, \frac{1}{2}, \frac{1}{4}$ and $\frac{1}{8}$. On the left we show the rescaled equilibration time $\tau_{(th)}$ as defined in (4.20), while on the right we show the same plot, but for the un-rescaled equilibration time $\alpha\tau_{(th)}$. The blue, purple, yellow and green curves correspond to $\rho_m = 0.9\rho_h, 0.99\rho_h, 0.995\rho_h$ and $0.999\rho_h$ respectively. Notice how the trends change sign from the left to the right plots.

Figure 8. The evolution of $\mathcal{L}_{2(th)}$ as a function of the boundary time of the two-point correlator. The plots are (from left to right, top to bottom) for $\alpha = 1, \frac{1}{2}, \frac{1}{4}$ and $\frac{1}{8}$, respectively. Each figure contains the plot for an equal time two-point function $\rho_m = 0.1\rho_h, 0.3\rho_h, 0.5\rho_h, 0.7\rho_h, 0.9\rho_h, 0.99\rho_h$ and $0.999\rho_h$, respectively. The plots for $0.9\rho_h, 0.99\rho_h$ and $0.999\rho_h$ are orange, bright blue, and red, respectively. We also plotted $p_{2(th)}$ in dashed lines, to compare with the equilibration of the two-point functions. We can see that the larger the separation of the two points (i.e., the depth ρ_m), the longer the thermalization time is in each case.

Figure 9. The thermalization measure of the perturbation of the entanglement entropy (as defined in (4.20)) for different-sized entangling regions. The evolution is a function of the rescaled boundary time τ_* . The plots are, from left to right, top to bottom, for $\rho_m = 0.1\rho_h$, $0.5\rho_h$, $0.9\rho_h$, $0.99\rho_h$ and $0.999\rho_h$. In each plot the thermalization measure is shown for quenching parameters $\alpha = 1$ (blue), $\alpha = \frac{1}{2}$ (purple), $\alpha = \frac{1}{4}$ (brown) and $\alpha = \frac{1}{8}$ (green). Note that the smaller α is, the longer thermalization takes, in this rescaled boundary time.

Figure 10. An alternative view of figure 9. The same plots are shown, but with the thermalization measures being functions of the un-rescaled boundary time $\alpha\tau_*$. In this case one can see that the smaller α is, the shorter thermalization of the entropy tends to take, from an absolute point of view.

Figure 11. We show the thermalization times of $S_{\Sigma(2)}$ for various values of ρ_m as a function of the inverse of the quenching parameter α , for $\alpha = 1, \frac{1}{2}, \frac{1}{4}$ and $\frac{1}{8}$. On the left we show the rescaled thermalization time $\tau_{(th)}$, while on the right we show the same plot, but for the un-rescaled thermalization time $\alpha\tau_{(th)}$. The blue, purple, yellow and green curves correspond to $\rho_m = 0.9\rho_h, 0.99\rho_h, 0.999\rho_h$ and $0.9999\rho_h$ respectively. Notice how the trends change sign from the left to the right plots.

Figure 12. The evolution of $S_{\Sigma(2)(th)}$ as a function of the boundary time. The plots are (from left to right, top to bottom) for $\alpha = 1, \frac{1}{2}, \frac{1}{4}$ and $\frac{1}{8}$, respectively. Each figure contains the plot for a minimal surface of height $\rho_m = 0.1\rho_h, 0.5\rho_h, 0.9\rho_h, 0.99\rho_h$ and $0.999\rho_h$, respectively. The plots for $0.9\rho_h, 0.99\rho_h$ and $0.999\rho_h$ are orange, bright blue, and red, respectively. We also plotted $p_{2(th)}$ in dashed lines for comparison. We can see that the larger the entangling surface Σ (i.e., the depth ρ_m), the longer the thermalization time of the entanglement entropy is in each case.

Figure 15. The thermalization times for the two-point correlator on the left and entanglement entropy on the right, respectively, as a function of the half-width of the correlator and entanglement regions, respectively. The blue, purple, yellow and green curves are for $\alpha = 1, \frac{1}{2}, \frac{1}{4},$ and $\frac{1}{8}$, respectively. The thermalization times in un-rescaled boundary time τ_* are functions of the un-rescaled separations and widths, respectively. We also show the thermalization times of the one-point correlator $\langle \mathcal{O}_3 \rangle$ for the various values of α as the horizontal dashed lines with the same colour scheme. Note that the thermalization time for the one-point function is nearly the same for $\alpha = \frac{1}{4}$ and $\alpha = \frac{1}{8}$.

Figure 17. Here we show the excitation and equilibration times of the integrand in (4.53) as a function of radius (both in un-rescaled coordinates). The blue, purple, yellow and green curves show the excitation (bottom) and equilibration boundary (top) times for $\alpha = 1, \frac{1}{2}, \frac{1}{4}$ and $\frac{1}{8}$, respectively.

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References

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