Natural and multi-natural inflation in axion landscape

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ABSTRACT: We propose a landscape of many axions, where the axion potential receives various contributions from shift symmetry breaking effects. We show that the existence of the axion with a super-Planckian decay constant is very common in the axion landscape for a wide range of numbers of axions and shift symmetry breaking terms, because of the accidental alignment of axions. The effective inflation model is either natural or multi-natural inflation in the axion landscape, depending on the number of axions and the shift symmetry breaking terms. The tension between BICEP2 and Planck could be due to small modulations to the inflaton potential or steepening of the potential along the heavy axions after the tunneling. The total duration of the slow-roll inflation our universe experienced is not significantly larger than 60 if the typical height of the axion potentials is of order $(10^{16-17} \text{ GeV})^4$.

KEYWORDS: Cosmology of Theories beyond the SM, Flux compactifications, Superstring Vacua

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1 Introduction

The discovery of the primordial B -mode polarization of cosmic microwave background (CMB) by BICEP2 [1] provides us with valuable information on the early universe. The measured tensor-to-scalar ratio reads $r = 0.20^{+0.07}_{-0.05}$, which, taken at face value, implies large-field inflation models such as quadratic chaotic inflation [2], natural inflation [3], or their extensions to polynomial [4, 5] or multi-natural inflation [6]. In particular, the inflaton field excursion exceeds the Planck scale, which places a tight constraint on the inflation model building.

The observed large tensor-to-scalar ratio, however, has a tension with the Planck results, $r < 0.11$ (95% CL) [7]. The tension can be relaxed if the scalar perturbations are suppressed at large scales [8], which might be a result of the steep potential after the false vacuum decay via bubble nucleation [9, 10]. Alternatively, the tension can be relaxed by a negative running of the spectral index [1, 11]. The running can be generated if there are small modulations to the inflaton potential [12, 13]. We shall see that the BICEP2 result and its apparent tension with Planck can be naturally explained in a landscape of many axions.

In the landscape paradigm [14, 15], there are numerous false vacua where eternal inflation occurs, continuously producing universes by bubble nucleation. Our universe is considered to be within a single bubble inside which slow-roll inflation took place after the tunneling event. This paradigm has various implications for cosmology and particle physics and seems to have gained further momentum after BICEP2. It however remains unanswered why and how the slow-roll inflation took place after the false vacuum decay. It might be due to some fine-tuning of the potential. Such fine-tuning may be justified because, most probably, there is an anthropic bound on the duration of slow-roll inflation

after the bubble nucleation [16]. Still, it is uncertain how a very flat inflaton potential extending beyond the Planck scale is realized in the landscape. Also, there is no clear connection between the slow-roll inflation and the landscape paradigm.

In this Letter we propose a landscape of axions where the eternal inflation occurs in the false vacua and the slow-roll inflation regime naturally appears after the tunneling events. See figure 1 for illustration of this concept. Most important, we find that there is very likely to be a direction along which the effective decay constant is super-Planckian, because of accidental alignment of axions known as the Kim-Nilles-Peloso (KNP) mechanism [17]. The important parameters that characterize the size and shape of the axion landscape are the number of axions, N_{axion} , and that of shift symmetry breaking terms, N_{source} . In the case of $N_{\text{source}} > N_{\text{axion}}$, the vacuum structure of the axions generates the so-called landscape, in which there are valleys and hills in the axion potential; most important, there exist many local vacua where the eternal inflation occurs, leading to a multiverse. On the other hand, in the case of $N_{\text{source}} \leq N_{\text{axion}}$, all the vacua are degenerate in energy, and we need to embed the axion landscape into the string landscape to induce eternal inflation. In both cases the KNP mechanism works and the super-Planckian decay constant can be generated from sub-Planckian ones. We will estimate the probability for obtaining a super-Planckian decay constant for a wide range of values of N_{axion} and N_{source} based on a simplified model. When $N_{\text{axion}} = N_{\text{source}}$, we will show that the enhancement of the decay constant by a factor of 10^3 happens with a probability of about 0.8%, 8%, and 24% for $N_{\text{axion}} = 10, 100$ and 300 , respectively. Thus, the existence of such a flat inflationary potential over super-Planckian field values is a built-in feature of the axion landscape. We will also study the other cases; when $N_{\text{source}} > N_{\text{axion}}$, it becomes less likely to obtain a large enhancement of the decay constant, whereas numerous local minima are generated. When $N_{\text{source}} < N_{\text{axion}}$, there appear massless (or extremely light) axions, which may play an important cosmological role.

Here let us mention the work relevant to the present study. Recently, it was shown in [18] that the KNP mechanism works for the case of $N_{\text{axion}} = N_{\text{source}} > 2$ with smaller values of anomaly coefficients. They estimated the probability for a given direction to have an effective super-Planckian decay constant, and they found that it was roughly given by the inverse of the enhancement factor. Our result looks significantly larger than their estimate, but this is because we have estimated a probability that the decay constant corresponding to *the lightest* direction happens to be super-Planckian for each configuration of the axion landscape. We obtained a consistent result when we used the same values of $N_{\text{axion}} = N_{\text{source}}$ adopted in [18].

The inflation dynamics along the plateau is given by either natural inflation [3] or multi-natural inflation [6], depending on N_{axion} and N_{source} as well as the height of each terms. In a sufficiently complicated landscape with $N_{\text{source}} \gtrsim N_{\text{axion}}$, we expect that the latter will be more generic. As we shall discuss shortly, the decay constant, inflaton mass, and duration of inflation etc. are determined by N_{axion} and N_{source} , i.e., the size and shape of the axion landscape. The axion landscape thus provides a unified picture of the slow-roll inflation in the landscape paradigm. In string theory, axions tend to be lighter than geometric moduli owing to gauge symmetries, and so, the axion landscape can be thought

Figure 1. Illustration of the axion landscape. The landscape consists of many axions with sinusoidal potentials of various height and periodicity. There is likely to be a flat direction with an effective super-Planckian decay constant because of the accidental alignment of axions, whereas the typical curvature at the false vacua is much larger than the Hubble parameter. The inflaton is one of the lightest axions, and the natural or multi-natural inflation takes place after the last Coleman-De-Luccia tunneling event.

of as a low-energy branch of the string landscape paradigm. In this case, the role of the axion landscape is to generate an axion with a super-Planckian decay constant.

2 Natural and multi-natural inflation

We must have a good control of the inflaton potential over more than about 10 times the reduced Planck scale for successful large-field inflation suggested by the BICEP2 result [1]. One way for realizing a sufficiently flat potential over large field ranges is to impose a shift symmetry on the inflaton:

$$\phi \rightarrow \phi + C, \quad (2.1)$$

where ϕ is an inflaton field and C is a real transformation parameter. In the string theory there appear many moduli fields through compactifications, and their axionic partners respect such shift symmetry and therefore axions tend to be lighter than the moduli fields. Thus, string axions are a good candidate for the inflaton.¹

In order to have a graceful exit of the inflation, we need to break the shift symmetry. One plausible way is to break the continuous shift symmetry down to a discrete one by

¹See, e.g., refs. [19–22] for attempts to realize natural inflation in the string theory.

adding a sinusoidal function:

$$V(\phi) = \Lambda^4 \left(1 - \cos \left(\frac{\phi}{f} \right) \right), \quad (2.2)$$

where f is the decay constant and Λ denotes a dynamically generated scale violating the continuous shift symmetry. Then, the natural inflation [3] takes place for a sufficiently large f . In particular, $f \gtrsim 5M_{Pl}$ is required by the Planck result [7], and a similar bound is obtained by a combined analysis of BICEP2 and Planck [23], where $M_{Pl} \simeq 2.4 \times 10^{18}$ GeV is the reduced Planck mass.

One can extend the natural inflation to include multiple sinusoidal functions of different height and decay constants;

$$V(\phi) = \sum_i \Lambda_i^4 \cos \left(\frac{\phi}{f_i} + \theta_i \right) + V_0, \quad (2.3)$$

where θ_i denotes a relative phase between different sinusoidal functions and a constant term V_0 is to make the cosmological constant vanish in the present vacuum. This is the multi-natural inflation [6, 24, 25].² This potential is naturally generated if the axion is coupled to multiple gauge theories and/or stringy instantons [29, 30]. Note that the potential contains many local vacua [6]. The multi-natural inflation can accommodate a wide range of values of the spectral index n_s and the tensor-to-scalar ratio r [6, 24, 25]. If there is a mild hierarchy of order 10 – 100 among the decay constants, it is possible to generate a sizable running of the spectral index, $dn_s/d \ln k \simeq -0.03 \sim -0.02$, which remains approximately constant over the CMB scales without any conflict with large-scale structure data [12, 13]. In the multi-natural inflation, at least one of the decay constants must be greater than the Planck scale to explain the BICEP2 result, although the lower bound on f_i is relaxed compared to the natural inflation.

3 Kim-Nilles-Peloso mechanism

The central issue for successful natural or multi-natural inflation is how to realize the effective decay constant greater than the Planck scale. There is an interesting possibility to generate such a large decay constant from sub-Planckian decay constants, proposed by Kim, Nilles and Peloso [17]. Their idea is very simple. Let us consider two axions with a shift symmetry, $\phi_i \rightarrow \phi_i + 2\pi f_i$. We assume that the shift symmetry is broken by the following potential:

$$V(\phi_1, \phi_2) = \Lambda_1^4 \cos \left(a_{11} \frac{\phi_1}{f_1} + a_{12} \frac{\phi_2}{f_2} \right) + \Lambda_2^4 \cos \left(a_{21} \frac{\phi_1}{f_1} + a_{22} \frac{\phi_2}{f_2} \right) + V_0 \quad (3.1)$$

where i, j run over 1, 2 and a_{ij} is an integer-valued anomaly coefficient which depends on the sources for the shift symmetry breaking. The typical value of the decay constants, f_1 and f_2 , are considered to be around the string scale $M_{\text{string}} = \mathcal{O}(10^{17})$ GeV. The point is

²A similar potential with an irrational ratio of the decay constants was considered in [26], and also recently in [27]. It was also studied in the context of curvatures in [28].

that, if $a_{11}/a_{12} = a_{21}/a_{22}$, one combination of ϕ_1 and ϕ_2 does not appear in the scalar potential, and therefore, there is a flat direction. It implies that, if $a_{11}/a_{12} \approx a_{21}/a_{22}$ but $a_{11}/a_{12} \neq a_{21}/a_{22}$, the flat direction is lifted, and its corresponding decay constant can be much larger than the typical value of f_i . In other words, a certain alignment of the axions appearing in the sinusoidal functions can generate a super-Planckian decay constant from sub-Planckian ones. The required enhancement is of order 10^{2-3} for the decay constant around the string scale.

The KNP mechanism is very useful to embed the natural and multi-natural inflation in UV theory such as string theory where the typical values of the decay constant is below the Planck scale. For instance, multi-natural inflation in string-inspired setting was discussed in [24, 25] based on the KNP mechanism. Recently, the extension of the KNP mechanism to multiple axions was proposed in [18], where they showed that the KNP mechanism works in the presence of multiple axions, and they estimated the probability to realize an effective large decay constant, f_{eff} in the case of $N_{\text{axion}} = N_{\text{source}}$, assuming that the other directions are sufficiently heavy. They showed that the probability for the alignment of axions is approximately given by the inverse of the enhancement f_{eff}/f_i . Later in this letter we will directly calculate the distributions of the axion mass for a wide range of values of N_{axion} and N_{source} , based on a simplified model of the axion landscape. There we will show that the required enhancement can be generically realized in the axion landscape.

4 False vacuum decay

In the landscape paradigm, eternal inflation occurs in numerous false vacua, creating various universes through the tunneling, leading to a multiverse. As pointed out in ref. [16], if we live in a bubble created from the false vacuum decay followed by the slow-roll inflation with the e-folding number $N_e = 50 - 60$, we may be able to observe a negative curvature as a remnant of the bubble nucleation. The detection of the negative spatial curvature will be more likely if there is a pressure toward shorter inflation. Also, if the inflaton potential after the bubble nucleation is sufficiently steep, scalar density perturbations at large scales can be suppressed [31, 32]. This could explain the low ℓ -anomaly of CMB. After BICEP2, the problem of the low- ℓ suppression was sharpened [10]. Recently, the landscape and its implications were also discussed in the sneutrino chaotic inflation model [33]. As we shall see shortly, these expected properties are built-in features of the axion landscape.

The Coleman-De-Luccia (CDL) instanton [34] requires a rather large curvature of the potential, $V'' \gg H^2$ in the false vacuum; otherwise the tunneling occurs as in the Hawking-Moss case [35]. In order to realize both the eternal inflation in the false vacuum, CDL instanton, and the subsequent slow-roll inflation in terms of a single scalar field, one needs a rather contrived functional form of the inflaton potential. If one considers multiple fields, on the other hand, we can naturally realize mass hierarchy between the false vacuum and the slow-roll inflation regime [36]. Still, the existence of the inflationary plateau had no direct relation with the landscape so far. This questions is also answered naturally in the axion landscape.

5 Axion landscape

Let us now consider an axion landscape. For simplicity we will focus on a landscape of axions with sub-Planckian decay constants, but the application to other cases is straightforward. The shift symmetries of axions are explicitly broken by various sources such as gaugino condensation and/or instanton:

$$V(\phi_i) = \sum_{i=1}^{N_{\text{source}}} \Lambda_i^4 \cos \left(\sum_{j=1}^{N_{\text{axion}}} a_{ij} \frac{\phi_j}{f_j} + \theta_i \right) + V_0 \quad (5.1)$$

with ϕ_i is an axion, a_{ij} integer-valued anomaly coefficients, and f_i the decay constant.³ The axion is assumed to have a periodicity,

$$\phi_i \rightarrow \phi_i + 2\pi f_i. \quad (5.2)$$

The potential height Λ_i is naively expected to be of order $M_{\text{string}} = \mathcal{O}(10^{17})$ GeV in the string theory, whereas some of them can be smaller if they are dynamically generated. In general, N_{source} does not necessarily coincide with N_{axion} . For the moment we assume N_{source} is comparable to N_{axion} , and we shall discuss the other cases and its cosmological implications later. Here and in what follows, N_{source} counts only the shift symmetry breaking terms which can affect the vacuum structure and/or the inflaton dynamics, and hierarchically small contributions are to be considered separately in the low energy. For a sufficiently large N_{source} , they form a complicated landscape with numerous local minima, where eternal inflation occurs, continuously creating new universes by bubble nucleation (cf. figure 1). On the other hand, when N_{source} is equal to or smaller than N_{axion} , all the vacua are degenerate in energy. This is not an obstacle for generating a direction with a super-Planckian decay constant. In order to induce eternal inflation, however, we would need to either introduce another kind of shift symmetry breaking to lift the degeneracy or embed the axion landscape into the string landscape which contains many local vacua. In the latter case, the role of the axion landscape is to provide an axion with the super-Planckian decay constant.

In the axion landscape, a super-Planckian decay constant for the inflaton is likely realized by accidental alignment of axions in the axion landscape. As will be shown below, the probability to obtain an enhancement of the decay constant by a factor of 10^3 is about 1%(10%) for $N_{\text{axion}} = N_{\text{source}} = 10(100)$. On the other hand, the probability decreases as N_{source} becomes larger than N_{axion} . The case of $N_{\text{axion}} = N_{\text{source}}$ was recently studied in detail in [18], and they showed that the KNP mechanism works for multiple axions with smaller values of the anomaly coefficients. Contrary to [18], however, we believe that no hierarchy in the potential height is needed to ensure the mass hierarchy between the inflaton and the other axions. This has an important implication for the duration of the inflation as we discuss later. Thus the KNP mechanism generically works for a wide range

³It is possible to add another functional form of the axion potential [37–39], but it does not significantly change our arguments, as long as there are sufficient number of axions which form the landscape like (5.1). In fact, such an additional potential will help to generate many false vacua.

of values of N_{axion} , N_{source} , and the potential height Λ_i^4 . The slow-roll inflation is a natural outcome of the axion landscape.

If the axion landscape has many local vacua, the CDL instantons are formed at the tunneling. This is because the typical decay constants are sub-Planckian and the axions other than the inflaton tend to be so heavy that the barrier between the local minima is narrow. After the tunneling, the universe will be dominated by the curvature term and the heavy axions for a while. The slow-roll inflation by the lightest axion starts when its energy density dominates the universe.

In order to get the feeling of the probability for a sufficiently flat direction to arise by the accidental alignment, let us consider a simplified model. We set $\Lambda_i = \Lambda$, $f_i = f$ and $\theta = 0$ in eq. (5.1).⁴ We take a_{ij} as an integer valued random matrix satisfying $-n \leq a_{ij} \leq n$ to scan various realization of the axion landscape. In order to find the flattest direction, we Taylor expand the cosine function up to the second order of axions. Then the mass matrix for axions $\{\phi_i\}$ is proportional to

$$M_{ij}^2 \propto A_{ij} \equiv \sum_{k=1}^{N_{\text{source}}} a_{ki} a_{kj}. \quad (5.3)$$

Of course, as the field values change, the contribution of each shift symmetry breaking term to the mass term changes. Since we are interested in the case that all the axions other than the lightest one are stabilized in one of the false vacua, such an expansion can be justified.

We numerically estimate the eigenvalue distribution of the integer-valued random matrix squared, A_{ij} . Let us denote the eigenvalues of A_{ij} as $0 < a_1^2 \leq a_2^2 \leq \dots \leq a_{N_{\text{axion}}}^2$.⁵ Here we exclude the case of $\det A = 0$, in which case there is always massless axion(s). This should be included in the case of $N_{\text{source}} < N_{\text{axion}}$, as some of the source terms are not independent (as far as the axion mass is concerned). The effective decay constant for each axion is enhanced by $\langle a \rangle / a_i$ with respect to its typical value f . For instance, the effective decay constant for the lightest axion is given by

$$f_{\text{eff}} \simeq f_i \frac{\langle a \rangle}{a_1}, \quad (5.4)$$

and similarly for the second or third lightest axions. Here $\langle a \rangle \equiv \sqrt{\langle a_i^2 \rangle}$ is the squared root of the averaged eigenvalues of A_{ij} . Note that $\langle a \rangle$ scales as $\sqrt{N_{\text{axion}}}$, which however does not change the relation (5.4) as it only affects the typical mass scale, not the decay constants.

Before going to the detail, we give the result on the integrated probability that the effective decay constant for the lightest axion is enhanced by more than f_{eff}/f_i , for the case

⁴In general, some of the phases are physical and cannot be absorbed by the redefinition of the parameters or the shift of axions. In fact, they will play an important role in the multi-natural inflation [6, 24, 25] to obtain a wide range of values of n_s and r .

⁵We focus on a situation that a_1 is smaller than a_2 , and identify the lightest axion with the inflaton. On the other hand, assisted inflation [40] or N-flation [41] will be possible if some of the eigenvalues are degenerate.

of $N_{\text{source}} = N_{\text{axion}}$:

$$\mathcal{P}(f_{\text{eff}}/f_i) \sim N_{\text{axion}} \left(\frac{f_i}{f_{\text{eff}}} \right) \quad (5.5)$$

as long as it is much smaller than unity. Here $\mathcal{P}(x)$ denotes the probability that the enhancement factor f_{eff}/f_i for the lightest axion becomes larger than x . This implies that the effective decay constant for the lightest axion is enhanced as N_{axion} increases.

In figure 2 we show the integrated probability distribution functions for the enhancement factor of the effective decay constant for the lightest, the second lightest, and the third lightest axions. We have generated 10^5 random matrices A_{ij} with $\det A \neq 0$ for $N_{\text{axion}} = 10$ and $n = 2$. For instance we can see that the probability that the lightest axion mass happens to be $10^3(10^2)$ times lighter than the averaged mass is about 0.8%(8%). In other words, the decay constant for the lightest axion is enhanced by a factor of 10^3 than the typical value with a probability of about 1% for $N_{\text{axion}} = 10$. Surprisingly, even for a relatively small axion landscape that consists of only 10 axions, we can realize the enhancement of 10^3 with a probability about 1%.⁶ As one can see from the figure, it is much more unlikely to have two (or more) relatively flat directions simultaneously. This suggests that our inflaton is the lightest axion in the axion landscape.

Note that our result on the probability is about one order of magnitude larger than the result of ref. [18]. This is because they estimated a probability for a given combination of axions to have a super-Planckian decay constant assuming the rest of axions are hierarchically heavier. Instead, we have evaluated all the mass eigenvalues for each realization of the axion landscape, and then estimated the probability for *the lightest* one to have super-Planckian decay constants. This corresponds to the probability that, for a given axion landscape, we find at least one flat direction with the super-Planckian decay constant. In addition, we can easily extend our analysis to the case of $N_{\text{source}} > N_{\text{axion}}$ as we directly estimate the mass eigenvalue distribution.

We show in figure 3 the integrated probability distribution function for f_{eff}/f in the case of $N_{\text{source}} = N_{\text{axion}}, N_{\text{axion}} + 1, N_{\text{axion}} + 2$. As the number of source terms increases, it becomes more difficult to have a light axion with a super-Planckian decay constant. Roughly, the probability distribution scales as $\mathcal{P}_{N_{\text{axion}}+1} \sim (\mathcal{P}_{N_{\text{axion}}})^2$ and $\mathcal{P}_{N_{\text{axion}}+2} \sim (\mathcal{P}_{N_{\text{axion}}})^3$. This can be intuitively understood as follows. Note that, if one replaces one of the rows of the random matrix A_{ij} with another random-valued row, the resultant matrix will be almost statistically independent from the initial one. Therefore, adding another source term is approximately equivalent to considering another sets of axions with $N_{\text{axion}} = N_{\text{source}}$. In order to have a suppression of the axion mass (or equivalently, the enhancement of the decay constant), the accidental cancellation must happen in the two sets of axions simultaneously. Therefore, we expect $\mathcal{P}_{N_{\text{axion}}+1} \sim (\mathcal{P}_{N_{\text{axion}}})^2$, and similarly for a larger value of the source terms.

Now let us go onto a larger axion landscape. To simplify our analysis we focus on the case of $N_{\text{axion}} = N_{\text{source}}$. We show in figure 4 the probability to have a flat direction with the decay constant more than 10^2 (upper), 10^3 (middle), 10^4 (bottom) times larger

⁶We are aware that it is non-trivial to interpret the probability because of the measure problem.

Figure 2. The integrated probability distribution functions, $\mathcal{P}(f_{\text{eff}}/f_i)$, for the lightest (solid), second lightest (dashed), and third lightest (dash dotted) mass eigenvalues, from right to left. We generated 10^5 random matrices with $N_{\text{axion}} = 10$ and $n = 2$. The probability for the enhancement by more than 10^3 is about 1%. It is rare that two (or three) flat directions arise simultaneously by the accidental alignment.

Figure 3. The integrated probability distribution functions, $\mathcal{P}(f_{\text{eff}}/f_i)$, for the case of $N_{\text{source}} = N_{\text{axion}}, N_{\text{axion}} + 1, N_{\text{axion}} + 2$, from right to left. We generated 10^5 random matrices and set $N_{\text{axion}} = 10$. As the number of shift symmetry breaking terms increases, it becomes more difficult to realize the suppression of the lightest mass eigenvalues (or equivalently, the enhancement of the decay constant).

than the typical value in one realization of the axion landscape, as a function of N_{axion} . We can see that the probabilities for the enhancement of 10^2 , 10^3 , and 10^4 are about 70%, 8%, and 0.8%, respectively for $N_{\text{axion}} \approx 100$, and they further increase in proportion to N_{axion} . Note that the enhancement by a factor of 10^4 is less likely unless the number

Figure 4. The probability to have a flat direction with the decay constant more than 10^2 (upper), 10^3 (middle), and 10^4 (bottom) times larger than the typical value in one realization of the axion landscape, as a function of $N_{\text{axion}} = N_{\text{source}}$.

of axions becomes much larger than $\mathcal{O}(100)$. If we compare our result at $N_{\text{axion}} = 10$ with that of [18], ours is about one order of magnitude larger, probably because we focus on the lightest direction; if we randomly choose one direction from the 10 directions, the probability to realize the enhancement decreases by a factor of 10. Thus, we conclude that the existence of the axion with a super-Planckian decay constant is very common in the axion landscape. A flat potential over the super-Planckian field values emerges from a complex vacuum structure in the axion landscape. The likely size of the decay constant sensitively depends on N_{axion} and N_{source} .

The effective potential for the inflaton generically contains a combination of various sinusoidal functions. Note that the height of the inflaton potential is generically comparable to the typical height in the axion landscape, although it can be suppressed if there is a hierarchy in the size of the shift symmetry breaking. This is the case if some of them are dynamically generated. If the inflaton potential is dominated by a single sinusoidal function, the inflaton dynamics is well approximated by the natural inflation (2.2). On the other hand, if there are multiple sinusoidal functions with slightly different decay constants and heights, it will be the multi-natural inflation (2.3). This is the case if $N_{\text{source}} > N_{\text{axion}}$. In particular, if N_{source} is slightly larger than N_{axion} and some of the source terms have hierarchically small height, the inflaton potential receives small modulations, leading to a sizable negative running of the spectral index [12, 13]. Thus, the running of the spectral index, if detected, will constrain the relation between N_{source} and N_{axion} .

Now let us consider the eternal inflation which occurs in the false vacua before the slow-roll inflation takes place. The tunneling rate is not sensitive to the value of the inflaton, since the couplings between the inflaton and the heavy axions are weak [36]. Therefore, the inflaton field value at the tunneling is generically deviated from the inflaton potential minimum by $\mathcal{O}(f_{\text{eff}})$, which determines the typical duration of the slow-roll inflation.⁷ The

⁷If the energy difference is small, the inflaton might be more or less stabilized near the potential minimum.

mass hierarchy between the false vacuum and the inflation plateau is also determined by f_{eff}/f_i , the value of which depends on the detailed properties of the axion landscape. It is a quantitative question how large hierarchy is generated or how long inflation typically lasts, but, based on our simple model of the axion landscape, it is likely that the decay constant $f = \mathcal{O}(10)M_{Pl}$ is generated from the typical decay constants $f_i = \mathcal{O}(10^{16-17})$ GeV with $N_{\text{axion}} \sim N_{\text{source}} = \mathcal{O}(10 - 100)$, and the typical e-folding number after the tunneling event is not significantly larger than 60. Intriguingly, the BICEP2 result suggests the inflation scale to be $V_{\text{inf}} \simeq (2.0 \times 10^{16} \text{ GeV})^4 \cdot (r/0.16)$, not far from $M_{\text{string}}^4 = \mathcal{O}((10^{17} \text{ GeV})^4)$, the potential height at false vacua naively expected in the string theory. The two scales can be even closer if the axion potentials are dynamically generated. Since the two scales are comparable, it implies that the inflaton field value at the horizon exit of cosmological scales is of order f_{eff} , which should be of order 10 times the Planck scale, M_{Pl} . That is to say, the typical duration of the inflation is not significantly different from 60.

It sensitively depends on the duration of inflation whether we will be able to observe the negative spatial curvature. The detection is more likely if there is a pressure toward shorter inflation. In the axion landscape, this is generically the case because the flat direction arises from the accidental alignments. In particular, the pressure toward shorter inflation will be significant if the number of axions is relatively small, or if the number of source terms tends to be (much) larger than the number of axions. On the other hand, the pressure might be weakened if there are many axions with $N_{\text{axion}} \sim N_{\text{source}}$, since the large effective decay constant can be generated more easily. Therefore, detection or non-detection of the negative spatial curvature by future observations will provide us with information on the size and shape of the axion landscape.

So far we have assumed $N_{\text{source}} \sim N_{\text{axion}}$. If $N_{\text{source}} \gtrsim N_{\text{axion}}$, it will be more difficult to realize a super-Planckian decay constant, as we have seen in figure 2. For $N_{\text{source}} < N_{\text{axion}}$, there appear massless (or extremely light) $N_{\text{axion}} - N_{\text{source}}$ axions in the low energy, while the above discussion on the inflation remains unchanged. They may behave as dark radiation or hot dark matter, ameliorating the tension between BICEP2 and Planck [42, 43]. They are produced through various mechanisms, e.g., coherent oscillations, thermal production, and non-thermal production from decays of long-lived states [44–46]. If they get tiny masses, stable axion dark matter can generate isocurvature fluctuations, which is strongly constrained by the BICEP2 result [47–49]. If there exist axions with masses at (more than) PeV scales, they may play a role in generating fluctuations as curvatons [50].

Another interesting implication of the axion landscape is that the decompactification problem during the inflation will be naturally evaded because the axions are the lightest moduli fields in the string theory owing to the shift symmetry. Although it is a non-trivial task to infer the supersymmetry breaking scale in the axion landscape, there is a lower bound on the gravitino mass, $m_{3/2} \gtrsim H_{\text{inf}} \sim 10^{14} \text{ GeV}$ [51], in a known set-up of moduli stabilization. Here $m_{3/2}$ is the gravitino mass, which is typically comparable to other heavy moduli masses. Note that there exist moduli fields heavier than the gravitino, which do

This situation can be avoided if the typical energy density in the false vacua is larger than the inflaton potential height.

not play any cosmological role in our context. In fact, such a high supersymmetry breaking is consistent with the Standard Model Higgs mass [52–54].

6 Reheating

For successful inflation, the inflaton energy is transferred into the Standard Model particles, and the baryon asymmetry is generated after the inflation. To this end, we shall discuss the reheating temperature and the leptogenesis after the inflaton decay. The mass of the inflaton m_ϕ in the natural or multi-natural inflation is given by

$$m_\phi \sim \frac{\Lambda^2}{f_{\text{eff}}} \sim 10^{13} \text{ GeV}, \quad (6.1)$$

where $\Lambda = \mathcal{O}(10^{16}) \text{ GeV}$ and the effective decay constant $f_{\text{eff}} = \mathcal{O}(10)M_{Pl}$. Since the inflaton is one of the lightest axions, there exists a coupling to the Standard Model gauge bosons: $\mathcal{L} = c(\phi/f_a)F_{\mu\nu}\tilde{F}^{\mu\nu}$. The decay width of the inflaton reads

$$\Gamma_\phi = N_g \frac{c^2}{4\pi} \frac{m_\phi^3}{f_a^2}, \quad (6.2)$$

where $N_g = 8 + 3 + 1$ counts the number of Standard Model gauge bosons. Note that the decay constant f_a can be different from the those in the scalar potential. Then, the reheating temperature T_R is estimated as

$$T_R \sim 4 \times 10^{10} \text{ GeV} \cdot \left(\frac{m_\phi}{10^{13} \text{ GeV}} \right)^{3/2} \left(\frac{f_a/c}{10^{17} \text{ GeV}} \right)^{-1}. \quad (6.3)$$

Here, we used $T_R = (90/\pi^2 g_*(T_R))\sqrt{\Gamma_\phi M_{Pl}}$ and $g_*(T_R) = 106.75$ counts the number of degree of the relativistic particles in plasma. Therefore successful thermal leptogenesis [55] is possible when the right-handed neutrinos is produced from the thermal bath:

$$\left. \frac{n_B}{s} \right|_{\text{max.}} \sim 10^{-10} \cdot \left(\frac{M_{N_1}}{10^{10} \text{ GeV}} \right) \quad \text{for } M_{N_1} \lesssim T_R, \quad (6.4)$$

where M_{N_1} is the mass of the lightest right-handed neutrino.

For $T_R \lesssim M_{N_i} \lesssim m_\phi/2$, non-thermal leptogenesis will be also possible, if there exists a direct coupling to the right-handed neutrinos, say, $\mathcal{L} \sim (\phi/f_a)M_{N_i}N_iN_i$, where N_i is the right-handed neutrinos. Then, the partial decay width reads

$$\Gamma_\phi^N \simeq \frac{1}{16\pi} \left(\frac{M_{N_i}}{f_a} \right)^2 m_\phi \quad (6.5)$$

and the reheating temperature does not change significantly. The right amount of the baryon asymmetry is then obtained through the decay of N_i produced from the inflaton and the sphaleron process:

$$\left. \frac{n_B}{s} \right|_{\text{max.}} \sim 5 \times 10^{-10} \cdot \left(\frac{T_R/m_\phi}{10^{-3}} \right) \left(\frac{M_{N_i}}{10^{12} \text{ GeV}} \right) \left(\frac{Br}{10^{-2}} \right). \quad (6.6)$$

Here, Br is the branching fraction of the decay mode of $\phi \rightarrow 2N_i$.

7 Conclusions

We have proposed a landscape of many axions where the axion potential receives various contributions from shift symmetry breaking effects. If the number of the shift symmetry breaking, N_{source} , is large enough, there are valleys and hills in the axion potential; eternal inflation occurs in the local minima, continuously creating new universes via the CDL tunneling. On the other hand, if $N_{\text{source}} \leq N_{\text{axion}}$, all the vacua are degenerate in energy. In this case, eternal inflation will be possible if one introduces another kind of shift symmetry breaking or if one embed the axion landscape into the string landscape with a large number of local minima.

Interestingly, there is very likely to be a direction along which the effective decay constant exceeds the Planck scale owing to the accidental alignment of axions, i.e., the KNP mechanism. Therefore, in the axion landscape, the existence of the slow-roll inflation regime is a natural outcome of the complicated vacuum structure. We have also argued that the effective inflation model in the axion landscape will be either natural inflation or multi-natural inflation, depending the values of N_{axion} and N_{source} . In the latter case, a wide range of (n_s, r) as well as the running of the spectral index can be realized. The size and shape of the axion landscape are parametrized by N_{axion} and N_{source} , which determines the effective super-Planckian decay constant, and therefore the typical duration of the slow-roll inflation. In a certain case, there might be a strong pressure toward shorter inflation, and it will be more likely that we can measure the negative spatial curvature as a remnant of the CDL tunneling. Conversely, non-detection of the negative curvature will constrain the size and shape of the axion landscape. Also, the size of density perturbations and the inflaton potential height will be useful to extract information on the axion landscape. A more quantitative study on this issue will be given elsewhere.

Note added. After completing this work, there appeared the papers [56–59], in which the alignment of two axions was discussed based on the KNP mechanism as well. We also note that the charge assignment studied in refs [56, 58] leads to the suppression of the lighter axion mass, similarly to the seesaw mechanism for the light neutrino mass.

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References

- [1] BICEP2 collaboration, P.A.R. Ade et al., *Detection of B-Mode Polarization at Degree Angular Scales by BICEP2*, *Phys. Rev. Lett.* **112** (2014) 241101 [[arXiv:1403.3985](#)] [[INSPIRE](#)].
- [2] A.D. Linde, *Chaotic Inflation*, *Phys. Lett. B* **129** (1983) 177 [[INSPIRE](#)].
- [3] K. Freese, J.A. Frieman and A.V. Olinto, *Natural inflation with pseudo - Nambu-Goldstone bosons*, *Phys. Rev. Lett.* **65** (1990) 3233 [[INSPIRE](#)].
- [4] K. Nakayama, F. Takahashi and T.T. Yanagida, *Polynomial Chaotic Inflation in the Planck Era*, *Phys. Lett. B* **725** (2013) 111 [[arXiv:1303.7315](#)] [[INSPIRE](#)].
- [5] K. Nakayama, F. Takahashi and T.T. Yanagida, *Polynomial Chaotic Inflation in Supergravity*, *JCAP* **08** (2013) 038 [[arXiv:1305.5099](#)] [[INSPIRE](#)].
- [6] M. Czerny and F. Takahashi, *Multi-Natural Inflation*, *Phys. Lett. B* **733** (2014) 241 [[arXiv:1401.5212](#)] [[INSPIRE](#)].
- [7] PLANCK collaboration, P.A.R. Ade et al., *Planck 2013 results. XXII. Constraints on inflation*, [arXiv:1303.5082](#) [[INSPIRE](#)].
- [8] V. Miranda, W. Hu and P. Adshead, *Steps to Reconcile Inflationary Tensor and Scalar Spectra*, [arXiv:1403.5231](#) [[INSPIRE](#)].
- [9] B. Freivogel, M. Kleban, M.R. Martinez and L. Susskind, *Observational Consequences of a Landscape: Epilogue*, [arXiv:1404.2274](#) [[INSPIRE](#)].
- [10] R. Bousso, D. Harlow and L. Senatore, *Inflation After False Vacuum Decay: New Evidence from BICEP2*, [arXiv:1404.2278](#) [[INSPIRE](#)].
- [11] C. Cheng, Q.-G. Huang and W. Zhao, *Constraints on the extensions to the base Λ CDM model from BICEP2, Planck and WMAP*, *Sci. China Phys. Mech. Astron.* **57** (2014) 1460 [[arXiv:1404.3467](#)] [[INSPIRE](#)].
- [12] T. Kobayashi and F. Takahashi, *Running Spectral Index from Inflation with Modulations*, *JCAP* **01** (2011) 026 [[arXiv:1011.3988](#)] [[INSPIRE](#)].
- [13] M. Czerny, T. Kobayashi and F. Takahashi, *Running Spectral Index from Large-field Inflation with Modulations Revisited*, [arXiv:1403.4589](#) [[INSPIRE](#)].
- [14] R. Bousso and J. Polchinski, *Quantization of four form fluxes and dynamical neutralization of the cosmological constant*, *JHEP* **06** (2000) 006 [[hep-th/0004134](#)] [[INSPIRE](#)].
- [15] L. Susskind, *The Anthropic landscape of string theory*, [hep-th/0302219](#) [[INSPIRE](#)].
- [16] B. Freivogel, M. Kleban, M. Rodriguez Martinez and L. Susskind, *Observational consequences of a landscape*, *JHEP* **03** (2006) 039 [[hep-th/0505232](#)] [[INSPIRE](#)].
- [17] J.E. Kim, H.P. Nilles and M. Peloso, *Completing natural inflation*, *JCAP* **01** (2005) 005 [[hep-ph/0409138](#)] [[INSPIRE](#)].
- [18] K. Choi, H. Kim and S. Yun, *Natural Inflation with Multiple Sub-Planckian Axions*, [arXiv:1404.6209](#) [[INSPIRE](#)].
- [19] T.W. Grimm, *Axion inflation in type-II string theory*, *Phys. Rev. D* **77** (2008) 126007 [[arXiv:0710.3883](#)] [[INSPIRE](#)].
- [20] R. Kallosh, N. Sivanandam and M. Soroush, *Axion Inflation and Gravity Waves in String Theory*, *Phys. Rev. D* **77** (2008) 043501 [[arXiv:0710.3429](#)] [[INSPIRE](#)].

- [21] R. Blumenhagen and E. Plauschinn, *Towards Universal Axion Inflation and Reheating in String Theory*, [arXiv:1404.3542](#) [INSPIRE].
- [22] T.W. Grimm, *Axion Inflation in F-theory*, [arXiv:1404.4268](#) [INSPIRE].
- [23] K. Freese and W.H. Kinney, *Natural Inflation: Consistency with Cosmic Microwave Background Observations of Planck and BICEP2*, [arXiv:1403.5277](#) [INSPIRE].
- [24] M. Czerny, T. Higaki and F. Takahashi, *Multi-Natural Inflation in Supergravity*, *JHEP* **05** (2014) 144 [[arXiv:1403.0410](#)] [INSPIRE].
- [25] M. Czerny, T. Higaki and F. Takahashi, *Multi-Natural Inflation in Supergravity and BICEP2*, [arXiv:1403.5883](#) [INSPIRE].
- [26] T. Banks, M. Dine and N. Seiberg, *Irrational axions as a solution of the strong CP problem in an eternal universe*, *Phys. Lett. B* **273** (1991) 105 [[hep-th/9109040](#)] [INSPIRE].
- [27] R. Kallosh, A. Linde and B. Vercnocke, *Natural Inflation in Supergravity and Beyond*, [arXiv:1404.6244](#) [INSPIRE].
- [28] F. Takahashi, *The Spectral Index and its Running in Axionic Curvaton*, *JCAP* **06** (2013) 013 [[arXiv:1301.2834](#)] [INSPIRE].
- [29] E. Witten, *Nonperturbative superpotentials in string theory*, *Nucl. Phys. B* **474** (1996) 343 [[hep-th/9604030](#)] [INSPIRE].
- [30] R. Blumenhagen, M. Cvetič, S. Kachru and T. Weigand, *D-Brane Instantons in Type II Orientifolds*, *Ann. Rev. Nucl. Part. Sci.* **59** (2009) 269 [[arXiv:0902.3251](#)] [INSPIRE].
- [31] D. Yamauchi, A. Linde, A. Naruko, M. Sasaki and T. Tanaka, *Open inflation in the landscape*, *Phys. Rev. D* **84** (2011) 043513 [[arXiv:1105.2674](#)] [INSPIRE].
- [32] R. Bousso, D. Harlow and L. Senatore, *Inflation after False Vacuum Decay: Observational Prospects after Planck*, [arXiv:1309.4060](#) [INSPIRE].
- [33] H. Murayama, K. Nakayama, F. Takahashi and T.T. Yanagida, *Sneutrino Chaotic Inflation and Landscape*, [arXiv:1404.3857](#) [INSPIRE].
- [34] S.R. Coleman and F. De Luccia, *Gravitational Effects on and of Vacuum Decay*, *Phys. Rev. D* **21** (1980) 3305 [INSPIRE].
- [35] S.W. Hawking and I.G. Moss, *Fluctuations in the Inflationary Universe*, *Nucl. Phys. B* **224** (1983) 180 [INSPIRE].
- [36] A.D. Linde, *Inflation with variable Omega*, *Phys. Lett. B* **351** (1995) 99 [[hep-th/9503097](#)] [INSPIRE].
- [37] E. Silverstein and A. Westphal, *Monodromy in the CMB: Gravity Waves and String Inflation*, *Phys. Rev. D* **78** (2008) 106003 [[arXiv:0803.3085](#)] [INSPIRE].
- [38] L. McAllister, E. Silverstein and A. Westphal, *Gravity Waves and Linear Inflation from Axion Monodromy*, *Phys. Rev. D* **82** (2010) 046003 [[arXiv:0808.0706](#)] [INSPIRE].
- [39] N. Kaloper and L. Sorbo, *A Natural Framework for Chaotic Inflation*, *Phys. Rev. Lett.* **102** (2009) 121301 [[arXiv:0811.1989](#)] [INSPIRE].
- [40] A.R. Liddle, A. Mazumdar and F.E. Schunck, *Assisted inflation*, *Phys. Rev. D* **58** (1998) 061301 [[astro-ph/9804177](#)] [INSPIRE].
- [41] S. Dimopoulos, S. Kachru, J. McGreevy and J.G. Wacker, *N-flation*, *JCAP* **08** (2008) 003 [[hep-th/0507205](#)] [INSPIRE].

- [42] E. Giusarma, E. Di Valentino, M. Lattanzi, A. Melchiorri and O. Mena, *Relic Neutrinos, thermal axions and cosmology in early 2014*, [arXiv:1403.4852](#) [[INSPIRE](#)].
- [43] J.-F. Zhang, Y.-H. Li and X. Zhang, *Sterile neutrinos help reconcile the observational results of primordial gravitational waves from Planck and BICEP2*, [arXiv:1403.7028](#) [[INSPIRE](#)].
- [44] M. Cicoli, J.P. Conlon and F. Quevedo, *Dark Radiation in LARGE Volume Models*, *Phys. Rev. D* **87** (2013) 043520 [[arXiv:1208.3562](#)] [[INSPIRE](#)].
- [45] T. Higaki and F. Takahashi, *Dark Radiation and Dark Matter in Large Volume Compactifications*, *JHEP* **11** (2012) 125 [[arXiv:1208.3563](#)] [[INSPIRE](#)].
- [46] T. Higaki, K. Nakayama and F. Takahashi, *Moduli-Induced Axion Problem*, *JHEP* **07** (2013) 005 [[arXiv:1304.7987](#)] [[INSPIRE](#)].
- [47] T. Higaki, K.S. Jeong and F. Takahashi, *Solving the Tension between High-Scale Inflation and Axion Isocurvature Perturbations*, *Phys. Lett. B* **734** (2014) 21 [[arXiv:1403.4186](#)] [[INSPIRE](#)].
- [48] D.J.E. Marsh, D. Grin, R. Hlozek and P.G. Ferreira, *Tensor Detection Severely Constrains Axion Dark Matter*, *Phys. Rev. Lett.* **113** (2014) 011801 [[arXiv:1403.4216](#)] [[INSPIRE](#)].
- [49] L. Visinelli and P. Gondolo, *Axion cold dark matter in view of BICEP2 results*, *Phys. Rev. Lett.* **113** (2014) 011802 [[arXiv:1403.4594](#)] [[INSPIRE](#)].
- [50] M. Kawasaki, T. Kobayashi and F. Takahashi, *Non-Gaussianity from Axionic Curvaton*, *JCAP* **03** (2013) 016 [[arXiv:1210.6595](#)] [[INSPIRE](#)].
- [51] R. Kallosh and A.D. Linde, *Landscape, the scale of SUSY breaking and inflation*, *JHEP* **12** (2004) 004 [[hep-th/0411011](#)] [[INSPIRE](#)].
- [52] A. Hebecker, A.K. Knochel and T. Weigand, *A Shift Symmetry in the Higgs Sector: Experimental Hints and Stringy Realizations*, *JHEP* **06** (2012) 093 [[arXiv:1204.2551](#)] [[INSPIRE](#)].
- [53] L.E. Ibáñez, F. Marchesano, D. Regalado and I. Valenzuela, *The Intermediate Scale MSSM, the Higgs Mass and F-theory Unification*, *JHEP* **07** (2012) 195 [[arXiv:1206.2655](#)] [[INSPIRE](#)].
- [54] A. Hebecker, A.K. Knochel and T. Weigand, *The Higgs mass from a String-Theoretic Perspective*, *Nucl. Phys. B* **874** (2013) 1 [[arXiv:1304.2767](#)] [[INSPIRE](#)].
- [55] M. Fukugita and T. Yanagida, *Baryogenesis Without Grand Unification*, *Phys. Lett. B* **174** (1986) 45 [[INSPIRE](#)].
- [56] S.H.H. Tye and S.S.C. Wong, *Helical Inflation and Cosmic Strings*, [arXiv:1404.6988](#) [[INSPIRE](#)].
- [57] R. Kappl, S. Krippendorff and H.P. Nilles, *Aligned Natural Inflation: Monodromies of two Axions*, [arXiv:1404.7127](#) [[INSPIRE](#)].
- [58] I. Ben-Dayan, F.G. Pedro and A. Westphal, *Hierarchical Axion Inflation*, [arXiv:1404.7773](#) [[INSPIRE](#)].
- [59] C. Long, L. McAllister and P. McGuirk, *Aligned Natural Inflation in String Theory*, *Phys. Rev. D* **90** (2014) 023501 [[arXiv:1404.7852](#)] [[INSPIRE](#)].