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AdS$_5$ backgrounds with 24 supersymmetries

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Abstract: We prove a non-existence theorem for smooth AdS_5 solutions with connected, compact without boundary internal space that preserve strictly 24 supersymmetries. In particular, we show that $D = 11$ supergravity does not admit such solutions, and that all such solutions of IIB supergravity are locally isometric to the $AdS_5 \times S^5$ maximally supersymmetric background. Furthermore, we prove that (massive) IIA supergravity also does not admit such solutions, provided that the homogeneity conjecture for massive IIA supergravity is valid. In the context of AdS/CFT these results imply that if gravitational duals for strictly $\mathcal{N} = 3$ superconformal theories in 4-dimensions exist, they are either singular or their internal spaces are not compact.

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Contents

1 Introduction

Warped AdS_5 backgrounds of 10- and 11-dimensional supergravity theories are of particular interest within the AdS/CFT correspondence as they are dual to 4-dimensional superconformal theories, see [1] for a review. The most celebrated example of such correspondence is the statement that IIB superstring theory on the maximally supersymmetric $AdS_5 \times S^5$ background is dual to $\mathcal{N} = 4$ supersymmetric gauge theory in four dimensions [2]. AdS spaces have also been used in supergravity compactifications, for a review see [3].

To establish such dualities to other 4-dimensional superconformal theories with less than maximal supersymmetry requires the construction of AdS_5 supergravity backgrounds preserving less than 32 supersymmetries. Recently it has been shown in [4–6] that AdS_5 backgrounds preserve 8, 16, 24 or 32 supersymmetries in type II 10-dimensional supergravities and in 11-dimensional supergravity.[1] The maximally supersymmetric AdS_5 backgrounds have been classified in [7] where it has been shown that no such backgrounds exist

[1]There are no supersymmetric AdS_5 backgrounds in either heterotic or type I supergravities.

in either 11-dimensional or (massive) IIA supergravities, and all maximally supersymmetric AdS_5 backgrounds in IIB supergravity are locally isometric to the previously known $AdS_5 \times S^5$ solution of the theory, see [8] and reference within. To our knowledge there is no classification of AdS_5 backgrounds preserving 16 or 24 supersymmetries. The geometry of AdS_5 solutions preserving 8 supersymmetries has been investigated in [9, 10], after assuming that the fields are invariant under the $\mathfrak{so}(4,2)$ symmetry of AdS_5 together with some additional restrictions[2] on the form of Killing spinors. Moreover, many AdS_5 solutions have been found, see for example [11]–[23]. In [4–6] a different approach to investigate the geometry of AdS backgrounds was proposed, which was based on earlier work on black hole near horizon geometries [24] which has the advantage that all additional restrictions are removed and the only assumption that remains is the requirement for the fields to be invariant under the $\mathfrak{so}(4,2)$ symmetry of AdS_5.

In this paper, we shall demonstrate, under the assumptions we describe in detail below, that there are no AdS_5 solutions in 11-dimensional and (massive) IIA supergravities that preserve 24 supersymmetries. Furthermore we shall show that all AdS_5 solutions of IIB supersgravity that preserve 24 supersymmetries are locally isometric to the maximally supersymmetric $AdS_5 \times S^5$ background.

One application of our results is in AdS/CFT and in particular on the existence of gravitational duals for strictly $\mathcal{N} = 3$ superconformal theories in four dimensions. It is known for sometime that the field content and component actions of $\mathcal{N} = 3$ and $\mathcal{N} = 4$ superconformal theories with rigid supersymmetry are the same. As a result $\mathcal{N} = 3$ superconformal symmetry classically enhances to $\mathcal{N} = 4$. Quantum mechanically, the picture is more involved as the quantization of these theories with manifest $\mathcal{N} = 3$ and $\mathcal{N} = 4$ supersymmetry will require the use of techniques like harmonic superspace [25, 26], and these are different for these two theories. Nevertheless the interpretation of the equivalence of the classical actions is that perturbatively the two quantum theories are indistinguishable.[3] Therefore if a theory exists with strictly $\mathcal{N} = 3$ superconformal symmetry, it must be intrinsically non-perturbative. The properties of such $\mathcal{N} = 3$ superconformal theories have been investigated in [28] and an F-theory construction for such a theory has been proposed in [29]. In this context our results imply that, unlike the $\mathcal{N} = 4$ superconformal theories, there are no smooth gravitational duals, with compact without boundary internal spaces, for strictly $\mathcal{N} = 3$ superconformal theories in four dimensions.

The proof of the above result utilizes the near horizon approach of [4–6] for solving the Killing spinor equations (KSEs) of supergravity theories for AdS backgrounds as well as a technique developed for the proof of the homogeneity conjecture in [30]. Furthermore, to prove our results we make certain smoothness and global assumptions. In particular apart

[2] Typically it is assumed that the Killing spinors factorize as Killing spinors on AdS and Killing spinors along the internal space. A factorization of this type has been investigated in [4–6] and it was found that it imposes more restrictions on the backgrounds than those required for invariance under the isometries of AdS. Therefore the generality of the factorization approaches must be re-investigated on a case by case basis.

[3] In fact it may be possible to prove this by demonstrating via Ward identity techniques like those in [27] that $\mathcal{N} = 3$ superconformal symmetry quantum mechanically always enhances to $\mathcal{N} = 4$. We would like to thank Paul Howe for suggesting this.

JHEP06(2016)126

from implementing the $\mathfrak{so}(4,2)$ symmetry on the fields, we also assume that the warped $AdS_5 \times_w M^{D-5}$ backgrounds, for $D=10$ or $D=11$, satisfy the following conditions:[4] (i) All the fields are smooth, and (ii) the internal space M^{D-5} is connected[5] and compact without boundary. Both these additional restrictions, apart from the connectness of M^{D-5}, can be replaced with the assertion that the data are such that the Hopf maximum principle applies. These assumptions are essential as otherwise there are for example AdS_5 backgrounds in 11-dimensions which preserve more than 16 supersymmetries, see also section 2.

This paper is organized as follows. In section 2, we prove the non-existence of AdS_5 backgrounds preserving 24 supersymmetries in 11-dimensional supergravity, and in section 3, we demonstrate the same result for both standard and massive IIA supergravities. In section 4, we show that the AdS_5 backgrounds that preserve 24 supersymmetries in IIB supergravity are locally isometric to the maximally supersymmetric background $AdS_5 \times S^5$. In section 5, we give our conclusions and explore an application to AdS/CFT. Furthermore, in appendix A we briefly summarize some of our conventions, and in appendix B for completeness we present a technique we use to derive our results which has been adapted from the proof of the homogeneity conjecture.

2 $AdS_5 \times_w M^6$ solutions in D=11

We begin by briefly summarizing the general structure of warped AdS_5 solutions in 11-dimensional supergravity, as determined in [4], whose conventions we shall follow throughout this section. Then we shall present the proof that there are no such solutions preserving 24 supersymmetries. The metric and 4-form are given by

$$\begin{aligned} ds^2 &= 2du(dr+rh) + A^2\left(dz^2 + e^{2z/\ell}\sum_{a=1}^{2}(dx^a)^2\right) + ds^2(M^6)\,, \\ F &= X\,, \end{aligned} \tag{2.1}$$

where we have written the solution as a near-horizon geometry [24], with

$$h = -\frac{2}{\ell}dz - 2A^{-1}dA\,, \tag{2.2}$$

(u, r, z, x^1, x^2) are the coordinates of the AdS_5 space, A is the warp factor which is a function on M^6, and X is a closed 4-form on M^6. A and X depend only on the coordinates of M^6, ℓ is the radius of AdS_5.

The 11-dimensional Einstein equation implies that

$$D^k\partial_k \log A = -\frac{4}{\ell^2}A^{-2} - 5\partial^k \log A\, \partial_k \log A + \frac{1}{144}X^2\,, \tag{2.3}$$

where D is the Levi-Civita connection on M^6. The remaining components of the Einstein and gauge field equations are listed in [4], however we shall only require (2.3) for the analysis

[4] We also assume the validity of the homogeneity conjecture for massive IIA supergravity. This has not been proven as yet but it is expected to hold.

[5] In fact M^{D-5} is required to be path connected but all manifolds are path connected if they are connected since they are locally path connected. From now on, we shall assume that M^{D-5} is always connected.

of the $N = 24$ solutions. In particular, (2.3) implies that A is everywhere non-vanishing on M^6, on assuming that M^6 is connected and all fields are smooth.

We adopt the following frame conventions; $\mathbf{e}^i$ is an orthonormal frame for M^6, and

$$\mathbf{e}^+ = du\,, \qquad \mathbf{e}^- = dr + rh\,, \qquad \mathbf{e}^z = A dz\,, \qquad \mathbf{e}^a = A e^{z/\ell} dx^a\,. \tag{2.4}$$

We use this frame in the investigation of KSEs below.

2.1 The Killing spinors

The Killing spinors of AdS_5 backgrounds are given by

$$\begin{aligned}\epsilon = {}& \sigma_+ - \ell^{-1}\sum_{a=1}^{2} x^a \Gamma_{az}\tau_+ + e^{-\frac{z}{\ell}}\tau_+ + \sigma_- + e^{\frac{z}{\ell}}\left(\tau_- - \ell^{-1}\sum_{a=1}^{2} x^a\Gamma_{az}\sigma_-\right)\\ & -\ell^{-1} u A^{-1}\Gamma_{+z}\sigma_- - \ell^{-1} r A^{-1} e^{-\frac{z}{\ell}}\Gamma_{-z}\tau_+\,,\end{aligned} \tag{2.5}$$

where we have used the light-cone projections

$$\Gamma_\pm \sigma_\pm = 0\,, \quad \Gamma_\pm\tau_\pm = 0\,, \tag{2.6}$$

and $\sigma_\pm$ and $\tau_\pm$ are 16-component spinors that depend only on the coordinates of M^6. We do not assume that the Killing spinors factorize as Killing spinors on AdS_5 and Killing spinors on M^6.

The remaining independent Killing spinor equations (KSEs) are:

$$D_i^{(\pm)}\sigma_\pm = 0\,, \quad D_i^{(\pm)}\tau_\pm = 0\,, \tag{2.7}$$

and

$$\Xi^{(\pm)}\sigma_\pm = 0\,, \quad \Xi^{(\mp)}\tau_\pm = 0\,, \tag{2.8}$$

where

$$\begin{aligned} D_i^{(\pm)} &= D_i \pm \frac{1}{2}\partial_i \log A - \frac{1}{288}\Gamma\!\!\not{X}_i + \frac{1}{36}\not{X}_i\,,\\ \Xi^{(\pm)} &= -\frac{1}{2}\Gamma_z\Gamma^i\partial_i \log A \mp \frac{1}{2\ell}A^{-1} + \frac{1}{288}\Gamma_z\not{X}\,. \end{aligned} \tag{2.9}$$

In particular algebraic KSEs (2.8) imply that σ_+ and τ_+ cannot be linearly dependent. For our Clifford algebra conventions see also appendix A.

2.2 Counting the Killing spinors

In order to count the number of supersymmetries, note that if σ_+ is a solution of the σ_+ KSEs, then so is $\Gamma_{12}\sigma_+$. Furthermore, $\tau_+ = \Gamma_z\Gamma_1\sigma_+$ and $\tau_+ = \Gamma_z\Gamma_2\sigma_+$ are solutions to the τ_+ KSEs. The spinors $\sigma_+, \Gamma_{12}\sigma_+, \Gamma_z\Gamma_1\sigma_+, \Gamma_z\Gamma_2\sigma_+$ are linearly independent. The positive chirality spinors also generate negative chirality spinors σ_-, τ_- which satisfy the appropriate KSEs. This is because if σ_+, τ_+ is a solution, then so is

$$\sigma_- = A\Gamma_-\Gamma_z\sigma_+\,, \quad \tau_- = A\Gamma_-\Gamma_z\tau_+\,, \tag{2.10}$$

and also conversely, if σ_-, τ_- is a solution, then so is

$$\sigma_+ = A^{-1}\Gamma_+\Gamma_z\sigma_- \,, \quad \tau_+ = A^{-1}\Gamma_+\Gamma_z\tau_- \,. \tag{2.11}$$

So for a generic $AdS_5 \times_w M^6$ solution, all of the Killing spinors are generated by the σ_+ spinors, each of which gives rise to 8 linearly independent spinors via the mechanism described here. The solutions therefore preserve $8k$ supersymmetries, where k is equal to the number of σ_+ spinors.

2.3 Non-existence of $N = 24$ AdS_5 solutions in D=11

To consider the AdS_5 solutions preserving 24 supersymmetries, we begin by setting

$$\Lambda = \sigma_+ + \tau_+ \,, \tag{2.12}$$

and defining

$$W_i = A\langle\Lambda, \Gamma_{z12}\Gamma_i\Lambda\rangle \,. \tag{2.13}$$

Then (2.7) implies that

$$D_{(i}W_{j)} = 0 \,, \tag{2.14}$$

so W is an isometry of M^6. In addition, the algebraic conditions (2.8) imply that

$$\frac{1}{288}\langle\Lambda, \Gamma \not{X}_i\Lambda\rangle - \frac{1}{2} \parallel \Lambda \parallel^2 A^{-1}D_iA - \ell^{-1}A^{-1}\langle\tau_+, \Gamma_i\Gamma_z\sigma_+\rangle = 0 \,. \tag{2.15}$$

Also, (2.7) implies that

$$D_i \parallel \Lambda \parallel^2 = - \parallel \Lambda \parallel^2 A^{-1}D_iA + \frac{1}{144}\langle\Lambda, \Gamma \not{X}_i\Lambda\rangle \,. \tag{2.16}$$

Combining (2.15), and (2.16) we have

$$D_i \parallel \Lambda \parallel^2 - 2\ell^{-1}A^{-1}\langle\tau_+, \Gamma_i\Gamma_z\sigma_+\rangle = 0 \,. \tag{2.17}$$

In addition (2.7) implies that

$$D^i\Big(A\langle\tau_+, \Gamma_i\Gamma_z\sigma_+\rangle\Big) = 0 \,. \tag{2.18}$$

Hence, on taking the divergence of (2.17), we find

$$D^iD_i \parallel \Lambda \parallel^2 + 2A^{-1}D^iAD_i \parallel \Lambda \parallel^2 = 0 \,. \tag{2.19}$$

A maximum principle argument then implies that $\parallel \Lambda \parallel^2$ is constant. Substituting these conditions back into (2.16), we find the condition

$$i_WH = 6 \parallel \Lambda \parallel^2 dA \,, \tag{2.20}$$

where

$$H = \star_6 X\,, \tag{2.21}$$

and $\star_6$ denotes the Hodge dual on M^6.

To prove a non-existence theorem for $N = 24$ solutions, we consider spinors of the type

$$\Lambda = \sigma_+ + \tau_+\ . \tag{2.22}$$

For a $N = 24$ solution, there are 12 linearly independent spinors of this type, because of the algebraic conditions (2.8). Next, consider the condition (2.20). This implies that

$$i_W dA = 0\,, \tag{2.23}$$

where W is the isometry generated by Λ as defined in (2.13).

A straightforward modification of the reasoning used in [30], which we describe in appendix B, implies that for $N = 24$ solutions, the vector fields dual to the 1-form bilinears W generated by the Λ spinors span the tangent space of M^6. Then the condition $i_W dA = 0$ implies that A is constant, and furthermore, (2.20) implies that $i_W H = 0$, which also implies that $H = 0$, and so $X = 0$.

However, the Einstein equation (2.3) admits no AdS_5 solutions for which $dA = 0$ and $X = 0$, so there can be no $N = 24$ AdS_5 solutions.

We should remark that the two assumptions we have made on the fields to derive this result are essential. This is because any AdS_{d+1} background can locally be written as a warped product $ds^2(AdS_{d+1}) = dy^2 + A^2(y)ds^2(AdS_d)$ for some function A which has been determined in e.g. [31]. For $d = 2$, this has previously been established in [32]. As a result the maximally supersymmetric $AdS_7 \times S^4$ solution of 11-dimensional supergravity can be seen as a warped AdS_5 background. This appears to be a contradiction to our result. However, the internal space M^6 in this case is non-compact and so it does not satisfy the two assumptions we have made.

3 $AdS_5 \times_w M^5$ solutions in (massive) IIA supergravity

As in the 11-dimensional supergravity investigated in the previous sections, there are no $N = 24$ AdS_5 backgrounds in (massive) IIA supergravity. We shall use the formalism and follow the conventions of [6] in the analysis that follows. Imposing invariance of the background under the symmetries of AdS_5 all the fluxes are magnetic, ie their components along AdS_5 vanish. In particular the most general AdS_5 background is

$$\begin{aligned} ds^2 &= 2du(dr + rh) + A^2\left(dz^2 + e^{2z/\ell}\sum_{a=1}^{2}(dx^a)^2\right) + ds^2(M^5)\,, \\ G &= G\,, \quad H = H\,, \quad F = F\,, \quad \Phi = \Phi\,, \quad S = S\,, \quad h = -\frac{2}{\ell}dz - 2A^{-1}dA\,, \end{aligned} \tag{3.1}$$

where we have denoted the 10-dimensional fluxes and their components along M^5 with the same symbol, A is the warp factor, Φ is the dilaton and S is the cosmological constant dressed with the dilaton. A, S and Φ are functions of M^5, while G , H and F are the 4-form, 3-form and a 2-form fluxes, respectively, which have support only on M^5. The coordinates of AdS_5 are (u, r, z, x^a) and we introduce the frame $(\mathbf{e}^+, \mathbf{e}^-, \mathbf{e}^z, \mathbf{e}^a)$ as in (2.4).

The fields satisfy a number of field equations and Bianchi identities which can be found in [6]. Those relevant for the analysis that follows are the field equation for the dilaton and the field equation for G

$$
\begin{aligned}
D^2\Phi &= -5A^{-1}\partial^i A\partial_i\Phi + 2(d\Phi)^2 + \frac{5}{4}S^2 + \frac{3}{8}F^2 - \frac{1}{12}H^2 + \frac{1}{96}G^2\,, && (3.2)\\
\nabla^\ell G_{ijk\ell} &= -5A^{-1}\partial^\ell A G_{ijk\ell} + \partial^\ell\Phi G_{ijk\ell}\,, && (3.3)
\end{aligned}
$$

respectively, and the Einstein equations both along AdS_5 and M^5

$$
\begin{aligned}
D^2\ln A &= -4\ell^{-2}A^{-2} - 5A^{-2}(dA)^2 + 2A^{-1}\partial_i A\partial^i\Phi + \frac{1}{96}G^2 + \frac{1}{4}S^2 + \frac{1}{8}F^2, && (3.4)\\
R^{(5)}_{ij} &= 5\nabla_i\nabla_j\ln A + 5A^{-2}\partial_i A\partial_j A + \frac{1}{12}G^2_{ij} - \frac{1}{96}G^2\delta_{ij} && (3.5)\\
&\quad - \frac{1}{4}S^2\delta_{ij} + \frac{1}{4}H^2_{ij} + \frac{1}{2}F^2_{ij} - \frac{1}{8}F^2\delta_{ij} - 2\nabla_i\nabla_j\Phi\,,
\end{aligned}
$$

respectively, where D is the Levi-Civita connection of M^5 and $R^{(5)}_{ij}$ is the Ricci tensor of M^5. The former is seen as the field equation for the warp factor A.

3.1 Killing spinor equations

The killing spinors of IIA AdS_5 backgrounds are given as in (2.5) where now $\sigma_\pm$ and $\tau_\pm$ are 16-component spinors that depend only on the coordinates of M^5. The remaining independent conditions are the gravitino KSEs

$$
\nabla^{(\pm)}_i\sigma_\pm = 0\,, \quad \nabla^{(\pm)}_i\tau_\pm = 0\,, \tag{3.6}
$$

the dilatino KSEs

$$
\mathcal{A}^{(\pm)}\sigma_\pm = 0\,, \quad \mathcal{A}^{(\pm)}\tau_\pm = 0\,, \tag{3.7}
$$

and the algebraic KSEs

$$
\Xi_\pm\sigma_\pm = 0\,, \qquad \Xi_\pm\tau_\pm = \mp\ell^{-1}\tau_\pm\,, \tag{3.8}
$$

where

$$
\begin{aligned}
\nabla^{(\pm)}_i &= D_i + \Psi^{(\pm)}_i\,,\\
\mathcal{A}^{(\pm)} &= \partial\!\!\!/\Phi + \frac{1}{12}H\!\!\!/\Gamma_{11} + \frac{5}{4}S + \frac{3}{8}F\!\!\!/\Gamma_{11} + \frac{1}{96}G\!\!\!/\,,\\
\Xi_\pm &= \mp\frac{1}{2\ell} + \frac{1}{2}\partial\!\!\!/A\Gamma_z - \frac{1}{8}AS\Gamma_z - \frac{1}{16}AF\!\!\!/\Gamma_z\Gamma_{11} - \frac{1}{192}AG\!\!\!/\Gamma_z\,, && (3.9)
\end{aligned}
$$

and where D is the spin connection on M^5 and

$$
\Psi^{(\pm)}_i = \pm\frac{1}{2A}\partial_i A + \frac{1}{8}H\!\!\!/_i\Gamma_{11} + \frac{1}{8}S\Gamma_i + \frac{1}{16}F\!\!\!/\Gamma_i\Gamma_{11} + \frac{1}{192}G\!\!\!/\Gamma_i\,, \tag{3.10}
$$

see appendix A for our Clifford algebra conventions. The counting of supersymmetries is exactly the same as in the D=11 supergravity described in the previous sections.

JHEP06(2016)126

3.2 $N = 24$ AdS_5 solutions in (massive) IIA supergravity

Before we proceed with the analysis, the homogeneity conjecture[6] [30] together with the results [33, 34] on the classification of (massive) IIA backgrounds imply that both Φ and S are constant functions over the whole spacetime which we shall assume from now on. Next let us set

$$\Lambda = \sigma_+ + \tau_+ \,, \tag{3.11}$$

and define

$$W_i = A\langle \Lambda, \Gamma_{zxy}\Gamma_i \Lambda\rangle \ . \tag{3.12}$$

Then (3.6) implies that

$$D_{(i}W_{j)} = 0 \,, \tag{3.13}$$

so W is an isometry of M^5.

After some straightforward computation using the gravitino KSEs, one finds

$$D_i \parallel \Lambda \parallel^2 = -A^{-1}\partial_i A \parallel \Lambda \parallel^2 - \frac{1}{4} S\langle \Lambda, \Gamma_i \Lambda\rangle - \frac{1}{8}\langle \Lambda, \Gamma\!\!\!/ F_i \Gamma_{11}\Lambda\rangle - \frac{1}{96}\langle \Lambda, \Gamma\!\!\!/ G_i \Lambda\rangle \ . \tag{3.14}$$

On the other hand (3.8) gives

$$\left(\partial\!\!\!/ A \Gamma_z - \frac{1}{4} A S \Gamma_z - \frac{1}{8} A F\!\!\!/ \Gamma_z \Gamma_{11} - \frac{1}{96} A G\!\!\!/ \Gamma_z\right)\Lambda = -\ell^{-1}\tau_+ + \ell^{-1}\sigma_+ \ . \tag{3.15}$$

Using this, (3.14) can be written as

$$D_i \parallel \Lambda \parallel^2 = 2\ell^{-1}A^{-1}\langle \tau_+, \Gamma_i \Gamma_z \sigma_+\rangle \ . \tag{3.16}$$

Furthermore using (3.6), one can show that

$$D^i\left(A\langle \tau_+, \Gamma_i \Gamma_z \sigma_+\rangle\right) = 0 \ . \tag{3.17}$$

Taking the covariant derivative of (3.16) and using the above equation, one finds that

$$D^i D_i \parallel \Lambda \parallel^2 + 2A^{-1} D^i A D_i \parallel \Lambda \parallel^2 = 0 \ . \tag{3.18}$$

This in turn implies after using the maximum principle that $\parallel \Lambda \parallel^2$ is constant.

Using the constancy of $\parallel \Lambda \parallel^2$, (3.14) and (3.16) imply that

$$-A^{-1}\partial_i A \parallel \Lambda \parallel^2 - \frac{1}{4} S\langle \Lambda, \Gamma_i \Lambda\rangle - \frac{1}{8}\langle \Lambda, \Gamma\!\!\!/ F_i \Gamma_{11}\Lambda\rangle - \frac{1}{96}\langle \Lambda, \Gamma\!\!\!/ G_i \Lambda\rangle = 0 \,, \tag{3.19}$$

and

$$\langle \tau_+, \Gamma_i \Gamma_z \sigma_+\rangle = 0 \ . \tag{3.20}$$

[6] Strictly speaking the homogeneity conjecture has not been proven for massive IIA supergravity, but it is expected to hold.

JHEP06(2016)126

Next taking the difference of the two identities below

$$\langle \tau_+, \Xi_+ \sigma_+ \rangle = 0\,, \quad \langle \sigma_+, (\Xi_+ + \ell^{-1}\tau_+ \rangle = 0\,, \tag{3.21}$$

and upon using (3.20), we find

$$\langle \tau_+, \sigma_+ \rangle = 0\,, \tag{3.22}$$

ie τ_+ and σ_+ are orthogonal.

To continue, multiply $\Xi_+\Lambda = -\ell^{-1}\tau_+$ with Γ_{xy}, and using the fact $\Gamma_{xy}\tau_+$ is again a type τ_+ Killing spinor, and the equation above, one obtains that

$$W^i \partial_i A = 0\ . \tag{3.23}$$

As straightforward modification of the argument used in [30] to prove the homogeneity conjecture, see also appendix B, one can show that the vector fields W span the tangent spaces of M^5. As a result, the above equation implies that A is constant.

Next using the dilatino KSE (3.7) to eliminate the G-dependent term in (3.19) and that $A = \text{const}$, one finds

$$4S\langle \Lambda, \Gamma_i \Lambda \rangle + \langle \Lambda, \Gamma \not{F}_i \Gamma_{11} \Lambda \rangle + \frac{1}{3} \langle \Lambda, \Gamma \not{H}_i \Gamma_{11} \Lambda \rangle = 0\ . \tag{3.24}$$

In what follows, we shall investigate the standard and massive IIA supergravities separately.

3.2.1 Standard IIA supergravity with $S = 0$

In the case for which $S = 0$, the dilatino KSEs (3.7) imply that

$$\langle \Lambda, \not{G} \Gamma_{11} \Lambda \rangle = 0\,, \tag{3.25}$$

or equivalently, $W \wedge G = 0$. As the W span the tangent space of M^5, it follows that $G = 0$. Then, using the dilatino KSE (3.7) to eliminate the F terms from (3.24), we obtain

$$\langle \Lambda, \Gamma \not{H}_i \Gamma_{11} \Lambda \rangle = 0\,, \tag{3.26}$$

which implies that $W \wedge H = 0$. As the W span the tangent space of M^5, it follows that $H = 0$ also. The dilaton field equation (3.3) then implies that $F = 0$ as well. However, for $S = 0$, $G = 0$, $H = 0$ and $F = 0$, the the warp factor field equation (3.4) becomes inconsistent, and so there are no AdS_5 solutions in standard IIA supergravity that preserve 24 supersymmetries.

3.2.2 Massive IIA supergravity with $S \neq 0$

On writing $G = \star_5 X$, where X is a 1-form on M^5, the condition

$$\frac{5}{4} S\langle \Lambda, \Gamma_{11} \Lambda \rangle + \frac{1}{96} \langle \Lambda, \not{G} \Gamma_{11} \Lambda \rangle = 0\,, \tag{3.27}$$

which is derived from the dilatino KSE (3.7), can be rewritten as

$$\frac{5}{4} S\langle \Lambda, \Gamma_{11} \Lambda \rangle - \frac{1}{4} A^{-1} i_W X = 0\ . \tag{3.28}$$

Furthermore, the G field equation implies that $dX = 0$, and we assume[7] that $\mathcal{L}_W G = 0$ which implies $\mathcal{L}_W X = 0$. This condition, together with $dX = 0$, gives that $i_W X$ is constant. Then it follows from (3.28) that $\langle \Lambda, \Gamma_{11} \Lambda \rangle$ is also constant.

On differentiating the condition $\langle \Lambda, \Gamma_{11} \Lambda \rangle = \text{const}$ using the gravitino KSEs, we obtain the condition

$$-\frac{1}{4} F_{ij} \langle \Lambda, \Gamma^j \Lambda \rangle + \frac{1}{24} \langle \Lambda, \Gamma_{11} \not{G}_i \Lambda \rangle = 0\,, \tag{3.29}$$

and hence

$$X^i F_{ij} \langle \Lambda, \Gamma^j \Lambda \rangle = 0\ . \tag{3.30}$$

However, using an argument directly analogous to that used to show that the vector fields W span the tangent space of M^5, it follows that the vectors $\langle \Lambda, \Gamma^j \Lambda \rangle \partial_j$ also span the tangent space of M^5, see appendix B. Therefore,

$$i_X F = 0\ . \tag{3.31}$$

Next, act on the right-hand-side of the dilatino equation (3.7) with $\not{X} \Gamma_{11}$ and take the inner product with Λ. On making use of $i_X F = 0$, we find the condition

$$\langle \Lambda, X_{\ell_1} H_{\ell_2 \ell_3 \ell_4} \Gamma^{\ell_1 \ell_2 \ell_3 \ell_4} \Lambda \rangle = 0\,, \tag{3.32}$$

and hence

$$\langle \Lambda, \Gamma_{11} \Gamma_{xyz} \Gamma_q \Lambda \rangle \epsilon^{q \ell_1 \ell_2 \ell_3 \ell_4} X_{\ell_1} H_{\ell_2 \ell_3 \ell_4} = 0\ . \tag{3.33}$$

Again, as the vectors $\langle \Lambda, \Gamma_{11} \Gamma_{xyz} \Gamma^j \Lambda \rangle \partial_j$ span the tangent space of M^5, this condition implies that

$$X \wedge H = 0\ . \tag{3.34}$$

Another useful condition is to note that $\mathcal{L}_W X = 0$ implies that

$$\mathcal{L}_W (D^i X_i) = 0\,, \tag{3.35}$$

and as the W span the tangent space of M^5, it follows that $D^i X_i$ must be constant. However the integral of $D^i X_i$ over M^5 vanishes, and hence it follows that

$$D^i X_i = 0\,, \tag{3.36}$$

ie X is co-closed. As it is also closed, X and so G are harmonic. This condition, together with $dX = 0$, imply that one can write

$$D^2 X^2 = 2 D^i X^j D_i X_j + 2 X^j (D_i D_j - D_j D_i) X^i = 2 D^i X^j D_i X_j + 2 X^i X^j R^{(5)}_{ij}\ . \tag{3.37}$$

[7] The invariance of G under the vector fields constructed as Killing spinor bilinears has not been proven for massive IIA in complete generality, but it is expected to hold.

On using the Einstein equation (3.5), together with the conditions $i_X F = 0$, $X \wedge H = 0$, we find

$$D^2 X^2 = 2D^i X^j D_i X_j + X^2 \Big(-\frac{1}{48} G^2 - \frac{1}{2} S^2 - \frac{1}{4} F^2 + \frac{1}{6} H^2 \Big) \,, \tag{3.38}$$

which can be written as

$$D^2 X^2 = 2D^i X^j D_i X_j + X^2 \Big(2S^2 + \frac{3}{2} F^2 \Big) \,, \tag{3.39}$$

on using the dilaton equation (3.3) to eliminate the G^2 term. As the right-hand-side of this expression is a sum of non-negative terms, an application of the maximum principle implies that X^2 is constant[8] and

$$X^2 S^2 = 0 \ . \tag{3.40}$$

As $S \neq 0$, it follows that $X^2 = 0$, and hence $G = 0$. Then (3.27) implies that

$$\langle \Lambda, \Gamma_{11} \Lambda \rangle = 0 \,, \tag{3.41}$$

for all Killing spinors Λ. However, this is a contradiction.

To see this, let the 12-dimensional vector space spanned by the Killing spinors Λ be denoted by K. Then the above condition implies that

$$\langle \Lambda_1, \Gamma_{11} \Lambda_2 \rangle = 0 \,, \tag{3.42}$$

for all $\Lambda_1, \Lambda_2 \in K$. Denoting

$$\Gamma_{11} K = \{ \Gamma_{11} \Lambda : \Lambda \in K \} \,, \tag{3.43}$$

the condition (3.42) implies that $\Gamma_{11} K \subseteq K^\perp$, where

$$K^\perp = \{ \Psi : \langle \Psi, \Lambda \rangle = 0 \text{ for all } \Lambda \in K \} \ . \tag{3.44}$$

The dimension of space of all Majorana Spin$(9,1)$ spinors ζ satisfying the lightcone projection $\Gamma_+ \zeta = 0$ is 16. As K has dimension 12, $K^\perp$ has dimension 4. As $\Gamma_{11} K$ is 12-dimensional it cannot be included in $K^\perp$ as required by the assumption (3.41). Therefore there are no AdS_5 solutions in massive IIA supergravity which preserve 24 supersymmetries.

We would like to remark that the proof of this result is considerably simpler if M^5 is simply connected. As has already been proven, G is harmonic. On a simply connected M^5, G vanishes. In such a case, (3.27) again implies (3.41). Then the non-existence of such AdS_5 backgrounds follows from the argument produced above that (3.41) cannot hold for all Killing spinors.

[8]The condition $X^2 = \text{const}$ also follows from $\mathcal{L}_W X^2 = 0$ together with homogeneity.

JHEP06(2016)126

4 $AdS_5 \times_w M^5$ solutions in IIB supergravity

The active fields of $AdS_5 \times_w M^5$ IIB backgrounds as well as the relevant field and KSEs have been determined in [5]. In particular, in the conventions of [5], the metric and other form field strengths are

$$\begin{aligned} ds^2 &= 2du(dr + rh) + A^2 \left(dz^2 + e^{2z/\ell} \sum_{a=1}^{2} (dx^a)^2 \right) + ds^2(M^5)\,, \\ G &= H, \quad P = \xi, \quad F = Y \left(A^3 e^{\frac{2z}{\ell}} du \wedge (dr + rh) \wedge dz \wedge dx \wedge dy - \mathrm{dvol}(M^5) \right), \end{aligned} \tag{4.1}$$

where again we have written the background as a near-horizon geometry [24], with

$$h = -\frac{2}{\ell} dz - 2A^{-1} dA\,, \tag{4.2}$$

A is the warp factor which is a smooth function on M^5, G is the complex 3-form, P encodes the (complexified) axion/dilaton gradients, F is the real self-dual 5-form and Y is a real scalar. The AdS_5 coordinates are (u, r, z, x^a) and we introduce the frame $(\mathbf{e}^+, \mathbf{e}^-, \mathbf{e}^z, \mathbf{e}^a)$ as in (2.4).

For the analysis that follows, we shall use the Bianchi identities

$$d(A^5 Y) = 0, \qquad dH = iQ \wedge H - \xi \wedge \bar{H}\,, \tag{4.3}$$

and the 10-dimensional Einstein equation along AdS_5 which gives the field equation

$$A^{-1} \nabla^2 A = 4Y^2 + \frac{1}{48} \parallel H \parallel^2 - \frac{4}{\ell^2} A^{-2} - 4A^{-2} (dA)^2\,, \tag{4.4}$$

for the warp factor A. The remaining Bianchi identities and bosonic field equations, which are not necessary for the investigation of $N = 24$ solutions, can be found in [5]. We also assume the same regularity assumptions as for the eleven dimensional solutions, and remark that (4.4) implies that A is nowhere vanishing on M^5.

4.1 The Killing spinors

Solving the KSEs of IIB supergravity for $AdS_5 \times_w M^5$ backgrounds along AdS_5, one finds that the Killing spinors can be written as in (2.5), where now $\sigma_\pm$ and $\tau_\pm$ are Weyl Spin$(9,1)$ spinors which depend only on the coordinates of M^5 that obey in addition the lightcone projections $\Gamma_\pm \sigma_\pm = \Gamma_\pm \tau_\pm = 0$.

The remaining independent KSEs are the gravitino parallel transport equations

$$D_i^{(\pm)} \sigma_\pm = 0, \qquad D_i^{(\pm)} \tau_\pm = 0\,, \tag{4.5}$$

where

$$D_i^{(\pm)} = D_i \pm \frac{1}{2} \partial_i \log A - \frac{i}{2} Q_i \pm \frac{i}{2} Y \Gamma_i \Gamma_{xyz} + \left(-\frac{1}{96} \Gamma \not{H}_i + \frac{3}{32} \not{H}_i \right) C*\,, \tag{4.6}$$

together with the dilatino KSEs

$$\left(\frac{1}{24}\not{H} + \not{\xi}C*\right)\sigma_\pm = 0, \qquad \left(\frac{1}{24}\not{H} + \not{\xi}C*\right)\tau_\pm = 0\,, \tag{4.7}$$

and some additional algebraic conditions which arise from the integration of the KSEs along the AdS_5 subspace

$$\Xi^{(\pm)}\sigma_\pm = 0, \qquad \left(\Xi^{(\pm)} \pm \ell^{-1}\right)\tau_\pm = 0\,, \tag{4.8}$$

where

$$\Xi^{(\pm)} = \mp\frac{1}{2\ell} - \frac{1}{2}\Gamma_z\not{\partial}A \pm \frac{i}{2}AY\Gamma_{xy} + \frac{1}{96}A\Gamma_z\not{H}C*\,, \tag{4.9}$$

and C is the charge conjugation matrix. Again, we have not made any assumptions on the form of the Killing spinors.

The counting of the Killing spinors, and the way in which one can construct the $\sigma_\pm$, $\tau_\pm$ spinors from each other proceeds in exactly in the same way as for the $D = 11$ AdS_5 solutions. So, again, for a generic $AdS_5 \times_w M^5$ solution, all of the Killing spinors are generated by the σ_+ spinors, each of which gives rise to 8 linearly independent spinors. The solutions therefore preserve $8k$ supersymmetries, where k is equal to the number of σ_+ spinors.

4.2 $N = 24$ AdS_5 solutions in IIB

To proceed with the analysis first note that as a consequence of the homogeneity conjecture proven in [30] is that the solutions with 24 supersymmetries must be locally homogeneous, with

$$\xi = 0\,. \tag{4.10}$$

Then, we set

$$\Lambda = \sigma_+ + \tau_+\,, \tag{4.11}$$

and define

$$W_i = A\langle\Lambda, \Gamma_{zxy}\Gamma_i\Lambda\rangle\,. \tag{4.12}$$

Then (4.5) implies that

$$D_{(i}W_{j)} = 0\,, \tag{4.13}$$

so W is an isometry of M^5. Next, using (4.5), we find

$$D_i \parallel \Lambda \parallel^2 = - \parallel \Lambda \parallel^2 A^{-1}D_iA + \frac{1}{48}\mathrm{Re}\langle\Lambda, \Gamma\not{H}_iC*\Lambda\rangle\,. \tag{4.14}$$

Furthermore, the algebraic condition (4.8) implies that

$$\frac{1}{48}\not{H}C*\Lambda = \left(A^{-1}\Gamma^jD_jA - iY\Gamma_{xyz}\right)\Lambda + \ell^{-1}A^{-1}\Gamma_z\left(\sigma_+ - \tau_+\right)\,. \tag{4.15}$$

On substituting this condition back into (4.14) we find

$$D_i \parallel \Lambda \parallel^2 = 2\ell^{-1} A^{-1} \mathrm{Re}\langle \tau_+, \Gamma_i \Gamma_z \sigma_+ \rangle \ . \tag{4.16}$$

However, (4.5) also implies that

$$D^i \left(A \mathrm{Re}\langle \tau_+, \Gamma_i \Gamma_z \sigma_+ \rangle \right) = 0 \ . \tag{4.17}$$

So combining this condition with (4.16), we find

$$D^i D_i \parallel \Lambda \parallel^2 + 2A^{-1} D^i A D_i \parallel \Lambda \parallel^2 = 0 \ . \tag{4.18}$$

A maximum principle argument then implies that $\parallel \Lambda \parallel^2$ is constant. Then (4.14) and (4.16) imply

$$- \parallel \Lambda \parallel^2 A^{-1} D_i A + \frac{1}{48} \mathrm{Re}\langle \Lambda, \Gamma \not{H}_i C * \Lambda \rangle = 0 \,, \tag{4.19}$$

or, equivalently

$$\mathrm{Re}\langle \tau_+, \Gamma_i \Gamma_z \sigma_+ \rangle = 0 \ . \tag{4.20}$$

Next, we shall show that the spinors σ_+, τ_+ are orthogonal with respect to the inner product Re $<,>$. To see this, note that (4.8) implies that

$$\langle \tau_+, \Xi^{(+)} \sigma_+ \rangle = 0, \qquad \langle \sigma_+, (\Xi^{(+)} + \ell^{-1}) \tau_+ \rangle = 0 \ . \tag{4.21}$$

On expanding out, and subtracting these two identities, one finds that the real and imaginary parts of the resulting expression imply

$$\ell^{-1} \mathrm{Re}\langle \tau_+, \sigma_+ \rangle + \mathrm{Re}\langle \tau_+, \Gamma_z \Gamma^i D_i A \sigma_+ \rangle = 0 \,, \tag{4.22}$$

and

$$Y \mathrm{Re}\langle \tau_+, \Gamma_{xy} \sigma_+ \rangle = 0 \,, \tag{4.23}$$

respectively. On substituting (4.20) into (4.22), we find that

$$\mathrm{Re}\langle \tau_+, \sigma_+ \rangle = 0 \ . \tag{4.24}$$

For $N = 24$ solutions there are 6 linearly independent σ_+ spinors, and 6 linearly independent τ_+ spinors, hence the spinors of the type $\Lambda = \sigma_+ + \tau_+$ span a 12 dimensional vector space over $\mathbb{R}$, which we shall denote by K.

It is also particularly useful to note that the algebraic condition (4.8) implies

$$\begin{aligned} &\frac{1}{2\ell} \langle \Lambda, \Gamma_{xy} (\tau_+ - \sigma_+) \rangle - \frac{1}{2} \langle \Lambda, \Gamma_{xyz} \Gamma^i D_i A \Lambda \rangle \\ &\qquad - \frac{i}{2} A Y \parallel \Lambda \parallel^2 + \frac{A}{96} \langle \Lambda, \Gamma_{xyz} \not{H} C * \Lambda \rangle = 0 \ . \end{aligned} \tag{4.25}$$

On taking the real part of this expression, one finds

$$W^i D_i A = 0\,, \tag{4.26}$$

where we have used the identity $\langle \Lambda, \Gamma_{xyz}\Gamma_{ijk} C * \Lambda\rangle = 0$.

The condition (4.26) implies that

$$dA = 0\ . \tag{4.27}$$

This is because, by a straightforward adaptation of the analysis in [30], it follows that the isometries W generated by the spinors $\Lambda \in K$ span the tangent space of M^5, see also appendix B. So A is constant, and the condition (4.19) implies that

$$\mathrm{Re}\langle \Lambda, \Gamma \not{H}_i C * \Lambda\rangle = 0\ . \tag{4.28}$$

To proceed further, take the divergence of this expression. On making use of the Bianchi identity for H given in (4.3), together with the KSE (4.5), we find the following condition:

$$\mathrm{Re}\Big\langle \Lambda, \Big(\frac{9}{8} H_{\ell_1\ell_2 i}\bar{H}_{\ell_3\ell_4}{}^{i}\Gamma^{\ell_1\ell_2\ell_3\ell_4} - \frac{3}{4} H_{\ell_1 mn}\bar{H}_{\ell_2}{}^{mn}\Gamma^{\ell_1\ell_2} + \frac{1}{4} H_{\ell_1\ell_2\ell_3}\bar{H}^{\ell_1\ell_2\ell_3}\Big)\Lambda\Big\rangle = 0\,, \tag{4.29}$$

where $\bar{H}$ is the complex conjugate of H. Furthermore, the algebraic condition (4.7) implies that

$$\mathrm{Re}\langle \Lambda, \frac{1}{24}\bar{\not{H}}\not{H}\Lambda\rangle = 0\ . \tag{4.30}$$

On expanding this expression out, and adding it to (4.29), one obtains the condition

$$\mathrm{Re}\langle \Lambda, H_{\ell_1\ell_2 i}\bar{H}_{\ell_3\ell_4}{}^{i}\Gamma^{\ell_1\ell_2\ell_3\ell_4}\Lambda\rangle = 0\,, \tag{4.31}$$

or equivalently

$$W^i \epsilon_i{}^{\ell_1\ell_2\ell_3\ell_4} H_{\ell_1\ell_2 j}\bar{H}_{\ell_3\ell_4}{}^{j} = 0\ . \tag{4.32}$$

Again, as the W isometries span the tangent space of M^5, one obtains

$$H_{[\ell_1\ell_2|i|}\bar{H}_{\ell_3\ell_4]}{}^{i} = 0\ . \tag{4.33}$$

Furthermore, on substituting this condition back into

$$\langle C * \Lambda, \bar{\not{H}}\not{H}\Lambda\rangle = 0\,, \tag{4.34}$$

which follows from (4.7), we find

$$\langle C * \Lambda, \Lambda\rangle \parallel H \parallel^2 = 0\ . \tag{4.35}$$

So either $H = 0$, or $\langle C * \Lambda, \Lambda\rangle = 0$ for all $\Lambda \in K$. We shall prove that $\langle C * \Lambda, \Lambda\rangle = 0$ cannot be satisfied for all Λ.

JHEP06(2016)126

Indeed, suppose that $\langle C * \Lambda, \Lambda \rangle = 0$ for all $\Lambda \in K$. We remark that $\langle C * \Lambda_1, \Lambda_2 \rangle$ is symmetric in Λ_1, Λ_2, and so $\langle C * \Lambda, \Lambda \rangle = 0$ for all $\Lambda \in K$ implies that

$$\langle C * \Lambda_1, \Lambda_2 \rangle = 0\,, \tag{4.36}$$

for all $\Lambda_1, \Lambda_2 \in K$. If we define

$$\bar{K} = \{C * \Lambda : \Lambda \in K\}, \qquad K^\perp = \{\Psi : \mathrm{Re}\langle \Psi, \Lambda \rangle = 0 \text{ for all } \Lambda \in K\}\,, \tag{4.37}$$

then the condition (4.36) implies that $\bar{K} \subset K^\perp$. However, this is not possible, because $\bar{K}$ is 12 dimensional, whereas $K^\perp$ is 4-dimensional. So, one cannot have $\langle C * \Lambda, \Lambda \rangle = 0$ for all $\Lambda \in K$.

It follows that

$$H = 0 \tag{4.38}$$

and hence the spinors Λ satisfy

$$D_i \Lambda = \left(\frac{i}{2} Q_i - \frac{i}{2} Y \Gamma_i \Gamma_{xyz} \right) \Lambda\,, \tag{4.39}$$

for constant Y, $Y \neq 0$, with

$$Y^2 = \frac{1}{\ell^2 A^2}\,, \tag{4.40}$$

as a consequence of (4.4). The integrability condition of (4.39) implies that

$$\Big(R_{ijmn} - Y^2 (\delta_{im}\delta_{jn} - \delta_{in}\delta_{jm}) \Big) \Gamma^{mn} \Lambda = 0\,, \tag{4.41}$$

where we have used the Bianchi identity $dQ = 0$. Then (4.41) gives that

$$\mathrm{Re}\Big\langle \Lambda, \Gamma_{xyz} \Big(R_{ijmn} - Y^2 (\delta_{im}\delta_{jn} - \delta_{in}\delta_{jm}) \Big) \Gamma^n \Lambda \Big\rangle = 0\,, \tag{4.42}$$

or equivalently

$$W^n \Big(R_{ijmn} - Y^2 (\delta_{im}\delta_{jn} - \delta_{in}\delta_{jm}) \Big) = 0\ . \tag{4.43}$$

As the isometries W span the tangent space of M^5, it follows that

$$R_{ijmn} = Y^2 (\delta_{im}\delta_{jn} - \delta_{in}\delta_{jm})\,, \tag{4.44}$$

and hence M^5 is locally isometric to the round S^5.

It follows that all (sufficiently regular) AdS_5 solutions with $N = 24$ supersymmetries are locally isometric to $AdS_5 \times S^5$, with constant axion and dilaton, and $G = 0$. This establishes that there are no distinct local geometries for IIB $AdS_5 \times M^5$ backgrounds that preserve strictly 24 supersymmetries.

5 Concluding remarks

We have proven, under some assumptions, a non-existence theorem for $AdS_5 \times_w M^{D-5}$, $D = 10, 11$, backgrounds that preserve strictly 24 supersymmetries in all 10- and 11-dimensional supergravity theories. In particular we have demonstrated that such backgrounds cannot exist in 11-dimensional and (massive) IIA supergravities, and all such IIB backgrounds must be locally isometric to the maximally supersymmetric $AdS_5 \times S^5$ solution of the theory.

Our assumptions are that the fields must be smooth and the internal space M^{D-5} must be connected, compact and without boundary. Alternatively, these assumptions can be summarized by saying that the data are such that the maximum principle applies. It turns out that these assumptions are required to establish our results. It is known that if the compactness assumption for M^6 is removed, then the maximally supersymmetric $AdS_7 \times S^4$ solution of 11-dimensional supergravity can be written locally as a warped $AdS_5 \times_w M^6$ solution. This would appear to be a contradiction to our result for eleven dimensions, but for such a solution M^6 is not compact [31]. Because of this, it is not apparent that the smoothness and global assumptions on M^{D-5} can be removed. This in particular leaves open the possibility that there are $AdS_5 \times_w M^{D-5}$ backgrounds in 10- and 11-dimensional supergravities but such backgrounds would either be singular or M^{D-5} will not be compact and without boundary. Another possibility for constructing AdS_5 backgrounds in IIB with 24 supersymmetries is to take appropriate orbifolds of the maximally supersymmetric $AdS_5 \times S^5$ solution of the theory. Though such a possibility cannot be ruled out, it is unlikely. It is also supported by the results of [28], that there are no relevant $\mathcal{N} = 3$ deformations of $\mathcal{N} = 4$ theory.

The existence of a smooth AdS_5 background with compact without boundary internal space in a 10- or 11-dimensional supergravity theory with distinct local geometry from that of maximally supersymmetric backgrounds would have raised the expectation that it should have been the AdS/CFT dual to a 4-dimensional $\mathcal{N} = 3$ superconformal theory. This would have been in parallel with the well known duality that string theory on $AdS_5 \times S^5$ is AdS/CFT dual to $\mathcal{N} = 4$ $\mathrm{U}(N)$ gauge theory. Because both $\mathcal{N} = 3$ and $\mathcal{N} = 4$ theories have the same classical action, it is believed that in perturbation theory the two theories are indistinguishable. Though such a proof is not known, it may be possible to demonstrate this by proving that quantum mechanically $\mathcal{N} = 3$ Ward identities imply, using for example techniques similar to [27], that the symmetry enhances to $\mathcal{N} = 4$. In any case assuming that perturbatively the two theories cannot be distinguished, the possibility that remains is that if a theory exists with strictly $\mathcal{N} = 3$ superconformal symmetry, it has to be intrinsically non-perturbative. The properties of such a theory have been investigated in [28] and non-perturbative constructions have been proposed in [29]. Our results prove that the gravitational duals of strictly $\mathcal{N} = 3$ superconformal theories, if they exist, cannot be smooth with compact without boundary internal spaces. This is unlike the gravitational duals of many other superconformal theories that preserve more than 16 supersymmetries. It would be of interest to understand why this is the case.

JHEP06(2016)126

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A Notation and conventions

Our form conventions are as follows. Let ω be a k-form, then

$$\omega = \frac{1}{k!}\omega_{i_1\dots i_k}dx^{i_1}\wedge\cdots\wedge dx^{i_k}\,,\quad \omega^2_{ij}=\omega_{i\ell_1\dots\ell_{k-1}}\omega_j{}^{\ell_1\dots\ell_{k-1}}\,,\quad \omega^2=\omega_{i_1\dots i_k}\omega^{i_1\dots i_k}\,. \tag{A.1}$$

We also define

$$\not{\omega}=\omega_{i_1\dots i_k}\Gamma^{i_1\dots i_k}\,,\quad \not{\omega}_{i_1}=\omega_{i_1 i_2\dots i_k}\Gamma^{i_2\dots i_k}\,,\quad \not{\Gamma}\!\!\!\!\omega_{i_1}=\Gamma_{i_1}{}^{i_2\dots i_{k+1}}\omega_{i_2\dots i_{k+1}}\,, \tag{A.2}$$

where the Γ_i are the Dirac gamma matrices.

The inner product $\langle\cdot,\cdot\rangle$ we use on the space of spinors is that for which space-like gamma matrices are Hermitian while time-like gamma matrices are anti-hermitian, ie the Dirac spin-invariant inner product is $\langle\Gamma_0\cdot,\cdot\rangle$. For more details on our conventions see [4–6].

B Homogeneity of internal spaces

In this appendix, we prove that for the $N=24$ AdS_5 solutions in eleven-dimensional supergravity, the isometries on M^6 generated by the Λ spinors via

$$W=\langle\Lambda,\Gamma^i\Gamma_{xyz}\Lambda\rangle\,\partial_i\,, \tag{B.1}$$

span the tangent space of M^6. The proof for this is a straightforward adaptation of a similar result used in the proof of the homogeneity conjecture [30]. To begin, let K denote the 12-dimensional vector space spanned by the Killing spinors Λ.

Define the map $\varphi: K\otimes K\to TM^6$ by

$$\varphi(\Lambda_1,\Lambda_2)=\langle\Lambda_1,\Gamma^i\Gamma_{xyz}\Lambda_2\rangle\partial_i\,. \tag{B.2}$$

As $\varphi(\Lambda_1,\Lambda_2)=\varphi(\Lambda_2,\Lambda_1)$, it follows that the W span $T(M^6)$ iff φ is surjective. However, φ is surjective iff the only vector $V\in T(M^6)$ satisfying

$$V^i\varphi(\Lambda_1,\Lambda_2)_i=0\,, \tag{B.3}$$

for all $\Lambda_1,\Lambda_2\in K$ is $V=0$, i.e. the perpendicular complement of the image of φ is trivial.

Suppose, for a contradiction, that the perpendicular complement of the image of φ is not trivial. Then there exists nonzero $V \in T(M^6)$ such that

$$V^i \Gamma_i \Gamma_{xyz} \Lambda \in K^\perp \,, \tag{B.4}$$

for all $\Lambda \in K$, where

$$K^\perp = \{\Psi : \langle \Psi, \Lambda \rangle = 0 \;\; \text{for all } \Lambda \in K\} \; . \tag{B.5}$$

Observe that $K \oplus K^\perp$ is a 16-dimensional vector space spanned by the Majorana Spin(10, 1) spinors ζ that satisfy the lightcone projection $\Gamma_+ \zeta = 0$. Thus as K is 12-dimensional, $K^\perp$ is a 4-dimensional subspace.

As $V \neq 0$, the kernel of the map $V^i \Gamma_i \Gamma_{xyz} : K \to K^\perp$ is zero and so it is injective. However this is not possible as the image $V^i \Gamma_i \Gamma_{xyz}(K)$ is 12-dimensional while $K^\perp$ is 4-dimensional. Thus the hypothesis that $V \neq 0$ is not valid and φ is surjective, and so the vectors W span the tangent space of M^6.

The argument for the AdS_5 backgrounds of massive IIA supergravity is the same as that described above upon replacing M^6 with M^5. It also generalizes for the AdS_5 solutions in IIB supergravity, after replacing the norm $<,>$ with Re $<,>$, and M^6 with M^5 throughout.

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