

RG flows in 6D $N = (1,0)$ SCFT from $SO(4)$ half-maximal 7D gauged supergravity

Parinya Karndumri

*String Theory and Supergravity Group, Department of Physics, Faculty of Science,
Chulalongkorn University, 254 Phayathai Road, Pathumwan, Bangkok 10330, Thailand*

E-mail: parinya.ka@hotmail.com

ABSTRACT: We study $N = 2$ seven-dimensional gauged supergravity coupled to three vector multiplets with $SO(4)$ gauge group. The resulting gauged supergravity contains 10 scalars consisting of the dilaton and 9 vector multiplet scalars parametrized by $SO(3,3)/SO(3) \times SO(3)$ coset manifold. The maximally supersymmetric AdS_7 vacuum with unbroken $SO(4)$ symmetry is identified with a $(1,0)$ SCFT in six dimensions. We find one new supersymmetric AdS_7 critical point preserving $SO(3)_{\text{diag}} \subset SO(3) \times SO(3) \sim SO(4)$ and study a holographic RG flow interpolating between the $SO(4)$ and the new $SO(3)$ supersymmetric critical points. The RG flow is driven by a vacuum expectation value of a dimension-four operator and describes a deformation of the UV $(1,0)$ SCFT to another supersymmetric fixed point in the IR. In addition, a number of non-supersymmetric critical points are identified, and some of them are stable with all scalar masses above the BF bound. RG flows to non-conformal $N = (1,0)$ Super Yang-Mills with $SO(2) \times SO(2)$ and $SO(2)$ symmetries are also investigated. Some of these flows have physically acceptable IR singularities since the scalar potential is bounded above. These provide physical RG flows from $(1,0)$ SCFT to non-conformal field theories in six dimensions.

KEYWORDS: Gauge-gravity correspondence, AdS-CFT Correspondence, Supergravity Models

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Contents

1	Introduction	1
2	$N = 2$, $SO(4)$ gauged supergravity in seven dimensions	2
3	Critical points of $N = 2$, $SO(4)$ seven-dimensional gauged supergravity	4
3.1	Critical points on $SO(3)_{\text{diag}}$ scalars	5
3.2	Critical points on scalar manifold with smaller residual symmetry	7
4	Supersymmetric RG flows	7
4.1	An RG flow to a supersymmetric $SO(3)$ fixed point	8
4.2	RG flows to non-conformal field theories	9
4.2.1	Flows to $SO(2) \times SO(2)$, $6D$ Super Yang-Mills	9
4.2.2	Flows to $SO(2)$, $6D$ Super Yang-Mills	11
5	Conclusions	14

1 Introduction

The AdS/CFT correspondence has attracted a lot of attention during the past twenty years. The original proposal in [1] discussed many examples in various dimensions. These examples included the duality between M-theory on $AdS_7 \times S^4$ and $(2, 0)$ superconformal field theory (SCFT) in six dimensions. The $AdS_7 \times S^4$ geometry can arise from the near horizon limit of M5-brane. In term of $N = 4$ seven-dimensional gauged supergravity with $SO(5)$ gauge group, the AdS_7 geometry corresponds to the maximally supersymmetric vacuum of the gauged supergravity, see for example [2].

In this paper, we will explore AdS_7/CFT_6 correspondence with sixteen supercharges. The dual SCFT to the AdS_7 background in this case would be $(1, 0)$ six-dimensional SCFT. Six-dimensional gauge theories with $N = (1, 0)$ supersymmetry are interesting in many aspects. In [3], it has been shown that the theories admit non-trivial RG fixed points. Examples of these field theories also arise in string theory [4], see also a review in [5]. After the AdS/CFT correspondence, a supergravity dual of a $(1, 0)$ field theory with E_8 global symmetry has been proposed in [6]. The dual gravity background has been identified with the orbifolds of $AdS_7 \times S^4$ geometry in M-theory. The operator spectrum of the $(1, 0)$ six-dimensional SCFT has been matched with the Kaluza-Klein spectrum in [7, 8].

Like in lower dimensions, it is more convenient to study AdS_{d+1}/CFT_d correspondence in the framework of $(d+1)$ -dimensional gauged supergravity. A consistent reduction ansatz can eventually be used to uplift the lower dimensional results to string/M theory in ten or eleven dimensions. A suitable framework in the holographic study of the above $(1, 0)$ field

theories is the half-maximal gauged supergravity in seven dimensions coupled to n vector multiplets. The supergravity theory has $N = 2$ or sixteen supercharges in exact agreement with the number of supercharges in six-dimensional $(1, 0)$ superconformal symmetry. This has been proposed long time ago in [9]. With the pure gauged supergravity and critical points found in [10] and [11], holographic RG flows to a non-supersymmetric IR fixed point and to a non-conformal $(1, 0)$ gauge theory have been studied in [12] and [13].

Pure $N = 2$ gauged supergravity in seven dimensions admit only two AdS_7 vacua with one being maximally supersymmetric and the other one being stable non-supersymmetric. To obtain more AdS_7 critical points, matter coupled supergravity theory is needed. This has been constructed in [14] but without the topological mass term for the 3-form field which is a dual of the 2-form field in the supergravity multiplet. Without this term, the scalar potential of the matter coupled gauged supergravity does not admit any critical point but a domain wall as can be verified by looking at the scalar potential explicitly given in [14]. Although mistakenly claimed in [15] that the topological mass term is not possible, the theory indeed admits this term as shown in [16] in which the full Lagrangian and supersymmetry transformations of this massive gauged supergravity have been given. This provides the starting point for the present work.

In this paper, we are interested in the gauged supergravity with $SO(4)$ gauge group. This requires three vector multiplets since six gauge fields are needed in order to implement the $SO(4)$ gauging. The theory can be obtained from a truncation of the maximal $N = 4$ gauged supergravity [17]. In addition to the dilaton, there are extra nine scalars from the vector multiplets parametrized by $SO(3, 3)/SO(3) \times SO(3) \sim SL(4, \mathbb{R})/SO(4)$ coset manifold. We will explore the scalar potential of this theory in the presence of topological mass term and identify some of its critical points. The critical points will correspond to new IR fixed point of the $(1, 0)$ SCFT identified with the maximally supersymmetric critical point with $SO(4)$ symmetry. We will also study RG flows between these critical points as well as RG flows to non-conformal field theories.

The paper is organized as follow. We briefly review the matter coupled gauged supergravity in seven dimensions and give relevant formulae which will be used throughout the paper in section 2. Some critical points of seven-dimensional gauged supergravity with $SO(4)$ gauge group are explored in section 3. A number of supersymmetric and non-supersymmetric critical points and the corresponding scalar masses will also be given in this section. In section 4, we study supersymmetric deformations of the UV $N = (1, 0)$ SCFT to a new superconformal fixed point in the IR and to non-conformal SYM in six dimensions. Both types of the solutions can be analytically obtained. The paper is closed with some conclusions and comments on the results in section 5.

2 $N = 2$, $SO(4)$ gauged supergravity in seven dimensions

We begin with a description of $N = 2$ gauged supergravity coupled to n vector multiplets. All notations are the same as those of [16]. The gravity multiplet in seven-dimensional $N = 2$ supersymmetry contains the following field content

$$\text{gravity multiplet : } (e_\mu^m, \psi_\mu^A, A_\mu^i, \chi^A, B_{\mu\nu}, \sigma). \tag{2.1}$$

A vector multiplet has the field content $(A_\mu, \lambda^A, \phi^i)$. Indices A, B label the doublet of the $\text{USp}(2)_R \sim \text{SU}(2)_R$ R-symmetry. Curved and flat space-time indices are denoted by μ, ν, \dots and m, n, \dots , respectively. $B_{\mu\nu}$ and σ are a two-form and the dilaton fields. For supergravity theory coupled to n vector multiplets, there are n copies of $(A_\mu, \lambda^A, \phi^i)^r$ labeled by an index $r = 1, \dots, n$, and indices $i, j = 1, 2, 3$ label triplets of $\text{SU}(2)_R$. The $3n$ scalars ϕ^{ir} are parametrized by $\text{SO}(3, n)/\text{SO}(3) \times \text{SO}(n)$ coset manifold. The corresponding coset representative will be denoted by

$$L = (L_I^i, L_I^r), \quad I = 1, \dots, n + 3. \quad (2.2)$$

The inverse of L is given by $L^{-1} = (L^I_i, L^I_r)$ where $L^I_i = \eta^{IJ} L_{Ji}$ and $L^I_r = \eta^{IJ} L_{Jr}$. Indices i, j and r, s are raised and lowered by δ_{ij} and δ_{rs} , respectively while the full $\text{SO}(3, n)$ indices I, J are raised and lowered by $\eta_{IJ} = \text{diag}(-\dots + \dots)$. There are some relations involving components of L and are given by

$$\begin{aligned} \eta_{IJ} &= -L_I^i L^i_J + L_I^r L^r_J, & L^i &= L_{Ii}, \\ L^i_I L^I_j &= -\delta^i_j, & L^i_I L^I_j &= -\delta^{ij}. \end{aligned} \quad (2.3)$$

Gaugings are implemented by promoting a global symmetry $\tilde{G} \subset \text{SO}(3, n)$ to a gauge symmetry. Consistency of the gauging imposes a condition on the \tilde{G} structure constants $f_{IJ}{}^K$

$$f_{IK}{}^L \eta_{LJ} + f_{JK}{}^L \eta_{LI} = 0 \quad (2.4)$$

meaning that η_{IJ} is invariant under the adjoint action of \tilde{G} . General semisimple gauge groups take the form of $\tilde{G} \sim G_0 \times H \subset \text{SO}(3, n)$ with G_0 being one of the six possibilities: $\text{SO}(3)$, $\text{SO}(3, 1)$, $\text{SL}(3, \mathbb{R})$, $\text{SO}(2, 1)$, $\text{SO}(2, 2)$ and $\text{SO}(2, 2) \times \text{SO}(2, 1)$ and H being compact with $\dim H \leq (n + 3 - \dim G_0)$.

In this paper, we are interested in the $\text{SO}(4)$ gauged supergravity corresponding to $G_0 = \text{SO}(3)$ and $H = \text{SO}(3)$. To obtain AdS_7 vacua, we need to consider the gauged supergravity with a topological mass term for a 3-form potential. The 3-form field is a dual of the 2-form $B_{\mu\nu}$. With all modifications to the Lagrangian and supersymmetry transformations as given in [16], the bosonic Lagrangian involving only scalars and the metric can be written as

$$e^{-1} \mathcal{L} = \frac{1}{2} R - \frac{5}{8} \partial_\mu \sigma \partial^\mu \sigma - \frac{1}{2} P^{\mu ir} P_{\mu ir} - V \quad (2.5)$$

where the scalar potential is given by

$$V = \frac{1}{4} e^{-\sigma} \left(C^{ir} C_{ir} - \frac{1}{9} C^2 \right) + 16h^2 e^{4\sigma} - \frac{4\sqrt{2}}{3} h e^{\frac{3\sigma}{2}} C. \quad (2.6)$$

The constant h characterizes the topological mass term. The quantities appearing in the above equations are defined by

$$\begin{aligned} P_\mu^{ir} &= L^{Ir} \left(\delta_I^K \partial_\mu + f_{IJ}{}^K A_\mu^J \right) L^i_K, & C_{rsi} f_{IJ}{}^K L^I_r L^J_s L_{Ki}, \\ C_{ir} &= \frac{1}{\sqrt{2}} f_{IJ}{}^K L^I_j L^J_k L_{Kr} \epsilon^{ijk}, & C = -\frac{1}{\sqrt{2}} f_{IJ}{}^K L^I_i L^J_j L_{Kk} \epsilon^{ijk}. \end{aligned} \quad (2.7)$$

We also need fermionic supersymmetry transformations with all fields but scalars vanishing. These are given by

$$\delta\psi_\mu = 2D_\mu\epsilon - \frac{\sqrt{2}}{30}e^{-\frac{\sigma}{2}}C\gamma_\mu\epsilon - \frac{4}{5}he^{2\sigma}\gamma_\mu\epsilon, \quad (2.8)$$

$$\delta\chi = -\frac{1}{2}\gamma^\mu\partial_\mu\sigma\epsilon + \frac{\sqrt{2}}{30}e^{-\frac{\sigma}{2}}C\epsilon - \frac{16}{5}e^{2\sigma}h\epsilon, \quad (2.9)$$

$$\delta\lambda^r = -i\gamma^\mu P_\mu^{ir}\sigma^i\epsilon - \frac{i}{\sqrt{2}}e^{-\frac{\sigma}{2}}C^{ir}\sigma^i\epsilon \quad (2.10)$$

where $SU(2)_R$ indices on spinors are suppressed. σ^i are the usual Pauli matrices.

In the remaining of this section, we focus on $n = 3$ case with $\tilde{G} = SO(4) \sim SO(3) \times SO(3)$. The first $SO(3)$ factor is identified with the $SU(2)_R$ R-symmetry. To give an explicit parametrization of $SO(3,3)/SO(3) \times SO(3)$ coset, we define thirty-six 6×6 matrices

$$(e_{ab})_{cd} = \delta_{ac}\delta_{bd}, \quad a, b, \dots = 1, \dots, 6. \quad (2.11)$$

Non-compact generators of $SO(3,3)$ are identified as

$$Y_{ir} = e_{i,r+3} + e_{r+3,i}, \quad r = 1, \dots, 3. \quad (2.12)$$

Accordingly, $SO(3) \times SO(3)$ generators can be written as

$$\begin{aligned} SO(3)_R : \quad & J_{ij} = e_{ij} - e_{ji}, \\ SO(3) : \quad & J_{rs} = e_{rs} - e_{sr}. \end{aligned} \quad (2.13)$$

In this case, the structure constants for the gauge group are given by

$$f_{IJK} = (g_1\epsilon_{ijk}, g_2\epsilon_{rst}) \quad (2.14)$$

where g_1 and g_2 are coupling constants of $SO(3)_R$ and $SO(3)$, respectively.

3 Critical points of $N = 2$, $SO(4)$ seven-dimensional gauged supergravity

In this section, we will compute the scalar potential of the $SO(4)$ gauged supergravity and study some of its critical points. Although complicated, it is possible to compute the scalar potential for all of the ten scalars. However, the long expression would make any analysis more difficult. Consequently, we will proceed by studying the scalar potential on a subset of the ten scalars as originally proposed in [18]. In this approach, the scalar potential is computed on a scalar submanifold which is invariant under some subgroup H_0 of the full gauge symmetry $SO(4)$. This submanifold consists of all scalars which are singlet under the unbroken subgroup H_0 . All critical points found on this submanifold are essentially critical points of the potential on the full scalar manifold. This can be seen by expanding the full potential to first order in scalar fluctuations which in turn contain both H_0 singlets and H_0 non-singlets. By a simple group theory argument, the non-singlet fluctuations cannot lead to H_0 singlets at first order. Their coefficients, variations of the potential with respect to non-singlet scalars, must accordingly vanish. This proves to be more convenient and more efficient. However, the truncation is consistent only when all relevant H_0 singlet scalars are included on the chosen submanifold. With only some of these singlets, the consistency is not guaranteed.

3.1 Critical points on $\text{SO}(3)_{\text{diag}}$ scalars

We begin with the most simplest case namely the potential on $\text{SO}(3)_{\text{diag}} \subset \text{SO}(3) \times \text{SO}(3)$ corresponding to the non-compact generator $Y_s = Y_{11} + Y_{22} + Y_{33}$. The coset representative is then parametrized by

$$L = e^{\phi Y_s}. \tag{3.1}$$

The scalar potential is given by

$$V = \frac{1}{32} e^{-\sigma} \left[(g_1^2 + g_2^2) (\cosh(6\phi) - 9 \cosh(2\phi)) - 8g_1 g_2 \sinh^3(2\phi) + 8 \left[g_2^2 - g_1^2 + 64h^2 e^{5\sigma} + 32e^{\frac{5\sigma}{2}} h (g_1 \cosh^2 \phi - g_2 \sinh^3 \phi) \right] \right]. \tag{3.2}$$

Notice that there is no critical point when $h = 0$ as mentioned before. In this case, the $\text{SO}(4)$ supergravity admits a half-supersymmetric domain wall as a vacuum solution. For $\phi = 0$, the above potential is the potential of pure $N = 2$ gauged supergravity with $\text{SO}(3)$ gauge group studied in [10] and [11]. There are two critical points in the pure gauged supergravity. One of them preserves all of the supersymmetry while the other completely breaks supersymmetry. In our conventions, they are given by

$$\sigma = \frac{2}{5} \ln \left[-\frac{g_1}{16h} \right] \quad \text{and} \quad \sigma = \frac{2}{5} \ln \left[-\frac{g_1}{8h} \right]. \tag{3.3}$$

It can be readily verified by using supersymmetry transformations of ψ_μ , χ and λ^r that the first one is supersymmetric. We can bring the supersymmetric point to $\sigma = 0$ by choosing $g_1 = -16h$ and find that the two critical points are now given by

$$\begin{aligned} \sigma = 0, & & V_0 = -240h^2 \\ \text{and } \sigma = \frac{2}{5} \ln 2, & & V_0 = -160(2^{\frac{3}{5}})h^2 \end{aligned} \tag{3.4}$$

where V_0 denotes the value of the cosmological constant.

Although non-supersymmetric, the second critical point has been shown to be stable in [11]. In the presence of matter scalars, this is however not the case. This can be seen from the scalar masses given below.

$\text{SO}(3) \times \text{SO}(3)$	$m^2 L^2$
$(\mathbf{1}, \mathbf{1})$	12
$(\mathbf{3}, \mathbf{3})$	-12

The AdS_7 radius L in our conventions is given by $L = \sqrt{\frac{-15}{V_0}} = \frac{1}{4h}$. The $(\mathbf{1}, \mathbf{1})$ scalar correspond to σ , and $(\mathbf{3}, \mathbf{3})$ is the nine scalars in $\text{SO}(3, 3)/\text{SO}(3) \times \text{SO}(3)$. The BF bound in seven dimensions is $m^2 L^2 \geq -9$. Therefore, the non-supersymmetric critical point of pure gauged supergravity is unstable in the matter coupled theory. This is very similar to the six-dimensional $N = (1, 1)$ gauged supergravity pointed out in [19].

Scalar masses at the supersymmetric point are given in the table below.

SO(3) × SO(3)	$m^2 L^2$
(1 , 1)	-8
(3 , 3)	-8

In the dual (1, 0) SCFT, these scalars correspond to dimension-4 operators via the relation $m^2 L^2 = \Delta(\Delta - 6)$.

There is one non-trivial supersymmetric point at

$$\sigma = -\frac{1}{5} \ln \left[\frac{g_2^2 - 256h^2}{g_2^2} \right], \quad \phi = \frac{1}{2} \ln \left[\frac{g_2 - 16h}{g_2 + 16h} \right],$$

$$V_0 = -\frac{240g_2^{\frac{8}{5}}h^2}{(g_2^2 - 256h^2)^{\frac{4}{5}}}. \tag{3.5}$$

At this point, scalar masses are computed as follow.

SO(3) _{diag}	$m^2 L^2$	Δ
1	-8	4
1	40	10
3	0	6
5	16	8

In the table, we have decomposed all of the ten scalars in representations of the SO(3)_{diag} residual symmetry. This can be done by the following decomposition. Under SO(3)_R × SO(3), the nine scalars transform as (**3**, **3**). They then transform as **3** × **3** = **1** + **3** + **5** under SO(3)_{diag}. Notice that the **3** scalars are massless corresponding to Goldstone bosons of the symmetry breaking SO(3) × SO(3) → SO(3)_{diag}.

There is one non-supersymmetric critical point given by

$$\sigma = \frac{1}{5} \ln \left[\frac{4g_2^2}{g_2^2 - 256h^2} \right], \quad \phi = \frac{1}{2} \ln \left[\frac{g_2 - 16h}{g_2 + 16h} \right],$$

$$V_0 = -\frac{160(2^{\frac{3}{5}})g_2^{\frac{8}{5}}h^2}{(g_2^2 - 256h^2)^{\frac{4}{5}}}. \tag{3.6}$$

This critical point is stable as can be seen from the mass spectrum below.

SO(3) _{diag}	$m^2 L^2$	Δ
1	12	$3 + \sqrt{21}$
1	36	$3 + 3\sqrt{5}$
3	0	6
5	0	6

For $g_2 = g_1$, we also find another non-supersymmetric critical point given by

$$\sigma = \frac{1}{10} \left[\sqrt{2} \ln 8 + 4 \ln(1 - 2^{-\sqrt{2}}) \right], \quad \phi = -\frac{1}{2} \ln 2, \quad V_0 = -246.675h^2. \quad (3.7)$$

This critical point is however unstable. Scalar masses at this point are given below.

$\text{SO}(3)_{\text{diag}}$	$m^2 L^2$
1	-4.278
1	16.059
3	0
5	-14.282

We can see that the mass of **5** scalars violates the BF bound.

3.2 Critical points on scalar manifold with smaller residual symmetry

To find other critical points, we can consider smaller residual symmetries. Breaking $\text{SO}(3)_{\text{diag}}$ to $\text{SO}(2)_{\text{diag}}$, we find that there are two singlets from $\text{SO}(3,3)/\text{SO}(3) \times \text{SO}(3)$ with the coset representative

$$L = e^{\phi_1(Y_{11}+Y_{22})} e^{\phi_2 Y_{33}}. \quad (3.8)$$

This gives the scalar potential, with $g_1 = -16h$,

$$\begin{aligned} V = \frac{1}{8} e^{-\sigma} & \left[2(g_2^2 + 64h^2(e^{5\sigma} - 4)) - 2(g_2^2 + 256h^2) \cosh(2\phi_1) \right. \\ & - 64he^{\frac{5\sigma}{2}} (16h \cosh^2 \phi_1 \cosh \phi_2 + g_2 \sinh^2 \phi_1 \sinh \phi_2) \\ & \left. + \sinh^2(2\phi_1) [(g_2^2 + 256h^2) \cosh(2\phi_2) + 32g_2h \sinh(2\phi_2)] \right]. \quad (3.9) \end{aligned}$$

This potential does not admit any supersymmetric critical points unless $\phi_1 = \phi_2$ which is the previously found $\text{SO}(3)_{\text{diag}}$ point. When $\phi_1 = 0$, the above scalar submanifold preserves $\text{SO}(2) \times \text{SO}(2)$ symmetry, but there is no critical point except for $\phi_2 = 0$. We are not able to obtain any new critical points from the above potential.

We now move to scalar fields invariant under $\text{SO}(2)_R \subset \text{SO}(3)_R$. There are three singlets corresponding to Y_{11}, Y_{12} and Y_{13} . Denoting the associated scalars by $\phi_i, i = 1, 2, 3$, we find a simple potential

$$V = -\frac{1}{2} g_1^2 e^{-\sigma} + 16h^2 e^{4\sigma} + g_1 h e^{\frac{3}{2}\sigma - \phi_1 - \phi_2 - \phi_3} (1 + e^{2\phi_1})(1 + e^{2\phi_2})(1 + e^{2\phi_3}) \quad (3.10)$$

which does not admit any non-trivial critical points.

4 Supersymmetric RG flows

We now consider domain wall solutions interpolating between critical points identified in the previous section. These solutions will generally have an interpretation in terms of

RG flows in the dual field theories in six dimensions. We are mainly interested in supersymmetric RG flows which can be obtained from solving BPS equations coming from supersymmetry variations of fermionic fields ψ_μ , χ and λ^r . A stable non-supersymmetric AdS_7 critical point also admits a well-defined dual CFT, but in most cases, finding the corresponding flow solutions requires a numerical analysis. Accordingly, we will not consider non-supersymmetric flows in this paper.

4.1 An RG flow to a supersymmetric SO(3) fixed point

There is one supersymmetric AdS_7 critical point with SO(3) symmetry. In this subsection, we will find the domain wall solution interpolating between this point and the trivial critical point at $\sigma = \phi = 0$.

Using the standard domain wall metric

$$ds^2 = e^{2A(r)} dx_{1,5}^2 + dr^2 \tag{4.1}$$

where $dx_{1,5}^2$ is the flat metric in six-dimensional space-time and the projection condition $\gamma_r \epsilon = \epsilon$, we can derive the following BPS equations

$$\phi' = \frac{1}{8} e^{-\frac{\sigma}{2} - 3\phi} (e^{4\phi} - 1) (g_1 + g_2 + e^{2\phi} g_1 - e^{2\phi} g_2), \tag{4.2}$$

$$\sigma' = \frac{1}{20} \left[e^{-\frac{\sigma}{2} - 3\phi} (g_2 (e^{2\phi} - 1)^3 - g_1 (1 + e^{2\phi})^3) - 128 h e^{2\sigma} \right], \tag{4.3}$$

$$A' = \frac{1}{40} e^{-\frac{\sigma}{2} - 3\phi} \left[g_2 (e^{2\phi} - 1)^3 - g_1 (1 + e^{2\phi})^3 \right] + \frac{4}{5} h e^{2\sigma} \tag{4.4}$$

where ' denotes $\frac{d}{dr}$. The above equations do not involve $\delta\psi_r$ equation which will give the Killing spinor condition on ϵ as usual. The above equations clearly admit two critical points. To find the solution, we combine equations (4.2) and (4.3) to

$$\frac{d\sigma}{d\phi} = \frac{2 \left[g_2 (e^{2\phi} - 1)^3 - g_1 (1 + e^{2\phi})^3 - 128 h e^{\frac{\sigma}{2} + 3\phi} \right]}{5 (e^{4\phi} - 1) (g_1 + g_2 + (g_1 - g_2) e^{2\phi})} \tag{4.5}$$

whose solution is given by

$$\sigma = \frac{2}{5} \ln \left[\frac{e^\phi (g_1 + g_2 + (g_1 - g_2) e^{2\phi})}{32 h (12 C_1 (e^{2\phi} - 1) - 1)} \right]. \tag{4.6}$$

In order for the solution to interpolate between the two critical points, we need to fix the integration constant to be $C_1 = \frac{(g_1 - g_2)^2}{48 g_1 g_2}$. We then find the solution for σ

$$\sigma = \frac{2}{5} \ln \left[-\frac{g_1 g_2 e^\phi}{8 h (g_1 + g_2 + (g_2 - g_1) e^{2\phi})} \right]. \tag{4.7}$$

Introducing a new radial coordinate \tilde{r} via $\frac{d\tilde{r}}{dr} = e^{-\frac{\sigma}{2}}$, we can solve equation (4.2) and find the solution for ϕ

$$g_1 g_2 \tilde{r} = 2 g_1 \tan^{-1} e^\phi + 2 \sqrt{g_2^2 - g_1^2} \tanh^{-1} \left[e^\phi \sqrt{\frac{g_2 - g_1}{g_2 + g_1}} \right] + g_2 \ln \left[\frac{1 - e^\phi}{1 + e^\phi} \right] \tag{4.8}$$

where we have neglected an additive integration constant to \tilde{r} . Taking the combination (4.4) + $\frac{1}{8}$ × (4.3) and changing the variable from r to ϕ , we find

$$\frac{dA}{d\phi} + \frac{1}{8} \frac{d\sigma}{d\phi} = \frac{g_2(e^{2\phi} - 1)^3 - g_1(1 + e^{2\phi})^3}{4(e^{4\phi} - 1)(g_1 + g_2 + (g_1 - g_2)e^{2\phi})}. \quad (4.9)$$

The solution is easily found to be

$$A = \frac{1}{8} \left[2\phi - \sigma - 2 \ln \left(2 - 2e^{4\phi} \right) + 2 \ln \left(g_1 + g_2 + (g_1 - g_2)e^{2\phi} \right) \right]. \quad (4.10)$$

Near the UV point $\sigma \sim 0$ and $\phi \sim 0$ with $g_1 = -16h$, we find

$$\sigma \sim \phi \sim e^{-16hr} = e^{-\frac{4r}{L}}, \quad L = \frac{1}{4h} \quad (4.11)$$

since $\tilde{r} \sim r$ near $\sigma \sim 0$. The flow is then driven by vacuum expectation values (vev) of relevant operators of dimension $\Delta = 4$. In the IR, we find that the solution behaves as

$$\sigma \sim e^{-\frac{4r}{L}}, \quad \phi \sim e^{\frac{4r}{L}}, \quad L = \frac{(g_2^2 - 256h^2)^{\frac{2}{5}}}{4hg_2^{\frac{4}{5}}}. \quad (4.12)$$

From this, we see that the operator dual to ϕ acquires an anomalous dimension and has dimension 10 in the IR. This is consistent with the value of $m^2 L^2$ given previously.

4.2 RG flows to non-conformal field theories

A supersymmetric flow to non-conformal field theory in pure gauged supergravity has been studied in [13]. We will study similar solutions in the matter coupled gauged supergravity. These solutions would be a generalization of the solution given in [13].

4.2.1 Flows to $SO(2) \times SO(2)$, 6D Super Yang-Mills

We first consider $SO(2)_R$ singlets scalars. With $\gamma_r \epsilon = \epsilon$, the BPS equations for these three singlets, denoted by ϕ_i , $i = 1, 2, 3$, σ and A are given by

$$\phi'_1 = \frac{1}{2} e^{-\frac{\sigma}{2} - \phi_1} g_1 (e^{2\phi_1} - 1), \quad (4.13)$$

$$\phi'_2 = \frac{1}{2} e^{-\frac{\sigma}{2} - \phi_2} g_1 (e^{2\phi_2} - 1), \quad (4.14)$$

$$\phi'_3 = \frac{1}{2} e^{-\frac{\sigma}{2} - \phi_3} g_1 (e^{2\phi_3} - 1), \quad (4.15)$$

$$\sigma' = -\frac{1}{20} g_1 e^{-\frac{\sigma}{2} - \phi_1 - \phi_2 - \phi_3} (1 + e^{2\phi_1})(1 + e^{2\phi_2})(1 + e^{2\phi_3}) - \frac{32}{5} h e^{2\sigma}, \quad (4.16)$$

$$A' = -\frac{1}{40} g_1 e^{-\frac{\sigma}{2} - \phi_1 - \phi_2 - \phi_3} (1 + e^{2\phi_1})(1 + e^{2\phi_2})(1 + e^{2\phi_3}) + \frac{4}{5} h e^{2\sigma}. \quad (4.17)$$

The above equations clearly admit only one critical point at $\phi_i = 0$.

For $\phi_1 = \phi_2 = 0$, the solution will preserve $SO(2)_R \times SO(2)$ symmetry. This is easily seen to be a consistent truncation. The solution to the above equations is given by

$$\begin{aligned}\phi_3 &= \pm \ln \left[\frac{1 + e^{g_1 \tilde{r} + C_1}}{1 - e^{g_1 \tilde{r} + C_1}} \right], \\ \sigma &= \frac{2}{5} \phi_3 - \frac{2}{5} \ln \left[-\frac{16h}{g_1} \left[4C_2 (e^{2\phi_3} - 1) - 1 \right] \right], \\ A &= \frac{1}{8} \left[2\phi_3 - \sigma - 2 \ln(e^{2\phi_3} - 1) \right]\end{aligned}\tag{4.18}$$

where as in the previous case \tilde{r} is related to r via $\frac{d\tilde{r}}{dr} = e^{-\frac{\sigma}{2}}$.

Near the UV point, the asymptotic behavior of ϕ_3 and σ is given by

$$\phi_3 \sim \sigma \sim e^{-16hr}, \quad A \sim 4hr \sim \frac{r}{L}.\tag{4.19}$$

In the IR, we will consider $\phi_3 > 0$ and $\phi_3 < 0$, separately. For $\phi_3 > 0$, there is a singularity when $\phi_3 \rightarrow \infty$ as $16h\tilde{r} \sim C_1$. With $C_2 \neq 0$, we find

$$\begin{aligned}\phi_3 &\sim -\ln(16h\tilde{r} - C_1), \quad \sigma \sim \frac{2}{5} \ln(16h\tilde{r} - C_1), \\ A &\sim -\frac{1}{8}(2\phi_3 + \sigma) = \frac{1}{5} \ln(16h\tilde{r} - C_1).\end{aligned}\tag{4.20}$$

As $16h\tilde{r} \sim C_1$, we find the relation between r and \tilde{r} to be $16hr - C = \frac{5}{6}(16h\tilde{r} - C_1)^{\frac{6}{5}}$ with C being another integration constant. As expected from the general DW/QFT correspondence [20–22], the metric in the IR takes the form of a domain wall

$$ds^2 = (16hr - C)^{\frac{1}{3}} dx_{1,5}^2 + dr^2\tag{4.21}$$

where the multiplicative constant has been absorbed in the rescaling of the x^μ coordinates.

Flows to non-conformal field theories usually encounter singularities in the IR. As can be seen from the above metric, there is a singularity at $16hr \sim C$. The criterion for determining whether a given singularity is physical or not has been given in [23]. The condition rules out naked time-like singularities which are clearly unphysical. According to the criterion of [23], the IR singularity in the solution is acceptable if the scalar potential is bounded above. One way to understand this criterion has been given in [24] for four-dimensional gauge theories. We will follow this argument and briefly discuss the meaning of the criterion in [23] in the context of six-dimensional field theories. Near the IR singularity, scalars ϕ_i , assumed to be canonical ones, and the metric warped factor A behave as

$$\phi_i \sim B_i \ln(r - r_0), \quad A \sim \kappa \ln(r - r_0)\tag{4.22}$$

where we have chosen the integration constant so that the singularity occurs at r_0 . In the IR, the bulk action for these scalars mainly contains the kinetic terms since the potential is irrelevant. This is because the potential diverges logarithmically, but the kinetic terms go like $(r - r_0)^{-2}$. According to the AdS/CFT correspondence, the one point function or the vacuum expectation value of operators O_i dual to ϕ_i is given by $\langle O_i \rangle = \frac{\delta S}{\delta \phi_i}$. Using

$$S = \frac{1}{2} \int d^6 x dr e^{\delta A} \partial_r \phi_i \partial^r \phi_i,\tag{4.23}$$

we find

$$\langle O_i \rangle = \frac{\delta S}{\delta \phi_i} \sim e^{6A} \partial_r \phi_i \sim B_i (r - r_0)^{6\kappa - 1}. \quad (4.24)$$

We can see that $\langle O_i \rangle$ diverges for $\kappa < \frac{1}{6}$. We then expect that solutions with $\kappa < \frac{1}{6}$ will be excluded. In four dimensions, it has been shown that this is related to the fact that the scalar potential becomes unbounded above. In the present case, we will see in the solutions given below that this is the case namely all solutions with $\kappa < \frac{1}{6}$ have $V \rightarrow \infty$.

It can be checked by using the scalar potential given in (3.10) that as $16h\tilde{r} \sim C_1$, the solution in (4.20) gives $V \rightarrow -\infty$. The solution is then physical and describes a supersymmetric RG flow from (1, 0) SCFT to six-dimensional SYM with $SO(2) \times SO(2)$ symmetry.

For $C_2 = 0$, the solution becomes

$$\begin{aligned} \phi_3 &\sim -\ln(16h\tilde{r} - C_1), & \sigma &\sim -\frac{2}{5} \ln(16h\tilde{r} - C_1), \\ ds^2 &= (16hr - C)^{\frac{3}{4}} dx_{1,5}^2 + dr^2. \end{aligned} \quad (4.25)$$

This is also physical since it leads to $V \rightarrow -\infty$.

For $\phi_3 < 0$ and $16h\tilde{r} \sim C_1$, the above solutions give, for any values of C_2 ,

$$\begin{aligned} \phi_3 &\sim \ln(16h\tilde{r} - C_1), & \sigma &\sim \frac{2}{5} \ln(16h\tilde{r} - C_1), \\ ds^2 &= (16hr - C)^{\frac{1}{3}} dx_{1,5}^2 + dr^2 \end{aligned} \quad (4.26)$$

which give rise to $V \rightarrow -\infty$. This solution is then physically acceptable.

The solution with all $\phi_i \neq 0$ turns out to be very difficult to find although the above BPS equations suggest that $\phi_1 = \phi_2 = \phi_3$. Most probably, a numerical analysis might be needed. Therefore, we will not further investigate this case.

4.2.2 Flows to $SO(2)$, 6D Super Yang-Mills

As a final example, we consider RG flows to non-conformal theories from $SO(2)_{\text{diag}}$ singlet scalars corresponding to $Y_{11} + Y_{22}$ and Y_{33} . The relevant BPS equations are given by

$$\phi'_1 = \frac{1}{8} e^{-\frac{\sigma}{2} - 2\phi_1 - \phi_2} (e^{4\phi_1} - 1) \left[g_1 + g_2 + (g_1 - g_2) e^{2\phi_2} \right], \quad (4.27)$$

$$\phi'_2 = \frac{1}{8} e^{-\frac{\sigma}{2} - 2\phi_1 - \phi_2} \left[g_1 (1 + e^{2\phi_1})^2 (e^{2\phi_2} - 1) - g_2 (1 + e^{2\phi_2}) (e^{2\phi_1} - 1)^2 \right], \quad (4.28)$$

$$\begin{aligned} \sigma' &= \frac{1}{20} e^{-\frac{\sigma}{2} - 2\phi_1 - \phi_2} \left[g_2 (e^{2\phi_2} - 1) (e^{2\phi_1} - 1)^2 - g_1 (1 + e^{2\phi_1})^2 (1 + e^{2\phi_2}) \right. \\ &\quad \left. - 128h e^{\frac{5\sigma}{2} + 2\phi_1 + \phi_2} \right], \end{aligned} \quad (4.29)$$

$$\begin{aligned} A' &= \frac{1}{40} e^{-\frac{\sigma}{2} - 2\phi_1 - \phi_2} \left[g_2 (e^{2\phi_2} - 1) (e^{2\phi_1} - 1)^2 - g_1 (1 + e^{2\phi_1})^2 (1 + e^{2\phi_2}) \right. \\ &\quad \left. + 32h e^{\frac{5\sigma}{2} + 2\phi_1 + \phi_2} \right]. \end{aligned} \quad (4.30)$$

These equations reduce to the $SO(3)_{\text{diag}}$ case when $\phi_2 = \phi_1$. If we set $\phi_2 = 0$, consistency requires that $\phi_1 = 0$. For $\phi_1 = 0$, the solution has $SO(2)_R \times SO(2)$ symmetry. This gives rise to the same solution studied above.

Since there are no interesting truncations, we now consider a solution to the above equations with $\phi_1, \phi_2 \neq 0$. Finding the solution for a general value of g_2 turns out to be difficult. However, for $g_2 = g_1 = -16h$, we can find an analytic solution. The first step in finding this solution is to combine (4.27) and (4.28) into a single equation

$$\frac{d\phi_2}{d\phi_1} = \frac{1 + e^{4\phi_1} - 2e^{2\phi_1 + \phi_2}}{1 - e^{4\phi_1}} \quad (4.31)$$

which is solved by

$$\phi_2 = \phi_1 - \frac{1}{2} \ln \left[\frac{8C_2 - 1 + e^{4\phi_1}}{8C_2} \right]. \quad (4.32)$$

Changing to a new radial coordinate \tilde{r} via $\frac{d\tilde{r}}{dr} = e^{-\frac{\sigma}{2} - \phi_2}$, we obtain the solution to equation (4.27)

$$\phi_1 = \pm \frac{1}{2} \ln \left[\frac{1 + e^{C_1 - 16h\tilde{r}}}{1 - e^{C_1 - 16h\tilde{r}}} \right]. \quad (4.33)$$

To find the solution for σ , we change to another new coordinate R via $\frac{dR}{dr} = -e^{-\frac{\sigma}{2} - \phi_2 - 2\phi_1}$. Equations (4.27), (4.28) and (4.29) can be combined to

$$\frac{5}{2} \frac{d\sigma}{dR} + 2 \frac{d\phi_1}{dR} + \frac{d\phi_2}{dR} = -16h \left(1 - e^{\frac{5}{2}\sigma + 2\phi_1 + \phi_2} \right) \quad (4.34)$$

which gives

$$\sigma = -\frac{2}{5} \left[2\phi_1 + \phi_2 + \ln \left(1 - C_3 e^{16hR} \right) \right]. \quad (4.35)$$

Combing (4.29) and (4.30), we find an equation for A as a function of R

$$\frac{dA}{dR} - \frac{1}{2} \frac{d\sigma}{dR} = -4e^{\frac{5}{2}\sigma + 2\phi_1 + \phi_2} \quad (4.36)$$

whose solution, after using σ solution, is given by

$$A = \frac{\sigma}{2} + \frac{1}{4} \ln \left[C_3 - e^{-16hR} \right]. \quad (4.37)$$

As in the previous case, we separately consider the two possibilities for $\phi_1 > 0$ and $\phi_1 < 0$.

For $\phi_1 > 0$, we can find the relation between R and \tilde{r} by using the relation $\frac{dR}{d\tilde{r}} = -e^{-2\phi_1(\tilde{r})}$. This results in

$$8hR = 8h\tilde{r} - \ln \left[2(e^{C_1} + e^{16h\tilde{r}}) \right]. \quad (4.38)$$

In term of \tilde{r} , the σ and A solutions become

$$\sigma = -\frac{2}{5} \left[2\phi_1 + \phi_2 + \ln \left[1 - \frac{C_3 e^{16h\tilde{r}}}{4(e^{C_1} + e^{16h\tilde{r}})^2} \right] \right], \quad (4.39)$$

$$A = \frac{\sigma}{2} + \frac{1}{4} \ln \left[C_3 - 4e^{-16h\tilde{r}} (e^{C_1} + e^{16h\tilde{r}})^2 \right]. \quad (4.40)$$

Near the IR singularity at $16h\tilde{r} \sim C_1$, we have $\phi_2 \sim -\phi_1$ for all values of C_2 . In the IR, the solution behaves differently for $C_3 = 16e^{C_1}$ and $C_3 \neq 16e^{C_1}$. This is because

the logarithmic term in (4.39) and (4.40) diverges, in this limit, when $C_3 = 16e^{C_1}$. For $C_3 \neq 16e^{C_1}$, we find

$$\begin{aligned} \phi_1 \sim -\phi_2 &\sim -\frac{1}{2} \ln(16h\tilde{r} - C_1), & \sigma &\sim -\frac{2}{5}\phi_1 \sim \frac{1}{5} \ln(16h\tilde{r} - C_1), \\ A &\sim \frac{\sigma}{2} \sim \frac{1}{10} \ln(16h\tilde{r} - C_1), & ds^2 &= (16hr - C)^{\frac{1}{8}} dx_{1,5}^2 + dr^2. \end{aligned} \quad (4.41)$$

This gives rise to $V \rightarrow \infty$ which is physically unacceptable.

However, if $C_3 = 16e^{C_1}$, the solution becomes

$$\begin{aligned} \sigma &\sim -\frac{3}{5} \ln(16h\tilde{r} - C_1), & A &\sim \frac{1}{5} \ln(16h\tilde{r} - C_1), \\ ds^2 &= (16hr - C)^{\frac{1}{3}} dx_{1,5}^2 + dr^2. \end{aligned} \quad (4.42)$$

This gives $V \rightarrow -\infty$, so this singularity is acceptable. We see that flows with $\phi_1 > 0$ are physical provided that $C_3 = 16e^{C_1}$.

For $\phi_1 < 0$, the solution $\phi_1 = -\frac{1}{2} \ln \left[\frac{1+e^{C_1-16h\tilde{r}}}{1+e^{C_1-16h\tilde{r}}} \right]$ gives

$$8hR = 8h\tilde{r} - \ln \left[2(e^{C_1} - e^{16h\tilde{r}}) \right]. \quad (4.43)$$

Accordingly, the solutions for σ and A become

$$\sigma = -\frac{2}{5} \left[2\phi_1 + \phi_2 + \ln \left[1 - \frac{C_3 e^{16h\tilde{r}}}{4(e^{C_1} - e^{16h\tilde{r}})^2} \right] \right], \quad (4.44)$$

$$A = \frac{\sigma}{2} + \frac{1}{4} \ln \left[C_3 - 4e^{-16h\tilde{r}}(e^{C_1} - e^{16h\tilde{r}})^2 \right]. \quad (4.45)$$

In this case, the logarithmic term in (4.45) diverges as $16h\tilde{r} \sim C_1$ when $C_3 = 0$, but the logarithmic term in (4.44) vanishes. When $C_3 \neq 0$, the situation is reversed. Unlike the $\phi_1 > 0$ case, the value of C_2 is important since there are two possibilities $\phi_1 = \mp\phi_2$ depending $C_2 = \frac{1}{8}$ or $C_2 \neq \frac{1}{8}$.

We begin with the first case with $C_2 = \frac{1}{8}$ and $C_3 = 0$. The IR behavior of the solution is given by

$$\begin{aligned} \phi_1 \sim -\phi_2 &\sim \frac{1}{2} \ln(16h\tilde{r} - C_1), & \sigma &\sim \frac{1}{5} \ln(16h\tilde{r} - C_1), \\ A &\sim \frac{3}{5} \ln(16h\tilde{r} - C_1), & 16hr - C &= \frac{5}{3} (16h\tilde{r} - C_1)^{\frac{3}{5}}. \end{aligned} \quad (4.46)$$

The metric becomes

$$ds^2 = (16hr - C)^2 dx_{1,5}^2 + dr^2. \quad (4.47)$$

When $C_3 \neq 0$, the solution in the IR becomes

$$\begin{aligned} \phi_1 \sim -\phi_2 &\sim \frac{1}{2} \ln(16h\tilde{r} - C_1), & \sigma &\sim \frac{3}{5} \ln(16h\tilde{r} - C_1), \\ A &\sim \frac{3}{10} \ln(16h\tilde{r} - C_1), & ds^2 &= (16hr - C)^{\frac{3}{4}} dx_{1,5}^2 + dr^2. \end{aligned} \quad (4.48)$$

Both of them lead to $V \rightarrow -\infty$. Therefore, the solution with $\phi_1 < 0$ and $C_2 = \frac{1}{8}$ is physical for all values of C_3 .

For $C_2 \neq \frac{1}{8}$, we find, with $C_3 = 0$, the IR behavior of the solution

$$\begin{aligned} \phi_1 \sim \phi_2 \sim \frac{1}{2} \ln(16h\tilde{r} - C_1), \quad \sigma \sim -\frac{6}{5} \ln(16h\tilde{r} - C_1), \\ ds^2 = (16hr - C)^{-\frac{2}{5}} dx_{1,5}^2 + dr^2, \end{aligned} \tag{4.49}$$

and, for $C_3 \neq 0$,

$$\begin{aligned} \phi_1 \sim \phi_2 \sim \frac{1}{2} \ln(16h\tilde{r} - C_1), \quad \sigma \sim \frac{1}{5} \ln(16h\tilde{r} - C_1), \\ ds^2 = (16hr - C)^{\frac{1}{8}} dx_{1,5}^2 + dr^2. \end{aligned} \tag{4.50}$$

Both of them lead to $V \rightarrow \infty$. We then conclude that flows with $\phi_1 < 0$ and $C_2 \neq \frac{1}{8}$ are not physical for any C_3 .

It could be very interesting to have interpretations of these results in terms of six-dimensional gauge theories.

5 Conclusions

We have studied some critical points of $N = 2$, $SO(4)$ gauged supergravity in seven dimensions. We have found one new supersymmetric AdS_7 critical point with $SO(3)$ symmetry. Recently, many new $AdS_7 \times M_3$ solutions have been identified in massive type IIA theory [25]. It would be interesting to see whether the new supersymmetric AdS_7 obtained here could be related to the classification in [25]. We have also found a number of non-supersymmetric AdS_7 critical points and checked their stability by computing all of the scalar masses. We have found that although the non-supersymmetric critical point originally found in pure gauged supergravity has been shown to be stable, it is unstable in the presence of vector multiplet scalars. On the other hand, new stable non-supersymmetric points are discovered here and should correspond to new non-trivial IR fixed points of the $(1,0)$ SCFT.

An analytic RG flow solution interpolating between the $SO(3)$ supersymmetric critical point and the trivial point with $SO(4)$ symmetry has also been given. To the best of the author's knowledge, this is the first example of holographic RG flows between two supersymmetric fixed points of the $(1,0)$ field theory in six dimensions. We have further studied supersymmetric flows to non-conformal field theories and identified the physical flows. These would provide more general flow solutions than those considered in [12] and [13] and could be useful in a holographic study of the dynamics of six-dimensional gauge theories similar to the analysis of [26]. Finding a field theory interpretation of the gravity solutions obtained in this paper is also interesting.

We end the paper with a short comment on a more general situation with n vector multiplets. The $(1,0)$ field theory with E_8 symmetry considered in [6] would need $n = 248 + 3$ vector multiplets. The resulting gauge group in this case is $SO(4) \times E_8$. The total

$3 \times (248 + 3)$ scalars, living on $SO(3, 248 + 3)/SO(3) \times SO(248 + 3)$ coset manifold, and the dilaton transform as $(\mathbf{3}, \mathbf{3}, \mathbf{1})$, $(\mathbf{3}, \mathbf{1}, \mathbf{248})$ and $(\mathbf{1}, \mathbf{1}, \mathbf{1})$ under $SO(3)_R \times SO(3) \times E_8$. We have considered only $(\mathbf{3}, \mathbf{3}, \mathbf{1})$ and $(\mathbf{1}, \mathbf{1}, \mathbf{1})$ scalars which are E_8 singlets. It is also interesting to consider scalars in $(\mathbf{3}, \mathbf{1}, \mathbf{248})$ representation. Our solutions given in this paper are of course solutions of the theory with $SO(4) \times E_8$ gauge group by the group theory argument of [18].

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