

# Supersymmetric geometries of IIA supergravity I

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ABSTRACT: IIA supergravity backgrounds preserving one supersymmetry locally admit four types of Killing spinors distinguished by the orbits of  $\text{Spin}(9,1)$  on the space of spinors. We solve the Killing spinor equations of IIA supergravity with and without cosmological constant for Killing spinors representing two of these orbits, with isotropy groups  $\text{Spin}(7)$  and  $\text{Spin}(7) \ltimes \mathbb{R}^8$ . In both cases, we identify the geometry of spacetime and express the fluxes in terms of the geometry. We find that the geometric constraints on backgrounds with a  $\text{Spin}(7) \ltimes \mathbb{R}^8$  invariant Killing spinor are identical to those found for heterotic backgrounds preserving one supersymmetry.

KEYWORDS: Supergravity Models, Superstring Vacua

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**1 Introduction**

The supersymmetric solutions of 10- and 11-dimensional supergravity theories have found widespread applications in compactifications, string solitons, black holes, dualities and in the AdS/CFT correspondence. Many examples of such solutions have been constructed, and some progress has been made to identify the geometry of all such solutions. All the maximally supersymmetric solutions has been classified up to a discrete identification [1]. The Killing spinor equations (KSEs) of  $D = 11$  [2–4] and IIB supergravity [5, 6] have been solved for one Killing spinor, and some near maximally supersymmetric solutions have been classified [7–11]. Far more significant progress has been made in heterotic supergravity

where the KSEs have been solved in all cases, and the possible fractions of supersymmetry and the geometry of all the backgrounds have been identified [12, 13]. In IIA supergravity less progress has been made. It is known that the only maximally supersymmetric solution up to a discrete identification is Minkowski spacetime and that all backgrounds that preserve 31 supersymmetries are actually maximally supersymmetric [14].

In this paper, we initiate the solution of KSEs of IIA supergravity for backgrounds which preserve one supersymmetry. As we demonstrate, there are four different cases to consider. In this work, we solve the KSEs for two of the four cases. The final aim is to bring the status of the classification of supersymmetric solutions of IIA supergravity to a similar standard to that of  $D = 11$  and IIB supergravities.

To solve the KSEs of IIA supergravity, we shall use spinorial geometry [4]. This relies on using the gauge symmetry of the KSEs to bring the Killing spinor into a canonical form. Then utilizing a realization of spinors in terms of forms, one can express the KSEs in terms of a linear system, with the unknowns being the components of the fluxes and of the spin connection. The linear system can then be solved to express the fluxes in terms of the geometry and to find the conditions on the geometry. The conditions on the geometry are usually expressed as a linear relation between the components of the spin connection. The final results can be organized in irreducible representations of the isotropy group of the Killing spinor.

The gauge group of IIA supergravity is  $\text{Spin}(9, 1)$  and the IIA supersymmetry parameter is in the 32-dimensional Majorana representation  $\Delta_{32}$ .  $\text{Spin}(9, 1)$  has several orbits on  $\Delta_{32}$  each giving a distinct local geometry.<sup>1</sup> There are four such orbits with isotropy groups  $\text{Spin}(7)$ ,  $\text{Spin}(7) \times \mathbb{R}^8$ ,  $\text{SU}(4)$  and  $G_2 \times \mathbb{R}^8$ . Here, we shall solve the KSEs for the first two of the orbits and the other two cases will be reported elsewhere. A representative of the first two orbits is

$$\epsilon = f(1 + e_{1234}) + g(e_5 + e_{12345}) . \tag{1.1}$$

If  $f, g \neq 0$  the spinor represents the  $\text{Spin}(7)$  orbit while if either  $f = 0$  or  $g = 0$ , the spinor represents the  $\text{Spin}(7) \times \mathbb{R}^8$  orbit. In the former case, one can further choose without loss of generality that  $f = \pm g$ . In the latter case, the  $f = 0$  and  $g = 0$  cases are symmetric, so without loss of generality we shall take  $g = 0$  and also set  $f = 1$  with a gauge transformation.

The existence of a  $\text{Spin}(7)$  or  $\text{Spin}(7) \times \mathbb{R}^8$  invariant Killing spinor imposes rather weak conditions on the geometry and these conditions have been summarized in equations (3.3) and (4.2), respectively. In particular,  $\text{Spin}(7)$  backgrounds admit two commuting vector fields one of which is timelike and Killing while the other is spacelike. The timelike Killing vector field leaves all the fields and the Killing spinor invariant. A more detailed analysis of the geometric conditions can be found in section 3.1, and the expression of all the fields in terms of the geometry can be found in section 3.2. It turns out that not all components

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<sup>1</sup>Because the holonomy of the supercovariant connection is  $\text{SL}(32, \mathbb{R})$ , and thus much larger than  $\text{Spin}(9, 1)$ , one expects that globally the Killing spinor will change orbit under  $\text{Spin}(9, 1)$  from patch to patch. Nevertheless solving the KSEs for each orbit captures the local geometry.

of the fluxes are expressed in terms of the geometry. We have also verified that our results contain those of [15] that describe the solution of the KSEs of common sector backgrounds admitting a Killing spinor with isotropy group Spin(7). The inclusion of the common sector Spin(7) backgrounds into the IIA Spin(7) ones is not a surprise as in both cases the Killing spinor representative is the same.

The geometric conditions imposed on the spacetime as a consequence of the existence of a Spin(7)  $\times$   $\mathbb{R}^8$  invariant Killing spinor are the same as those found in [12] for heterotic backgrounds admitting a Spin(7)  $\times$   $\mathbb{R}^8$  invariant Killing spinor. These are the most general heterotic backgrounds preserving one supersymmetry. Furthermore, the conditions imposed by the KSEs on the IIA dilaton and 3-form flux are the same as those of heterotic supergravity. Although a relation between the IIA Spin(7)  $\times$   $\mathbb{R}^8$  backgrounds and the heterotic Spin(7)  $\times$   $\mathbb{R}^8$  ones was expected as they both have the same Killing spinor representative, the property we proved, i.e. that the conditions on the geometry and the expression for the 3-form flux are the same in both cases, was not expected. As a result all solutions of IIA supergravity with a Spin(7)  $\times$   $\mathbb{R}^8$  invariant Killing spinor are extensions of those of heterotic supergravity that include the addition of 0-, 2- and 4-form fluxes.<sup>2</sup> We also give the expression of the remaining IIA fluxes in terms of the geometry. The conditions on the geometry as well as the expression of all the fields in terms of the geometry can be found in sections 4.1 and 4.2, respectively.

This paper has been organized as follows. In section 2, we state the KSEs, choose representatives for the Killing spinors and investigate some of the global properties of spacetime. In section 3, we solve the KSEs of IIA supergravity for Spin(7) invariant Killing spinors and give the conditions on the geometry of spacetime. In section 4, we solve the KSEs of IIA supergravity for Spin(7)  $\times$   $\mathbb{R}^8$  invariant spinors and give the conditions on the geometry of spacetime. In section 5, we state our conclusions. In appendices A and B, we collect various useful formulae such as the integrability conditions of the IIA KSEs, the expression for the supercovariant curvature as well as a selection of properties of Spin(7) representations. In appendix C, we give the form spinor bilinears of (1.1). In appendices D and E, we give the solutions of the linear systems for the Spin(7) and Spin(7)  $\times$   $\mathbb{R}^8$  cases, respectively.

## 2 Killing spinor equations and Killing spinor representatives

### 2.1 Killing spinor equations

The Killing spinor equations of type IIA supergravity [16–20] are the vanishing condition of the supersymmetry variations of the fermions evaluated at the locus where all fermions vanish. In the conventions of [21], the KSEs are given by the vanishing conditions of

$$\begin{aligned} \mathcal{D}_M \epsilon \equiv & \nabla_M \epsilon + \frac{1}{8} H_{MP_1 P_2} \Gamma^{P_1 P_2} \Gamma_{11} \epsilon + \frac{1}{8} e^\Phi \tilde{S} \Gamma_M \epsilon \\ & + \frac{1}{16} e^\Phi \tilde{F}_{P_1 P_2} \Gamma^{P_1 P_2} \Gamma_M \Gamma_{11} \epsilon + \frac{1}{8 \cdot 4!} e^\Phi \tilde{G}_{P_1 \dots P_4} \Gamma^{P_1 \dots P_4} \Gamma_M \epsilon, \end{aligned}$$

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<sup>2</sup>Using this, we conclude that there are IIA solutions with strictly one supersymmetry. Such a solution with a Spin(7)  $\times$   $\mathbb{R}^8$  invariant Killing spinor is a pp-wave propagating on an 8-dimensional manifold with Spin(7) holonomy.

$$\begin{aligned} \mathcal{A}\epsilon \equiv & \partial_P \Phi \Gamma^P \epsilon + \frac{1}{12} H_{P_1 P_2 P_3} \Gamma^{P_1 P_2 P_3} \Gamma_{11} \epsilon + \frac{5}{4} e^\Phi \tilde{S} \epsilon \\ & + \frac{3}{8} e^\Phi \tilde{F}_{P_1 P_2} \Gamma^{P_1 P_2} \Gamma_{11} \epsilon + \frac{1}{4 \cdot 4!} e^\Phi \tilde{G}_{P_1 \dots P_4} \Gamma^{P_1 \dots P_4} \epsilon, \end{aligned} \tag{2.1}$$

where  $\nabla$  is the spin connection,  $H$  is the NS-NS 3-form field strength,  $\tilde{S}, \tilde{F}, \tilde{G}$  are the RR  $k$ -form field strengths, and  $\Phi$  is the dilaton. For later convenience, we set

$$S = e^\Phi \tilde{S}, \quad F = e^\Phi \tilde{F}, \quad G = e^\Phi \tilde{G}. \tag{2.2}$$

The spinor  $\epsilon$  is in the Majorana representation of  $\text{Spin}(9, 1)$ . The first and second equations in (2.1) are associated with the gravitino and dilatino supersymmetry transformations, respectively. In particular, the first KSE is a parallel transport equation with respect to the supercovariant connection  $\mathcal{D}$  of the spin bundle while  $\mathcal{A}$  is an algebraic condition on the spinor  $\epsilon$ . In what follows, we shall seek solutions to the conditions  $\mathcal{D}\epsilon = \mathcal{A}\epsilon = 0$  without making simplifying assumptions on the fields, the Killing spinor or the geometry of spacetime.

## 2.2 Choice of Killing spinors

The holonomy group<sup>3</sup> of the supercovariant connection  $\mathcal{D}$  of generic type IIA backgrounds is  $\text{SL}(32, \mathbb{R})$ , while the gauge group of the KSEs is  $\text{Spin}(9, 1)$ . The former can be seen from the expression of the supercovariant curvature in appendix A. Backgrounds that are related by a gauge transformation have the same geometry, up to a choice of frame. Because of this, the different types of geometries that appear in supersymmetric backgrounds can be locally labeled by the orbits of the gauge group on the space of spinors. The gauge algebra acts on the Majorana representation of  $\text{Spin}(9, 1)$  and has four distinct orbits with isotropy algebras  $\text{Spin}(7)$ ,  $\text{Spin}(7) \times \mathbb{R}^8$ ,  $\text{SU}(4)$  and  $G_2 \times \mathbb{R}^8$ . The general analysis how all these orbits arise can be found in [26]. Here, we shall explain how the first two arise that we use for our analysis.

The Majorana representation  $\Delta_{32}$  of  $\text{Spin}(9, 1)$  decomposes into a chiral and an anti-chiral Majorana-Weyl representation as  $\Delta_{32} = \Delta_{16}^+ \oplus \Delta_{16}^-$ . So the Killing spinor can be written as  $\epsilon = \epsilon^+ + \epsilon^-$ . It is known that  $\text{Spin}(9, 1)$  has a single orbit in  $\Delta_{16}^+$  with isotropy algebra  $\text{Spin}(7) \times \mathbb{R}^8$ . Thus the  $\text{Spin}(7) \times \mathbb{R}^8$  orbit arises by asserting that  $\epsilon = \epsilon^+$ . In such case, a representative for the orbit can be chosen as

$$\epsilon = 1 + e_{1234}, \tag{2.3}$$

where we have used a realization of spinors in terms of forms in a basis described in either [12] or [26]. For the other 3 orbits  $\epsilon^- \neq 0$ . To see how that  $\text{Spin}(7)$  arises, observe that under  $\text{Spin}(7) \subset \text{Spin}(7) \times \mathbb{R}^8$ , the isotropy group of  $\epsilon^+$ , the  $\Delta_{16}^-$  representation decomposes as  $\Delta_{16}^- = \Delta_1 \oplus \Delta_7 \oplus \Delta_8$ . The  $\text{Spin}(7)$  orbit of  $\text{Spin}(9, 1)$  arises by asserting that  $\epsilon^+$  takes values in  $\Delta_{16}^+$  and  $\epsilon^-$  takes values in  $\Delta_1$ . A representative for the orbit can be chosen as

$$\epsilon = f(1 + e_{1234}) + g(e_5 + e_{12345}). \tag{2.4}$$

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<sup>3</sup>The holonomy group of the supercovariant connection of  $D = 11$  [22, 23] and IIB supergravities [24, 25] is also in  $\text{SL}(32, \mathbb{R})$ .

When  $g \neq 0$ , we can use the gauge symmetry, which is generated by boosts along the 5-th direction, to set  $f = \pm g$ , which leads to a significant simplification of the solution to the Killing spinor equations. We shall solve the KSEs for  $f = g$ . The solution corresponding to  $f = -g$  is obtained from the  $f = g$  one by, for each term, adding a sign for every plus and minus index appearing.<sup>4</sup>

### 2.3 Some global aspects

The existence of a Killing spinor, and in particular of a parallel spinor with respect to  $\mathcal{D}$ , implies that there is a global nowhere vanishing section of the spin bundle  $\mathcal{S}$ . The mere existence of a nowhere vanishing section of  $\mathcal{S}$  does not impose a topological condition on the spacetime manifold  $M$  as  $\mathcal{S}$  has rank 32 which is much larger than the dimension of  $M$ . So it always admits a nowhere vanishing section. Nevertheless,  $\mathcal{D}$  is a specific connection and the existence of a parallel section of  $\mathcal{S}$  with respect to  $\mathcal{D}$  may impose some additional conditions which are not apparent. For example, if the Killing spinor is restricted to lie in a certain sub-bundle of  $\mathcal{S}$ , because of a so far unknown property of the theory, then both the topology and geometry of  $M$  can be restricted. For example in the two cases we investigate here, if in addition we restrict the Killing spinor to have isotropy group  $\text{Spin}(7) \times \mathbb{R}^8$  or  $\text{Spin}(7)$  *everywhere* on the manifold  $M$ , then the structure group of  $M$  reduces to these isotropy groups. However in general, since the holonomy group of  $\mathcal{D}$  is  $\text{SL}(32, \mathbb{R})$ , even if the Killing spinor is chosen in one chart to lie in a given orbit of  $\text{Spin}(9, 1)$  it is expected that parallel transport will change the type of orbit when it crosses to another chart and so the structure group may not reduce. In the analysis that follows, we shall assume that there is a chart such that the isotropy group of the Killing spinor is either  $\text{Spin}(7) \times \mathbb{R}^8$  or  $\text{Spin}(7)$  and use this to solve the KSEs. Of course our results can be extended to the whole spacetime provided that the spinors maintain their type of orbit everywhere on spacetime.

## 3 Solution of the KSEs for the Spin(7) backgrounds

Having identified the Killing spinor, one can easily solve the KSEs using spinorial geometry. We have not given the linear system that arises from the KSEs. Instead, we have presented its solution in appendix D. This expresses the fluxes in terms of the geometry and identifies the conditions that restrict the geometry of spacetime. The latter are expressed as relations between the components of the spin connection. The results in appendix D have been written in  $\text{SU}(4)$  representations but they can be re-organized in irreducible  $\text{Spin}(7)$  representations.

### 3.1 Geometry of spacetime

To identify the geometry of spacetime, it is convenient to use the spinor bilinears associated with the  $\text{Spin}(7)$  invariant spinor  $\epsilon$ . These are explicitly stated in appendix B. To continue,

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<sup>4</sup>This corresponds to a reflection of time and one spatial coordinate (the 0 and 5 directions) which might not be a symmetry of the theory as the transformation is not part of the component of  $\text{Spin}(9, 1)$  which is connected to the identity element. However, here we just view the transformation as a solution generating transformation.

choose the basis in the space of form spinor bilinears<sup>5</sup> given by

$$K = f^2 e^0, \quad X = e^5, \quad \phi, \quad (3.1)$$

where  $\phi$  is the fundamental Spin(7) self-dual 4-form. Clearly  $K$  is time-like,  $X$  is space-like<sup>6</sup> and  $\phi$  is transverse to both  $K$  and  $X$ , i.e.  $i_K \phi = i_X \phi = 0$ .

It follows that the tangent space  $TM$  of spacetime  $M$  has two preferred directions and therefore decomposes, everywhere that  $f \neq 0$ , as  $TM = I^2 \oplus E$ , where  $I^2$  is a trivial bundle and  $E$  is a rank 8 bundle which consists of the directions transverse to  $X$  and  $K$ . Introducing a frame adapted to this splitting of  $M$  as  $(e^0, e^5, e^i)$ , the spin connection  $\nabla$  decomposes in various components. We define

$$\nabla_i^{(8)} Y^j = \partial_i Y^j + \Omega_{i,j}^k Y^k \quad (3.2)$$

i.e.  $\nabla^{(8)}$  denotes the component of  $\nabla$  for which both spacetime and frame indices are restricted along  $E$ .

The conditions on the geometry of spacetime imposed by the solution of KSEs for a spin(7) invariant Killing spinor are

$$\begin{aligned} \mathcal{L}_K g &= 0, \quad \mathcal{L}_K \epsilon = 0, \\ \mathcal{L}_X \epsilon &= -\frac{f^{-4}}{4} K_A (i_X dK)_B \Gamma^{AB} \epsilon + (X \log f^2) \epsilon - \frac{1}{4} X_A (i_X dX)_B \Gamma^{AB} \epsilon, \\ \partial_5 \Phi &= \frac{1}{2} \theta_5 + \partial_5 \log f^2, \\ \partial_i \Phi &= \frac{3}{4} \theta_i + \frac{3}{4} \partial_i \log f^2 - \frac{1}{4} (de^5)_{5i}, \end{aligned} \quad (3.3)$$

where the spinorial Lie derivative is defined as

$$\mathcal{L}_K \epsilon = \nabla_K \epsilon + \frac{1}{4} \nabla_A K_B \Gamma^{AB} \epsilon = 0, \quad (3.4)$$

and similarly for  $\mathcal{L}_X$ , and

$$\theta_i = -\frac{1}{36} \nabla^{(8)m} \phi_{mk_1 k_2 k_3} \phi^{k_1 k_2 k_3}{}_i, \quad \theta_5 = -\frac{1}{42} \phi^{k_1 k_2 k_3 k_4} \nabla_{k_1} \phi_{5 k_2 k_3 k_4}. \quad (3.5)$$

The 1-form  $\theta_i$  is defined in analogy to the Lee form for manifolds with a Spin(7) structure. The remaining conditions which arise from the KSEs express the IIA fluxes in terms of the geometry and the corresponding expression will be given in the next section.

The first geometric condition in (3.3) implies that  $K$  is a Killing vector. In fact  $K$  leaves all the fields of the theory invariant, i.e.

$$\mathcal{L}_K \Phi = \mathcal{L}_K S = \mathcal{L}_K F = \mathcal{L}_K H = \mathcal{L}_K G = 0. \quad (3.6)$$

In addition, the second condition in (3.3) implies that the Killing spinor  $\epsilon$  is invariant under the motion generated by  $K$ .  $X$  is not Killing but commutes with  $K$

$$[X, K] = 0, \quad (3.7)$$

<sup>5</sup>Note that the bilinear in fact is  $f^2 e^5$ , but it is convenient to instead choose  $X = e^5$  for the basis.

<sup>6</sup>From now on, we shall denote both the 1-forms  $K$  and  $X$  and their associated vectors with the same symbol.

as a consequence of the first three conditions in (3.3). As a result one can adapt coordinates on the spacetime independently for both  $K$  and  $X$ . The third condition in (3.3) also implies that  $X$  leaves the Killing spinor  $\epsilon$  invariant up to possible rotations which always have an  $X$  or  $K$  direction. The remaining two conditions in (3.3) can be seen as a generalization of the well-known conformal balance condition which arises from the dilatino KSE in the context of heterotic supergravity [12]. However, the numerical coefficient is not the standard one.

It is straightforward to do a Gray-Hervella type of decomposition for 10-dimensional manifolds with a Spin(7) structure. In such a decomposition two of the classes would have been represented by  $\theta_i$  and  $\theta_5$ , so the conditions in (3.3) imply that both are restricted. Note that the Lee form  $\theta_i$  that appears also represents one of the Gray-Hervella classes of 8-dimensional manifolds with a Spin(7) structure [27, 28].

### 3.2 Fields in terms of geometry

To express the field strengths of supersymmetric IIA backgrounds in terms of the geometry, it is convenient to first decompose the space of forms along the  $e^0$  and  $e^5$  directions and the rest, and then further decompose the directions transverse to  $e^0$  and  $e^5$  in terms of irreducible representations of  $\mathfrak{spin}(7)$ . For the latter, we use that the space of 2-, 3- and 4-forms on  $\mathbb{R}^8$  decompose in irreducible Spin(7) representations as

$$\Lambda^2(\mathbb{R}^8) = \Lambda^7 \oplus \Lambda^{21}, \quad \Lambda^3(\mathbb{R}^8) = \Lambda^8 \oplus \Lambda^{48}, \quad \Lambda^4(\mathbb{R}^8) = \Lambda^1 \oplus \Lambda^7 \oplus \Lambda^{27} \oplus \Lambda_-^{35}, \quad (3.8)$$

where  $\Lambda_-^{35}$  denotes the 35-dimensional subspace of anti-self dual 4-forms.

Using this, we can write

$$F = F_{(0)} e^0 \wedge e^5 + e^0 \wedge F_{0(1)} + e^5 \wedge F_{5(1)} + F_{(2)}, \quad (3.9)$$

where

$$F_{(2)} = \frac{1}{2} F_{ij} e^i \wedge e^j = F^7 + F^{21} \quad (3.10)$$

and similarly

$$\begin{aligned} H &= e^0 \wedge e^5 \wedge H_{(1)} + e^0 \wedge (H_{0(2)}^7 + H_{0(2)}^{21}) + e^5 \wedge (H_{5(2)}^7 + H_{5(2)}^{21}) + H_{(3)}, \\ H_{(3)} &= \frac{1}{3!} H_{ijk} e^i \wedge e^j \wedge e^k = H^8 + H^{48}, \\ G &= e^0 \wedge e^5 \wedge (G_{(2)}^7 + G_{(2)}^{21}) + e^0 \wedge (G_{0(3)}^8 + G_{0(3)}^{48}) + e^5 \wedge (G_{5(3)}^8 + G_{5(3)}^{48}) + G_{(4)}, \\ G_{(4)} &= \frac{1}{4!} G_{ijkl} e^i \wedge e^j \wedge e^k \wedge e^l = G^1 + G^7 + G^{27} + G^{35}, \end{aligned} \quad (3.11)$$

where the number in the parenthesis is the degree of the forms in the directions transverse to  $(e^0, e^5)$ .

Typically, the KSEs express some of the components of the fluxes in the above decompositions in terms of the geometry. As we shall see several components remain unconstrained. In particular, we find that

$$\begin{aligned} F &= - \left( S + \frac{1}{2} \theta_5 \right) e^0 \wedge e^5 + e^0 \wedge \left( -\frac{3}{4} \theta + \frac{1}{4} d_{(8)} \log f^2 + \frac{1}{4} (de^5)_{5(1)} \right) \\ &\quad - e^5 \wedge (de^0)_{5(1)} + \frac{1}{96} \phi_i^{k_1 k_2 k_3} \nabla_0 \phi_{j k_1 k_2 k_3} e^i \wedge e^j + F^{21} \end{aligned} \quad (3.12)$$



$$\begin{aligned}
H &= -d(e^0 \wedge e^5) + e^5 \wedge \tilde{H}_{5(2)}^{21} + H_{(3)} \\
G &= e^0 \wedge e^5 \wedge ((de^0)_{(2)}^7 - \tilde{H}_{5(2)}^{21} - F_{(2)}^{21}) - e^0 \wedge H_{(3)} \\
&\quad + e^5 \wedge \star_{(8)} \left( \left( \frac{1}{4} [d_{(8)} \log f^2 + (de^5)_{5(1)}] - \frac{3}{4} \theta \right) \wedge \phi + d\phi \right) + G_{(4)}, \quad (3.13)
\end{aligned}$$

where  $\tilde{H}_{5(2)}^{21} = H_{5(2)}^{21} + (de^0)_{(2)}^{21}$  and<sup>7</sup>

$$\begin{aligned}
G_{(4)} &= -\frac{1}{14} (\theta_5 + 4\partial_5 \log f^2) \phi + \frac{1}{4} i_\phi de_{(2)}^5 + G^{27} \\
&\quad - \frac{1}{21 \cdot 4!} \left( \phi_{(i_1}{}^{k_1 k_2 k_3} \nabla_m \phi_{5 k_1 k_2 k_3} + \frac{21}{4} \theta_5 \delta_{m i_1} \right) \phi^m{}_{i_2 i_3 i_4} e^{i_1} \wedge e^{i_2} \wedge e^{i_3} \wedge e^{i_4} \quad (3.14)
\end{aligned}$$

where  $d_{(8)}$  and  $\star_{(8)}$  are the exterior derivative and the Hodge star operation along the directions transverse to  $(e^0, e^5)$ , respectively.

The components that have not been expressed in terms of the geometry, like  $F^{21}$ ,  $H_{(3)}$  and others, are not constrained by the KSEs. The above expressions for the field strengths together with the conditions on the geometry are the full content of the KSEs admitting as a solution a Spin(7) invariant spinor.

As a further confirmation of our result, one can compare them with those found in [15] for the common sector. It is straightforward to verify that when the additional fields of IIA supergravity are set to zero, both the geometric conditions and the expressions of the fluxes in terms of the geometry become those of common sector backgrounds which admit one Spin(7)-invariant Killing spinor.

## 4 Solution of the KSEs of Spin(7) $\times$ $\mathbb{R}^8$ backgrounds

### 4.1 Geometry of spacetime

As in the previous case, to describe the geometry one finds that the Killing spinor bilinears of a Spin(7)  $\times$   $\mathbb{R}^8$  invariant spinor are

$$e^-, \quad e^- \wedge \phi. \quad (4.1)$$

These have been computed in appendix B, where  $\phi$  is the fundamental self-dual 4-form of Spin(7).

The conditions imposed on the geometry by the existence of a Spin(7)  $\times$   $\mathbb{R}^8$  Killing spinor are

$$\mathcal{L}_K g = 0, \quad \nabla_K \epsilon = 0, \quad dK \in \mathfrak{spin}(7, \mathbb{R}) \oplus_{\mathfrak{s}} \mathbb{R}^8. \quad (4.2)$$

and

$$2\partial_i \Phi = \theta_i - (de^-)_{-i}, \quad (4.3)$$

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<sup>7</sup>We have defined  $i_\phi \psi = \frac{k}{3! \cdot k!} \phi^m{}_{i_1 i_2 i_3} \psi_{m i_4 \dots i_{k+2}} e^{i_1} \wedge \dots \wedge e^{i_{k+2}}$  and  $\star_{(8)} \psi = \frac{1}{k! \cdot (8-k)!} \psi_{j_1 \dots j_k} \epsilon^{j_1 \dots j_k}{}_{i_1 \dots i_{8-k}} e^{i_1} \wedge \dots \wedge e^{i_{8-k}}$ .

where  $K = e^-$  and  $\theta$  is the Spin(7) Lee form as in (3.5). These conditions imply that  $\mathcal{L}_K \epsilon = 0$  as in the Spin(7) case.

The first condition in (4.2) implies that  $K$  is Killing. In fact  $K$  leaves all the fields invariant including the dilaton, i.e.

$$\mathcal{L}_K \Phi = \mathcal{L}_K S = \mathcal{L}_K F = \mathcal{L}_K H = \mathcal{L}_K G = 0 . \quad (4.4)$$

All the conditions on the geometry that arise from the KSEs are identical to those found in [12] in the context of heterotic backgrounds preserving one supersymmetry. In particular, (4.3) is precisely the condition found for the solution of the dilatino KSE in heterotic theory. Furthermore, we shall see below that the 3-form field strength is precisely that one finds as a solution of the KSEs for heterotic backgrounds preserving one supersymmetry. Therefore in this case, the Killing spinor of the IIA theory is parallel with respect to a connection  $\hat{\nabla}$  with skew-symmetric torsion,

$$\hat{\nabla} \epsilon = 0 , \quad (4.5)$$

where the skew-symmetric torsion is the 3-form field strength  $H$ .

## 4.2 Fields in terms of geometry

To express the form field strengths in terms of the geometry of spacetime, we introduce a light-cone frame  $(e^-, e^+, e^i)$  as for the heterotic backgrounds in [12]. Then we decompose the fields as in the previous Spin(7) case with the  $(e^0, e^5)$  frames replaced by  $(e^-, e^+)$ . Using these decompositions, the solution of the linear system can be organized as

$$\begin{aligned} F &= S e^- \wedge e^+ + e^- \wedge F_{-(1)} + F_{(2)} , \\ H &= d e^- \wedge e^+ - \frac{1}{4 \cdot 4!} \phi_i^{k_1 k_2 k_3} \nabla_- \phi_{j k_1 k_2 k_3} e^- \wedge e^i \wedge e^j + e^- \wedge H_{-(2)}^{21} + H_{(3)} , \\ G &= F_{(2)} \wedge e^- \wedge e^+ + e^- \wedge \left( \frac{1}{7} \star_{(8)} (F_{-(1)} \wedge \phi) + G_{-(3)}^{48} \right) + G_{(4)} , \end{aligned} \quad (4.6)$$

where

$$\begin{aligned} H_{(3)} &= - \star_{(8)} d\phi + \star_{(8)} (\theta \wedge \phi) , \\ G_{(4)} &= \frac{1}{4} i_\phi F_{(2)}^7 + G^{27} . \end{aligned} \quad (4.7)$$

The fields that have not been expressed in terms of the geometry, like  $F_{-(1)}$ ,  $F_{(2)}$  and others, are not restricted by the KSEs.

As has already been mentioned, these solutions are an extension of the heterotic backgrounds which preserve one supersymmetry and satisfy  $dH = 0$ . So we can take any solution of the KSEs of the heterotic theory, including those that preserve more than one supersymmetry, and add on them the additional IIA field strengths as described in (4.6). In such a case, the solution of the KSEs of IIA supergravity will be automatically satisfied. Then to find a supergravity solution, it will remain to solve the IIA field equations and Bianchi identities.

## 5 Conclusions

There are four types of supersymmetric IIA backgrounds that preserve one supersymmetry which are locally characterized by the four orbits of  $\text{Spin}(9,1)$  in the 32-dimensional Majorana spinor representation. We have solved the KSEs for two of these orbits with isotropy groups  $\text{Spin}(7)$  and  $\text{Spin}(7) \ltimes \mathbb{R}^8$  without making any additional assumptions on the fields. We have found all geometric conditions imposed on the spacetime and the expression of the fluxes in terms of the geometry. We have also verified that when our results for backgrounds with a  $\text{Spin}(7)$  invariant Killing spinor are restricted to the common sector, then they coincide with those found in [15] for the common sector backgrounds admitting a  $\text{Spin}(7)$  invariant Killing spinor. Furthermore, the IIA solutions of the KSEs with a  $\text{Spin}(7) \ltimes \mathbb{R}^8$  invariant Killing spinor are an extension of the heterotic backgrounds preserving at least one supersymmetry. In particular, the IIA KSEs imply exactly the same conditions on the geometry of spacetime and give the same expression for the 3-form field strength  $H$  in terms of the geometry as the heterotic KSEs. As a result all solutions of the heterotic KSEs can be embedded in the IIA backgrounds which admit a  $\text{Spin}(7) \ltimes \mathbb{R}^8$  invariant Killing spinor.

To find all solutions of the KSEs of type II and  $D = 11$  supergravities remains an open problem. It is not known in general what fractions of supersymmetry supergravity backgrounds preserve or what their geometry is. For a suggestion about the former see [29]. The only 10- or 11-dimensional supergravity for which both the possible fractions of preserved supersymmetry and the geometry of the backgrounds is known is the heterotic theory [12, 13]. However the techniques applied to solve the problem in the heterotic theory are not directly applicable in the type II and  $D = 11$  supergravities. Nevertheless, there are methods that can be applied to find the geometry of supersymmetric backgrounds of type II and  $D = 11$  theories with a very small number of supersymmetries [4–6]. Moreover, it is possible to classify the backgrounds which preserve a large number of supersymmetries [7–10]. To complete the task to find the geometry of all IIA backgrounds that preserve one supersymmetry, it remains to solve the KSEs for the remaining two orbits with isotropy groups  $\text{SU}(4)$  and  $G_2 \ltimes \mathbb{R}^8$ . This will be presented elsewhere.

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## A Conventions and integrability conditions

Our IIA supergravity conventions including the form of KSEs, apart from relabeling some of the fields, are similar to those of [21]. For spinors, we use the same conventions as those employed in type IIB supergravity in the context of spinorial geometry, cf. [5], e.g. we choose  $\Gamma_{11} = -\Gamma_{01\dots 9}$ .

It is well-known that the integrability conditions of the KSEs imply some of the field equations and Bianchi identities. Because of this, it is useful to derive the field equations and Bianchi identities from the integrability conditions of the KSEs.<sup>8</sup> For this, the integrability conditions of the KSEs (2.1) can be written as

$$[\mathcal{D}_M, \mathcal{A}] \epsilon = 0 \tag{A.1}$$

and

$$[\mathcal{D}_M, \mathcal{D}_N] \epsilon \equiv \mathcal{R}_{MN} \epsilon = 0 \tag{A.2}$$

where the explicit expression for the supercovariant curvature  $\mathcal{R}_{MN}$  is given below. It is a well-known property of supergravity that after a judicious  $\Gamma$ -trace of the above integrability conditions, one gets a linear expression in terms of the field equations and Bianchi identities of the theory. In particular, we find<sup>9</sup>

$$\begin{aligned} \mathcal{I} \epsilon &= \Gamma^M [\mathcal{D}_M, \mathcal{A}] \epsilon \\ &= \left( F\Phi - FG_{(3)}\Gamma^{(3)} + BG_{(5)}\Gamma^{(5)} \right) \epsilon \\ &\quad + \left( -3FF_{(1)}\Gamma^{(1)} + FH_{(2)}\Gamma^{(2)} + BF_{(3)}\Gamma^{(3)} + 2BH_{(4)}\Gamma^{(4)} \right) \Gamma_{11} \epsilon \end{aligned} \tag{A.3}$$

and

$$\begin{aligned} \mathcal{I}_M \epsilon &= \Gamma^N [\mathcal{D}_M, \mathcal{D}_N] \epsilon \\ &= \left( -\frac{1}{2}E_{M(1)}\Gamma^{(1)} - \frac{1}{4}E_P{}^P\Gamma_M + \frac{1}{2}F\Phi\Gamma_M + FG_{(3)}\Gamma^{(3)}{}_M - 5BG_{M(4)}\Gamma^{(4)} \right) \epsilon \\ &\quad + \left( FH_{M(1)}\Gamma^{(1)} + FF_{(1)}\Gamma^{(1)}{}_M - BF_{M(2)}\Gamma^{(2)} + \frac{1}{3}BH_{M(3)}\Gamma^{(3)} + BH_{(4)}\Gamma^{(4)}{}_M \right) \Gamma_{11} \epsilon \end{aligned} \tag{A.4}$$

where we have defined

$$\begin{aligned} E_{MN} &:= R_{MN} - \frac{1}{12}G_{M(3)}G_N{}^{(3)} + \frac{1}{96}g_{MN}G_{(4)}G^{(4)} + \frac{1}{4}g_{MN}S^2 \\ &\quad - \frac{1}{4}H_{M(2)}H_N{}^{(2)} - \frac{1}{2}F_{MP}F_N{}^P + \frac{1}{8}g_{MN}F_{(2)}F^{(2)} + 2D_M\partial_N\phi, \end{aligned} \tag{A.5}$$

$$F\Phi := \square\Phi - 2(\partial\Phi)^2 - \frac{3}{8}F_{(2)}F^{(2)} - \frac{1}{96}G_{(4)}G^{(4)} + \frac{1}{12}H_{(3)}H^{(3)} - \frac{5}{4}S^2, \tag{A.6}$$

$$\begin{aligned} FH_{MN} &:= \frac{1}{4} \left( D^P H_{MNP} - 2(\partial^P \Phi) H_{MNP} - \frac{1}{2} G_{MN(2)} F^{(2)} \right. \\ &\quad \left. - F_{MNS} + \frac{1}{1152} \epsilon_{MN(4)(4)} G^{(4)} G^{(4)} \right), \end{aligned} \tag{A.7}$$

$$FF_M := \frac{1}{4} \left( D^P F_{MP} - (\partial^P \Phi) F_{MP} + \frac{1}{6} G_{M(3)} H^{(3)} \right), \tag{A.8}$$

$$FG_{M_1 M_2 M_3} := \frac{1}{4!} \left( D^P G_{M_1 M_2 M_3 P} - (\partial^P \Phi) G_{M_1 M_2 M_3 P} - \frac{1}{144} \epsilon_{M_1 M_2 M_3 (3)(4)} H^{(3)} G^{(4)} \right) \tag{A.9}$$

$$BH_{M_1 \dots M_4} := \frac{1}{4!} D_{[M_1} H_{M_2 M_3 M_4]}, \tag{A.10}$$

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<sup>8</sup>In order to perform the necessary computations the Mathematica package GAMMA [30] has been used.

<sup>9</sup>In the formulae below ( $n$ ) denotes  $n$  contracted indices.

$$BF_{M_1 M_2 M_3} := \frac{3}{8}(D_{[M_1} F_{M_2 M_3]} - (\partial_{[M_1} \Phi) F_{M_2 M_3]} - \frac{1}{3} H_{M_1 M_2 M_3} S), \quad (\text{A.11})$$

$$BG_{M_1 \dots M_5} := \frac{1}{4 \cdot 4!}(D_{[M_1} G_{M_2 \dots M_5]} - (\partial_{[M_1} \Phi) G_{M_2 \dots M_5]} - 2F_{[M_1 M_2} H_{M_3 M_4 M_5]}) \quad (\text{A.12})$$

where

$$S = e^\Phi \tilde{S}, \quad F = e^\Phi \tilde{F}, \quad G = e^\Phi \tilde{G}. \quad (\text{A.13})$$

For completeness, the formula for the supercovariant curvature is

$$R_{MN} = [\mathcal{D}_A, \mathcal{D}_B] \epsilon = R_{MN}^1 \epsilon + R_{MN}^2 \Gamma_{11} \epsilon, \quad (\text{A.14})$$

where

$$\begin{aligned} R_{MN}^1 = & \frac{1}{32} S^2 \Gamma_{MN} - \frac{1}{4} \Gamma_{[M} D_{N]} S + \frac{1}{64} \Gamma_{MN} F_{(2)} F^{(2)} - \frac{1}{16} \Gamma_{[M}^{P_1} F_{N]}^{P_2} F_{P_1 P_2} \\ & - \frac{1}{128} \Gamma_{MN}^{(2)(\tilde{2})} F_{(2)} F_{(\tilde{2})} + \frac{1}{32} \Gamma_{[M}^{P_1 P_2 P_3} F_{N] P_1} F_{P_2 P_3} + \frac{1}{384} \Gamma_{MN}^{(4)} S G_{(4)} \\ & - \frac{1}{96} \Gamma_{[M}^{(4)} D_{N]} G_{(4)} + \frac{1}{768} \Gamma_{MN} G_{(4)} G^{(4)} - \frac{1}{96} \Gamma_{[M}^{(3)} S G_{N](3)} - \frac{1}{24} \Gamma^{(3)} D_{[M} G_{N](3)} \\ & + \frac{1}{96} \Gamma^{P(3)} G_{MNP}{}^Q G_{(3)Q} - \frac{1}{96} \Gamma_{[M}^P G_{N]}^{(3)} G_{P(3)} - \frac{1}{256} \Gamma_{MN}^{(2)(\tilde{2})} G_{(2)Q_1 Q_2} G_{(\tilde{2})}{}^{Q_1 Q_2} \\ & + \frac{1}{64} \Gamma_{[M}^{P_1 P_2 P_3} G_{N] P_1}{}^{Q_1 Q_2} G_{P_2 P_3 Q_1 Q_2} + \frac{1}{192} \Gamma_{[M}^{(2)(3)} G_{N](2)}{}^Q G_{(3)Q} \\ & + \frac{1}{18432} \Gamma_{MN}^{(4)(\tilde{4})} G_{(4)} G_{(\tilde{4})} - \frac{1}{2304} \Gamma_{[M}^{(3)(4)} G_{N](3)} G_{(4)} - \frac{1}{16} \Gamma_{[M} F_{(2)} H_{N](2)} \\ & + \frac{1}{32} \Gamma_{[M}^{(2)(\tilde{2})} H_{N](2)} F_{(\tilde{2})} - \frac{1}{16} \Gamma^{P_1 P_2 P_3} F_{[M|P_1} H_{N]P_2 P_3} - \frac{1}{8} \Gamma^{P_1 P_2} H_{MP_1}{}^Q H_{NP_2}{}^Q \\ & - \frac{1}{8} \Gamma^P F_P{}^Q H_{MNQ} + \frac{1}{4} \Gamma^{(2)} R_{MN(2)} \end{aligned} \quad (\text{A.15})$$

and

$$\begin{aligned} R_{MN}^2 = & -\frac{1}{8} \Gamma_{[M}^{(2)} D_{N]} F_{(2)} + \frac{1}{16} \Gamma_{[M}^P S F_{N]P} - \frac{1}{4} \Gamma^P D_{[M} F_{N]P} + \frac{1}{16} S F_{MN} \\ & - \frac{1}{96} \Gamma_{[M}^{(3)} F_{N]}{}^Q G_{(3)Q} + \frac{1}{384} \Gamma_{[M}^{P(4)} F_{N]P} G_{(4)} + \frac{1}{384} \Gamma^{(4)} F_{MN} G_{(4)} \\ & - \frac{1}{96} \Gamma_{MN}{}^{P(3)} F_P{}^Q G_{(3)Q} + \frac{1}{32} \Gamma_{[M}^P F^{(2)} G_{N]P(2)} - \frac{1}{192} \Gamma_{[M}^{(2)(3)} F_{(2)} G_{N](3)} \\ & + \frac{1}{32} \Gamma_{[M}^{P_1 P_2 P_3} F_{P_1}{}^Q G_{N]P_2 P_3} + \frac{1}{32} F^{(2)} G_{MN(2)} - \frac{1}{64} \Gamma^{(2)(\tilde{2})} F_{(2)} G_{MN(\tilde{2})} \\ & + \frac{1}{16} \Gamma_{[M}^{(2)} S H_{N](2)} + \frac{1}{4} \Gamma^{(2)} D_{[M} H_{N](2)} + \frac{1}{384} \Gamma_{[M}^{(2)(4)} H_{N](2)} G_{(4)} \\ & + \frac{1}{96} \Gamma^{(2)(3)} H_{(2)[M} G_{N](3)} - \frac{1}{16} \Gamma^P G_{P[M}^{(2)} H_{N](2)} - \frac{1}{32} \Gamma_{[M}^{(2)} H_{N]}^{(\tilde{2})} G_{(2)(\tilde{2})} \\ & - \frac{1}{48} \Gamma^{(3)} H_{MN}{}^Q G_{(3)Q} \end{aligned} \quad (\text{A.16})$$

It can be easily seen from this that the holonomy of the supercovariant connection for generic IIA backgrounds is contained in  $\text{SL}(32, \mathbb{R})$ .

## B Spin(7) formulae

### B.1 Fundamental form

The linear system that arises from the solution of the KSEs in the context of spinorial geometry is written in SU(4) representations. However, this can be re-organized in Spin(7) representations. For this we have used the fundamental self-dual Spin(7) invariant 4-form

$$\phi = \frac{1}{4!} \phi_{i_1 i_2 i_3 i_4} e^{i_1} \wedge e^{i_2} \wedge e^{i_3} \wedge e^{i_4} = \text{Re } \chi - \frac{1}{2} \omega \wedge \omega, \quad (\text{B.1})$$

where

$$\omega = -(e^1 \wedge e^6 + \dots e^4 \wedge e^9), \quad \chi = (e^1 + ie^6) \wedge \dots (e^4 + ie^9).$$

In the Hermitian basis

$$e^a = \frac{1}{\sqrt{2}}(e^a + ie^{a+5}), \quad e^{\bar{a}} = \frac{1}{\sqrt{2}}(e^a - ie^{a+5}), \quad a = 1, \dots, 4 \quad (\text{B.2})$$

of SU(4) representations, we have

$$\omega_{\alpha\bar{\beta}} = -i\delta_{\alpha\bar{\beta}}, \quad \chi_{\alpha_1\alpha_2\alpha_3\alpha_4} = 4\epsilon_{\alpha_1\alpha_2\alpha_3\alpha_4}, \quad (\text{B.3})$$

and

$$\phi_{\alpha\bar{\beta}\gamma\bar{\delta}} = \delta_{\alpha\bar{\beta}}\delta_{\gamma\bar{\delta}} - \delta_{\gamma\bar{\beta}}\delta_{\alpha\bar{\delta}}, \quad \phi_{\alpha_1\alpha_2\alpha_3\alpha_4} = 2\epsilon_{\alpha_1\alpha_2\alpha_3\alpha_4}. \quad (\text{B.4})$$

The contractions of  $\phi$  are

$$\begin{aligned} \phi_{i_1 l_1 l_2 l_3} \phi^{j_1 l_1 l_2 l_3} &= 42\delta_i^j, \\ \phi_{i_1 i_2 l_1 l_2} \phi^{j_1 j_2 l_1 l_2} &= -4\phi_{i_1 i_2}^{j_1 j_2} + 12\delta_{[i_1 i_2]}^{j_1 j_2}, \\ \phi_{i_1 i_2 i_3 l} \phi^{j_1 j_2 j_3 l} &= -9\delta_{[i_1}^{j_1} \phi_{i_2 i_3]}^{j_2 j_3} + 6\delta_{[i_1 i_2 i_3]}^{j_1 j_2 j_3} \end{aligned} \quad (\text{B.5})$$

where

$$\delta_{[j_1 j_2 \dots j_n]}^{i_1 i_2 \dots i_n} = \delta^{[i_1}_{[j_1} \delta^{i_2}_{j_2} \dots \delta^{i_n]}_{j_n]}. \quad (\text{B.6})$$

Using that  $\phi$  is constant in the local Lorentz frame we get

$$\nabla_A \phi_{B_1 B_2 B_3 B_4} = 4\Omega_{A, [B_1}^C \phi_{C | B_2 B_3 B_4]}, \quad (\text{B.7})$$

which can be used to express the spin connection in terms of  $\nabla\phi$  as follows:

$$g^{AB_4} \nabla_A \phi_{B_1 B_2 B_3 B_4} = -\Omega^C{}_{,C1} \phi^l{}_{B_1 B_2 B_3} + 3\Omega_{l_1, l_2 [B_1} \phi^{l_1 l_2}{}_{B_2 B_3]}, \quad (\text{B.8})$$

$$\frac{1}{4!} \phi^{j_1 j_2 j_3 j_4} \nabla_{j_1} \phi_{i j_2 j_3 j_4} = -\Omega^l{}_{,li} - \frac{1}{2} \Omega_{l_1, l_2 l_3} \phi^l{}_{i l_1 l_2 l_3}, \quad (\text{B.9})$$

$$\frac{1}{3!} \phi^{j_1 j_2 j_3 j_4} \nabla_{j_1} \phi_{a j_2 j_3 j_4} = -7\Omega^l{}_{,la}, \quad (\text{B.10})$$

$$\frac{1}{4!} \phi_k{}^{j_1 j_2 j_3} \nabla_A \phi_{i j_1 j_2 j_3} = \Omega_{A, ik} - \frac{1}{2} \Omega_{A, l_1 l_2} \phi^{l_1 l_2}{}_{ik}, \quad (\text{B.11})$$

$$\frac{1}{3!}\phi_k^{j_1j_2j_3}\nabla_A\phi_{bj_1j_2j_3} = 7\Omega_{A,bk}, \quad (\text{B.12})$$

$$\begin{aligned} \frac{1}{8}\phi_{[k}^{l_1l_2l_3}\nabla_{|l_1|}\phi_{j_1j_2]l_2l_3} &= -\Omega_{[j_1,j_2k]} + \frac{1}{2}\Omega_{[j_1,l_1l_2]}\phi^{l_1l_2}_{j_2k]} \\ &\quad -\frac{3}{4}\Omega_{l_1,l_2[j_1}\phi^{l_1l_2}_{j_2k]} + \frac{1}{4}\Omega^{l_1,l_1l_2}\phi^{l_2}_{j_1j_2k}, \end{aligned} \quad (\text{B.13})$$

$$\frac{1}{3!}\phi_k^{l_1l_2l_3}\nabla_{l_1}\phi_{ail_2l_3} = \Omega_{i,ak} + \delta_{ik}\Omega^l_{,la} - \frac{2}{3}\Omega_{l_1,l_2a}\phi^{l_1l_2}_{ik}, \quad (\text{B.14})$$

where  $a, b, \dots \in \{0, 5\}$  or  $\{-, +\}$ .

## B.2 Decomposition of Spin(7) representations

As we have explained, the fields and geometric conditions can be decomposed in irreducible Spin(7) representations. In this paper, we have used the decomposition of  $\Lambda^2(\mathbb{R}^8)$ ,  $\Lambda^3(\mathbb{R}^8)$  and  $\Lambda^4(\mathbb{R}^8)$  in terms of Spin(7) representations. The result is stated in (3.8). To perform the computation, we have used the projections

$$\begin{aligned} \psi_{ij}^{(7)} &\equiv \frac{1}{4}\left(\psi_{[ij]} - \frac{1}{2}\phi_{ijkl}\psi^{kl}\right) \\ \psi_{ij}^{(21)} &\equiv \frac{1}{4}\left(3\psi_{[ij]} + \frac{1}{2}\phi_{ijkl}\psi^{kl}\right), \end{aligned} \quad (\text{B.15})$$

associated to the decomposition of  $\psi \in \Lambda^2(\mathbb{R}^8) = \Lambda^7 \oplus \Lambda^{21}$  with  $\Lambda^{21} = \mathfrak{spin}(7)$ .

We have also found convenient to write the decompositions expressed in (3.8) as

$$\psi^2 = \frac{1}{2}v^a(\gamma_a)_{ij}e^i \wedge e^j + \frac{1}{2}\chi_{ij}e^i \wedge e^j, \quad \chi_{ij}\gamma_a^{ij} = 0, \quad (\text{B.16})$$

$$\psi^3 = \frac{1}{3!}w_m\phi^m_{ijk}e^i \wedge e^j \wedge e^k + \frac{1}{3!}v_{ai}\gamma_{jk}^ae^i \wedge e^j \wedge e^k, \quad v_{ai}\gamma^{ai}_j = 0, \quad (\text{B.17})$$

$$\psi^4 = z\phi + \frac{1}{4!}v^a\gamma_{aim}\phi^m_{jkl}e^i \wedge e^j \wedge e^k \wedge e^l + \frac{1}{4!}z_{ab}\gamma_{ij}^a\gamma_{kl}^be^i \wedge e^j \wedge e^k \wedge e^l \quad (\text{B.18})$$

$$+ \frac{1}{4!}w_{im}\phi^m_{jkl}e^i \wedge e^j \wedge e^k \wedge e^l, \quad z_{ab} = z_{(ab)}, \quad z_a^a = 0; \quad w_{ij} = w_{(ij)}, \quad w_i^i = 0, \quad (\text{B.19})$$

where we have used that the Spin(7) gamma matrices  $\gamma_a$ ,  $a, b = 1, \dots, 7$ , give an isomorphism between  $\mathbb{R}^7$  and  $\Lambda^7$ , i.e.  $\gamma_a$  as 2-forms in  $\Lambda^2(\mathbb{R}^8)$  lie in the 7-dimensional representation. It is easy to observe then from the above expressions that the **48** representation is associated with a gamma-traceless gravitino, the **28** representation is the symmetric traceless vector representation, and the **35** representation is the symmetric traceless spinor representation.

## C The form spinor bilinears of Spin(7) and Spin(7) $\times$ $\mathbb{R}^8$ spinors

The IIA spinors are Majorana, so for the computation of the bilinears one can use either the Majorana or Dirac inner products. The conventions for these can be found in [5]. In IIA supergravity apart from the Killing spinor  $\epsilon$ , one can define another globally defined spinor  $\tilde{\epsilon} = \Gamma_{11}\epsilon$ . Considering the spinor  $\epsilon$  in (1.1), the form bilinears of  $\epsilon$  and  $\tilde{\epsilon}$  are a 0-form

$$\sigma(\epsilon, \tilde{\epsilon}) = -2fg, \quad (\text{C.1})$$

two 1-forms

$$\begin{aligned}\kappa(\epsilon, \epsilon) &= f^2(e^0 - e^5) + g^2(e^0 + e^5), \\ \kappa(\epsilon, \tilde{\epsilon}) &= -f^2(e^0 - e^5) + g^2(e^0 + e^5),\end{aligned}\tag{C.2}$$

a 2-form

$$\omega(\epsilon, \epsilon) = 2fge^0 \wedge e^5,\tag{C.3}$$

a 4-form

$$\zeta(\epsilon, \tilde{\epsilon}) = -2fg\phi,\tag{C.4}$$

and two 5-forms

$$\begin{aligned}\tau(\epsilon, \epsilon) &= f^2(e^0 - e^5) \wedge \phi + g^2(e^0 + e^5) \wedge \phi, \\ \tau(\epsilon, \tilde{\epsilon}) &= -f^2(e^0 - e^5) \wedge \phi + g^2(e^0 + e^5) \wedge \phi,\end{aligned}\tag{C.5}$$

where  $\phi$  is the invariant Spin(7) 4-form defined in appendix B and we have normalized the Killing spinor with an additional factor of  $1/\sqrt{2}$ .

## D The $N = 1$ Spin(7) solution

In this appendix we present the solution of the Killing spinor equations for one Killing spinor with stability subgroup Spin(7). The solution is organised in terms of irreducible  $SU(4) \subset Spin(7)$  representations. This is due to the way that a basis is constructed in the space of spinors in the context of spinorial geometry, which we use to solve the KSEs. Note that all representations below are irreducible, i.e. for the (1,1) and (1,2) representations the trace part is projected out. In particular, we have the trivial  $SU(4)$  representation relations

$$df_0 = 0, \quad df_5 = -\frac{f}{2}\Omega_{0,05}, \quad d\Phi_0 = 0, \quad H_{0\gamma}{}^\gamma = \Omega_{,5\gamma}{}^\gamma - \Omega_{\gamma,5}{}^\gamma,\tag{D.1}$$

$$d\Phi_5 = -\frac{1}{2}\Omega_{,5\gamma}{}^\gamma - \Omega_{0,05} - \frac{1}{2}\Omega_{\gamma,5}{}^\gamma, \quad H_{5\gamma}{}^\gamma = -2\Omega_{\gamma,0}{}^\gamma,\tag{D.2}$$

$$\epsilon^{\tilde{\gamma}_1 \tilde{\gamma}_2 \tilde{\gamma}_3 \tilde{\gamma}_4} G_{\tilde{\gamma}_1 \tilde{\gamma}_2 \tilde{\gamma}_3 \tilde{\gamma}_4} = 3G_{\gamma_1 \gamma_2}{}^{\gamma_1 \gamma_2} - 9\Omega_{,5\gamma}{}^\gamma + 24\Omega_{0,05} + 15\Omega_{\gamma,5}{}^\gamma,\tag{D.3}$$

$$F_\gamma{}^\gamma = -2\Omega_{\gamma,0}{}^\gamma, \quad F_{05} = -S + \frac{1}{2}\Omega_{,5\gamma}{}^\gamma + \frac{1}{2}\Omega_{\gamma,5}{}^\gamma,\tag{D.4}$$

$$G_{05\gamma}{}^\gamma = 2\Omega_{\gamma,0}{}^\gamma, \quad \Omega_{,0\gamma}{}^\gamma = -\Omega_{\gamma,0}{}^\gamma, \quad \Omega_{0,\gamma}{}^\gamma = \Omega_{\gamma,0}{}^\gamma,\tag{D.5}$$

$$\Omega_{5,\gamma}{}^\gamma = -\frac{1}{2}\Omega_{,5\gamma}{}^\gamma + \frac{1}{2}\Omega_{\gamma,5}{}^\gamma, \quad \Omega_{5,05} = 0,\tag{D.6}$$

the fundamental  $SU(4)$  or (1,0) representation relations

$$df_\alpha = -\frac{f}{2}\Omega_{0,0\alpha}, \quad \epsilon_\alpha^{\tilde{\gamma}_1 \tilde{\gamma}_2 \tilde{\gamma}_3} H_{\tilde{\gamma}_1 \tilde{\gamma}_2 \tilde{\gamma}_3} = -3G_{0\alpha\gamma}{}^\gamma - 6\Omega_{\alpha,05},\tag{D.7}$$

$$d\Phi_\alpha = \frac{1}{2}\epsilon_\alpha^{\tilde{\gamma}_1 \tilde{\gamma}_2 \tilde{\gamma}_3} \Omega_{\tilde{\gamma}_1, \tilde{\gamma}_2 \tilde{\gamma}_3} - \Omega_{, \alpha\gamma}{}^\gamma - \frac{3}{4}\Omega_{0,0\alpha} + \frac{1}{4}\Omega_{5,5\alpha} - \frac{1}{2}\Omega_{\alpha,\gamma}{}^\gamma,\tag{D.8}$$

$$\epsilon_\alpha^{\tilde{\gamma}_1 \tilde{\gamma}_2 \tilde{\gamma}_3} G_{5\tilde{\gamma}_1 \tilde{\gamma}_2 \tilde{\gamma}_3} = -3\Omega_{0,0\alpha} - 3\Omega_{5,5\alpha} - 6\Omega_{\alpha,\gamma}{}^\gamma, \quad H_{05\alpha} = \Omega_{0,0\alpha} - \Omega_{5,5\alpha},\tag{D.9}$$

$$\epsilon_\alpha^{\tilde{\gamma}_1 \tilde{\gamma}_2 \tilde{\gamma}_3} G_{0\tilde{\gamma}_1 \tilde{\gamma}_2 \tilde{\gamma}_3} = 3G_{0\alpha\gamma}{}^\gamma + 6\Omega_{\alpha,05}, \quad H_{\alpha\gamma}{}^\gamma = -G_{0\alpha\gamma}{}^\gamma,\tag{D.10}$$



$$F_{0\alpha} = -\frac{1}{2}\epsilon_{\alpha}^{\bar{\gamma}_1\bar{\gamma}_2\bar{\gamma}_3}\Omega_{\bar{\gamma}_1,\bar{\gamma}_2\bar{\gamma}_3} + \Omega^{\gamma}_{,\alpha\gamma} - \frac{1}{4}\Omega_{0,0\alpha} - \frac{1}{4}\Omega_{5,5\alpha} + \frac{1}{2}\Omega_{\alpha,\gamma}^{\gamma}, \quad (\text{D.11})$$

$$G_{5\alpha\gamma}^{\gamma} = -\frac{1}{2}\epsilon_{\alpha}^{\bar{\gamma}_1\bar{\gamma}_2\bar{\gamma}_3}\Omega_{\bar{\gamma}_1,\bar{\gamma}_2\bar{\gamma}_3} + \Omega^{\gamma}_{,\alpha\gamma} + \frac{3}{4}\Omega_{0,0\alpha} + \frac{3}{4}\Omega_{5,5\alpha} - \frac{3}{2}\Omega_{\alpha,\gamma}^{\gamma}, \quad (\text{D.12})$$

$$F_{5\alpha} = 2\Omega_{\alpha,05}, \quad \Omega_{0,5\alpha} = -\Omega_{\alpha,05}, \quad \Omega_{5,0\alpha} = -\Omega_{\alpha,05}, \quad (\text{D.13})$$

the  $4 \otimes \bar{4}$  or (1,1)-traceless representation relations

$$H_{0\alpha\bar{\beta}} \stackrel{\circ}{=} \Omega_{\bar{\beta},5\alpha} - \Omega_{\alpha,5\bar{\beta}}, \quad H_{5\alpha\bar{\beta}} \stackrel{\circ}{=} -F_{\alpha\bar{\beta}} - G_{05\alpha\bar{\beta}} - 2\Omega_{\alpha,0\bar{\beta}}, \quad (\text{D.14})$$

$$G_{\alpha\gamma\bar{\beta}}^{\gamma} \stackrel{\circ}{=} \Omega_{\bar{\beta},5\alpha} + \Omega_{\alpha,5\bar{\beta}}, \quad \Omega_{\bar{\beta},0\alpha} \stackrel{\circ}{=} -\Omega_{\alpha,0\bar{\beta}}, \quad (\text{D.15})$$

the symmetric bi-fundamental representation relations

$$\epsilon_{(\alpha_1}^{\bar{\gamma}_1\bar{\gamma}_2\bar{\gamma}_3}G_{\alpha_2)\bar{\gamma}_1\bar{\gamma}_2\bar{\gamma}_3} = 6\Omega_{(\alpha_1,\alpha_2)5}, \quad \Omega_{(\alpha_1,\alpha_2)0} = 0 \quad (\text{D.16})$$

the skew-symmetric bi-fundamental representation relations

$$H_{0\alpha_1\alpha_2}^- = -2\Omega_{5,\alpha_1\alpha_2}^-, \quad H_{5\alpha_1\alpha_2}^- = -2\Omega_{0,\alpha_1\alpha_2}^-, \quad G_{05\alpha_1\alpha_2}^- = 2\Omega_{0,\alpha_1\alpha_2}^-, \quad (\text{D.17})$$

$$F_{\alpha_1\alpha_2}^- = -2\Omega_{0,\alpha_1\alpha_2}^-, \quad G_{\alpha_1\alpha_2\gamma}^- \stackrel{\gamma}{=} 4\Omega_{5,\alpha_1\alpha_2}^-, \quad H_{0\alpha_1\alpha_2}^+ = -2\Omega_{\alpha_1,5\alpha_2}^+, \quad (\text{D.18})$$

$$H_{5\alpha_1\alpha_2}^+ = -F_{\alpha_1\alpha_2}^+ - G_{05\alpha_1\alpha_2}^+ - 2\Omega_{\alpha_1,0\alpha_2}^+, \quad (\text{D.19})$$

$$\Omega_{\alpha_1,0\alpha_2}^- = \Omega_{0,\alpha_1\alpha_2}^-, \quad \Omega_{\alpha_1,5\alpha_2}^- = \Omega_{5,\alpha_1\alpha_2}^-, \quad (\text{D.20})$$

and the traceless (1,2) representation relations

$$H_{\alpha\beta_1\beta_2} \stackrel{\circ}{=} -G_{0\alpha\beta_1\beta_2}, \quad (\text{D.21})$$

$$G_{5\alpha\beta_1\beta_2} \stackrel{\circ}{=} \frac{2}{3}\epsilon_{\beta_1\beta_2}^{\gamma_1\gamma_2}\Omega_{\alpha,\gamma_1\gamma_2} + \frac{2}{3}\epsilon_{\beta_1\beta_2}^{\gamma_1\gamma_2}\Omega_{\gamma_1,\alpha\gamma_2} - 2\Omega_{\alpha,\beta_1\beta_2}, \quad (\text{D.22})$$

where in all the above relations that involve (1,1) and (1,2) traceless representations, we have denoted the equality of the traceless parts with  $\stackrel{\circ}{=}$  and suppressed the trace parts.

The solution described above in terms of SU(4) representations can be rewritten after some computation in terms of Spin(7) representations. Using that and some of the results of appendix C, the final result can be expressed as in section 3. In addition, although the above equations involve explicit components of the spin connections and thus may appear non-covariant, this is not the case. All the components of the connection that appear in the solution of the KSEs above are part of the co-torsion in a Gray-Hervella type of analysis and therefore transform like tensors.

## E The $N = 1$ Spin(7) $\times$ $\mathbb{R}^8$ solution

In this appendix we present the solution to the Killing spinor equations for one Killing spinor with stability subgroup Spin(7)  $\times$   $\mathbb{R}^8$ . As in the Spin(7) case, the solution is organised in terms of irreducible SU(4)  $\subset$  Spin(7) representations. In particular have, the trivial SU(4) representation relations are

$$d\Phi_+ = 0, \quad H_{-\gamma}^{\gamma} = 2\Omega_{-\gamma}^{\gamma}, \quad H_{+\gamma}^{\gamma} = 0, \quad (\text{E.1})$$

$$F_{\gamma}^{\gamma} = -\frac{1}{24}\epsilon^{\bar{\gamma}_1\bar{\gamma}_2\bar{\gamma}_3\bar{\gamma}_4}G_{\bar{\gamma}_1\bar{\gamma}_2\bar{\gamma}_3\bar{\gamma}_4} + \frac{1}{24}\epsilon^{\gamma_1\gamma_2\gamma_3\gamma_4}G_{\gamma_1\gamma_2\gamma_3\gamma_4}, \quad (\text{E.2})$$

$$G_{\gamma_1\gamma_2}^{\gamma_1\gamma_2} = \frac{1}{6}\epsilon^{\bar{\gamma}_1\bar{\gamma}_2\bar{\gamma}_3\bar{\gamma}_4}G_{\bar{\gamma}_1\bar{\gamma}_2\bar{\gamma}_3\bar{\gamma}_4} + \frac{1}{6}\epsilon^{\gamma_1\gamma_2\gamma_3\gamma_4}G_{\gamma_1\gamma_2\gamma_3\gamma_4}, \quad (\text{E.3})$$

$$G_{-+\gamma}^{\gamma} = -\frac{1}{24}\epsilon^{\bar{\gamma}_1\bar{\gamma}_2\bar{\gamma}_3\bar{\gamma}_4}G_{\bar{\gamma}_1\bar{\gamma}_2\bar{\gamma}_3\bar{\gamma}_4} + \frac{1}{24}\epsilon^{\gamma_1\gamma_2\gamma_3\gamma_4}G_{\gamma_1\gamma_2\gamma_3\gamma_4}, \quad (\text{E.4})$$

$$F_{-+} = S, \quad \Omega_{\gamma,+}^{\gamma} = 0, \quad \Omega_{-,-+} = 0, \quad \Omega_{+,\gamma}^{\gamma} = 0, \quad \Omega_{+,-+} = 0, \quad (\text{E.5})$$

the (1,0) representation relations are

$$d\Phi_{\alpha} = \frac{1}{6}\epsilon_{\alpha}^{\bar{\gamma}_1\bar{\gamma}_2\bar{\gamma}_3}H_{\bar{\gamma}_1\bar{\gamma}_2\bar{\gamma}_3} - \Omega_{\alpha,\gamma}^{\gamma} + \Omega_{-,+ \alpha}, \quad H_{\alpha\gamma}^{\gamma} = 2\Omega_{\alpha,\gamma}^{\gamma}, \quad (\text{E.6})$$

$$H_{-+\alpha} = 2\Omega_{-,+ \alpha}, \quad F_{-\alpha} = \frac{1}{3}\epsilon_{\alpha}^{\bar{\gamma}_1\bar{\gamma}_2\bar{\gamma}_3}G_{-\bar{\gamma}_1\bar{\gamma}_2\bar{\gamma}_3} - G_{-\alpha\gamma}^{\gamma}, \quad (\text{E.7})$$

$$F_{+\alpha} = 0, \quad G_{+\alpha\gamma}^{\gamma} = 0, \quad (\text{E.8})$$

$$\Omega^{\gamma}_{,\alpha\gamma} = -\frac{1}{4}\epsilon_{\alpha}^{\bar{\gamma}_1\bar{\gamma}_2\bar{\gamma}_3}H_{\bar{\gamma}_1\bar{\gamma}_2\bar{\gamma}_3} + \frac{1}{2}\epsilon_{\alpha}^{\bar{\gamma}_1\bar{\gamma}_2\bar{\gamma}_3}\Omega_{\bar{\gamma}_1,\bar{\gamma}_2\bar{\gamma}_3} + \Omega_{\alpha,\gamma}^{\gamma}, \quad (\text{E.9})$$

$$\Omega_{\alpha,-+} = \Omega_{-,+ \alpha}, \quad \Omega_{+,+ \alpha} = 0, \quad G_{+\gamma_1\gamma_2\gamma_3} = 0, \quad (\text{E.10})$$

the (1,1) trace-less relations are

$$H_{+\alpha\bar{\beta}} \stackrel{\circ}{=} -2\Omega_{\alpha,+ \bar{\beta}}, \quad F_{\alpha\bar{\beta}} \stackrel{\circ}{=} G_{-+\alpha\bar{\beta}}, \quad G_{\alpha\gamma\bar{\beta}}^{\gamma} \stackrel{\circ}{=} 0, \quad \Omega_{\bar{\beta},+\alpha} \stackrel{\circ}{=} -\Omega_{\alpha,+ \bar{\beta}}, \quad (\text{E.11})$$

the symmetric bi-fundamental representation relations

$$\Omega_{(\alpha_1,\alpha_2)+} = 0, \quad \epsilon_{(\alpha_1}^{\bar{\gamma}_1\bar{\gamma}_2\bar{\gamma}_3}G_{\alpha_2)\bar{\gamma}_1\bar{\gamma}_2\bar{\gamma}_3} = 0, \quad (\text{E.12})$$

the skew-symmetric bi-fundamental representation relations

$$H_{-\alpha_1\alpha_2}^{-} = 2\Omega_{-,\alpha_1\alpha_2}^{-}, \quad H_{+\alpha_1\alpha_2}^{-} = 0, \quad F_{\alpha_1\alpha_2}^{-} = G_{-+\alpha_1\alpha_2}^{-}, \quad (\text{E.13})$$

$$G_{\alpha_1\alpha_2\gamma}^{-\gamma} = -2G_{-+\alpha_1\alpha_2}^{-}, \quad H_{+\alpha_1\alpha_2}^{+} = -2\Omega_{\alpha_1,+ \alpha_2}^{+}, \quad F_{\alpha_1\alpha_2}^{+} = G_{-+\alpha_1\alpha_2}^{+}, \quad (\text{E.14})$$

$$\Omega_{\alpha_1,+ \alpha_2}^{-} = 0, \quad \Omega_{+,\alpha_1\alpha_2}^{-} = 0, \quad (\text{E.15})$$

and the traceless (1,2) representation relations

$$H_{\alpha\bar{\beta}_1\bar{\beta}_2} \stackrel{\circ}{=} -\frac{2}{3}\epsilon_{\bar{\beta}_1\bar{\beta}_2}^{\gamma_1\gamma_2}\Omega_{\alpha,\gamma_1\gamma_2} + -\frac{2}{3}\epsilon_{\bar{\beta}_1\bar{\beta}_2}^{\gamma_1\gamma_2}\Omega_{\gamma_1,\alpha\gamma_2} + 2\Omega_{\alpha,\bar{\beta}_1\bar{\beta}_2}, \quad (\text{E.16})$$

$$G_{+\alpha\bar{\beta}_1\bar{\beta}_2} \stackrel{\circ}{=} 0, \quad (\text{E.17})$$

where in all the above relations that involve (1,1) and (1,2) traceless representations, we have denoted the equality of the traceless parts with  $\stackrel{\circ}{=}$  and suppressed the trace parts. The solution described above in terms of SU(4) representations can be rewritten in terms of Spin(7) representations using the relations in appendix C, and the final result is given in section 4. In addition, as has been mentioned in the previous appendix, all the components of the connection that appear in the solution of the KSEs above are part of the co-torsion in a Gray-Hervella type of analysis and therefore transform like tensors leading to covariant expressions.

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## References

- [1] J.M. Figueroa-O’Farrill and G. Papadopoulos, *Maximally supersymmetric solutions of ten-dimensional and eleven-dimensional supergravities*, *JHEP* **03** (2003) 048 [[hep-th/0211089](https://arxiv.org/abs/hep-th/0211089)] [[INSPIRE](https://inspirehep.net/literature/59411)].
- [2] J.P. Gauntlett and S. Pakis, *The Geometry of  $D = 11$  Killing spinors*, *JHEP* **04** (2003) 039 [[hep-th/0212008](https://arxiv.org/abs/hep-th/0212008)] [[INSPIRE](https://inspirehep.net/literature/59411)].
- [3] J.P. Gauntlett, J.B. Gutowski and S. Pakis, *The Geometry of  $D = 11$  null Killing spinors*, *JHEP* **12** (2003) 049 [[hep-th/0311112](https://arxiv.org/abs/hep-th/0311112)] [[INSPIRE](https://inspirehep.net/literature/60411)].
- [4] J. Gillard, U. Gran and G. Papadopoulos, *The Spinorial geometry of supersymmetric backgrounds*, *Class. Quant. Grav.* **22** (2005) 1033 [[hep-th/0410155](https://arxiv.org/abs/hep-th/0410155)] [[INSPIRE](https://inspirehep.net/literature/60411)].
- [5] U. Gran, J. Gutowski and G. Papadopoulos, *The Spinorial geometry of supersymmetric IIB backgrounds*, *Class. Quant. Grav.* **22** (2005) 2453 [[hep-th/0501177](https://arxiv.org/abs/hep-th/0501177)] [[INSPIRE](https://inspirehep.net/literature/60411)].
- [6] U. Gran, J. Gutowski and G. Papadopoulos, *The  $G_2$  spinorial geometry of supersymmetric IIB backgrounds*, *Class. Quant. Grav.* **23** (2006) 143 [[hep-th/0505074](https://arxiv.org/abs/hep-th/0505074)] [[INSPIRE](https://inspirehep.net/literature/60411)].
- [7] U. Gran, J. Gutowski, G. Papadopoulos and D. Roest,  *$N = 31$  is not IIB*, *JHEP* **02** (2007) 044 [[hep-th/0606049](https://arxiv.org/abs/hep-th/0606049)] [[INSPIRE](https://inspirehep.net/literature/60411)].
- [8] U. Gran, J. Gutowski, G. Papadopoulos and D. Roest,  *$N = 31$ ,  $D = 11$* , *JHEP* **02** (2007) 043 [[hep-th/0610331](https://arxiv.org/abs/hep-th/0610331)] [[INSPIRE](https://inspirehep.net/literature/60411)].
- [9] U. Gran, J. Gutowski, G. Papadopoulos and D. Roest, *IIB solutions with  $N > 28$  Killing spinors are maximally supersymmetric*, *JHEP* **12** (2007) 070 [[arXiv:0710.1829](https://arxiv.org/abs/0710.1829)] [[INSPIRE](https://inspirehep.net/literature/60411)].
- [10] U. Gran, J. Gutowski and G. Papadopoulos, *Classification of IIB backgrounds with 28 supersymmetries*, *JHEP* **01** (2010) 044 [[arXiv:0902.3642](https://arxiv.org/abs/0902.3642)] [[INSPIRE](https://inspirehep.net/literature/60411)].
- [11] J.M. Figueroa-O’Farrill and S. Gadhia,  *$M$ -theory preons cannot arise by quotients*, *JHEP* **06** (2007) 043 [[hep-th/0702055](https://arxiv.org/abs/hep-th/0702055)] [[INSPIRE](https://inspirehep.net/literature/60411)].
- [12] U. Gran, P. Lohrmann and G. Papadopoulos, *The Spinorial geometry of supersymmetric heterotic string backgrounds*, *JHEP* **02** (2006) 063 [[hep-th/0510176](https://arxiv.org/abs/hep-th/0510176)] [[INSPIRE](https://inspirehep.net/literature/60411)].
- [13] U. Gran, G. Papadopoulos, D. Roest and P. Sloane, *Geometry of all supersymmetric type-I backgrounds*, *JHEP* **08** (2007) 074 [[hep-th/0703143](https://arxiv.org/abs/hep-th/0703143)] [[INSPIRE](https://inspirehep.net/literature/60411)].
- [14] I.A. Bandos, J.A. de Azcarraga and O. Varela, *On the absence of BPS preonic solutions in IIA and IIB supergravities*, *JHEP* **09** (2006) 009 [[hep-th/0607060](https://arxiv.org/abs/hep-th/0607060)] [[INSPIRE](https://inspirehep.net/literature/60411)].
- [15] U. Gran, P. Lohrmann and G. Papadopoulos, *Geometry of type-II common sector  $N = 2$  backgrounds*, *JHEP* **06** (2006) 049 [[hep-th/0602250](https://arxiv.org/abs/hep-th/0602250)] [[INSPIRE](https://inspirehep.net/literature/60411)].
- [16] M. Huq and M.A. Namazie, *Kaluza-Klein Supergravity in Ten-dimensions*, *Class. Quant. Grav.* **2** (1985) 293 [Erratum *ibid.* **2** (1985) 597] [[INSPIRE](https://inspirehep.net/literature/60411)].
- [17] F. Giani and M. Pernici,  *$N = 2$  supergravity in ten-dimensions*, *Phys. Rev. D* **30** (1984) 325 [[INSPIRE](https://inspirehep.net/literature/60411)].

- [18] I.C.G. Campbell and P.C. West,  *$N = 2$   $D = 10$  Nonchiral Supergravity and Its Spontaneous Compactification*, *Nucl. Phys. B* **243** (1984) 112 [[INSPIRE](#)].
- [19] L.J. Romans, *Massive  $N = 2a$  Supergravity in Ten-Dimensions*, *Phys. Lett. B* **169** (1986) 374 [[INSPIRE](#)].
- [20] E.A. Bergshoeff, J. Hartong, P.S. Howe, T. Ortín and F. Riccioni, *IIA/IIB Supergravity and Ten-forms*, *JHEP* **05** (2010) 061 [[arXiv:1004.1348](#)] [[INSPIRE](#)].
- [21] E.A. Bergshoeff, M. de Roo, S.F. Kerstan, T. Ortín and F. Riccioni, *IIA ten-forms and the gauge algebras of maximal supergravity theories*, *JHEP* **07** (2006) 018 [[hep-th/0602280](#)] [[INSPIRE](#)].
- [22] C. Hull, *Holonomy and symmetry in M-theory*, [hep-th/0305039](#) [[INSPIRE](#)].
- [23] M.J. Duff and J.T. Liu, *Hidden space-time symmetries and generalized holonomy in M-theory*, *Nucl. Phys. B* **674** (2003) 217 [[hep-th/0303140](#)] [[INSPIRE](#)].
- [24] G. Papadopoulos and D. Tsimpis, *The Holonomy of the supercovariant connection and Killing spinors*, *JHEP* **07** (2003) 018 [[hep-th/0306117](#)] [[INSPIRE](#)].
- [25] G. Papadopoulos and D. Tsimpis, *The Holonomy of IIB supercovariant connection*, *Class. Quant. Grav.* **20** (2003) L253 [[hep-th/0307127](#)] [[INSPIRE](#)].
- [26] U. Gran, J. Gutowski and G. Papadopoulos, *Invariant Killing spinors in 11D and type-II supergravities*, *Class. Quant. Grav.* **26** (2009) 155004 [[arXiv:0802.2040](#)] [[INSPIRE](#)].
- [27] M. Fernandez, *A classification of Riemannian manifolds with structure  $Spin(7)$* , *Ann. Mat. Pura Appl.* **143** (1982) 101.
- [28] F. Cabrera, *Riemannian manifolds with  $Spin(7)$ -structure*, *Publ. Math. Debrecen* **46** (1995) 271.
- [29] M.J. Duff, *M theory on manifolds of  $G_2$  holonomy: The First twenty years*, [hep-th/0201062](#) [[INSPIRE](#)].
- [30] U. Gran, *GAMMA: A Mathematica package for performing gamma matrix algebra and Fierz transformations in arbitrary dimensions*, [hep-th/0105086](#) [[INSPIRE](#)].