

Exploring 6D origins of 5D supergravities with Chern-Simons terms

Federico Bonetti,^a Thomas W. Grimm^a and Stefan Hohenegger^b

^aMax-Planck-Institut für Physik,
Föhringer Ring 6, 80805 Munich, Germany

^bDepartment of Physics, CERN — Theory Division,
CH-1211 Geneva 23, Switzerland

E-mail: bonetti@mpp.mpg.de, grimm@mpp.mpg.de,
stefan.hohenegger@cern.ch

ABSTRACT: We consider five-dimensional supergravity theories with eight or sixteen supercharges with Abelian vector fields and ungauged scalars. We address the question under which conditions these theories can be interpreted as effective low energy descriptions of circle reductions of anomaly free six-dimensional theories with (1,0) or (2,0) supersymmetry. We argue that classical and one-loop gauge- and gravitational Chern-Simons terms are instrumental for this question.

KEYWORDS: Chern-Simons Theories, Superstring Vacua, Supersymmetric Effective Theories

ARXIV EPRINT: [1303.2661](https://arxiv.org/abs/1303.2661)

Contents

1	Introduction	1
2	Six-dimensional origin of five-dimensional theories	2
2.1	$\mathcal{N} = 2$ supersymmetric theories	2
2.2	$\mathcal{N} = 4$ supersymmetric theories	5
3	Conclusions	6

1 Introduction

In this note we address the question whether it is possible to determine if a given five-dimensional supergravity theory can be understood as the effective low-energy description of an anomaly-free six-dimensional supergravity theory on a circle. Our investigation is motivated by the following considerations. On general grounds, it is an interesting problem to study the constraints that gravity places on low-energy quantum field theories. For instance, even-dimensional chiral theories are subject to the requirement of cancellation of gravitational anomalies. In the spirit of [1–4], one can maybe look for analogue constraints in odd-dimensional theories by exploring classes of models that cannot be seen as a circle reduction of an anomaly-free even-dimensional theory. More specifically, the study of five-dimensional quantum field theories with coupling to gravity has recently attracted a lot of attention, partly related to the attempt to find an effective world-volume action for multiple M5-branes [5–10]. Given the great number of new insights, it would be desirable to classify those theories which are consistent at the quantum level. This is a formidable task and therefore it is advantageous to first try to understand a subset of these theories, namely those that come from a circle reduction from six dimensions (see figure 1). Of course, not all consistent five-dimensional theories arise in such a circle compactification. Well-known examples include Calabi-Yau threefold reductions of M-theory that in general do not admit a six-dimensional lift if the threefold is not elliptically fibered [11–13].

Deciding upon this question is generically a highly non-trivial task, for various reasons. On the one hand, in order to extract the low-energy effective action of a six-dimensional theory on a circle one needs not only to perform a classical dimensional reduction, but also to integrate out massive excitations such as Kaluza-Klein modes. Five-dimensional quantum effects due to these massive excitations can make a direct comparison to a possible higher-dimensional action prohibitively difficult. On the other hand, the structure of six-dimensional supergravities is quite rich and is not completely under control. The study of non-Abelian interactions among self-dual tensors, in particular, remains an open problem in the context of $(2, 0)$ theories and has been investigated in $(1, 0)$ models in the regime where gravity is decoupled [14–16].

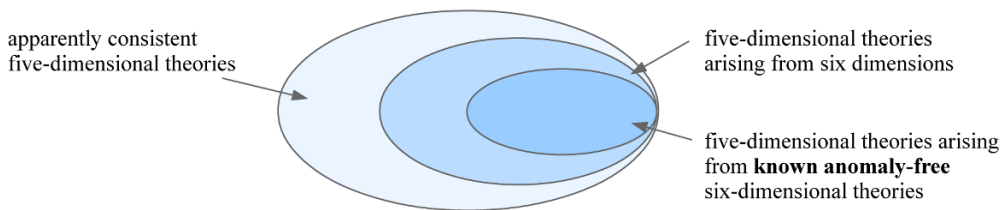


Figure 1. Five dimensional effective low-energy theories coupled to gravity which arise through compactification of anomaly-free six-dimensional theories form a subset of all apparently quantum-consistent theories.

Even if we do not have control over the full class of six-dimensional supergravities, we can still formulate non-trivial conditions for a given five-dimensional theory to be lifted to a specific subset of six-dimensional models. Moreover, there are objects at the quantum level of the theory that are robust under dimensional reduction. Anomalies, and in particular gravitational ones, are examples of such objects, since they are mostly sensitive to more general features of the theory rather than intricate details of the action [17]. In this note, we discuss the possibility to study them using classical and one-loop gauge and gravitational Chern-Simons terms in the theory obtained by compactification on a circle. Reversing the logic, we try to argue that a careful study of Chern-Simons terms in a generic five-dimensional gauge theory allows to obtain non-trivial information about the spectrum (and thus also about the quantum-consistency) of a potential six-dimensional parent theory.

The two setups that we investigate admit eight and sixteen supercharges, respectively. Firstly, we suppose we are given a five-dimensional Abelian action with eight supercharges and we explore the possibility to lift it to a (1,0) theory with simple gauge group. We find that non-trivial necessary conditions can be formulated in terms of the Chern-Simons sector only. Secondly, we take an Abelian theory with sixteen supercharges and we search for a possible lift to an Abelian (2,0) theory. As before, a necessary condition on the Chern-Simons couplings, accompanied by suitable kinetic terms to fix the normalization of the fields, is found.

2 Six-dimensional origin of five-dimensional theories

2.1 $\mathcal{N} = 2$ supersymmetric theories

The minimal amount of supersymmetry in five-dimensions consists of eight real supercharges and will be referred to as $\mathcal{N} = 2$ supersymmetry. We consider minimal supergravity coupled to n Abelian vector multiplets and a number of massless neutral hypermultiplets. The supersymmetric action of such a theory contains the topological couplings

$$S_{CS}^{(5)} = \frac{1}{(2\pi)^2} \int \left[k_{ABC} A^A \wedge F^B \wedge F^C + \kappa_A A^A \wedge \text{Tr} (R \wedge R) \right], \quad (2.1)$$

where A^A , $A = 1, \dots, n + 1$ denotes collectively the graviphoton and the vectors from the vector multiplets, $F^A = dA^A$ are the corresponding Abelian field strengths, and R is the curvature two-form. Supersymmetrizations of the second term are discussed in [18, 19].

If an $\mathcal{N} = 2$ theory can be seen as the circle reduction of a six-dimensional theory, it has to come from a $(1, 0)$ theory: for one, if the six-dimensional theory had more supersymmetry, we would find more than eight supercharges in five dimensions.¹ For another, it seems impossible to lift the five-dimensional gravitino of an $\mathcal{N} = 2$ theory to a consistent, interacting six-dimensional theory with no supersymmetry. Note that a five-dimensional theory with massless $U(1)$ gauge fields can arise as low energy effective action of a possibly non-Abelian six-dimensional theory on a circle. This is what happens when the gauge group is broken to the five-dimensional Coulomb branch by giving a VEV to the scalars in the five-dimensional vector multiplets. For simplicity, in the following we study the possibility to lift the five-dimensional theory to a non-Abelian $(1, 0)$ with simple gauge group G . The generalization to semi-simple G is straightforward. The inclusion of $U(1)$ factors is also possible, but would make the analysis of the six-dimensional action and anomalies more involved.

The first step in the search for a parent six-dimensional theory is to determine if the five-dimensional spectrum can be lifted to six-dimensions. Five-dimensional hypermultiplets directly lift to six-dimensional hypermultiplets, which are allowed in the $(1, 0)$ theory. To understand the possible lift of the vector sector to six dimensions one has to divide the $n + 1$ five-dimensional vector fields A^B into three sets:

- the vector A^0 that lifts to the Kaluza-Klein vector in the reduction of the six-dimensional metric on a circle;
- the vectors A^α , $\alpha = 1, \dots, T + 1$ that lift to components of T six-dimensional tensor multiplets and a single tensor in the supergravity multiplet;
- the vectors A^i , $i = 1, \dots, \text{rank}(G)$ that lift to Cartan elements of six-dimensional gauge group G .

Furthermore, to allow for a consistent six-dimensional parent theory, the constants k_{ABC} and κ_A in (2.1) have to split in such a way to accommodate the following Chern-Simons terms for the above mentioned classes of vector fields

$$S_{CS}^{(5)} = \frac{1}{(2\pi)^2} \int \left[-\frac{1}{2} \Omega_{\alpha\beta} A^0 F^\alpha F^\beta + \frac{1}{2} b^\alpha \Omega_{\alpha\beta} C_{ij} A^\beta F^i F^j - \frac{1}{8} a^\alpha \Omega_{\alpha\beta} A^\beta \text{Tr} R^2 \right] \quad (2.2)$$

$$+ \frac{1}{(2\pi)^2} \int \left[k_0 A^0 F^0 F^0 + k_{ij} A^0 F^i F^j + k_{ijk} A^i F^j F^k + \kappa_0 A^0 \text{Tr} R^2 \right],$$

where we suppressed wedge products for brevity. As discussed for example in [4, 13], only the Chern-Simons terms in the first line can be lifted to a classical six-dimensional action, while the terms in the second line cannot be obtained by classical reduction on a circle. As it has been shown in [20], however, such terms can arise at the one-loop level by integrating out

¹Here we consider only simple compactifications on a circle. In particular, we do not discuss any compactification mechanism which (partially) breaks supersymmetry.

massive spin-1/2, spin 3/2, or two-forms charged under A^0 or A^i . It is precisely the interplay between these two subsets of Chern-Simons terms that allows us to formulate necessary conditions for the five-dimensional theory to come from an anomaly-free (1, 0) theory.

Let us recall briefly the six-dimensional interpretation of the first line of terms in (2.2). The constant symmetric matrix $\Omega_{\alpha\beta}$ has signature (1, T) and is identified with the $SO(1, T)$ invariant metric associated to the moduli space $SO(1, T)/SO(T)$ of the scalars in the tensor multiplets in six-dimensions. The matrix C_{ij} is identified with the Cartan matrix of the gauge group G . The constant vectors b^α and a^α contain crucial information about the anomaly of the six-dimensional parent theory. Indeed, they are the coefficient of the Green-Schwarz terms that cancel factorisable anomalies. Note also that the vector b^α determines the kinetic term of six-dimensional vectors.

As mentioned above, the requirement of anomaly cancellation in the parent (1, 0) theory allows us to formulate necessary conditions on the Chern-Simons terms for the lift to six-dimensions to be possible. In the following, we focus on six-dimensional gravitational anomalies, since they do not depend on many details of the charged hypermultiplet spectrum in six dimensions. Recall that gravitational anomalies are canceled provided that [4]

$$H - V = 273 - 29T, \quad a^\alpha \Omega_{\alpha\beta} a^\beta = 9 - T, \quad (2.3)$$

where T, V, H are the number of six-dimensional tensor multiplets, vector multiplets, and hypermultiplets, respectively. To check the first condition in (2.3) directly we would need to know the number of hypermultiplets H in six dimensions. This number, however, is in general different from the number of neutral massless hypermultiplets in five dimensions, since some charged hypermultiplets become massive after breaking of the gauge group, and therefore do not appear in the five-dimensional effective action.

This problem can be circumvented by studying the Chern-Simons terms in (2.2). In particular, the couplings k_0 and κ_0 encode information about the gravitational anomaly cancellation conditions (2.3). To see this, recall that each massless field in the six-dimensional theory gives rise to a Kaluza-Klein tower of massive modes in five dimensions. They are all minimally coupled to the Kaluza-Klein vector A^0 , with charge proportional to the Kaluza-Klein level. The massive modes of six-dimensional chiral fermions and (anti)self-dual tensors are capable of generating the couplings k_0, κ_0 in (2.2) by running in five-dimensional one-loop diagrams. The total value of k_0, κ_0 is obtained by summing the contribution of all Kaluza-Klein modes of all relevant fields.² This sum yields [20]

$$k_0 = \frac{1}{24}(T - 9), \quad \kappa_0 = \frac{1}{24}(12 - T). \quad (2.4)$$

These expressions hold under the assumption that the first condition in (2.3) is satisfied, but they only involve the number T of tensor multiplets of the theory, which can be read off from range of the α indices in (2.2). Combining (2.4) with the second condition in (2.3) we get the following necessary conditions for the Chern-Simons terms (2.2) to be lifted to

²The sum over Kaluza-Klein levels is regularized using the Riemann zeta function. For instance $\sum_n n \rightarrow \zeta(-1) = -1/12$.

six-dimensional theory free of gravitational anomalies:

$$24 k_0 = -a^\alpha \Omega_{\alpha\beta} a^\beta = T - 9, \quad 24 \kappa_0 = a^\alpha \Omega_{\alpha\beta} a^\beta + 3 = 12 - T. \quad (2.5)$$

These equations encode three independent requirements and cannot be trivially satisfied by rescaling A^0 and A^α .

One can formulate similar tests on the Chern-Simons coefficients in (2.2) to check if the candidate parent theory is free of purely gauge anomalies. Such conditions involve a comparison between $b^\alpha \Omega_{\alpha\beta} b^\beta$ and the coupling k_{ijk} , which contains crucial information about the six-dimensional charged hypermultiplet spectrum [21, 22]. While it was only shown for specific examples [22], and not yet in general, that the knowledge of the Chern-Simons coefficients allows to check cancellation of six-dimensional gauge anomalies, we believe that such a statement should hold in general. In a similar way, we suspect that conditions involving $a^\alpha \Omega_{\alpha\beta} b^\beta$ and the Chern-Simons coupling k_{ij} can be used to test if the six-dimensional theory is free of mixed gauge-gravitational anomalies.

2.2 $\mathcal{N} = 4$ supersymmetric theories

We can apply the strategy outlined so far also to five-dimensional theories with sixteen supercharges, denoted $\mathcal{N} = 4$. We restrict to the theory of n Abelian vector multiplets coupled to supergravity. Recall that the $\mathcal{N} = 4$ supergravity multiplet contains six vectors. Five of them form the **5** representation of the $SO(5)_R$ R-symmetry group, while the sixth one is a singlet.³ The singlet will be denoted A^0 , and the remaining ones together with the n gauge fields from the vector multiplets are denoted A^A , $A = 1, \dots, n + 5$. The collective index A is a fundamental $SO(5, n)$ index. The associated constant metric is denoted η_{AB} . With this notation the topological sector of the action reads

$$S_{CS}^{(5)} = \frac{1}{(2\pi)^2} \int \left[-\frac{1}{2} \eta_{AB} A^0 \wedge F^B \wedge F^C + \kappa_0 A^0 \wedge \text{Tr} (R \wedge R) \right]. \quad (2.6)$$

To the best of our knowledge it has not been shown that the gravitational Chern-Simons coupling can be supersymmetrized. We will see, however, that in some circumstances it can be generated at the quantum level from a six-dimensional theory with sixteen supercharges on a circle. We thus expect it to be an admissible coupling in the five-dimensional $\mathcal{N} = 4$ action.

In contrast to the $\mathcal{N} = 2$ case, the Chern-Simons sector of an $\mathcal{N} = 4$ theory is too simple to provide any test that cannot be trivially satisfied by means of rescaling of A^0 , A^A . Therefore, we also need to record some kinetic terms in order to fix this ambiguity. This requires some additional notation. Each vector multiplet contributes five scalars to the spectrum. These $5n$ scalars parametrize the coset space $SO(5, n)/SO(5) \times SO(n)$. This is conveniently described in terms of matrices L_A^i, L_A^I , where i, I are fundamental indices of $SO(5), SO(n)$ respectively. These matrices satisfy

$$\eta_{AB} = \delta_{ij} L_A^i L_B^j - \delta_{IJ} L_A^I L_B^J, \quad G_{AB} = \delta_{ij} L_A^i L_B^j + \delta_{IJ} L_A^I L_B^J, \quad (2.7)$$

³This structure is fixed by identifying the five-dimensional gravity multiplet.

where G_{AB} is a non-constant, positive-definite matrix that enters the gauge coupling function. The needed kinetic terms are

$$S_{\text{kin}}^{(5)} = \frac{1}{(2\pi)^2} \int \left[R * 1 - \frac{1}{2} d\sigma \wedge * d\sigma - \frac{1}{2} e^{2\sigma/\sqrt{6}} G_{AB} F^A \wedge * F^B - \frac{1}{2} e^{-4\sigma/\sqrt{6}} F^0 \wedge * F^0 \right], \quad (2.8)$$

in which σ is the scalar in the gravity multiplet. The sum $S_{CS}^{(5)} + S_{\text{kin}}^{(5)}$ can be supersymmetrized since it coincides with part of the standard form of the five-dimensional $\mathcal{N} = 4$ action as found e.g. in [23], up to field redefinitions.⁴

The five-dimensional $\mathcal{N} = 4$ theory under examination can come from circle reduction of a (2, 0) or (1, 1) theory. Since (1, 1) theories are non-chiral, we cannot use anomalies as a check of the quantum consistency of the candidate parent theory. For this reason, in the rest of this section we formulate necessary conditions for the lift of the five-dimensional theory to a (2, 0) theory, and we do not give conditions for the lift to a (1, 1) theory. Furthermore, since a six-dimensional action for non-Abelian (2, 0) is not known, we explore the possibility to lift the five-dimensional theory to an Abelian (2, 0) theory.

Recall that such a theory has only tensors as matter multiplets. Cancellation of gravitational anomalies requires a number $T = 21$ of them. This implies that the five-dimensional theory must have exactly 26 vectors in addition to the singlet A^0 . This provides a first elementary check on (2.6). A far less trivial check comes from the gravitational Chern-Simons coupling κ_0 . It cannot be generated by reduction of the classical Abelian (2, 0) action on a circle, and it is rather generated by one-loop diagrams in which massive Kaluza-Klein modes run in the loop. This coupling has been computed in [20] with the result

$$\kappa_0 = \frac{1}{4}. \quad (2.9)$$

If in $S_{CS}^{(5)} + S_{\text{kin}}^{(5)}$ a different value of κ_0 appears, the theory cannot be lifted to an Abelian (2, 0) theory.

3 Conclusions

In this work we explored the space of supersymmetric five-dimensional effective theories. We addressed the question whether or not a given theory can effectively arise from an anomaly-free six-dimensional theory on a circle at low energies. By focusing on a certain class of six-dimensional theories we formulated explicit constraints on the spectrum and supersymmetry content of the six-dimensional theory in terms of the five-dimensional Chern-Simons couplings. We note that our findings based on Chern-Simons terms alone cannot be viewed as a classification of all five-dimensional theories that can arise in a circle compactification in the spirit of figure 1. However, we provided a setup in which this question can be posed systematically and checked for a given example. Therefore,

⁴More precisely, we have performed an overall rescaling of the action, together with the redefinitions $\sigma_{\text{there}} = \sigma_{\text{here}}/\sqrt{2}$, $A_{\text{there}}^0 = A_{\text{here}}^0/\sqrt{2}$, $A_{\text{there}}^A = A_{\text{here}}^A/\sqrt{2}$. Our form of the action is best suited for comparison between tree-level and one-loop terms. It is such that the action and the vectors both have period 2π . It has been inferred by deriving $S_{CS}^{(5)}$ from M-theory on $K3 \times T^2$ making use of the effective action discussed in [24].

we see our work as a first step towards a systematic analysis of consistency conditions for five-dimensional quantum field theories in the presence of gravity.

It is an interesting task to extend our approach to more general six-dimensional theories. In particular, one might ask to which extent our results can be used to explore the possibility of a lift to a non-Abelian $(2, 0)$ theory. To address this question, a remark is due. There are two familiar realization of $(2, 0)$ theories in string theory and M-theory. On the one hand, they are the world-volume theory of a stack of M5 branes. On the other hand, these theories can arise from Type IIB on a singular K3. In the former setup gravitational anomalies on the world-volume of the stack of M5 branes can be canceled by anomaly inflow from the eleven-dimensional bulk [25, 26]. In this way, the number of tensor multiplets is not restricted to be 21. Note instead that in the Type IIB setup anomaly inflow is not available, and indeed the theory possesses 21 tensor multiplets even in presence of non-Abelian interactions among the tensors. We can argue that the condition (2.9) is still valid in this case, while it is probably not required if we allow six-dimensional gravitational anomalies to be canceled by inflow from some higher-dimensional bulk theory. To systematically approach the non-Abelian theory from five-dimensions remains an exciting challenge and might yield intriguing insights about the nature of $(2, 0)$ theories.

One might also hope to apply the same strategy to theories in other dimensions. In particular, the study of circle compactifications from four to three dimensions can be motivated by the duality of F-theory and M-theory compactifications and the match of their effective actions. In the three-dimensional theory Chern-Simons terms are also generated at one loop. It was shown in [22, 27] that they capture information about the four-dimensional chiral spectrum and its anomalies. Focusing as in five dimensions on the Coulomb branch, the Chern-Simons terms are specified by a constant matrix Θ_{AB} for the coupling $\int \Theta_{AB} A^A \wedge F^B$. These encode both the four-dimensional gaugings of axions, as well as the one-loop contributions from integrated out massive matter. As in five dimensions this matter includes modes that become massive in the Coulomb branch and fields that are Kaluza-Klein modes. However, in contrast to five dimensions one cannot infer all relevant information for the four-dimensional Green-Schwarz mechanism from the Chern-Simons terms alone [27, 28]. The four-dimensional analogs of a^α, b^α introduced in (2.2) do not appear in Chern-Simons terms and one needs to extend the analysis to other couplings of the effective action. Including these couplings one could proceed in a similar manner as in the five-dimensional case and check if a given three-dimensional theory can effectively arise from a four-dimensional anomaly-free theory.

Finally, let us note that it is significantly more complicated to apply the presented strategy to compactifications that are not on circles, but on general higher-dimensional geometries. To make any concrete statements about the underlying theory one would need finer information about the effective actions and their corrections. Moreover, when turning to string theory, also massive extended modes can arise, correct the effective theory, and induce dualities.⁵ Formulating the criteria which allow an effective theory to arise from string theory is a giant mountain dwarfing the hill climbed in this work.

⁵Higher-dimensional theories might be distinguished by the representations of the massive modes [29], but this does not imply that a distinction can be made on the level of the low-energy effective theories.

Acknowledgments

We gratefully acknowledge interesting discussions with Ignatios Antoniadis, Denis Klevers, Dieter Lüst, and Wati Taylor. This research was supported by a grant of the Max Planck Society.

Open Access. This article is distributed under the terms of the Creative Commons Attribution License which permits any use, distribution and reproduction in any medium, provided the original author(s) and source are credited.

References

- [1] C. Vafa, *The string landscape and the swampland*, [hep-th/0509212](#) [[INSPIRE](#)].
- [2] H. Ooguri and C. Vafa, *On the geometry of the string landscape and the swampland*, *Nucl. Phys. B* **766** (2007) 21 [[hep-th/0605264](#)] [[INSPIRE](#)].
- [3] V. Kumar and W. Taylor, *String universality in six dimensions*, *Adv. Theor. Math. Phys.* **15** (2011) 325 [[arXiv:0906.0987](#)] [[INSPIRE](#)].
- [4] W. Taylor, *TASI lectures on supergravity and string vacua in various dimensions*, [arXiv:1104.2051](#) [[INSPIRE](#)].
- [5] M.R. Douglas, *On $D = 5$ super Yang-Mills theory and $(2, 0)$ theory*, *JHEP* **02** (2011) 011 [[arXiv:1012.2880](#)] [[INSPIRE](#)].
- [6] N. Lambert, C. Papageorgakis and M. Schmidt-Sommerfeld, *$M5$ -branes, $D4$ -branes and quantum $5D$ super-Yang-Mills*, *JHEP* **01** (2011) 083 [[arXiv:1012.2882](#)] [[INSPIRE](#)].
- [7] N. Lambert, C. Papageorgakis and M. Schmidt-Sommerfeld, *Deconstructing $(2, 0)$ proposals*, [arXiv:1212.3337](#) [[INSPIRE](#)].
- [8] F. Bonetti, T.W. Grimm and S. Hohenegger, *Non-Abelian tensor towers and $(2, 0)$ superconformal theories*, [arXiv:1209.3017](#) [[INSPIRE](#)].
- [9] P.-M. Ho, K.-W. Huang and Y. Matsuo, *A non-Abelian self-dual gauge theory in $5 + 1$ dimensions*, *JHEP* **07** (2011) 021 [[arXiv:1104.4040](#)] [[INSPIRE](#)].
- [10] K.-W. Huang, *Non-Abelian chiral 2-form and $M5$ -branes*, [arXiv:1206.3983](#) [[INSPIRE](#)].
- [11] A. Cadavid, A. Ceresole, R. D'Auria and S. Ferrara, *Eleven-dimensional supergravity compactified on Calabi-Yau threefolds*, *Phys. Lett. B* **357** (1995) 76 [[hep-th/9506144](#)] [[INSPIRE](#)].
- [12] S. Ferrara, R. Minasian and A. Sagnotti, *Low-energy analysis of M and F theories on Calabi-Yau threefolds*, *Nucl. Phys. B* **474** (1996) 323 [[hep-th/9604097](#)] [[INSPIRE](#)].
- [13] F. Bonetti and T.W. Grimm, *Six-dimensional $(1, 0)$ effective action of F -theory via M -theory on Calabi-Yau threefolds*, *JHEP* **05** (2012) 019 [[arXiv:1112.1082](#)] [[INSPIRE](#)].
- [14] H. Samtleben, E. Sezgin and R. Wimmer, *$(1, 0)$ superconformal models in six dimensions*, *JHEP* **12** (2011) 062 [[arXiv:1108.4060](#)] [[INSPIRE](#)].
- [15] H. Samtleben, E. Sezgin, R. Wimmer and L. Wulff, *New superconformal models in six dimensions: gauge group and representation structure*, *PoS(CORFU2011)071* [[arXiv:1204.0542](#)] [[INSPIRE](#)].

- [16] H. Samtleben, E. Sezgin and R. Wimmer, *Six-dimensional superconformal couplings of non-Abelian tensor and hypermultiplets*, *JHEP* **03** (2013) 068 [[arXiv:1212.5199](#)] [[INSPIRE](#)].
- [17] L. Álvarez-Gaumé and E. Witten, *Gravitational anomalies*, *Nucl. Phys. B* **234** (1984) 269 [[INSPIRE](#)].
- [18] K. Hanaki, K. Ohashi and Y. Tachikawa, *Supersymmetric completion of an R^2 term in five-dimensional supergravity*, *Prog. Theor. Phys.* **117** (2007) 533 [[hep-th/0611329](#)] [[INSPIRE](#)].
- [19] M. Ozkan and Y. Pang, *Supersymmetric completion of Gauss-Bonnet combination in five dimensions*, *JHEP* **03** (2013) 158 [[arXiv:1301.6622](#)] [[INSPIRE](#)].
- [20] F. Bonetti, T.W. Grimm and S. Hohenegger, *One-loop Chern-Simons terms in five dimensions*, [arXiv:1302.2918](#) [[INSPIRE](#)].
- [21] K.A. Intriligator, D.R. Morrison and N. Seiberg, *Five-dimensional supersymmetric gauge theories and degenerations of Calabi-Yau spaces*, *Nucl. Phys. B* **497** (1997) 56 [[hep-th/9702198](#)] [[INSPIRE](#)].
- [22] T.W. Grimm and H. Hayashi, *F-theory fluxes, chirality and Chern-Simons theories*, *JHEP* **03** (2012) 027 [[arXiv:1111.1232](#)] [[INSPIRE](#)].
- [23] G. Dall'Agata, C. Herrmann and M. Zagermann, *General matter coupled $N = 4$ gauged supergravity in five-dimensions*, *Nucl. Phys. B* **612** (2001) 123 [[hep-th/0103106](#)] [[INSPIRE](#)].
- [24] E. Witten, *On flux quantization in M-theory and the effective action*, *J. Geom. Phys.* **22** (1997) 1 [[hep-th/9609122](#)] [[INSPIRE](#)].
- [25] D. Freed, J.A. Harvey, R. Minasian and G.W. Moore, *Gravitational anomaly cancellation for M-theory five-branes*, *Adv. Theor. Math. Phys.* **2** (1998) 601 [[hep-th/9803205](#)] [[INSPIRE](#)].
- [26] J.A. Harvey, R. Minasian and G.W. Moore, *Non-Abelian tensor multiplet anomalies*, *JHEP* **09** (1998) 004 [[hep-th/9808060](#)] [[INSPIRE](#)].
- [27] M. Cvetič, T.W. Grimm and D. Klevers, *Anomaly cancellation and Abelian gauge symmetries in F-theory*, *JHEP* **02** (2013) 101 [[arXiv:1210.6034](#)] [[INSPIRE](#)].
- [28] T.W. Grimm and W. Taylor, *Structure in 6D and 4D $N = 1$ supergravity theories from F-theory*, *JHEP* **10** (2012) 105 [[arXiv:1204.3092](#)] [[INSPIRE](#)].
- [29] M. Abou-Zeid, B. de Wit, D. Lüst and H. Nicolai, *Space-time supersymmetry, IIA/B duality and M-theory*, *Phys. Lett. B* **466** (1999) 144 [[hep-th/9908169](#)] [[INSPIRE](#)].