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# On T-duality of $R^2$-corrections to DBI action at all orders of gauge field

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Abstract: Recently, it has been observed that in a T-duality invariant world-volume theory in flat spacetime, all orders of gauge field strength and all orders of the D-brane velocity appear in two specific matrices. Using these two matrices, we construct the world-volume couplings of two massless NSNS states at order $\alpha'^2$ and all orders of the velocity and the gauge field strength, by requiring them to be invariant under the linear T-duality. The standard extension $F \to F + P[B]$, then produces all orders of the pull-back of B-field into the action. We compare the resulting couplings for zero velocity and gauge field strength, with the $\alpha'^2$ terms of the disk-level S-matrix element of two massless NSNS vertex operators in the presence of a constant background B-field. We have found an exact agreement.

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# 1 Introduction

D-branes are non-perturbative objects in superstring theory which play the central role in exploring different aspects of the theory, from statistical computation of black hole entropy [1] to realization of the AdS/CFT correspondence [2] or appearance of noncommutative geometry in string theory [3]. These objects should be described by the supersymmetric extension of the cubic string field theory [4] which includes massless and all infinite tower of the massive excitations of the open strings. In the Wilsonian effective action, however, the effect of all massive states appears in the higher derivatives of the massless fields.

At long wavelength limit, the higher derivative terms can be ignored and D-branes are completely described by Dirac-Born-Infeld (DBI) action [5, 6] which includes constant metric, B-field and dilaton, as well as the first derivative of the massless NS fields. A non-Abelian extension for DBI action has been proposed in [7]. The leading higher derivative corrections to the Abelian DBI action should include acceleration which appears through the second fundamental form $\Omega$ (see e.g., [8]) and the first derivative of the gauge field strength in the NS part, as well as the first and the second derivatives of the metric, B-field and dilaton in the NSNS part. The first derivative of the metric appears also through $\Omega$ and the second derivatives of the metric appears through the curvature terms.

The leading higher derivative corrections of the metric have been found in [9] by requiring the consistency of the effective action with the $O(\alpha'^2)$ terms of the corresponding disk-level scattering amplitude [10, 11]. For totally-geodesic embeddings of the world-volume in the ambient spacetime ($\Omega = 0$), the corrections to the DBI action in string frame for vanishing gauge field and B-field and for a constant dilaton is the following action [9]:[1]

$$S = \frac{\pi^2\alpha'^2 T_p}{48}\int d^{p+1}x\, e^{-\phi}\sqrt{-\widetilde{G}}\Big(R_{abcd}R^{abcd} - 2\hat{R}_{ab}\hat{R}^{ab} - R_{abij}R^{abij} + 2\hat{R}_{ij}\hat{R}^{ij}\Big), \quad (1.1)$$

[1]In our index notation the Greek letters $(\mu, \nu, \cdots)$ are the indices of the space-time coordinates, the Latin letters $(a, b, c, \cdots)$ are the world-volume indices and the letters $(i, j, k, \cdots)$ are corresponding to the normal coordinates of the D-brane.



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where $\hat{R}_{ab} = \tilde{G}^{cd}R_{cadb}$, $\hat{R}_{ij} = \tilde{G}^{cd}R_{cidj}$ and $\tilde{G}_{ab}$ is the pull-back of the bulk metric, $\tilde{G}_{ab} = \partial_a X^\mu \partial_b X^\nu g_{\mu\nu}$. In above equation, the Riemann curvatures are the projections of the bulk Riemann tensors into the world-volume or the transverse space. For example:

$$R_{abcd} = \partial_a X^\alpha \partial_b X^\beta \partial_c X^\mu \partial_d X^\nu R_{\alpha\beta\mu\nu}\,. \tag{1.2}$$

The above action includes all orders of the velocity through the pull-back operator in the static gauge, *i.e.,* $X^a = \sigma^a$ and $X^i = \lambda^i$ . The acceleration terms ($\Omega \neq 0$) have been also found in [9] (see also [12]). The consistency of these couplings with T-duality may include all orders of dilaton, B-field and $F$.

In [15, 16], it has been shown that the consistency of the couplings in (1.1) at zero velocity with linear T-duality transformation, requires $\nabla H$ and $\nabla\nabla\phi$ couplings to appear as:

$$\begin{aligned} S = \frac{\pi^2\alpha'^2 T_p}{48}\int d^{p+1}x\, e^{-\phi}\sqrt{-\tilde{G}}\Big( & R_{abcd}R^{abcd} - 2\hat{\mathcal{R}}_{ab}\hat{\mathcal{R}}^{ab} - R_{abij}R^{abij} + 2\hat{\mathcal{R}}_{ij}\hat{\mathcal{R}}^{ij} \\ & -\frac{1}{6}\nabla_a H_{ijk}\nabla^a H^{ijk} - \frac{1}{3}\nabla_i H_{abc}\nabla^i H^{abc} + \frac{1}{2}\nabla_a H_{bci}\nabla^a H^{bci}\Big)\,, \end{aligned} \tag{1.3}$$

where $\hat{\mathcal{R}}_{\mu\nu} = \hat{R}_{\mu\nu} + \nabla_\mu\nabla_\nu\phi$ and $H$ is the field strength of the B-field. The consistency of the action (1.1) at zero velocity and non-zero $\Omega$ with nonlinear T-duality should also include $H^4$ couplings in which we are not interested in this paper. Such couplings have been found in [13, 14] for O-planes. In this paper, we are interested to include all orders of the constant Abelian gauge field strength $F$ in the above action. This can be done by releasing the assumption in [15] that the velocity is zero, because the world-volume transverse scalars transform to the gauge fields under the T-duality transformations.

Recently, it has been observed in [26] that in a T-duality invariant world-volume theory in flat spacetime, all orders of gauge field strength and all orders of the D-brane velocity appear in the following two matrices:

$$\begin{aligned} G^{\mu\nu} &= \partial_a X^\mu \partial_b X^\nu\; G^{ab}\,, \\ \Theta^{\mu\nu} &= \partial_a X^\mu \partial_b X^\nu\; \Theta^{ab}\,, \end{aligned} \tag{1.4}$$

where $G^{ab}$ and $\Theta^{ab}$ are the symmetric and antisymmetric parts of $(\frac{1}{\eta+\partial\lambda^i\partial\lambda_i+F})^{ab}$, respectively. They transform into each other under T-duality [26]. Using these matrices, in the next section we construct world-volume $R^2, R\phi, \phi^2$, $R\nabla H, \phi H$ and $(\nabla H)^2$ couplings by requiring them to be invariant under the linear T-duality and to reduce to (1.1) when $F = 0$. In section three, in order to confirm our result with the S-matrix element, we use the standard extension $F \to F + P[B]$, to include all pull-back of B-field into the action. We then compare the resulting couplings for zero velocity and gauge field strength with the $\alpha'^2$ terms of the disk-level S-matrix element of two massless NSNS vertex operators in the presence of constant background B-field [27, 28]. We will show an exact agreement.

## 2 T-duality constraint

In this section, we are going to impose the constraint that the effective action should be invariant under linear T-duality transformations, to include constant gauge field strength into the action (1.3). Let us first review the strategy presented in [15] and [17]. As we know under T-duality the Neumann and Dirichlet boundary conditions are exchanging [19], this is due to the fact that a compact direction that is lying in the world-volume of the D-brane in original picture, becomes a transverse direction after T-duality. Symbolically if we consider $S$ as an action for D-brane and $y$ as a compact direction then under T-duality we will have:

$$S_{D_p}[(a,y),i] \xleftrightarrow{\text{T-duality}} S_{D_{p-1}}[a,(i,y)]\,, \tag{2.1}$$

where $a$ represents the tangent coordinate to the D-brane and $i$ stands for normal coordinate. The $y$-index on the left-hand side is a tangent coordinate whereas on the right-hand side it is a normal coordinate.

The T-duality transformation for gauge field is $\tilde{A}_y = \lambda^y$ which is a linear transformation. However, the T-duality transformation for NSNS fields are nonlinear [20–25]. Assuming NSNS fields as perturbations around the flat space, one can find the relevant linear T-duality transformations for the closed string fields, *i.e.*,

$$\begin{aligned}
\tilde{\phi} &= \phi - \frac{1}{2}h_{yy}\,, & \tilde{h}_{yy} &= -\,h_{yy}\,, & \tilde{h}_{\mu\nu} &= h_{\mu\nu}\,,\\
\tilde{h}_{\mu y} &= B_{\mu y}\,, & \tilde{B}_{\mu y} &= h_{\mu y}\,, & \tilde{B}_{\mu\nu} &= B_{\mu\nu}\,.
\end{aligned} \tag{2.2}$$

One can use the T-duality implied by (2.1) as a constraint to find some new couplings from the known ones, in fact this is the method have been used in [15] and [17].

Before going through the T-duality consideration we note that the curvature terms, and those terms with $\nabla\nabla\phi$ and $\nabla H$ in equation (1.3) are the projections of the corresponding bulk tensors into the world-volume or the transverse space. By using the projection operator onto the vector space normal to the D-brane defined as:

$$\perp^{\mu\nu} = \sum_{\mathcal{I}=p+1}^{9} n^{\mu}_{\mathcal{I}} n^{\nu}_{\mathcal{I}} = G^{\mu\nu} - \tilde{G}^{\mu\nu}\,, \tag{2.3}$$

where $n^{\mu}_{\mathcal{I}}$ is the normal vector to the D-brane (see e.g., [8, 18]) and $\tilde{G}^{\mu\nu} = \partial_a X^\mu \partial_b X^\nu \tilde{G}^{ab}$ is the first fundamental form, one can transform the transverse indices in the action (1.3) into the bulk space-time coordinates. As an example:

$$\begin{aligned}
\hat{R}_{ij}\hat{R}^{ij} &= \perp^{\mu\nu}\perp^{\alpha\beta}\hat{R}_{\alpha\mu}\hat{R}_{\beta\nu} = \perp^{\mu\nu}\perp^{\alpha\beta}\tilde{G}^{ab}\partial_a X^\rho \partial_b X^\lambda R_{\alpha\rho\mu\lambda}\tilde{G}^{cd}\partial_c X^\sigma \partial_d X^\gamma R_{\beta\sigma\nu\gamma}\\
&= \perp^{\mu\nu}\perp^{\alpha\beta}\tilde{G}^{\rho\lambda}\tilde{G}^{\sigma\gamma}R_{\alpha\rho\mu\lambda}R_{\beta\sigma\nu\gamma}\,.
\end{aligned} \tag{2.4}$$

Apart from the overall constant factor, the Lagrangian density in the action (1.3) can be rewritten as:

$$\begin{aligned}
\mathcal{L} = {} & \tilde{G}^{\alpha\beta}\tilde{G}^{\nu\mu}\left(2R_{\alpha}{}^{\rho}{}_{\beta}{}^{\kappa}R_{\nu\rho\mu\kappa} - R_{\alpha\nu}{}^{\rho\kappa}R_{\beta\mu\rho\kappa}\right) - 2\tilde{G}^{\alpha\beta}\tilde{G}^{\nu\mu}\tilde{G}^{\rho\kappa}\left(2R_{\alpha\nu\beta}{}^{\lambda}R_{\rho\mu\kappa\lambda} - R_{\alpha\nu\rho}{}^{\lambda}R_{\beta\mu\kappa\lambda}\right)\\
& -\frac{1}{3}\tilde{G}^{\alpha\beta}\nabla_\alpha H^{\gamma\delta\rho}\,\nabla_\beta H_{\gamma\delta\rho} + \tilde{G}^{\alpha\beta}\tilde{G}^{\gamma\delta}\nabla_\gamma H_\alpha{}^{\rho\kappa}\nabla_\delta H_{\beta\rho\kappa} - \frac{2}{3}\tilde{G}^{\alpha\beta}\tilde{G}^{\gamma\delta}\tilde{G}^{\rho\kappa}\nabla_\lambda H_{\beta\delta\kappa}\nabla^\lambda H_{\alpha\gamma\rho}\\
& +4R_{\alpha\lambda\beta\mu}\tilde{G}^{\alpha\beta}\nabla^\mu\nabla^\lambda\phi - 8R_{\lambda\beta\mu\nu}\tilde{G}^{\alpha\beta}\tilde{G}^{\lambda\mu}\nabla^\nu\nabla_\alpha\phi + 2\nabla_\beta\nabla_\alpha\phi\nabla^\beta\nabla^\alpha\phi\\
& -4\tilde{G}^{\alpha\beta}\nabla_\mu\nabla_\beta\phi\nabla^\mu\nabla_\alpha\phi\,.
\end{aligned} \tag{2.5}$$

Writing in this form, all the Riemann curvatures, $\nabla H$ and $\nabla\nabla\phi$ will be the bulk tensors. Moreover, in this form, the transverse scalar fields appear in the action just through the first fundamental form $\tilde{G}^{\mu\nu}$. To find the scalar couplings, it is sufficient to break the indices of $\tilde{G}^{\mu\nu}$ to tangent and normal indices and to go to the static gauge *i.e.*, $X^a = \sigma^a$ and $X^i = \lambda^i$ as follows:

$$\begin{aligned}\tilde{G}^{ia} &= \tilde{G}^{ab}\partial_b\lambda^i\,,\\ \tilde{G}^{ij} &= \tilde{G}^{ab}\partial_a\lambda^i\partial_b\lambda^j\,,\end{aligned}\tag{2.6}$$

where $\tilde{G}^{ab}$ is the inverse of the pull-back metric in the static gauge $\tilde{G}_{ab} = g_{ab} + \partial_a\lambda^i\partial_b\lambda_i + 2g_{i\{a}\partial_{b\}}\lambda^i$.

The bulk metric in $\tilde{G}^{\mu\nu}$ is not flat, so the Lagrangian density (2.5) is covariant and contains all orders of the D-brane velocity. It would be desirable to include all orders of gauge field in a covariant expression. That expression may be found by expanding $\tilde{G}^{\mu\nu}$ in (2.5) in terms of different orders of $g$ and velocity, and then include appropriate B-field and $F$ to make each term to be invariant under the T-duality. To restrict the calculation for finding only $F$-terms, we assume the bulk metric in $\tilde{G}^{\mu\nu}$ to be flat metric. Then the extension $\tilde{G}^{\mu\nu} \to G^{\mu\nu}$ where $G^{\mu\nu}$ is given in (1.4), produces the following couplings:

$$\begin{aligned}\mathcal{L} =\; & G^{\alpha\beta}G^{\nu\mu}(2R_{\alpha}{}^{\rho}{}_{\beta}{}^{\kappa}R_{\nu\rho\mu\kappa} - R_{\alpha\nu}{}^{\rho\kappa}R_{\beta\mu\rho\kappa}) - 2G^{\alpha\beta}G^{\nu\mu}G^{\rho\kappa}\big(2R_{\alpha\nu\beta}{}^{\lambda}R_{\rho\mu\kappa\lambda} - R_{\alpha\nu\rho}{}^{\lambda}R_{\beta\mu\kappa\lambda}\big)\\ & -\frac{1}{3}G^{\alpha\beta}\nabla_{\alpha}H^{\gamma\delta\rho}\,\nabla_{\beta}H_{\gamma\delta\rho} + G^{\alpha\beta}G^{\gamma\delta}\nabla_{\gamma}H_{\alpha}{}^{\rho\kappa}\nabla_{\delta}H_{\beta\rho\kappa} - \frac{2}{3}G^{\alpha\beta}G^{\gamma\delta}G^{\rho\kappa}\nabla_{\lambda}H_{\beta\delta\kappa}\nabla^{\lambda}H_{\alpha\gamma\rho}\\ & +4R_{\alpha\lambda\beta\mu}G^{\alpha\beta}\nabla^{\mu}\nabla^{\lambda}\phi - 8R_{\lambda\beta\mu\nu}G^{\alpha\beta}G^{\lambda\mu}\nabla^{\nu}\nabla_{\alpha}\phi + 2\nabla_{\beta}\nabla_{\alpha}\phi\nabla^{\beta}\nabla^{\alpha}\phi\\ & -4G^{\alpha\beta}\nabla_{\mu}\nabla_{\beta}\phi\nabla^{\mu}\nabla_{\alpha}\phi\,.\end{aligned}\tag{2.7}$$

Obviously this action reduces to (2.5) when $F \to 0$. When the indices of $G^{\mu\nu}$ are not the $y$-index, the above expression is invariant under linear T-duality. The matrix $G^{\mu\nu}$ has even number of $F$, so the above couplings are also invariant under the parity.

However, the above action is not invariant under the linear T-duality when the indices of $G^{\mu\nu}$ are the $y$-index. It has been shown in [26] that under T-duality $G^{\mu\nu}$ transforms to $\Theta^{\mu\nu}$. So in the T-duality invariant theory there must be new couplings involving $\Theta^{\mu\nu}$ as well. Using the fact that $G^{\mu\nu}$ has even number of $F$ and $\Theta^{\mu\nu}$ has odd number of $F$, the invariance under parity requires the new terms contracted by even number of $\Theta$ in $R^2, R\phi$ and $H^2$ terms and odd number in $RH$ and $\phi H$ couplings.[2] Moreover, the couplings $R^2$ and $H^2$ in (2.7) in which the indices of $G^{\mu\nu}$ can be the $y$-index, contain two or three $G^{\mu\nu}$s. As a result, T-duality requires the new terms with structure $R^2$ and $H^2$ to have two or three $\Theta$ and/or $G$. The couplings $R\phi$ in which the indices of $G^{\mu\nu}$ can be the $y$-index, contain one or two $G^{\mu\nu}$s. As a result, T-duality requires the new terms with structure $R\phi$ to have two $\Theta$s, and the terms with structure $H\phi$ to have one, two or three $\Theta$ and/or $G$. Note

[2] In fact we have observed that for example by considering a term such as $RR\Theta$, the T-duality transformation leads to $RHG$ couplings, which reduce to $RH$ couplings in vanishing $F$ limit, but such terms do not exist in (2.5), so T-duality requires parity invariance.

that the indices of $G^{\mu\nu}$ in $\phi^2$ term can not be the $y$-index, so there is no new coupling with structure $\phi^2$.

All possible independent $R^2$ and $R\phi$ terms are then the followings:

$$\begin{aligned}\mathcal{L}_1 = {}& \Theta^{\alpha\beta}\Theta^{\kappa\lambda}\big(\alpha_1 R_{\alpha\kappa}{}^{\mu\nu}R_{\beta\lambda\mu\nu} + \alpha_2 R_\alpha{}^\mu{}_\kappa{}^\nu R_{\beta\mu\lambda\nu} + \alpha_3 R_{\alpha\beta}{}^{\mu\nu}R_{\kappa\lambda\mu\nu} + \rho R_{\beta\kappa\lambda\mu}\nabla^\mu\nabla_\alpha\phi\big)\\ &+ \Theta^{\alpha\beta}\Theta^{\kappa\lambda}G^{\mu\nu}\big(\alpha_4 R_{\alpha\kappa\mu}{}^\rho R_{\beta\lambda\nu\rho} + \alpha_5 R_{\alpha\mu\kappa}{}^\rho R_{\beta\nu\lambda\rho} + \alpha_6 R_{\alpha\beta\mu}{}^\rho R_{\kappa\lambda\nu\rho} + \alpha_7 R_{\alpha\kappa\beta}{}^\rho R_{\lambda\mu\nu\rho}\big)\,,\end{aligned} \tag{2.8}$$

where $\alpha_1, \cdots, \alpha_7$ and $\rho$ are constants. Similarly for $H^2$ terms we have:

$$\begin{aligned}\mathcal{L}_2 = {}& \Theta^{\alpha\beta}\Theta^{\gamma\delta}\big(\beta_1\nabla_\beta H_\delta{}^{\mu\nu}\nabla_\gamma H_{\alpha\mu\nu} + \beta_2\nabla_\nu H_{\beta\delta}{}^\mu\nabla^\nu H_{\alpha\gamma\mu}\\ &\qquad + \beta_3\nabla_\gamma H_\alpha{}^{\mu\nu}\nabla_\delta H_{\beta\mu\nu} + \beta_4\nabla_\nu H_{\gamma\delta}{}^\mu\nabla^\nu H_{\alpha\beta\mu}\big)\\ &+ \Theta^{\alpha\beta}\Theta^{\gamma\delta}G^{\mu\nu}\big(\beta_5\nabla_\beta H_{\delta\nu}{}^\lambda\nabla_\gamma H_{\alpha\mu\lambda} + \beta_6\nabla_\gamma H_{\alpha\mu}{}^\lambda\nabla_\delta H_{\beta\nu\lambda} + \beta_7\nabla_\beta H_{\alpha\mu}{}^\lambda\nabla_\delta H_{\gamma\nu\lambda}\\ &\qquad + \beta_8\nabla_\mu H_{\alpha\gamma}{}^\lambda\nabla_\nu H_{\beta\delta\lambda} + \beta_9\nabla_\mu H_{\alpha\beta}{}^\lambda\nabla_\nu H_{\gamma\delta\lambda} + \beta_{10}\nabla_\beta H_{\alpha\gamma}{}^\lambda\nabla_\nu H_{\delta\mu\lambda}\\ &\qquad + \beta_{11}\nabla_\gamma H_{\alpha\beta}{}^\lambda\nabla_\nu H_{\delta\mu\lambda} + \beta_{12}\nabla_\lambda H_{\gamma\delta\nu}\nabla^\lambda H_{\alpha\beta\mu} + \beta_{13}\nabla_\lambda H_{\beta\delta\nu}\nabla^\lambda H_{\alpha\gamma\mu}\big)\,,\end{aligned} \tag{2.9}$$

where $\beta_1, \cdots, \beta_{13}$ are unknown coefficients. The new possible $RH$ and $\phi H$ couplings are:

$$\begin{aligned}\mathcal{L}_3 = {}& \Theta^{\alpha\beta}G^{\gamma\delta}\big(\sigma_1 R_{\beta\delta\mu\nu}\nabla_\alpha H_\gamma{}^{\mu\nu} + \sigma_2 R_{\alpha\beta\mu\nu}\nabla_\delta H_\gamma{}^{\mu\nu} + \sigma_3 R_{\gamma\mu\delta\nu}\nabla^\nu H_{\alpha\beta}{}^\mu\\ &\qquad + \sigma_4 R_{\beta\mu\delta\nu}\nabla^\nu H_{\alpha\gamma}{}^\mu + \sigma_5 R_{\beta\delta\mu\nu}\nabla_\gamma H_\alpha{}^{\mu\nu}\big)\\ &+ \Theta^{\alpha\beta}G^{\gamma\delta}G^{\mu\nu}\big(\sigma_6 R_{\beta\delta\nu\rho}\nabla_\alpha H_{\gamma\mu}{}^\rho + \sigma_7 R_{\delta\mu\nu\rho}\nabla_\beta H_{\alpha\gamma}{}^\rho + \sigma_8 R_{\delta\mu\nu\rho}\nabla_\gamma H_{\alpha\beta}{}^\rho\\ &\qquad + \sigma_9 R_{\beta\mu\nu\rho}\nabla_\delta H_{\alpha\gamma}{}^\rho + \sigma_{10} R_{\alpha\beta\nu\rho}\nabla_\delta H_{\gamma\mu}{}^\rho + \sigma_{11} R_{\beta\delta\nu\rho}\nabla_\mu H_{\alpha\gamma}{}^\rho\\ &\qquad + \sigma_{12} R_{\beta\nu\delta\rho}\nabla_\mu H_{\alpha\gamma}{}^\rho\big)\\ &+ \Theta^{\alpha\beta}\Theta^{\gamma\delta}\Theta^{\mu\nu}\big(\sigma_{13} R_{\delta\mu\nu\rho}\nabla_\beta H_{\alpha\gamma}{}^\rho + \sigma_{14} R_{\beta\delta\nu\rho}\nabla_\mu H_{\alpha\gamma}{}^\rho\\ &\qquad + \sigma_{15} R_{\beta\delta\nu\rho}\nabla^\rho H_{\alpha\gamma\mu} + \sigma_{16} R_{\delta\mu\nu\rho}\nabla_\gamma H_{\alpha\beta}{}^\rho\big)\\ &+ \gamma_1\Theta^{\alpha\beta}\Theta^{\gamma\delta}\Theta^{\theta\kappa}\nabla_\gamma\nabla_\alpha\phi\nabla_\delta H_{\beta\theta\kappa} + \gamma_2\Theta^{\alpha\beta}\nabla_\delta H_{\alpha\beta\gamma}\nabla^\delta\nabla^\gamma\phi\\ &+ \gamma_3\Theta^{\alpha\beta}G^{\gamma\delta}\nabla_\delta H_{\beta\gamma}{}^\theta\nabla_\theta\nabla_\alpha\phi + \gamma_4\Theta^{\alpha\beta}G^{\gamma\delta}\nabla_\delta H_{\alpha\beta}{}^\theta\nabla_\theta\nabla_\gamma\phi\\ &+ \gamma_5\Theta^{\alpha\beta}G^{\gamma\delta}\nabla_\theta H_{\alpha\beta\delta}\nabla^\theta\nabla_\gamma\phi\,,\end{aligned} \tag{2.10}$$

here again $\sigma_1, \cdots, \sigma_{16}$ and $\gamma_1, \cdots, \gamma_5$ are unknown coefficients. In writing the above terms we have considered independency by taking care of the Bianchi identities (for more details see appendix A). In this regard (2.8)–(2.10) are the most general Lagrangians. In our calculation we are going to assume constant $G$ and $\Theta$ and to work with the second order of perturbations, so the terms with coefficients $\beta_3, \beta_6, \sigma_{15}$ and $\gamma_1$ are total derivatives. Hence, we ignore them, *i.e.*, $\beta_3 = \beta_6 = \sigma_{15} = \gamma_1 = 0$. On the other hand, for some specific relations between the coefficients of some of the above terms, there might be total derivatives which should be dropped. We will find such terms after imposing the T-duality constraint.

In order to fix the unknown coefficients in above Lagrangians by the linear T-duality (2.2), we need to expand the metric around the flat background as $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$ and keep terms up to the second order of perturbation expansion. Moreover one needs to expand $G^{\mu\nu}$ and $\Theta^{\mu\nu}$ up to the third order of velocity and/or $F$ in the static gauge.

Performing these steps for (2.7)–(2.10), we compute both sides of relation (2.1). Next we apply the linear T-duality on left(right) hand side and identify it with right(left) hand side to fix the unknown coefficients. After all we find the following relations:

$$\begin{aligned}
&\alpha_2 = 0\,, && \alpha_3 = -\frac{\alpha_1}{2}\,, && \alpha_5 = -4\,, && && \\
&\alpha_6 = 2 - \frac{\alpha_4}{2}\,, && \alpha_7 = 8\,, && \rho = -8\,; && && \\
&\beta_1 = 0\,, && \beta_2 = 0\,, && \beta_4 = 1\,, && \beta_7 = \frac{\beta_{10}}{2} + \beta_{11} + \beta_5\,, && \\
&\beta_8 = \frac{\beta_{10}}{4}\,, && \beta_9 = \frac{\beta_{11}}{4}\,, && \beta_{12} = -2 - \frac{\beta_{11}}{4}\,, && \beta_{13} = 2 - \frac{\beta_{10}}{4}\,; && \\
&\sigma_2 = \frac{\sigma_1}{2}\,, && \sigma_3 = -4\,, && \sigma_4 = 0\,, && \sigma_5 = 4\,, && \sigma_6 = 16 + 2\sigma_{10}\,, \\
&\sigma_7 = \sigma_{11}\,, && \sigma_8 = -8\,, && \sigma_9 = 16 - \sigma_{11}\,, && \sigma_{12} = -16\,, && \sigma_{14} = 8 + \frac{\sigma_{13}}{2}\,; \\
&\gamma_2 = 2\,, && \gamma_3 = 2\gamma_4\,, && \gamma_5 = -4 - \gamma_4\,. && &&
\end{aligned} \tag{2.11}$$

In obtaining the above results we have used the integration by part and assumed $G^{\mu\nu}$ and $\Theta^{\mu\nu}$ are constant. Substituting (2.11) into the Lagrangian (2.8) for $R^2$ and $R\phi$ terms, we have found:

$$\begin{aligned}
\mathcal{L}_1 = &\Theta^{\alpha\beta}\Theta^{\gamma\delta}G^{\rho\kappa}\big(-4R_{\alpha\rho\gamma}{}^{\lambda}R_{\beta\kappa\delta\lambda} + 2R_{\alpha\beta\rho}{}^{\lambda}R_{\gamma\delta\kappa\lambda} + 8R_{\alpha\gamma\beta}{}^{\lambda}R_{\delta\rho\kappa\lambda}\big) \\
&-8\Theta^{\alpha\beta}\Theta^{\kappa\lambda}R_{\beta\kappa\lambda\theta}\nabla^{\theta}\nabla_{\alpha}\phi.
\end{aligned} \tag{2.12}$$

We have also found that the unfixed coefficients, $\alpha_1$ and $\alpha_4$ appear in the following expressions:

$$\begin{aligned}
&\alpha_1\Theta^{\alpha\beta}\Theta^{\gamma\delta}\left(R_{\alpha\gamma}{}^{\rho\kappa}R_{\beta\delta\rho\kappa} - \frac{1}{2}R_{\alpha\beta}{}^{\rho\kappa}R_{\gamma\delta\rho\kappa}\right) \\
&\qquad + \alpha_4\Theta^{\alpha\beta}\Theta^{\gamma\delta}G^{\rho\kappa}\left(R_{\alpha\gamma\rho}{}^{\lambda}R_{\beta\delta\kappa\lambda} - \frac{1}{2}R_{\alpha\beta\rho}{}^{\lambda}R_{\gamma\delta\kappa\lambda}\right).
\end{aligned} \tag{2.13}$$

However, they are total derivatives up to the second order of the perturbations. Since our calculations are valid up to the second order of the perturbations, we can ignore these terms, *i.e.*, $\alpha_1 = \alpha_4 = 0$. For $H^2$ terms, we have found

$$\mathcal{L}_2 = \Theta^{\alpha\beta}\Theta^{\mu\nu}\nabla_{\sigma}H_{\mu\nu}{}^{\rho}\nabla^{\sigma}H_{\alpha\beta\rho} + 2\Theta^{\alpha\beta}\Theta^{\mu\nu}G^{\rho\sigma}\left(\nabla_{\lambda}H_{\beta\nu\sigma}\nabla^{\lambda}H_{\alpha\mu\rho} - \nabla_{\lambda}H_{\mu\nu\sigma}\nabla^{\lambda}H_{\alpha\beta\rho}\right). \tag{2.14}$$

We have also showed that the constants $\beta_5, \beta_{10}$ and $\beta_{11}$ are the coefficients of total derivative terms, so we set them to zero. Finally for $RH$ and $\phi H$ couplings in (2.10) we get:

$$\begin{aligned}
\mathcal{L}_3 = &8\Theta^{\alpha\beta}G^{\gamma\delta}G^{\mu\nu}\big(2R_{\beta\delta\nu\rho}\nabla_{\alpha}H_{\gamma\mu}{}^{\rho} - R_{\delta\mu\nu\rho}\nabla_{\gamma}H_{\alpha\beta}{}^{\rho} + 2R_{\beta\mu\nu\rho}\nabla_{\delta}H_{\alpha\gamma}{}^{\rho} - 2R_{\beta\nu\delta\rho}\nabla_{\mu}H_{\alpha\gamma}{}^{\rho}\big) \\
&+4\Theta^{\alpha\beta}G^{\gamma\delta}\big(R_{\beta\delta\mu\nu}\nabla_{\gamma}H_{\alpha}{}^{\mu\nu} - R_{\gamma\mu\delta\nu}\nabla^{\nu}H_{\alpha\beta}{}^{\mu}\big) \\
&+8\Theta^{\alpha\beta}\Theta^{\gamma\delta}\Theta^{\mu\nu}\big(R_{\delta\mu\nu\rho}\nabla_{\gamma}H_{\alpha\beta}{}^{\rho} + R_{\beta\delta\nu\rho}\nabla_{\mu}H_{\alpha\gamma}{}^{\rho}\big) + 2\Theta^{\alpha\beta}\nabla_{\delta}H_{\alpha\beta\gamma}\nabla^{\delta}\nabla^{\gamma}\phi \\
&-4\Theta^{\alpha\beta}G^{\gamma\delta}\nabla_{\theta}H_{\alpha\beta\delta}\nabla^{\theta}\nabla_{\gamma}\phi\,.
\end{aligned} \tag{2.15}$$

We have also found that the constants $\sigma_1, \sigma_{10}, \sigma_{11}, \sigma_{13}$ and $\gamma_4$ are again the coefficients of total derivative terms, so we set them to zero too.



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It is interesting to note that the consistency of the couplings (2.5) with the linear T-duality could uniquely fix all orders of constant $F$. They are given in (2.7), (2.12), (2.14) and (2.15). Similar observation has been made in [26] in making the world-volume transverse scalar couplings at order $\alpha'$ in the bosonic string theory to be consistent with T-duality. In the next section, we are going to compare the above couplings with the corresponding disk-level S-matrix element.

## 3 Comparison with S-matrix

In the previous section, we have found the couplings of two massless NSNS states at order $\alpha'^2$ to all orders of gauge field which appear through $G$ and $\Theta$. It is known that the pull-back of B-field can be included into the D-brane effective action via the replacement $F \to F + P[B]$. This combination is invariant under B-field gauge transformation as $P[B] \to P[B] - P[d\Lambda]$, $F \to F + P[d\Lambda]$. After this replacement, we set the velocity and gauge field strength $F$ to zero. This produces then the couplings of two NSNS states at order $\alpha'^2$ to all orders of constant B-field. Such couplings may be compared with the disk-level S-matrix element of two NSNS vertex operators in the presence of background B-field.

The S-matrix element of two NSNS vertex operators in the presence of background B-field has been calculated in [27, 28], *i.e.,*

$$A = -\frac{T_p\sqrt{-\det(\eta+B)}}{2}\frac{\Gamma(-t/2)\Gamma(-2s)}{\Gamma(1-t/2-2s)}\left(-2s\, a_1 + \frac{t}{2}\, a_2\right), \tag{3.1}$$

where $t = -2p_1\cdot p_2$ is the square of the momentum transfer in the transverse directions and $s = -\frac{1}{2}p_1\cdot D\cdot p_1$ is the momentum flow parallel to the world-volume of the D-brane. $D$ is defined such that for world-volume coordinates it is $D^{ab} = (\frac{\eta+B}{\eta-B})^{ab}$ and for transverse directions it is $D^{ij} = -\delta^{ij}$. The kinematic factors $a_1$ and $a_2$ are given by:

$$\begin{aligned}
a_1 &= \mathrm{Tr}(\varepsilon_1\cdot D)\, p_1\cdot\varepsilon_2\cdot p_1 - p_1\cdot\varepsilon_2\cdot D\cdot\varepsilon_1\cdot p_2 - p_1\cdot\varepsilon_2\cdot\varepsilon_1^T\cdot D^T\cdot p_1 - p_1\cdot\varepsilon_2^T\cdot\varepsilon_1\cdot D\cdot p_1\\
&\quad -\frac{1}{2}(p_1\cdot\varepsilon_2\cdot\varepsilon_1^T\cdot p_2 + p_2\cdot\varepsilon_1^T\cdot\varepsilon_2\cdot p_1) - s\,\mathrm{Tr}(\varepsilon_1\cdot\varepsilon_2^T) + \Big\{1\longleftrightarrow 2\Big\},\\
a_2 &= \mathrm{Tr}(\varepsilon_1\cdot D)\,(p_1\cdot\varepsilon_2\cdot D\cdot p_2 + p_2\cdot D\cdot\varepsilon_2\cdot p_1 + p_2\cdot D\cdot\varepsilon_2\cdot D\cdot p_2) + p_1\cdot D\cdot\varepsilon_1\cdot D\cdot\varepsilon_2\cdot D\cdot p_2\\
&\quad -\frac{1}{2}(p_2\cdot D\cdot\varepsilon_2\cdot\varepsilon_1^T\cdot D^T\cdot p_1 + p_1\cdot D^T\cdot\varepsilon_1^T\cdot\varepsilon_2\cdot D\cdot p_2) - s\,\mathrm{Tr}(\varepsilon_1\cdot D\cdot\varepsilon_2\cdot D)\\
&\quad +s\,\mathrm{Tr}(\varepsilon_1\cdot\varepsilon_2^T) + \mathrm{Tr}(\varepsilon_1\cdot D)\mathrm{Tr}(\varepsilon_2\cdot D)\,(s+t/4) + \Big\{1\longleftrightarrow 2\Big\}\ ,
\end{aligned} \tag{3.2}$$

where $\varepsilon_1$ and $\varepsilon_2$ are the polarizations of the NSNS states. In order to find the corresponding effective action at order $\alpha'^2$, we need to expand it in powers of $\alpha' p^2$ which is given as

$$A = -\frac{T_p\sqrt{-\det(\eta+B)}}{2}\Big(-2s\, a_1 + \frac{t}{2}\, a_2\Big)\left(\frac{1}{st} + \frac{\pi^2\alpha'^2}{24} + O(\alpha'^4)\right). \tag{3.3}$$

The leading order of this amplitude contains a t-channel and s-channel in addition to some contact terms. This order completely is described by supergravity action in the bulk plus the DBI action on the D-brane [27].[3] At all other orders, the amplitude just contains contact terms which are the effective couplings in the momentum space. We are interested in the $\alpha'^2$-contact terms.

In what follows we impose physical conditions for graviton ($\varepsilon_{\mu\nu} = \varepsilon_{\nu\mu}$) and B-field ($\varepsilon_{\mu\nu} = -\varepsilon_{\nu\mu}$) to find two-graviton, two-B-field and one-graviton-one-B-field couplings in the presences of background B-field. For two gravitons we can simplify $O(\alpha')^2$ part as follows:

$$\begin{aligned}
A \sim\ & s^2 \mathrm{Tr}(\varepsilon_1\!\cdot\!\varepsilon_2) - 2s\ \mathrm{Tr}(\varepsilon_1\!\cdot\! V_S) p_1\!\cdot\!\varepsilon_2\!\cdot\! p_1 + \frac{t}{4}\mathrm{Tr}(\varepsilon_1\!\cdot\! V_S) Tr(\varepsilon_2\!\cdot\! V_S)(4s+t)\\
&+st\mathrm{Tr}(\varepsilon_1\!\cdot\!\varepsilon_2\!\cdot\! V_S) + 2t\ p_1\!\cdot\! V_S\!\cdot\!\varepsilon_1\!\cdot\!\varepsilon_2\!\cdot\! V_S\!\cdot\! p_1 + 2s\ p_1\!\cdot\!\varepsilon_2\!\cdot\! V_S\!\cdot\!\varepsilon_1\!\cdot\! p_2 - st\mathrm{Tr}(\varepsilon_1\!\cdot\! V_S\!\cdot\!\varepsilon_2\!\cdot\! V_S)\\
&+4sp_1\!\cdot\!\varepsilon_2\!\cdot\!\varepsilon_1\!\cdot\! V_S\!\cdot\! p_1 - 2t\mathrm{Tr}(\varepsilon_1\!\cdot\! V_S) p_1\!\cdot\!\varepsilon_2\!\cdot\! V_S\!\cdot\! p_1 + 2t\mathrm{Tr}(\varepsilon_1\!\cdot\! V_S) p_1\!\cdot\! V_S\!\cdot\!\varepsilon_2\!\cdot\! V_S\!\cdot\! p_1\\
&-2t\ p_1\!\cdot\! V_S\!\cdot\!\varepsilon_2\!\cdot\! V_S\!\cdot\!\varepsilon_1\!\cdot\! V_S\!\cdot\! p_1 - st\mathrm{Tr}(\varepsilon_1\!\cdot\! V_A\!\cdot\!\varepsilon_2\!\cdot\! V_A) - 4t\ p_1\!\cdot\! V_A\!\cdot\!\varepsilon_2\!\cdot\! V_A\!\cdot\!\varepsilon_1\!\cdot\! V_S\!\cdot\! p_1\\
&+2t\mathrm{Tr}(\varepsilon_1\!\cdot\! V_S) p_1\!\cdot\! V_A\!\cdot\!\varepsilon_2\!\cdot\! V_A\!\cdot\! p_1 - 2t\ p_1\!\cdot\! V_A\!\cdot\!\varepsilon_2\!\cdot\! V_S\!\cdot\!\varepsilon_1\!\cdot\! V_A\!\cdot\! p_1 + \Big\{1 \longleftrightarrow 2\Big\}, \qquad (3.4)
\end{aligned}$$

where $V_S$ and $V_A$ are the symmetric and antisymmetric parts of $V = \frac{\eta+D^T}{2} = \frac{1}{\eta+B}$. The amplitude is invariant under parity ($V_S \to V_S$,$V_A \to -V_A$ ). We observe that the B-dependence appears as $\frac{1}{\eta+B}$ which is the same as the $F$-dependence in the T-duality invariant couplings that we have found in the previous section. More precisely, the parts of amplitude (3.4) that contain the symmetric matrix $V_S$, coincide with the $R^2$ terms in (2.7). The other terms containing the antisymmetric matrix $V_A$, also completely reproduce (2.12). This part of the S-matrix calculations has been done already in [28]. Our results confirm the computations of [28] after considering some identities for $R^2$ structures (for more details see appendix A).

The contact terms for two antisymmetric B-fields can be found by imposing the corresponding polarization tensors in (3.2), *i.e.*,

$$\begin{aligned}
A \sim\ & \frac{1}{2}s(4s+t)\mathrm{Tr}(\varepsilon_1\!\cdot\!\varepsilon_2) - 2s\ p_2\!\cdot\!\varepsilon_1\!\cdot\!\varepsilon_2\!\cdot\! p_1 + st\mathrm{Tr}(\varepsilon_1\!\cdot\! V_S\!\cdot\!\varepsilon_2) - 4s\ p_1\!\cdot\!\varepsilon_2\!\cdot\!\varepsilon_1\!\cdot\! V_S\!\cdot\! p_1\\
&-st\ \mathrm{Tr}(\varepsilon_1\!\cdot\! V_S\!\cdot\!\varepsilon_2\!\cdot\! V_S) + 2s\ p_2\!\cdot\!\varepsilon_1\!\cdot\! V_S\!\cdot\!\varepsilon_2\!\cdot\! p_1 - 2t\ p_1\!\cdot\! V_S\!\cdot\!\varepsilon_1\!\cdot\! V_S\!\cdot\!\varepsilon_2\!\cdot\! V_S\!\cdot\! p_1\\
&-st\ \mathrm{Tr}(\varepsilon_1\!\cdot\! V_A\!\cdot\!\varepsilon_2\!\cdot\! V_A) + 2t\ p_1\!\cdot\! V_A\!\cdot\!\varepsilon_2\!\cdot\!\varepsilon_1\!\cdot\! V_A\!\cdot\! p_1 + \frac{1}{4}t(2s+t)\mathrm{Tr}(\varepsilon_1\!\cdot\! V_A)\mathrm{Tr}(\varepsilon_1\!\cdot\! V_A)\\
&-2t\ \mathrm{Tr}(\varepsilon_1\!\cdot\! V_A) p_1\!\cdot\!\varepsilon_2\!\cdot\! V_A\!\cdot\! p_1 - 4t\ p_1\!\cdot\! V_A\!\cdot\!\varepsilon_2\!\cdot\! V_A\!\cdot\!\varepsilon_1\!\cdot\! V_S\!\cdot\! p_1\\
&-4t\ \mathrm{Tr}(\varepsilon_1\!\cdot\! V_A) p_1\!\cdot\! V_A\!\cdot\!\varepsilon_2\!\cdot\! V_S\!\cdot\! p_1 - 2t\ p_1\!\cdot\! V_A\!\cdot\!\varepsilon_2\!\cdot\! V_S\!\cdot\!\varepsilon_1\!\cdot\! V_A\!\cdot\! p_1 + \Big\{1 \longleftrightarrow 2\Big\}. \qquad (3.5)
\end{aligned}$$

Again we see that the part without $V_A$ is in agreement with $H^2$ terms in (2.7) and the remaining part is reproduced exactly by (2.14).

The amplitude for one graviton and one antisymmetric B-field has no counterpart at zero background B-field limit because it has odd number of $V_A$. Imposing physical

[3] This is similar to bosonic string computations [29].

conditions for graviton and B-field in amplitude (3.3), we have found

$$\begin{aligned} A \sim & -2st\ \mathrm{Tr}(\varepsilon_1\!\cdot\!\varepsilon_2\!\cdot\!V_A) + 4st\ \mathrm{Tr}(\varepsilon_1\!\cdot\!V_S\!\cdot\!\varepsilon_2\!\cdot\!V_A) - \frac{1}{2}t(4s+t)\ \mathrm{Tr}(\varepsilon_1\!\cdot\!V_S)\mathrm{Tr}(\varepsilon_1\!\cdot\!V_A) \\ & +4s\ p_1\!\cdot\!V_A\!\cdot\!\varepsilon_2\!\cdot\!\varepsilon_1\!\cdot\!p_2 + 4s\ p_1\!\cdot\!V_A\!\cdot\!\varepsilon_1\!\cdot\!\varepsilon_2 - 4s\ p_1\!\cdot\!\varepsilon_2\!\cdot\!V_A\!\cdot\!\varepsilon_1\!\cdot\!p_2 + 2s\ p_2\!\cdot\!\varepsilon_1\!\cdot\!p_2\mathrm{Tr}(\varepsilon_1\!\cdot\!V_A) \\ & -2t\ p_1\!\cdot\!V_S\!\cdot\!\varepsilon_1\!\cdot\!p_2\mathrm{Tr}(\varepsilon_2\!\cdot\!V_A) - 4t\ p_1\!\cdot\!V_A\!\cdot\!\varepsilon_2\!\cdot\!\varepsilon_1\!\cdot\!V_S\!\cdot\!p_1 + 2t\ p_1\!\cdot\!V_A\!\cdot\!\varepsilon_2\!\cdot\!p_1\mathrm{Tr}(\varepsilon_1\!\cdot\!V_S) \\ & -2t\ p_1\!\cdot\!V_S\!\cdot\!\varepsilon_1\!\cdot\!V_S\!\cdot\!p_1\mathrm{Tr}(\varepsilon_2\!\cdot\!V_A) + 4t\ p_1\!\cdot\!V_S\!\cdot\!\varepsilon_1\!\cdot\!V_A\!\cdot\!\varepsilon_2\!\cdot\!V_S\!\cdot\!p_1 \\ & -4t\ p_1\!\cdot\!V_A\!\cdot\!\varepsilon_2\!\cdot\!V_S\!\cdot\!p_1\mathrm{Tr}(\varepsilon_1\!\cdot\!V_S) + 4t\ p_1\!\cdot\!V_S\!\cdot\!\varepsilon_1\!\cdot\!V_S\!\cdot\!\varepsilon_2\!\cdot\!V_A\!\cdot\!p_1 - 4t\ p_1\!\cdot\!V_S\!\cdot\!\varepsilon_2\!\cdot\!V_S\!\cdot\!\varepsilon_1\!\cdot\!V_A\!\cdot\!p_1 \\ & +4t\ p_1\!\cdot\!V_A\!\cdot\!\varepsilon_1\!\cdot\!V_A\!\cdot\!\varepsilon_1\!\cdot\!V_A\!\cdot\!p_1 - 2t\ p_1\!\cdot\!V_A\!\cdot\!\varepsilon_1\!\cdot\!V_A\!\cdot\!p_1\mathrm{Tr}(\varepsilon_2\!\cdot\!V_A)\,. \end{aligned} \tag{3.6}$$

In this amplitude $\varepsilon_1$ is the polarization of graviton and $\varepsilon_2$ belongs to B-field. These contact terms reproduce exactly the couplings (2.15) in the momentum space. We have also replaced the dilaton polarization $\varepsilon_{\mu\nu} = \eta_{\mu\nu} + \ell_\mu p_\nu + \ell_\nu p_\mu$ where $\ell$ is an auxiliary vector satisfying $\ell p = 1$, in the amplitude and found exact agreement with the corresponding couplings in the previous section. This ends our illustration of precise agreement between the couplings that we have found in the previous section by the linear T-duality calculations and the S-matrix element of two NSNS vertex operators in the presence of background B-field.

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## A On Riemann polynomial identities

The method we have used here to construct the independent Riemann polynomials is according to work of [30] and using the Mathematica package xAct [31] which is projecting Riemann tensors onto their Young tableaux:

$$R_{\alpha\beta\mu\nu} \to \frac{1}{3}(2R_{\alpha\beta\mu\nu} - R_{\alpha\nu\beta\mu} + R_{\alpha\mu\beta\nu})\,. \tag{A.1}$$

For example it is easy to show that the identity $2R_{\alpha\beta\mu\nu}R^{\alpha\mu\beta\nu} = R_{\alpha\beta\mu\nu}R^{\alpha\beta\mu\nu}$ holds. Similarly there are other many identities for $R^2$ terms contracted with other tensors, which reduce the number of independent terms. As another example one may consider the following two terms independent:

$$x_1 R_{\alpha\delta}{}^{\theta\kappa} R_{\beta\gamma\theta\kappa} T^{\alpha\beta} T^{\gamma\delta} + x_2 R_{\alpha\delta}{}^{\theta\kappa} R_{\beta\theta\gamma\kappa} T^{\alpha\beta} T^{\gamma\delta}\,, \tag{A.2}$$

where T is an arbitrary tensor. But by projecting these terms one gets:

$$\begin{aligned} & \frac{2}{9}(2x_1+x_2)R_{\alpha\mu}{}^{\rho\kappa}R_{\beta\nu\rho\kappa}T^{\alpha\beta}T^{\nu\mu} + \frac{4}{9}(2x_1+x_2)R_{\alpha\mu}{}^{\rho\kappa}R_{\beta\rho\nu\kappa}T^{\alpha\beta}T^{\nu\mu} \\ & \quad + \frac{1}{9}(2x_1+x_2)R_\alpha{}^\rho{}_\mu{}^\kappa R_{\beta\rho\nu\kappa}T^{\alpha\beta}T^{\nu\mu} - \frac{1}{9}(2x_1+x_2)R_\alpha{}^\rho{}_\mu{}^\kappa R_{\beta\kappa\nu\rho}T^{\alpha\beta}T^{\nu\mu}\,, \end{aligned} \tag{A.3}$$

which means that the above terms are not independent and by $x_2 = -2x_1$ we have an identity. As another example consider the following three terms:

$$x_1 R_{\alpha\mu}{}^{\rho\kappa} R_{\beta\nu\rho\kappa} T^{\alpha\beta} T^{\mu\nu} + x_2 R_\alpha{}^\rho{}_\mu{}^\kappa R_{\beta\rho\nu\kappa} T^{\alpha\beta} T^{\mu\nu} + x_3 R_\alpha{}^\rho{}_\mu{}^\kappa R_{\beta\kappa\nu\rho} T^{\alpha\beta} T^{\mu\nu}\,, \qquad \text{(A.4)}$$

we can show that we have an identity when $x_2 = -2x_1$ and $x_3 = 2x_1$ . Similarly one may go further to find more independent structures by generalizing above procedure.

To find independent couplings for $\nabla H$, one should impose the Bianchi identity $dH = 0$. This can be done by writing $H$ in terms of $B$.



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