M-theory compactifications to three dimensions with M2-brane potentials

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ABSTRACT: We study a class of compactifications of M-theory to three dimensions that preserve $N = 2$ supersymmetry and which have the defining feature that a probe space-time filling M2 brane feels a non-trivial potential on the internal manifold. Using M-theory/F-theory duality such compactifications include the uplifts of 4-dimensional $N = 1$ type IIB compactifications with D3 potentials to strong coupling. We study the most general 8-dimensional manifolds supporting these properties, derive the most general flux that induces an M2 potential, and show that it is parameterised in terms of two real vectors. We study the supersymmetry equations when only this flux is present and show that over the locus where the M2 potential is non-vanishing the background takes the form of a Calabi-Yau three-fold fibered over a 2-dimensional base spanned by the flux vectors, while at the minima of the potential the flux vanishes. Allowing also for non-vanishing four-form flux with one leg in the internal directions we find that the Calabi-Yau three-fold in the fibration is replaced by an SU(3)-structure manifold with torsion classes satisfying $2W_4 = -W_5$.

Contents

1 Introduction

In this paper we study compactifications of M-theory to three dimensions that preserve $N = 2$ supersymmetry and which induce a potential for space-filling M2-branes. Though interesting as three-dimensional vacua in themselves, our primary motivation for studying them originates in F-theory [1]. Four-dimensional $N = 1$ F-theory vacua can be defined as dual to a particular limit of $N = 2$ M-theory compactifications to three-dimensions. This definition relies on the assumption that the 8-dimensional manifold on which M-theory

is compactified is elliptically fibered. Then the appropriate limit where four-dimensional physics is recovered is that of a vanishing fibre. This duality is most often used to construct F-theory backgrounds as dual to M-theory compactifications on elliptically fibered Calabi-Yau manifolds.[1] Such compactifications can exhibit realistic particle physics models and have been under intensive study in recent years [4–6]. A well-understood generalisation of such models is to include a particular $(2, 2)$ background four-form flux on the M-theory side, which is dual to both background closed-string and brane fluxes on the F-theory side. Such fluxes play a crucial role in moduli stabilisation and in generating chirality in particle physics models. It is known that the backreaction of such fluxes deforms the background only to conformal CY [7, 8], so that the relevant compactifications are warped $N = 2$ compactifications of M-theory to three dimensions. This class of compactifications have the property that space-filling probe M2-branes on the M-theory side, which are dual to space-filling probe D3-branes on the F-theory side, are BPS at all points of the (conformal) CY and therefore feel no potential [8].

There are a number of interesting departures from such backgrounds which can be characterised by the fact that a potential is generated for (probe) D3-branes. The most familiar are backgrounds of type IIB string theory that support gaugino condensation on D7-branes, or certain types of D3-instantons that contribute to the superpotential, as used for moduli stabilisation in the KKLT and Large volume scenarios [9, 10]. The fact that these induce a potential for D3-branes can be shown by performing a 1-loop string calculation [11] or through gravitational back-reaction [12]. As well as being used for moduli stabilisation, there is a direct cosmological use for type IIB backgrounds that induce D3-potentials for inflation [13, 14]. Such backgrounds also play an important role in particle physics model building. In F-theory models where all the generations are realised on a single matter curve the Yukawa coupling at a point of intersection is rank one [15, 16] unless an appropriate deformation of the geometry which induces a D3 potential by flux [15] or non-perturbative effects [17] is present.[2] Since all the listed interesting IIB/F-theory backgrounds have potentials for D3-branes they are not within the better understood class of warped CY compactifications. Although an approach of neglecting the backreaction of the effects that induced the D3 potential on the geometry and continuing to use a CY background may be a valid approximation for some purposes, an understanding of the backreaction is essential for applications where the effects are large or when a treatment of the background in a 10-dimensional sense, rather than a 4-dimensional effective theory, is important. Studying such backgrounds, particularly at strong coupling, using F-theory/M-theory duality should therefore involve some understanding of three-dimensional M-theory backgrounds which preserve $N = 2$ supersymmetry and have a potential for M2-branes.

Having identified the presence of a D3/M2 brane potential as the defining feature we are interested in, it is essential to understand what are the properties of the background geometry and flux which induce such a potential. There is a rather general and neat answer to this question in type IIB supergravity. The key property of the background is

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[1]Though see recent work on manifolds with Spin(7) holonomy [2, 3].

[2]Interestingly, such backgrounds also induce non-commutative deformations on the world-volume theories of the 7-branes [13, 15, 17].

the structure group of the metric once any flux/branes are back-reacted. Recall that a nowhere vanishing spinor on a manifold reduces its structure group. Since the presence of a nowhere vanishing internal spinor is a direct requirement for the background to preserve some supersymmetry a reduced structure group typically characterises supersymmetric backgrounds [18]. In particular the most general type IIB (supergravity) backgrounds which preserve four-dimensional $N = 1$ supersymmetry have an SU(3) × SU(3) structure group [19, 20].[3] Here each SU(3) is associated to a spinor on the manifold, but the two spinors are not everywhere orthogonal (which would lead to SU(2)-structure) or parallel (leading to SU(3)-structure) but rather the angle between them varies over the manifold. Now an interesting result of [21, 22] is that backgrounds which support potentials for D3-branes are those which have such an SU(3) × SU(3) structure group, and further, that the minimum for the potential, where the D3-branes are BPS, occurs exactly on the loci where the two spinors become parallel, yielding a 'local' SU(3)-structure.[4] This general result was checked for the particular case of D7 gaugino condensation in [23] which confirmed that their backreaction indeed changes the structure group from SU(3) to SU(3) × SU(3). It was also confirmed to some extent in [24, 25] for the case of D3-potentials induced by non-Imaginary-Self-Dual background flux, specifically by showing that the backreaction of D7 gaugino-condensation can locally be viewed as such a background flux.[5]

One aim of this paper is to develop analogous relations between structure groups and M2-brane potentials in M-theory. The relation between structure groups and non-vanishing spinors in 8 dimensions is rather different from the more familiar 6 and 7 dimensions. Consider a compactification on an 8-dimensional manifold X_8. A requirement for preserving some supersymmetry in 3-dimensions is the existence of a nowhere vanishing Majorana spinor on X_8. This is so that the 11-dimensional M-theory supersymmetry spinor decomposes into a product of a 3-dimensional and 8-dimensional Majorana spinor. However, the existence of a nowhere vanishing Majorana spinor in 8 dimensions does not imply a reduction of the structure group.[6] This only occurs in the presence of nowhere vanishing Majorana-Weyl spinors: a single such spinor implies Spin(7)-structure, while two such spinors imply G_2 or SU(4)-structure if they have the opposite or same relative chirality respectively. Now the number of supersymmetries preserved by an M2-brane in a background is given by the number of independent covariantly constant Majorana-Weyl spinors of fixed chirality [27]. Since the manifolds with fixed structure group have a fixed number of Majorana-Weyl spinors of fixed chiralities M2-branes are BPS over the whole space and preserve the supersymmetries of the background. Therefore a connection between structure groups and M2 potentials for 8-dimensional manifolds is not obvious.

There is a useful way to think about the M2-potential in terms of 8-dimensional local structure groups, i.e. submanifolds of X_8 over which the spinors satisfy certain properties

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[3]See also the analogous dynamical SU(2)-structure type II backgrounds [29].

[4]We will use the notion of a 'local' structure group often in the paper. We define it as the structure group that would result should the properties of the spinors on a local submanifold where the local structure group is defined be extended to the full space.

[5]See also [26] for further investigations along this direction.

[6]The stabiliser group of a Majorana spinor is G_2. However we use the notion of a global structure group as maximal over the manifold, while the G_2 stabiliser may enhance over certain loci.

such as being non-vanishing. In this paper we are interested in $N = 2$ vacua and so require two covariantly constant, and therefore nowhere vanishing, Majorana spinors. The requirement of Majorana spinors rather than Majorana-Weyl spinors leads to the most general 8-dimensional backgrounds that preserves $N = 2$ in three dimensions from M-theory, in this sense they are the analogs of $SU(3) \times SU(3)$ structure 6-dimensional backgrounds in IIB. The two nowhere vanishing Majorana spinors can be decomposed into 4 Majorana-Weyl spinors, but any of these four may vanish on certain loci. In the generic point on X_8 all four are non-vanishing and we have a local $SU(3)$-structure on X_8, over certain loci one of the Majorana-Weyl components in each Majorana spinor may vanish so that we are left with two Majorana-Weyl spinors of same or opposite chirality leading to local $SU(4)$ or G_2-structures. M2-branes are calibrated by a Killing Majorana-Weyl spinor of fixed chirality and therefore on the generic $SU(3)$-structure loci they preserve no supersymmetry, on G_2 loci they preserve $N = 1$ supersymmetry and on $SU(4)$ loci they preserve $N = 2$ supersymmetry. They therefore feel a potential in such backgrounds with minima at $SU(4)$ and G_2-structure loci. Such backgrounds are therefore of interest following our original motivation and much of this paper is dedicated to exploring their properties.

In [28] it was shown that one can induce a connection between the existence of a Majorana spinor on X_8 and global G-structures by defining an auxiliary 9-dimensional manifold, Y_9, which is just the product of the 8-dimensional one with a circle $Y_9 = X_8 \times S_1$. Now the existence of an 8-dimensional nowhere vanishing Majorana spinor on X_8 induces a nowhere vanishing Majorana spinor on Y_9 and this is known to imply a reduction of the structure group of Y_9 to $\mathrm{Spin}(7)$. Therefore it is quite natural to work with this auxiliary 9-dimensional manifold when studying the supersymmetry properties of the background. As with the case of one spinor, two nowhere vanishing Majorana spinors do not induce a reduction of the structure group in 8 dimensions. It is possible to consider again an uplift to Y_9, but although the structure group is reduced to at least $\mathrm{Spin}(7)$, there is in general no further reduction on Y_9 due to the second spinor. Nonetheless, the 9-dimensional approach is useful for treating the 8-dimensional local structure groups in a unified way and we will utilise it in this work.

So far we have discussed only the geometry part of the compactification and not the energy-momentum tensor that sources it. In this work we will study the background flux that can source M2-brane potentials. In relation to the previous discussion of physics sources for a D3-potential in IIB, this flux can be thought of either as non-trivial background flux or, in the spirit of [24, 25], as flux that is accounting for the backreaction of localised sources. We will be able to give the form of the most general flux that generates an M2-potential in terms of 8-dimensional $SU(3)$-structure geometric objects and two real one-forms that parameterise the flux. For the simple case where only this type of flux and four-form flux with one leg in the internal directions, which we henceforth refer to as 1-form flux, are present the supersymmetry equations simplify considerably and we are able to present them as differential relations on the $SU(3)$-structure forms and extract some key properties. We find that for this limited flux configuration the compactification must be to 3-dimensional Minkowski space. If we further impose the vanishing of the 1-form flux we find that the M2-potential is only along the directions parameterised by the two singlet

vectors of the 8-dimensional SU(3)-structure. The torsion classes are such that on the generic SU(3)-structure locus the manifold can be described as a 6-dimensional Calabi-Yau fibered over a 2-dimensional base which is spanned by the singlet vectors, and over which the M2-branes have a non-trivial potential. While over the special SU(4) and G_2-structure loci the flux vanishes. The more involved background where we also allow for non-vanishing 1-form flux leads to a similar configuration but the 6-dimensional fibre is not Calabi-Yau but has non-vanishing torsion classes (which satisfy the relation $2W_4 = -W_5$). We will also present the supersymmetry equations in differential form for the most general flux configurations that have other, non M2-potential inducing, fluxes turned on. Though this substantially more complicated system is difficult to analyse in as much detail.

The outline of the paper is as follows. In section 2 we study the geometric properties of the background using G-structures. In particular we describe 8-dimensional manifolds with varying structure groups and M2-potentials. In section 3 we study the supersymmetry equations and formulate them in a way compatible with the 9-dimensional geometry. In particular we identify the flux which is responsible for the M2-potentials, and parameterise it in terms of SU(3)-structure objects. In section 4 we study the implication of the supersymmetry equations for backgrounds supporting such a flux. We summarise our results in section 5.

Note 1: the results presented in this paper rely on quite lengthy calculations. Although most of these calculations can be done by hand we made extensive use of symbolic calculation applications which are able to manipulate tensors and/or gamma matrices, in order to check and derive some of our results. We acknowledge the use of the following resources: Mathematica [30], MathTensor [31], Cadabra, [32, 33], Gamma [34] and xTensor [35].

Note 2: this paper has some overlap with a project which was initiated together with M. Babalic, I. Coman and C. Lazaroiu to whom we thank for insights in the subject of M-theory compactifications to three dimensions.

2 N=2, D=3 M-theory compactifications and G-structures

2.1 Supersymmetric compactification of M-theory

The effective action of M-theory is described by eleven dimensional supergravity consisting of the following fields: metric g_{MN}, three-form potential C with corresponding field strength $G = dC$ and gravitino Ψ_M. The action can then be written in the following way [36]

$$S_{11} = \frac{1}{2} \int d^{11}x \sqrt{-g} \left(R - \frac{1}{2} G \wedge *G - \frac{1}{6} C \wedge G \wedge G \right) . \tag{2.1}$$

We shall be interested in supersymmetric flux backgrounds. They correspond to fluxes for which the background gravitino vanishes together with its supersymmetry variation with 11-dimensional (Majorana) spinor parameter ϵ

$$\delta \Psi_M = \nabla_M \epsilon - \frac{1}{288} \left(\Gamma_M{}^{NPQR} - 8\delta_M^N \Gamma^{PQR} \right) G_{NPQR} \epsilon = 0 . \tag{2.2}$$

The matrices Γ_M are taken to satisfy the eleven-dimensional Clifford algebra with metric signature $(-, +, \ldots, +)$.

The supersymmetry equations should be supplemented by the Bianchi identity and equations of motion for G which read[7]

$$dG = 0 \, , \tag{2.3}$$

$$d \star G = -\frac{1}{2} G \wedge G + 2\pi T_6 X_8 \, . \tag{2.4}$$

The last term in (2.4) corresponds to a higher derivative gravitation correction [40], where T_6 is the M5-brane tension and X_8 is a known combination of the first and second Pontrjagin forms. This correction is important as it allows the support of solutions to three-dimensional Minkowski space in the presence of background flux for compact smooth manifolds. Equation (2.4) is usually imposed as integrated over the manifold in which case the first term vanishes and the last term gives the Euler number of the manifold, This leads to the familiar D3/M2 tadpole cancellation constraint.[8]

The compactification Ansatz is chosen by imposing a 3-8 split of the 11 dimensional manifold. We choose a metric which is a warped product of 3-dimensional space-time and an 8-dimensional Euclidean manifold.

$$ds_{11}^2 = e^{2\Delta} \left(ds_{2,1}^2 + ds_8^2 \right) \, . \tag{2.5}$$

The 11-dimensional index M decomposes into an external 3-dimensional index $\mu = 1, 2, 3$, and an internal 8-dimensional index $\alpha = 1, \ldots, 8$. For the 4-form field strength G we choose the most general ansatz compatible with Lorentz invariance

$$G = e^{3\Delta} \left(\tilde{f} \wedge \mathrm{Vol}_3 + F \right) \, , \tag{2.6}$$

where $\tilde{f}$ is a one-form and F is a 4-form on the internal manifold, while Vol_3 is the volume element of the external space-time.

The eleven-dimensional Clifford algebra is decomposed according to the following equations

$$\begin{aligned} \Gamma_\mu &= e^\Delta \left(\gamma_\mu \otimes \gamma_9 \right) \, , \\ \Gamma_\alpha &= e^\Delta \left(\mathbb{1} \otimes \gamma_\alpha \right) \, , \end{aligned} \tag{2.7}$$

with the 2×2 matrices $\{ \gamma_\mu \, ; \, \mu = 1, 2, 3 \}$ generating the three-dimensional Clifford algebra $\mathrm{Cl}(2,1)$. An explicit representation can be given in terms of the Pauli matrices. The 16×16 matrices γ_α are taken to be real and symmetric. They generate the eight-dimensional Clifford algebra $\mathrm{Cl}(8,0)$.

We can decompose the 11-dimensional supersymmetry parameter ϵ, which is an 11-dimensional Majorana spinor, according to the 3-8 split as

$$\epsilon = e^{-\frac{\Delta}{2}} \eta \otimes \xi \, , \tag{2.8}$$

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[7]It is usually the case that the equations of motion and the supersymmetry equations imply the Einstein equations, though we will not prove it here for the particular class of backgrounds under consideration.

[8]Note that for manifolds with 8-dimensional G_2-structure the Euler number is forced to vanish [41].

where η is a 3-dimensional Majorana spinor, while ξ is an 8-dimensional Majorana spinor. Each non-vanishing spinor ξ defines by the relation above one spinor η in three dimensions. Therefore for $N = 2$ supersymmetry we need two spinors ξ_i on the internal manifold. Note that in 8 dimensions there exist Majorana-Weyl spinors. However we do not impose any chirality condition on the internal spinors as from the supersymmetry equation it is clear that only the Majorana condition is necessary. Imposing the Weyl property is an additional constraint. In most studies of M-theory compactifications to 3 dimensions so far the Majorana-Weyl condition was imposed for simplicity but, as emphesised in [28, 37], this is not the most general case.

2.2 General 8-dimensional manifolds preserving $N = 2$ supersymmetry

In this section we present a detailed description of the manifolds on which we compactify M-theory. As explained before, such manifolds admit two independent, nowhere vanishing Majorana spinors $\xi_{1,2}$. We shall see further that the supersymmetry conditions imply that the norm of these spinors is constant [37] and therefore without loss of generality we shall assume that the two spinors are orthonormal

$$\xi_i^T \xi_j = \delta_{ij} \ , \quad i,j = 1,2 \ . \tag{2.9}$$

Since in 8 dimensions the Majorana and Weyl conditions are compatible, we can split the two spinors into spinors of definite chirality

$$\xi_i = (\xi_+)_i + (\xi_-)_i \ . \tag{2.10}$$

However, the Majorana-Weyl components $(\xi_\pm)_i$ are no longer required to have constant norm and moreover they can even vanish at certain points. The only requirement is the unit norm of the spinors ξ_i

$$||(\xi_+)_i||^2 + ||(\xi_-)_i||^2 = 1 \ . \tag{2.11}$$

In the case that all Majorana-Weyl components are everywhere non-vanishing we are actually dealing with an 8-dimensional manifold with SU(3) structure which preserves (a maximum of) $N = 4$ supersymmetry in 3 dimensions. If some of the Majorana-Weyl components vanish identically over the entire internal manifold, while the others are non-vanishing, then we are dealing with manifolds with G_2 structure or manifolds with SU(4) structure depending on the relative chirality of the non-vanishing spinors. We see that the fact that the Majorana-Weyl components are allowed to vanish at certain points implies that such manifolds do not admit a global reduction of the structure group. At generic points they look like SU(3) structure manifolds, while at special points they look like G_2 or SU(4) structure manifolds.[9]

Note that we did not consider the case where only one of the four Majorana-Weyl components vanishes. This is because on an 8-dimensional manifold once we are given three linearly independent, non-vanishing, Majorana-Weyl spinors which are not of the

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[9]It would be interesting to explore connections between these backgrounds and SU(4) × SU(4) backgrounds as studied in [42, 43], as well as the Spin(8) × Spin(8) backgrounds of [44].

same chirality, one can define a fourth spinor such that we end up with two spinors of one chirality and two of the other. To prove this, suppose that we have an eight-dimensional manifold which has two spinors, ξ_1 and ξ_2 of positive chirality and one, χ_1 of negative chirality. The two spinors of positive chirality define a SU(4) structure and therefore one finds an almost complex structure which is given by

$$J_{\alpha\beta} = \xi_1^T \gamma_{\alpha\beta} \xi_2 \, . \tag{2.12}$$

We can define a fourth spinor χ_2 which is of negative chirality

$$\chi_2 = \frac{1}{2} J^{\alpha\beta} \gamma_{\alpha\beta} \chi_1 \, , \tag{2.13}$$

and which has non-vanishing norm. This spinor is obviously orthogonal to $\xi_{1,2}$ and because the matrix $\gamma_{\alpha\beta}$ is antisymmetric in its spinorial indices, it is also orthogonal to χ_1. Therefore, in this way we have 4 non-vanishing Majorana-Weyl spinors, two of positive and two of negative chirality, which define a SU(3) structure in 8 dimensions.

2.2.1 9-dimensional uplifts to $Y_9 = X_8 \times S_1$

It was pointed out in [28] that we can associate the existence of a nowhere vanishing Majorana spinor on X_8 to a reduced structure group by uplifting it to an auxiliary 9-dimensional manifold, Y_9, defined as the direct product of the 8-dimensional manifold under consideration X_8 and a circle $Y_9 = X_8 \times S_1$. For the case of 2 Majorana spinors this procedure is not as effective since, as we will show, the structure group does not reduce further on Y_9. However, uplifting to 9-dimensions is still a useful procedure because it will allow us to describe the local structure groups of X_8 in a unified way. In particular the 8-dimensional structure groups, and the related M2 potential, will be mapped to an angle between two 9-dimensional vectors.

We therefore go on to study 9-dimensional manifolds supporting two nowhere vanishing Majorana spinors. It is most common to study such manifolds from the perspective of spinor bilinears which can be constructed out of the spinors. Since 9-dimensional Euclidean gamma matrices can be chosen real (and therefore symmetric) the only spinor bilinears which can be defined are the following

$$
\begin{aligned}
(V_1)_m = \xi_1^T \gamma_m \xi_1 \, , \qquad & (V_2)_m = \xi_2^T \gamma_m \xi_2 \, , \qquad && (V_3)_m = \xi_1^T \gamma_m \xi_2 = \xi_2^T \gamma_m \xi_1 \, , \\
K_{mn} = \xi_1^T \gamma_{mn} \xi_2 = -\xi_2^T \gamma_{mn} \xi_1 \, , \qquad & && \Psi_{mnp} = \xi_1^T \gamma_{mnp} \xi_2 = -\xi_2^T \gamma_{mnp} \xi_1 \, , \\
(\Phi_1)_{mnpq} = \xi_1^T \gamma_{mnpq} \xi_1 \, , \qquad & (\Phi_2)_{mnpq} = \xi_2^T \gamma_{mnpq} \xi_2 \, , \qquad && (\Phi_3)_{mnpq} = \xi_1^T \gamma_{mnpq} \xi_2 \, .
\end{aligned}
\tag{2.14}
$$

Here γ_m are 9-dimensional gamma matrices, or generators of Cl(9, 0), with $m = 1, \ldots, 9$. These forms are not independent as certain products of such bilinears can be expressed in terms of linear combinations of these bilinears. The complete set of relations which the forms above satisfy is given in appendix A.

For an efficient description of such manifolds it is important to know the number of independent vectors. The forms defined above include three vectors. The Fierz relations

for the vectors imply

$$||V_1||^2 = ||V_2||^2 = 1 \ , \qquad ||V_3||^2 = \frac{1}{2}\left(1 - \alpha\right) \ , \tag{2.15}$$

$$V_1 \cdot V_2 \equiv \alpha \ , \qquad\qquad V_1 \cdot V_3 = V_2 \cdot V_3 = 0 \ . \tag{2.16}$$

Here we define the usual contraction $V \cdot U = V^m U_m$. Therefore the number of independent vectors is governed by a real parameter α which is the scalar product of the vectors V_1 and V_2. Since the vectors V_1 and V_2 are of unit norm, this parameter α is in fact the cosine of the angle between these vectors and therefore can take values in the interval $[-1, 1]$. At generic values of this parameter, all the three vectors are independent and of non-vanishing norm. This is the case of a local SU(3) structure. We denote this a 'local structure' because α varies over X_8 and so particular values of it define certain submanifolds. Of course the notion of structure group only has a global meaning. However the terminology of a local structure is well defined by the properties of the spinors over the submanifold and will be used extensively in this work. For $\alpha = -1$ the vectors V_1 and V_2 are no longer independent (they are anti-parallel) while V_3 has unit norm. Therefore we are left with two independent unit vectors. This is the case of a G_2 structure. Finally, if $\alpha = 1$, the vectors V_1 and V_2 are parallel while V_3 vanishes. This is the case of an SU(4) structure. These local 9-dimensional structure groups dictated by the parameter α are directly inherited by the 8-dimensional manifold. In particular the physics we are interested in, which is the potential for a probe space-filling M2-brane, is therefore directly related to the variation of α over the internal manifold. We will show this in more detail in section 3.3.

Before going into more details about these manifolds it will be useful to understand the special cases of $\alpha = \pm 1$.

2.2.2 Loci of G_2 structure: $\alpha = -1$

On such loci, the Fierz relations presented in appendix simplify and we find

$$V_1 = -V_2 = V \ , \qquad ||V||^2 = ||V_3||^2 = 1 \ , \tag{2.17}$$

$$K = V \wedge V_3 \ , \tag{2.18}$$

$$V \lrcorner \Psi = V_3 \lrcorner \Psi = 0 \ , \tag{2.19}$$

$$\Phi_1 = -V_3 \wedge \Psi - *(V \wedge V_3 \wedge \Psi) \ , \tag{2.20}$$

$$\Phi_2 = V_3 \wedge \Psi - *(V \wedge V_3 \wedge \Psi) \ , \tag{2.21}$$

$$\Phi_3 = V \wedge \Psi \ . \tag{2.22}$$

This description looks much like a G_2 structure in 7-dimensions (which is given in terms of the 3-form Ψ) and two additional vectors which lift this G_2 structure to 9-dimensions. This is expected from the decomposition of the fundamental of SO(9) under G_2

$$\mathbf{9} \to \mathbf{7} \oplus \mathbf{1} \oplus \mathbf{1} \ . \tag{2.23}$$

A linear combination of these vectors gives the direction along the auxiliary circle used to uplift to 9-dimensions.

2.2.3 Loci of SU(4)-structure: $\alpha = +1$

The Fierz relations for the case $\alpha = 1$ read

$$V_1 = V_2 = V \ , \qquad ||V||^2 = 1 \ , \qquad\qquad ||V_3||^2 = 0 \ , \qquad (2.24)$$

$$\Psi = K \wedge V \ , \qquad \Phi_+ = -K \wedge K \ , \qquad K_{m[n} \left(\Phi_-\right)^m{}_{pq]} = 2 \left(\Phi_3\right)_{npq} \ , \qquad (2.25)$$

where we define $\Phi_\pm = \Phi_1 \pm \Phi_2$. Moreover it can be shown that when restricted to the subspace which is orthogonal to V, K is an almost complex structure and it is clear that we can organise this orthogonal space as a space of SU(4) structure with Φ_- and $2\Phi_3$ playing the role of the real and imaginary parts of the complex four-form. The additional vector field should be understood as the direction which we added to go to the auxiliary nine-dimensional manifold.

2.3 Parametrisation in terms of a SU(3)-structure

The most interesting case is of when the angle α, between the vectors V_1 and V_2 varies. This occurs over the generic patch on the manifold which manifests a local SU(3)-structure. Over this locus we have that $\alpha \neq \pm 1$, and we will assume this in our present analysis and return to the limit points later. In dealing with the more complicated SU(3)-structure case we are guided by the idea that, analogous to the SU(4) and G_2-structure loci, we expect to be able to describe it in terms of an (uplifted) 6-dimensional SU(3)-structure. In order to unveil this structure let us first define

$$V_\pm = V_1 \pm V_2 \ . \qquad (2.26)$$

Clearly V_+ and V_- are mutually orthogonal and, since both V_1 and V_2 are orthogonal to V_3, they are also orthogonal to V_3. However, these vectors no longer have unit norm but we find

$$||V_\pm||^2 = 2(1 \pm \alpha) \ . \qquad (2.27)$$

The next step is to decompose all the forms in (2.14) into forms of lower or equal rank forms which are orthogonal to these vectors. Let us use as an example the decomposition of K. We write

$$K = J + aV_+ \wedge V_- + bV_+ \wedge V_3 + cV_- \wedge V_3 \ , \qquad (2.28)$$

where a, b and c are coefficients which should be derived from imposing that J satisfies $V_\pm \lrcorner J = V_3 \lrcorner J = 0$. From the Fierz relations with one free index we see that V_+ is already orthogonal to K and therefore $a = b = 0$. In order to find the coefficient c we contract with V_- and use the Fierz relations with one free index and obtain

$$K = J + \frac{1}{1-\alpha} V_- \wedge V_3 \ . \qquad (2.29)$$

The other cases work in a similar way. However, when considering forms of higher rank, the number of terms that can be written on the right hand side increases rendering the

calculation rather tedious. We find, eventually, the following equations

$$\Psi = \varphi + \frac{1}{1+\alpha} J \wedge V_+ + \frac{1}{2(1-\alpha)} V_+ \wedge V_- \wedge V_3 \,, \tag{2.30}$$

$$\Phi_+ = -\frac{2}{1+\alpha} J \wedge J - \frac{2}{1+\alpha} \rho \wedge V_+ - \frac{2}{1-\alpha} J \wedge V_- \wedge V_3 \,, \tag{2.31}$$

$$\Phi_- = \frac{4}{1-\alpha} \varphi \wedge V_3 + \frac{2}{1-\alpha} \rho \wedge V_- + \frac{2}{1+\alpha} J \wedge V_+ \wedge V_3 \,, \tag{2.32}$$

$$\Phi_3 = -\frac{1}{1-\alpha} \varphi \wedge V_- + \frac{2}{1-\alpha} \rho \wedge V_3 - \frac{1}{2(1+\alpha)} J \wedge V_+ \wedge V_- \,. \tag{2.33}$$

In the above the three-form ρ is not independent, but can be expressed as

$$\rho_{mnp} = J_{rm} \varphi_{np}{}^r \,. \tag{2.34}$$

Using the Fierz relation which involves K and Ψ one can check that the r.h.s. of the above equation is indeed antisymmetric in all three indices as should be for the components of a 3-form (see eq. (B.2)). Note also that ρ and φ are orthogonal to the vectors V_i.

Using the Fierz relations in appendix A it is not very hard to check the parametrisation above. The terms which contain at least one vector field can be immediately verified by projecting the entire relation on the corresponding vector and using the fact that vectors $V_\pm$ and V_3 are orthogonal. The only remaining problem is to determine the top forms which are orthogonal to the vectors. There are two different cases above. In (2.30), we denote this top form by φ and we shall further discuss its properties. In the remaining relations, the top form is no longer an independent form, but is given in terms of J, as in (2.31), or it simply vanishes as in (2.32) and (2.33). The way to decide whether or not one can write additional terms in (2.31)–(2.33) is by computing the norms of the r.h.s. and l.h.s. of these relations. It is not difficult to check that these norms precisely agree, and therefore if we were to add some arbitrary form to these relations, such forms would automatically have zero norm and thus vanish on an Euclidean space.

Naively the equations (2.29)–(2.33) diverge at the special points $\alpha = \pm 1$. To show that actually there are no divergences in the SU(3) parameterisation in the SU(4) or G_2 loci limits, we can extract the leading behaviour with respect to $(1 \pm \alpha)$ of the relevant SU(3)-structure forms

$$V_3 \sim (1-\alpha)^{\frac{1}{2}} \,, \qquad\qquad V_- \sim (1-\alpha)^{\frac{1}{2}} \,, \quad V_+ \sim (1+\alpha)^{\frac{1}{2}} \,,$$
$$\rho \sim [(1-\alpha)(1+\alpha)]^{\frac{1}{2}} \,, \quad \varphi \sim (1-\alpha)^{\frac{1}{2}} \,, \quad J \sim (1+\alpha)^{\frac{1}{2}} \,. \tag{2.35}$$

These relations can be inferred from the norms of these objects which we compute in appendix. The objects that define the geometry are the 9-dimensional bilinears. These are perfectly smooth over the full range of α, though they take particular, different, forms over the SU(3), SU(4) and G_2 loci.

Note that apart from J, φ and ρ, which satisfy (2.34), no other form orthogonal to the vectors appears. We can in principle use in the parametrisation only φ and J, but it is more intuitive to consider also ρ as these forms are used in general to describe a SU(3)

structure. Indeed, the results obtained agree precisely with what is expected from a SU(3)-structure in six dimensions plus three additional directions orthogonal to it. However, in order to establish the SU(3)-structure behind we still have to check a few more relations which the forms J and φ satisfy. Using the symmetric relation for K given in eq. (A.67) of the appendix we can compute the similar identity for J

$$
\begin{aligned}
J_{mn}J^n{}_p &= -\frac{1}{2}(1+\alpha)\delta_{mp} + \frac{1}{4}(V_+)_m(V_+)_p + \frac{1+\alpha}{4(1-\alpha)}(V_-)_m(V_-)_p + \frac{1+\alpha}{1-\alpha}(V_3)_m(V_3)_p \\
&= \frac{1}{2}(1+\alpha)\left[-\delta_{mp} + (P_+)_{mp} + (P_-)_{mp} + (P_3)_{mp}\right] ,
\end{aligned} \tag{2.36}
$$

where $P_{\pm,3}$ denote the projectors on the directions $+$, $-$ and 3 respectively. It follows that by an appropriate normalisation J can be indeed viewed as the almost complex structure on the 6-dimensional subspace orthogonal to $V_\pm$ and V_3.

Making use of the Fierz identities listed in the appendix and of the eqs. (2.29)–(2.33) one can show that the following identities must hold

$$
J \lrcorner \varphi = J \lrcorner \rho = 0 , \tag{2.37}
$$

$$
J \wedge \varphi = J \wedge \rho = 0 , \tag{2.38}
$$

$$
\varphi \wedge \rho = *(V_+ \wedge V_- \wedge V_3) , \tag{2.39}
$$

$$
J \wedge J \wedge J = \frac{3(1+\alpha)}{2(1-\alpha)} *(V_+ \wedge V_- \wedge V_3) . \tag{2.40}
$$

Furthermore, it is possible to show that eqs. (2.29)–(2.40) together with eq. (B.28) imply all the Fierz identities for the bilinear forms listed in the appendix. By construction, the converse is also true.

It is useful to construct from φ and ρ the following holomorphic and anti-holomorphic (with respect J) three-forms

$$
\Omega = \varphi + i\sqrt{\frac{2}{1+\alpha}}\rho , \qquad \bar{\Omega} = \varphi - i\sqrt{\frac{2}{1+\alpha}}\rho . \tag{2.41}
$$

Indeed, it is easy to see that these forms obey the relations

$$
J_m{}^n\Omega_{npq} = i\sqrt{\frac{1+\alpha}{2}}\Omega_{mpq} , \qquad J_m{}^n\bar{\Omega}_{npq} = -i\sqrt{\frac{1+\alpha}{2}}\bar{\Omega}_{mpq} , \tag{2.42}
$$

and therefore, up to some normalisation, Ω can be seen as a $(3,0)$ form with respect to the almost complex structure J. It should now be clear that following a suitable normalisation the forms J and Ω (or its real components φ and ρ) can be seen as the forms defining an SU(3)-structure on the space orthogonal to the vectors $V_\pm$ and V_3. Note that the normalisation we have depends on the parameter α and care must be taken over the $\alpha = \pm 1$ points where a reparameterisation in terms of SU(4) or G_2 structures is more suitable.

3 Supersymmetry conditions

The G-structure technology introduced in the previous section is ideal for studying the supersymmetry equations (2.2). In this section we rewrite the supersymmetry constraints as differential constraints on the appropriate forms and use these to extract general properties of any solutions, in particular with respect to supporting a background with varying α.

The supersymmetry variation of the gravitino (2.2) splits into internal and external components which read [37–39]

$$D_\alpha \xi = \nabla_\alpha \xi + A_\alpha \xi = 0 \,, \qquad Q\xi = 0 \,, \tag{3.1}$$

where we defined

$$A_\alpha = \lambda \gamma_\alpha \gamma_9 + \frac{1}{24} F_{\alpha\beta\gamma\delta} \gamma^{\beta\gamma\delta} + \frac{1}{4} \tilde{f}_\beta \gamma_\alpha{}^\beta \gamma_9 \,, \tag{3.2}$$

$$Q = -\lambda \gamma_9 + \frac{1}{2} \partial_\alpha \Delta \gamma^\alpha - \frac{1}{288} F_{\alpha\beta\gamma\delta} \gamma^{\alpha\beta\gamma\delta} - \frac{1}{6} \tilde{f}_\alpha \gamma^\alpha \gamma_9 \,, \tag{3.3}$$

and the covariant derivative ∇_α is taken now with respect to the 8-dimensional metric defined in (2.5). The parameter λ is the cosmological constant for the 3-dimensional external space. These equations are valid for spinors ξ which live on an 8-dimensional manifold. As discussed previously we are interested in studying the background on a 9-dimensional manifold $Y_9 = X_8 \times S_1$. To do this we uplift on a circle by adding a ninth direction so that now we have an index range $m = 1, \ldots, 9$. We uplift the gamma matrices by considering γ_9 together with the other gamma matrices γ_α, as the generators of the Clifford algebra $\mathrm{Cl}(9,0)$. The analysis of the geometry and G-structures performed above does not identify a particular direction and so we would like to rewrite the uplifted supersymmetry equations also in an SO(9) covariant way. We therefore introduce a constant vector field θ such that

$$\gamma_9 = \theta^m \gamma_m \,, \quad m = 1, \ldots, 9 \,. \tag{3.4}$$

With this the supersymmetry equations (3.1) have the same form where now

$$A_m = \lambda \theta^n \gamma_{mn} + \frac{1}{24} F_{mnpq} \gamma^{npq} + \frac{1}{4} \tilde{f}_n \theta_p \gamma_m{}^{np} \,, \tag{3.5}$$

$$Q = -\lambda \theta^m \gamma_m + \frac{1}{2} \partial_n \Delta \gamma^n - \frac{1}{288} F_{mnpq} \gamma^{mnpq} - \frac{1}{6} \tilde{f}_m \theta_n \gamma^{mn} \,, \tag{3.6}$$

and we must impose the independence of physics quantities on the ninth direction

$$\theta \cdot \tilde{f} = 0 \,, \qquad \theta \lrcorner F = 0 \,, \qquad \theta \cdot d\Delta = 0 \,. \tag{3.7}$$

These equations now hold after an arbitrary SO(9) rotation which no longer identifies θ with the index value $m = 9$.

Note the following simple consequences of the above. Firstly, the restrictions on the fluxes imply that one component of A, i.e. $\theta^m A_m$, identically vanishes. This means that also the spinor ξ does not depend on this direction. Secondly, since we chose the 8-dimensional gamma matrices to be real and symmetric (this also holds for γ_9), A_m is a totally antisymmetric matrix. Contracting (3.1) with ξ^T from the left we find

$$0 = \xi^T \nabla_m \xi = \frac{1}{2} \nabla_m \left(\xi^T \xi \right) = \frac{1}{2} \partial_m ||\xi||^2 \,, \tag{3.8}$$

and therefore the supersymmetry equations impose that the internal spinors must have constant norms as we already anticipated in the previous section.

The auxiliary uplift direction should be given by a linear combination of the 9-dimensional vectors. This can be seen from the analysis of the various particular cases. In particular for the SU(4) case, in 8-dimensions we expect no singlet vector field and so the vector field which survives should be interpreted as the additional ninth direction. In the G_2 case in 8 dimensions, we expect a single vector field which is a singlet under the structure group. In 9 dimensions we found two such vector fields and therefore a certain linear combination of them should be interpreted as the additional direction. Finally, in the case of a SU(3) structure we expect two singlet vector fields in eight dimensions, while we found three of them in 9 dimensions. Again, one combination of them should precisely give the additional direction. Thus we can write

$$\theta = \frac{1}{2(1+\alpha)}(\theta \cdot V_+)V_+ + \frac{1}{2(1-\alpha)}(\theta \cdot V_-)V_- + \frac{2}{1-\alpha}(\theta \cdot V_3)V_3 \, . \tag{3.9}$$

Finally it is worth noting the practical matter that uplifting to 9-dimensions does not add any further complexity to the equations. This is because we are working with Majorana, rather than Majorana-Weyl, spinors in 8 dimensions which implies that they do not have any nice properties under γ_9. Therefore including it in a higher dimensional Clifford algebra is useful, in particular because the complete set of gamma matrices means the Hodge star acts simply allowing us to write all the bilinears in terms of forms of degree 4 or less.

3.1 Equations for $N = 2$ supersymmetry in 3D

For $N = 2$ supersymmetry in 3 dimensions, the supersymmetry equations (3.1) have to be satisfied by two spinors on the internal manifold, $\xi_{1,2}$. In order to see what sort of constraints these equations impose on the 9-dimensional manifolds discussed in the previous section we should first rewrite them in terms of the spinor bilinears (2.14). Using the definitions (2.14) and (3.1) we can compute the derivatives of all the spinor bilinears to find [39]

$$\nabla_m V_{i\,n} = 2\lambda \theta_n V_{i\,m} - 2\lambda(\theta \cdot V_i)\delta_{mn} - \frac{1}{12}F_{mpqr}\Phi_{i\,n}{}^{pqr} + \frac{1}{2}\Phi_{i\,mnpq}\tilde{f}^p\theta^q \tag{3.10}$$

$$\begin{aligned}\nabla_m K_{np} = &-4\lambda K_{m[n}\theta_{p]} + 4\delta_{m[n}K_{p]q}\theta^q + \frac{1}{2}F_{m[n}{}^{qr}\Psi_{p]qr} + \Psi_{m[n}{}^q\theta_{p]}\tilde{f}_q \\ &- \Psi_{m[n}{}^q\tilde{f}_{p]}\theta_q + \delta_{m[n}\Psi_{p]qr}\tilde{f}^q\theta^r\end{aligned} \tag{3.11}$$

$$\begin{aligned}\nabla_m \Psi_{npr} = &6\lambda \Psi_{m[np}\theta_{r]} - 6\lambda\delta_{m[n}\Psi_{pr]q}\theta^q + \frac{1}{12}F_m{}^{stu}(*\Psi)_{nprstu} \\ &+ \frac{3}{2}F_{m[np}{}^q K_{r]q} - \frac{1}{2}(*\Psi)_{mnprst}\tilde{f}^s\theta^t - 3K_{m[n}\tilde{f}_p\theta_{r]} \\ &+ 3\delta_{m[n}\tilde{f}_p K_{r]q}\theta^q - 3\delta_{m[n}\theta_p K_{r]q}\tilde{f}^q \, ,\end{aligned} \tag{3.12}$$

$$\begin{aligned}\nabla_m \Phi_{i\,npqr} = &-8\lambda\Phi_{i\,m[npq}\theta_{r]} + 8\lambda\delta_{m[n}\Phi_{i\,pqr]s}\theta^s + F_{m[n}{}^{st}(*\Phi_i)_{pqr]st} \\ &- 2F_{m[npq}V_{i\,r]} + 2(*\Phi_i)_{m[npq}{}^s\theta_{r]}\tilde{f}_s - 2(*\Phi_i)_{m[npq}{}^s\tilde{f}_{r]}\theta_s \\ &+ 2\delta_{m[n}(*\Phi_i)_{pqr]st}\tilde{f}^s\theta^t - 12\delta_{m[n}V_{i\,p}\tilde{f}_q\theta_{r]} \, ,\end{aligned} \tag{3.13}$$

where the subscript i stands for any of $1, 2, 3, +$ or $-$. It is interesting to note that the equations above imply that the derivatives along θ vanish identically upon using the condition on the fluxes (3.7).

We should now consider the constraints which come from the external gravitino variation. These equations should be projected on a basis of spinors in order to find an equivalent set of equations. In 8 dimensions it is easy to find a basis of spinors in terms of Majorana-Weyl singlet spinors, but since the Majorana-Weyl components of the spinors we consider may vanish at certain points, we will not be able to use a basis constructed in such a way globally. Instead we shall project the supersymmetry equations on a larger set of spinors — not linearly independent — which are constructed by multiplying the spinors $\xi_{1,2}$ by arbitrary elements of the Clifford algebra. Specifically we shall project the spinor equations (3.1) on spinors of the form

$$\xi_{i\,A} = \gamma_A \xi_i \,, \qquad i = 1, 2 \,, \qquad \gamma_A \in \mathrm{Cl}(9, 0) \,. \tag{3.14}$$

In practice we shall see that taking $\gamma_A = \mathbb{1}$, γ_m is enough and the other constraints are just consequences of these ones. Therefore we shall use the following equations

$$-\lambda(\theta \cdot V_i) + \frac{1}{2}(V_i \cdot d\Delta) - \frac{1}{12} F \lrcorner \Phi_i = 0 \tag{3.15}$$

$$\tilde{f}_m \theta_n K^{mn} = 0 \tag{3.16}$$

$$-2\lambda\theta + d\Delta - \frac{1}{12} * (F \wedge \Phi_+) + \frac{1}{6} V_+ \lrcorner (\tilde{f} \wedge \theta) = 0 \tag{3.17}$$

$$-\frac{1}{12} * (F \wedge \Phi_{-,3}) + \frac{1}{6} V_{-,3} \lrcorner (\tilde{f} \wedge \theta) = 0 \tag{3.18}$$

$$2\lambda\theta \lrcorner K - d\Delta \lrcorner K + \frac{1}{6} \Psi \lrcorner F - \frac{1}{3} (\tilde{f} \wedge \theta) \lrcorner \Psi = 0 \tag{3.19}$$

while the ones corresponding to other Clifford elements can be found in appendix C.

3.2 Flux induced variations of α

The key property of the backgrounds that we are interested in is the variation of the parameter α, since this is the property that signals an M2-brane potential. We will explain this relationship in more detail in section 3.3. In this section we are interested in determining which flux is responsible for inducing such a variation. In general not any flux will induce this variation, for example we know that $(2, 2)$ primitive flux as studied in the case of SU(4)-structure compactifications [7] will not induce such a variation. We will determine the flux which is relevant by using the supersymmetry equations to directly evaluate the variation of α.

The form of the supersymmetry equations as given in section 3.1 carries redundancies because the bilinears (2.14) are not independent, but satisfy various relations coming from the Fierz relations. In order to find a more tractable set of equations we should use a parametrisation for the forms (2.14) which already makes use of the Fierz relations. A particular parameterisation is the one given in (2.29)–(2.33) which is valid for the SU(3)-structure loci. We claim that this parameterisation is sufficient to capture all the variation of α. This follows from the simple reasoning that over the patches where it breaks down, the SU(4) and G_2 loci, α is constant by definition. Further it is also at a maximum or minimum value over these loci so that derivatives along directions leading away from the loci also vanish. Therefore over such loci $d\alpha = 0$ and all the variation is within the SU(3)-structure locus.

Using the supersymmetry equations (3.10), the variation of α can be generally computed as

$$\nabla_m \alpha = \frac{1}{2} \nabla \left(V_+^n V_{+\,n} \right) = V_+^n \nabla_m V_{+\,n} = -\frac{1}{12} V_+^n F_{mpqr} \Phi_{+\,n}{}^{pqr} + \frac{1}{2} \Phi_{+\,mnpq} \tilde{f}^p \theta^q V_+^n \ . \quad (3.20)$$

Making use of the SU(3) parametrisation of the form Φ_+, (2.31), and of the fact that the forms J, ρ, V_- and V_3 are all orthogonal to V_+, together with the fact that θ should be a combination of the vector fields $V_\pm$ and V_3, we find that the second term above does not contribute to the variation of the parameter α and so

$$\nabla_m \alpha = -\frac{1}{3} \rho_{pqr} F_m{}^{pqr} \ . \quad (3.21)$$

Given again all the orthogonality properties of the SU(3)-structure forms we find that the flux which is responsible for the variation of α can be written as

$$F = \tilde{h} \wedge \rho + \tilde{g} \wedge \varphi \ , \quad (3.22)$$

where at this stage $\tilde{h}$ and $\tilde{g}$ are arbitrary one-forms on the internal manifold that parameterise the flux.

Note that ρ and φ are (depending on the conventions) the real and imaginary part of the holomorphic 3-form which can be defined on a manifold with SU(3)-structure. Over the SU(4) locus the flux therefore lifts to either $(4,0)+(0,4)$ or $(3,1)+(1,3)$ flux. Note that both these fluxes are forbidden by supersymmetry in the case of pure SU(4)-structure where only $(2,2)$ primitive fluxes are allowed [7], which is consistent with the understanding that the variation of α vanishes over that locus.

3.3 M2-brane potentials and supersymmetry

At this stage it is worth going into some more detail regarding the relation between a potential for probe space-time filling M2-branes and the background geometry. In type IIB compactified on manifolds with $\mathrm{SU}(3) \times \mathrm{SU}(3)$ structure, we know that space-time-filling D3 branes become supersymmetric at points where the manifold locally looks like a manifold with SU(3) structure (i.e. the two spinors defining the $\mathrm{SU}(3) \times \mathrm{SU}(3)$ structure are parallel) [21, 22]. In this section we will find similar results for M2-branes in terms of local structure groups.

Let us briefly recall some well-known facts about supersymmetry and M2 branes. We will utilise the form of the M2-brane action given in [47]

$$S_{M2} = -T_{M2} \int d^3 \zeta \sqrt{-G} - \frac{T_{M2}}{6} \int d^3 \zeta \epsilon^{ijk} A_{kji} + i \frac{T_{M2}}{2} \int d^3 \zeta \sqrt{-G} \bar{y} (1 - \Gamma_{M2}) \Gamma^i \tilde{D}_i y \ , \quad (3.23)$$

where G_{ij} is the induced metric on the brane, T_{M_2} is the tension of the brane, A_{ijk} is the pull-back of the supergravity 3-form on the brane, $\tilde{D}_i$ is the pull-back of the supercovariant derivative and y is an 11-dimensional Majorana spinor. Γ_{M2} is the brane chirality operator which is given by

$$\Gamma_{M2} = \frac{1}{6\sqrt{-G}} \epsilon^{ijk} \Gamma_i \Gamma_j \Gamma_k \ . \quad (3.24)$$

Note that $\Gamma_{M2}^2 = 1$ so that its eigenvalues are ± 1 and we can define projectors on the corresponding subspaces as

$$P_\pm = \frac{1}{2}(1 \pm \Gamma_{M2}) \ . \tag{3.25}$$

The action (3.23) is invariant under local κ transformations [47]

$$\delta_\kappa y = (1 + \Gamma_{M2})\kappa + \mathcal{O}\left(y^2\right) \ , \qquad \delta_\kappa X^M = \frac{i}{2}\bar{y}\Gamma^M(1 + \Gamma_{M2})\kappa + \mathcal{O}\left(y^3\right) \tag{3.26}$$

where κ is a 32-component spinor which may depend on the coordinates on the worldvolume of the M2 brane, ζ^i.

For a background with Killing spinor ϵ satisfying (2.2), supersymmetry transformations act as

$$\delta_\epsilon y = \epsilon + \mathcal{O}\left(y^2\right) \ , \qquad \delta_\epsilon X^M = -\frac{i}{2}\bar{y}\Gamma^M\epsilon + \mathcal{O}\left(y^3\right) \ . \tag{3.27}$$

A bosonic brane configuration ($y = 0$) is supersymmetric only if $\delta_\epsilon y = 0$ and we see that this can not be satisfied unless $\epsilon = 0$. However $\delta_\epsilon y = \epsilon$ is compatible with supersymmetry only if this transformation can be compensated by a κ transformation [27]. Therefore the only supersymmetry generators which are not broken by the bosonic brane configuration are those which can be written as

$$\epsilon = \delta_\kappa y = (1 + \Gamma_{M2})\kappa = 2P_+\kappa \ . \tag{3.28}$$

This is equivalent to requiring [27]

$$P_-\epsilon = (1 - \Gamma_{M2})\epsilon = 0 \ . \tag{3.29}$$

For the compactification to 3 dimensions with space-time-filling M2-branes, the M2 brane chirality operator decomposes according to the the split of gamma matrices (2.7), as

$$\Gamma_{M2} = \mathbf{1} \otimes \gamma_9 \tag{3.30}$$

where γ_9 is the chirality operator on the 8d manifold. Using the spinor decomposition for the compactification to 3 dimensions (2.8) we find we find that the condition above is equivalent to

$$\gamma_9\xi = \xi \ . \tag{3.31}$$

We see that the condition that the M2 brane is supersymmetric requires Killing spinors of definite chirality.

With this result in mind consider probe M2 branes in our supergravity background at the different local SU(3), SU(4) or G_2-structure loci. Generally we have that there are 2 Majorana Killing spinors. Over G_2 loci, $\alpha = -1$, these become Majorana-Weyl Killing spinors of opposite chirality. Therefore M2 branes preserve $N = 1$ supersymmetry on these loci. Over SU(4) loci, $\alpha = +1$, the Killing spinors are Majorana-Weyl of same chirality and so M2 branes either preserve $N = 2$ supersymmetry or no supersymmetry. However, for a given fixed chirality of the two Majorana-Weyl spinors either M2 or anti-M2 branes preserve $N = 2$ supersymmetry, and we define M2 branes as the objects which

preserve the supersymmetries over SU(4) loci.[10] Over the SU(3) loci, $-1 < \alpha < 1$, we have two Killling Majorana spinors which contain 4 non-vanishing Majorana-Weyl component spinors. We have three possibilities: either all of the Majorana-Weyl components are Killing individually, in which case the M2 branes preserves $N = 2$ supersymmetry, or two Majorana-Weyl components of one Majorana spinor are Killing while the components of the other are not, in which case the M2 preserves $N = 1$ supersymmetry, or none of the components are Killing in which case an M2 brane is non-supersymmetric. In appendix D we show that the flux which induces a varying α (3.22) precisely implies that the last possibility is realised and M2 branes preserve no supersymmetry. Indeed it is rather simple to see that this should be the case for any background which does not have a global SU(3) or G_2-structure since if any Majorana-Weyl components were covariantly constant by themselves their norm could be set to unity over the full manifold, thereby implying that they are nowhere vanishing and induce a global SU(3) or G_2-structure.

On general grounds we expect that supersymmetric loci are minima of the potential. This means that on the generic (non-supersymmetric) locus a probe M2 brane should feel a potential which drives it to a supersymmetric locus.

Let us be more explicit about the condition for $N = 2$ supersymmetry of the M2 brane. The condition (3.31) has to be satisfied for two spinors ξ_1 and ξ_2, which means that the two spinors are actually Majorana-Weyl of positive chirality and therefore define a SU(4) structure. In the language used in section 2.2 this means that the vectors V_1 and V_2 are equal and therefore

$$V_- = V_3 = 0 \,. \tag{3.32}$$

Conversely, the condition $V_- = V_3 = 0$ implies that we are dealing with a SU(4) point and therefore the two spinors are Majorana-Weyl.[11] Finally, it is interesting to note that, in the spirit of the analysis in [21, 22] of D3 superpotentials, the condition (3.32) hints that some combination of the 1-forms V_- and V_3 may be associated to the derivative of a world-volume potential.

4 Analysis of the special flux

In the previous section we identified the particular flux that sources the variation of α. In this section we study backgrounds that can support this flux. As an initial investigation we restrict the 4-form flux F to be solely composed of the flux of interest so that it takes the form (3.22), while the warp-factor Δ and 1-form flux f are unconstrained. We leave a complete investigation allowing also for other types of 4-form fluxes in the background for future work.

---

[10]Note that there exists the interesting possibility of multiple SU(4) loci with different chiralities in which case neither M2 or anti-M2 can preserve supersymmetry on all of loci. We will not consider such configurations in detail in this work.

[11]In principle the chirality of the spinors can not be determined. However, if this is negative, and therefore, the condition (3.31) is not satisfied, this would be a point where anti M2 branes are supersymmetric.

It is useful to decompose the 4-form flux along the three vectors as

$$
\begin{aligned}
F = {} & h \wedge \rho + g \wedge \varphi \\
& + \frac{1}{2(1+\alpha)} h_+ V_+ \wedge \rho + \frac{1}{2(1-\alpha)} h_- V_- \wedge \rho + \frac{2}{(1-\alpha)} h_3 V_3 \wedge \rho \\
& + \frac{1}{2(1+\alpha)} g_+ V_+ \wedge \varphi + \frac{1}{2(1-\alpha)} g_- V_- \wedge \varphi + \frac{2}{(1-\alpha)} g_3 V_3 \wedge \varphi \, .
\end{aligned}
\tag{4.1}
$$

In the above we defined

$$
h_i = V_i \cdot \tilde{h} \equiv V_i^m \tilde{h}_m \, , \qquad g_i = V_i \cdot \tilde{g} \equiv V_i^m \tilde{g}_m \, , \quad i = \pm, 3 \, ,
\tag{4.2}
$$

while g and h are defined as the components of $\tilde{g}$ and $\tilde{h}$ orthogonal to the vectors $V_\pm$ and V_3. We therefore decompose the fluxes h, g and $\tilde{f}$ as

$$
\tilde{h} = h + \frac{1}{2(1+\alpha)} h_+ V_+ + \frac{1}{2(1-\alpha)} h_- V_- + \frac{2}{(1-\alpha)} h_3 V_3 \, ,
\tag{4.3}
$$

$$
\tilde{g} = g + \frac{1}{2(1+\alpha)} g_+ V_+ + \frac{1}{2(1-\alpha)} g_- V_- + \frac{2}{(1-\alpha)} g_3 V_3 \, ,
\tag{4.4}
$$

$$
\tilde{f} = f + \frac{1}{2(1+\alpha)} f_+ V_+ + \frac{1}{2(1-\alpha)} f_- V_- + \frac{2}{(1-\alpha)} f_3 V_3 \, ,
\tag{4.5}
$$

where f, $f_\pm$ and f_3 are defined in analogy with g and h by their relation to the vectors. Note that there are no divergences at $\alpha = \pm 1$ in the expressions above, which can be seen using the limits (2.35). Recall that we are still formally working on a nine-dimensional manifold and in order not to alter the physics we have to impose (3.7) on the fluxes above. Since θ is a linear combination of the vectors, this condition imposes the orthogonality of the one-form fluxes $\tilde{h}$, $\tilde{g}$ and $\tilde{f}$ on θ

$$
\theta \lrcorner \tilde{h} = \theta \lrcorner \tilde{g} = \theta \lrcorner \tilde{f} = 0 \, .
\tag{4.6}
$$

With these definitions, and using the various relations in appendix B, we find that (3.21) yields

$$
d\alpha = -(1-\alpha)(1+\alpha)h - 2(1-\alpha)g \lrcorner J - (1-\alpha)h_+ V_+ - (1+\alpha)h_- V_- - 4(1+\alpha)h_3 V_3
\tag{4.7}
$$

Note that, as expected, $d\alpha = 0$ on the SU(4) and G_2 loci.

To analyse the supersymmetry conditions for this particular flux Ansatz we shall start with the constraints (3.15)–(3.19). Inserting the SU(3) parametrisation (2.30)–(2.33) and

the flux Ansatz above, by using the relations in appendix B we find

$$0 = -\lambda(\theta \cdot V_+) + \frac{1}{2}(V_+ \cdot d\Delta) - \frac{1}{6}(1-\alpha)h_+ \,, \tag{4.8a}$$

$$0 = -\lambda(\theta \cdot V_-) + \frac{1}{2}(V_- \cdot d\Delta) + \frac{1}{6}(1+\alpha)h_- + \frac{2}{3}g_3 \,, \tag{4.8b}$$

$$0 = -\lambda(\theta \cdot V_3) + \frac{1}{2}(V_3 \cdot d\Delta) + \frac{1}{6}(1+\alpha)h_3 - \frac{1}{6}g_- \,, \tag{4.8c}$$

$$0 = (\theta \cdot V_3)f_- - f_3(\theta \cdot V_-) \,, \tag{4.8d}$$

$$0 = d\Delta - 2\lambda\theta - \frac{1}{3}(g_- V_3 - g_3 V_-) + \frac{1}{6}f_+\theta - \frac{1}{6}(\theta \cdot V_+)\tilde{f} \,, \tag{4.8e}$$

$$0 = -h_+ V_- + h_- V_+ - 2g_+ V_3 + 2g_3 V_+ + f_-\theta - (\theta \cdot V_-)\tilde{f} \,, \tag{4.8f}$$

$$0 = -2h_+ V_3 + 2h_3 V_+ + g_+ V_- - g_- V_+ + 2f_3\theta - 2(\theta \cdot V_3)\tilde{f} \,, \tag{4.8g}$$

$$
\begin{aligned}
0 = {} & \frac{2\lambda}{1-\alpha}(\theta \cdot V_-)V_3 - \frac{2\lambda}{1-\alpha}(\theta \cdot V_3)V_- - d\Delta\lrcorner J - \frac{1}{1-\alpha}(d\Delta \cdot V_-)V_3 + \frac{1}{1-\alpha}(d\Delta \cdot V_3)V_- \\
& + \frac{1}{6}(1-\alpha)\tilde{h}\lrcorner J - \frac{1}{6}(1-\alpha)\tilde{g} - \frac{1}{12}\frac{1-\alpha}{1+\alpha}g_+ V_+ - \frac{1}{3}g_3 V_3 - \frac{1}{12}g_- V_- \\
& + \frac{(\theta \cdot V_+)}{3(1+\alpha)}\tilde{f}\lrcorner J - \frac{1}{6(1-\alpha)}(\tilde{f} \wedge \theta)\lrcorner(V_+ \wedge V_- \wedge V_3) \,. \tag{4.8h}
\end{aligned}
$$

Let us look more carefully at equation (4.8f). Contracting it with V_- we immediately find that $h_+ = 0$. By contracting with V_3 and using (4.8d) we find that $g_+ = 0$. Furthermore, projecting equations (4.8f) and (4.8g) on V_+ and on the 6-dimensional space orthogonal to the vectors $V_\pm$ and V_3 one obtains the following equations

$$g_3 = -\frac{1}{2}h_- - \frac{1}{4(1+\alpha)}f_-(\theta \cdot V_+) + \frac{1}{4(1+\alpha)}(\theta \cdot V_-)f_+ \,, \tag{4.9}$$

$$g_- = 2h_3 + \frac{1}{1+\alpha}f_3(\theta \cdot V_+) - \frac{1}{1+\alpha}(\theta \cdot V_3)f_+ \,, \tag{4.10}$$

and

$$(\theta \cdot V_-)f = 0 \,, \qquad (\theta \cdot V_3)f = 0 \,. \tag{4.11}$$

Now, using (3.9), the relations coming from the orthogonality of θ on the fluxes $\tilde{f}$ and $\tilde{g}$ become

$$h_-(\theta \cdot V_-) + 4h_3(\theta \cdot V_3) = 0 \,, \tag{4.12}$$

$$h_3(\theta \cdot V_-) - h_-(\theta \cdot V_3) = 0 \,, \tag{4.13}$$

where we used (4.9), (4.10) and (4.8d). These equations can be viewed as a linear system for h_- and h_3 which has a non-trivial solution only if the corresponding determinant vanishes. Therefore, there are two cases to consider. One of them must have $\theta \cdot V_- = \theta \cdot V_3 = 0$ and the other one has to satisfy $h_- = h_3 = 0$ and, following (4.11), $f = 0$. We shall focus on the first solutions as we want to study loci with $d\alpha \neq 0$. Indeed, it is easy to show that in the latter case one must have that $d\alpha = 0$. In order to prove it one needs also the identity

$$g\lrcorner J + \frac{1}{2}(1+\alpha)h = 0 \,, \tag{4.14}$$

resulting from eq. (4.8h) after contraction with J.

Let us continue with the case $\theta \cdot V_- = \theta \cdot V_3 = 0$. Making use of eq. (3.9) we obtain that θ has to be in the direction of V_+. We can therefore write

$$\theta = \frac{(\theta \cdot V_+)}{2(1+\alpha)} V_+ , \qquad (4.15)$$

and the fact that the physics should not depend on θ now transfers to V_+. Notice that this is consistent with the expectation that the flux should vanish over G_2 structure loci since in this case we have $V_+ = 0$ but the auxiliary direction θ must be non-vanishing.

Together with equation (4.8a), (4.15) immediately implies that $\lambda = 0$, and therefore all such compactifications are to 3-dimensional Minkowski space. Furthermore, equations (4.8b), (4.8c), (4.8f) and (4.8g) allow to solve for the projections of $d\Delta$ on V_- and V_3 in terms of f_- and f_3. Altogether the equations (4.8) become

$$0 = \lambda = f_+ = g_+ = h_+ = d\Delta \cdot V_+ , \qquad (4.16\text{a})$$

$$d\Delta \cdot V_- = \frac{1-\alpha}{3} h_- + \frac{\theta \cdot V_+}{3(1+\alpha)} f_- , \qquad d\Delta \cdot V_3 = \frac{1-\alpha}{3} h_3 + \frac{\theta \cdot V_+}{3(1+\alpha)} f_3 , \quad (4.16\text{b})$$

$$g_3 = -\frac{1}{2} h_- - \frac{\theta \cdot V_+}{4(1+\alpha)} f_- , \qquad g_- = 2h_3 + \frac{\theta \cdot V_+}{1+\alpha} f_3 , \qquad (4.16\text{c})$$

$$0 = d\Delta - \frac{1}{3}(g_- V_3 - g_3 V_-) - \frac{1}{6}(\theta \cdot V_+)\tilde{f} , \qquad (4.16\text{d})$$

$$0 = -d\Delta \lrcorner J + \frac{1}{6}(1-\alpha)(h \lrcorner J - g) + \frac{(\theta \cdot V_+)}{3(1+\alpha)} f \lrcorner J . \qquad (4.16\text{e})$$

Contracting (4.16d) with J we find

$$d\Delta \lrcorner J = \frac{(\theta \cdot V_+)}{6} f \lrcorner J , \qquad (4.17)$$

and then (4.16e) becomes

$$g - h \lrcorner J = \frac{(\theta \cdot V_+)}{1+\alpha} f \lrcorner J . \qquad (4.18)$$

4.1 The case $\tilde{f} = 0$

At this point we split the analysis to two cases, the simpler case $\tilde{f} = 0$ is studied in this section, while the more general case is studied in section 4.2. For this section we therefore set $f = f_+ = f_- = f_3 = 0$.

Let us analyse more closely equation (4.18). Since J, up to normalisation, acts like an almost complex structure on the 6-dimensional space orthogonal to the vectors $V_\pm$ and V_3, this equation tell us that a particular combination of the fluxes g and h is holomorphic with respect to the almost complex structure J. It is not hard to see that the complex flux defined as

$$\mathfrak{h} = h + i\sqrt{\frac{2}{1+\alpha}} g , \qquad \bar{\mathfrak{h}} = h - i\sqrt{\frac{2}{1+\alpha}} g \qquad (4.19)$$

is holomorphic in that it obeys

$$J_m{}^n \mathfrak{h}_n = i\sqrt{\frac{1+\alpha}{2}} \mathfrak{h}_m , \qquad J_m{}^n \bar{\mathfrak{h}}_n = -i\sqrt{\frac{1+\alpha}{2}} \bar{\mathfrak{h}}_m . \qquad (4.20)$$

It is useful to use this flux combination together with objects which again have well-defined behavior when contracted with J. In particular, we can construct a $(4,0)$ form by taking the exterior product of (2.41) with (4.19). Since J only acts on a 6-dimensional subspace, a $(4,0)$ form in the sense defined above must identically vanish. In particular we have (B.34)

$$0 = \mathfrak{h} \wedge \Omega = h \wedge \varphi - \frac{2}{1+\alpha} g \wedge \rho + i\sqrt{\frac{2}{1+\alpha}} (h \wedge \rho + g \wedge \varphi) = 0 . \qquad (4.21)$$

Note that the imaginary part is proportional to the projection of the flux F orthogonal to the vectors. The fact that this part of the flux vanishes implies that the variation of α only depends on the projection of the $\tilde{h}$ and $\tilde{g}$ fluxes along the vectors V_- and V_3.

The equations (4.16) now become

$$d\Delta \cdot V_- = \frac{1-\alpha}{3} h_- , \quad d\Delta \cdot V_3 = \frac{1-\alpha}{3} h_3 , \quad g_3 = -\frac{1}{2} h_- , \quad g_- = 2h_3 , \quad h \lrcorner J - g = 0 \qquad (4.22)$$

with all the rest of the flux components being zero. All the above relations greatly simplify the differential equations which now become

$$d\alpha = -(1+\alpha)h_- V_- - 4(1+\alpha)h_3 V_3 \qquad (4.23)$$

$$dV_+ = \frac{1}{2} h_- V_+ \wedge V_- + 2h_3 V_+ \wedge V_3 , \quad dV_- = 2h_3 V_- \wedge V_3 , \quad dV_3 = -\frac{1}{2} h_- V_- \wedge V_3 , \quad (4.24)$$

$$dK = -h_- J \wedge V_- - 4h_3 J \wedge V_3 , \qquad (4.25)$$

$$d\Psi = \frac{2}{1+\alpha} J \wedge V_+ \wedge \left(\frac{1}{4} h_- V_- + h_3 V_3\right) - \frac{2}{1-\alpha} \rho \wedge (h_- V_3 - h_3 V_-)$$
$$+ \frac{1-5\alpha}{1-\alpha} \varphi \wedge \left(\frac{1}{4} h_- V_- + h_3 V_3\right) , \qquad (4.26)$$

where we made extensive use of the relations (B.33) for $k = h$ which, in the case $\tilde{f} = 0$, implies $\tilde{k} = g$. Note that the exterior derivative of J is the same as the derivative of K above, while for the derivative of φ we find

$$d\varphi = \frac{2}{1-\alpha} \rho \wedge (h_3 V_- - h_- V_3) + \frac{1-5\alpha}{1-\alpha} \varphi \wedge \left(\frac{1}{4} h_- V_- + h_3 V_3\right) . \qquad (4.27)$$

In the above formula the term which is in the direction of V_+ in $d\Psi$ drops out. This is precisely as it should be, as φ has no components along V_+ and moreover its derivative along θ (which is identified with V_+ in this case) vanishes. With a bit of more work one can compute the exterior derivative of ρ as well. We find

$$d\rho = \frac{3-7\alpha}{1-\alpha} \rho \wedge \left(\frac{1}{4} h_- V_- + h_3 V_3\right) + \frac{1+\alpha}{1-\alpha} \varphi \wedge (h_- V_3 - h_3 V_-) . \qquad (4.28)$$

At a first glance it seems that $d\varphi$ and $d\rho$ do not combine nicely into $d\Omega$, but one has to take into account that $d\Omega$ contains an additional term of the form $d\alpha \wedge \rho$ due to the α-dependent factor in front of ρ in the definition of Ω. With this we find

$$d\Omega = \frac{1-5\alpha}{1-\alpha} \Omega \wedge \left(\frac{1}{4} h_- V_- + h_3 V_3\right) + i\frac{1+\alpha}{1-\alpha} \sqrt{\frac{2}{1+\alpha}} \Omega \wedge (h_- V_3 - h_3 V_-) . \qquad (4.29)$$

It is important to notice that the exterior derivatives of the SU(3)-structure forms do not have components strictly orthogonal to the vectors. Therefore one can conclude that the intrinsic torsion classes for the 6-dimensional manifold orthogonal to the vectors vanish and therefore this is a Calabi-Yau manifold. We conclude that in the case $\tilde{f} = 0$ the supersymmetry equations require that, over the SU(3) locus, the 8-dimensional manifold is a fibration of a 6-dimensional CY manifold over a 2-dimensional base spanned by the vectors V_- and V_3. While over any G_2 or SU(4) loci the flux vanishes.

For the equations (4.23)–(4.26) to be nilpotent we must impose a constraint on the derivative of the flux to be along the vectors V_- and V_3 and to satisfy

$$dh_- \wedge V_- + 4dh_3 \wedge V_3 = 0 . \tag{4.30}$$

Further to this the Bianchi Identity (2.3) imposes the constraint

$$dh_3 \wedge V_- - dh_- \wedge V_3 = -\frac{7+\alpha}{4(1-\alpha)} \left(h_-^2 + 4h_3^2 \right) V_- \wedge V_3 . \tag{4.31}$$

It is simple to solve the first order differential equations (4.30) and (4.31) which fixes the form of the two independent flux components in terms of α

$$h_i = c_i \left(1 - \alpha\right) \left(1 + \alpha\right)^{-\frac{3}{4}} , \quad i = -, 3 . \tag{4.32}$$

Here the c_i are some undetermined constants.[12]

Finally let us note that using the normalisation of V_+ we can write

$$V_+ = \sqrt{2(1+\alpha)}\theta , \tag{4.33}$$

taking θ to be constant we derive

$$dV_+ = \frac{1}{2(1+\alpha)}d\alpha \wedge V_+ , \tag{4.34}$$

which precisely agree with dV_+ found in (4.23). This is consistent with the interpretation that the auxiliary 9-dimensional manifold is a direct product of the original 8-dimensional manifold and a circle.

4.2 The case $\tilde{f} \neq 0$

Let us now return to the study of the $\tilde{f} \neq 0$ case. Compared to the case $\tilde{f} = 0$ we see that now the complex fluxes (4.19) are no longer (anti)holomorphic. Rather one should replace $h \rightarrow h + \frac{(\theta \cdot V_+)}{1+\alpha} f$ in order to obtain a holomorphic combination. This means that following the same argument of constructing a $(4, 0)$ form on the space orthogonal to the vectors introduced in the previous section, we now find

$$h \wedge \rho + g \wedge \varphi = -\frac{(\theta \cdot V_+)}{1 + \alpha} f \wedge \rho , \tag{4.35}$$

and so the flux F has also a component which is orthogonal to the vectors.

---

[12]We expect them to be fixed within string theory through requiring flux quantisation.

For the variation of α we find

$$d\alpha = (1 - \alpha)(\theta \cdot V_+)f - (1 + \alpha)(h_- V_- + 4h_3 V_3) . \tag{4.36}$$

One can compute again the derivatives of the forms and we find

$$dV_+ = \frac{1 - \alpha}{2(1 + \alpha)}(\theta \cdot V_+)f \wedge V_+ + 2h_3 V_+ \wedge V_3 + \frac{1}{2}h_- V_+ \wedge V_- , \tag{4.37}$$

$$dV_- = \frac{2(\theta \cdot V_+)}{(1 - \alpha)(1 + \alpha)}f_3 V_- \wedge V_3 + 2h_3 V_- \wedge V_3 - \frac{1}{2}(\theta \cdot V_+)f \wedge V_- , \tag{4.38}$$

$$dV_3 = -\frac{(\theta \cdot V_+)}{2(1 - \alpha)(1 + \alpha)}f_- V_- \wedge V_3 - \frac{1}{2}h_- V_- \wedge V_3 - \frac{1}{2}(\theta \cdot V_+)f \wedge V_3 , \tag{4.39}$$

$$dK = -h_- J \wedge V_- - 4h_3 J \wedge V_3 - \frac{\alpha}{1 + \alpha}(\theta \cdot V_+)f \wedge J - \frac{\theta \cdot V_+}{2(1 - \alpha)}f \wedge V_- \wedge V_3$$
$$- \frac{\theta \cdot V_+}{2(1 + \alpha)(1 - \alpha)}f_- J \wedge V_- - 2\frac{\theta \cdot V_+}{(1 + \alpha)(1 - \alpha)}f_3 J \wedge V_3 . \tag{4.40}$$

$d\Psi$ can again be computed directly from its covariant derivative. Again, when deriving $d\varphi$ the terms in the direction of V_+ cancel and we are left with

$$d\varphi = \frac{3}{2}(\theta \cdot V_+)\varphi \wedge f + \frac{(\theta \cdot V_+)}{2(1 - \alpha)}(f \lrcorner \rho) \wedge V_- \wedge V_3 + \frac{1 - 5\alpha}{1 - \alpha}\varphi \wedge \left(\frac{1}{4}h_- V_- + h_3 V_3\right)$$
$$+ \frac{3(\theta \cdot V_+)}{(1 - \alpha)(1 + \alpha)}\varphi \wedge \left(\frac{1}{4}f_- V_- + f_3 V_3\right) - \frac{2}{1 - \alpha}\rho \wedge (h_- V_3 - h_3 V_-) \tag{4.41}$$
$$- \frac{(\theta \cdot V_+)}{2(1 - \alpha)(1 + \alpha)}\rho \wedge (f_- V_3 - f_3 V_-) .$$

$d\rho$ can be computed from various combinations of spinor bilinears (e.g. $d\rho = \frac{1}{4}(V_+ \lrcorner \Phi_+)$) and we find

$$d\rho = \frac{2\alpha + 1}{1 + \alpha}(\theta \cdot V_+)\rho \wedge f + \frac{(\theta \cdot V_+)}{2(1 - \alpha)}(f \lrcorner \varphi) \wedge V_- \wedge V_3 + \frac{3 - 7\alpha}{1 - \alpha}\rho \wedge \left(\frac{1}{4}h_- V_- + h_3 V_3\right)$$
$$+ \frac{3(\theta \cdot V_+)}{(1 - \alpha)(1 + \alpha)}\rho \wedge \left(\frac{1}{4}f_- V_- + f_3 V_3\right) + \frac{1 + \alpha}{1 - \alpha}\varphi \wedge (h_- V_3 - h_3 V_-) \tag{4.42}$$
$$+ \frac{(\theta \cdot V_+)}{4(1 - \alpha)}\varphi \wedge (f_- V_3 - f_3 V_-) .$$

Again, taking into account the factors which multiply ρ in the definition of Ω we find for the latter

$$d\Omega = \frac{3}{2}(\theta \cdot V_+)\Omega \wedge f + i\sqrt{\frac{2}{1 + \alpha}}\frac{(\theta \cdot V_+)}{2(1 - \alpha)}(f \lrcorner \Omega) \wedge V_- \wedge V_3 + \frac{1 - 5\alpha}{1 - \alpha}\Omega \wedge \left(\frac{1}{4}h_- V_- + h_3 V_3\right)$$
$$+ \frac{3(\theta \cdot V_+)}{(1 - \alpha)(1 + \alpha)}\Omega \wedge \left(\frac{1}{4}f_- V_- + f_3 V_3\right) + i\sqrt{\frac{2}{1 + \alpha}}\frac{1 + \alpha}{1 - \alpha}\Omega \wedge (h_- V_3 - h_3 V_-) \tag{4.43}$$
$$+ i\sqrt{\frac{2}{1 + \alpha}}\frac{(\theta \cdot V_+)}{2(1 - \alpha)(1 + \alpha)}\Omega \wedge (f_- V_3 - f_3 V_-) .$$

As in the case of $f = 0$ it will be instructive to find the torsion classes of the manifold with SU(3)-structure which is orthogonal to the vectors. Projecting on the 6-dimensional space the above derivatives become

$$P_6(dJ) = -\frac{\alpha}{1+\alpha}(\theta \cdot V_+)J \wedge f , \qquad (4.44)$$

$$P_6(d\Omega) = \frac{3}{2}(\theta \cdot V_+)\Omega \wedge f . \qquad (4.45)$$

We see that now these derivatives are non-zero and in order to read off the torsion classes of the 6-dimensional manifold with SU(3)-structure we need to first normalise the forms J and Ω. First we normalise J to be a proper almost complex structure (i.e. its square to be -1). This is achieved with the following rescaling

$$J' = \sqrt{\frac{2}{1+\alpha}}J , \qquad (4.46)$$

Furthermore we define

$$\varphi' = \sqrt{\frac{2}{1-\alpha}}\varphi , \qquad (4.47)$$

thus the rescaling of ρ is the following

$$(\rho')_{mnp} = J'_{rm}(\varphi')_{npr} = \frac{2}{\sqrt{(1-\alpha)(1+\alpha)}}\rho . \qquad (4.48)$$

We then find the following expressions for the projections of dJ' and $d\varphi'$ on the 6 dimensional space orthogonal to the vectors

$$P_6(dJ') = -\frac{(\theta \cdot V_+)}{2}J' \wedge f , \qquad (4.49)$$

$$P_6(d\varphi') = (\theta \cdot V_+)\varphi' \wedge f . \qquad (4.50)$$

According to [45] this means that the only non-vanishing torsion classes are W_4 and W_5 which are given by

$$W_4 = \frac{1}{2}J' \lrcorner (dJ') = -\frac{1}{2}(\theta \cdot V_+)f , \qquad (4.51)$$

$$W_5 = \frac{1}{2}\varphi' \lrcorner (d\varphi') = (\theta \cdot V_+)f , \qquad (4.52)$$

and therefore satisfy $2W_4 + W_5 = 0$. Note that this relation also featured in the conditions found in [46] for non-Kähler backgrounds in heterotic string compactifications.

5 Summary

In this paper we studied $N = 2$ compactifications of M-theory to three dimensions which have the defining property that a potential is induced for probe space-time filling M2-branes. Such backgrounds are specifically relevant for many model building applications, ranging from moduli stabilisation to flavour physics, all of which rely on backgrounds that involve potentials for space-time filling probe D3-branes on the F-theory side, which by the F-theory/M-theory duality implies an M2-potential. We showed that it is possible to translate the requirement of an M2-potential to a specific property of the background geometry. Specifically, that the 8-dimensional background should uplift over a trivial circle fibration to a 9-dimensional manifold in which the M2-potential can be related to variations

of the angle α between two vectors. In terms of 8-dimensional geometry the variation of α implies that one can define a varying 'local' structure group, so that over a generic point the manifold exhibits an 8-dimensional SU(3)-structure, while over special loci this changes to SU(4)-structure or G_2-structure.

We studied the supersymmetry equations over such backgrounds and wrote them as differential constraints on the 9-dimensional forms. We identified a specific 4-form flux that sources variations of the angle α over the space, and showed that it vanishes over the special SU(4)-structure and G_2-structure loci, while over the generic SU(3)-structure locus it is parameterised by two real one-forms h and g. We went on to study backgrounds which support this particular 4-form flux as well as a further possible four-form flux with one leg in the internal directions, but no other 4-form fluxes. We showed that in this restricted case, over the generic SU(3)-structure locus the background takes the form of a 6-dimensional SU(3)-structure manifold with torsion classes W_4 and W_5, satisfying $2W_4 = -W_5$, fibered over a 2-dimensional base. In the case where the additional 1-leg flux is turned off we showed that the geometry is a 6-dimensional Calabi-Yau fibered over a 2-dimensional base which supports the flux and over which α varies. Since a major motivation for our work is an application to F-theory, and this requires that the background supports an elliptic fibration, it is encouraging that the simplest solutions are based on a Calabi-Yau fibration since it is well known how to construct elliptically fibered Calabi-Yau threefolds.

The analysis of the supersymmetry equations performed in this work can form a guide for finding full explicit solutions of M-theory. This would involve also solving, if they are not automatically implied, Einstein's equations. Further, although we found solutions to the Bianchi identity for the simplified flux backgrounds, one would need to solve the equations of motions for the C_3 field. It may be that imposing the full set of requirements for a stable solution on an explicit background, and possibly a realistic vacuum, would imply the need to also incorporate further fluxes, perhaps the analogs of the primitive $(2, 2)$ flux. Note that the backgrounds that arise in the simplified flux cases discussed above are naturally similar to the backgrounds studied in [48], and the approach presented in that work of a two-stage reduction may be useful for finding full solutions. We plan to return to these aspects in future work [49].

As discussed in the introduction, M2 potentials can be sourced by non-perturbative effects and it would be interesting, perhaps along the lines of [24, 25], to develop a map between such non-perturbative effects and the flux presented in this work. A related extension of our work would be a more detailed understanding of the full form of the potential that is induced for the M2 branes. Along the same lines, a more detailed study of the applications to the phenomenological aims presented: moduli stabilisation, inflation, flavour physics, would be interesting.

Acknowledgments

A Fierz identities

Fierz relations represent identities of products of gamma matrices with reshuffled indices. They emerge as a direct consequence of the the fact that the elements of the Clifford algebra in d-dimensions form a basis for the square matrices $\left(2^{\left[\frac{d}{2}\right]} \times 2^{\left[\frac{d}{2}\right]}\right)$. Thus any square matrix M can be expanded in a basis of gamma matrices in the following way

$$M = \frac{1}{2^{\left[\frac{d}{2}\right]}} \sum_A \text{Tr}\left(M\gamma^A\right)\gamma_A. \tag{A.1}$$

Fierz identities take a simpler form in the case of Majorana spinors. We restrict to this case and assume that the gamma matrices are real and symmetric. In order to write the general quadratic Fierz identity one can define the following matrix

$$M_{cd} = (\gamma_A)_{ac}(\gamma_B)_{bd} \tag{A.2}$$

for arbitrary fixed spinorial indices a, b and use eq. (A.1) to obtain

$$(\gamma_A)_{ac}(\gamma_B)_{bd} = \frac{1}{2^{\left[\frac{d}{2}\right]}} \sum_C \left(\gamma_A \gamma_C \gamma_B^T\right)_{ab}\left(\gamma^C\right)_{cd}. \tag{A.3}$$

The equation above can be used to generate all the necessary Fierz identities. For instance, by taking $\gamma_A = \gamma_B = \mathbb{1}$ one obtains the well-known completeness relation for gamma matrices. Above, we have chosen to reshuffle the indices b and c. Similarly one can obtain relations with other indices reshuffled.

We shall use the tensor form of the Fierz identities in eq. (A.3) which is obtained by contracting with the invariant spinors on the nine-dimensional manifold. The relations we obtain are exhaustive as γ_A, γ_B run over all the basis elements of the Clifford algebra.

An equivalent approach is to start with the completeness relation for the gamma matrices

$$(\mathbb{1})_{ac}(\mathbb{1})_{bd} = \frac{1}{2^{\left[\frac{d}{2}\right]}} \sum_C (\gamma_C)_{ab}\left(\gamma^C\right)_{cd} \tag{A.4}$$

and contract with arbitrary spinors. In our case the spinors are chosen as the invariant spinors on the nine-dimensional manifold multiplied by arbitrary elements of the Clifford algebra. These spinors do not form a basis on the space of spinors as they are not all linearly independent, but it is clear that they form a generating set. Therefore, the relations we obtain are exhaustive and they are equivalent to the original statement that the gamma matrices form a basis.

In this appendix we summarize the results obtained by performing a linear analysis of the system of equations generated in the way described above. We list the Fierz identities according to the number of space-time free indices. We also split the results into identities which have completely antisymmetric free indices and the ones which have symmetries corresponding to other Young tableaux. For the antisymmetric identities, the maximum number of free indices which produces new results is four, as other relations with more antisymmetric free indices can be obtained by contracting with the nine-dimensional ϵ symbol. For identities which have other symmetries of the free indices, we also stop at four indices as in our calculations we do not need further relations.

A.1 Completely antisymmetric Fierz identities

No free indices.

$$||V_1||^2 = 1 \qquad\qquad \langle V_1, V_2 \rangle = \alpha \tag{A.5}$$

$$||V_2||^2 = 1 \qquad\qquad \langle V_1, V_3 \rangle = 0 \tag{A.6}$$

$$||V_3||^2 = \frac{1}{2}(1-\alpha) \qquad\qquad \langle V_2, V_3 \rangle = 0 \tag{A.7}$$

$$||K||^2 = \frac{1}{2}(5+3\alpha) \tag{A.8}$$

$$||\Psi||^2 = \frac{1}{2}(11-3\alpha) \tag{A.9}$$

$$||\Phi_1||^2 = 14 \qquad\qquad \langle \Phi_1, \Phi_2 \rangle = -1-\alpha \tag{A.10}$$

$$||\Phi_2||^2 = 14 \qquad\qquad \langle \Phi_1, \Phi_3 \rangle = 0 \tag{A.11}$$

$$||\Phi_3||^2 = \frac{1}{2}(15+\alpha) \qquad\qquad \langle \Phi_2, \Phi_3 \rangle = 0 \,. \tag{A.12}$$

Notice that the real parameter α can take values only in the interval

$$\alpha \in [-1, 1] \,. \tag{A.13}$$

One free index.

$$V_1^m K_m{}^r = V_3^r \qquad\qquad \frac{1}{3!}\Psi^{mnp}(\Phi_1)_{mnp}{}^r = 7V_3^r \tag{A.14}$$

$$V_2^m K_m{}^r = -V_3^r \qquad\qquad \frac{1}{3!}\Psi^{mnp}(\Phi_2)_{mnp}{}^r = -7V_3^r \tag{A.15}$$

$$V_3^m K_m{}^r = -\frac{1}{2}(V_1^r - V_2^r) \qquad\qquad \frac{1}{3!}\Psi^{mnp}(\Phi_3)_{mnp}{}^r = -\frac{7}{2}(V_1^r - V_2^r) \tag{A.16}$$

$$\frac{1}{2!}K^{mn}\Psi_{mn}{}^r = 2(V_1^r + V_2^r) \tag{A.17}$$

$$\frac{1}{4!}(\Phi_1)^{mnpq}(*\Phi_1)_{mnpq}{}^r = 14V_1^r \qquad \frac{1}{4!}(\Phi_1)^{mnpq}(*\Phi_2)_{mnpq}{}^r = -(V_1^r + V_2^r) \tag{A.18}$$

$$\frac{1}{4!}(\Phi_2)^{mnpq}(*\Phi_2)_{mnpq}{}^r = 14V_2^r \qquad \frac{1}{4!}(\Phi_1)^{mnpq}(*\Phi_3)_{mnpq}{}^r = 7V_3^r \tag{A.19}$$

$$\frac{1}{4!}(\Phi_3)^{mnpq}(*\Phi_3)_{mnpq}{}^r = 4(V_1^r + V_2^r) \qquad \frac{1}{4!}(\Phi_2)^{mnpq}(*\Phi_3)_{mnpq}{}^r = 7V_3^r \,. \tag{A.20}$$

Two free indices.

$$V_1^m \Psi_m{}^{rs} = K^{rs} - 2V_1^{[r}V_3^{s]} \qquad \frac{1}{2}K^{mn}(\Phi_1)_{mn}{}^{rs} = -3K^{rs} + 6V_1^{[r}V_3^{s]} \tag{A.21}$$

$$V_2^m \Psi_m{}^{rs} = K^{rs} + 2V_2^{[r}V_3^{s]} \qquad \frac{1}{2}K^{mn}(\Phi_2)_{mn}{}^{rs} = -3K^{rs} - 6V_2^{[r}V_3^{s]} \tag{A.22}$$

$$V_3^m \Psi_m{}^{rs} = -V_1^{[r}V_2^{s]} \qquad\qquad \frac{1}{2}K^{mn}(\Phi_3)_{mn}{}^{rs} = 3V_1^{[r}V_2^{s]} \tag{A.23}$$

$$\frac{1}{3!}\Psi^{mnp}(*\Phi_1)_{mnp}{}^{rs} = -3K^{rs} - 8V_1^{[r}V_3^{s]} \tag{A.24}$$

$$\frac{1}{3!}\Psi^{mnp}(*\Phi_2)_{mnp}{}^{rs} = -3K^{rs} + 8V_2^{[r}V_3^{s]} \tag{A.25}$$

$$\frac{1}{3!}\Psi^{mnp}(*\Phi_3)_{mnp}{}^{rs} = -4V_1^{[r}V_2^{s]} \tag{A.26}$$

$$\frac{1}{3!}(\Phi_1)^{mnp[r}(\Phi_1)_{mnp}{}^{s]} = 0 \qquad \frac{1}{3!}(\Phi_1)^{mnp[r}(\Phi_2)_{mnp}{}^{s]} = -V_1^{[r}V_2^{s]} \tag{A.27}$$

$$\frac{1}{3!}(\Phi_2)^{mnp[r}(\Phi_2)_{mnp}{}^{s]} = 0 \qquad \frac{1}{3!}(\Phi_1)^{mnp[r}(\Phi_3)_{mnp}{}^{s]} = 4K^{rs} - V_1^{[r}V_3^{s]} \tag{A.28}$$

$$\frac{1}{3!}(\Phi_3)^{mnp[r}(\Phi_3)_{mnp}{}^{s]} = 0 \qquad \frac{1}{3!}(\Phi_2)^{mnp[r}(\Phi_3)_{mnp}{}^{s]} = -4K^{rs} - V_2^{[r}V_3^{s]} \tag{A.29}$$

$$K^{m[r}K_m{}^{s]} = 0 \tag{A.30}$$

$$\Psi^{mn[r}\Psi_{mn}{}^{s]} = 0 \,. \tag{A.31}$$

Three free indices.

$$V_1^m(\Phi_1)_m{}^{rst} = 0 \qquad\qquad V_2^m(\Phi_1)_m{}^{rst} = 6V_3^{[r}K^{st]} + 2K^{m[r}\Psi_m{}^{st]} \tag{A.32}$$

$$V_1^m(\Phi_2)_m{}^{rst} = -6V_3^{[r}K^{st]} + 2K^{m[r}\Psi_m{}^{st]} \qquad V_2^m(\Phi_2)_m{}^{rst} = 0 \tag{A.33}$$

$$V_1^m(\Phi_3)_m{}^{rst} = \Psi^{rst} - 3V_1^{[r}K^{st]} \qquad\qquad V_2^m(\Phi_3)_m{}^{rst} = -\Psi^{rst} + 3V_2^{[r}K^{st]} \tag{A.34}$$

$$V_3^m(\Phi_1)_m{}^{rst} = -\Psi^{rst} + 3V_1^{[r}K^{st]} \qquad \frac{1}{2}K^{mn}(*\Phi_1)_{mn}{}^{rst} = -\Psi^{rst} - 6V_1^{[r}K^{st]} \tag{A.35}$$

$$V_3^m(\Phi_2)_m{}^{rst} = \Psi^{rst} - 3V_2^{[r}K^{st]} \qquad \frac{1}{2}K^{mn}(*\Phi_2)_{mn}{}^{rst} = -\Psi^{rst} - 6V_2^{[r}K^{st]} \tag{A.36}$$

$$V_3^m(\Phi_3)_m{}^{rst} = -K^{m[r}\Psi_m{}^{st]} \qquad \frac{1}{2}K^{mn}(*\Phi_3)_{mn}{}^{rst} = -6V_3^{[r}K^{st]} \tag{A.37}$$

$$\frac{1}{2}\Psi^{mn[r}(\Phi_1)_{mn}{}^{st]} = -2\Psi^{rst} + 3V_1^{[r}K^{st]} \tag{A.38}$$

$$\frac{1}{2}\Psi^{mn[r}(\Phi_2)_{mn}{}^{st]} = -2\Psi^{rst} + 3V_2^{[r}K^{st]} \tag{A.39}$$

$$\frac{1}{2}\Psi^{mn[r}(\Phi_3)_{mn}{}^{st]} = 3V_3^{[r}K^{st]} \tag{A.40}$$

$$\frac{1}{3!}(\Phi_1)^{mnp[r}(*\Phi_1)_{mnp}{}^{st]} = 0 \qquad \frac{1}{3!}(\Phi_1)^{mnp[r}(*\Phi_2)_{mnp}{}^{st]} = 4V_3^{[r}K^{st]} \tag{A.41}$$

$$\frac{1}{3!}(\Phi_2)^{mnp[r}(*\Phi_2)_{mnp}{}^{st]} = 0 \qquad \frac{1}{3!}(\Phi_1)^{mnp[r}(*\Phi_3)_{mnp}{}^{st]} = 2\Psi^{rst} + 2V_1^{[r}K^{st]} \tag{A.42}$$

$$\frac{1}{3!}(\Phi_3)^{mnp[r}(*\Phi_3)_{mnp}{}^{st]} = 0 \qquad \frac{1}{3!}(\Phi_2)^{mnp[r}(*\Phi_3)_{mnp}{}^{st]} = -2\Psi^{rst} - 2V_1^{[r}K^{st]} \tag{A.43}$$

$$\frac{1}{3!}(\Phi_2)^{mnp[r}(*\Phi_1)_{mnp}{}^{st]} = -4V_3^{[r}K^{st]} \tag{A.44}$$

$$\frac{1}{3!}(\Phi_3)^{mnp[r}(*\Phi_1)_{mnp}{}^{st]} = -2\Psi^{rst} - 2V_1^{[r}K^{st]} \tag{A.45}$$

$$\frac{1}{3!}(\Phi_3)^{mnp[r}(*\Phi_2)_{mnp}{}^{st]} = 2\Psi^{rst} + 2V_2^{[r}K^{st]} \,. \tag{A.46}$$

Four free indices.

$$V_1^m(*\Phi_1)_m{}^{rsuv} = (\Phi_1)^{rsuv} \tag{A.47}$$

$$V_1^m(*\Phi_2)_m{}^{rsuv} = -(\Phi_1)^{rsuv} - 8V_3^{[r}\Psi^{suv]} - 6K^{[rs}K^{uv]} \tag{A.48}$$

$$V_1^m(*\Phi_3)_m{}^{rsuv} = (\Phi_3)^{rsuv} - 4V_1^{[r}\Psi^{suv]} \tag{A.49}$$

$$V_2^m (*\Phi_1)_m{}^{rsuv} = -(\Phi_2)^{rsuv} + 8V_3^{[r}\Psi^{suv]} - 6K^{[rs}K^{uv]} \tag{A.50}$$

$$V_2^m (*\Phi_2)_m{}^{rsuv} = (\Phi_2)^{rsuv} \tag{A.51}$$

$$V_2^m (*\Phi_3)_m{}^{rsuv} = (\Phi_3)^{rsuv} + 4V_2^{[r}\Psi^{suv]} \tag{A.52}$$

$$V_3^m (*\Phi_1)_m{}^{rsuv} = 4V_1^{[r}\Psi^{suv]} \tag{A.53}$$

$$V_3^m (*\Phi_2)_m{}^{rsuv} = -4V_2^{[r}\Psi^{suv]} \tag{A.54}$$

$$V_3^m (*\Phi_3)_m{}^{rsuv} = \frac{1}{2}(\Phi_1 + \Phi_2)^{rsuv} + 3K^{[rs}K^{uv]} \tag{A.55}$$

$$K^{m[r} (\Phi_1)_m{}^{suv]} = (\Phi_3)^{rsuv} - 3V_1^{[r}\Psi^{suv]} \tag{A.56}$$

$$K^{m[r} (\Phi_2)_m{}^{suv]} = -(\Phi_3)^{rsuv} - 3V_2^{[r}\Psi^{suv]} \tag{A.57}$$

$$K^{m[r} (\Phi_3)_m{}^{suv]} = -\frac{1}{2}(\Phi_1 - \Phi_2)^{rsuv} - 3V_3^{[r}\Psi^{suv]} \tag{A.58}$$

$$\frac{1}{2}\Psi^{mn[r} (*\Phi_1)_{mn}{}^{suv]} = (\Phi_3)^{rsuv} + 2V_1^{[r}\Psi^{suv]} \tag{A.59}$$

$$\frac{1}{2}\Psi^{mn[r} (*\Phi_2)_{mn}{}^{suv]} = -(\Phi_3)^{rsuv} + 2V_2^{[r}\Psi^{suv]} \tag{A.60}$$

$$\frac{1}{2}\Psi^{mn[r} (*\Phi_3)_{mn}{}^{suv]} = -\frac{1}{2}(\Phi_1 - \Phi_2)^{rsuv} + 2V_3^{[r}\Psi^{suv]} \tag{A.61}$$

$$\frac{1}{2}K^{mn} (*\Psi)_{mn}{}^{rsuv} = -\frac{1}{2}(\Phi_1 + \Phi_2)^{rsuv} + 3K^{[rs}K^{uv]} \tag{A.62}$$

$$\Psi^{m[rs}\Psi_m{}^{uv]} = -\frac{1}{2}(\Phi_1 + \Phi_2)^{rsuv} - 2K^{[rs}K^{uv]} \tag{A.63}$$

$$\frac{1}{2}(\Phi_1)^{mn[rs}(\Phi_1)_{mn}{}^{uv]} = -2(\Phi_1)^{rsuv} \qquad\qquad \frac{1}{2}(\Phi_1)^{mn[rs}(\Phi_2)_{mn}{}^{uv]} = -2K^{[rs}K^{uv]} \tag{A.64}$$

$$\frac{1}{2}(\Phi_2)^{mn[rs}(\Phi_2)_{mn}{}^{uv]} = -2(\Phi_2)^{rsuv} \qquad\qquad \frac{1}{2}(\Phi_1)^{mn[rs}(\Phi_3)_{mn}{}^{uv]} = -(\Phi_3)^{rsuv} \tag{A.65}$$

$$\frac{1}{2}(\Phi_3)^{mn[rs}(\Phi_3)_{mn}{}^{uv]} = -\frac{1}{2}(\Phi_1 + \Phi_2)^{rsuv} + K^{[rs}K^{uv]} \qquad \frac{1}{2}(\Phi_2)^{mn[rs}(\Phi_3)_{mn}{}^{uv]} = -(\Phi_3)^{rsuv}\,. \tag{A.66}$$

A.2 Fierz identities with symmetric part

We list here the Fierz identities which can have a symmetric part, that is the ones which lie in the tensorial algebra. We restrict only to necessary identities, that is the ones involving the forms V_3, K, Ψ and Φ_3. The rest can be obtained from these ones making also use of the antisymmetric identities already given earlier in the appendix.

Two indices.

$$K^{mr}K_m{}^s = \frac{1}{2}(1 + \alpha)\delta^{rs} - V_1^{(r}V_2^{s)} + V_3^r V_3^s \tag{A.67}$$

$$\Psi^{mnr}\Psi_{mn}{}^s = (4 - 2\alpha)\delta^{rs} + 6V_1^{(r}V_2^{s)} - 6V_3^r V_3^s \tag{A.68}$$

$$(\Phi_3)^{mnpr}(\Phi_3)_{mnp}{}^s = 3(7 + \alpha)\delta^{rs} - 24V_1^{(r}V_2^{s)} - 18V_3^r V_3^s\,. \tag{A.69}$$

Three indices.

$$K^{mr}\Psi_m{}^{st} = \delta^{r[s}\left(V_1^{t]} + V_2^{t]}\right) + K^{m[r}\Psi_m{}^{st]} \tag{A.70}$$

$$\Psi^{mnr}(\Phi_3)_{mn}{}^{st} = -6\delta^{r[s}\left(V_1^{s]} - V_2^{s]}\right) + 12V_3^{[r}K^{st]} - 6V_3^r K^{st} \tag{A.71}$$

$$\frac{1}{3!}(\Phi_3)^{mnpr}(*\Phi_3)_{mnp}{}^{st} = 4\delta^{r[s}\left(V_1^{t]} + V_2^{t]}\right). \tag{A.72}$$

Four indices.

$$\Psi^{mrs}\Psi_{muv} = (1-\alpha)\delta_{uv}^{rs} + 2\delta_{[u}^{[r}\left(V_1^{s]}V_{2v]} + V_{1v]}V_2^{s]} - 2V_3^{s]}V_{3v]}\right) \tag{A.73}$$

$$-\frac{1}{2}(\Phi_1 + \Phi_2)^{rs}{}_{uv} - 3K^{[rs}K^{uv]} + K^{rs}K_{uv} \tag{A.74}$$

$$\frac{1}{2}(\Phi_3)^{mnrs}(\Phi_3)_{mnuv} = (3+\alpha)\delta_{uv}^{rs} - 4\delta_{[u}^{[r}\left(V_1^{s]}V_{2v]} + V_{1v]}V_2^{s]} + V_3^{s]}V_{3v]}\right) \tag{A.75}$$

$$-\frac{1}{2}(\Phi_1 + \Phi_2)^{rs}{}_{uv} + 3K^{[rs}K^{uv]} - 2K^{rs}K_{uv}. \tag{A.76}$$

B Relations satisfied by the SU(3) structure forms

In the main text we derived the parametrisation of the spinor bilinears (2.14) in terms of forms defining a SU(3) structure in six dimensions and three additional vectors $V_\pm$ and V_3. Here we shall give more details about the relations which these forms satisfy, including also brief indications on how to derive such relations. The crucial relation which we shall use almost everywhere is the symmetric relation obtained by contracting J with itself over one index which can be derived from (A.67)

$$J_{mn}J^n{}_p = -\frac{1}{2}(1+\alpha)\delta_{mp} + \frac{1}{4}(V_+)_m(V_+)_p + \frac{1+\alpha}{4(1-\alpha)}(V_-)_m(V_-)_p + \frac{1+\alpha}{1-\alpha}(V_3)_m(V_3)_p$$

$$= \frac{1}{2}(1+\alpha)\left(-\delta_{mp} + (P_+)_{mp} + (P_-)_{mp} + (P_3)_{mp}\right). \tag{B.1}$$

In the main text we claimed that ρ defined as in (2.34) is totally antisymmetric. Using the Fierz relations (with 3 free indices) which give the contractions of the vectors $V_{1,2}$ with the forms $\Phi_{1,2}$ we can construct the object $V_+\lrcorner\Phi_+$. Using (2.31) we find

$$J_{mn}\varphi^m{}_{pq} = K_{m[n}\Psi^m{}_{pq]}. \tag{B.2}$$

Let us continue by computing the norms of the SU(3) forms. Note that since we have done an orthogonal decomposition in terms of the vector fields, the terms on the r.h.s. of (2.29)–(2.33) are independent, in the sense that total contractions of different terms vanish by definition. Using the norms of the vector fields which were listed at the beginning of this appendix, we can immediately derive the norm of J as

$$J\lrcorner J = \frac{1}{2}J_{mn}J^{mn} = \frac{3}{2}(1+\alpha). \tag{B.3}$$

Taking the square of (2.30) we find in a similar way the norm of φ

$$\varphi\lrcorner\varphi = \frac{1}{6}\varphi_{mnp}\varphi^{mnp} = 2(1-\alpha). \tag{B.4}$$

From (2.34) and using (B.1) we find

$$\rho \lrcorner \rho = \frac{1}{6} \rho_{mnp} \rho^{mnp} = (1 - \alpha)(1 + \alpha). \tag{B.5}$$

Using the Fierz relation involving the contraction of K and Ψ over two indices, we immediately find

$$J \lrcorner \varphi = 0 = J \lrcorner \rho \tag{B.6}$$

where the second equality holds due to the fact that ρ in (2.34) is totally antisymmetric. Using the above relation and the Fierz identity involving the contraction of Ψ and $\Phi_{1,2}$ over three indices, we obtain

$$\varphi \lrcorner \rho = 0 . \tag{B.7}$$

Most of the other relations we shall need involve a Hodge $*$ operation and are somehow more complicated. Let us look at the Fierz relation which gives the contraction of Φ_3 with $*\Phi_3$ over four indices. This can be rewritten in form notation as

$$*(\Phi_3 \wedge \Phi_3) = 4V_+ . \tag{B.8}$$

Using (2.33) we find

$$\frac{4}{(1-\alpha)^2} \varphi \wedge \rho \wedge V_- \wedge V_3 - \frac{2}{(1-\alpha)(1+\alpha)} \rho \wedge J \wedge V_+ \wedge V_- \wedge V_3 = 4 * V_+ . \tag{B.9}$$

Clearly, the second term on the l.h.s. must vanish, as it contains V_+ while on the r.h.s. we only find $*V_+$. J and ρ are orthogonal to the vectors, and the only way this term can vanish is if

$$\rho \wedge J = 0 . \tag{B.10}$$

The remaining relation can be rewritten, by contracting V_- and V_3, as

$$\varphi \wedge \rho = *(V_+ \wedge V_- \wedge V_3) . \tag{B.11}$$

From Fierz identities involving the contraction of $\Phi_{1,2}$ with Φ_3 we find similar relations, and again, by contracting the appropriate vectors we find

$$J \wedge J \wedge J = \frac{3}{2} \frac{1+\alpha}{1-\alpha} * (V_+ \wedge V_- \wedge V_3) . \tag{B.12}$$

Further relations with Hodge star can be derived form these ones by contracting with appropriate forms and making use of the orthogonality conditions and of the norms of the various quantities. We find

$$*J = \frac{1}{2(1+\alpha)(1-\alpha)} J \wedge J \wedge V_+ \wedge V_- \wedge V_3 \tag{B.13}$$

$$*\varphi = \frac{1}{(1+\alpha)(1-\alpha)} \rho \wedge V_+ \wedge V_- \wedge V_3 \tag{B.14}$$

$$*\rho = -\frac{1}{2(1+\alpha)} \varphi \wedge V_+ \wedge V_- \wedge V_3 . \tag{B.15}$$

Other useful relations which can be derived easily from the ones already written so far are

$$*(J \wedge V_+ \wedge V_- \wedge V_3) = (1-\alpha)J \wedge J \tag{B.16}$$

$$*(J \wedge V_- \wedge V_3) = \frac{(1-\alpha)}{2(1+\alpha)} J \wedge J \wedge V_+ \tag{B.17}$$

$$*(J \wedge V_+ \wedge V_3) = -\frac{1}{2} J \wedge J \wedge V_- \tag{B.18}$$

$$*(J \wedge V_+ \wedge V_-) = 2 J \wedge J \wedge V_3 \tag{B.19}$$

and

$$*(\rho \wedge V_+ \wedge V_3) = \frac{1}{2}(1+\alpha)\varphi \wedge V_- \qquad *(\rho \wedge V_3) = \frac{1}{4}\varphi \wedge V_+ \wedge V_- \tag{B.20}$$

$$*(\rho \wedge V_- \wedge V_3) = -\frac{1}{2}(1-\alpha)\varphi \wedge V_+ \qquad *(\rho \wedge V_+) = \frac{1+\alpha}{1-\alpha}\varphi \wedge V_- \wedge V_3 \tag{B.21}$$

$$*(\rho \wedge V_+ \wedge V_-) = -2(1+\alpha)\varphi \wedge V_3 \qquad *(\rho \wedge V_-) = -\varphi \wedge V_+ \wedge V_3 \,. \tag{B.22}$$

One can also compute the Hodge duals of the original spinor bilinear forms and express them in terms of the SU(3) parametrisation

$$*K = \frac{1}{2(1+\alpha)(1-\alpha)} J \wedge J \wedge V_+ \wedge V_- \wedge V_3 + \frac{1}{2(1+\alpha)(1-\alpha)} \varphi \wedge \rho \wedge V_+ \tag{B.23}$$

$$*\Psi = \frac{1}{(1+\alpha)(1-\alpha)} \rho \wedge V_+ \wedge V_- \wedge V_3 + \frac{1}{(1+\alpha)(1-\alpha)} J \wedge J \wedge V_- \wedge V_3 + \frac{1}{2(1-\alpha)} \varphi \wedge \rho \tag{B.24}$$

$$*\Phi_+ = -\frac{2}{(1+\alpha)(1-\alpha)} J \wedge V_+ \wedge V_- \wedge V_3 - \frac{2}{1-\alpha}\varphi \wedge V_- \wedge V_3 - \frac{1}{1+\alpha} J \wedge J \wedge V_+ \tag{B.25}$$

$$*\Phi_- = -\frac{2}{(1+\alpha)(1-\alpha)} \rho \wedge V_+ \wedge V_- - \frac{2}{1-\alpha}\varphi \wedge V_+ \wedge V_3 - \frac{1}{1+\alpha} J \wedge J \wedge V_- \tag{B.26}$$

$$*\Phi_3 = -\frac{2}{(1+\alpha)(1-\alpha)} \rho \wedge V_+ \wedge V_3 + \frac{1}{2(1-\alpha)}\varphi \wedge V_+ \wedge V_- - \frac{1}{1+\alpha} J \wedge J \wedge V_3 \,. \tag{B.27}$$

From the symmetric Fierz identity (A.68) involving the contraction of Ψ with itself over one index we can derive a similar relation for φ

$$\varphi^{mrs}\varphi_{mtu} = (1-\alpha)\delta^{rs}_{tu} + 2\frac{1-\alpha}{1+\alpha}J^r{}_{[t}J_{u]}{}^s - \frac{1-\alpha}{1+\alpha}\delta^{[r}_{[t}V_{+u]}V^{s]}_+ - \delta^{[r}_{[t}V_{-u]}V^{s]}_- - 4\delta^{[r}_{[t}V_{3u]}V^{s]}_3$$
$$+ \frac{1}{2(1+\alpha)}V_{+[t}V_{-u]}V^{[r}_+V^{s]}_- + \frac{2}{1+\alpha}V_{+[t}V_{3u]}V^{[r}_+V^{s]}_3 + \frac{2}{1-\alpha}V_{-[t}V_{3u]}V^{[r}_-V^{s]}_3 \,. \tag{B.28}$$

By contracting this relation with J we can find similar relations for ρ or ρ and φ

$$\rho_m{}^{rs}\varphi^m{}_{tu} = 2(1-\alpha)\delta^{[r}_{[t}J_{u]}{}^{s]} + \frac{1-\alpha}{1+\alpha}J^{[r}{}_{[t}V_{+u]}V^{s]}_+ + J^{[r}{}_{[t}V_{-u]}V^{s]}_- + 4J^{[r}{}_{[t}V_{3u]}V^{s]}_3 \tag{B.29}$$

$$\rho^{mrs}\rho_{mtu} = \frac{1}{2}(1-\alpha)(1+\alpha)\delta^{rs}_{tu} + (1-\alpha)J^r{}_{[t}J_{u]}{}^s - \frac{1}{2}(1-\alpha)\delta^{[r}_{[t}V_{+u]}V^{s]}_+ - \frac{1}{2}(1+\alpha)\delta^{[r}_{[t}V_{-u]}V^{s]}_-$$
$$-2(1+\alpha)\delta^{[r}_{[t}V_{3u]}V^{s]}_3 + \frac{1}{4}V_{+[t}V_{-u]}V^{[r}_+V^{s]}_- + V_{+[t}V_{3u]}V^{[r}_+V^{s]}_3 + \frac{1+\alpha}{1-\alpha}V_{-[t}V_{3u]}V^{[r}_-V^{s]}_3 \,. \tag{B.30}$$

Finally, by contracting a pair of indices in the above relations it is easy to find

$$\varphi_{mnr}\varphi^{mns} = 2(1-\alpha)\delta_r^s - \frac{1-\alpha}{1+\alpha}V_{+r}V_+^s - V_{-r}V_-^s - 4V_{3r}V_3^s \ ;$$

$$\varphi_{mnr}\rho^{mns} = 2(1-\alpha)J_r{}^s \tag{B.31}$$

$$\rho_{mnr}\rho^{mns} = (1-\alpha)(1+\alpha)\delta_r^s - \frac{1-\alpha}{2}V_{+r}V_+^s - \frac{1+\alpha}{2}V_{-r}V_-^s - 2(1+\alpha)V_{3r}V_3^s \ .$$

Before we end this section, let us note the following fact which is useful during the calculations. Consider a 1-form k which is orthogonal to all the vector fields, i.e. $k \cdot V_i = 0$, and define $\tilde{k} = k \lrcorner J$. By contracting J we find the equivalent relation

$$\tilde{k} \lrcorner J = -\frac{1+\alpha}{2}k \ . \tag{B.32}$$

Furthermore, by contracting with φ and taking into account the definition of ρ from (2.34) we find

$$\tilde{k} \lrcorner \varphi + k \lrcorner \rho = 0 \ , \qquad \frac{1+\alpha}{2}k \lrcorner \varphi - \tilde{k} \lrcorner \rho = 0 \ . \tag{B.33}$$

Taking now the exterior product with ρ and φ we find

$$\tilde{k} \wedge \varphi + (k \lrcorner \varphi) \wedge J = 0 \ , \qquad\qquad \tilde{k} \wedge \rho + (k \lrcorner \rho) \wedge J = 0$$

$$-\left(\tilde{k} \lrcorner \rho\right) \wedge J + \frac{1+\alpha}{2}k \wedge \rho = 0 \ , \qquad -\left(\tilde{k} \lrcorner \varphi\right) \wedge J + \frac{1+\alpha}{2}k \wedge \varphi = 0$$

where we used the identity $(k \lrcorner \varphi) \wedge J - \varphi \wedge (k \lrcorner J) = k \lrcorner (\varphi \wedge J) = 0$ and similar ones for ρ and $\tilde{k}$. Adding up the equations in the same column in such a way to obtain the combinations in (B.33) we find

$$\tilde{k} \wedge \varphi + k \wedge \rho = 0 \ , \qquad \tilde{k} \wedge \rho - \frac{1+\alpha}{2}k \wedge \varphi = 0 \ . \tag{B.34}$$

These relations can be intuitively understood in a simple way. From the 1-forms k and $\tilde{k}$ we can construct a complex $(1,0)$ form

$$\Bbbk = k + i\sqrt{\frac{2}{1+\alpha}}\tilde{k} \ . \tag{B.35}$$

Then the expressions in (B.34) are nothing but the real and imaginary components of the $(4,0)$ form $\Bbbk \wedge \Omega$. But this form should vanish identically since it only lives on the 6-dimensional space orthogonal to the vectors and this gives the relations in (B.34).

C Supersymmetry equations

We summarize in this appendix the supersymmetry algebraic constraints arising from the variation of the external components of the gravitino. For the $N = 2$ flux background that we consider one has to satisfy the equations

$$Q\xi_j = 0 \qquad j = 1, 2 \tag{C.1}$$

where the operator Q is given in eq. (3.6) for the auxiliary 9d manifold Y_9. We translate the equations above into constraints on the fluxes $\tilde{f}$ and F involving the spinor bilinears defined in eq. (2.14). Specifically, we contract eq. (C.1) with the following generating set of the spinorial representation

$$\gamma_A \xi_i \qquad i = 1,2 \qquad \text{and} \qquad \gamma_A \in \{\mathbb{1}, \gamma_m, \gamma_{mn}, \gamma_{mnp}, \gamma_{mnpq}\}\,. \qquad (C.2)$$

It is then convenient to represent the algebraic constraints in eq. (C.1) in the the following equivalent form

$$\xi_i^T \left[Q\gamma_A \pm \gamma_A^T Q^T \right] \xi_j = 0 \qquad i,j = 1,2\,. \qquad (C.3)$$

After inserting the expression of Q given in eq. (3.6) and expanding the products of gamma matrices one can express the resulting equations in terms of the spinor bilinears in eq. (2.14). The result is the following (a number of these expressions first appeared in [39])

$$-\lambda(\theta \cdot V_i) + \frac{1}{2}(V_i \cdot d\Delta) - \frac{1}{12}F_\lrcorner \Phi_i = 0\,; \qquad \tilde{f}_m \theta_n K^{mn} = 0 \qquad (C.4)$$

$$-2\lambda\theta + d\Delta - \frac{1}{12} * (F \wedge \Phi_+) + \frac{1}{6}V_{+\lrcorner}\left(\tilde{f} \wedge \theta\right) = 0 \qquad (C.5)$$

$$-\frac{1}{12} * (F \wedge \Phi_{-,3}) + \frac{1}{6}V_{-,3\lrcorner}\left(\tilde{f} \wedge \theta\right) = 0 \qquad (C.6)$$

$$2\lambda\theta_\lrcorner K - d\Delta_\lrcorner K + \frac{1}{6}\Psi_\lrcorner F - \frac{1}{3}\left(\tilde{f} \wedge \theta\right)_\lrcorner \Psi = 0 \qquad (C.7)$$

$$2\lambda\theta_{[m}V_{+\,n]} - \partial_{[m}\Delta V_{+\,n]} + \frac{1}{36}F_{[m}{}^{pqr}\Phi_{+\,n]pqr} - \frac{1}{6}\Phi_{+\,mnpq}\tilde{f}^p\theta^q + \frac{2}{3}\tilde{f}_{[m}\theta_{n]} = 0 \qquad (C.8)$$

$$2\lambda\theta_{[m}(V_{-\,3})_{n]} - \partial_{[m}\Delta(V_{-\,3})_{n]} + \frac{1}{36}F_{[m}{}^{klp}(\Phi_{-\,3})_{n]klp} - \frac{1}{6}(\Phi_{-\,3})_{mnkl}\tilde{f}^k\theta^l = 0 \qquad (C.9)$$

$$-2\lambda\theta_\lrcorner \Psi + d\Delta_\lrcorner \Psi + \frac{1}{6} * (F \wedge \Psi) + \frac{1}{6}K_\lrcorner F + \frac{1}{3}(\theta_\lrcorner K) \wedge \tilde{f} - \frac{1}{3}\left(\tilde{f}_\lrcorner K\right) \wedge \theta = 0 \qquad (C.10)$$

$$-\lambda\theta_k\Phi_{i\,mnpk} + \frac{1}{2}\partial_k\Delta\Phi_{i\,mnpk} - \frac{1}{24}F_{[p}{}^{klq}(*\Phi_i)_{mn]klq} + \frac{1}{12}F_{mnpk}V_i^k$$
$$-\frac{1}{6} * \left(\Phi_i \wedge \tilde{f} \wedge \theta\right)_{mnp} + \frac{1}{6}\left(V_i \wedge \tilde{f} \wedge \theta\right)_{mnp} = 0 \qquad (C.11)$$

$$-2\lambda(K \wedge \theta)_{mnp} + K \wedge d\Delta_{mnp} + \frac{1}{6} * (F \wedge K)_{mnp} + \frac{1}{4}\Psi_{[m}{}^{kl}F_{np]kl}$$
$$-\frac{1}{3}\left[(\theta_\lrcorner \Psi) \wedge \tilde{f}\right]_{mnp} + \frac{1}{3}\left[\left(\tilde{f}_\lrcorner \Psi\right) \wedge \theta\right]_{mnp} = 0 \qquad (C.12)$$

$$-\lambda * (\Phi_+ \wedge \theta)_{mnpq} + \frac{1}{2} * (\Phi_+ \wedge d\Delta)_{mnpq} - \frac{1}{12} * (F \wedge V_+)_{mnpq}$$
$$+\frac{1}{4}F_{[mn}{}^{rs}(\Phi_+)_{pq]rs} - \frac{1}{6}F_{mnpq} + \frac{1}{6}(\theta_\lrcorner \Phi_+) \wedge \tilde{f} - \frac{1}{6}\left(\tilde{f}_\lrcorner \Phi_+\right) \wedge \theta = 0 \qquad (C.13)$$

$$-\lambda * (\Phi_{-3} \wedge \theta)_{mnpq} + \frac{1}{2} * (\Phi_{-3} \wedge d\Delta)_{mnpq} - \frac{1}{12} * (F \wedge V_{-3})_{mnpq}$$
$$+\frac{1}{4}F_{[mn}{}^{rs}(\Phi_{-3})_{pq]rs} + \frac{1}{6}(\theta_\lrcorner \Phi_{-3}) \wedge \tilde{f} - \frac{1}{6}\left(\tilde{f}_\lrcorner \Phi_{-3}\right) \wedge \theta = 0 \qquad (C.14)$$

$$-2\lambda(\Psi \wedge \theta)_{mnpq} + (\Psi \wedge d\Delta)_{mnpq} + \frac{1}{9}(*\Psi)_{[mnp}{}^{rst}F_{q]rst}$$
$$+\frac{2}{3}K_{[m}{}^{r}F_{npq]r} + \frac{1}{3} * \left(\Psi \wedge \tilde{f} \wedge \theta\right)_{mnpq} + \frac{1}{3}\left(K \wedge \tilde{f} \wedge \theta\right)_{mnpq} = 0\,. \qquad (C.15)$$

Notice that since we are using a set of generators $\{\gamma_A\xi_i\}$ instead of a basis, the equations above are not independent. In fact, as it is done explicitly for the special flux analysed in the paper, one only needs the constraints arising from contraction with ξ_i and $\gamma_m\xi_i$.

D Killing properties of the Majorana-Weyl components

Given that a background supports a covariantly constant Majorana spinor ξ, the requirement for its Majorana-Weyl components to also solve the Killing spinor equation is

$$[\mathcal{D}_m, \theta^r\gamma_r]\xi = 0 \, , \tag{D.1}$$

$$[Q, \theta^r\gamma_r]\xi = 0 \, . \tag{D.2}$$

Here $\mathcal{D}_m$ and Q are defined in (3.1), and we recall that the eight-dimensional chirality matrix γ_9 was given in terms of the 9-dimensional basis as $\gamma_9 = \theta^r\gamma_r$. One can easily show that the commutators with γ_9 are expressed as

$$[A_m, \theta^r\gamma_r] = 2\lambda\left(\gamma_m - \theta_m\theta^n\gamma_n\right) + \frac{1}{12}F^{mnpq}\theta^r\gamma_{npqr} \, , \tag{D.3}$$

$$[Q, \theta^r\gamma_r] = \partial^n\Delta\theta^r\gamma_{nr} + \frac{1}{36}F^{mnpq}\theta_{[m}\gamma_{npq]} + \frac{2}{3}\tilde{f}^m\theta^n\theta_{[m}\gamma_{n]} \, . \tag{D.4}$$

We now impose the orthogonality of the auxiliary direction θ on the fluxes and the fact that $\lambda = 0$ for our specific choice of flux

$$[A_m, \theta^r\gamma_r] = \frac{1}{12}F^{mnpq}\theta^r\gamma_{npqr} \, , \tag{D.5}$$

$$[Q, \theta^r\gamma_r] = \partial^n\Delta\theta^r\gamma_{nr} - \frac{1}{3}\tilde{f}^m\gamma_m \, . \tag{D.6}$$

Let us examine further the first condition. For this we multiply it by $\theta^s\gamma_s$ from the left and find after some simple gamma matrix manipulations

$$F^{mnpq}\gamma_{npq}\xi = 0 \, , \tag{D.7}$$

where ξ can be any (or both) of the Majorana spinors $\xi_{1,2}$, which define our background. For definiteness we shall consider that this equation is valid for $\xi = \xi_1$. Multiplying this from the left with ξ_2^T and using (2.14) we obtain

$$F^{mnpq}\Psi_{npq} = 0 \, . \tag{D.8}$$

Using (2.30), the fact that for backgrounds which admit varying parameter α the ninth direction θ is parallel to V_+ and the independence of the flux on this ninth direction (V_+), we find

$$\varphi_\lrcorner F = 0 \, . \tag{D.9}$$

This relation however can only be compatible with (3.21) if $F = 0$ and therefore we conclude that for backgrounds in which the parameter α varies, none of the Majorana-Weyl components can satisfy by itself the Killing spinor equation.

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