Towards the holographic dual of $\mathcal{N} = 2$ SYK

Stefan Förste, Joshua Kames-King and Max Wiesner

*Bethe Center for Theoretical Physics and Physikalisches Institut der Universität Bonn,
Nussallee 12, 53115 Bonn, Germany*

E-mail: forste@th.physik.uni-bonn.de, jvakk@yahoo.com,
wiesner@physik.uni-bonn.de

ABSTRACT: The gravitational part of the holographic dual to the SYK model has been conjectured to be Jackiw-Teitelboim (JT) gravity. In this paper we construct an AdS_2 background in $\mathcal{N} = (2, 2)$ JT gravity and show that the gravitational dynamics are — as in the $\mathcal{N} = 0$ and $\mathcal{N} = 1$ cases — fully captured by the extrinsic curvature as an effective boundary action. This boundary term is given by the super-Schwarzian of the $\mathcal{N} = 2$ SYK model, thereby providing further evidence of the JT/SYK duality. The chirality of this SYK model is reproduced by the inherent chirality of axial $\mathcal{N} = (2, 2)$ supergravity.

KEYWORDS: 2D Gravity, AdS-CFT Correspondence, Extended Supersymmetry

ARXIV EPRINT: [1712.07398](https://arxiv.org/abs/1712.07398)

Contents

1	Introduction	1
2	$\mathcal{N} = (2, 2)$ supergravity	2
3	$\mathcal{N} = (2, 2)$ Jackiw-Teitelboim action	5
4	Determination of the superconformal factor	8
5	Effective action: appearance of the super-Schwarzian	11
6	Consistency check	15
7	Summary and conclusions	16

1 Introduction

The Sachdev-Ye-Kitaev (SYK) model [1–7] is conformally (i.e. reparametrisation) invariant in the IR. The breaking of conformal symmetry results in an effective Lagrangian for time reparametrisations which is given by the Schwarzian. Models without random couplings sharing this property have been constructed and studied in e.g. [8–18]. There are various other mutations of the SYK model, for instance, higher dimensional analogs have been proposed in [19–21], complex versions are studied e.g. in [19, 22, 23], more than one flavour and non Abelian global symmetries have been investigated in [24–27]. For the present paper the supersymmetric versions constructed in [28] are most relevant. For variations and further aspects of supersymmetric SYK models see [29–41]. Now, superreparametrisations are an exact symmetry only in the infrared limit, and their breaking gives rise to an effective super-Schwarzian action.

The holographic dual is believed to contain some version of dilatonic 2d gravity arising quite universally in compactifications from higher dimensions [42]. Moreover, near the horizon of an AdS_2 black hole, corresponding to the IR of SYK, solutions of the dilaton are nearly constant. Approximating to linear order in non constant contributions leads to Jackiw-Teitelboim (JT) gravity [43, 44], for a recent review see [45]. JT gravity has been corroborated as the gravitational dual of the SYK model e.g. by deriving the Schwarzian as an effective action for the UV regulator curve [46–49]. Liouville theory instead of JT gravity has been considered in [50]. A three dimensional holographic dual has, however, been advocated in a series of papers [51–53]. Corrections to JT gravity have been recently proposed in [54]. Gathering information about the holographic dual beyond the gravitational sector has been the subject of [55, 56].

To start extending these investigations to supersymmetric versions of SYK it was shown in [57] that an $\mathcal{N} = (1, 1)$ supersymmetric version of JT gravity [58] supplemented with the appropriate boundary term leads to the $\mathcal{N} = 1$ super-Schwarzian as an effective action for the UV regulator curve. In the present paper we will extend this further to $\mathcal{N} = (2, 2)$ JT gravity.

The paper is organised as follows. In section 2 we collect some results on $\mathcal{N} = (2, 2)$ supergravity in superconformal gauge. Section 3 deals with the $\mathcal{N} = (2, 2)$ extension of JT gravity. The Gibbons-Hawking-York term is added. In section 4 the superconformal gauge is solved for AdS_2 as a supersymmetric background. The main result, the super-Schwarzian as effective Lagrangian of the boundary curve, is obtained in section 5. In section 6, a consistency check will be performed. Our results are summarised in section 7.

2 $\mathcal{N} = (2, 2)$ supergravity

In this section, we collect some information about extended $\mathcal{N} = (2, 2)$ supersymmetry and supergravity in two dimensions. Useful references are [59–64]. The two dimensional rigid $\mathcal{N} = (2, 2)$ superspace is given by the coset space [60]

$$\frac{(2, 2)\text{Supergroup}}{\text{Lorentz} \otimes U_A(1) \otimes U_V(1)},$$

with coordinates

$$z^\pi = (z, \theta^+, \bar{\theta}^+; \bar{z}, \theta^-, \bar{\theta}^-)$$

and covariant derivatives

$$\begin{aligned} \partial_z, \quad D_+ &= \frac{\partial}{\partial \theta^+} + \frac{1}{2} \bar{\theta}^+ \partial_z, & \bar{D}_+ &= \frac{\partial}{\partial \bar{\theta}^+} + \frac{1}{2} \theta^+ \partial_z, \\ \partial_{\bar{z}}, \quad D_- &= \frac{\partial}{\partial \theta^-} + \frac{1}{2} \bar{\theta}^- \partial_{\bar{z}}, & \bar{D}_- &= \frac{\partial}{\partial \bar{\theta}^-} + \frac{1}{2} \theta^- \partial_{\bar{z}}. \end{aligned}$$

They satisfy the anticommutation relations

$$\{D_+, \bar{D}_+\} = \partial_z, \quad \{D_-, \bar{D}_-\} = \partial_{\bar{z}}.$$

There are two versions of minimal $\mathcal{N} = 2$ supergravity, which can be obtained from the nonminimal $U_A(1) \otimes U_V(1)$ by gauging either the $U_A(1)$ or $U_V(1)$ factor of the tangent space symmetry group [61]. Here we will focus on the axial version of minimal $\mathcal{N} = 2$ sugra with gauged $U_A(1)$, which can also be obtained by dimensionally reducing $\mathcal{N} = 1$ sugra in $d = 4$. Accordingly, the tangent space symmetry group consists of the 2D Lorentz group and the gauged $U_A(1)$ factor.

The spinorial covariant derivatives in the minimal theory are given by

$$\nabla_\alpha = E_\alpha + \Omega_\alpha \mathcal{J} + \Sigma_\alpha \mathcal{Y}, \tag{2.1}$$

where $\alpha = \pm$ is a flat space spinor index and $\mathcal{J}, \mathcal{Y}$ are respectively the Lorentz and $U_A(1)$ generators with corresponding connections Ω_α and Σ_α . For the complex conjugates and vector derivatives similar relations hold.

The Lorentz and $U_A(1)$ generators form together with the four supercharges $Q^+, Q^-, \bar{Q}^+, \bar{Q}^-$ our SUSY algebra [60]:

$$\begin{aligned}
 [Q_+, \mathcal{Y}] &= -Q_+, & [\bar{Q}_+, \mathcal{Y}] &= \bar{Q}_+, \\
 [Q_-, \mathcal{Y}] &= Q_-, & [\bar{Q}_-, \mathcal{Y}] &= -\bar{Q}_-, \\
 [Q_+, \mathcal{J}] &= \frac{i}{2}Q_+, & [\bar{Q}_+, \mathcal{J}] &= \frac{i}{2}\bar{Q}_+, \\
 [Q_-, \mathcal{J}] &= -\frac{i}{2}Q_-, & [\bar{Q}_-, \mathcal{J}] &= -\frac{i}{2}\bar{Q}_-.
 \end{aligned} \tag{2.2}$$

Therefore Majorana constraints are implemented by the following constraint on Weyl spinors in Euclidean space:

$$(Q_+)^* = \bar{Q}_-, \quad (Q_-)^* = \bar{Q}_+. \tag{2.3}$$

For convenience, we introduce the following linear combination of the tangent group generators:

$$M \equiv \mathcal{J} - \frac{i}{2}\mathcal{Y}, \tag{2.4}$$

$$\bar{M} \equiv \mathcal{J} + \frac{i}{2}\mathcal{Y}. \tag{2.5}$$

In order to get a physical sugra theory, torsion constraints have to be imposed. In our case, the relevant constraints are given by

$$\begin{aligned}
 \{\nabla_\pm, \nabla_\pm\} &= 0, \quad \text{and} \quad \{\nabla_+, \nabla_-\} = -\frac{1}{2}\bar{R}\bar{M}, \\
 \{\bar{\nabla}_\pm, \bar{\nabla}_\pm\} &= 0, \quad \text{and} \quad \{\bar{\nabla}_+, \bar{\nabla}_-\} = -\frac{1}{2}RM, \\
 \{\nabla_+, \bar{\nabla}_-\} &= 0, \quad \text{and} \quad \{\bar{\nabla}_+, \nabla_-\} = 0,
 \end{aligned} \tag{2.6}$$

where R is the chiral and $\bar{R}$ the anti-chiral curvature supermultiplet. These supermultiplets contain in their two θ component the usual bosonic scalar curvature $\mathcal{R}$ as well as the $U_A(1)$ field strength $\mathcal{F}$ as can be most easily displayed in a Wess-Zumino gauge [62]: the components of the supercurvature multiplets can be expressed through the components of the supergravity multiplet, namely the vielbein e_a^m , the gravitini ψ_a^α and the two auxiliary fields S and $\bar{S}$. These fields are defined by the leading components of the vector covariant derivatives

$$\begin{aligned}
 \nabla_a| &= \mathbf{D}_a + \psi_a^\alpha \nabla_\alpha| + \psi_a^{\dot{\alpha}} \nabla_{\dot{\alpha}}| \\
 &= \mathbf{D}_a + \psi_a^\alpha \partial_\alpha + \psi_a^{\dot{\alpha}} \partial_{\dot{\alpha}},
 \end{aligned} \tag{2.7}$$

where $|$ sets $\theta^+ = \theta^- = \bar{\theta}^+ = \bar{\theta}^- = 0$ and

$$\mathbf{D}_a = e_a + \Omega_a \mathcal{J} + \Sigma_a \mathcal{Y}. \tag{2.8}$$

The leading components of the curvature supermultiplets are given by

$$R| = S, \quad \bar{R}| = \bar{S}. \quad (2.9)$$

The higher order components can be determined by looking at

$$\begin{aligned} [\nabla_l, \nabla_{\bar{l}}] = & -\frac{i}{2} [(\nabla_- R)\nabla_+ + (\nabla_+ R)\nabla_- + (\bar{\nabla}_- \bar{R})\bar{\nabla}_+ + (\bar{\nabla}_+ \bar{R})\bar{\nabla}_-] \\ & + \frac{1}{2} \left[\bar{\nabla}_- \bar{\nabla}_+ \bar{R} + \frac{i}{2} \bar{R} R \right] \bar{M} + \frac{1}{2} \left[\nabla_- \nabla_+ R + \frac{i}{2} R R \right] M. \end{aligned} \quad (2.10)$$

In the following, we will make use of the notation

$$\bar{\nabla}^2 \equiv \bar{\nabla}_+ \bar{\nabla}_-, \quad \nabla^2 \equiv \nabla_+ \nabla_-. \quad (2.11)$$

We can insert (2.7) into the commutator $[\nabla_l, \nabla_{\bar{l}}]$, use

$$[\nabla_l, \nabla_{\bar{l}}]| = [\nabla_l|, \nabla_{\bar{l}}|] + \psi_l^\alpha \nabla_\alpha \nabla_{\bar{l}}| + \psi_{\bar{l}}^{\dot{\alpha}} \nabla_{\dot{\alpha}} \nabla_l| - \psi_l^\alpha \nabla_\alpha \nabla_l| - \psi_{\bar{l}}^{\dot{\alpha}} \nabla_{\dot{\alpha}} \nabla_{\bar{l}}|, \quad (2.12)$$

and read off the other components of R and $\bar{R}$. The calculation is rather tedious and since we are only interested in a certain classical background solution, we set the gravitini to zero and the only relevant component of the supercurvature is the $\theta^+ \theta^-$ component. This component depends on the $U_A(1)$ field strength $\mathcal{F}$ and the scalar curvature $\mathcal{R}$ and is of the form

$$\left(\nabla^2 R + \frac{i}{2} \bar{R} R \right) \Big| = -i(\mathcal{R} + i\mathcal{F}), \quad (2.13)$$

$$\left(\bar{\nabla}^2 \bar{R} + \frac{i}{2} \bar{R} R \right) \Big| = -i(\mathcal{R} - i\mathcal{F}), \quad (2.14)$$

if the gravitini are set to zero.

Coming back to the torsion constraints (2.6), these are most easily solved in superconformal gauge in terms of a chiral field σ and an anti-chiral field $\bar{\sigma}$. The solution of the torsion constraints is then given by

$$\begin{aligned} \nabla_+ &= e^{\bar{\sigma}} (D_+ + i(D_+ \sigma) \bar{M}), \\ \nabla_- &= e^{\bar{\sigma}} (D_- - i(D_- \sigma) \bar{M}), \\ \bar{\nabla}_+ &= e^{\sigma} (\bar{D}_+ + i(\bar{D}_+ \bar{\sigma}) M), \\ \bar{\nabla}_- &= e^{\sigma} (\bar{D}_- - i(\bar{D}_- \bar{\sigma}) M). \end{aligned} \quad (2.15)$$

The vector derivatives are

$$\begin{aligned} \nabla_l = \{ \nabla_+, \bar{\nabla}_+ \} &= e^{\sigma + \bar{\sigma}} [(\partial_z + 2(D_+ \sigma) \bar{D}_+ + 2(\bar{D}_+ \bar{\sigma}) D_+) \\ &\quad + i(\partial_z \bar{\sigma} + 2(D_+ \sigma)(\bar{D}_+ \bar{\sigma})) \bar{M} + i(\partial_z \sigma + 2(\bar{D}_+ \bar{\sigma})(D_+ \sigma)) \bar{M}] , \end{aligned} \quad (2.16)$$

$$\begin{aligned} \nabla_{\bar{l}} = \{ \nabla_-, \bar{\nabla}_- \} &= e^{\sigma + \bar{\sigma}} [(\partial_{\bar{z}} + 2(D_- \sigma) \bar{D}_- + 2(\bar{D}_- \bar{\sigma}) D_-) \\ &\quad - i(\partial_{\bar{z}} \bar{\sigma} + 2(D_- \sigma)(\bar{D}_- \bar{\sigma})) \bar{M} - i(\partial_{\bar{z}} \sigma + 2(\bar{D}_- \bar{\sigma})(D_- \sigma)) \bar{M}] . \end{aligned} \quad (2.17)$$

The connection and vielbein components can be read off by comparing our expressions for the covariant derivatives with (2.1). One gets for the Lorentz connection

$$\begin{aligned}
 \Omega_l &= i\partial_z(\sigma + \bar{\sigma})e^{\sigma+\bar{\sigma}}, \\
 \Omega_{\bar{l}} &= -i\partial_{\bar{z}}(\sigma + \bar{\sigma})e^{\sigma+\bar{\sigma}}, \\
 \Omega_+ &= ie^{\bar{\sigma}}(D_+\sigma) \quad \text{and} \quad \bar{\Omega}_+ = ie^{\sigma}(\bar{D}_+\bar{\sigma}), \\
 \Omega_- &= -ie^{\bar{\sigma}}(D_-\sigma) \quad \text{and} \quad \bar{\Omega}_- = -ie^{\sigma}(\bar{D}_-\bar{\sigma}).
 \end{aligned} \tag{2.18}$$

The holomorphic part of the vielbein is given by

$$E_A{}^\pi = \begin{pmatrix} (1 + (D_+\sigma)\theta^+ + (\bar{D}_+\bar{\sigma})\bar{\theta}^+)e^{\sigma+\bar{\sigma}} & 2e^{\sigma+\bar{\sigma}}(\bar{D}_+\bar{\sigma}) & 2e^{\sigma+\bar{\sigma}}(D_+\sigma) \\ \frac{1}{2}e^{\bar{\sigma}}\bar{\theta}^+ & e^{\bar{\sigma}} & 0 \\ \frac{1}{2}e^{\sigma}\theta^+ & 0 & e^{\sigma} \end{pmatrix} \tag{2.19}$$

and the inverse

$$E_\pi{}^A = \begin{pmatrix} e^{-(\sigma+\bar{\sigma})} & -2(\bar{D}_+\bar{\sigma})e^{-\bar{\sigma}} & -2(D_+\sigma)e^{-\sigma} \\ -\frac{\bar{\theta}^+}{2}e^{-(\sigma+\bar{\sigma})} & e^{-\bar{\sigma}}(1 + \bar{\theta}^+(\bar{D}_+\bar{\sigma})) & e^{-\sigma}\theta^+(\bar{D}_+\bar{\sigma}) \\ -\frac{\theta^+}{2}e^{-(\sigma+\bar{\sigma})} & e^{-\bar{\sigma}}\bar{\theta}^+(D_+\sigma) & e^{-\sigma}(1 + \theta^+(D_+\sigma)) \end{pmatrix}, \tag{2.20}$$

with analogous expressions for the antiholomorphic part. Finally, the supercurvature is given by

$$R = 4ie^{2\sigma}(\bar{D}_+\bar{D}_-\bar{\sigma}), \tag{2.21}$$

$$\bar{R} = 4ie^{2\bar{\sigma}}(D_+D_-\sigma), \tag{2.22}$$

which are thus respectively a chiral and an anti-chiral superfield.

3 $\mathcal{N} = (2, 2)$ Jackiw-Teitelboim action

In the following, we want to consider the $\mathcal{N} = (2, 2)$ generalisation to JT gravity. First, we consider the action for pure supergravity supplemented with a Gibbons-Hawking-York term,

$$S = -\frac{\Phi_0}{16\pi G_N} \left[\int_{\mathcal{M}} d^2z d^2\theta \mathcal{E}^{-1} R + \int_{\mathcal{M}} d^2z d^2\bar{\theta} \bar{\mathcal{E}}^{-1} \bar{R} + 2 \int_{\partial\mathcal{M}} du d\vartheta K + 2 \int_{\partial\mathcal{M}} du d\bar{\vartheta} \bar{K} \right]. \tag{3.1}$$

Here, respectively R and $\bar{R}$ are the chiral and anti-chiral curvature superfields (2.21), (2.22), $\mathcal{E}$ and $\bar{\mathcal{E}}$ are the chiral and anti-chiral density which are needed to get well-defined (anti-)chiral integrals. We comment on the projection to x -space at the end of this section. Furthermore, Φ_0 is a constant which can be interpreted as a constant dilaton, K is the extrinsic curvature associated to the chiral bulk supercurvature and $\bar{K}$ is the anti-chiral extrinsic curvature coming from the anti-chiral bulk supercurvature. These two extrinsic

curvatures can be calculated from the $\mathcal{N} = (1, 1)$ expressions [57]

$$K = \frac{T^A \bar{D}_T n_A}{T^A T_A}, \quad (3.2)$$

$$\bar{K} = \frac{T^A D_T n_A}{T^A T_A}, \quad (3.3)$$

where $A = l, \bar{l}$. Furthermore, T is the tangent vector along the boundary, n the normal vector satisfying $T^A n_A = 0$ and $n^A n_A = 1$ and the derivatives $\bar{D}_T$ and D_T are defined as

$$D_T n_A = D n_A + \left(D z^\xi \Omega_\xi \mathcal{J} \right) n_A, \quad (3.4)$$

$$\bar{D}_T n_A = \bar{D} n_A + \left(\bar{D} z^\xi \Omega_\xi \mathcal{J} \right) n_A. \quad (3.5)$$

The supersymmetric generalisations (3.2) and (3.3) of the extrinsic curvature are chosen such that transformations of the derivatives D and $\bar{D}$, which replace the derivative ∂_u in the bosonic extrinsic curvature, cancel the Berezinian of the (anti-) chiral superspace measure (cf. [28]). In the following, it will be useful to express K and $\bar{K}$ as a more general boundary superfield $\mathcal{K}$ in order to couple the extrinsic curvature to superfields without definite chirality (as e.g. the dilaton at the boundary). We therefore define the overall extrinsic curvature $\mathcal{K}$ through the condition

$$\int_{\partial\mathcal{M}} du d\vartheta d\bar{\vartheta} \mathcal{K} \stackrel{!}{=} \int_{\partial\mathcal{M}} du d\vartheta K + \int_{\partial\mathcal{M}} du d\bar{\vartheta} \bar{K}. \quad (3.6)$$

One could also try to directly find an expression for $\mathcal{K}$ by searching for a generalisation of the derivative ∂_u appearing in the bosonic extrinsic curvature which cancels the Berezinian of the full $d = 1$ superspace measure upon transformations. However, there is no obvious expression involving the covariant derivatives D and $\bar{D}$ and generalising ∂_u that satisfies this condition since the Berezinian of the full superspace measure equals one. Therefore, we have to take the detour and calculate $\mathcal{K}$ via K and $\bar{K}$.

Now, we can use $\mathcal{K}$ to define the supersymmetric $\mathcal{N} = (2, 2)$ generalisation of the JT action. This action reads

$$S = -\frac{1}{16\pi G_N} \left[\int_{\mathcal{M}} d^2 z d^2 \theta \mathcal{E}^{-1} \Phi(R - \alpha) + \int_{\mathcal{M}} d^2 z d^2 \bar{\theta} \mathcal{E}^{-1} \bar{\Phi}(\bar{R} - \alpha) + 2 \int_{\partial\mathcal{M}} du d\vartheta d\bar{\vartheta} (\Phi_b + \bar{\Phi}_b) \mathcal{K} \right], \quad (3.7)$$

where Φ and $\bar{\Phi}$ are respectively the chiral and anti-chiral dilaton superfields, which serve as Lagrange multipliers imposing the constraints $R = \bar{R} = \alpha$. As we will soon see, the choice $\alpha = -2$ corresponds to an AdS background which we will use from now on. Moreover, Φ_b and $\bar{\Phi}_b$ are the respective boundary values of the chiral and anti-chiral dilatons. Imposing the supercurvature constraints yields the effective action for the boundary degrees of freedom

$$S_{\text{eff}} = -\frac{1}{8\pi G_N} \int_{\partial\mathcal{M}} du d\vartheta d\bar{\vartheta} (\Phi_b + \bar{\Phi}_b) \mathcal{K}. \quad (3.8)$$

One can check that the action above is indeed a supersymmetric generalisation of the bosonic JT action by considering the action in x -space, i.e. performing the integrals over the Grassmann variables. To do this, one has to know how to deal with the chiral density. The procedure to find the expression for the chiral density is explained in detail in [62]: if a Lagrangian $\mathcal{L}$ is considered, the chiral projection has to take the form

$$\int d^2z d^2\theta d^2\bar{\theta} E^{-1} \mathcal{L} = \int d^2z d^2\theta \mathcal{E}^{-1} \bar{\nabla}^2 \mathcal{L}|_{\bar{\theta}=0} \quad (3.9)$$

$$= \int d^2z e^{-1} [\nabla^2 + X^+ \nabla_+ + X^- \nabla_- + Y] \bar{\nabla}^2 \mathcal{L}|_{\theta=\bar{\theta}=0}, \quad (3.10)$$

where $e = \det(e_a{}^m)$ and the coefficients X^+ , X^- and Y have to be determined. They can be found from the requirement that the transformation of the full superspace integral to the x -space integral should not depend on whether one has a chiral integral or an anti-chiral integral in the intermediate step. As in [62] this condition can be implemented for e.g. the kinetic term of a chiral field with $\mathcal{L} = \bar{\Phi}\Phi$ by choosing X^+ , X^- and Y s.t. the resulting x -space integral is symmetric in barred and unbarred quantities.

The calculation is tedious and since we are interested in a classical background solution, we set the gravitini to zero again. In that case, we obtain

$$X^+ = X^- = 0, \quad Y = \frac{i}{2} \bar{R}| = \frac{i}{2} \bar{S}, \quad (3.11)$$

$$\bar{X}^+ = \bar{X}^- = 0, \quad \bar{Y} = \frac{i}{2} R| = \frac{i}{2} S, \quad (3.12)$$

where $\bar{X}^+$, $\bar{X}^-$ and $\bar{Y}$ are the corresponding quantities for the anti-chiral density projection formula.

Having the explicit formula for the (anti-)chiral projections, we can now proceed to find the x -space action of our particular supergravity setup. Let us for the moment only consider the bulk part of the action. We start with the supersymmetric Einstein-Hilbert action which now reads:

$$\begin{aligned} S_{EH} &= -\frac{\Phi_0}{16\pi G_N} \left[\int_{\mathcal{M}} d^2z d^2\theta \mathcal{E}^{-1} R + \int_{\mathcal{M}} d^2z d^2\bar{\theta} \bar{\mathcal{E}}^{-1} \bar{R} \right] \\ &= -\frac{\Phi_0}{16\pi G_N} \left[\int_{\mathcal{M}} d^2z e^{-1} \left(\nabla^2 + \frac{i}{2} \bar{S} \right) R| + \int_{\mathcal{M}} d^2z e^{-1} \left(\bar{\nabla}^2 + \frac{i}{2} S \right) \bar{R}| \right] \\ &= -\frac{\Phi_0}{16\pi G_N} \left[\int_{\mathcal{M}} d^2z e^{-1} (-i(\mathcal{R} + i\mathcal{F})) + \int_{\mathcal{M}} d^2z e^{-1} (-i(\mathcal{R} - i\mathcal{F})) \right] \\ &= +\frac{i\Phi_0}{8\pi G_N} \int_{\mathcal{M}} d^2z e^{-1} \mathcal{R}, \end{aligned} \quad (3.13)$$

where we made use of (2.13), (2.14). Thus, this part of the action, together with the extrinsic curvature term just gives the Euler characteristic of $\mathcal{M}$ times an overall prefactor.

The second part of the bulk action is given by the JT term, which reads (using the

chiral projection formula)

$$\begin{aligned}
 S_{JT} &= -\frac{1}{16\pi G_N} \left[\int_{\mathcal{M}} d^2z d^2\theta \mathcal{E}^{-1} \Phi(R+2) + \int_{\mathcal{M}} d^2z d^2\bar{\theta} \bar{\mathcal{E}}^{-1} \bar{\Phi}(\bar{R}+2) \right] \\
 &= -\frac{1}{16\pi G_N} \left[\int_{\mathcal{M}} d^2z e^{-1} \left(\nabla^2 + \frac{i}{2} \bar{S} \right) \Phi(R+2) + \int_{\mathcal{M}} d^2z e^{-1} \left(\bar{\nabla}^2 + \frac{i}{2} S \right) \bar{\Phi}(\bar{R}+2) \right] \\
 &= \frac{i}{16\pi G_N} \int_{\mathcal{M}} d^2z e^{-1} [\varphi(\mathcal{R}+i\mathcal{F}) + \bar{\varphi}(\mathcal{R}-i\mathcal{F}) - \bar{S}\varphi - S\bar{\varphi} + iB(S+2) + i\bar{B}(\bar{S}+2)] ,
 \end{aligned} \tag{3.14}$$

where we used (2.13) and (2.14) as well as the component expansion of the dilaton superfield

$$\Phi = \varphi + \theta^\alpha \lambda_\alpha + \theta^+ \theta^- B, \quad \text{and} \quad \bar{\Phi} = \bar{\varphi} + \bar{\theta}^\alpha \bar{\lambda}_\alpha + \bar{\theta}^+ \bar{\theta}^- \bar{B}. \tag{3.15}$$

If we consider the variations of this JT action w.r.t. the auxiliary supergravity fields S and $\bar{S}$, we get the relations

$$B = i\bar{\varphi} \quad \text{and} \quad \bar{B} = i\varphi. \tag{3.16}$$

Further variations w.r.t. the auxiliary dilaton fields B and $\bar{B}$ yield the bosonic JT action

$$S_{JT} = \frac{i}{16\pi G_N} \int_{\mathcal{M}} d^2z e^{-1} [\varphi(\mathcal{R} + i\mathcal{F} + 2) + \bar{\varphi}(\mathcal{R} - i\mathcal{F} + 2)], \tag{3.17}$$

which upon variation w.r.t. φ and $\bar{\varphi}$ gives indeed an AdS background with vanishing field strength $\mathcal{F}$.

Finally, variations w.r.t. the vielbein give an energy momentum tensor similar to the bosonic case in [48]. Thus, one possible solution for φ and $\bar{\varphi}$ is given by the dilaton solution found in that reference. This implies in particular that $\varphi = \bar{\varphi}$.

4 Determination of the superconformal factor

A crucial step for calculating the extrinsic curvature is to find an expression for the (anti-) chiral superconformal field σ ($\bar{\sigma}$), which can be done in two different ways: on the one hand, one can consider (2.21), (2.22) and solve for σ and $\bar{\sigma}$ using the constraint $R = \bar{R} = -2$. On the other hand one can calculate σ and $\bar{\sigma}$ using the Killing spinors of AdS_2 . Since the final result for the extrinsic curvature and thus the effective boundary action crucially depends on the result for σ and $\bar{\sigma}$, we will present both ways in order to check our findings.

First, we solve the supercurvature constraints (2.21) and (2.22) for the superconformal factors σ and $\bar{\sigma}$. Since σ is a chiral superfield it can be written in the chiral basis $z_c = z + \frac{1}{2}\theta^+\bar{\theta}^+$ and $\bar{z}_c = \bar{z} + \frac{1}{2}\theta^-\bar{\theta}^-$ as

$$\sigma = \phi(z_c, \bar{z}_c) + \theta^+ \theta^- w(z, \bar{z}). \tag{4.1}$$

Here, ϕ and w are functions of the superspace variables which we will determine later on. Accordingly, the anti-chiral field $\bar{\sigma}$ can be written in terms of the anti-chiral basis $z_{ac} = z - \frac{1}{2}\theta^+\bar{\theta}^+$ and $\bar{z}_{ac} = \bar{z} - \frac{1}{2}\theta^-\bar{\theta}^-$ as

$$\bar{\sigma} = \bar{\phi}(z_{ac}, \bar{z}_{ac}) + \bar{\theta}^+ \bar{\theta}^- \bar{w}(z, \bar{z}). \tag{4.2}$$

According to (2.22) $\bar{R} = -2$ yields

$$w = \frac{-i}{2} e^{-2\bar{\phi}}, \quad (4.3)$$

$$0 = 2w\bar{w} - \partial_z \partial_{\bar{z}} \phi. \quad (4.4)$$

The second equation has the form of a Liouville equation. Since we are interested in an AdS background geometry, we impose

$$\bar{\phi} = \phi = -\frac{1}{2} \log \left(\frac{1}{2y} \right), \quad (4.5)$$

where $z = t + iy$. With this input, (4.4) can be solved by setting

$$\bar{w} = \frac{-i}{2} e^{-2\phi}. \quad (4.6)$$

Note that this implies $R = -2$, in accordance with the remaining supercurvature constraint.

After expanding the chiral basis, the superconformal factors will be given by

$$\sigma = -\frac{1}{2} \log \left(\frac{1}{\beta y} \right) + \frac{i}{8y} \theta^+ \bar{\theta}^+ - \frac{i}{8y} \theta^- \bar{\theta}^- - \frac{i}{4y} \theta^+ \theta^- - \frac{1}{32y^2} \theta^- \bar{\theta}^- \theta^+ \bar{\theta}^+, \quad (4.7)$$

$$\bar{\sigma} = -\frac{1}{2} \log \left(\frac{1}{\beta y} \right) - \frac{i}{8y} \theta^+ \bar{\theta}^+ + \frac{i}{8y} \theta^- \bar{\theta}^- - \frac{i}{4y} \bar{\theta}^+ \bar{\theta}^- - \frac{1}{32y^2} \theta^- \bar{\theta}^- \theta^+ \bar{\theta}^+. \quad (4.8)$$

We see that σ and $\bar{\sigma}$ are not complex conjugates of each other, thereby making AdS₂ a non-unitary background. For our further deliberations we should corroborate the result for the superconformal factors. More precisely, the result of non-unitarity should be confirmed by other means. We opt to perform a short classification of $\mathcal{N} = (2, 2)$ supersymmetric backgrounds by calculation of Killing spinors. The following analysis closely follows the steps given in appendix D of [64]. Since we chose different conventions for our superspace it is useful to re-derive their results for our setup.

Recall, that due to conformal flatness of two-dimensional supergravity, the background geometry is entirely encoded in the conformal factors $e^{-2\sigma}$ and $e^{-2\bar{\sigma}}$. Thus, the relevant fields that we have to consider in order to determine the Killing spinors ϵ and $\bar{\epsilon}$ of supersymmetry variations are just the chiral σ field and the anti-chiral $\bar{\sigma}$ field. To obtain the background geometry, the fermionic components of σ and $\bar{\sigma}$ are set to zero.

As in [64] the standard restriction for supersymmetry can be written as

$$\partial_z \epsilon^- = \partial_z \bar{\epsilon}^- = 0, \quad \text{and} \quad \partial_{\bar{z}} \epsilon^+ = \partial_{\bar{z}} \bar{\epsilon}^+ = 0. \quad (4.9)$$

Further restrictions on the Killing spinors come from the requirement, that the fermionic components of the conformal factors $e^{-2\sigma}$ and $e^{-2\bar{\sigma}}$ remain zero under local supersymmetry transformations. In the following, we will work in the chiral basis as introduced above (4.1). With our conventions these conditions can be written as

$$\partial_{z_c} \left(\bar{\epsilon}^+ e^{-2\phi} \right) - 2\epsilon^- w e^{-2\phi} = 0, \quad \partial_{\bar{z}_c} \left(\bar{\epsilon}^- e^{-2\phi} \right) + 2\epsilon^+ w e^{-2\phi} = 0, \quad (4.10)$$

$$\partial_{z_{ac}} \left(\epsilon^+ e^{-2\bar{\phi}} \right) - 2\bar{\epsilon}^- \bar{w} e^{-2\bar{\phi}} = 0, \quad \partial_{\bar{z}_{ac}} \left(\epsilon^- e^{-2\bar{\phi}} \right) + 2\bar{\epsilon}^+ \bar{w} e^{-2\bar{\phi}} = 0, \quad (4.11)$$

where we inserted $\sigma = \phi + \theta^+ \theta^- w$ and $\bar{\sigma} = \bar{\phi} + \bar{\theta}^+ \bar{\theta}^- \bar{w}$. The classification of backgrounds preserving different numbers of supercharges can now be carried out along the lines of [64].

For a background preserving one supercharge with a particular $U_A(1)$ charge, we can e.g. choose the Killing spinor $(\epsilon_1^+, \bar{\epsilon}_1^-)$ to be non-zero with the other Killing spinor components zero. Solving (4.10), (4.11) algebraically, we get

$$w = \frac{1}{2} \frac{\bar{\epsilon}_1^-}{\epsilon_1^+} \partial_{\bar{z}_c} (2\phi - \log \bar{\epsilon}_1^-) , \quad (4.12)$$

$$\bar{w} = -\frac{1}{2} \frac{\epsilon_1^+}{\bar{\epsilon}_1^-} \partial_{z_{ac}} (2\bar{\phi} - \log \epsilon_1^+) . \quad (4.13)$$

If the background should also preserve a second supercharge of the opposite $U_A(1)$ charge, there should also exist a second non-zero Killing spinor $(\epsilon_2^-, \bar{\epsilon}_2^+)$. Solving (4.10), (4.11) with this Killing spinor yields results similar to (4.12), (4.13). Consistency of the two solutions requires

$$(\bar{\epsilon}_1^- \epsilon_2^- \partial_{\bar{z}_{ac}} + \bar{\epsilon}_2^+ \epsilon_1^+ \partial_{z_{ac}}) (2\bar{\phi} + \log \epsilon_1^+ \epsilon_2^-) = 0 , \quad (4.14)$$

with a similar expression for z_c and ϕ . Thus, $\bar{\phi}$ is invariant under the vector $v = \bar{\epsilon}_1^- \epsilon_2^- \partial_{\bar{z}_{ac}} + \bar{\epsilon}_2^+ \epsilon_1^+ \partial_{z_{ac}}$ up to a superconformal transformation.

As shown in e.g. [63], maximally four supercharges are preserved if and only if the background space is maximally symmetric and the $U_A(1)$ gauge field has zero field strength. Thus, with (4.12), (4.13) the w and $\bar{w}$ fields can be expressed in terms of the bosonic conformal factor ϕ . Since we are interested in an AdS background, we know that $\phi = -\frac{1}{2} \log \left(\frac{1}{2y_c} \right)$ and $\bar{\phi} = -\frac{1}{2} \log \left(\frac{1}{2y_{ac}} \right)$. In that case indeed a set of four Killing spinors satisfying (4.14) can be found. These are

$$\zeta \equiv \begin{pmatrix} \epsilon^+ \\ \bar{\epsilon}^- \end{pmatrix} = \frac{1}{2} \begin{pmatrix} -1 \\ 1 \end{pmatrix} , \quad \bar{\zeta} \equiv \begin{pmatrix} \epsilon^- \\ \bar{\epsilon}^+ \end{pmatrix} = \frac{1}{2} \begin{pmatrix} -1 \\ 1 \end{pmatrix} , \quad (4.15)$$

and

$$\eta \equiv \begin{pmatrix} \epsilon^+ \\ \bar{\epsilon}^- \end{pmatrix} = \frac{1}{2} \begin{pmatrix} -z \\ \bar{z} \end{pmatrix} , \quad \bar{\eta} \equiv \begin{pmatrix} \epsilon^- \\ \bar{\epsilon}^+ \end{pmatrix} = \frac{1}{2} \begin{pmatrix} -\bar{z} \\ z \end{pmatrix} , \quad (4.16)$$

where we used a bar to distinguish Killing spinors with opposite $U_A(1)$ charge. Indeed, with these four Killing spinors, one can now calculate the three Killing vectors $\zeta \gamma^\mu \bar{\zeta} \partial_\mu$, $\eta \gamma^\mu \bar{\eta} \partial_\mu$ and $\eta \gamma^\mu \bar{\zeta} \partial_\mu$ to be

$$L_{-1} \equiv \zeta \gamma^\mu \bar{\zeta} \partial_\mu = -\frac{1}{2} (\partial_z + \partial_{\bar{z}}) , \quad (4.17)$$

$$L_0 \equiv \eta \gamma^\mu \bar{\eta} \partial_\mu = -\frac{1}{2} (z \partial_z + \bar{z} \partial_{\bar{z}}) , \quad (4.18)$$

$$L_1 \equiv \eta \gamma^\mu \bar{\zeta} \partial_\mu = -\frac{1}{2} (z^2 \partial_z + \bar{z}^2 \partial_{\bar{z}}) , \quad (4.19)$$

which are precisely the Killing vectors of AdS_2 . At the boundary ($y \rightarrow 0$) they correctly reduce to the global conformal transformations

$$L_0 \rightarrow -t\partial_t, \quad L_{-1} \rightarrow -\partial_t, \quad L_1 \rightarrow -t^2\partial_t. \quad (4.20)$$

Now one can take any Killing spinor out of (4.15), (4.16) to calculate w and $\bar{w}$. The results for σ and $\bar{\sigma}$ are

$$\sigma = -\frac{1}{2} \log \left(\frac{1}{2y_c} \right) - \frac{i}{4y_c} \theta^+ \theta^-, \quad (4.21)$$

$$\bar{\sigma} = -\frac{1}{2} \log \left(\frac{1}{2y_{ac}} \right) - \frac{i}{4y_{ac}} \bar{\theta}^+ \bar{\theta}^-. \quad (4.22)$$

These results obtained by considering the Killing spinors perfectly coincide with our solution for σ and $\bar{\sigma}$ (4.7), (4.8) as obtained from the requirement $\bar{R} = R = -2$. In particular, this shows again that our background is non-unitary since σ and $\bar{\sigma}$ are not complex conjugates of each other.

5 Effective action: appearance of the super-Schwarzian

Only boundary curves of constant arc length are considered in the calculation of the effective action [48, 57],

$$\frac{du^2 + \bar{\vartheta} d\vartheta du + \vartheta d\bar{\vartheta} du + \frac{1}{2} \vartheta \bar{\vartheta} d\vartheta d\bar{\vartheta}}{4\epsilon^2} = \left(dz^\xi E_\xi^l dz^\pi E_\pi^{\bar{l}} \right) \Big|_{\text{pull-back}}. \quad (5.1)$$

This results in the constraints

$$Dz = \frac{1}{2} (\bar{\theta}^+ (D\theta^+) + \theta^+ (D\bar{\theta}^+)) , \quad \bar{D}z = \frac{1}{2} (\theta^+ (\bar{D}\bar{\theta}^+) + \bar{\theta}^+ (\bar{D}\theta^+)) , \quad (5.2)$$

$$D\bar{z} = \frac{1}{2} (\theta^- (D\bar{\theta}^-) + \bar{\theta}^- (D\theta^-)) , \quad \bar{D}\bar{z} = \frac{1}{2} (\bar{\theta}^- (\bar{D}\theta^-) + \theta^- (\bar{D}\bar{\theta}^-)) , \quad (5.3)$$

where we defined one dimensional supercovariant derivatives as in [28] (up to factors of one half due to differences in conventions),

$$D = \frac{\partial}{\partial \vartheta} + \frac{\bar{\vartheta}}{2} \frac{\partial}{\partial u} , \quad \bar{D} = \frac{\partial}{\partial \bar{\vartheta}} + \frac{\vartheta}{2} \frac{\partial}{\partial u} \quad (5.4)$$

In addition to (5.2) and (5.3), we impose the following chirality conditions

$$D\theta^- = D\bar{\theta}^+ = \bar{D}\theta^+ = \bar{D}\bar{\theta}^- = 0. \quad (5.5)$$

Equations (5.2), (5.3) and (5.5) are equivalent to the $\mathcal{N} = 2$ superconformal transformations of [28]. The bulk variables correspond to super-reparametrisations of the boundary. Furthermore, the conformal factor has to satisfy

$$e^{2(\sigma+\bar{\sigma})} = 4\epsilon^2 [(D\theta^+)(D\bar{\theta}^-)(\bar{D}\theta^-)(\bar{D}\bar{\theta}^+)] . \quad (5.6)$$

Together with (4.7), (4.8) this leads to

$$y = \text{Im } z = \epsilon \left[(D\theta^+)(D\bar{\theta}^-)(\bar{D}\theta^-)(\bar{D}\bar{\theta}^+) \right]^{1/2} + \frac{i}{4} (\theta^+ \theta^- + \bar{\theta}^+ \bar{\theta}^-) . \quad (5.7)$$

Now, we can calculate the chiral and anti-chiral part of the extrinsic curvature given in (3.2) and (3.3). The tangent vector for the boundary can be evaluated using

$$T^l = (\partial_u z^\xi) E_\xi^l , \quad (5.8)$$

$$T^{\bar{l}} = (\partial_u z^\xi) E_\xi^{\bar{l}} , \quad (5.9)$$

leading to

$$T^l = e^{-(\sigma+\bar{\sigma})} \left[(D\theta^+)(\bar{D}\bar{\theta}^+) \right] , \quad (5.10)$$

$$T^{\bar{l}} = e^{-(\sigma+\bar{\sigma})} \left[(D\bar{\theta}^-)(\bar{D}\theta^-) \right] , \quad (5.11)$$

and

$$n_l = -\frac{i}{2} \left(\frac{(D\bar{\theta}^-)(\bar{D}\theta^-)}{(D\theta^+)(\bar{D}\bar{\theta}^+)} \right)^{1/2} , \quad (5.12)$$

$$n_{\bar{l}} = +\frac{i}{2} \left(\frac{(D\theta^+)(\bar{D}\bar{\theta}^+)}{(D\bar{\theta}^-)(\bar{D}\theta^-)} \right)^{1/2} . \quad (5.13)$$

Hence, the contribution to the anti-chiral extrinsic supercurvature $\bar{K}$ which does not include the connection is given by

$$\frac{T^l Dn_l + T^{\bar{l}} Dn_{\bar{l}}}{T^2} = i\epsilon \left[\frac{(\bar{\theta}^+)' }{(\bar{D}\bar{\theta}^+)} - \frac{(\theta^-)' }{(\bar{D}\theta^-)} \right] , \quad (5.14)$$

with a similar expression for the chiral extrinsic supercurvature K . Here, the prime indicates derivatives with respect to u . For the part of $\bar{K}$ containing the connection, we first observe that the Lorentz generators applied to n_l and $n_{\bar{l}}$ give

$$[\mathcal{J}, n_{\bar{l}}] = in_{\bar{l}} , \quad [\mathcal{J}, n_l] = -in_l . \quad (5.15)$$

Thus, the contribution to $\bar{K}$ containing the connection part is given by

$$\begin{aligned} & \frac{1}{T^2} T^A (Dz^\xi \Omega_\xi) J n_A \\ &= -2\epsilon (Dz^\xi \Omega_\xi) \end{aligned} \quad (5.16)$$

$$= \frac{1}{2 \left[(D\theta^+)(D\bar{\theta}^-)(\bar{D}\theta^-)(\bar{D}\bar{\theta}^+) \right]^{1/2}} \left[(D\theta^+)(\bar{\theta}^+ - \theta^-) + (D\bar{\theta}^-)(\theta^- - \bar{\theta}^+) \right] . \quad (5.17)$$

Having the general expression for the extrinsic curvature, we want to make contact to the boundary theory. The $\mathcal{N} = (2, 2)$ supersymmetry of the bulk reduces to $\mathcal{N} = (1, 1)$ supersymmetry on the boundary. We therefore need an expression for the bulk variables

at the boundary in terms of the boundary degrees of freedom. To zeroth order the solution to (5.1) after imposing (5.2), (5.3) reads

$$\theta^+ = \bar{\theta}^- = \xi, \quad \theta^- = \bar{\theta}^+ = \bar{\xi}, \quad \text{Im} z = \epsilon(D\xi)(\bar{D}\bar{\xi}). \quad (5.18)$$

In that case, (5.5) reduces to

$$D\bar{\xi} = \bar{D}\xi = 0. \quad (5.19)$$

We also need the corrections in ϵ to these solutions. We choose the ansatz

$$\theta^+ = \xi - i\epsilon\rho, \quad \bar{\theta}^+ = \bar{\xi} + i\epsilon\bar{\rho}, \quad (5.20)$$

$$\theta^- = \bar{\xi} - i\epsilon\bar{\rho}, \quad \bar{\theta}^- = \xi + i\epsilon\rho. \quad (5.21)$$

With this ansatz, (5.2), (5.3) can be solved by

$$\rho = -\xi' \quad \text{and} \quad \bar{\rho} = \bar{\xi}'. \quad (5.22)$$

Thus, the boundary solution of $\theta^+, \theta^-, \bar{\theta}^+, \bar{\theta}^-$ has the form of a Taylor expansion of e.g. $\theta^+(u + i\epsilon, \vartheta, \bar{\vartheta})$ around $\epsilon = 0$, with $\theta^+(u, \vartheta, \bar{\vartheta}) = \xi$ and similarly for the other Grassmann coordinates.

With this solution, the extrinsic curvature is given by

$$\bar{K} = -2\epsilon^2 \left[\frac{\bar{\xi}''}{\bar{D}\bar{\xi}} - \frac{\bar{\xi}'(\bar{D}\bar{\xi}')}{(\bar{D}\bar{\xi})^2} + \left(\frac{(D\xi')\bar{\xi}'}{(D\xi)(\bar{D}\bar{\xi})} \right) \right], \quad (5.23)$$

$$K = -2\epsilon^2 \left[\frac{\xi''}{D\xi} - \frac{\xi'(D\xi')}{(D\xi)^2} + \left(\frac{(\bar{D}\bar{\xi}')\xi'}{(D\xi)(\bar{D}\bar{\xi})} \right) \right]. \quad (5.24)$$

The next step is to find the overall extrinsic curvature $\mathcal{K}$ as defined in (3.6). The following identity proves useful

$$\int du d\vartheta d\bar{\vartheta} f(u, \vartheta, \bar{\vartheta}) = \int du_c d\vartheta (\bar{D}f) \left(u + \frac{\vartheta\bar{\vartheta}}{2}, \vartheta \right) = - \int du_{ac} d\bar{\vartheta} (Df) \left(u - \frac{\vartheta\bar{\vartheta}}{2}, \bar{\vartheta} \right), \quad (5.25)$$

where $du \simeq du_{c/ac} = d(u \pm \frac{1}{2}\vartheta\bar{\vartheta})$ denote chiral and anti-chiral measures. To check this formula it is best to expand the superfield f into components

$$f(u, \vartheta, \bar{\vartheta}) = g(u) + \vartheta\bar{\zeta}(u) + \bar{\vartheta}\zeta(u) + \vartheta\bar{\vartheta}h(u). \quad (5.26)$$

Then one gets

$$\int du d\vartheta d\bar{\vartheta} f(u, \vartheta, \bar{\vartheta}) = - \int du h. \quad (5.27)$$

whereas for the second integral one gets

$$\int du_c d\vartheta (\bar{D}f) \left(u + \frac{\vartheta\bar{\vartheta}}{2}, \vartheta \right) = \int du_c d\vartheta \left[\zeta(u_c) + \vartheta \left(\frac{g'(u_c)}{2} - h(u_c) \right) \right] = - \int du h. \quad (5.28)$$

In the second step the first term has been killed by the $d\vartheta$ integral and the second term gave rise to a du_c integral over a derivative, leaving only the third contribution in agreement with (5.27). The other equality in (5.25) can be seen analogously.

Note that $\bar{K}$ and K in (5.23), (5.24) can be expressed as derivatives,

$$\bar{K} = -2\epsilon^2 D \left(\frac{(\bar{D}\bar{\xi}')}{\bar{D}\bar{\xi}} - \frac{\xi'\bar{\xi}'}{(D\xi)(\bar{D}\bar{\xi})} \right), \quad (5.29)$$

$$K = -2\epsilon^2 \bar{D} \left(\frac{(D\xi')}{D\xi} + \frac{\xi'\bar{\xi}'}{(D\xi)(\bar{D}\bar{\xi})} \right). \quad (5.30)$$

This result can now be plugged into the Gibbons-Hawking-York term in (3.1):

$$S_{\text{GH}} = -\frac{1}{8\pi G_N \epsilon^2} \int_{\partial\mathcal{M}} dud\bar{v} \left[-2\epsilon^2 D \left(\frac{(\bar{D}\bar{\xi}')}{\bar{D}\bar{\xi}} - \frac{\xi'\bar{\xi}'}{(D\xi)(\bar{D}\bar{\xi})} \right) \right] \\ - \frac{1}{8\pi G_N \epsilon^2} \int_{\partial\mathcal{M}} dudv \left[-2\epsilon^2 \bar{D} \left(\frac{(D\xi')}{D\xi} + \frac{\xi'\bar{\xi}'}{(D\xi)(\bar{D}\bar{\xi})} \right) \right], \quad (5.31)$$

where the $1/\epsilon^2$ factor arises due to the ϵ factor in the flat space vielbein (5.1) [57]. With (5.25) each integral in the above expression can be expressed as an integral over the total superspace. Comparing with (3.6) yields

$$\mathcal{K} = 2\epsilon^2 \text{Schw}(t, \xi, \bar{\xi}; u, v, \bar{v}). \quad (5.32)$$

where

$$\text{Schw}(t, \xi, \bar{\xi}; u, v, \bar{v}) = \frac{(\bar{D}\bar{\xi}')}{\bar{D}\bar{\xi}} - \frac{(D\xi')}{D\xi} - 2 \frac{\xi'\bar{\xi}'}{(D\xi)(\bar{D}\bar{\xi})} \quad (5.33)$$

denotes the super-Schwarzian. With this expression for the extrinsic curvature, we get for the effective boundary action in (3.8)

$$S_{\text{eff}} = -\frac{1}{4\pi G_N} \int_{\partial\mathcal{M}} dudv d\bar{v} (\Phi_b + \bar{\Phi}_b) \text{Schw}(t, \xi, \bar{\xi}; u, v, \bar{v}), \quad (5.34)$$

which can be further simplified by noting that only the leading components φ of the dilaton supermultiplets contribute at the boundary, since for the two θ components of the dilaton, at the boundary we have at zeroth order in ϵ the relation

$$(\Phi + \bar{\Phi})_b \supset i(\varphi\theta^+\theta^- + \varphi\bar{\theta}^+\bar{\theta}^-)_b = i\varphi_b(\xi\bar{\xi} + \bar{\xi}\xi) = 0, \quad (5.35)$$

where φ_b is the value of the leading component at the boundary and we used that the chiral and the anti-chiral dilaton superfields have the same leading component on-shell

$$S_{\text{eff}} = -\frac{1}{2\pi G_N} \int_{\partial\mathcal{M}} dudv d\bar{v} \varphi_b \text{Schw}(t, \xi, \bar{\xi}; u, v, \bar{v}). \quad (5.36)$$

We close this section by briefly commenting on the physical interpretation of the action considered here (cf. e.g. [28, 45] for the bosonic case). The entire action reads

$$S = -\frac{\Phi_0}{16\pi G_N} \left[\int_{\mathcal{M}} d^2z d^2\theta \mathcal{E}^{-1} R + \int_{\mathcal{M}} d^2z d^2\bar{\theta} \bar{\mathcal{E}}^{-1} \bar{R} + 2 \int_{\partial\mathcal{M}} dudv d\bar{v} \mathcal{K} \right] \\ - \frac{\bar{\Phi}_0}{16\pi G_N} \left[\int_{\mathcal{M}} d^2z d^2\theta \mathcal{E}^{-1} \Phi(R+2) + \int_{\mathcal{M}} d^2z d^2\bar{\theta} \bar{\mathcal{E}}^{-1} \bar{\Phi}(\bar{R}+2) \right. \\ \left. + 2 \int_{\partial\mathcal{M}} dudv d\bar{v} (\Phi_b + \bar{\Phi}_b) \mathcal{K} \right]. \quad (5.37)$$

The manifold $\mathcal{M}$ is obtained by cutting out a line given by coordinates $t(u, \vartheta, \bar{\vartheta})$, $y(u, \vartheta, \bar{\vartheta})$, $\theta^+(u, \vartheta, \bar{\vartheta})$, $\theta^-(u, \vartheta, \bar{\vartheta})$, $\bar{\theta}^+(u, \vartheta, \bar{\vartheta})$ and $\bar{\theta}^-(u, \vartheta, \bar{\vartheta})$ corresponding to the boundary $\partial\mathcal{M}$. The parameters u , ϑ , $\bar{\vartheta}$ are the coordinates of the one-dimensional $\mathcal{N} = (1, 1)$ superspace at this boundary. Different boundaries are related via superreparametrisations of the one-dimensional superspace. We saw earlier that the first line of (5.37) is just topological and corresponds to the Euler characteristic of $\mathcal{M}$ which is not altered by superreparametrisations. Thus, we have a large symmetry group consisting of all superreparametrisations that do not violate the chirality constraints on the superfields in (5.2)–(5.5). We derived in (5.20)–(5.22), under the assumption of the chirality constraints, that our reparametrisations satisfy

$$\theta^+ = \xi(u, \vartheta, \bar{\vartheta}) + i\epsilon\xi'(u, \vartheta, \bar{\vartheta}), \quad \bar{\theta}^+ = \bar{\xi}(u, \vartheta, \bar{\vartheta}) + i\epsilon\bar{\xi}'(u, \vartheta, \bar{\vartheta}), \quad (5.38)$$

$$\theta^- = \bar{\xi}(u, \vartheta, \bar{\vartheta}) - i\epsilon\bar{\xi}'(u, \vartheta, \bar{\vartheta}), \quad \bar{\theta}^- = \xi(u, \vartheta, \bar{\vartheta}) - i\epsilon\xi'(u, \vartheta, \bar{\vartheta}), \quad (5.39)$$

$$y = \epsilon(D\xi)(\bar{D}\bar{\xi}) - \frac{\epsilon}{2}(\xi\bar{\xi}' + \bar{\xi}\xi'). \quad (5.40)$$

Since superreparametrisations of the one-dimensional superspace in general map a given boundary to a completely different one, we have a spontaneous breaking of the entire superreparametrisation symmetry to the subgroup $SU(1, 1|1)$ of global reparametrisations that leave the boundaries invariant [28]. As in the bosonic case the other reparametrisations can be interpreted as Goldstone modes.

The part of the action in (5.37) involving the dilaton now explicitly breaks this symmetry since its boundary term gives a non-zero action for the superreparametrisations, namely the Schwarzian action in (5.36) which vanishes only for $SU(1, 1|1)$ reparametrisations [28].

6 Consistency check

Here, we perform the following consistency check. The Gibbons-Hawking-York term should ensure that Dirichlet conditions on field variations should not lead to further boundary conditions on 2d fields. Hence, any solution to the bulk equations should be viable. In particular it can be seen that plugging a bulk solution of the dilaton into the effective action (5.35) and taking variations with respect to superreparametrisations yields zero. For cases with less supersymmetry this has been done in [48, 57]. In particular, in [57] superreparametrisations have been expressed in terms of unconstrained bosonic and fermionic degrees of freedom with respect to which the variation has been considered. Already there, this procedure turned out to give rather lengthy expressions. In the $\mathcal{N} = 2$ case the complexity of this calculation grows further [65]. Fortunately, there is a shortcut which could have been used also in [57]. The underlying trick can be found e.g. in chapter 4 of [66]. Using the anomalous chain rule, the variation of the Schwarzian can be linearised. For $\mathcal{N} = 2$ the details are as follows,

$$\begin{aligned} \delta\text{Schw}(t, \xi, \bar{\xi}; u, \vartheta, \bar{\vartheta}) &\equiv \text{Schw}(t + \delta t, \xi + \delta\xi, \bar{\xi} + \delta\bar{\xi}; u, \vartheta, \bar{\vartheta}) - \text{Schw}(t, \xi, \bar{\xi}; u, \vartheta, \bar{\vartheta}) \\ &= D\xi\bar{D}\bar{\xi}\text{Schw}(t + \delta t, \xi + \delta\xi, \bar{\xi} + \delta\bar{\xi}; t, \xi, \bar{\xi}) \\ &= D\xi\bar{D}\bar{\xi}(\partial_t\bar{D}\xi\delta\bar{\xi} - \partial_t D\xi\delta\xi). \end{aligned} \quad (6.1)$$

The next steps are to replace bosonic derivatives by anti-commutators of supercovariant derivatives and use

$$D_\xi = (D\xi)^{-1} D \quad , \quad \bar{D}_\xi = (\bar{D}\bar{\xi})^{-1} \bar{D}.$$

For the variation of (5.36) one gets

$$\delta S \sim \int du d\vartheta d\bar{\vartheta} \left\{ \delta\bar{\xi} \bar{D} \left(\frac{1}{\bar{D}\bar{\xi}} D \left(\frac{1}{D\xi} \bar{D} (\varphi_b D\xi) \right) \right) - \delta\xi D \left(\frac{1}{D\xi} \bar{D} \left(\frac{1}{\bar{D}\bar{\xi}} D (\varphi_b \bar{D}\bar{\xi}) \right) \right) \right\} \quad (6.2)$$

From the discussion in section 3 one can see that φ_b is given by the $\mathcal{N} = 0$ solution [48], multiplied with appropriate powers of ϵ , evaluated at the boundary

$$\varphi_b = \frac{\alpha + \beta t + \gamma t^2}{D\xi \bar{D}\bar{\xi}}, \quad (6.3)$$

with

$$2Dt = \bar{\xi} D\xi \quad , \quad 2\bar{D}t = \xi \bar{D}\bar{\xi}. \quad (6.4)$$

Plugging this into (6.2) and using the chirality conditions (5.19) yields

$$\delta S_{\text{eff}} \sim \gamma \int du d\vartheta d\bar{\vartheta} (\xi \delta\bar{\xi} - \bar{\xi} \delta\xi). \quad (6.5)$$

With (5.25) and the variation of (6.4) this can be brought into the form

$$\delta S_{\text{eff}} \sim \int du d\vartheta \bar{D}\delta t = \int du \delta t' = 0. \quad (6.6)$$

7 Summary and conclusions

In the present paper, we argued that the gravitational part of the holographic dual to the $\mathcal{N} = 2$ supersymmetric SYK model is given by an $\mathcal{N} = (2, 2)$ supersymmetric JT action. We elaborated on the construction of this supersymmetric extension including also a Gibbons-Hawking-York boundary term. The main part of the calculation was done in superconformal gauge. The superconformal factors can be determined in two ways giving the same result. First, one can solve the constraints of constant supercurvatures. On the other hand imposing the existence of four unbroken supersymmetries together with an AdS_2 metric yields the same superconformal factors. Symmetry breaking patterns due to a UV regulator curve match those of the SYK model. Further, we showed that the effective Lagrangian of those curves is given by the super-Schwarzian in agreement with the result [28] for SYK. The chirality of the SYK model emerges from the chirality of the two separate extrinsic curvature fields in our gravitational setup. As a consistency check we plugged a known dilaton solution into the effective boundary action. Its variation with respect to super-reparametrisations vanishes.

It would be interesting to see whether there are corrections towards deviation from JT. For $\mathcal{N} = 0$ such corrections have been proposed in [54]. Also the reconstruction of a more complete holographic dual along the lines of [55, 56] should be extended to supersymmetric

models. In particular the addition of matter and its backreaction should be considered. The dilaton equations of motion will be affected [42]. Insisting on a constant dilaton along the boundary will change the boundary supercurve. It is expected that the same change of the boundary curve will be obtained when supplementing the super-Schwarzian by a term coupling the boundary values of matter to the boundary curve [48]. A further subject of future research will be to study if JT supergravity admits more general dilaton bulk solutions than the one considered here. Less supercharges than in the constant dilaton case will be preserved.

Supersymmetric JT gravity might also be considered in the context of $d = 4$ black hole physics with extended supersymmetry. In particular, the qualitative difference between $\mathcal{N} = 1$ and $\mathcal{N} = 2$ SYK, the non-perturbative SUSY breaking of the former [28], should be relevant in this setting.

Acknowledgments

We thank Iris Golla and Hans Jockers for useful discussions. This work was supported by the SFB-Transregio TR33 “The Dark Universe” (Deutsche Forschungsgemeinschaft) and by “Bonn-Cologne Graduate School for Physics and Astronomy” (BCGS).

Open Access. This article is distributed under the terms of the Creative Commons Attribution License ([CC-BY 4.0](https://creativecommons.org/licenses/by/4.0/)), which permits any use, distribution and reproduction in any medium, provided the original author(s) and source are credited.

References

- [1] S. Sachdev and J. Ye, *Gapless spin fluid ground state in a random, quantum Heisenberg magnet*, *Phys. Rev. Lett.* **70** (1993) 3339 [[cond-mat/9212030](#)] [[INSPIRE](#)].
- [2] A. Kitaev, *A simple model of quantum holography*, talk given at the *KITP Program: entanglement in strongly-correlated quantum matter*, April 6–July 2, University of California, U.S.A. (2015), [part 1](#) and [part 2](#) available online.
- [3] S. Sachdev, *Holographic metals and the fractionalized Fermi liquid*, *Phys. Rev. Lett.* **105** (2010) 151602 [[arXiv:1006.3794](#)] [[INSPIRE](#)].
- [4] S. Sachdev, *Bekenstein-Hawking entropy and strange metals*, *Phys. Rev. X* **5** (2015) 041025 [[arXiv:1506.05111](#)] [[INSPIRE](#)].
- [5] J. Polchinski and V. Rosenhaus, *The spectrum in the Sachdev-Ye-Kitaev model*, *JHEP* **04** (2016) 001 [[arXiv:1601.06768](#)] [[INSPIRE](#)].
- [6] A. Jevicki and K. Suzuki, *Bi-local holography in the SYK model: perturbations*, *JHEP* **11** (2016) 046 [[arXiv:1608.07567](#)] [[INSPIRE](#)].
- [7] J. Maldacena and D. Stanford, *Remarks on the Sachdev-Ye-Kitaev model*, *Phys. Rev. D* **94** (2016) 106002 [[arXiv:1604.07818](#)] [[INSPIRE](#)].
- [8] R. Gurau, *The complete $1/N$ expansion of a SYK-like tensor model*, *Nucl. Phys. B* **916** (2017) 386 [[arXiv:1611.04032](#)] [[INSPIRE](#)].
- [9] E. Witten, *An SYK-like model without disorder*, [arXiv:1610.09758](#) [[INSPIRE](#)].

- [10] I.R. Klebanov and G. Tarnopolsky, *Uncolored random tensors, melon diagrams and the Sachdev-Ye-Kitaev models*, *Phys. Rev. D* **95** (2017) 046004 [[arXiv:1611.08915](#)] [[INSPIRE](#)].
- [11] C. Krishnan, S. Sanyal and P.N. Bala Subramanian, *Quantum chaos and holographic tensor models*, *JHEP* **03** (2017) 056 [[arXiv:1612.06330](#)] [[INSPIRE](#)].
- [12] C. Krishnan, K.V.P. Kumar and S. Sanyal, *Random matrices and holographic tensor models*, *JHEP* **06** (2017) 036 [[arXiv:1703.08155](#)] [[INSPIRE](#)].
- [13] C. Peng, *Vector models and generalized SYK models*, *JHEP* **05** (2017) 129 [[arXiv:1704.04223](#)] [[INSPIRE](#)].
- [14] P. Narayan and J. Yoon, *SYK-like Tensor Models on the Lattice*, *JHEP* **08** (2017) 083 [[arXiv:1705.01554](#)] [[INSPIRE](#)].
- [15] C. Krishnan and K.V.P. Kumar, *Towards a finite- N hologram*, *JHEP* **10** (2017) 099 [[arXiv:1706.05364](#)] [[INSPIRE](#)].
- [16] S. Choudhury et al., *Notes on melonic $O(N)^{q-1}$ tensor models*, [arXiv:1707.09352](#) [[INSPIRE](#)].
- [17] C. Krishnan, K.V. Pavan Kumar and D. Rosa, *Contrasting SYK-like models*, *JHEP* **01** (2018) 064 [[arXiv:1709.06498](#)] [[INSPIRE](#)].
- [18] J. Ben Geloun and V. Rivasseau, *A renormalizable SYK-type tensor field theory*, [arXiv:1711.05967](#) [[INSPIRE](#)].
- [19] R.A. Davison et al., *Thermoelectric transport in disordered metals without quasiparticles: the Sachdev-Ye-Kitaev models and holography*, *Phys. Rev. B* **95** (2017) 155131 [[arXiv:1612.00849](#)] [[INSPIRE](#)].
- [20] M. Berkooz, P. Narayan, M. Rozali and J. Simón, *Comments on the random Thirring model*, *JHEP* **09** (2017) 057 [[arXiv:1702.05105](#)] [[INSPIRE](#)].
- [21] G. Turiaci and H. Verlinde, *Towards a 2d QFT analog of the SYK model*, *JHEP* **10** (2017) 167 [[arXiv:1701.00528](#)] [[INSPIRE](#)].
- [22] K. Bulycheva, *A note on the SYK model with complex fermions*, *JHEP* **12** (2017) 069 [[arXiv:1706.07411](#)] [[INSPIRE](#)].
- [23] R. Bhattacharya, S. Chakrabarti, D.P. Jatkar and A. Kundu, *SYK model, chaos and conserved charge*, *JHEP* **11** (2017) 180 [[arXiv:1709.07613](#)] [[INSPIRE](#)].
- [24] D.J. Gross and V. Rosenhaus, *A generalization of Sachdev-Ye-Kitaev*, *JHEP* **02** (2017) 093 [[arXiv:1610.01569](#)] [[INSPIRE](#)].
- [25] J. Yoon, *SYK models and SYK-like tensor models with global symmetry*, *JHEP* **10** (2017) 183 [[arXiv:1707.01740](#)] [[INSPIRE](#)].
- [26] Y. Chen, H. Zhai and P. Zhang, *Tunable quantum chaos in the Sachdev-Ye-Kitaev model coupled to a thermal bath*, *JHEP* **07** (2017) 150 [[arXiv:1705.09818](#)] [[INSPIRE](#)].
- [27] P. Zhang, *Dispersive Sachdev-Ye-Kitaev model: band structure and quantum chaos*, *Phys. Rev. B* **96** (2017) 205138 [[arXiv:1707.09589](#)] [[INSPIRE](#)].
- [28] W. Fu, D. Gaiotto, J. Maldacena and S. Sachdev, *Supersymmetric Sachdev-Ye-Kitaev models*, *Phys. Rev. D* **95** (2017) 026009 [*Addendum ibid.* **D 95** (2017) 069904] [[arXiv:1610.08917](#)] [[INSPIRE](#)].

- [29] T. Nishinaka and S. Terashima, *A note on Sachdev–Ye–Kitaev like model without random coupling*, *Nucl. Phys. B* **926** (2018) 321 [[arXiv:1611.10290](#)] [[INSPIRE](#)].
- [30] N. Sannomiya, H. Katsura and Y. Nakayama, *Supersymmetry breaking and Nambu–Goldstone fermions with cubic dispersion*, *Phys. Rev. D* **95** (2017) 065001 [[arXiv:1612.02285](#)] [[INSPIRE](#)].
- [31] C. Peng, M. Spradlin and A. Volovich, *A supersymmetric SYK-like tensor model*, *JHEP* **05** (2017) 062 [[arXiv:1612.03851](#)] [[INSPIRE](#)].
- [32] T. Li, J. Liu, Y. Xin and Y. Zhou, *Supersymmetric SYK model and random matrix theory*, *JHEP* **06** (2017) 111 [[arXiv:1702.01738](#)] [[INSPIRE](#)].
- [33] D. Stanford and E. Witten, *Fermionic localization of the Schwarzian theory*, *JHEP* **10** (2017) 008 [[arXiv:1703.04612](#)] [[INSPIRE](#)].
- [34] T.G. Mertens, G.J. Turiaci and H.L. Verlinde, *Solving the Schwarzian via the conformal bootstrap*, *JHEP* **08** (2017) 136 [[arXiv:1705.08408](#)] [[INSPIRE](#)].
- [35] T. Kanazawa and T. Wettig, *Complete random matrix classification of SYK models with $\mathcal{N} = 0, 1$ and 2 supersymmetry*, *JHEP* **09** (2017) 050 [[arXiv:1706.03044](#)] [[INSPIRE](#)].
- [36] J. Murugan, D. Stanford and E. Witten, *More on supersymmetric and 2d analogs of the SYK model*, *JHEP* **08** (2017) 146 [[arXiv:1706.05362](#)] [[INSPIRE](#)].
- [37] J. Yoon, *Supersymmetric SYK model: bi-local collective superfield/supermatrix formulation*, *JHEP* **10** (2017) 172 [[arXiv:1706.05914](#)] [[INSPIRE](#)].
- [38] C. Peng, M. Spradlin and A. Volovich, *Correlators in the $\mathcal{N} = 2$ Supersymmetric SYK Model*, *JHEP* **10** (2017) 202 [[arXiv:1706.06078](#)] [[INSPIRE](#)].
- [39] N. Hunter-Jones, J. Liu and Y. Zhou, *On thermalization in the SYK and supersymmetric SYK models*, *JHEP* **02** (2018) 142 [[arXiv:1710.03012](#)] [[INSPIRE](#)].
- [40] N. Hunter-Jones and J. Liu, *Chaos and random matrices in supersymmetric SYK*, [arXiv:1710.08184](#) [[INSPIRE](#)].
- [41] P. Narayan and J. Yoon, *Supersymmetric SYK model with global symmetry*, [arXiv:1712.02647](#) [[INSPIRE](#)].
- [42] A. Almheiri and J. Polchinski, *Models of AdS_2 backreaction and holography*, *JHEP* **11** (2015) 014 [[arXiv:1402.6334](#)] [[INSPIRE](#)].
- [43] R. Jackiw, *Lower dimensional gravity*, *Nucl. Phys. B* **252** (1985) 343 [[INSPIRE](#)].
- [44] C. Teitelboim, *Gravitation and Hamiltonian structure in two space-time dimensions*, *Phys. Lett. B* **126** (1983) 41.
- [45] G. Sárosi, *AdS_2 holography and the SYK model*, [arXiv:1711.08482](#) [[INSPIRE](#)].
- [46] G. Turiaci and H. Verlinde, *On CFT and quantum chaos*, *JHEP* **12** (2016) 110 [[arXiv:1603.03020](#)] [[INSPIRE](#)].
- [47] K. Jensen, *Chaos in AdS_2 holography*, *Phys. Rev. Lett.* **117** (2016) 111601 [[arXiv:1605.06098](#)] [[INSPIRE](#)].
- [48] J. Maldacena, D. Stanford and Z. Yang, *Conformal symmetry and its breaking in two dimensional Nearly Anti-de-Sitter space*, *PTEP* **2016** (2016) 12C104 [[arXiv:1606.01857](#)] [[INSPIRE](#)].

- [49] M. Cvetič and I. Papadimitriou, *AdS₂ holographic dictionary*, *JHEP* **12** (2016) 008 [Erratum *ibid.* **01** (2017) 120] [[arXiv:1608.07018](#)] [[INSPIRE](#)].
- [50] G. Mandal, P. Nayak and S.R. Wadia, *Coadjoint orbit action of Virasoro group and two-dimensional quantum gravity dual to SYK/tensor models*, *JHEP* **11** (2017) 046 [[arXiv:1702.04266](#)] [[INSPIRE](#)].
- [51] S.R. Das, A. Jevicki and K. Suzuki, *Three dimensional view of the SYK/AdS duality*, *JHEP* **09** (2017) 017 [[arXiv:1704.07208](#)] [[INSPIRE](#)].
- [52] S.R. Das, A. Ghosh, A. Jevicki and K. Suzuki, *Three dimensional view of arbitrary q SYK models*, [arXiv:1711.09839](#) [[INSPIRE](#)].
- [53] S.R. Das, A. Ghosh, A. Jevicki and K. Suzuki, *Space-time in the SYK model*, [arXiv:1712.02725](#) [[INSPIRE](#)].
- [54] A. Kitaev and S.J. Suh, *The soft mode in the Sachdev-Ye-Kitaev model and its gravity dual*, [arXiv:1711.08467](#) [[INSPIRE](#)].
- [55] D.J. Gross and V. Rosenhaus, *The bulk dual of SYK: cubic couplings*, *JHEP* **05** (2017) 092 [[arXiv:1702.08016](#)] [[INSPIRE](#)].
- [56] D.J. Gross and V. Rosenhaus, *All point correlation functions in SYK*, *JHEP* **12** (2017) 148 [[arXiv:1710.08113](#)] [[INSPIRE](#)].
- [57] S. Förste and I. Golla, *Nearly AdS₂ SUGRA and the super-Schwarzian*, *Phys. Lett. B* **771** (2017) 157 [[arXiv:1703.10969](#)] [[INSPIRE](#)].
- [58] A.H. Chamseddine, *Superstrings in arbitrary dimensions*, *Phys. Lett. B* **258** (1991) 97 [[INSPIRE](#)].
- [59] P.S. Howe and G. Papadopoulos, *N = 2, D = 2 supergeometry*, *Class. Quant. Grav.* **4** (1987) 11 [[INSPIRE](#)].
- [60] P.C. West, *Introduction to supersymmetry and supergravity*, World Scientific, Singapore (1990).
- [61] M.T. Grisaru and M.E. Wehlau, *Prepotentials for (2,2) supergravity*, *Int. J. Mod. Phys. A* **10** (1995) 753 [[hep-th/9409043](#)] [[INSPIRE](#)].
- [62] M.T. Grisaru and M.E. Wehlau, *Superspace measures, invariant actions and component projection formulae for (2,2) supergravity*, *Nucl. Phys. B* **457** (1995) 219 [[hep-th/9508139](#)] [[INSPIRE](#)].
- [63] C. Closset and S. Cremonesi, *Comments on $\mathcal{N} = (2,2)$ supersymmetry on two-manifolds*, *JHEP* **07** (2014) 075 [[arXiv:1404.2636](#)] [[INSPIRE](#)].
- [64] J. Gomis et al., *Anomalies, conformal manifolds and spheres*, *JHEP* **03** (2016) 022 [[arXiv:1509.08511](#)] [[INSPIRE](#)].
- [65] J. Kames-King, *Constructing the supergravity dual of the $\mathcal{N} = 2$ SYK model*, Master Thesis, Bonn University, Bonn, Germany (2018), to appear.
- [66] R. Blumenhagen, D. Lüst and S. Theisen, *Basic concepts of string theory*, Springer, Germany (2013).