Twistor origin of the superstring

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ABSTRACT: After introducing a d=10 pure spinor λ^α, the Virasoro constraint $\partial x^m \partial x_m = 0$ can be replaced by the twistor-like constraint $\partial x^m (\gamma_m \lambda)_\alpha = 0$. Quantizing this twistor-like constraint leads to the pure spinor formalism for the superstring where the fermionic superspace variables θ^α and their conjugate momenta come from the ghosts and antighosts of the twistor-like constraint.

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1 Introduction

The conventional manner to obtain the superstring from the bosonic string is to generalize the worldsheet conformal invariance generated by the Virasoro constraint $\partial x^m \partial x_m = 0$ to the worldsheet N=1 superconformal invariance generated by the super-Virasoro constraint $\partial x^m \psi_m = 0$ where ψ^m is the fermionic worldsheet superpartner of x^m. This Ramond-Neveu-Schwarz (RNS) construction of the superstring [1, 2] was developed in the 1970's, and although it is based on the simple geometrical idea of generalizing conformal invariance to superconformal invariance, it has the disadvantage that spacetime supersymmetry is only present after including both periodic and antiperiodic conditions for ψ^m and performing a GSO projection [3] which truncates out states constructed from an even number of ψ^m variables. This lack of manifest spacetime supersymmetry makes it difficult to compute scattering amplitudes involving fermionic states and has prevented the RNS formalism from being used to describe Ramond-Ramond backgrounds.

In the 1980's, Green and Schwarz developed a new formalism [4, 5] for the superstring in which spacetime supersymmetry is manifest and is constructed using a spacetime spinor variable θ^α intead of the spacetime vector variable ψ^m of the RNS formalism. In addition to worldsheet conformal invariance, their superstring action contains a fermionic symmetry called "kappa symmetry" [6] which replaces the N=1 worldsheet superconformal invariance of the RNS formalism. However, the structure of kappa symmetry prevented quantization of the Green-Schwarz formalism except in light-cone gauge, which complicates the computation of scattering amplitudes and the quantum description of Ramond-Ramond backgrounds.

Starting in 2000, a new formalism for the superstring has been developed in which spacetime supersymmetry is manifest and which can be easily quantized in a covariant

manner [7]. In addition to the fermionic spinor variable θ^α of the Green-Schwarz formalism, this new formalism includes a bosonic spinor variable λ^α which satisfies the d=10 "pure spinor" constraint

$$\lambda \gamma^m \lambda = 0 \tag{1.1}$$

for $m = 0$ to 9. Unlike the RNS and Green-Schwarz formalisms, it has been successfully used to compute multiloop amplitudes involving both bosonic and fermionic states [8–10] and to prove the quantum consistency of Ramond-Ramond backgrounds such as $AdS_5 \times S^5$ [11–13].

However, unlike the RNS formalism, the geometric origin of this new formalism was not understood. Physical states and scattering amplitudes are defined using a gauge-fixed action and a nilpotent BRST operator Q constructed from the Green-Schwarz variables and the pure spinor λ^α as

$$Q = \int dz \ \lambda^\alpha d_\alpha, \tag{1.2}$$

where d_α is the fermionic Green-Schwarz-Siegel constraint [14] which generates kappa symmetry. But despite several attempts [15–18], this pure spinor BRST operator was not obtained in a simple manner by gauge-fixing a worldsheet reparameterization invariant action.

In this paper, an elegant geometrical origin for this formalism will be proposed and the pure spinor BRST operator of (1.2) will be obtained by gauge-fixing a simple worldsheet reparameterization invariant action. Surprisingly, this reparameterization invariant action will be constructed entirely from bosonic worldsheet variables, and the fermionic worldsheet variables θ^α and their conjugate momenta will come from ghosts and antighosts associated with the gauge fixing.

The bosonic variables in the worldsheet action will consist of the usual d=10 spacetime vector variable x^m together with a spacetime spinor variable λ^α satisfying the d=10 pure spinor constraint $\lambda \gamma^m \lambda = 0$. The pure spinor constraint implies that only 11 of the 16 components of λ^α are independent, and after Wick-rotation to Euclidean signature, λ^α parameterizes the eleven-dimensional complex space $\frac{\text{SO}(10)}{\text{U}(5)} \times C^*$ where C^* is the complex plane minus the origin [19].

Instead of generalizing the Virasoro constraint $T = -\frac{1}{2} \partial x^m \partial x_m = 0$ to a super-Virasoro constraint as in the RNS formalism, the Virasoro constraint $T = 0$ will instead by replaced by the twistor-like constraint

$$C_\alpha = -\frac{1}{2} \partial x^m (\gamma_m \lambda)_\alpha = 0. \tag{1.3}$$

Note that $C_\alpha = 0$ implies $T = 0$ since T is equal to $\frac{1}{\lambda^\beta \overline{\lambda}_\beta} C_\alpha (\gamma_m \overline{\lambda})^\alpha \partial x^m$ where $\overline{\lambda}_\alpha$ is any spinor satisfying $\lambda^\alpha \overline{\lambda}_\alpha \neq 0$. As discussed by several authors [20–29], pure spinors are the natural generalization to higher dimensions of d=4 Penrose twistors [30], and (1.3) is the d=10 stringy version [31–41] of the d=4 twistor constraint $(\frac{\partial}{\partial \tau} x_{a\dot{a}})\lambda^a = 0$ where $a, \dot{a} = 1$ to 2 and $x_{a\dot{a}}(\tau)$ is a d=4 light-like trajectory.

The worldsheet reparameterization invariant action for x^m and λ^α will be

$$S = \int d^2z (\det e)\left(\frac{1}{2}\nabla x^m \overline{\nabla} x_m + w_\alpha \overline{\nabla}\lambda^\alpha + L^\alpha C_\alpha + \lambda^\alpha \overline{\Lambda}_\alpha\right) \tag{1.4}$$

where $\nabla = e_-^J \partial_J$, $\overline{\nabla} = e_+^J \partial_J$, $e_\pm^J$ is the usual two-dimensional vierbein, and L^α is a Lagrange multiplier which enforces the constraint $C_\alpha = 0$. In addition, the term $\lambda^\alpha \overline{\Lambda}_\alpha$ has been included in the Lagrangian where $\overline{\Lambda}_\alpha$ is a bosonic pure spinor of opposite chirality to λ^α. If $\overline{\Lambda}_\alpha$ is interpreted (in Euclidean signature) as the complex conjugate to λ^α, the term $\lambda^\alpha \overline{\Lambda}_\alpha$ concentrates the functional integration over λ^α to the region near $\lambda^\alpha = 0$ and eliminates the divergence coming from functional integration over the non-compact zero modes of λ^α.

To quantize this action, one needs to gauge-fix the invariances generated by the constraint C_α of (1.3). But because $\lambda \gamma^m \lambda = 0$, only 5 of the 16 components of C_α are independent. To gauge-fix, one should first restrict λ^α to a patch of pure spinor space where $\overline{\lambda}_\alpha \lambda^\alpha \neq 0$ for some fixed constant pure spinor $\overline{\lambda}_\alpha$. On this patch of pure spinor space, one can restrict the Lagrange multiplier L^α to satisfy the 11 independent constraints $L\gamma^{mn}\overline{\lambda} = 0$, and the remaining 5 components of L^α can be gauge-fixed in the usual manner to produce 5 fermionic Faddeev-Popov ghosts and antighosts, f^α and m_α, which satisfy the constraints $f\gamma^{mn}\overline{\lambda} = 0$ and $m\gamma^{mn}\lambda = 0$.

On this same patch of pure spinor space, one can similarly gauge-fix $\overline{\Lambda}_\alpha$ to be proportional to the constant pure spinor $\overline{\lambda}_\alpha$. This gauge-fixing procedure produces additional fermionic Faddeev-Popov ghosts and antighosts, g_α and n^α, which because of the pure spinor constraint on $\overline{\Lambda}_\alpha$, are constrained to satisfy $g\gamma^m \overline{\lambda} = 0$ and $n\gamma^m \lambda = 0$ so they each have 11 independent components.

Note that there are no additional Faddeev-Popov ghosts and antighosts coming from gauge-fixing worldsheet reparameterization invariance to conformal gauge since the Virasoro constraint $T = 0$ is already implied by the twistor-like constraint of (1.3). This explains why the b ghost satisfying $\{Q, b\} = T$ is not a fundamental worldsheet variable in the pure spinor formalism, but is a composite operator constructed out of the other variables.

Although the constraints on the fermionic ghosts (f^α, g_α) and antighosts (m_α, n^α) depend on the choice of patch of pure spinor space, one can define an unconstrained fermionic spinor variable θ^α and its conjugate momentum p_α as

$$\theta^\alpha = f^\alpha + n^\alpha \quad \text{and} \quad p_\alpha = g_\alpha + m_\alpha \tag{1.5}$$

which are independent of the choice of $\overline{\lambda}_\alpha$. In terms of these unconstrained fermionic variables, the Faddeev-Popov ghost contribution to the action is

$$\int d^2z (m_\alpha \overline{\partial} f^\alpha + n^\alpha \overline{\partial} g_\alpha) = \int d^2z \, p_\alpha \overline{\partial}\theta^\alpha, \tag{1.6}$$

so the gauge-fixed action is

$$S = \int d^2z \left(\frac{1}{2}\partial x^m \overline{\partial} x_m + w_\alpha \overline{\partial}\lambda^\alpha + p_\alpha \overline{\partial}\theta^\alpha\right). \tag{1.7}$$

And the resulting BRST operator is

$$Q = \int dz \left(\lambda^\alpha p_\alpha + C_\alpha \theta^\alpha - \frac{1}{8}(\lambda\gamma^m\theta)(\theta\gamma_m\partial\theta) \right) = \int dz \, \lambda^\alpha d_\alpha, \qquad (1.8)$$

where d_α is the supersymmetric Green-Schwarz-Siegel constraint [14] and the term $-\frac{1}{8}(\lambda\gamma^m\theta)(\theta\gamma_m\partial\theta)$ in Q comes from the non-abelian constraint algebra $[C_\alpha, C_\beta] = \frac{1}{8}(\gamma^m\lambda)_{[\alpha}(\gamma_m\nabla\lambda)_{\beta]}$.

So the gauge-fixed action and BRST operator of the pure spinor formalism are obtained by quantizing the simple worldsheet reparameterization invariant action of (1.4). Since the fermionic worldsheet variables in the gauge-fixed action come from Faddeev-Popov ghosts and antighosts, a natural question is how fermionic variables can appear in light-cone gauge where ghosts are absent. This question can be studied by simplifying to the d=10 superparticle [42] where the twistor-like constraint of (1.3) reduces to $C_\alpha = -\frac{1}{2}P^m(\gamma_m\lambda)_\alpha = 0$. Although C_α has 5 independent components, it implies a single mass-shell constraint $P^2 = 0$ for the x^m dependence. And since λ^α dependence is fixed by the $\lambda^\alpha\overline{\Lambda}_\alpha$ term in the Lagrangian to be near $\lambda^\alpha = 0$, there are 4 components of C_α which overconstrain the classical worldsheet variables. These 4 extra constraints of C_α lead to 4 fermionic variables together with their conjugate momenta which are the usual 8 light-cone Green-Schwarz fermions.

It is interesting to point out that this same phenomenon occurs for the d=11 pure spinor description of the superparticle [43–46] which describes d=11 supergravity. In this case, the bosonic variables are x^M for $M = 0$ to 10 and λ^A for $A = 1$ to 32 where λ^A satisfies the pure spinor constraint $\lambda\gamma^M\lambda = 0$ that reduces its 32 components to 23 independent components. The twistor-like constraint $C_A = -\frac{1}{2}P^M(\gamma_M\lambda)_A = 0$ has 9 independent components, and implies the d=11 mass-shell constraint $P^2 = 0$. So there are 8 components of C_A which overconstrain the classical variables and lead to 8 fermionic variables and their conjugate momenta in light-cone gauge.

After describing the worldsheet reparameterization invariant action of (1.4) and its gauge invariances in sections 2 and 3 of this paper, the gauge-fixing procedure on a patch where $\overline{\lambda}_\alpha\lambda^\alpha \neq 0$ will be discussed in section 4. In section 5, the "minimal" version of the pure spinor formalism will be derived using this procedure, and in section 6, the "non-minimal" version of the pure spinor formalism [47, 48] will be derived by upgrading $\overline{\lambda}_\alpha$ from a constant pure spinor to a worldsheet variable.

Finally, a conjecture for generalizing this procedure to curved Type II supergravity backgrounds including Ramond-Ramond fields will be proposed in section 7. Since all fermionic variables in the worldsheet action arise from Faddeev-Popov ghosts, Ramond-Ramond background fields will not directly appear in the reparameterization invariant action and will only appear after performing the gauge-fixing procedure. The absence of physical Ramond-Ramond fields from the classical action implies that there is non-trivial BRST cohomology at nonzero ghost number where the ghosts (f^α, g_α) and antighosts (m_α, n^α) are defined to carry ghost-number $+1$ and -1. This fact is not surprising since the patch-independent variables θ^α and p_α of (1.5) do not have well-defined ghost number when ghost number is defined in terms of (f^α, g_α) and (m_α, n^α).

2 Worldsheet action

The worldsheet variables in the reparameterization invariant action will include the space-time x^m variables ($m = 0$ to 9), the left-moving bosonic pure spinor λ^α variables ($\alpha = 1$ to 16) and their conjugate momenta w_α, and the right-moving bosonic pure spinor $\widehat{\lambda}^{\hat\alpha}$ variables and their conjugate momenta $\widehat{w}_{\hat\alpha}$. Because of the pure spinor constraints

$$\lambda\gamma^m\lambda = \widehat{\lambda}\gamma^m\widehat{\lambda} = 0, \tag{2.1}$$

the conjugate momenta w_α and $\widehat{w}_{\hat\alpha}$ can only appear in combinations which are invariant under the gauge transformations $\delta w_\alpha = c^m(\gamma_m\lambda)_\alpha$ and $\delta\widehat{w}_{\hat\alpha} = \widehat{c}^m(\gamma_m\widehat{\lambda})_{\hat\alpha}$ for arbitrary c^m and $\widehat{c}^m$. Note that $(x^m, \lambda^\alpha, \widehat{\lambda}^{\hat\alpha})$ are worldsheet scalars, and w_α and $\widehat{w}_{\hat\alpha}$ carry conformal weight $(1,0)$ and $(0,1)$ respectively. For the Type IIA (or Type IIB) superstring, the $\hat\alpha$ index on right-moving spinors denotes the opposite (or same) spacetime chirality as the unhatted α index on left-moving spinors. And the heterotic superstring is obtained by replacing the right-moving sector with the same right-moving sector as in the RNS heterotic formalism.

The Type II worldsheet action in a flat background is

$$S = \int d^2z \,(\det e)\left[\frac{1}{2}\nabla x^m\overline{\nabla}x_m + w_\alpha\overline{\nabla}\lambda^\alpha + \widehat{w}_{\hat\alpha}\nabla\widehat{\lambda}^{\hat\alpha}\right. \tag{2.2}$$
$$\left. + L^\alpha C_\alpha + \overline{\Lambda}_\alpha\lambda^\alpha + \widehat{L}^{\hat\alpha}\widehat{C}_{\hat\alpha} + \widehat{\overline{\Lambda}}_{\hat\alpha}\widehat{\lambda}^{\hat\alpha} + \frac{1}{4}(L\gamma^m\lambda)(\widehat{L}\gamma_m\widehat{\lambda})\right]$$

where $\nabla = e_-^J\partial_J$, $\overline{\nabla} = e_+^J\partial_J$, $e_\pm^J$ is the worldsheet vielbein for $J = 1$ to 2, C_α and $\widehat{C}_{\hat\alpha}$ are the twistor-like constraints

$$C_\alpha = -\frac{1}{2}\nabla x^m(\gamma_m\lambda)_\alpha, \quad \widehat{C}_{\hat\alpha} = -\frac{1}{2}\overline{\nabla} x^m(\gamma_m\widehat{\lambda})_{\hat\alpha}, \tag{2.3}$$

L^α and $\widehat{L}^{\hat\alpha}$ are Lagrange multipliers of conformal weight $(0,1)$ and $(1,0)$, and $\overline{\Lambda}_\alpha$ and $\widehat{\overline{\Lambda}}_{\hat\alpha}$ are Lagrange multipliers of conformal weight $(1,1)$.

Just as λ^α and $\widehat{\lambda}^{\hat\alpha}$ are pure spinors satisfying the constraint of (2.1), the Lagrange multipliers $\overline{\Lambda}_\alpha$ and $\widehat{\overline{\Lambda}}_{\hat\alpha}$ will also be required to be pure spinors satisfying the constraints

$$\overline{\Lambda}\gamma^m\overline{\Lambda} = \widehat{\overline{\Lambda}}\gamma^m\widehat{\overline{\Lambda}} = 0, \tag{2.4}$$

so that $\overline{\Lambda}_\alpha$ and $\widehat{\overline{\Lambda}}_{\hat\alpha}$ each have 11 independent complex components. After Wick rotation to Euclidean signature, pure spinors parameterize the complex space $\frac{\mathrm{SO}(10)}{\mathrm{U}(5)} \times C^*$ where C^* denotes the complex plane minus the origin. So all components of a pure spinor cannot be simultaneously zero. To globally paramaterize pure spinors, one therefore needs to divide the space into 16 patches $\mathcal{O}_\alpha$ for $\alpha = 1$ to 16 where, on the patch $\mathcal{O}_\alpha$, the component λ^α and $\overline{\Lambda}_\alpha$ of the pure spinors are required to be nonvanishing [19].

In addition to acting as a Lagrange multiplier for the nonzero modes of λ^α, the zero modes of $\overline{\Lambda}_\alpha$ can be interpreted as a regulator for the zero modes of λ^α. In other words, if the zero modes of $\overline{\Lambda}_\alpha$ are interpreted (after Wick rotation) as the complex conjugate

of the λ^α zero modes, the term $\lambda^\alpha \overline{\Lambda}_\alpha$ in the action acts as a Gaussian regulator for the functional integration over these non-compact pure spinor zero modes. Note that the pure spinor constraint on $\overline{\Lambda}_\alpha$ implies that it cannot be used to remove all λ^α dependence from the action of (2.2). For example, the shift

$$\overline{\Lambda}_\alpha \to \overline{\Lambda}_\alpha + \overline{\nabla} w_\alpha + \frac{1}{2}\nabla x^m (\gamma_m L)_\alpha \qquad (2.5)$$

which would naively remove λ^α dependence from the action is not allowed since it does not preserve the constraint of (2.4).

To simplify notation, the right-moving sector will be ignored for the rest of this paper when it plays an identical role to the left-moving sector.

3   Gauge invariances

Because of the first-class constraint $C_\alpha = -\frac{1}{2}\nabla x^m (\gamma_m \lambda)_\alpha$, the worldsheet action of (2.2) is invariant under the gauge transformation

$$\delta x^m = \frac{1}{2}\lambda\gamma^m f, \quad \delta w_\alpha = -\frac{1}{2}\nabla x^m (\gamma_m f)_\alpha + \frac{1}{4}(\widehat{L}\gamma^m\widehat{\lambda})(\gamma_m f)_\alpha, \quad \delta L^\alpha = \overline{\nabla} f^\alpha, \quad (3.1)$$

$$\delta\overline{\Lambda}_\alpha = \frac{1}{16(\lambda\overline{\Lambda})}(\gamma^m\gamma^n\overline{\Lambda})_\alpha[\nabla(\lambda\gamma_m f)(\lambda\gamma_n L) - \nabla(\lambda\gamma_m L)(\lambda\gamma_n f)], \qquad (3.2)$$

where f^α is an arbitrary infinitesimal parameter and the variation of $\delta\overline{\Lambda}_\alpha$ is necessary since $[C_\alpha, C_\beta] = \frac{1}{8}(\gamma_m\lambda)_{[\alpha}(\gamma_n\nabla\lambda)_{\beta]}$ implies that

$$\lambda^\alpha\delta\overline{\Lambda}_\alpha = \frac{1}{8}[(\lambda\gamma^m L)\nabla(\lambda\gamma_m f) - (\lambda\gamma^m f)\nabla(\lambda\gamma_m L)]. \qquad (3.3)$$

Although (3.3) does not uniquely determine (3.2), it will be later argued that any other $\delta\overline{\Lambda}_\alpha$ that satisfies (3.3) will lead to the same BRST operator up to a similarity transformation.

The gauge invariance $x^m \sim x^m + \frac{1}{2}\lambda\gamma^m f$ of (3.1) is the d=10 generalization of the d=4 twistor symmetry [30]

$$x^{a\dot{a}} \sim x^{a\dot{a}} + \lambda^a f^{\dot{a}} \quad \text{where} \quad a,\dot{a} = 1 \text{ to } 2 \qquad (3.4)$$

that identifies points on a self-dual plane and leaves the twistor variable $\mu^{\dot{a}} = x^{a\dot{a}}\lambda_a$ invariant. So as discussed in [20–29], the d=10 pure spinor variable λ^α plays a similar role to the d=4 twistor variable λ^a of Penrose.

The worldsheet action of (2.2) is also invariant under the gauge transformation generated by λ^α which is

$$\delta w_\alpha = g_\alpha, \quad \delta\overline{\Lambda}_\alpha = \overline{\nabla}g_\alpha + \frac{1}{2(\lambda\overline{\Lambda})}(g\gamma^m\overline{\nabla\Lambda})(\gamma_m\lambda)_\alpha, \qquad (3.5)$$

where g_α is an arbitrary infinitesimal parameter of conformal weight $(1,0)$ satisfying $(g\gamma^m\overline{\Lambda}) = 0$ and the second term in $\delta\overline{\Lambda}_\alpha$ is needed so that $\delta\overline{\Lambda}\gamma^m\overline{\Lambda} = 0$. Furthermore, since $\lambda\gamma^m\lambda = 0$, (2.2) is invariant under the gauge transformations

$$\delta L^\alpha = c_{mn}(\gamma^{mn}\lambda)^\alpha \qquad (3.6)$$

for arbitrary c^{mn}, which implies that 11 of the 16 components of L^α can be gauged away.

Finally, the worldsheet action is invariant under the usual worldsheet reparameterizations generated by the Virasoro constraint

$$T = -\frac{1}{2}\nabla x^m \nabla x_m - w_\alpha \nabla \lambda^\alpha. \tag{3.7}$$

However, these reparameterizations are already included as a special case of the previous gauge transformations. This can be seen from the fact that the Virasoro constraint of (3.7) can be expressed as a linear combination of the other constraints $C_\alpha = -\frac{1}{2}\nabla x^m (\gamma_m \lambda)_\alpha$ and λ^α as

$$T = C_\alpha \frac{\nabla x^m (\gamma_m \overline{\Lambda})^\alpha}{(\lambda \overline{\Lambda})} + \nabla \lambda^\alpha \frac{(\lambda \gamma_{mn} w)(\gamma^{mn}\overline{\Lambda})_\alpha + 2(\lambda w)\overline{\Lambda}_\alpha}{8(\lambda \overline{\Lambda})}. \tag{3.8}$$

So all dependence of the action of (2.2) on off-diagonal components of the worldsheet vierbein can be removed by an appropriate shift of the Lagrange multipliers $(L^\alpha, \overline{\Lambda}_\alpha)$ and $(\widehat{L}^{\hat\alpha}, \widehat{\overline{\Lambda}}_{\hat\alpha})$.

4 Gauge fixing

After shifting the Lagrange multipliers to eliminate the off-diagonal components of the worldsheet vierbien, the worldsheet action can be expressed in conformal gauge where $e_\pm^J$ is proportional to $\delta_\pm^J$ so that $\nabla \to \partial$ and $\overline{\nabla} \to \overline{\partial}$. One then needs to fix the gauge invariances of (3.1), (3.5) and (3.6). The first step to perform this gauge fixing is to restrict the pure spinor λ^α to a patch $\mathcal{O}_\alpha$ where one of its components is required to be nonzero. This patch can be defined by introducing a *constant* pure spinor $\overline{\lambda}_\alpha$ and requiring that $\overline{\lambda}_\alpha \lambda^\alpha$ is nonzero on the patch. Different choices of the constant pure spinor $\overline{\lambda}_\alpha$ correspond to different patches $\mathcal{O}_\alpha$, and consistency of the gauge fixing will require that the resulting gauge-fixed action and BRST operator are independent of the choice of $\overline{\lambda}_\alpha$.

On the patch where $\overline{\lambda}_\alpha \lambda^\alpha$ is nonzero, the gauge invariance of (3.6) implies that one can gauge fix $L\gamma^{mn}\overline{\lambda} = 0$, which fixes 11 of the 16 components of L^α. The remaining 5 components of L^α will be gauge-fixed to zero using the invariance of (3.1) in which the gauge parameter f^α is also constrained to satisfy

$$f\gamma^{mn}\overline{\lambda} = 0. \tag{4.1}$$

Finally, the gauge parameter g_α of (3.5) can be used to gauge-fix the Lagrange multipler $\overline{\Lambda}_\alpha$ to satisfy

$$\overline{\Lambda}_\alpha = \epsilon \overline{\lambda}_\alpha \tag{4.2}$$

where ϵ is a constant. Note that $\overline{\Lambda}_\alpha$ cannot be gauge-fixed to zero since it is a pure spinor taking values in $\frac{SO(10)}{U(5)} \times C^*$. In the gauge of (4.2), g_α satisfies the constraint $g\gamma^m\overline{\lambda} = 0$.

One can now follow the standard BRST procedure where the gauge parameters f^α and g_α are interpreted as fermionic ghosts, and fermionic antighosts m_α and n^α are introduced due to the gauge-fixing of the Lagrange multipliers L^α and $\overline{\Lambda}_\alpha$. But because the Virasoro constraint T can be expressed in terms of C_α and λ^α as in (3.8), there is no

need to introduce the usual Virasoro ghost and antighost, c and b, from gauge-fixing the reparameterization invariance.[1]

The resulting gauge-fixed action is

$$S = S_0 - \int d^2z \, Q(m_\alpha L^\alpha + n^\alpha(\overline{\Lambda}_\alpha - \epsilon\overline{\lambda}_\alpha)) - \int d^2z \, \widehat{Q}(\widehat{m}_{\hat\alpha}\widehat{L}^{\hat\alpha} + \widehat{n}^{\hat\alpha}(\widehat{\overline{\Lambda}}_{\hat\alpha} - \widehat{\overline{\epsilon}}\widehat{\overline{\lambda}}_{\hat\alpha}))$$

$$= \int d^2z \Bigg[\frac{1}{2}\partial x^m \overline{\partial} x_m + w_\alpha \overline{\partial}\lambda^\alpha + \widehat{w}_{\hat\alpha}\partial\widehat{\lambda}^{\hat\alpha}$$

$$+ L^\alpha C_\alpha + \overline{\Lambda}_\alpha \lambda^\alpha + \widehat{L}^{\hat\alpha}\widehat{C}_{\hat\alpha} + \widehat{\overline{\Lambda}}_{\hat\alpha}\widehat{\lambda}^{\hat\alpha} + \frac{1}{4}(L\gamma^m\lambda)(\widehat{L}\gamma_m\widehat{\lambda})$$

$$- M_\alpha L^\alpha - N^\alpha(\overline{\Lambda}_\alpha - \epsilon\overline{\lambda}_\alpha) + m_\alpha \overline{\partial}f^\alpha$$

$$+ n^\alpha \left(\overline{\partial}g_\alpha + \frac{1}{8}(\gamma^m L)_\alpha \partial(\lambda\gamma_m f) - \frac{1}{8}(\gamma^m f)\partial(\lambda\gamma^m L) \right)$$

$$- \widehat{M}_{\hat\alpha}\widehat{L}^{\hat\alpha} - \widehat{N}^{\hat\alpha}(\widehat{\overline{\Lambda}}_{\hat\alpha} - \widehat{\overline{\epsilon}}\widehat{\overline{\lambda}}_{\hat\alpha}) + \widehat{m}_{\hat\alpha}\partial\widehat{f}^{\hat\alpha}$$

$$+ \widehat{n}^{\hat\alpha}\left(\partial\widehat{g}_{\hat\alpha} + \frac{1}{8}(\gamma^m\widehat{L})_{\hat\alpha}\overline{\partial}(\widehat{\lambda}\gamma_m\widehat{f}) - \frac{1}{8}(\gamma^m\widehat{f})\overline{\partial}(\widehat{\lambda}\gamma^m\widehat{L}) \right) \Bigg] \qquad (4.5)$$

where S_0 is the action of (2.2) in conformal gauge, (M_α, N^α) are bosonic Nakanishi-Lautrup fields associated with the gauge-fixing of L^α and $\overline{\Lambda}_\alpha$, and

$$Q = \int dz[\lambda^\alpha g_\alpha + C_\alpha f^\alpha - \frac{1}{8}(n\gamma^m f)\partial(\lambda\gamma_m f)] \qquad (4.6)$$

is the BRST operator which generates the BRST transformations

$$Qx^m = \frac{1}{2}\lambda\gamma^m f, \quad Qw_\alpha = g_\alpha + \ldots, \quad QL^\alpha = \overline{\partial}f^\alpha, \qquad Qf^\alpha = 0,$$

$$Qg_\alpha = -\frac{1}{8}(\gamma^m f)_\alpha \partial(\lambda\gamma_m f), \qquad Qm_\alpha = M_\alpha, \qquad Qn^\alpha = N^\alpha,$$

$$Q\overline{\Lambda}_\alpha = \overline{\partial}g_\alpha + \frac{1}{16(\lambda\overline{\Lambda})}(\gamma^m\gamma^n\overline{\Lambda})_\alpha[\partial(\lambda\gamma_m f)(\lambda\gamma_n L) - \partial(\lambda\gamma_m L)(\lambda\gamma_n f)]. \qquad (4.7)$$

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[1] If desired, one can treat the invariances generated by T as independent symmetries if one also includes the gauge-for-gauge invariances implied by the relation of (3.8). In this case, the gauge-fixing procedure will generate the usual fermionic (b, c) Virasoro ghosts of conformal wieght $(2, -1)$ together with a set of bosonic ghost-for-ghosts (β, γ) which also carry conformal weight $(2, -1)$. Although it will not be verified here, it is expected that these ghosts and ghost-for-ghosts will contribute to the BRST operator the terms

$$Q = Q_0 + \int dz[\gamma(b - B) + c(T - b\partial c - \beta\partial\gamma - \partial(\beta\gamma)] \qquad (4.3)$$

where $Q_0 = \int dz(\lambda^a d_\alpha + \overline{w}^\alpha r_\alpha)$ is the usual non-minimal pure spinor BRST operator and

$$B = d_\alpha \frac{(\partial x^m + \frac{1}{2}\theta\gamma^m\partial\theta)(\gamma_m\overline{\lambda})^\alpha}{(\lambda\overline{\lambda})} + \partial\theta^\alpha \frac{(\lambda\gamma_{mn}w)(\gamma^{mn}\overline{\lambda})_\alpha + 2(\lambda w)\overline{\lambda}_\alpha}{8(\lambda\overline{\lambda})} + \ldots \qquad (4.4)$$

is the composite ghost satisfying $\{Q_0, B\} = T$ with ... denoting terms depending on the non-minimal variables (r_α, s^α). Note that $Q = e^U(Q_0 + \gamma b)e^{-U}$ where $U = \int dz(cB - c\partial c\beta)$ and that the structure of B in (4.4) resembles the structure of (3.8).

Since $L\gamma^{mn}\overline{\lambda} = 0$ and $\overline{\Lambda}\gamma^m\overline{\Lambda} = 0$ imply that only 5 components of L^α and 11 components of $\overline{\Lambda}_\alpha$ are independent, one can choose the antighosts and Nakanashi-Lautrup fields to satisfy the constraints

$$\lambda\gamma^{mn}m = \lambda\gamma^{mn}M = 0 \quad \text{and} \quad \lambda\gamma^m n = \lambda\gamma^m N = 0. \tag{4.8}$$

5 Gauge-fixed pure spinor formalism

After integrating out the Lagrange multipliers and Nakanashi-Lautrup fields, one obtains the equations

$$L^\alpha = 0, \qquad\qquad\qquad\qquad\qquad \overline{\Lambda}_\alpha - \epsilon\overline{\lambda}_\alpha = 0,$$
$$M_\alpha = C_\alpha + \frac{1}{8}(\gamma^m n)_\alpha\partial(\lambda\gamma_m f) + \frac{1}{8}(\gamma^m\lambda)_\alpha\partial(n\gamma_m f), \qquad N^\alpha = \lambda^\alpha, \tag{5.1}$$

and the action

$$S = \int d^2z\left(\frac{1}{2}\partial x^m\overline{\partial}x_m + w_\alpha\overline{\partial}\lambda^\alpha + \widehat{w}_{\hat\alpha}\partial\widehat{\lambda}^{\hat\alpha} + m_\alpha\overline{\partial}f^\alpha + n^\alpha\overline{\partial}g_\alpha + \widehat{m}_{\hat\alpha}\partial\widehat{f}^{\hat\alpha} + \widehat{n}^{\hat\alpha}\partial\widehat{g}_{\hat\alpha}\right). \tag{5.2}$$

Since $\overline{\lambda}_\alpha$ appears in the action of (5.2) and in the BRST operator of (4.6) through the constraints on the ghosts and antighosts, this gauge fixing naively appears to depend on the choice of patch $\mathcal{O}_\alpha$. However, after a cleverly chosen field redefinition, all dependence on $\overline{\lambda}_\alpha$ can be eliminated from the action and the BRST operator, and one can take the limit $\epsilon \to 0$ in the gauge-fixing condition $\overline{\Lambda}_\alpha = \epsilon\overline{\lambda}_\alpha$.

The field redefinition involves defining a new unconstrained fermionic variable θ^α and its conjugate momentum p_α in terms of the constrained variables $(f^\alpha, g_\alpha, m_\alpha, n^\alpha)$ as

$$\theta^\alpha = f^\alpha + n^\alpha \quad \text{and} \quad p_\alpha = e^R(g_\alpha + m_\alpha)e^{-R} \tag{5.3}$$

where

$$R = -\frac{1}{24}\int dz[(n\gamma^m\partial n)(n\gamma_m f) + 3(n\gamma^m\partial f)(n\gamma_m f)]. \tag{5.4}$$

Note that

$$\lambda^\alpha p_\alpha = e^R(\lambda^\alpha g_\alpha)e^{-R} = \lambda^\alpha g_\alpha + \frac{1}{8}(\lambda\gamma^m f)(n\gamma_m\partial n) + \frac{1}{4}(\lambda\gamma^m f)(n\gamma_m\partial f), \tag{5.5}$$

and if one had chosen a different $\delta\overline{\Lambda}_\alpha$ in (3.2) which also satisfied (3.3), the similarity transformation R of (5.4) would be modified in a manner to leave the BRST operator invariant when expressed in terms of θ^α and p_α.

It is easy to verify that all 16 components of θ^α and p_α in (5.3) are unconstrained since the 5 independent components of f^α and m_α are in different directions from the 11 independent components of g_α and n^α. However, since (f^α, g_α) and (m_α, n^α) are ghosts and antighosts which carry conventional ghost number $+1$ and -1, θ^α and p_α of (5.3) do not have well-defined ghost number with respect to the conventional definition. Nevertheless, one can define a new ghost number where $(x^m, \theta^\alpha, p_\alpha)$ carry zero ghost number and $(\lambda^\alpha, w_\alpha)$ carry ghost number $(+1, -1)$. With respect to this new ghost number, the worldsheet

action will carry zero ghost number and the BRST operator will carry $+1$ ghost number as desired.

After a suitable shift of w_α to absorb terms proportional to $\overline{\partial}\lambda^\alpha$, the action and BRST operator of (5.2) and (4.6) can be simply expressed in terms of θ^α and p_α of (5.3) as

$$S = \int d^2z \left(\frac{1}{2}\partial x^m \overline{\partial} x_m + w_\alpha \overline{\partial}\lambda^\alpha + \widehat{w}_{\hat{\alpha}}\partial\widehat{\lambda}^{\hat{\alpha}} + p_\alpha \overline{\partial}\theta^\alpha + \widehat{p}_{\hat{\alpha}}\partial\widehat{\theta}^{\hat{\alpha}}\right), \tag{5.6}$$

$$Q = \int dz \left(\lambda^\alpha p_\alpha - \frac{1}{2}\partial x^m (\lambda\gamma_m\theta) - \frac{1}{8}(\lambda\gamma^m\theta)(\theta\gamma_m\partial\theta)\right) = \int dz \lambda^\alpha d_\alpha \tag{5.7}$$

where

$$d_\alpha = p_\alpha - \frac{1}{2}\partial x^m (\gamma_m\theta)_\alpha - \frac{1}{8}(\gamma^m\theta)_\alpha(\theta\gamma_m\partial\theta) \tag{5.8}$$

is the spacetime supersymmetric Green-Schwarz-Siegel constraint. So one recovers the spacetime supersymmetric gauge-fixed action and BRST operator of the "minimal" pure spinor formalism which is manifestly independent of the choce of $\overline{\lambda}_\alpha$.

6 Gauge-fixed non-minimal pure spinor formalism

To obtain the non-minimal pure spinor formalism [47, 48] from gauge fixing, one upgrades the constant pure spinor $\overline{\lambda}_\alpha$ to a worldsheet variable and constrains its conjugate momentum $\overline{w}^\alpha$ to vanish by adding the term

$$\int d^2z (\overline{w}^\alpha \overline{\nabla}\overline{\lambda}_\alpha + \overline{w}^\alpha H_\alpha) \tag{6.1}$$

to the action of (5.6) where H_α is a Lagrange multiplier for the constraint $\overline{w}^\alpha = 0$. Since only 11 components of $\overline{w}^\alpha$ are independent, the Lagrange multiplier needs to be constrained to satisfy $H\gamma^m\overline{\lambda} = 0$.

When expressed in terms of θ^α and p_α, the action of (5.6) is independent of $\overline{\lambda}_\alpha$ in the limit where the constant ϵ of $\overline{\Lambda}_\alpha = \epsilon\overline{\lambda}_\alpha$ is taken to zero. To obtain the gauge-fixed nonminimal formalism, one leaves ϵ nonzero and defines the non-minimal contribution to the BRST transformations of (4.7) as

$$Q\overline{\lambda}_\alpha = -r_\alpha, \quad QH_\alpha = \overline{\nabla}r_\alpha, \quad Qs^\alpha = S^\alpha, \tag{6.2}$$

where r_α is the fermionic ghost constrained to satisfy $r\gamma^m\overline{\lambda} = 0$, and s^α and S^α are the antighost and Nakanishi-Lautrup field associated to H_α. Note that since θ^α and p_α are defined to be independent of $\overline{\lambda}_\alpha$, their BRST transformations do not involve r_α and are

$$Q\theta^\alpha = \lambda^\alpha, \quad Qp_\alpha = C_\alpha - \frac{1}{8}[(\theta\gamma^m\partial\theta)(\gamma_m\lambda)_\alpha - \partial(\lambda\gamma^m\theta)(\gamma_m\theta)_\alpha - 2(\lambda\gamma^m\theta)(\gamma_m\partial\theta)_\alpha]. \tag{6.3}$$

After gauge-fixing $H_\alpha = 0$, the resulting gauge-fixed action and BRST operator are

$$S = \int d^2z \left[\frac{1}{2}\partial x^m \overline{\partial} x_m + w_\alpha \overline{\partial}\lambda^\alpha + \widehat{w}_{\hat{\alpha}}\partial\widehat{\lambda}^{\hat{\alpha}} + p_\alpha \overline{\partial}\theta^\alpha + \widehat{p}_{\hat{\alpha}}\partial\widehat{\theta}^{\hat{\alpha}} \right. \tag{6.4}$$

$$\left. + \overline{w}^\alpha \overline{\partial}\overline{\lambda}_\alpha + \widehat{\overline{w}}^{\hat{\alpha}}\partial\widehat{\overline{\lambda}}_{\hat{\alpha}} + s^\alpha \overline{\partial}r_\alpha + \widehat{s}^{\hat{\alpha}}\partial\widehat{r}_{\hat{\alpha}} + \epsilon(\lambda^\alpha\overline{\lambda}_\alpha + \theta^\alpha r_\alpha) + \widehat{\epsilon}(\widehat{\lambda}^{\hat{\alpha}}\widehat{\overline{\lambda}}_{\hat{\alpha}} + \widehat{\theta}^{\hat{\alpha}}\widehat{r}_{\hat{\alpha}})\right],$$

$$Q = \int dz (\lambda^\alpha d_\alpha + \overline{w}^\alpha r_\alpha), \tag{6.5}$$

where the term $\epsilon(\lambda^\alpha\overline{\lambda}_\alpha + \theta^\alpha r_\alpha) = \epsilon(\lambda^\alpha\overline{\lambda}_\alpha + n^\alpha r_\alpha)$ in (6.4) comes from the gauge-fixing term $-Q(n^\alpha(\overline{\Lambda}_\alpha - \epsilon\overline{\lambda}_\alpha))$ in (4.5). Equations (6.4) and (6.5) are the gauge-fixed action and BRST operator of the non-minimal pure spinor formalism [47, 48] where the term $e^{-\int d^2z\epsilon(\lambda^\alpha\overline{\lambda}_\alpha + \theta^\alpha r_\alpha)}$ in e^{-S} plays the role of a BRST-invariant regulator for integration over the zero modes of the pure spinors.

7 Generalization to curved backgrounds

The natural conjecture for generalizing the worldsheet reparameterization invariant action of (2.2) to a curved Type II target-space background is

$$S = \int d^2z(\det e)\left[\frac{1}{2}(g_{mn}(x) + b_{mn}(x))\nabla x^m\overline{\nabla}x^n + w_\alpha\overline{\nabla}\lambda^\alpha + \widehat{w}_{\hat\alpha}\nabla\widehat{\lambda}^{\hat\alpha}\right. \tag{7.1}$$

$$+\Omega_m{}^{np}(x)\overline{\nabla}x^m(w\gamma_{np}\lambda) + \widehat{\Omega}_m{}^{np}(x)\nabla x^m(\widehat{w}\gamma_{np}\widehat{\lambda})$$

$$+R_{mnpq}(x)(w\gamma^{mn}\lambda)(\widehat{w}\gamma^{pq}\widehat{\lambda}) + L^\alpha C_\alpha + \overline{\Lambda}_\alpha\lambda^\alpha + \widehat{L}^{\hat\alpha}\widehat{C}_{\hat\alpha}$$

$$\left.+\widehat{\overline{\Lambda}}_{\hat\alpha}\widehat{\lambda}^{\hat\alpha} + \frac{1}{4}(L\gamma^m\lambda)(\widehat{L}\gamma_m\widehat{\lambda})\right]$$

where $g_{mn}(x)$ and $b_{mn}(x)$ are the target-space metric and Kalb-Ramond field,

$$\Omega_m{}^{np} = \Gamma_m{}^{np} + H_m{}^{np} \quad\text{and}\quad \widehat{\Omega}_m{}^{np} = \Gamma_m{}^{np} - H_m{}^{np} \tag{7.2}$$

are the left and right-moving connections constructed as in the RNS action from the Christoffel connection $\Gamma_m{}^{np}$ and the torsion $H_{mnp} = \partial_{[m}B_{np]}$, R_{mnpq} is the Riemann curvature tensor, $\gamma^m_{\alpha\beta} = E^m_a(x)\gamma^a_{\alpha\beta}$ where $a = 0$ to 9 is a tangent-space index and E^m_a is the target-space vierbein satisfying $\eta^{ab}E^m_aE^n_b = g^{mn}$, and

$$C_\alpha = -\frac{1}{2}\nabla x^m(\gamma_m\lambda)_\alpha \quad\text{and}\quad \widehat{C}_{\hat\alpha} = -\frac{1}{2}\overline{\nabla}x^m(\gamma_m\widehat{\lambda})_{\hat\alpha} \tag{7.3}$$

are the twistor-like constraints in the curved background.

Surprisingly, the action of (7.1) has the same structure as the RNS worldsheet action if one replaces the left and right-moving pure spinor Lorentz currents $(w\gamma^{mn}\lambda)$ and $(\widehat{w}\gamma^{mn}\widehat{\lambda})$ in (7.1) with the left and right-moving RNS Lorentz currents $\psi^m\psi^n$ and $\widehat{\psi}^m\widehat{\psi}^n$ and replaces the Lagrange multipliers $(L\gamma^m\lambda)$ and $(\widehat{L}\gamma^m\widehat{\lambda})$ in (7.1) with $\xi\psi^m$ and $\widehat{\xi}\widehat{\psi}^m$ where ξ and $\widehat{\xi}$ are the RNS worldsheet gravitini and ψ^m and $\widehat{\psi}^m$ are the RNS fermionic vectors. Just as the structure of the RNS action is determined by worldsheet supersymmetry, the structure of (7.1) is determined by the requirement that C_α and $\widehat{C}_{\hat\alpha}$ in (7.3) generate symmetries of the action.

Although the Ramond-Ramond background fields do not appear in (7.1), one expects that consistency of the gauge-fixing procedure will require that they appear in both the BRST transformations and in the gauge-fixed action. To be more specific, one needs to follow the procedure of (5.3) and construct $(\theta^\alpha, p_\alpha)$ and $(\widehat{\theta}^{\hat\alpha}, \widehat{p}_{\hat\alpha})$ variables in terms of the Fadeev-Popov ghosts and antighosts such that $(\theta^\alpha, p_\alpha)$ and $(\widehat{\theta}^{\hat\alpha}, \widehat{p}_{\hat\alpha})$ are independent of the

choice of patch of pure spinor space. It is expected that this construction will necessarily involve the Ramond-Ramond background fields and will imply equations of motion for all of the background fields. So instead of obtaining the equations of motion for the NS-NS background fields from quantum worldsheet superconformal invariance as in the RNS formalism, it is conjectured that the equations of motion for all of the background supergravity fields (including the Ramond-Ramond fields) will be obtained in this formalism by requiring that the gauge-fixed action and BRST operator are independent of the choice of patch of pure spinor space.

For example, for the Ramond-Ramond plane-wave background, the classical action of (7.1) is

$$
S = \int d^2z (det\, e) \Big[\frac{1}{2} \nabla x^m \overline{\nabla} x_m + \frac{1}{2} \mu^2 (\nabla x^+)(\overline{\nabla} x^+) x^j x^j + w_\alpha \overline{\nabla} \lambda^\alpha + \widehat{w}_{\hat\alpha} \nabla \widehat{\lambda}^{\hat\alpha} \tag{7.4}
$$
$$
+ \mu^2 (x^j \overline{\nabla} x^+ (w \gamma^{j+} \lambda) + x^j \nabla x^+ (\widehat{w} \gamma^{j+} \widehat{\lambda}) + (w \gamma^{j+} \lambda)(\widehat{w} \gamma^{j+} \widehat{\lambda}))
$$
$$
+ L^\alpha C_\alpha + \overline{\Lambda}_\alpha \lambda^\alpha + \widehat{L}^{\hat\alpha} \widehat{C}_{\hat\alpha} + \overline{\widehat{\Lambda}}_{\hat\alpha} \widehat{\lambda}^{\hat\alpha} + \frac{1}{4} (L \gamma^m \lambda)(\widehat{L} \gamma_m \widehat{\lambda}) \Big]
$$

where $j = 1$ to 8, $x^\pm = x^0 \pm x^9$, and μ^2 is the nonzero component R_{+j+j} of the curvature. Since the constraints C_α and $\widehat{C}_{\hat\alpha}$ of (7.3) are classically conserved, the action of (7.4) is invariant under local symmetries analogous to the flat background symmetries of (3.1) and (3.2). But combining the fermionic ghosts and antighosts for these symmetries into unconstrained patch-independent variables, $(\theta^\alpha, p_\alpha)$ and $(\widehat{\theta}^{\hat\alpha}, \widehat{p}_{\hat\alpha})$, is expected to be more complicated than in (5.3) and to require Ramond-Ramond coupling terms such as $\mu \int d^2z (p \gamma^{+1234} \widehat{p})$ in the action. The complete consistency of this gauge-fixing procedure is expected to lead to the conformally invariant pure spinor action for the plane-wave background of [49].

For a general curved background, the gauge-fixing procedure of section 4 and construction of patch-independent $(\theta^\alpha, p_\alpha)$ and $(\widehat{\theta}^{\hat\alpha}, \widehat{p}_{\hat\alpha})$ variables is expected to imply a gauge-fixed action and BRST operator which coincides with the pure spinor worldsheet action and BRST operator of [50]

$$
S = \int d^2z [(G_{MN}(x, \theta, \widehat{\theta}) + B_{MN}(x, \theta, \widehat{\theta})) \partial Z^M \overline{\partial} Z^N + \ldots], \tag{7.5}
$$
$$
Q = \int dz \, \lambda^\alpha d_\alpha, \quad \widehat{Q} = \int d\overline{z} \, \widehat{\lambda}^{\hat\alpha} \widehat{d}_{\hat\alpha}, \tag{7.6}
$$

where $[G_{MN}, B_{MN}, \ldots]$ are the Type II supergravity superfields described in [50], $Z^M = (x^m, \theta^\alpha, \widehat{\theta}^{\hat\alpha})$ are the N=2 d=10 superspace variables, and p_α and $\widehat{p}_{\hat\alpha}$ are the canonical momentum variables for θ^α and $\widehat{\theta}^{\hat\alpha}$ defined by

$$
p_\alpha = d_\alpha - B_{\alpha M}(\partial Z^M - \overline{\partial} Z^M) - \Omega_\alpha{}^{mn}(\lambda \gamma_{mn} w) - \widehat{\Omega}_\alpha{}^{mn}(\widehat{\lambda} \gamma_{mn} \widehat{w}), \tag{7.7}
$$
$$
\widehat{p}_{\hat\alpha} = \widehat{d}_{\hat\alpha} - B_{\hat\alpha M}(\partial Z^M - \overline{\partial} Z^M) - \Omega_{\hat\alpha}{}^{mn}(\lambda \gamma_{mn} w) - \widehat{\Omega}_{\hat\alpha}{}^{mn}(\widehat{\lambda} \gamma_{mn} \widehat{w}).
$$

It would of course be very important to verify these conjectures for the curved Type II supergravity background. The first step would be to study the physical states in an

open string background which should include both the super-Yang-Mills gluon and gluino. Since the gluino vertex operator is fermionic, it is absent from the reparameterization invariant action which only depends on bosonic worldsheet variables. This means that one should find non-trivial BRST cohomology at nonzero ghost number using the conventional definition of ghost number where the ghosts (f^α, g_α) and antighosts (m_α, n^α) carry ghost number $+1$ and -1. After understanding how this works for the open superstring, it should be straightforward to generalize to the Type II superstring by taking the left-right product of two open superstrings.

Acknowledgments

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