Warping the Kähler potential of F-theory/IIB flux compactifications

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ABSTRACT: We identify the low-energy Kähler potential of warped F-theory/IIB flux compactifications whose light spectrum includes universal, Kähler, axionic and mobile D3-brane moduli. The derivation is based on four-dimensional local superconformal symmetry and holomorphy of brane instanton contributions and it reproduces and generalises previous partial results. We compute the resulting kinetic terms, which show their explicit dependence on the warping. The Kähler potential satisfies the no-scale condition and produces, at leading order in the large volume expansion, a specific correction to the unwarped Kähler potential.

KEYWORDS: Flux compactifications, F-Theory, Supersymmetric Effective Theories

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1 Introduction and summary

Warped F-theory/IIB flux compactifications [1–7] play a prominent role in several phenomenologically inspired string constructions, for reviews see for instance [8, 9]. In particular, the flux allows to stabilise several moduli at tree-level and at the same time back-reacts on the geometry by generating a non-trivial warping, which can lead to interesting physical effects, as discussed e.g. in [5, 10–18].

However, a systematic understanding of the low-energy effective theory of such flux compactifications is, so far, still missing. The most common approach is to use the effective theory obtained from direct dimensional reduction by neglecting the back-reaction of fluxes and other sources, approximating the warping to a constant [19–23]. On the other hand, attempts to repeat a similar direct dimensional reduction by including a non-trivial warping, see for instance [10, 13, 24–29], are technically quite involved. These complications obscure the $N = 1$ supersymmetric structure, which should eventually show up in the effective theory. Indeed, conclusive general answers have been so far reached only in few particular sub-cases.

In this paper, following [30, 31], we use an alternative strategy to identify the effective Kähler potential of warped F-theory/IIB compactifications. This strategy is based on the interplay between ten-dimensional geometry, four-dimensional local superconformal symmetry and holomorphy of brane instantons. In section 2 we review the structure of the warped F-theory/IIB backgrounds and in section 3 we adapt the strategy of [30, 31] to them. As it happens in the constant warping approximation, in general the Kähler potential can only be defined implicitly. We omit from our discussion the moduli associated with the axio-dilaton, the complex structure and the position of seven-branes, which typically get a mass from the flux-induced tree-level superpotential. Hence, in section 4, we argue that the implicit Kähler potential has the very simple form

$$K = -3 \log \hat{a} + \text{const} \quad (1.1)$$

where $\hat{a}$ is a specific parametrisation of the universal modulus. The formula (1.1) is the warped counterpart of the analogous implicit formula $K_{\text{unwarped}} \simeq -2 \log V$, where V is the internal volume, obtained in the constant warping approximation [19–23].

In order to make (1.1) explicit, one should know how $\hat{a}$ depends on the chiral fields φ^A which properly parametrise the scalar sector of the $N = 1$ four-dimensional effective theory. This issue is addressed in section 5, focusing on the case in which only the universal, Kähler and mobile D3-brane moduli are present. By using probe supersymmetric brane instantons, we introduce a natural set of chiral fields φ^A and identify the relevant dependence thereof on the background moduli. This implicitly defines $\hat{a} = \hat{a}(\varphi, \bar{\varphi})$ and then, from (1.1), $K = K(\varphi, \bar{\varphi})$.

As we discuss in section 6, even though the Kähler potential (1.1) is defined only implicitly, it is actually possible to explicitly compute the associated low-energy kinetic terms. Furthermore, we show that the Kähler potential (1.1) is of no-scale type [32, 33]. This result is actually required by the physical consistency of the theory and then provides a strong consistency check.

Our results exhibit the precise contribution of mobile D3-branes, background fluxes and the remaining D3-charge sources to the effective Kähler potential. The contribution of D3-branes had been already considered before under certain restrictions or approximations [10, 13, 19, 27, 34]. On the other hand, to best of our knowledge, the contribution of fluxes is completely new. As discussed in section 7, by considering the large universal modulus limit, one can make more manifest the warping-induced correction to the Kähler potential with respect to the unwarped case. In particular, we explicitly identify the leading correction

as a function of the Kähler moduli, the D3-brane positions, the fluxes and the other D3-charge sources.

In section 8, going to a weakly-coupled regime for technical simplicity, we show how one can adopt the same approach to incorporate the chiral fields associated with purely axionic moduli, neglected so far. All the steps work similarly as in the previous sections. In particular, one can compute the explicit form of the kinetic terms and verify the no-scale condition.

Finally, appendix A contains some technical details and appendix B describes the application of the general results of the paper to a simple concrete class of $N = 1$ flux compactifications on $T^6/\mathbb{Z}_2$ [35], for which the Kähler potential can be made fully explicit.

In this paper we do not explicitly consider the incorporation of other sectors of the effective theory, as for instance the gauge sector and the chiral matter, and the implications of higher-order corrections to the leading ten-dimensional supergravity theory. The investigation of these aspects is left to the future.

2 Background structure

Let us review the structure of warped F-theory/IIB compactifications, by using the type IIB description of these vacua [3–6]. These vacua have an Einstein frame metric of the form

$$ds_{10}^2 = e^{2D} ds_{\mathbb{R}^{1,3}}^2 + e^{-2D} ds_X^2 \quad (2.1)$$

where $ds_{\mathbb{R}^{1,3}}^2$ is the flat Minkowski four-dimensional metric, the internal space X is Kähler and the warping e^{2D} is generically non-constant along the internal directions. Furthermore for F-theory backgrounds, namely in presence of 7-branes, the axio-dilaton $\tau \equiv C_0 + ie^{-\phi}$ varies holomorphically along X and is allowed to undergo non-trivial $SL(2, \mathbb{Z})$ monodromies. The Ricci tensor R^X associated with the internal Kähler metric ds_X^2 is related to the dilaton ϕ by the formula:

$$R_{i\bar{j}}^X = \partial_i \partial_{\bar{j}} \phi \quad (2.2)$$

In addition, there can be a non-trivial three-form flux $G_3 = F_3 - ie^{-\phi} H_3$, with $F_3 = dC_2 - C_0 H_3$, which must be imaginary-self-dual (ISD) on X , $*_X G_3 = iG_3$. In other words G_3 can have non-vanishing primitive $(2, 1)$ or $(0, 3)$ components. Supersymmetry, which we will mostly assume, requires the $(0, 3)$ component to be absent. In addition there can be supersymmetric world-volume fluxes supported on the world-volume of the seven branes. Furthermore, the R-R F_5 is also non-trivial and its form is directly linked to the warping by

$$F_5 = *_X de^{-4D} + dx^{0123} \wedge de^{4D} \quad (2.3)$$

where $*_X$ is computed by using the Kähler metric ds_X^2 .

The F_5 Bianchi identity is

$$dF_5 - H_3 \wedge F_3 = -Q_6^{\text{loc}} \quad (2.4)$$

where Q_6^{loc} denotes the localised D3-charge current

$$Q_6^{\text{loc}} = \ell_s^4 \sum_{I \in \text{D3's}} \delta_I^6 - \frac{1}{4} \ell_s^4 \sum_{O \in \text{O3's}} \delta_O^6 + \dots \quad (2.5)$$

We have introduced the string length $\ell_s \equiv 2\pi\sqrt{\alpha'}$ and the six-form currents δ_I^6 and δ_O^6 localised at the position of the I -th D3-brane and of the O -th O3-plane respectively.¹ In Q_6^{loc} the ellipses denote other localised sources of D3-brane charge, induced by fluxes and curvature corrections on the 7-branes. By combining (2.3) and (2.4) one obtains the following equation for the warp factor:

$$\Delta e^{-4D} = \frac{1}{2\text{Im}\tau} G_3 \cdot \bar{G}_3 + *_X Q_6^{\text{loc}} \quad (2.6)$$

where we have introduced the Laplacian $\Delta \equiv -\nabla_m \nabla^m$.

The supersymmetric structure of these backgrounds is characterised by a Weyl Killing spinor ϵ of the form

$$\epsilon = e^{\frac{D}{2}} \zeta_R \otimes \eta \quad (2.7)$$

where η is an internal Weyl spinor satisfying the normalisation condition $\eta^\dagger \eta = 1$, and ζ_R is an external right-handed spinor. The spinor η can be used to construct the holomorphic $(3,0)$ -form Ω and the Kähler form J as follows

$$J_{mn} = -i\eta^\dagger \gamma_{mn} \eta \quad \Omega_{mnp} = e^{\frac{\phi}{2}} \eta^T \gamma_{mnp} \eta \quad (2.8)$$

These satisfy the normalisation condition

$$\frac{1}{3!} J \wedge J \wedge J = \frac{i}{8} e^{-\phi} \Omega \wedge \bar{\Omega} = \text{dvol}_X \quad (2.9)$$

where the volume form dvol_X is computed by using the internal Kähler metric $\text{d}s_X^2$.²

It is important to observe that this description of the background is intrinsically redundant. Indeed, one can always perform a constant rescaling of the four-dimensional metric

$$\text{d}s_{\mathbb{R}^{1,3}}^2 \rightarrow e^{-2\omega} \text{d}s_{\mathbb{R}^{1,3}}^2 \quad (2.10)$$

if accompanied by $e^{2D} \rightarrow e^{2D+2\omega}$ and $\text{d}s_X^2 \rightarrow e^{2\omega} \text{d}s_X^2$. Furthermore, in the spinorial ansatz (2.7) one can perform a constant phase change $\zeta_R \rightarrow e^{\frac{i}{2}\alpha} \zeta_R$, if accompanied by $\eta \rightarrow e^{-\frac{i}{2}\alpha} \eta$. Under such reparametrizations Ω transforms as

$$\Omega \rightarrow e^{3\omega - i\alpha} \Omega \quad (2.11)$$

while $J \rightarrow e^{2\omega} J$.

3 Kähler potential from warped volume

In this section we discuss how the Kähler potential for warped F-theory/IIB compactifications is directly related to the warped volume of the internal space. The Kähler potential

¹In general, the p -form current δ_Σ^p associated with a $6-p$ surface Σ is such that, for any $(6-p)$ -form α on X , $\int_\Sigma \alpha = \int_X \alpha \wedge \delta_\Sigma^p$.

²In presence of 7-branes, Ω cannot be considered as a section of the canonical bundle $\mathcal{K}_X$. Rather, it is a section of $\mathcal{K}_X \otimes \mathcal{L}_Q$, where $\mathcal{L}_Q$ is a holomorphic line bundle associated with the non-trivial holomorphic τ . Supersymmetry requires that $\mathcal{L}_Q \simeq \mathcal{K}_X^{-1}$ so that Ω is indeed globally defined and no-where vanishing.

that we obtain has the same form of the Kähler potential proposed in [10, 13]. Our argument is based on local superconformal symmetry as in [30, 31]. It is morally very close to arguments presented in other papers, as for instance in [23, 36–39], in which the Kähler potential is also directly related to the volume-like pre-factor appearing in the dimensionally reduced Einstein-Hilbert term. On the other hand, the local superconformal symmetry provides a precise scheme in which this conclusion is reached in a neat and natural way.

3.1 The conformal Kähler potential

The backgrounds described in section 2 are characterised by several moduli, which in this section we collectively denote by u^a . In order to obtain the four-dimensional effective theory one should allow for various components of the ten-dimensional ansatz to be slowly varying functions of the external coordinates. In particular, the external flat metric $ds_{\mathbb{R}^{1,3}}^2$ and the moduli must be promoted to dynamical fields $g_{\mu\nu}(x)dx^\mu dx^\nu$ and $u^a(x)$ respectively. Then, the vacuum metric ansatz (2.1) must be generalised to

$$ds_{10}^2 = e^{2D(y;u)} g_{\mu\nu} dx^\mu dx^\nu + e^{-2D(y;u)} ds_X^2(y;u) + \dots \quad (3.1)$$

We have shown only the terms that correspond to the naive generalisation of (2.1). In fact, as discussed for instance in [13, 25, 26, 40], in presence of warping these two terms do not constitute a consistent KK ansatz by themselves, but must be supplemented by additional compensating terms (often referred to as ‘compensators’), which depend on the derivatives of the space-time dependent moduli $\partial_\mu u^a(x)$. These terms should then fill in the ellipses appearing on the right-hand side of (3.1).

Now, in presence of warping, a standard dimensional reduction is generically quite cumbersome, in particular because of the compensating terms. However, since the vacua we start from are supersymmetric, one already knows that the four-dimensional theory must be supersymmetric. Furthermore, crucially, in the dynamical ansatz (3.1), the scaling symmetry (2.10) is promoted to a gauge symmetry under arbitrary Weyl transformations $g_{\mu\nu} \rightarrow e^{-2\omega(x)} g_{\mu\nu}$. An analogous conclusion holds for the arbitrary phase transformation of the spinors entering the dynamical generalisation of (2.7). Hence, in particular (2.11) is generalised to

$$\Omega \rightarrow e^{3\omega(x) - i\alpha(x)} \Omega \quad (3.2)$$

We then see that the effective four-dimensional theory must be gauge invariant under such complexified Weyl transformation. This tells us that the effective theory can be naturally regarded as a superconformal supergravity. By definition, fields transforming by a phase $e^{w\omega(x) + i c\alpha(x)}$ under the above Weyl-chiral symmetry are said to have Weyl-chiral weight (w, c) . Hence Ω has Weyl-chiral weight $(3, -1)$. We refer to [41] for an exhaustive discussion on $N = 1$ superconformal supergravities.

One of the nice features of superconformal supergravity is that the Einstein-Hilbert term as well as the scalars’ kinetic terms all derive from a D-term of the schematic form

$$- 3 \int d^4\theta \mathcal{N}(\Phi, \bar{\Phi}) \quad (3.3)$$

Here Φ^I denote the conformal chiral multiplets and we refer to $\mathcal{N}(\Phi, \bar{\Phi})$ as the conformal Kähler potential. Consistency requires $\mathcal{N}(\Phi, \bar{\Phi})$ to have Weyl-chiral weight $(2, 0)$.

A discussion of the precise meaning of (3.3) can be found in [41]. Here we just need few bosonic terms which appear once (3.3) is written in components:

$$\frac{1}{2}\mathcal{N}R_4 + 3\mathcal{N}_{I\bar{J}}\partial_\mu\Phi^I\partial^\mu\bar{\Phi}^{\bar{J}} + \dots \quad (3.4)$$

where $\mathcal{N}_{I\bar{J}}(\Phi, \bar{\Phi}) \equiv \partial_I\bar{\partial}_{\bar{J}}\mathcal{N}(\Phi, \bar{\Phi})$ and we use the same symbol for chiral multiplets as well as for their scalar components.

By appropriately performing the KK reduction one should be able to recover the terms appearing in (3.4). Of course, in order to reconstruct the scalars' kinetic terms one should properly take into account the compensating terms, which contain the derivatives of the moduli $\partial_\mu u^a$. However, there is a clear short-cut. The scalars' kinetic terms are defined by the conformal Kähler potential $\mathcal{N}(\Phi, \bar{\Phi})$, which also appears as the prefactor of the Einstein term in (3.4). Since such term does not contain space-time derivatives of the chiral fields, one can obtain $\mathcal{N}$ by plugging the naive ansatz (3.1) in the Einstein term of the ten-dimensional type IIB action

$$\frac{2\pi}{\ell_s^8} \int \sqrt{-g_{10}} R_{10}, \quad (3.5)$$

simply ignoring the problematic compensating terms. In this way, one easily gets

$$\mathcal{N} = \frac{4\pi}{\ell_s^8} \int_X e^{-4D} \text{dvol}_X \quad (3.6)$$

Of course, this is not the end of the story, since (3.6) provides only a rather implicit formula for $\mathcal{N}(\Phi, \bar{\Phi})$.³

3.2 The conformal compensator and the Kähler potential

Part of the possible background moduli correspond to the deformations of the axio-dilaton τ , which encodes the value of the string coupling as well as the seven-brane positions, and of the holomorphic $(3,0)$ form Ω , which specifies the complex structure of the Kähler space X . The dual M-theory perspective elegantly unifies these data into the holomorphic $(4,0)$ form associated with the dual elliptically fibered Calabi-Yau four-fold. Although these moduli could in principle be included in the discussion, we assume them to be stuck at a given value. This condition can be indeed dynamically enforced by the presence of fluxes. This means that τ and Ω are fixed, up to the latter's overall normalisation, which cannot be fixed basically because of the gauge invariance (3.2). It is then convenient to isolate such degree of freedom by writing

$$\Omega \equiv \ell_s^6 Y^3 \Omega_0 \quad (3.7)$$

³In [30] a holomorphic parametrisation of the generic (massless and massive) fluctuations was introduced, which used the formalism of generalised complex geometry. By using this parametrisation, it was shown how the conformal Kähler potential (3.6) and an associated GVW-like superpotential [42] reproduce the *complete* set of ten-dimensional equations of general $N = 1$ compactifications to Mink₄ and AdS₄ found in [43].

where Y is a completely arbitrary complex parameter which transforms with Weyl-chiral weight $(1, -\frac{1}{3})$

$$Y \rightarrow e^{\omega(x) - \frac{1}{3}\alpha(x)} Y \quad (3.8)$$

while Ω_0 is some fiducial dimensionless holomorphic $(3, 0)$ form, satisfying a normalisation condition

$$\frac{i}{8} \int_X e^{-\phi} \Omega_0 \wedge \bar{\Omega}_0 = v_0 \quad (3.9)$$

with v_0 being a dimensionless constant.⁴ For instance, one may set $v_0 = 1$.

The complex parameter Y appears as a chiral field in the low-energy superconformal effective theory and can be used as conformal compensator to gauge fix the theory to a standard Poincaré supergravity. A detailed description of the procedure can be found in [41] and we will be sketchy.

First, one can isolate the dependence of $\mathcal{N}$ on the conformal compensator by writing it as

$$\mathcal{N} = |Y|^2 e^{-\frac{1}{3}K(\varphi, \bar{\varphi})} \quad (3.10)$$

where φ^A denote chiral fields of Weyl-chiral weight $(0, 0)$ parametrising the background moduli. Then, one can gauge-fix the superconformal symmetry by imposing

$$Y = M_P e^{\frac{1}{6}K(\varphi, \bar{\varphi})} \quad (3.11)$$

This implies that $\mathcal{N} \equiv M_P^2$ and the first term in (3.4) reduces to the standard Einstein term $\frac{1}{2}M_P^2 R_4$. On the other hand, after the gauge fixing (3.11) the kinetic terms appearing in (3.4) produce the more familiar kinetic terms

$$- M_P^2 K_{A\bar{B}}(\varphi, \bar{\varphi}) \partial_\mu \varphi^A \partial^\mu \bar{\varphi}^{\bar{B}} \quad (3.12)$$

where $K_{A\bar{B}} \equiv \partial_A \bar{\partial}_{\bar{B}} K$. Hence the function $K(\varphi, \bar{\varphi})$ introduced in (3.10) can be identified with the Kähler potential of standard $N = 1$ supergravity.

In order to identify the ten-dimensional expression for K we observe that the redefinition (3.7) is naturally associated with the redefinitions $e^{2D} \equiv \ell_s^2 |Y|^2 e^{2A}$, $ds_X^2 \equiv \ell_s^4 |Y|^2 ds_{X,0}^2$ and $J \equiv \ell_s^4 |Y|^2 J_0$, so that the ten-dimensional metric now reads

$$ds_{10}^2 = \ell_s^2 e^{2A} |Y|^2 ds_4^2 + \ell_s^2 e^{-2A} ds_{X,0}^2 \quad (3.13)$$

where the metric $ds_{X,0}^2$ still satisfies (2.2). Notice that $d\text{vol}_{X,0} \equiv \frac{1}{3!} J_0 \wedge J_0 \wedge J_0$ can be considered as a fiducial volume-form with normalisation fixed by (3.9):

$$\int_X d\text{vol}_{X,0} = \frac{1}{3!} \int_X J_0 \wedge J_0 \wedge J_0 = v_0 \quad (3.14)$$

In practice, this normalisation condition removes the overall rescaling from the set of possible deformations of the Kähler form J_0 . Such rescaling parametrises the universal modulus in standard unwarped compactifications. On the other hand, as we will review in the next

⁴The factor $e^{-\phi}$ is necessary to make the integrand a proper volume-form even in presence of $\text{SL}(2, \mathbb{Z})$ monodromies.

section, in warped backgrounds the universal modulus is encoded in the warp factor and then no physical degree of freedom is lost in the normalisation condition (3.14).

We can now write the (implicit) form of the Kähler potential $K(\varphi, \bar{\varphi})$. From (3.10) and the above field redefinitions one obtains

$$\begin{aligned} K(\varphi, \bar{\varphi}) &= -3 \log \left(4\pi \int_X e^{-4A} \text{dvol}_{X,0} \right) \\ &\equiv -3 \log \left(\frac{1}{3!} \int_X e^{-4A} J_0 \wedge J_0 \wedge J_0 \right) - 3 \log 4\pi \end{aligned} \quad (3.15)$$

This Kähler potential has the same form as the Kähler potential proposed in [10, 13].

4 Universal modulus and Kähler potential

In this section we show that the Kähler potential (3.15) admits a simple expression in terms of the universal modulus. So, we start by reviewing the origin of the universal modulus in warped F-theory/IIB compactifications. First notice that the rescaled warp-factor e^{2A} introduced in the previous section satisfies an equation formally identical to (2.6),

$$\Delta_0 e^{-4A} = \frac{1}{\ell_s^4} *_0 Q_6 \equiv \frac{1}{\ell_s^4} \left(\frac{1}{2\text{Im}\tau} G_3 \cdot \bar{G}_3 + *_0 Q_6^{\text{loc}} \right) \quad (4.1)$$

where all quantities are now computed by using the rescaled metric $ds_{X,0}^2$.

The universal modulus is readily identified by looking at (4.1) and noticing that it completely determines e^{-4A} up to an additive constant. One can identify the universal modulus a with such a constant, by writing

$$e^{-4A} = a + e^{-4A_0} \quad (4.2)$$

where e^{-4A_0} is a particular solution of (4.1). Notice that the split (4.2) is not unique as one could shift $a \rightarrow a + c$ and $e^{-4A_0} \rightarrow e^{-4A_0} - c$. One can fix such redundancy by imposing

$$\int_X e^{-4A_0} \text{dvol}_{X,0} = v_0^w \quad (4.3)$$

for an arbitrary constant v_0^w . Hence, recalling (3.14), we can write the Kähler potential (3.15) as follows

$$K = -3 \log(v_0 a + v_0^w) - 3 \log 4\pi \quad (4.4)$$

Consider now the particular solution $e^{-4\hat{A}_0}$ of (4.1) provided by the Green's operator of the Laplacian Δ_0 . The associated Green's function $G(y; y')$ satisfies the equation

$$\Delta_{0,y} G(y; y') = *_0 \delta_y^6(y') - \frac{1}{v_0} \quad (4.5)$$

The particular solution $e^{-4\hat{A}_0}$ is then given by

$$e^{-4\hat{A}_0(y)} = \frac{1}{\ell_s^4} \int_X G(y; y') Q_6(y') \quad (4.6)$$

By definition, the Green's operator vanishes on constant functions and maps coexact functions to coexact functions, see for instance [44]. This implies that $e^{-4\hat{A}_0(y)}$ defined by (4.6) is a coexact function on X , so that

$$\hat{v}_0^w \equiv \int_X e^{-4\hat{A}_0} \text{dvol}_{X,0} = \frac{1}{3!} \int_X e^{-4\hat{A}_0} J_0 \wedge J_0 \wedge J_0 \equiv 0 \quad (4.7)$$

The particular solution (4.6) is associated with a particular parametrisation $\hat{a}$ of the universal modulus, defined by

$$e^{-4A} = \hat{a} + e^{-4\hat{A}_0} \quad (4.8)$$

By using this specific split of e^{-4A} and (4.7), the Kähler potential (3.15) takes the form

$$K = -3 \log(v_0 \hat{a}) - 3 \log 4\pi \quad (4.9)$$

We then see that the Kähler potential (3.15) reduces to a simple function of just the universal modulus $\hat{a}$ (or a). On the other hand, the formula (4.9) (or (4.4)) is still implicit. In order to make it explicit we need to understand how the universal modulus $\hat{a}$ (or a) is parametrised in terms of appropriate holomorphic coordinates φ^A . This issue will be addressed in the following sections.

Notice that we have been careful in not dropping out additional constants in K , which are irrelevant in the computation of the Kähler metric. Nevertheless, such terms could be relevant for the computation of other quantities, as the potential

$$V_{4D} = \frac{1}{M_P^2} e^K \left(K^{A\bar{B}} D_A W D_{\bar{B}} \bar{W} - 3|W|^2 \right) \quad (4.10)$$

which depends on the normalisation of the superpotential W as well. For instance, with our choice of the Kähler potential, the classical GVW superpotential [5, 42] has the following specific normalisation [30]

$$W_{\text{tree}} = \frac{\pi M_P^3}{\ell_s^2} \int_X \Omega_0 \wedge G_3 \quad (4.11)$$

For later convenience, let us recall that (4.11) depends just on the axio-dilaton, complex-structure moduli and seven-brane moduli, and is at the origin of their stabilisation, which we assume. Hence, in the present paper W_{tree} can be considered as constant, $W_{\text{tree}} \equiv W_0$. For supersymmetric vacua, i.e. with $(2, 1)$ and primitive G_3 , $W_0 = 0$. On the other hand, as reviewed in section 2, supersymmetry is broken if the ISD G_3 has a non-vanishing $(0, 3)$ component, still preserving the ten-dimensional equations of motion. In this case $W_0 \neq 0$.

5 Decoding the Kähler potential

In order to interpret (4.9), or (4.4), as a Kähler potential, it is necessary to identify an appropriate holomorphic parametrisation φ^A of the background moduli. A key observation is that the background can be ‘probed’ by supersymmetric Euclidean D3-branes (or vertical M5-branes in the dual M-theory description). It is well known that these branes enter the path-integral as instantonic corrections and can generate new F-terms in the effective

action. These terms contain a universal factor $e^{-S_{\text{D3}}}$, where S_{D3} is the Euclidean on-shell D3-brane action. Crucially, by four-dimensional supersymmetry, S_{D3} must depend holomorphically on the chiral fields φ^A .⁵

Now, the bosonic Euclidean D3-brane action can be written as

$$S_{\text{D3}} = S_{\text{DBI}} - iS_{\text{CS}} \quad (5.1)$$

where S_{DBI} and S_{CS} are the standard Dirac-Born-Infeld (DBI) and Chern-Simons (CS) terms. In fact, in presence of warping and fluxes, the definition of S_{CS} is problematic, since the R-R potentials are not globally defined, see for instance [45] for a discussion in the analogous case of heterotic compactifications.

On the other hand, S_{DBI} has no such ambiguities and one can in principle compute its dependence on the background moduli. At the same time S_{DBI} must be the real part of a chiral field. Hence, it can be used to identify the dependence of the real part of a set of chiral fields on the background moduli. As we will see, this is sufficient to implicitly define $\hat{a}$, and then the Kähler potential (4.9), as functions of chiral fields.

In this section we apply this strategy by assuming that no two-form axionic moduli and seven-brane Wilson lines are present. This condition is better expressed in the dual elliptically fibered Calabi-Yau four-fold $\hat{X}$, by requiring that $b^3(\hat{X}) - b^3(X) = 0$, see e.g. [23]. The inclusion of axions will be discussed in section 8.

Hence, in addition to the universal modulus $\hat{a}$ discussed in section 4, we allow for three other kinds of moduli. The $h^{1,1}(X) - 1$ Kähler moduli describing the deformations of J_0 that preserve the normalisation condition (3.14),⁶ the $h^{2,2}(X) = h^{1,1}(X)$ moduli describing the C_4 axions, and the $6 \times N_{\text{D3}}$ (real) moduli describing the positions of the N_{D3} D3-branes in the internal space. The latter have a natural holomorphic parametrisation provided by the bulk complex coordinates z^i , $i = 1, 2, 3$. Hence, we denote the chiral fields describing the D3-brane positions by Z_I^i , with $I = 1, \dots, N_{\text{D3}}$. We then need to understand how the remaining moduli organise into chiral fields.

Let us introduce a set of integral closed two-forms ω^a , whose associated cohomology classes $[\omega_a]$ provide a basis of $H^2(X; \mathbb{Z})$, and expand the Kähler form J_0 in cohomology as follows

$$[J_0] = v_a [\omega^a] \quad (5.2)$$

Then the constraint (3.14) can be written as

$$\frac{1}{3!} v_a v_b v_c \mathcal{I}^{abc} = v_0 \quad (5.3)$$

where we have introduced the intersection matrix

$$\mathcal{I}^{abc} \equiv \int_X \omega^a \wedge \omega^b \wedge \omega^c \quad (5.4)$$

⁵Notice that the following discussion can be straightforwardly applied to seven-brane four-dimensional gauge-couplings as well. Indeed, by dimensionally reducing the effective action of a D7-branes (or, by $\text{SL}(2, \mathbb{Z})$ duality, of a more general seven-brane) wrapped on a divisor D , one readily gets the complexified gauge coupling $\tau_{\text{YM}} = \frac{1}{2\pi} S_{\text{D3}}$.

⁶We are implicitly assuming that the proper orientifold projections are satisfied and we are ignoring for simplicity the possible lift of Kähler moduli induced by flux primitivity conditions. These aspects can be easily taken into account, as in the concrete example on $T^6/\mathbb{Z}_2$ discussed in appendix B.

The constraint (5.3) identifies $h^{1,1} - 1$ Kähler moduli out of the $h^{1,1}$ parameters v_a . Hence we will refer to v_a as *constrained* Kähler moduli.

Now, a supersymmetric D3-brane must wrap a homographic four-cycle (an effective divisor) D , with anti-self-dual world-volume flux $\mathcal{F} = \frac{1}{2\pi}\ell_s^2 F_{D3} - B_2|_D$. These conditions are equivalent to requiring that the D3-brane is calibrated in the generalised sense of [46], so that the associated on-shell DBI action reduces to⁷

$$S_{\text{DBI}} = \pi \int_D e^{-4A} J_0 \wedge J_0 - \frac{\pi}{\ell_s^4} \int_D e^{-\phi} \mathcal{F} \wedge \mathcal{F} \quad (5.5)$$

Notice that the second term does not depend on the background moduli we are considering in this section. So, for the moment, it is a constant and we can ignore it.

Now, there are $h^{1,1}(X)$ independent divisors D^a and we choose the $[\omega^a] \in H^2(X; \mathbb{Z})$ to be Poincaré dual to such divisors:

$$[\omega^a] = \text{PD}_X(D^a) \quad (5.6)$$

By wrapping D3-branes along linear combinations of the divisors D^a , the associated S_{DBI} 's can detect all geometric moduli, i.e. universal modulus as well as all $h^{1,1}(X) - 1$ constrained Kähler moduli v_a . On the other hand S_{CS} would detect the $h^{1,1}(X)$ R-R axions, which should complexify the geometric moduli. However, since the Kähler potential (4.9) (or (4.4)) depends just on geometric moduli, we do not actually need the information captured by S_{CS} .

We can then focus on the integrals

$$I^a \equiv \frac{1}{2} \int_{D^a} e^{-4A} J_0 \wedge J_0 \quad (5.7)$$

According to the arguments presented above, these must correspond to $h^{1,1}(X)$ chiral fields ρ^a through a relation of the form

$$I^a = \text{Re} \rho^a + (\text{hol} + \overline{\text{hol}}) \quad (5.8)$$

where the $(\text{hol} + \overline{\text{hol}})$ contribution is the real part of some holomorphic quantity depending only on the remaining chiral fields.

Of course, deciding what to include in $(\text{hol} + \overline{\text{hol}})$ and what to absorb in the definition of $\text{Re} \rho^a$ is a matter of choice. Indeed, possible different choices are related by a redefinition of the form $\rho^a \rightarrow \rho^a + \text{hol}$, which preserves the holomorphic parametrisation of the chiral fields. In the following, our choice will be the minimal one. Namely, whenever we will be able to isolate a contribution to I^a of the form $\text{hol} + \overline{\text{hol}}$ (not depending on $\hat{a}$ and v_a) we will omit it from the definition of $\text{Re} \rho^a$.

⁷See [47–49] for more details on the supersymmetric structure of D3-brane instantons in F-theory compactifications, in the IIB framework adopted in the present paper.

5.1 Decoding the moduli dependence

In order to proceed, we need to decode the dependence of the integrals (5.7) on the background moduli. Let us first use the decomposition (4.8) and write I^a as

$$I^a \equiv \hat{a} \mathcal{V}^a(v) + \frac{1}{2} \int_{D^a} e^{-4\hat{A}_0} J_0 \wedge J_0 \quad (5.9)$$

where

$$\mathcal{V}^a(v) \equiv \frac{1}{2} \int_{D^a} J_0 \wedge J_0 = \frac{1}{2} \mathcal{I}^{abc} v_b v_c \quad (5.10)$$

The effect of warping is encoded in the second term on the r.h.s. of (5.9), which can depend on the constrained Kähler moduli v_a and on the D3-brane moduli Z_I^i . In order to learn more about such dependence let us write the second term on the r.h.s. of (5.9) as

$$\frac{1}{2} \int_X e^{-4\hat{A}_0} J_0 \wedge J_0 \wedge \delta_{D^a}^2 = \int_X e^{-4\hat{A}_0} (J_0 \lrcorner \delta_{D^a}^2) \text{dvol}_{X,0} \quad (5.11)$$

where $\delta_{D^a}^2$ is the two-form current (cf. footnote 1) associated with the divisor D^a and

$$J_0 \lrcorner \delta_{D^a}^2 \equiv \frac{1}{2} J_0^{mn} (\delta_{D^a}^2)_{mn} \quad (5.12)$$

Now, the localised term $J_0 \lrcorner \delta_{D^a}^2$ admits a nice explicit realisation in terms of the section $\zeta^a(z)$ of the line bundle $\mathcal{O}_X(D^a)$ whose vanishing locus identifies D^a : $D^a = \{\zeta^a = 0\}$. Indeed, the Poincaré-Lelong equation

$$i\partial\bar{\partial} \text{Re}(\log \zeta^a) = \pi \delta_{D^a}^2 \quad (5.13)$$

implies that

$$J_0 \lrcorner \delta_{D^a}^2 = -\frac{1}{2\pi} \Delta_0 \text{Re} \log \zeta^a(z) \quad (5.14)$$

Here we have used the fact that, since the internal space is Kähler, the Laplacian can be written as

$$\Delta_0 = -2g^{i\bar{j}} \partial_i \bar{\partial}_{\bar{j}} = -2J_0 \lrcorner (i\partial\bar{\partial}) \quad (5.15)$$

By plugging (5.14) into (5.11), one could be tempted to integrate by parts Δ_0 and then use (4.1). However, such procedure would not be admissible, since $\zeta^a(z)$ is a section of a non-trivial line bundle and then one would end with a meaningless expression.

In order to find a way out of this problem, let us choose the integral two-forms ω^a to be (1,1) and *harmonic*, which is always possible within their cohomology class. Since J_0 is harmonic, we can promote the cohomological identity (5.2) to the pointwise identity

$$J_0 = v_a \omega^a \quad (5.16)$$

It is important to realise that, in general, the ω^a 's actually depend on the constrained Kähler moduli v_a .

Now, by the $\partial\bar{\partial}$ -lemma we can introduce a set of $h^{1,1}(X)$ local ‘potentials’ $\kappa^a(z, \bar{z}; v)$, which generically depend also on the v_a 's, such that

$$\omega^a = i\partial\bar{\partial} \kappa^a \quad (5.17)$$

Clearly the κ^a 's cannot be extended to global functions, since the ω^a 's are non-trivial in cohomology. Indeed, $e^{-2\pi\kappa^a}$ can be identified with a metric on the line bundle $\mathcal{O}_X(D^a)$ associated with the divisor D^a , whose curvature is given by $2\pi\omega^a$, see e.g. [50]. More explicitly, let us consider a holomorphic transformation

$$\zeta^a(z) \rightarrow e^{2\pi\chi^a(z)}\zeta^a(z) \quad (5.18)$$

that relates the expressions of $\zeta^a(z)$ on different local patches. Then, $\kappa^a(z, \bar{z}; v)$ must correspondingly transform as

$$\kappa^a(z, \bar{z}; v) \rightarrow \kappa^a(z, \bar{z}; v) + \chi^a(z) + \bar{\chi}^a(\bar{z}) \quad (5.19)$$

Hence, the combination

$$\pi\kappa^a(z, \bar{z}; v) - \text{Re} \log \zeta^a(z) \equiv -\frac{1}{2} \log e^{-2\pi\kappa^a} |\zeta^a|^2 \quad (5.20)$$

is a globally well defined function on X .

Notice now that, since we have chosen the closed integral two-forms ω^a to be harmonic, the functions $J_0 \lrcorner \omega^a$ are harmonic too, i.e. they are constant along X (although in general they depend on the constrained Kähler moduli v_a). By setting $d^a(v) \equiv J_0 \lrcorner \omega^a$, we can rewrite (5.14) as

$$J_0 \lrcorner \delta_{D^a}^2 = \frac{1}{2\pi} \Delta_0 \left[\pi\kappa^a(z, \bar{z}; v) - \text{Re} \log \zeta^a(z) \right] + d^a(v) \quad (5.21)$$

We see that we have obtained a formula for $J_0 \lrcorner \delta_{D^a}^2$ in terms of the globally well defined function (5.20). By plugging (5.21) into (5.11), the contribution containing the constant $d^a(v)$ vanishes because of (4.7), and we are left with

$$\frac{1}{2\pi} \int_X e^{-4\hat{A}_0} \Delta_0 (\pi\kappa^a - \text{Re} \log \zeta^a) \text{dvol}_{X,0} \quad (5.22)$$

Now we are legitimised to integrate by parts the Laplacian and, by using (4.1), we arrive at the following remarkable identity

$$\frac{1}{2} \int_{D^a} e^{-4\hat{A}_0} J_0 \wedge J_0 = \frac{1}{2\pi\ell_s^4} \int_X (\pi\kappa^a - \text{Re} \log \zeta^a) Q_6 \quad (5.23)$$

We recall that the six-form Q_6 denotes the D3-brane charge density and must satisfy the tadpole condition

$$\int_X Q_6 = 0 \quad (5.24)$$

Hence, even though both κ^a and $\log \zeta^a$ are defined up to an additive constant, the identity (5.23) as well as all results that will follow from it are not affected by this ambiguity.

One can isolate the contribution of the mobile D3-branes to Q_6 by writing

$$Q_6 \equiv \ell_s^4 \sum_I \delta_I^6 + Q_6^{\text{bg}} \quad (5.25)$$

where Q_6^{bg} is the D3-charge 6-form induced by bulk fluxes, O3-planes as well as other localised D3-charge sources:

$$Q_6^{\text{bg}} = F_3 \wedge H_3 - \frac{1}{4} \ell_s^4 \sum_{O \in \text{O3's}} \delta_O^6 + \dots \quad (5.26)$$

By plugging (5.25) into (5.23) and inserting the result back in (5.9), we finally arrive at the following more explicit form for I^a :

$$I^a(v, Z, \bar{Z}) = \hat{a} \mathcal{V}^a(v) + h^a(v) + \frac{1}{2} \sum_I \kappa^a(Z_I, \bar{Z}_I; v) - \frac{1}{2\pi} \text{Re} \log \left[\prod_I \zeta^a(Z_I) \right] \quad (5.27)$$

where⁸

$$h^a(v) \equiv \frac{1}{2\pi \ell_s^4} \int_X (\pi \kappa^a - \text{Re} \log \zeta^a) Q_6^{\text{bg}} \quad (5.28)$$

We stress that I^a depends on the specific choice of the divisor D^a , since $h^a(v)$ depends on the associated holomorphic section $\zeta^a(z)$. The choice of a different divisor $\tilde{D}^a$, homologous to D^a , would correspond to a different section $\tilde{\zeta}^a(z)$, and then to a different warping-induced contribution:

$$\tilde{I}^a(v, Z, \bar{Z}) = I^a(v, Z, \bar{Z}) + \frac{1}{2\pi \ell_s^4} \int_X \text{Re}(\log \zeta^a - \log \tilde{\zeta}^a) Q_6 \quad (5.29)$$

The second term on the r.h.s. contributes to the $(\text{hol} + \overline{\text{hol}})$ part in the prescription (5.8) and can then be reabsorbed in a holomorphic redefinition of the chiral fields ρ^a .

5.2 Chiral fields and Kähler potential

We can now come back to the prescription (5.8) to express the set of chiral fields ρ^a , or more precisely their real part, in terms of the background moduli. Indeed, by (5.27) we are naturally led to the following identification

$$\text{Re} \rho^a = \hat{a} \mathcal{V}^a(v) + h^a(v) + \frac{1}{2} \sum_I \kappa^a(Z_I, \bar{Z}_I; v) \quad (5.30)$$

Notice that, by consistency, under the transformations (5.19) ρ^a must transform as

$$\rho^a \rightarrow \rho^a + \sum_I \chi^a(Z_I) \quad (5.31)$$

Hence the chiral fields ρ^a are non-trivially fibered over the moduli space $\mathcal{M}_{\text{D3}}$ of the D3-branes⁹ which can be identified with N_{D3} copies of the internal space X , modded out by

⁸Notice that $h^a(v)$ diverges to $-\infty$ if the divisor D^a touches some O3-planes. This is a consequence of the localized negative tension associated with the O3-planes. This is expected to be an artefact of the effective supergravity description, which should be cured by some higher-order or non-perturbative physical effect, as it happens for O7-branes in F-theory. In any case, one may judiciously choose the divisors D^a , in order to avoid this issue.

⁹See for instance [12].

the permutation group $S_{N_{D3}}$. In other words, the total moduli space $\mathcal{M}_{\text{tot}}$ is given by a non-trivial fibration over $\mathcal{M}_{D3}$.

Now, the equations (5.30) implicitly define $\hat{a}$ and v_a as functions of $\text{Re}\rho^a$, Z_I^i and $\bar{Z}_I^{\bar{i}}$. In particular, from (4.9), one can in principle write

$$K(\rho, \bar{\rho}, Z, \bar{Z}) = -3 \log [v_0 \hat{a}(\text{Re}\rho, Z, \bar{Z})] + c_0 \quad (5.32)$$

where

$$c_0 = -3 \log 4\pi \quad (5.33)$$

Of course, the main difficulty is the identification of the explicit form of the function $\hat{a}(\text{Re}\rho, Z, \bar{Z})$, for which unfortunately there is no general solution. Simple cases in which one can be more explicit are discussed in the following. Nevertheless, as we will see in section 6, one can compute the effective kinetic terms by using just (5.30) and (5.32).

In order to make the relation with the literature on unwarped compactifications more immediate, one can combine $\hat{a}$ and the constrained Kähler moduli v_a into the following $h^{1,1}(X)$ unconstrained moduli

$$\hat{v}_a = v_a \sqrt{a} \quad (5.34)$$

These can be considered as moduli of an *auxiliary* Kähler form $\hat{J} = \hat{v}_a \omega^a = \sqrt{a} J_0$, which would have a real geometrical interpretation only if one could consistently neglect $e^{-4\hat{A}_0}$ with respect to $\hat{a}$ in (4.8), i.e. only if the warping could be considered constant.

By using the moduli $\hat{v}_a$ the Kähler potential (5.32) can be written as

$$K(\rho, \bar{\rho}, Z, \bar{Z}) = -2 \log \left(\frac{1}{3!} \mathcal{I}^{abc} \hat{v}_a \hat{v}_b \hat{v}_c \right) - \log v_0 + c_0 \quad (5.35)$$

while the relations (5.30) become

$$\text{Re}\rho^a = \frac{1}{2} \mathcal{I}^{abc} \hat{v}_b \hat{v}_c + h^a(\hat{v}) + \frac{1}{2} \sum_I \kappa^a(Z_I, \bar{Z}_I; \hat{v}) \quad (5.36)$$

Here we have used the fact that we can write $\kappa^a(z, \bar{z}; v) = \kappa^a(z, \bar{z}; \hat{v})$ and $h^a(v) = h^a(\hat{v})$, since the harmonicity condition on ω^a does not change if it is defined by $\hat{J}$ instead of J_0 .

The description of the Kähler potential provided by (5.35) and (5.36) is formally closer to the one obtained in [19, 20, 23] in the constant warping approximation. Basically, the Kähler potential can be casted in the universal form (5.35), while the new ingredients like D3-branes and fluxes are completely encoded in the functional dependence $\hat{v}_a = \hat{v}_a(\text{Re}\rho, Z, \bar{Z})$ which should be obtained by inverting (5.36). In particular, in [19] an analogous contribution of the D3-branes was obtained by using the probe D3-brane effective action expanded around a given point. In our language, such contribution can be reproduced by expanding $Z_I^i \simeq Z_{(0)I}^i + \Phi_I^i$ and approximating $\kappa^a(Z_I, \bar{Z}_I; \hat{v}) \simeq -i\omega_{i\bar{j}}(Z_{(0)I}, \bar{Z}_{(0)I}) \Phi_I^i \bar{\Phi}_I^{\bar{j}}$.

On the other hand, to best of our knowledge, the contribution of the background fluxes and of the other localized D3-charge sources to the definition of the chiral coordinates had never been explicitly considered so far. This is completely encoded in the functions $h^a(v) \equiv h^a(\hat{v})$. As we will see in section 6, the presence of such terms is crucial in the derivation of the (warped) kinetic terms of the effective action.

As a final comment, notice that the relations (5.30), or (5.36), involve the background complex structure and, more implicitly, the seven-brane positions, whose induced D3-charge would contribute to Q_6^{bg} . In this paper we are assuming the corresponding moduli to be frozen. However, in any more complete treatment that includes (part of) these moduli as dynamical, they would mix with the remaining moduli in a non-trivial way, producing a non-diagonal Kähler potential, see also [19, 30, 31].

5.3 A comment on D3-brane instantons

Consider a supersymmetric Euclidean D3-brane wrapping a divisor D , associated with a holomorphic section $\zeta_D(z)$. In homology, we can expand $D \simeq n_a D^a$. Proceeding as in subsection 5.1 and using (5.30), one arrives at the identity

$$\frac{1}{2} \int_D e^{-4A} J_0 \wedge J_0 = n_a \text{Re} \rho^a - \frac{1}{2\pi} \text{Re} \sum_I \log \zeta_D(Z_I) + \frac{1}{2\pi \ell_s^4} \int_X \text{Re} (n_a \log \zeta^a - \log \zeta_D) Q_6^{\text{bg}} \quad (5.37)$$

Hence, the factor $e^{-S_{\text{D3}}}$ appearing in the associated non-perturbative F-terms is proportional to

$$\mathcal{A}^{\text{bg}} \left[\prod_I \zeta_D(Z_I) \right] e^{-2\pi n_a \rho^a} \quad (5.38)$$

where we have introduced

$$\mathcal{A}^{\text{bg}} = \exp \left[\frac{1}{\ell_s^4} \int_X (\log \zeta_D - n_a \log \zeta^a) Q_6^{\text{bg}} \right] \quad (5.39)$$

We have added an overall phase in order to complexify $\mathcal{A}^{\text{bg}}$ in a way naturally compatible with the bulk complex structure.

The combination (5.38) is well defined on the total moduli space $\mathcal{M}_{\text{tot}}$, since $\zeta_D(z)$ is a section of $\mathcal{O}_X(D) \simeq \mathcal{O}_X(n_a D^a)$, while from (5.31) it follows that $e^{-2\pi n_a \rho^a}$ transforms as a section of $\mathcal{O}_X(-n_a D^a) \simeq \mathcal{O}_X(-D)$.

In particular, the presence of the factor $\prod_I \zeta_D(Z_I)$, which encodes the dependence on the D3-brane moduli, is in agreement with what was anticipated in [51] from an argument based on monodromy and holomorphy. Hence our procedure gives, as particular by-product, an explicit derivation of such prefactor which is in the same spirit, although more direct and general, as the derivation presented in [34] for certain conifold backgrounds, see also [52].

On the other hand, the factor $\mathcal{A}^{\text{bg}}$ contains the effect of the back-reaction of the other background sources, see (5.26). It does not depend on the moduli we consider in the present paper. However, as $\prod_I \zeta_D(Z_I)$, it may become relevant in more general flux-compactifications that include more dynamical fields in the low-energy effective theory.¹⁰

¹⁰For instance, our results automatically incorporate the mechanism proposed in [53] to generate non-perturbatively induced Yukawa couplings.

5.4 The $h^{1,1} = 1$ case

Let us restrict to the particular case in which there are no constrained Kähler moduli v_a , i.e. $h^{1,1}(X) = 1$, so that $H^{1,1}(X)$ is generated by the Kähler form J_0 . It is then convenient to take it integrally quantised and choose as single integral two-form $\omega \equiv J_0$. This corresponds to setting the single constrained Kähler modulus $v \equiv 1$. Notice that in this case h is just a constant. Hence it can be omitted from (5.30), which then reduces to

$$\text{Re } \rho = 3v_0 \hat{a} + \frac{1}{2} \sum_I k(Z_I, \bar{Z}_I) \quad (5.40)$$

Here we have identified $\kappa(z, \bar{z})$ with the Kähler potential $k(z, \bar{z})$ associated with J_0 , defined by

$$i\partial\bar{\partial}k = J_0 \quad (5.41)$$

Inverting (5.40) to write $\hat{a} = \hat{a}(\text{Re } \rho, Z, \bar{Z})$, the Kähler potential (5.32) takes the form

$$K = -3 \log \left[\text{Re } \rho - \frac{1}{2} \sum_I k(Z_I, \bar{Z}_I) \right] + c_0 + 3 \log 3 \quad (5.42)$$

This is in agreement with the Kähler potential first proposed in [10] and further discussed in [13, 27, 34]. Notice that we could have started from (4.4) and used in (5.40) the more general universal modulus a instead of $\hat{a} = a + \frac{v_0^w}{v_0}$. This shift in the definition of $\text{Re } \rho$ would then modify the functional dependence of K as follows

$$K = -3 \log \left[\text{Re } \rho - \frac{1}{2} \sum_I k(Z_I, \bar{Z}_I) + 3v_0^w \right] + c_0 + 3 \log 3 \quad (5.43)$$

with v_0^w as in (4.3), consistently with the results of [26, 27].

6 Warped kinetic terms and no-scale condition

In this section we derive the kinetic terms of the chiral fields ρ^a and Z_I^i defined by the Kähler potential (5.32). Let us again collectively denote these fields by φ^A . Then, according to (3.12) we have to compute the matrix $K_{A\bar{B}} \equiv \partial_A \partial_{\bar{B}} K$. Since K is only implicitly defined, one needs to compute the derivatives of a and v_a with respect to ρ^a and Z_I^i by using (5.30). One of the main difficulties comes from the dependence of $\kappa^a(z, \bar{z}; v)$ and $h(v)$ on the constrained Kähler moduli v_a . Nevertheless, one can actually compute these derivatives. This is discussed in detail in appendix A and the final results are in (A.18) and (A.21). Here we just mention that a key ingredient in the derivation is a non-trivial interplay, physically due to the tadpole condition (5.24), between the contributions of $\kappa^a(Z_I, \bar{Z}_I; v)$ and $h(v)$.

As a preliminary step, we introduce the matrix

$$M_{\text{w}}^{ab} \equiv \int_X e^{-4A} J_0 \wedge \omega^a \wedge \omega^b \quad (6.1)$$

Notice that M_w^{ab} cannot be written in terms of topological intersection numbers, because of the warping e^{-4A} . Let us also define the warped metric

$$\mathcal{G}_{ab}^w \equiv \frac{1}{4v_0\hat{a}} \left[\frac{1}{2v_0\hat{a}} v_a v_b - (M_w^{-1})_{ab} \right] \quad (6.2)$$

and

$$\mathcal{A}_i^{aI}(Z_I, \bar{Z}_I; v) \equiv \frac{\partial \kappa^a(Z_I, \bar{Z}_I; v)}{\partial Z_I^i} \quad \bar{\mathcal{A}}_i^{aI}(Z_I, \bar{Z}_I; v) \equiv \frac{\partial \kappa^a(Z_I, \bar{Z}_I; v)}{\partial \bar{Z}_I^i} \quad (6.3)$$

Notice that under the transformations (5.19), we have

$$\begin{aligned} \mathcal{A}_i^{aI}(Z_I, \bar{Z}_I; v) &\rightarrow \mathcal{A}_i^{aI}(Z_I, \bar{Z}_I; v) + \frac{\partial \chi^a(Z_I)}{\partial Z_I^i} \\ \bar{\mathcal{A}}_i^{aI}(Z_I, \bar{Z}_I; v) &\rightarrow \bar{\mathcal{A}}_i^{aI}(Z_I, \bar{Z}_I; v) + \frac{\partial \bar{\chi}^a(\bar{Z}_I)}{\partial \bar{Z}_I^i} \end{aligned} \quad (6.4)$$

Now, by using (A.18) and (A.21) one can quite straightforwardly compute the Kähler metric $K_{A\bar{B}}$ associated with the Kähler potential (5.32). The result is

$$K_{A\bar{B}} \equiv \begin{pmatrix} \frac{\partial K}{\partial \rho^a \partial \bar{\rho}^b} & \frac{\partial K}{\partial \rho^a \partial \bar{Z}_I^j} \\ \frac{\partial K}{\partial Z_I^i \partial \bar{\rho}^b} & \frac{\partial K}{\partial Z_I^i \partial \bar{Z}_I^j} \end{pmatrix} = \begin{pmatrix} \mathcal{G}_{ab}^w & -\mathcal{G}_{ac}^w \bar{\mathcal{A}}_j^{cI} \\ -\mathcal{A}_i^{cI} \mathcal{G}_{cb}^w & \mathcal{G}_{cd}^w \mathcal{A}_i^{cI} \bar{\mathcal{A}}_j^{dI} + \frac{1}{2v_0\hat{a}} \delta^{IJ} g_{0,i\bar{j}}(Z_I, \bar{Z}_I) \end{pmatrix} \quad (6.5)$$

where the $g_{0,i\bar{j}}(z, \bar{z})$ is the Kähler metric of the internal space X .

By using (6.5) we can write the kinetic terms (3.12) in the form

$$\mathcal{L}_{\text{kin}} = -M_P^2 \mathcal{G}_{ab}^w \nabla_\mu \rho^a \nabla^\mu \bar{\rho}^b - \frac{1}{2v_0\hat{a}} M_P^2 \sum_I g_{0,i\bar{j}}(Z_I, \bar{Z}_I) \partial_\mu Z_I^i \partial^\mu \bar{Z}_I^{\bar{j}} \quad (6.6)$$

where we have introduced the covariant derivatives

$$\nabla_\mu \rho^a \equiv \partial_\mu \rho^a - \mathcal{A}_i^{aI} \partial_\mu Z_I^i \quad (6.7)$$

and $\nabla_\mu \bar{\rho}^a \equiv (\nabla_\mu \rho^a)^*$, which are invariant under the transformations (5.31), taking (6.4) into account.

Notice that the second term on the r.h.s. of (6.6) is easily reproduced by probe D3-branes. Indeed, focusing on a single D3-brane, from (4.9) and (3.11) it follows that $M_P^2 = 4\pi v_0 \hat{a} |Y|^2$ so that the second term appearing in the r.h.s. of (6.6) can be written as

$$-2\pi |Y|^2 g_{0,i\bar{j}}(Z, \bar{Z}; v) \partial_\mu Z^i \partial^\mu \bar{Z}^{\bar{j}} \quad (6.8)$$

which indeed coincides with the kinetic term obtained by expanding the DBI action of a probe D3-brane on the ten-dimensional metric (3.13). This provides a non-trivial check of our result.

Now, from (A.18) and (A.21) one can easily compute the first derivatives of the Kähler potential:

$$K_A = \left(\frac{\partial K}{\partial \rho^a}, \frac{\partial K}{\partial Z_I^i} \right) = \left(-\frac{v_a}{2v_0\hat{a}}, \frac{1}{2v_0\hat{a}} v_a \mathcal{A}_i^{aI} \right) \quad (6.9)$$

and $K_{\bar{A}} = (K_A)^*$. Furthermore, the inverse of the Kähler metric (6.5) takes the form:

$$K^{A\bar{B}} = \begin{pmatrix} \mathcal{G}_w^{ab} + 2v_0\hat{a} \sum_L g_0^{l\bar{m}}(Z_L, \bar{Z}_L) \mathcal{A}_l^{aL} \bar{\mathcal{A}}_m^{bL} & 2v_0\hat{a} \mathcal{A}_l^{aL} g_0^{l\bar{j}}(Z_J, \bar{Z}_J) \delta_{LJ} \\ 2v_0\hat{a} \delta_{IM} g_0^{i\bar{m}}(Z_I, \bar{Z}_I) \bar{\mathcal{A}}_m^{bM} & 2v_0\hat{a} \delta_{IJ} g_0^{i\bar{j}}(Z_I, \bar{Z}_I) \end{pmatrix} \quad (6.10)$$

where $g_0^{\bar{i}j}$ is the inverse of $g_{0,i\bar{j}}$ and

$$\mathcal{G}_w^{ab} \equiv \left(M_w^{ad} v_d v_c - 4v_0\hat{a} \delta_c^a \right) M_w^{cb} \quad (6.11)$$

is the inverse of $\mathcal{G}_{ab}^w$. Hence, from (6.9) and (6.10) one can straightforwardly check that

$$K^{A\bar{B}} K_A K_{\bar{B}} = 3 \quad (6.12)$$

That is, our warped Kähler potential is of *no-scale* type [32, 33].

In fact (6.12) had to be expected. Indeed supersymmetry can be broken, preserving the ten-dimensional equations of motion, if the classical superpotential (4.11) has a non-vanishing expectation value $W_0 \neq 0$. In particular, these non-supersymmetric vacua still have a Minkowski external space. The no-scale condition (6.12) makes such ten-dimensional supersymmetry breaking mechanism naturally consistent with the effective four-dimensional supergravity viewpoint. Hence, (6.12) provides a further consistency check for our results.

So far we have used only chiral multiplets in the low-energy effective theory. On the other hand, one could dualise the chiral fields ρ^a to linear multiplets. Let us denote the scalar components of the dual linear multiplets by l_a . They can be easily computed within the superconformal formalism, before imposing the gauge fixing condition (3.11) on the conformal compensator, and are given by [54, 55]

$$l_a = \frac{3}{4\pi} \frac{\partial \mathcal{N}}{\partial \text{Re} \rho^a} \quad (6.13)$$

From (3.10) it follows that $\mathcal{N} = 4\pi|Y|^2 v_0 \hat{a}$. Then, by using (A.18a) we obtain

$$l_a = |Y|^2 v_a \quad (6.14)$$

That is, the scalars l_a are just the components appearing in the expansion $J = \ell_s^2 l_a \omega^a$ of the non-normalised Kähler form $J \equiv \ell_s^2 |Y|^2 J_0$ introduced in (2.8). This shows that the scalars l_a have a more transparent geometrical interpretation, compared to the chiral fields ρ^a . The complexity of (5.30) could then be regarded as the result of performing the inverse duality from linear to chiral multiplets. See [20, 23] for a detailed discussion in the unwarped case and [31] for similar remarks in the case of generalised compactifications.

Finally, let us give a closer look at the warped matrix M_w^{ab} defined in (6.1), which encodes the non-trivial warping contribution to $\mathcal{G}_{ab}^w$. In particular, by splitting e^{-4A} as in (4.8), M_w^{ab} can be rewritten as

$$M_w^{ab} = \hat{a} \mathcal{I}^{abc} v_c + \int_X e^{-4\hat{A}_0} J_0 \wedge \omega^a \wedge \omega^b \quad (6.15)$$

The second term on the r.h.s. of this equation is what makes $\mathcal{G}_{ab}^w$ quantitatively different with respect to the corresponding metric in the constant-warping approximation [20, 23].

Even though we generically expect the second term on the r.h.s. of (6.15) to be non-trivial, there are some particular cases in which it actually vanishes. Indeed, recall that $e^{-4\hat{A}_0}$ satisfies the normalisation condition (4.7). Hence we see that the second term on the r.h.s. of (6.15) vanishes if

$$J_0 \wedge \omega^a \wedge \omega^b = C^{ab} J_0 \wedge J_0 \wedge J_0 \quad (6.16)$$

for some constants C^{ab} . The condition (6.16) is equivalent to requiring that the product of any harmonic (1,1)-form and any harmonic (2,2)-form gives a harmonic 6-form. This is for instance the case for the simple class of $N = 1$ compactifications on $X = T^6/\mathbb{Z}_2$ discussed in appendix B. More generically, such property is guaranteed if X is geometrically formal [56] that is, if the product of any pair of harmonic forms gives a harmonic form. A similar conclusion was reached in [29] by studying the kinetic terms of the C_4 axionic moduli on warped Calabi-Yau three-folds.

7 Large moduli limit

It can be sensible to consider a regime in which $\text{Re}\rho^a$ are all very large, $\text{Re}\rho^a \gg 1$. This is indeed the regime in which the supergravity description of string theory is expected to be fully legitimate. Hence, it is natural to look for a perturbative solution of the inverted relations $\hat{a} = \hat{a}(\text{Re}\rho, Z, \bar{Z})$ and $v_a = v_a(\text{Re}\rho, Z, \bar{Z})$ in powers of $(\text{Re}\rho)^{-1}$.

First expand $\hat{a}$ and v_a as follows

$$\hat{a} = \hat{a}^{(-1)} + \hat{a}^{(0)} + \mathcal{O}\left(\frac{1}{\text{Re}\rho}\right) \quad v_a = v_a^{(0)} + v_a^{(1)} + \mathcal{O}\left(\frac{1}{(\text{Re}\rho)^2}\right) \quad (7.1)$$

Furthermore, by expanding the constraint (5.3) up to first order, one gets

$$v_a^{(0)} v_b^{(0)} v_c^{(0)} \mathcal{I}^{abc} = 6v_0 \quad v_a^{(0)} v_b^{(0)} v_c^{(1)} \mathcal{I}^{abc} = 0 \quad (7.2)$$

Hence the leading order contribution $\text{Re}\rho^a = \frac{1}{2} \hat{a}^{(-1)} \mathcal{I}^{abc} v_b^{(0)} v_c^{(0)}$ to (5.30) implicitly gives $\hat{a}^{(-1)}$ and $v_a^{(0)}$ in terms of $\text{Re}\rho^a$. On the other hand, the next-to-leading order of (5.30) gives $a^{(0)}$ in terms of $v_a^{(0)}$:¹¹

$$\hat{a}^{(0)} = -\frac{1}{3v_0} h(v^{(0)}) - \frac{1}{6v_0} \sum_I k(Z_I, \bar{Z}_I; v^{(0)}) \quad (7.3)$$

where $k \equiv v_a \kappa^a$ is nothing but the Kähler potential associated with J_0 , as defined in (5.41), and we have introduced the quantities $M_{(0)}^{ab} \equiv \mathcal{I}^{abc} v_c^{(0)}$ and $h(v^{(0)}) \equiv v_a^{(0)} h^a(v^{(0)})$.

We can now consider $\hat{a}^{(-1)}$ and $v_a^{(0)}$ as the independent geometric moduli and suppress the superscripts for notational simplicity. Hence, by neglecting terms of order $(\text{Re}\rho)^{-1}$ in the logarithm, the Kähler potential (5.32) can be approximated by

$$K \simeq -3 \log \left[v_0 \hat{a} - \frac{1}{3} h(v) - \frac{1}{6} \sum_I k(Z_I, \bar{Z}_I; v) \right] + c_0 \quad (7.4)$$

¹¹One also obtains $v_a^{(1)} = \frac{1}{\hat{a}^{(-1)}} \left[\frac{1}{6v_0} v_a^{(0)} v_b^{(0)} - (M_{(0)}^{-1})_{ab} \right] \left[h^b(v^{(0)}) + \frac{1}{2} \sum_I \kappa^b(Z_I, \bar{Z}_I; v^{(0)}) \right]$.

where

$$h(v) \equiv \frac{1}{2\pi\ell_s^4} \int_X \left[\pi k(z, \bar{z}; v) - v_a \operatorname{Re} \log \zeta^a(z) \right] Q_6^{\text{bg}} \quad (7.5)$$

Alternatively, by using unconstrained Kähler parameters $\hat{v}_a = v_a \sqrt{\hat{a}}$ as in (5.34), associated with $\hat{J} = \hat{v}_a \omega^a$, the Kähler potential (7.4) can be written as

$$K \simeq -2 \log \left[\frac{1}{3!} \mathcal{I}^{abc} \hat{v}_a \hat{v}_b \hat{v}_c - \frac{1}{2} h(\hat{v}) - \frac{1}{4} \sum_I k(Z_I, \bar{Z}_I; \hat{v}) \right] - \log v_0 + c_0 \quad (7.6)$$

In (7.4) and (7.6) the geometric moduli $\hat{a}$ and v_a , or $\hat{v}_a$, must be regarded as functions of $\operatorname{Re} \rho^a$ alone, implicitly defined by

$$\operatorname{Re} \rho^a = \frac{1}{2} \hat{a} \mathcal{I}^{abc} v_b v_c \equiv \frac{1}{2} \mathcal{I}^{abc} \hat{v}_b \hat{v}_c \quad (7.7)$$

In (7.4), or (7.6), the effect of D3-branes and fluxes appears directly in the Kähler potential, while the relation (7.7) between geometric moduli $\hat{v}_a$ and holomorphic moduli ρ^a is formally the same as for constant warping compactifications [20, 23]. The modification due to D3-branes is of the same form proposed in [10]. On the other hand, the effect of the fluxes and other D3-charge sources is encoded in $h(\hat{v})$. Notice that if we rescale $\hat{v}_a \rightarrow \lambda \hat{v}_a$, then $k(z, \bar{z}; \hat{v})$ and $h(\hat{v})$ scale like homogeneous functions of degree one. Hence, by expanding the logarithm, we see that the overall correction to the Kähler potential scales with λ^{-2} . In other words, the first order correction scales like the $V^{-\frac{2}{3}}$, where $V \equiv \frac{1}{3!} \mathcal{I}^{abc} \hat{v}_a \hat{v}_b \hat{v}_c$ can be identified with the internal volume of the corresponding constant warping compactification. This scaling behaviour was also argued in [29] by studying the dimensionally reduced theory for axionic moduli.

8 Including axions

In the previous sections we assumed the absence of chiral fields corresponding to purely axionic moduli. In fact, they can be included by using the same strategy followed so far. The possible axionic chiral fields are more easily identified in the dual elliptically fibered Calabi-Yau four-fold $\hat{X}$, on which they are counted by $b^3(\hat{X}) - b^3(X)$, see [23] for a detailed discussion. In such description we should use probe Euclidean M5-branes wrapping vertical divisors and, unfortunately, the M5-brane effective theory on non-trivial supergravity backgrounds is not very manageable [57, 58]. On the other hand, in the type IIB framework the axionic chiral fields are less easily identified, but one can use the Euclidean D3-brane action, which is under better control.

To avoid such technical complications and outline the general strategy to include axionic moduli, we will restrict to F-theory backgrounds admitting a weakly coupled description, with D7-branes and O7-planes and an almost constant axio-dilaton τ . In this limit one can work in the Calabi-Yau double cover three-fold $\tilde{X}$ and distinguish between closed and open string axionic chiral fields.

8.1 Closed string axionic chiral fields

Let us first focus on the closed string moduli. Introduce a set of harmonic $(1,1)$ -forms χ_α that are odd under the orientifold involution and form a basis for $H_-^{1,1}(\tilde{X})$. We can then fix a certain reference $B_2^{(0)}$ and $C_2^{(0)}$ satisfying the flux equations, and write $B_2 = B_2^{(0)} + \Delta B_2$ and $C_2 = C_2^{(0)} + \Delta C_2$, with ΔB_2 and ΔC_2 being closed harmonic $(1,1)$ -forms. We can then expand ΔB_2 and ΔC_2 as follows

$$\Delta B_2 = \ell_s^2 b^\alpha \chi_\alpha \quad \Delta C_2 = \ell_s^2 c^\alpha \chi_\alpha \quad (8.1)$$

The moduli b^α and c^α are naturally combined into the following set of $h_-^{1,1}(\tilde{X})$ chiral fields:

$$\beta^\alpha = c^\alpha - \tau b^\alpha \quad (8.2)$$

In order to include them in the above discussion, we need to take into account the second term appearing in the on-shell supersymmetric DBI-action (5.5). By considering a Euclidean D3-brane wrapping an even divisor $D \subset \tilde{X}$ (that is, such that $\sigma(D) = D$), the second term on the r.h.s. of (5.5) can be written as

$$-\frac{\pi}{2\ell_s^4} \int_D e^{-\phi} \mathcal{F}_{(0)} \wedge \mathcal{F}_{(0)} - \left(\frac{\pi}{\ell_s^2} \int_D \mathcal{F}_{(0)} \wedge \chi_\alpha \right) \text{Im} \beta^\alpha + \frac{\pi}{\text{Im} \tau} \mathcal{M}_{\alpha\beta} \text{Im} \beta^\alpha \text{Im} \beta^\beta \quad (8.3)$$

where $\mathcal{M}_{\alpha\beta} \equiv -\frac{1}{2} \int_D \chi_\alpha \wedge \chi_\beta$ and we have included an overall factor $\frac{1}{2}$ to take into account that we are working in the double cover $\tilde{X}$.

Taking (8.3) into account, the definition of I^a given in (5.7) (written in the covering space) should be modified into

$$I^a = \frac{1}{4} \int_D e^{-4A} J_0 \wedge J_0 - \left(\frac{1}{2\ell_s^2} \int_D \mathcal{F}_{(0)} \wedge \chi_\alpha \right) \text{Im} \beta^\alpha + \frac{1}{2\text{Im} \tau} \mathcal{M}_{\alpha\beta} \text{Im} \beta^\alpha \text{Im} \beta^\beta \quad (8.4)$$

where we have omitted the contribution of the first term in (8.3), since moduli independent. The second term in (8.4) can be written as $(\text{hol} + \overline{\text{hol}})$. Hence, according to the general prescription (5.8), it can be ignored as well. Repeating the discussion which lead to (5.30), we then arrive at the identification

$$\text{Re} \rho^a = \hat{a} \mathcal{V}^a(v) + h^a(v) + \frac{1}{2} \sum_I \kappa^a(Z_I, \bar{Z}_I; v) + \frac{1}{2\text{Im} \tau} \mathcal{M}_{\alpha\beta}^a \text{Im} \beta^\alpha \text{Im} \beta^\beta \quad (8.5)$$

where

$$\mathcal{M}_{\alpha\beta}^a \equiv -\frac{1}{2} \int_{\tilde{X}} \omega^a \wedge \chi_\alpha \wedge \chi_\beta \quad (8.6)$$

As for the Kähler potential, it still takes the form (5.32), up to allowing $\hat{a}$ to depend also on $\text{Im} \beta^a$. So (8.5) is the only modification we need to consider. In particular, the axionic contribution to (8.5) is identical to the corresponding contribution in the constant warping approximation [20].

8.2 Wilson lines

Finally, we incorporate the open string axions as well. Let us focus on a D7-brane wrapping an orientifold invariant divisor $\Sigma = \sigma(\Sigma)$. Then, the (orientifold odd) gauge field A_{D7} supported on Σ can be split into $A_{D7}^{(0)} + \Delta A_{D7}$, where $A_{D7}^{(0)}$ is some fixed background gauge field and ΔA_{D7} is an arbitrary harmonic one-form, which can be regarded as a Wilson line. The corresponding chiral fields $a^{\mathcal{I}}$ can be obtained by expanding the $(0,1)$ component $\Delta A_{D7}^{0,1}$ in a basis of harmonic $(0,1)$ -forms $\eta_{\mathcal{I}} \in H^{0,1}(\Sigma)$:

$$\Delta A_{D7}^{0,1} = 2\pi a^{\mathcal{I}} \eta_{\mathcal{I}} \quad (8.7)$$

Now, the gauge field A_{D7} couples to a Euclidean D3-brane wrapping a divisor D through its coupling with the chiral scalar β supported on the intersection curve $\mathcal{C} = \Sigma \cap D$ [49]:

$$\frac{i}{8\pi} \int_{\mathcal{C}} \left[\partial\beta \wedge \bar{\partial}\beta - 2\partial\beta \wedge (A_{D3}^{0,1} - A_{D7}^{0,1}) + A_{D3}^{1,0} \wedge A_{D3}^{0,1} - 2A_{D3}^{1,0} \wedge A_{D7}^{0,1} + A_{D7}^{1,0} \wedge A_{D7}^{0,1} \right] \quad (8.8)$$

This two-dimensional action contributes to S_{D3} . In order to identify the modification of (8.5) due to Wilson lines, we use the same strategy as above and we require S_{D3} to depend homomorphically on the background chiral fields. The terms that do not depend on A_{D7} can be ignored. Furthermore, the terms that are linear on $A_{D7}^{0,1}$ can be ignored as well, since they depend holomorphically on the Wilson lines $a^{\mathcal{I}}$. On the other hand, the real quadratic term

$$\frac{i}{8\pi} \int_{\mathcal{C}} A_{D7}^{1,0} \wedge A_{D7}^{0,1} \quad (8.9)$$

must be taken into account. More specifically, by using the expansion (8.7), the definition of $\text{Re}\rho^a$ given in (8.5) must be further modified into

$$\text{Re}\rho^a = \hat{a} \mathcal{V}^a(v) + h^a(v) + \frac{1}{2} \sum_I \kappa^a(Z_I, \bar{Z}_I; v) + \frac{1}{2\text{Im}\tau} \mathcal{M}_{\alpha\beta}^a \text{Im}\beta^\alpha \text{Im}\beta^\beta + \frac{1}{2} \mathcal{C}_{\mathcal{I}\mathcal{J}}^a a^{\mathcal{I}} \bar{a}^{\mathcal{J}} \quad (8.10)$$

where

$$\mathcal{C}_{\mathcal{I}\mathcal{J}}^a = -\frac{i}{2} \int_{\mathcal{C}^a} \eta_{\mathcal{I}} \wedge \bar{\eta}_{\mathcal{J}} \quad (8.11)$$

with $\mathcal{C}^a \equiv D^a \cap \Sigma$. The formula (8.10) can be obviously extended to include the contribution of additional D7-branes. Notice that the contribution of the Wilson lines to (8.10) has the same form of the contribution identified in [21] by using probe D7-branes on unwarped compactifications.

8.3 Complete Kähler potential for $h_+^{1,1} = 1$

The new relations (8.10) should be partially inverted in order to find $\hat{a}$ as function of the chiral fields ρ^a , Z_I^i , β^α and $a^{\mathcal{I}}$, and to properly interpret the Kähler potential as a function of the chiral coordinates. The case $h_+^{1,1} = 1$ is the simplest one in which one can explicitly invert (8.10). Indeed, proceeding as in section 5.4 one arrives at the Kähler potential

$$K = -3 \log \left[\text{Re}\rho - \frac{1}{2} \sum_I k(Z_I, \bar{Z}_I) - \frac{1}{2\text{Im}\tau} \mathcal{M}_{\alpha\beta} \text{Im}\beta^\alpha \text{Im}\beta^\beta - \frac{1}{2} \mathcal{C}_{\mathcal{I}\mathcal{J}} a^{\mathcal{I}} \bar{a}^{\mathcal{J}} \right] + c_0 + 3 \log 3 \quad (8.12)$$

If we had used in (8.10) the more general universal modulus a instead of $\hat{a}$, we would have obtained the Kähler potential

$$K = -3 \log \left[\text{Re} \rho - \frac{1}{2} \sum_I k(Z_I, \bar{Z}_I) - \frac{1}{2 \text{Im} \tau} \mathcal{M}_{\alpha\beta} \text{Im} \beta^\alpha \text{Im} \beta^\beta - \frac{1}{2} \mathcal{C}_{\mathcal{I}\mathcal{J}} a^{\mathcal{I}} \bar{a}^{\mathcal{J}} + 3 v_0^w \right] + c_0 + 3 \log 3 \quad (8.13)$$

with v_0^w defined in (4.3). Ignoring the contribution of mobile D3-branes and D7-brane Wilson lines, this matches the Kähler potential obtained in [31], using a strategy similar to the one followed in the present paper, which was then confirmed by a direct dimensional reduction in [29].

8.4 Kähler metric and no-scale structure

In order to compute the Kähler metric one must use the derivatives of the geometric moduli discussed in appendix A.3. The final result is

$$\begin{aligned} \mathcal{L}_{\text{kin}} = & -M_{\text{P}}^2 \mathcal{G}_{ab}^w \nabla_\mu \rho^a \nabla^\mu \bar{\rho}^b - \frac{1}{2 v_0 \hat{a}} M_{\text{P}}^2 \sum_I g_{0,i\bar{j}}(Z_I, \bar{Z}_I) \partial_\mu Z_I^i \partial^\mu \bar{Z}_I^{\bar{j}} \\ & - \frac{1}{4 v_0 \hat{a} \text{Im} \tau} M_{\text{P}}^2 v_a \mathcal{M}_{\alpha\beta}^a \partial_\mu \beta^\alpha \partial^\mu \bar{\beta}^\beta - \frac{1}{2 v_0 \hat{a}} M_{\text{P}}^2 v_a \mathcal{C}_{\mathcal{I}\mathcal{J}}^a \partial_\mu a^{\mathcal{I}} \partial^\mu \bar{a}^{\mathcal{J}} \end{aligned} \quad (8.14)$$

where $\mathcal{G}_{ab}^w$ is as in (6.2) and the covariant derivatives are now defined as follows:

$$\nabla_\mu \rho^a \equiv \partial_\mu \rho^a - \mathcal{A}_i^{aI} \partial_\mu Z_I^i + \frac{i}{\text{Im} \tau} \mathcal{M}_{\alpha\beta}^a \text{Im} \beta^\alpha \partial_\mu \beta^\beta - \mathcal{C}_{\mathcal{I}\mathcal{J}}^a \bar{a}^{\mathcal{J}} \partial_\mu a^{\mathcal{I}} \quad (8.15)$$

and $\nabla_\mu \bar{\rho}^a \equiv (\nabla_\mu \rho^a)^*$.

By ignoring D3-branes, in the constant warping approximation the kinetic terms (8.14) are consistent with the results of [21]. In particular, as a direct check of the numerical factors, the last term on the r.h.s. of (8.14) can be easily reproduced by dimensionally reducing the DBI action for a probe D7-brane on the metric (3.13).

Furthermore, one can verify that the no-scale condition (6.12) is still satisfied, as required by physical consistency.

Finally, by performing the large moduli expansion discussed in section 7, one can obtain the appropriate modification of (7.4) or, equivalently, of (7.6). For instance, the latter reads

$$\begin{aligned} K \simeq & -2 \log \left[\frac{1}{3!} \mathcal{I}^{abc} \hat{v}_a \hat{v}_b \hat{v}_c - \frac{1}{2} h(\hat{v}) - \frac{1}{4} \sum_I k(Z_I, \bar{Z}_I; \hat{v}) \right. \\ & \left. - \frac{1}{4 \text{Im} \tau} \hat{v}_a \mathcal{M}_{\alpha\beta}^a \text{Im} \beta^\alpha \text{Im} \beta^\beta - \frac{1}{4} \hat{v}_a \mathcal{C}_{\mathcal{I}\mathcal{J}}^a a^{\mathcal{I}} \bar{a}^{\mathcal{J}} \right] - \log v_0 + c_0 \end{aligned} \quad (8.16)$$

where $\hat{v}_a$ are functions of $\text{Re} \rho^a$ implicitly determined by $\text{Re} \rho^a = \frac{1}{2} \mathcal{I}^{abc} \hat{v}_b \hat{v}_c$.

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A Derivatives of geometric moduli

The relations (5.30), (8.5) and (8.10) implicitly define the geometric moduli $\hat{a}$ and v_a as functions of the chiral fields ρ^a , Z_I^i , β^a and a^T . Even though in general one cannot make such dependence explicit, it is possible to compute the derivatives of $\hat{a}$ and v_a with respect to the chiral fields.

A.1 Useful preliminary formulas

Once we have chosen the integral two-forms ω^a to be $(1, 1)$ and harmonic, we can expand the normalised Kähler form as

$$J_0 = v_a \omega^a \quad (\text{A.1})$$

Notice that the ω^a 's, being harmonic, depend on the constrained Kähler moduli v_a . Now, since any Kähler structure deformation should preserve (2.2), one can easily adapt the discussion for Ricci-flat Calabi-Yau spaces [59] to argue that any infinitesimal Kähler structure deformation must take the form

$$\delta J_0 = \delta v_a \omega^a \quad (\text{A.2})$$

By consistency, this implies that

$$v_a \delta \omega^a = 0 \quad (\text{A.3})$$

Moreover the normalisation condition (5.3) implies that

$$\mathcal{I}^{abc} v_a v_b \delta v_c = 0 \quad (\text{A.4})$$

where $\mathcal{I}^{abc}$ are the intersection numbers defined in (5.4).

Another useful identity is

$$\frac{1}{2} J_0 \wedge J_0 \wedge \omega^a = \frac{1}{3!} (J_0 \lrcorner \omega^a) J_0 \wedge J_0 \wedge J_0 = d^a(v) \text{dvol}_{X,0} \quad (\text{A.5})$$

where

$$d^a(v) \equiv J_0 \lrcorner \omega^a \equiv \frac{1}{2v_0} \mathcal{I}^{abc} v_b v_c \quad (\text{A.6})$$

The constraint (A.4) implies $\delta v_a d^a = 0$ and then

$$J_0 \wedge J_0 \wedge \delta J_0 = \delta v_a J_0 \wedge J_0 \wedge \omega^a = 0 \quad (\text{A.7})$$

In other words, the normalisation condition (5.3) requires δJ_0 to be primitive.

In the following computations one of the main subtleties comes from the dependence of the potentials $\kappa^a(z, \bar{z}; v)$, defined by (5.17), on the constrained Kähler moduli v_a . Under a deformation δv_a , we can write

$$\delta \omega^a = i \partial \bar{\partial} \delta \kappa^a \quad (\text{A.8})$$

where, since the cohomology class of ω^a is fixed, the variations $\delta\kappa^a$ are *globally* defined functions on X . Now, by using (A.8) and (A.2), the constancy of (A.6) implies the following equation for $\delta\kappa^a$

$$\Delta_0\delta\kappa^a = -2(\delta d^a + \delta v_b \omega^a \lrcorner \omega^b) \quad (\text{A.9})$$

Consistently, the r.h.s. is a coexact function since from (A.6) one can check that

$$\delta d^a = \frac{1}{v_0} \mathcal{I}^{abc} v_b \delta v_c = -\frac{1}{v_0} \delta v_b \int_X \omega^a \lrcorner \omega^b \text{dvol}_{X,0} \quad (\text{A.10})$$

eq. (A.9) can be integrated, up to an irrelevant constant along X , by means of the Green's function $G(y, y')$ introduced in section 4:

$$\delta\kappa^a(y; v) = 2\delta v_b \int_{X, y'} G(y; y') (J_0 \wedge \omega^a \wedge \omega^b)(y') \quad (\text{A.11})$$

Notice that from (A.7) it follows that $v_a \delta\kappa^a = 0$, consistently with (A.3).

Now, recalling the definition (5.28) of $h^a(v)$, observe that

$$\delta h^a(v) + \frac{1}{2} \sum_I \delta\kappa^a(Z_I, \bar{Z}_I; v) = \frac{1}{2\ell_s^4} \int_X \delta\kappa^a Q_6 \quad (\text{A.12})$$

Combining this equation with (A.11) and (4.6), one can deduce the following crucial formula:¹²

$$\delta h^a(v) + \frac{1}{2} \sum_I \delta\kappa^a(Z_I, \bar{Z}_I; v) = \delta v_b \int_X e^{-4\hat{A}_0} J_0 \wedge \omega^a \wedge \omega^b \quad (\text{A.13})$$

A.2 Derivatives in absence of axionic chiral fields

We are now ready to compute the derivatives of the background geometric moduli with respect to the chiral fields. Take first the derivatives of (5.30) with respect to $\text{Re}\rho^b$. By using (A.13) one gets

$$\delta_b^a = \frac{\partial \hat{a}}{\partial \text{Re}\rho^b} \mathcal{V}^a + \mathcal{I}_w^{acd} v_c \frac{\partial v_d}{\partial \text{Re}\rho^b} \quad (\text{A.14})$$

where we have introduced the warped ‘intersection numbers’

$$\mathcal{I}_w^{abc} \equiv \int_X e^{-4A} \omega^a \wedge \omega^b \wedge \omega^c \quad (\text{A.15})$$

Notice that $\mathcal{I}_w^{abc}$ is not a topological quantity and that by (A.7) we have

$$\mathcal{I}_w^{abc} v_a v_b \delta v_b = 0 \quad (\text{A.16})$$

Let us also define

$$M_w^{ab} \equiv \int_X e^{-4A} J_0 \wedge \omega^a \wedge \omega^b \equiv \mathcal{I}_w^{abc} v_c \quad (\text{A.17})$$

¹²In deducing the formula (A.13) we have neglected the possible dependence on the Kähler moduli of the higher-order curvature corrections appearing in Q_6 . Taking such dependence into account would require to work beyond the second derivative order in the effective action.

Then from (A.14) one can derive

$$\frac{\partial \hat{a}}{\partial \text{Re} \rho^a} = \frac{v_a}{3v_0} \quad (\text{A.18a})$$

$$\frac{\partial v_a}{\partial \text{Re} \rho^b} = (M_w^{-1})_{ab} - \frac{1}{6v_0 \hat{a}} v_a v_b \quad (\text{A.18b})$$

Indeed, (A.18a) can be obtained by contracting (A.14) with v_a and using (A.16). Then, one can determine (A.18b) by substituting (A.18a) in (A.14) and by observing that

$$M_w^{ab} v_b = 2\hat{a} \mathcal{V}^a(v) \quad (\text{A.19})$$

which follows from (A.5) and (4.7).

Starting back from (5.30), taking its derivative with respect to Z_I^i and using again (A.13) one gets

$$0 = \frac{\partial \hat{a}}{\partial Z_I^i} \mathcal{V}^a + M_w^{ab} \frac{\partial v_b}{\partial Z_I^i} + \frac{1}{2} \frac{\partial \kappa^a(Z_I, \bar{Z}_I; v)}{\partial Z_I^i} \quad (\text{A.20})$$

Then, proceeding as in the previous case, we arrive at

$$\frac{\partial \hat{a}}{\partial Z_I^i} = -\frac{v_a}{6v_0} \frac{\partial \kappa^a(Z_I, \bar{Z}_I; v)}{\partial Z_I^i} \quad (\text{A.21a})$$

$$\frac{\partial v_a}{\partial Z_I^i} = \frac{1}{2} \left[\frac{1}{6v_0 \hat{a}} v_a v_b - (M_w^{-1})_{ab} \right] \frac{\partial \kappa^b(Z_I, \bar{Z}_I; v)}{\partial Z_I^i} \quad (\text{A.21b})$$

A.3 Derivatives in presence of axionic chiral fields

The discussion of the previous subsection can be repeated starting from (8.5) or more generically (8.10). Let collectively denote the additional chiral fields β^a and $a^{\mathcal{I}}$ with $\phi^{\mathbb{A}}$ and rewrite (8.5) or (8.10) in the general form

$$\text{Re} \rho^a = \hat{a} \mathcal{V}^a(v) + \frac{1}{2} \sum_I \kappa^a(Z_I, \bar{Z}_I; v) + h^a(v) + N^a(\phi, \bar{\phi}) \quad (\text{A.22})$$

where $N^a(\phi, \bar{\phi})$ does not depend on the chiral fields ρ^a and Z_I^i . Proceeding as in the previous subsection one again arrives at (A.18) and (A.21). In addition, one also gets

$$\frac{\partial \hat{a}}{\partial \phi^{\mathbb{A}}} = -\frac{1}{3v_0} v_a N_{\mathbb{A}}^a \quad (\text{A.23a})$$

$$\frac{\partial v_a}{\partial \phi^{\mathbb{A}}} = \left[\frac{1}{6v_0 \hat{a}} v_a v_b - (M_w^{-1})_{ab} \right] N_{\mathbb{A}}^b \quad (\text{A.23b})$$

where $N_{\mathbb{A}} \equiv \frac{\partial N^a}{\partial \phi^{\mathbb{A}}}$.

B A simple example

In this appendix we apply our results to a simple concrete class of $N = 1$ compactifications on $X = T^6/\mathbb{Z}_2$ [35], for which the Kähler potential can be made fully explicit. The internal

six-torus is factorised, $T^6 = T^2 \times T^2 \times T^2$, and the corresponding complex coordinates (z^1, z^2, z^3) are periodically identified according to the rule $z^i \simeq z^i + n^i + \lambda m^i$ (no sum over i), where $n^i, m^i \in \mathbb{Z}$ and $\lambda \equiv e^{\frac{2\pi i}{3}}$ defines the complex structure modulus of the three tori. The axio-dilaton takes the value $\tau = e^{\frac{2\pi i}{3}}$ too. The $\mathbb{Z}_2$ orientifold involution is defined by $z^i \rightarrow -z^i$. One can choose a supersymmetric G_3 flux of the form

$$G_3 \sim dz^1 \wedge dz^2 \wedge d\bar{z}^3 + dz^2 \wedge dz^3 \wedge d\bar{z}^1 + dz^3 \wedge dz^1 \wedge d\bar{z}^2 \quad (\text{B.1})$$

The flux quantisation condition fixes the number of D3-branes. In [35] one can find an example with $N_{\text{D3}} = 10$.

On $T^6/\mathbb{Z}_2$ one has $h_+^{1,1} = 9$, but the primitivity requirement $J_0 \wedge G_3 = 0$ imposes additional conditions. Such conditions are not explicitly taken into account in the rest of the present paper but, as this appendix shows, they can be easily incorporated.

Even though one would generically expect six non-trivial conditions from $J_0 \wedge G_3 = 0$, the simplicity of (B.1) implies that only three conditions are effective. These select six out of the nine independent integral even $(1, 1)$ forms, which can be chosen as follows:

$$\begin{aligned} \omega^1 &= \frac{i}{\text{Im} \lambda} dz^1 \wedge d\bar{z}^1, & \omega^2 &= \frac{i}{\text{Im} \lambda} dz^2 \wedge d\bar{z}^2, & \omega^3 &= \frac{i}{\text{Im} \lambda} dz^3 \wedge d\bar{z}^3, \\ \omega^4 &= \frac{1}{\text{Im} \lambda} \text{Im}(d\bar{z}^2 \wedge dz^3), & \omega^5 &= \frac{1}{\text{Im} \lambda} \text{Im}(d\bar{z}^3 \wedge dz^1), & \omega^6 &= \frac{1}{\text{Im} \lambda} \text{Im}(d\bar{z}^1 \wedge dz^2) \end{aligned} \quad (\text{B.2})$$

The non-vanishing intersection numbers $\mathcal{I}^{abc} = \frac{1}{2} \int_{T^6} \omega^a \wedge \omega^b \wedge \omega^c$ are given by

$$\mathcal{I}^{123} = 4, \quad \mathcal{I}^{456} = 1, \quad \mathcal{I}^{144} = \mathcal{I}^{255} = \mathcal{I}^{366} = -2 \quad (\text{B.3})$$

Then the constraint (5.3) takes the form

$$4v_1v_2v_3 + v_4v_5v_6 - v_1v_4^2 - v_2v_5^2 - v_3v_6^2 = v_0 \quad (\text{B.4})$$

Since the Kähler form $J_0 = v_a \omega^a$ as well as the two-forms ω^a have constant coefficients, ω^a are automatically harmonic. In particular they do not depend on the constrained Kähler moduli and the potentials

$$\begin{aligned} \kappa^1 &= \frac{z^1 \bar{z}^1}{\text{Im} \lambda} & \kappa^2 &= \frac{z^2 \bar{z}^2}{\text{Im} \lambda} & \kappa^3 &= \frac{z^3 \bar{z}^3}{\text{Im} \lambda} \\ \kappa^4 &= \frac{\text{Re}(z^2 \bar{z}^3)}{\text{Im} \lambda} & \kappa^5 &= \frac{\text{Re}(z^3 \bar{z}^1)}{\text{Im} \lambda} & \kappa^6 &= \frac{\text{Re}(z^1 \bar{z}^2)}{\text{Im} \lambda} \end{aligned} \quad (\text{B.5})$$

do not depend on the moduli v_a either. This implies the drastic simplification that the functions $h^a(v)$ defined in (5.28) are constant, so that they can be ignored in (5.30).

Hence, the geometric moduli $\hat{a}$ and v_a are related to the chiral fields ρ^a and Z_I^i by

$$\begin{aligned}
\hat{a}(4v_2v_3 - v_4^2) &= \text{Re}\rho^1 - \frac{1}{2\text{Im}\lambda} \sum_I Z_I^1 \bar{Z}_I^1 \equiv T^1 \\
\hat{a}(4v_1v_3 - v_5^2) &= \text{Re}\rho^2 - \frac{1}{2\text{Im}\lambda} \sum_I Z_I^2 \bar{Z}_I^2 \equiv T^2 \\
\hat{a}(4v_1v_2 - v_6^2) &= \text{Re}\rho^3 - \frac{1}{2\text{Im}\lambda} \sum_I Z_I^3 \bar{Z}_I^3 \equiv T^3 \\
\hat{a}(v_5v_6 - 2v_1v_4) &= \text{Re}\rho^4 - \frac{1}{2\text{Im}\lambda} \sum_I \text{Re}(Z_I^2 \bar{Z}_I^3) \equiv T^4 \\
\hat{a}(v_4v_6 - 2v_2v_5) &= \text{Re}\rho^5 - \frac{1}{2\text{Im}\lambda} \sum_I \text{Re}(Z_I^3 \bar{Z}_I^1) \equiv T^5 \\
\hat{a}(v_4v_5 - 2v_3v_6) &= \text{Re}\rho^6 - \frac{1}{2\text{Im}\lambda} \sum_I \text{Re}(Z_I^1 \bar{Z}_I^2) \equiv T^6
\end{aligned} \tag{B.6}$$

where we have introduced the set of real functions

$$T^a(\text{Re}\rho, Z, \bar{Z}) \equiv \text{Re}\rho^a - \frac{1}{2} \sum_I \kappa^a(Z_I, \bar{Z}_I) \tag{B.7}$$

By using (B.4) one can partially invert the relations (B.6) to find the function $\hat{a}(\text{Re}\rho, Z, \bar{Z})$ and then write the Kähler potential (5.32) as follows:

$$K = -\log \left[T^1 T^2 T^3 + 2T^4 T^5 T^6 - T^1 (T^4)^2 - T^2 (T^5)^2 - T^3 (T^6)^2 \right] - \log \frac{v_0}{4} + c_0 \tag{B.8}$$

So for these simple models one can find a fully explicit Kähler potential. On the other hand, these models are too simple to exhibit non-trivial warping effects due to the fluxes.

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