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On the renormalization of the electroweak chiral Lagrangian with a Higgs

M.B. Gavela,[a] K. Kanshin,[b] P.A.N. Machado[a] and S. Saa[a]

[a] *Departamento de Física Teórica and Instituto de Física Teórica, IFT-UAM/CSIC, Universidad Autónoma de Madrid, Cantoblanco, 28049, Madrid, Spain*

[b] *Dipartimento di Fisica e Astronomia "G. Galilei", Università di Padova and INFN, Sezione di Padova, Via Marzolo 8, I-35131 Padua, Italy*

E-mail: belen.gavela@uam.es, kanshin@pd.infn.it, pedro.machado@uam.es, sara.saa@uam.es

Abstract: We consider the scalar sector of the effective non-linear electroweak Lagrangian with a light "Higgs" particle. For a leading order Lagrangian, the complete one-loop off-shell renormalization procedure is performed, including the effects of a finite Higgs mass. This determines the complete set of independent chiral invariant scalar counterterms required for consistency; these include bosonic operators often disregarded. A novel general parametrization of the Goldstone boson matrix is proposed, which reduces to the various usual ones for specific values of its parameter. Furthermore, new counterterms involving the Higgs field which are apparently chiral non-invariant are identified in the perturbative analysis. A redefinition of the Goldstone boson fields which absorbs all chiral non-invariant counterterms is then explicitly determined. The physical results translate into renormalization group equations which may be useful when comparing future Higgs data at different energies.

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1 Introduction

The field of particle physics is at a most interesting cross-roads, in which the fantastic discovery of a light Higgs particle has not been accompanied up to now by any sign of new exotic resonances. If the situation persists, either the so-called electroweak hierarchy problem should stop being considered a problem, with the subsequent revolution and abandon of the historically successful paradigm that fine-tunings call for physical explanations -recall for instance the road to the prediction and discovery of the charm particle, or a questioning of widespread expectations about the nature of physics at the TeV is called for.

Indeed, the experimental lack of resonances other than the Higgs particle casts serious questions on the most popular beyond the Standard Model (BSM) scenarios devised to confront the electroweak hierarchy problem, such as low-energy supersymmetry. While there is still much space for the latter to appear in data to come, it is becoming increasingly pertinent to explore an alternative solution: the possibility that the lightness of the Higgs is due to its being a pseudo-Goldstone boson of some strongly interacting physics, whose scale would be higher than the electroweak one. After all, all previously known pseudoscalar particles are understood as Goldstone or pseudo-Goldstone bosons, as for instance the pion and the other scalar mesons, or the longitudinal components of the W and Z gauge bosons.

A light Higgs as a pseudo-Goldstone boson was proposed already in the 80's [1, 2]. The initial models assumed a strong dynamics corresponding to global symmetry groups such as SU(5) with a characteristic scale Λ_s. One of the Goldstone bosons generated upon spontaneous breakdown of that symmetry was identified with the Higgs particle h, with a Goldstone boson scale f such that $\Lambda_s \leq 4\pi f$. The non-zero Higgs mass would result instead from an explicit breaking of the global symmetry at a lower scale, which breaks the electroweak symmetry and generates dynamically a potential for the Higgs particle [3]. The electroweak scale v, defined from the W gauge boson mass $m_W = gv/2$, does not need to coincide neither with the vacuum expectation value (vev) of the Higgs particle, nor with f, although a relation links them together. In these hybrid linear/non-linear constructions, a linear regime is recovered in the limit in which Λ_s — and thus f — goes to infinity.

The most successful modern variants of the same idea include SO(5) as strong group [4, 5], with the nice new feature that the Standard Model (SM) electroweak interactions themselves may suffice as agents of the explicit breaking. This avenue is being intensively explored, albeit significant fine-tunings in the fermionic sector [6] plague the models considered up to now.

A model-independent way to approach the low-energy impact of a pseudo-Goldstone nature of the Higgs particle is to use the effective Lagrangian for a non-linear realization of electroweak symmetry breaking (EWSB), as it befits the subjacent strong dynamics. While decades ago that effective Lagrangian was determined for the case of a heavy Higgs (that is, a Higgs absent from the low-energy spectrum), only in recent years the formulation has been extended to include a light Higgs particle h [7–13]. On the scalar sector, the effective Lagrangian is necessarily a hybrid construction: while the longitudinal components of $W^\pm$ and Z are well described by an expansion in derivatives, as it corresponds to Goldstone bosons, the insertions of h are generic polynomials.

Effective Lagrangians for BSM physics within a linear realization of EWSB have been extensively studied. They include the scalar h as part of an $SU(2)_L$ doublet. The major differences of the non-linear realization with respect to linear ones are: i) the substitution of the typical functional h dependence in powers of $(v + h)$, corresponding to the SM scalar doublet by a generic functional dependence on h/f; ii) an operator basis which in all generality differs from that in linear realizations.

This last point was recently clarified [14]. If the pseudo-Goldstone boson h is embedded in the high-energy strong dynamics as an electroweak doublet, the number of independent operators coincides with that in linear expansions, as does the relative weight of gauge couplings for fixed number of external h legs. If instead h was born as a Goldstone boson but it was not embedded in the strong dynamics as an electroweak doublet (e.g. if it is a SM singlet), the total number of operators is still as in the linear case but the operators are different: the relative weight of phenomenological gauge couplings, for a fixed number of external h legs, differs from that in the SM and in linear expansions. The best analysis tool then is the general non-linear effective Lagrangian, supplemented by model-dependent relations. Finally, h may not be a pseudo-Goldstone boson but a generic SM scalar singlet:

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e.g. a SM "impostor", a dilaton or any dark sector scalar singlet; the appropriate tool then is that of the non-linear effective Lagrangian with a light h and completely arbitrary coefficients. Note that this Lagrangian can in fact describe all cases mentioned, including the SM one, by setting constraints on its parameters appropriate to each case, and we will thus analyze it here in full generality.

It is important to stress that the choice of the leading order (LO) Lagrangian, for the hybrid (non-linear) expansion is to a large extent arbitrary. Once the LO Lagrangian is specified, counting rules may be explored in order to "guess" what couplings may appear at next to leading order (NLO), as it is done for instance in refs. [15, 16]. Naturally, the counting rule depends on the choice of LO Lagrangian. Nevertheless, the one-loop computation which will be explored here avoids precisely the need to define or discuss any rule: the renormalization procedure will show beyond any doubt what the counterterms required at NLO for that LO Lagrangian are, which is the main motivation for our work.

The one loop renormalization has been extensively pursued in the literature for the linear effective Lagrangian (see e.g. refs. [17–24]), and up to some extent for the non-linear one [25–32]. The latter exploration was restricted to on-shell analysis for phenomenological purposes, while the goal of this manuscript requires an off-shell treatment.

Given the complexity of the off-shell renormalization of the complete LO electroweak chiral Lagrangian involving all SM fields, it is a meaningful and important first step to consider a subsector of the Lagrangian. Although fermions and gauge bosons may play an important role in the physical impact of the NLO complete Lagrangian, we will focus here on the scalar sector of the non-linear theory (i.e. longitudinal components of the W and Z bosons plus h), and we will show that, in this case, the renormalization procedure requires all possible invariant scalar terms up to four derivatives. We implement here the complete off-shell renormalization procedure for the scalar sector, by considering the one-loop corrections to the LO scalar Lagrangian, and furthermore taking into account the finite Higgs mass. The off-shell procedure will allow:

- To guarantee that all counterterms required for consistency are identified, and that the corresponding basis of chirally invariant scalar operators is thus complete. It will follow that some operators often disregarded previously are mandatory when analyzing the bosonic sector by itself.

- To shed light on the expected size of the counterterm coefficients, in relation with current controversies on the application of "naive dimensional analysis" (NDA) [15, 33] for light h.

- To identify the renormalization group equations (RGE) for the bosonic sector of the chiral Lagrangian.

A complete one-loop off-shell renormalization of the electroweak chiral Lagrangian with a decoupled Higgs particle was performed in the seminal papers in ref. [34] (see also ref. [35] and references therein). Using the non-linear sigma model and a perturbative analysis, apparently chiral non-invariant divergences (NIDs) were shown to appear as counterterms of four-point functions for the "pion" fields, in other words, for the longitudinal components of the W and Z bosons. Physical consistency was guaranteed as those NIDs were shown to vanish on-shell and thus did not contribute to physical amplitudes. They were an artifact resulting from performing the computation in terms of a truncated expansion in the number of "pion" fields [36] for a non-linear theory, a procedure which inherently breaks chiral invariance in the intermediate steps, while not in the final result. There is thus no need to add counterterms to cancel the NIDs, but it is convenient to do so in order to check explicitly the consistency of the renormalization procedure. Possible ways to deal with NIDs are a redefinition of the pion fields, leading to their reabsorption as identified in ref. [34], a modification of the usual background field method [37], or a different quantization of the theory which yields non-standard Feynman rules [38]. In the present work, additional new NIDs in three and four-point functions involving the Higgs field will be shown to be present, and their reabsorption explored. Furthermore, a general parametrization of the pseudo-Goldstone boson matrix will be formulated, defining a parameter η which reduces to the various usual pion parametrizations for different values of η, and the non-physical character of all NIDs will be analyzed.

The resulting RGE restricted to the bosonic sector may eventually illuminate future experimental searches when comparing data to be obtained at different energy scales. The structure of the paper can be easily inferred from the table of Contents.

2 The Lagrangian

We will adopt the formulation in refs. [4, 8, 9, 11, 12] to describe in all generality a light scalar boson h in the context of a generic non-linear realization of EWSB. The Lagrangian describes h as a SM singlet scalar whose couplings do not need to match those of an SU(2) doublet. The focus of the present analysis will be set on the physics of the longitudinal components of the gauge bosons (denoted below as "pions" π) and of the h scalar, and only these degrees of freedom will be explicited below. The corresponding Lagrangian can be decomposed as

$$\mathcal{L} = \mathcal{L}_0 + \mathcal{L}_2 + \mathcal{L}_4 \,, \tag{2.1}$$

where the $\mathcal{L}_i$ subindex indicates number of derivatives:

$$\mathcal{L}_0 = -\,V(h)\,, \tag{2.2}$$

$$\mathcal{L}_2 = \frac{1}{2}\partial_\mu h \partial^\mu h\ \mathcal{F}_H(h) - \frac{v^2}{4}\mathrm{Tr}[\mathbf{V}_\mu \mathbf{V}^\mu]\ \mathcal{F}_C(h)\,, \tag{2.3}$$

$$\mathcal{L}_4 = \sum_i c_i \mathcal{P}_i\,. \tag{2.4}$$

We assume $\mathcal{L}_0 + \mathcal{L}_2$ as the leading order Lagrangian, that is, all possible scalar terms up to two derivatives. In eq. (2.3) we have omitted the two-derivative custodial breaking operator, because the size of its coefficient is phenomenologically very strongly constrained. In consequence, and as neither gauge nor Yukawa interactions are considered in this work, no custodial-breaking countertem will be required by the renormalization procedure to be present among the four-derivative operators in $\mathcal{L}_4$. Our analysis is thus restricted to the custodial-preserving sector.

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The $\mathcal{P}_i$ operators in eq. (2.4) are shown explicitly in table 1, with c_i being arbitrary constant coefficients; in the SM limit only a_C and b_C would survive, with $a_C = b_C = 1$. $V(h)$ in eq. (2.2) denotes a general potential for the h field, for which only up to terms quartic in h will be made explicit, with arbitrary coefficients μ_i and λ,

$$V \equiv \mu_1^3\, h + \frac{1}{2} m_h^2 h^2 + \frac{\mu_3}{3!} h^3 + \frac{\lambda}{4!} h^4 \,. \tag{2.5}$$

It will be assumed that the h field is the physical one, with $\langle h \rangle = 0$: the first term in $V(h)$ is provisionally kept in order to cancel the tadpole amplitude at one loop; we will clarify this point in section 3.1.

Note that we have computed only up to quadratic terms in the functions $\mathcal{F}_i(h)$. h^3 and h^4 terms in $\mathcal{F}_C(h)$ and $\mathcal{F}_H(h)$, as well as h^5 and h^6 in $V(h)$ also contribute to the renormalization of the relevant parameters at one-loop. Nevertheless, it is easy to see that they only generate contributions to the counterterms associated to the h and h^2 terms in $\mathcal{F}_C(h)$ and $\mathcal{F}_H(h)$ and terms h^3 and h^4 in $V(h)$, respectively. Therefore, they can be absorbed by the coefficients of those counterterms and thus disregarded. Besides, these contributions are expected to be small. $\mathcal{F}_{H,C}(h)$ will be thus parametrized as [8]

$$\mathcal{F}_{H,C}(h) \equiv 1 + 2a_{H,C} h/v + b_{H,C} h^2/v^2 \,, \tag{2.6}$$

while for all $\mathcal{P}_i(h)$ operators in table 1 the corresponding functions will be defined as[1]

$$c_i \mathcal{F}_i(h) \equiv c_i + 2a_i h/v + b_i h^2/v^2 \,. \tag{2.7}$$

Note that in these parametrizations the natural dependence on h/f expected from the underlying models has been traded by h/v: the relative $\xi \equiv v/f < 1$ normalization is thus implicitly reabsorbed in the definition of the constant coefficients, which are then expected to be small parameters, justifying the truncated expansion. The case of $\mathcal{F}_C(h)$ is special in that the v^2 dependence in front of the corresponding term in the Lagrangian implies a well known fine-tuning to obtain the correct M_W mass, with $a_C = b_C = 1$ in the SM limit. Furthermore, while present data set strong constraints on departures from SM expectations for the latter, a_H and b_H could still be large. Note as well that v/f is not by itself a physical observable from the point of view of the low-energy effective Lagrangian.

A further comment on $\mathcal{F}_H(h)$ may be useful: through a redefinition of the h field [39] it would be possible to absorb it completely. Nevertheless, this redefinition would affect all other couplings in which h participates and induce for instance corrections on fermionic couplings which are weighted by SM Yukawa couplings; it is thus pertinent not to disregard $\mathcal{F}_H(h)$ here, as otherwise consistency would require to include in the analysis the corresponding $\mathcal{F}_i(h)$ fermionic and gauge functions. If a complete basis including all SM fields is considered assigning individual arbitrary functions $\mathcal{F}_i(h)$ to all operators, it would then be possible to redefine away completely one $\mathcal{F}_i(h)$ without loss of generality: it is up

[1]The notation differs slightly from that in ref. [12]: for simplicity, redundant parameters have been eliminated via the replacements $\partial_\mu \mathcal{F}_i(h) \to \partial_\mu h\, \mathcal{F}_i(h)$, $\partial_\mu \mathcal{F}_i(h) \partial_\nu \mathcal{F}_i'(h) \to \partial_\mu h \partial_\nu h\, \mathcal{F}_i(h)$, and $\Box \mathcal{F}_i(h) \to \Box h\, \mathcal{F}_i(h)$.

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to the practitioner to decide which set of independent operators he/she may prefer, and to redefine away one of the functions, for instance $\mathcal{F}_H(h)$. For the time being, we keep explicit $\mathcal{F}_H(h)$ all through, for the sake of generality.[2]

In eqs. (2.3) and (2.4) $\mathbf{V}_\mu \equiv (\mathbf{D}_\mu \mathbf{U})\,\mathbf{U}^\dagger$, with $\mathbf{U}(x)$ being the customary dimensionless unitary matrix describing the longitudinal degrees of freedom of the three electroweak gauge bosons, which transforms under the accidental $\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R$ global symmetry of the SM scalar sector as

$$\mathbf{U}(x) \to L\,\mathbf{U}(x)\,R^\dagger\,, \tag{2.8}$$

where L, R denote the corresponding $\mathrm{SU}(2)_{L,R}$ transformations. Upon EWSB this symmetry is spontaneously broken to the vector subgroup. $\mathbf{V}_\mu$ is thus a vector chiral field belonging to the adjoint of the global $\mathrm{SU}(2)_L$ symmetry. The covariant derivative can be taken in what follows as given by its pure kinetic term $\mathcal{D}_\mu = \partial_\mu$, since the transverse gauge field components will not play a role in this paper.

We analyze next the freedom in defining the $\mathbf{U}$ matrix and work with a general parametrization truncated up to some order in π/v. On-shell quantities must be independent of the choice of parametrization for the $\mathbf{U}$ matrix [36], while it will be shown below that all NIDs depend instead on the specific parametrization chosen. The NIDs in which the h particle participates will turn out to offer a larger freedom to be redefined away than the pure pionic ones.

2.1 The Lagrangian in a general U parametrization

The nonlinear σ model can be written as [36]

$$\mathcal{L}_{\rm NL} = \frac{1}{2} D_\mu \boldsymbol{\pi} D^\mu \boldsymbol{\pi} = \frac{v^2}{4}\mathrm{Tr}[\partial_\mu \mathbf{U} \partial^\mu \mathbf{U}^\dagger] = \frac{1}{2} G_{ij}(\boldsymbol{\pi}^2)\partial_\mu \pi_i \partial^\mu \pi_j\,, \tag{2.9}$$

where D_μ is a derivative "covariant" under the non-linear chiral symmetry, $\mathbf{U}$ has been defined in eq. (2.8) and $\boldsymbol{\pi} = (\pi_1, \pi_2, \pi_3)$ represents the pion vector. In geometric language, $G_{ij}(\boldsymbol{\pi}^2)$ can be interpreted as the metric of a 3-sphere in which the pions live, and the freedom of parametrization is just a coordinate transformation (see ref. [34] and references therein). Indeed, Weinberg has shown [36] that different linear realizations of the chiral symmetry would lead to different metrics, which turn out to correspond to different $\mathbf{U}$ parametrizations; they are all equivalent with respect to the dynamics of the pion fields as the non-linear transformation induced on them is unique, and they are connected via redefinitions of the pion fields. In order to illustrate this correspondence explicitly, let us define general X and Y functions as follows:

$$\mathbf{U} \equiv X(z) + \frac{i\boldsymbol{\tau}\cdot\boldsymbol{\pi}}{v} Y(z), \qquad z = \boldsymbol{\pi}^2/v^2\,, \tag{2.10}$$

[2]Note that $\mathcal{F}_H(h)$ is not expected to be generated from the most popular composite Higgs models, as the latter break explicitly the chiral symmetry only via a potential for h externally generated, while $\mathcal{F}_H(h)$ would require derivative sources of explicit breaking of the chiral symmetry. A similar comment could be applied to $\mathcal{P}_{\Delta H}$.

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where $\boldsymbol{\tau}$ denotes the Pauli matrices, and v is the characteristic scale of the $\boldsymbol{\pi}$ Goldstone bosons. $X(z)$ and $Y(z)$ are related via the unitarity condition $\mathbf{U}\mathbf{U}^\dagger = \mathbf{1}$,

$$X(z) = \sqrt{1 - zY(z)^2}\,. \tag{2.11}$$

The G_{ij} metric can now be rewritten as

$$G_{ij}(\boldsymbol{\pi}^2) = Y(z)^2\delta_{ij} + 4\left(X'(z)^2 + zY'(z)^2 + Y(z)Y'(z)\right)\frac{\pi_i\pi_j}{v^2}, \tag{2.12}$$

where the primes indicate derivatives with respect the the z variable, and $Y(0) = \pm 1$ is required for canonically normalized pion kinetic terms.

The Lagrangian in eq. (2.9) is invariant under the transformation $Y \to -Y$, or equivalently $\boldsymbol{\pi} \to -\boldsymbol{\pi}$. It is easy to relate X and Y to the functions in Weinberg's analysis of chiral symmetry.[3] A Taylor expansion of $\mathbf{U}$ up to order $\boldsymbol{\pi}^{2N+2}$ bears N free parameters. *A priori* the present analysis requires to consider in $\mathscr{L}_2$ terms up to $\mathcal{O}(\boldsymbol{\pi}^6)$, as the latter may contribute to 4-point functions joining two of its pion legs into a loop. Nevertheless, the latter results in null contributions for massless pions, and in practice it will suffice to consider inside $\mathbf{U}$ up to terms cubic on the pion fields. We thus define a single parameter η which encodes all the parametrization dependence,

$$Y(z) \equiv 1 + \eta\, z + O(z^2)\,, \tag{2.13}$$

resulting in

$$\mathbf{U} = 1 - \frac{\boldsymbol{\pi}^2}{2v^2} - \left(\eta + \frac{1}{8}\right)\frac{\boldsymbol{\pi}^4}{v^4} + \frac{i(\boldsymbol{\pi\tau})}{v}\left(1 + \eta\frac{\boldsymbol{\pi}^2}{v^2}\right) + \ldots \tag{2.14}$$

Specific values of η can be shown to correspond to different parametrizations up to terms with four pions, for instance:

- $\eta = 0$ yields the square root parametrization: $\mathbf{U} = \sqrt{1 - \boldsymbol{\pi}^2/v^2} + i(\boldsymbol{\pi\tau})/v\,,$
- $\eta = -1/6$ yields the exponential one: $\mathbf{U} = \exp(i\boldsymbol{\pi}\cdot\tau/v)\,.$

The $\mathcal{L}_2$ Lagrangian can now be written in terms of pion fields. Using the $\mathcal{F}_i(h)$ expansions in eqs. (2.6) and (2.7) it results

$$\begin{aligned}\mathcal{L}_2 =& \frac{1}{2}\partial_\mu h\partial^\mu h\left(1 + 2a_H\frac{h}{v} + b_H\frac{h^2}{v^2}\right) \\ &+ \left\{\frac{1}{2}\partial_\mu\boldsymbol{\pi}\partial^\mu\boldsymbol{\pi} + \frac{(\boldsymbol{\pi}\partial_\mu\boldsymbol{\pi})^2}{2v^2} + \eta\left[\frac{\boldsymbol{\pi}^2(\partial_\mu\boldsymbol{\pi})^2}{v^2} + 2\frac{(\boldsymbol{\pi}\partial_\mu\boldsymbol{\pi})^2}{v^2}\right]\right\}\left(1 + 2a_C\frac{h}{v} + b_C\frac{h^2}{v^2}\right),\end{aligned} \tag{2.15}$$

where terms containing more than four fields are to be disregarded. The operators required by the renormalization procedure to be present in $\mathcal{L}_4$ as counterterms will be shown below to correspond to those on the left-hand side of table 1, which were already known to constitute an independent and complete set of bosonic four-derivative operators [11, 12]. The expansion up to four fields of the terms in $\mathcal{L}_4$ — eq. (2.4) — is shown on the right column of table 1.

[3] The $f(\boldsymbol{\pi}^2)$ function defined in ref. [36] is related to X and Y simply by $f(x) = X(x)/Y(x)$.

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$\mathcal{L}_4$ operators		Expansion in π fields
$c_6\mathcal{P}_6$	$c_6\left[\mathrm{Tr}(V_\mu V^\mu)\right]^2\,\mathcal{F}_6(h)$	$\frac{4c_6}{v^4}(\partial_\mu\boldsymbol{\pi}\partial^\mu\boldsymbol{\pi})^2$
$c_7\mathcal{P}_7$	$c_7\mathrm{Tr}(V_\mu V^\mu)\frac{1}{v}\Box h\,\mathcal{F}_7(h)$	$-\frac{2c_7}{v^3}\Box h(\partial_\nu\boldsymbol{\pi}\partial^\nu\boldsymbol{\pi})-\frac{4a_7}{v^4}(h\Box h)(\partial_\nu\boldsymbol{\pi}\partial^\nu\boldsymbol{\pi})$
$c_8\mathcal{P}_8$	$c_8\mathrm{Tr}(V_\mu V_\nu)\frac{1}{v^2}\partial^\mu h\partial^\nu h\,F_8(h)$	$-2\frac{c_8}{v^4}(\partial_\mu h\partial_\nu h)(\partial^\mu\boldsymbol{\pi}\partial^\nu\boldsymbol{\pi})$
$c_9\mathcal{P}_9$	$c_9\mathrm{Tr}[(\mathcal{D}_\mu V^\mu)^2]\,\mathcal{F}_9(h)$	$-\frac{2c_9}{v^4}\left[v^2(\Box\boldsymbol{\pi}\Box\boldsymbol{\pi})+2\eta\boldsymbol{\pi}^2(\Box\boldsymbol{\pi})^2+(1+4\eta)\,(\boldsymbol{\pi}\Box\boldsymbol{\pi})^2\right.$ $\left.+8\eta(\boldsymbol{\pi}\partial_\mu\boldsymbol{\pi})(\partial^\mu\boldsymbol{\pi}\Box\boldsymbol{\pi})+(2+4\eta)\,(\partial_\mu\boldsymbol{\pi})^2(\boldsymbol{\pi}\Box\boldsymbol{\pi})\right]$ $-\frac{4a_9}{v^3}h(\Box\boldsymbol{\pi}\Box\boldsymbol{\pi})-\frac{2b_9}{v^4}h^2(\Box\boldsymbol{\pi}\Box\boldsymbol{\pi})$
$c_{10}\mathcal{P}_{10}$	$c_{10}\mathrm{Tr}(V_\nu\mathcal{D}_\mu V^\mu)\frac{1}{v}\partial^\nu h\,\mathcal{F}_{10}(h)$	$\frac{-2c_{10}}{v^3}\partial^\nu h(\partial_\nu\boldsymbol{\pi}\Box\boldsymbol{\pi})+\frac{-4a_{10}}{v^4}h\partial^\nu h(\partial_\nu\boldsymbol{\pi}\Box\boldsymbol{\pi})$
$c_{11}\mathcal{P}_{11}$	$c_{11}\left[\mathrm{Tr}(V_\mu V_\nu)\right]^2\,\mathcal{F}_{11}(h)$	$\frac{4c_{11}}{v^4}(\partial_\mu\boldsymbol{\pi}\partial_\nu\boldsymbol{\pi})^2$
$c_{20}\mathcal{P}_{20}$	$c_{20}\mathrm{Tr}(V_\mu V^\mu)\frac{1}{v^2}\partial_\nu h\partial^\nu h\,\mathcal{F}_{20}(h)$	$-2\frac{c_{20}}{v^4}(\partial_\mu h\partial^\mu h)(\partial_\nu\boldsymbol{\pi}\partial^\nu\boldsymbol{\pi})$
$c_{\Box H}\mathcal{P}_{\Box H}$	$\frac{c_{\Box H}}{v^2}(\Box h\Box h)\,\mathcal{F}_{\Box H}(h)$	$\frac{c_{\Box H}}{v^2}\,(\Box h\Box h)+\frac{2a_{\Box H}}{v^3}h\,(\Box h\Box h)+\frac{b_{\Box H}}{v^4}h^2\,(\Box h\Box h)$
$c_{\Delta H}\mathcal{P}_{\Delta H}$	$\frac{c_{\Delta H}}{v^3}(\partial_\mu h\partial^\mu h)\Box h\,\mathcal{F}_{\Delta H}(h)$	$\frac{c_{\Delta H}}{v^3}(\partial_\mu h\partial^\mu h)\Box h+\frac{2a_{\Delta H}}{v^4}(\partial_\mu h\partial^\mu h)h\Box h$
$c_{DH}\mathcal{P}_{DH}$	$\frac{c_{DH}}{v^4}(\partial_\mu h\partial^\mu h)^2\,\mathcal{F}_{DH}(h)$	$\frac{c_{DH}}{v^4}(\partial_\mu h\partial^\mu h)^2$

Table 1. The two columns on the left show the operators required to be in $\mathcal{L}_4$, eq. (2.4), by the renormalization procedure. The right hand side gives the corresponding explicit expansion in terms of pion and h fields (up to four fields), following the $\mathbf{U}$ expansion in eq. (2.8) and the $\mathcal{F}_i(h)$ parametrization in eq. (2.7).

Counterterm Lagrangian. It is straightforward to obtain the counterterm Lagrangian via the usual procedure of writing the bare parameters and field wave functions in terms of the renormalized ones (details in appendix A),

$$\begin{aligned}\delta\mathcal{L}_0+\delta\mathcal{L}_2=&\frac{1}{2}\partial_\mu h\partial^\mu h\left(\delta_h+2\delta a_H\frac{h}{v}+\delta b_H\frac{h^2}{v^2}\right)-\frac{1}{2}\delta m_h^2h^2-\delta\mu_1^3h-\frac{\delta\mu_3}{3!}h^3-\frac{\delta\lambda}{4!}h^4\\&+\frac{1}{2}\partial_\mu\boldsymbol{\pi}\partial^\mu\boldsymbol{\pi}\left(\delta_\pi+2\delta a_C\frac{h}{v}+\delta b_C\frac{h^2}{v^2}\right)\\&+\left(\delta_\pi-\frac{\delta v^2}{v^2}\right)\frac{1}{2v^2}\left((\boldsymbol{\pi}\partial_\mu\boldsymbol{\pi})(\boldsymbol{\pi}\partial^\mu\boldsymbol{\pi})+2\eta\left(\boldsymbol{\pi}^2(\partial_\mu\boldsymbol{\pi})^2+2(\boldsymbol{\pi}\partial_\mu\boldsymbol{\pi})^2\right)\right).\end{aligned}\qquad(2.16)$$

$\delta\mathcal{L}_4$ is simply given by $\mathcal{L}_4$ with the replacement $c_i, a_i, b_i \to \delta c_i, \delta a_i, \delta b_i$, apart from operator $\mathcal{P}_9$, for which

$$\begin{aligned}\delta(c_9\mathcal{P}_9) \to &-\frac{2\delta c_9}{v^4}\left[(1+4\eta)(\boldsymbol{\pi}\Box\boldsymbol{\pi})^2 + 2(1+2\eta)(\boldsymbol{\pi}\Box\boldsymbol{\pi})(\partial_\mu\boldsymbol{\pi})^2\right.\\ &\left.+2\eta\boldsymbol{\pi}^2(\Box\boldsymbol{\pi})^2 + \eta(\Box\boldsymbol{\pi}\partial_\mu\boldsymbol{\pi})(\boldsymbol{\pi}\partial^\mu\boldsymbol{\pi})\right]\\ &-\frac{2}{v^2}\Box\boldsymbol{\pi}\Box\boldsymbol{\pi}\left[\left(\delta c_9 - \frac{\delta v^2}{v^2}\right) + \frac{2\delta a_9 h}{v} + \frac{\delta b_9 h^2}{v^2}\right].\end{aligned}$$

Among the Lagrangian parameters above, v plays the special role of being the characteristic scale of the Goldstone bosons (that is, of the longitudinal degrees of freedom of the electroweak bosons), analogous to the pion decay constant in QCD. It turns out that the counterterm coefficient $\delta v^2 = 0$ as shown below. We have left explicit the δv^2 dependence all through the paper, though, in case it may be interesting to apply our results to some scenario which includes sources of explicit chiral symmetry breaking in a context different than the SM one; it also serves as a check-point of our computations.

3 Renormalization of off-shell Green functions

We present in this section the results for the renormalization of the 1- 2-, 3-, and 4-point functions involving h and/or $\boldsymbol{\pi}$ in a general $\mathbf{U}$ parametrization, specified by the η parameter in eq. (2.14). Dimensional regularization is a convenient regularization scheme as it avoids quadratic divergences, some of which would appear to be chiral noninvariant, leading to further technical complications [38, 40]. Dimensional regularization is thus used below, as well as minimal subtraction scheme as renormalization procedure. The notation

$$\Delta_\varepsilon = +\frac{1}{16\pi^2}\frac{2}{\varepsilon}$$

will be adopted, while FeynRules, FeynArts, and FormCalc [41–45] will be used to compute one-loop amplitudes. Diagrams with closed pion loops give zero contribution for the case of massless pions under study, and any reference to them will be omitted below.

Table 2 provides and overview of which $\mathcal{L}_4$ operator coefficients contribute to amplitudes involving pions and/or h, up to 4-point vertices. It also serves as an advance over the results: all operators in (2.4) will be shown to be required by the renormalization procedure. Furthermore we have checked that they are all independent, and they can be thus chosen as a non-redundant scalar set to be embedded in a complete Lagrangian which should include fermions as well. Only when neglecting all fermion masses, the equations of motion (EOM) would allow to reduce the number of scalar operators singled out above, as some of them would become redundant in that limit. In all generality, no such reduction is appropriate unless those operators are traded by fermionic ones.

It is only when considering all possible couplings and all SM fields, that is, when aiming to analyze the complete electroweak non-linear effective Lagrangian, that the practitioner will have to repeat the "EOM check" to make a sound choice of what operators to keep in the complete basis involving all SM fields, in order to avoid redundancies. Only in that

	Amplitudes						
	$2h$	$3h$	$4h$	2π	$2\pi h$	$2\pi 2h$	4π
$\mathcal{P}_6$							c_6
$\mathcal{P}_7$					c_7	a_7	
$\mathcal{P}_8$						c_8	
$\mathcal{P}_9$				c_9	a_9	b_9	c_9
$\mathcal{P}_{10}$					c_{10}	a_{10}	
$\mathcal{P}_{11}$							c_{11}
$\mathcal{P}_{20}$						c_{20}	
$\mathcal{P}_{\Box H}$	$c_{\Box H}$	$a_{\Box H}$	$b_{\Box H}$				
$\mathcal{P}_{\Delta H}$		$c_{\Delta H}$	$a_{\Delta H}$				
$\mathcal{P}_{DH}$			c_{DH}				

Table 2. Illustration of which operators in $\mathcal{L}_4$ (see eq. (2.4) and table 1) contribute to 2-, 3-, and 4-point amplitudes involving pions and/or h fields. The specific operator coefficients contributing to each amplitude are indicated, following the $c_i\mathcal{F}_i$ expansion in eq. (2.7).

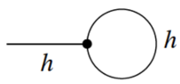

Figure 1. Diagram contributing to the Higgs 1-point function.

perspective it would be correct to trade some of the counterterms obtained here by fermionic or other couplings via the application of EOM, in the absence of further assumptions. For comparison with previous literature on the scalar sector, see section 5.

3.1 1-point functions

Because of chiral symmetry pions always come in even numbers in any vertex, unlike Higgs particles, thus tadpole contributions may be generated only for the latter. At tree-level it would suffice to set $\mu_1 = 0$ in $V(h)$ (eq. (2.2)) in order to insure $\langle h \rangle = 0$. At one-loop, a tadpole term is induced from the triple Higgs couplings μ_3 and a_H, though, via the Feynman diagram in figure 1. The counterterm required to cancel this contribution reads

$$\delta\mu_1^3 = m_h^2 \left(\frac{\mu_3}{2} - a_H \frac{m_h^2}{v} \right) \Delta_\varepsilon \,, \tag{3.1}$$

and has no impact on the rest of the Lagrangian.

3.2 2-point functions

Consider mass and wave function renormalization for the pion and h fields. Because of chiral symmetry no pion mass will be induced by loop corrections at any order, unlike for the h field, whose mass is not protected by that symmetry. The diagrams contributing to the pion self-energy are shown in figure 2. The divergent part of the amplitudes, $\Pi^{ij}_{\rm div}(p^2)\Delta_\varepsilon$,

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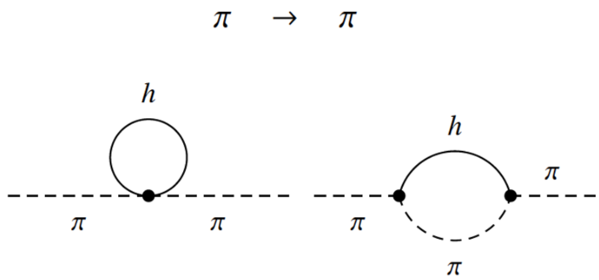

Figure 2. Diagrams contributing to the π self-energy.

and the counterterm structure are given by

$$\Pi^{ij}_{\rm div}(p^2) = \left[p^2\left(a_C^2 - b_C\right)\frac{m_h^2}{v^2} + p^4\frac{a_C^2}{v^2}\right]\delta_{ij}\,, \tag{3.2}$$

$$\Pi^{ij}_{ctr}(p^2) = \left[p^2\delta_\pi - p^4\frac{4}{v^2}\left(\delta c_9 - \frac{\delta v^2}{v^2}\right)\right]\delta_{ij}\,. \tag{3.3}$$

In an off-shell renormalization scheme, it is necessary to match all the momenta structure of the divergent amplitude with that of the counterterms, which leads to the following determination

$$\begin{aligned} \delta_\pi &= -\left(a_C^2 - b_C\right)\frac{m_h^2}{v^2}\Delta_\varepsilon\,, \\ \delta c_9 - \frac{\delta v^2}{v^2} &= \frac{a_C^2}{4}\Delta_\varepsilon\,. \end{aligned} \tag{3.4}$$

It follows that the π wave function renormalization has no divergent part whenever $a_C^2 = b_C$, which happens for instance in the case of the SM ($a_C = b_C = 1$). Note as well that the absence of a constant term in eq. (3.2) translates into massless pions at 1-loop level, as mandated by chiral symmetry at any loop order. Furthermore, the p^4 term stems from the $h - \pi$ coupling a_C, which is an entire new feature compared to the nonlinear σ model renormalization. This term demands the presence of a $\Box\boldsymbol{\pi}\Box\boldsymbol{\pi}$ counterterm in the $\mathcal{L}_4$ Lagrangian, as expected by naive dimensional analysis.

Turning to the Higgs particle, the diagrams contributing to its self-energy are shown in figure 3, with the divergent part and the required counterterm structure given by

$$\begin{aligned} \Pi_{\rm div}(p^2) =& p^4\frac{(3a_C^2 + a_H^2)}{2v^2} + p^2\left(-\frac{\mu_3}{v}a_H + \frac{m_h^2\left(5a_H^2 - b_H\right)}{v^2}\right) \\ &+ \left(\frac{1}{2}\mu_3^2 + \frac{1}{2}m_h^2\left(\lambda - 8\frac{\mu_3}{v}a_H\right) + \frac{m_h^4\left(6a_H^2 - b_H\right)}{v^2}\right), \end{aligned} \tag{3.5}$$

$$\Pi_{ctr}(p^2) = p^4\frac{2\delta c_{\Box H}}{v^2} + p^2\delta_h - \delta m_h^2\,. \tag{3.6}$$

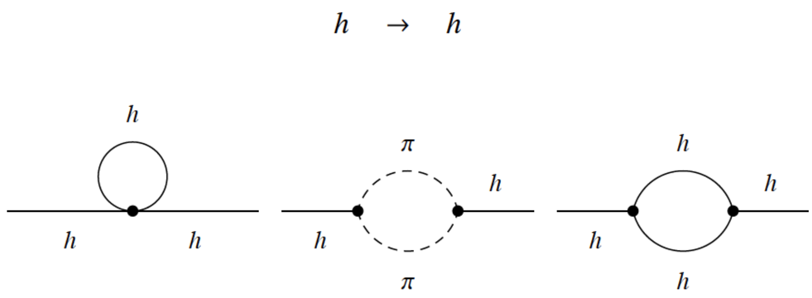

Figure 3. Diagrams contributing to the Higgs self-energy.

It follows that the required counterterms are given by

$$
\begin{aligned}
\delta_h &= \left[\frac{\mu_3}{v}a_H + \frac{m_h^2\left(b_H - 5a_H^2\right)}{v^2}\right]\Delta_\varepsilon, \\
\delta m_h^2 &= \left[\frac{1}{2}\mu_3^2 + \frac{1}{2}m_h^2\left(\lambda - 8\frac{\mu_3}{v}a_H\right) + \frac{m_h^4\left(6a_H^2 - b_H\right)}{v^2}\right]\Delta_\varepsilon, \qquad (3.7) \\
\delta c_{\Box H} &= -\frac{1}{4}\left(3a_C^2 + a_H^2\right)\Delta_\varepsilon.
\end{aligned}
$$

This result implies that a non-vanishing a_C (as in the SM limit) and/or a_H leads to a p^4 term in the counterterm Lagrangian, requiring a $\Box h \Box h$ term in $\mathcal{L}_4$. In this scheme, a Higgs wave function renormalization is operative only in deviations from the SM with non-vanishing a_H and/or b_H.

3.3 3-point functions

The computational details for the 3- and 4-point functions will not be explicitly shown as they are not particularly illuminating.[4] Vertices with an odd number of legs necessarily involve at least one Higgs particle.

***hhh*.** Let us consider first the hhh amplitude at one loop. The relevant diagrams to be computed are displayed in figure 4. As h behaves as a generic singlet, the vertices involving uniquely external h legs which appear in the Lagrangian eq. (2.1) will span all possible momentum structures that can result from one-loop amplitudes. Hence any divergence emerging on amplitudes involving only external h particles will be easily absorbable. The specific results for the counterterms emerging from $\mathcal{L}_0$ and $\mathcal{L}_2$ can be found in appendix A.

$\pi\pi h$. The diagrams for $\pi\pi h$ amplitudes are shown in figure 5. The one-loop divergences are studied in detail in appendix A; for instance, it turns out that neither δa_C nor δa_9 are induced in the SM limit. Chiral symmetry restricts the possible structures spanned by the pure π and $h - \pi$ operators. Because of this, it turns out that part of the divergent amplitude induced by the last diagram in figure 5 cannot be cast as a function of the $\mathcal{L}_2$ and $\mathcal{L}_4$ operators, that is, it cannot be reabsorbed by chiral-invariant counterterms, and

[4]See appendix A for details and ref. [46] for an exhaustive description.

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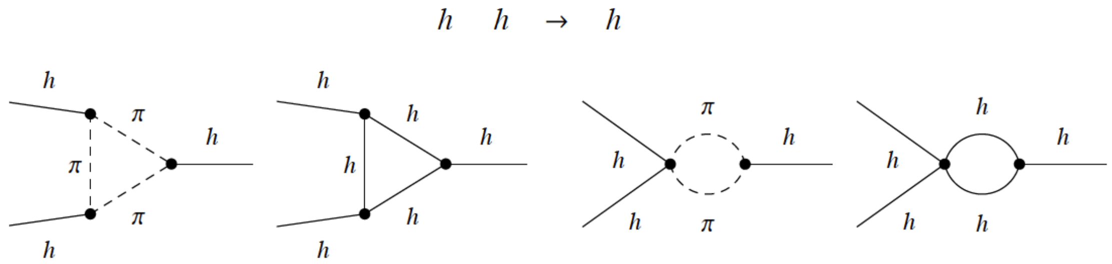

Figure 4. Diagrams contributing to the $hh \to h$ amplitude, not including diagrams obtained by crossing.

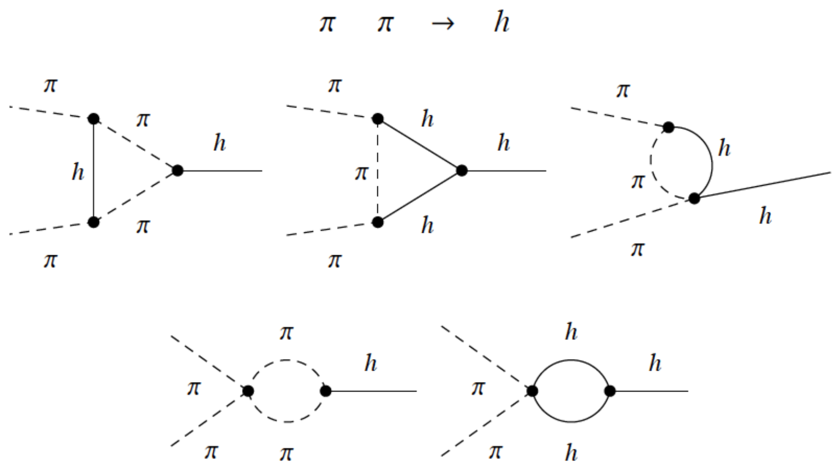

Figure 5. Diagrams contributing to the $\pi\pi \to h$ scattering amplitude, not including diagrams obtained by crossing.

furthermore its coefficient depends on the pion parametrization used: an apparent non chiral-invariant divergence has been identified. NIDs are an artifact of the apparent breaking of chiral symmetry when the one-loop analysis is treated in perturbation theory [36] and have no physical impact as they vanish for on-shell amplitudes. While long ago NIDs had been isolated in perturbative analysis of four-pion vertices in the non-linear sigma model [34], the result obtained here is a new type of NIDs: a three-point function involving the Higgs particle, corresponding to the chiral non-invariant operator

$$\mathcal{O}_1^{\mathrm{NID}} = -a_C \left(\frac{3}{2} + 5\eta\right) \frac{\Delta_\varepsilon}{v^3}\, \boldsymbol{\pi}\Box\boldsymbol{\pi}\Box h\,. \tag{3.8}$$

This coupling cannot be reabsorbed as part of a chiral invariant counterterm, but its contribution to on-shell amplitudes indeed vanishes. It is interesting to note that while the renormalization conditions of all physical parameters turn out to be independent of the choice of $\mathbf{U}$ parametrization, as they should, NIDs exhibit instead an explicit η dependence, as illustrated by eq. (3.8). This pattern will be also present in the renormalization of 4-point functions, developed next.

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3.4 4-point functions

The analysis of this set of correlation functions turns out to be tantalizing when comparing the results for mixed $\pi - h$ vertices with those for pure pionic ones.[5]

$\boldsymbol{\pi\pi hh}$**.** The computation of the $\pi\pi \to hh$ one-loop amplitude shows that the renormalization procedure requires the presence of all possible chiral invariant $hh\pi\pi$ counterterms in the Lagrangian, in the most general case.

Furthermore, we have identified new NIDs in $hh\pi\pi$ amplitudes:

$$\begin{aligned}
\mathcal{O}_2^{\rm NID} &= +(2a_C^2 - b_C)\left(\frac{3}{2}+5\eta\right)\frac{\Delta_\varepsilon}{v^4}\,\boldsymbol{\pi}\Box\boldsymbol{\pi}\,h\Box h\,,\\
\mathcal{O}_3^{\rm NID} &= +(a_C^2 - b_C)\left(\frac{3}{2}+5\eta\right)\frac{\Delta_\varepsilon}{v^4}\,\boldsymbol{\pi}\Box\boldsymbol{\pi}\,\partial_\mu h\partial^\mu h\,,\\
\mathcal{O}_4^{\rm NID} &= -2a_C^2\left(\frac{3}{2}+5\eta\right)\frac{\Delta_\varepsilon}{v^4}\,\boldsymbol{\pi}\partial_\mu\boldsymbol{\pi}\,\partial^\mu h\Box h\,.
\end{aligned}\tag{3.9}$$

While these NIDs differ from that for the three-point function in eq. (3.8) in their counterterm structure, they all share an intriguing fact: to be proportional to the factor $(3/2+5\eta)$. Therefore a proper choice of parametrization, i.e. $\eta = -3/10$, removes all mixed $h - \pi$ NIDs. That value of η is of no special significance as fas as we know, and in fact there is no choice of parametrization that can avoid *all* noninvariant divergences, as proved next.

$\boldsymbol{\pi\pi\pi\pi}$**.** Consider now $\pi\pi \to \pi\pi$ amplitudes. Only two counterterms are necessary to reabsorb chiral-invariant divergences, namely δc_6 and δc_{11}. In this case, we find no other NIDs than those already present in the nonlinear σ model [34], which stemmed from the insertion in the loop of the four-pion vertex (whose coupling depends on η). Our analysis shows that the four-π NIDs read:

$$\begin{aligned}
\mathcal{O}_5^{\rm NID} &= +\left(9\eta^2+5\eta+\frac{3}{4}\right)\frac{\Delta_\varepsilon}{v^4}\,(\boldsymbol{\pi}\Box\boldsymbol{\pi})^2,\\
\mathcal{O}_6^{\rm NID} &= +\left[1+4\eta+\left(\frac{1}{2}+\eta\right)a_C^2\right]\frac{\Delta_\varepsilon}{v^4}\,(\boldsymbol{\pi}\Box\boldsymbol{\pi})(\partial_\mu\boldsymbol{\pi}\partial^\mu\boldsymbol{\pi}),\\
\mathcal{O}_7^{\rm NID} &= +2\eta^2\frac{\Delta_\varepsilon}{v^4}\,\boldsymbol{\pi}^2(\Box\boldsymbol{\pi})^2,\\
\mathcal{O}_8^{\rm NID} &= +2\eta\left(a_C^2-1\right)\frac{\Delta_\varepsilon}{v^4}\,(\Box\boldsymbol{\pi}\partial_\mu\boldsymbol{\pi})(\boldsymbol{\pi}\partial^\mu\boldsymbol{\pi}).
\end{aligned}\tag{3.10}$$

As expected, the parametrization freedom — the dependence on the η parameter — appears only in NIDs, and never on chiral-invariant counterterms, as the latter describe physical processes. Furthermore, the contribution of all NIDs to on-shell amplitudes vanishes as expected.[6] Finally, the consideration of the ensemble of three and four-point NIDs in eqs. (3.8), (3.9) and (3.10) shows immediately that no parametrization can remove *all*

[5] It provides in addition nice checks of the computations; for instance we checked explicitly in the present context that the consistency of the renormalization results for four-point functions requires $\delta_v^2 = 0$.

[6] This is not always seen when taken individually. For instance, the contribution of $\mathcal{O}_4^{\rm NID}$ to the $hh\pi\pi$ amplitude is cancelled by that of $\mathcal{O}_1^{\rm NID}$, which corrects the $h\pi\pi$ vertex.

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NIDs: it is possible to eliminate those involving h,[7] but no value of η would remove all pure pionic ones.

***hhhh*.** The renormalization procedure for $hh \to hh$ amplitudes is straightforward. It results in contributions to $\delta a_{\Delta H}$, δc_{DH}, and δb_H. Interestingly, appendix B illustrates that large coefficients are present in some terms of the RGE for b_H and λ; this might *a priori* translate into measurable effects when comparing data at different scales, if ever deviations from the SM predictions are detected, see section 4.

There is a particularity of the off-shell renormalization scheme which deserves to be pointed out. A closer look at the counterterms reveals that, in the SM case, that is

$$a_C = b_C = 1, \quad a_H = b_H = 0, \quad \mu_3 = 3\frac{m_h^2}{v} \quad \textit{and} \quad \lambda = 3\frac{m_h^2}{v^2}, \tag{3.11}$$

several BSM operator coefficients do not vanish. Although at first this might look counterintuitive, when calculating physical amplitudes the contribution of these non-vanishing operator coefficients all combine in such a way that the overall BSM contribution indeed cancels. The same pattern propagates to the renormalization group equations discussed in section 4.

3.5 Dealing with the apparent non-invariant divergences

For the nonlinear σ model the issue of NIDs was analyzed long ago [34, 37, 40, 47–49]). In that case, it was finally proven that a nonlinear redefinition of the pion field which includes space-time derivatives could reabsorb them [34]. This method reveals a deeper rationale in understanding the issue, as Lagrangians related by a field redefinition are equivalent, even when it involves derivatives [50–53]. Consequently, if via a pion field redefinition[8]

$$\boldsymbol{\pi} \to \boldsymbol{\pi}\, f(\boldsymbol{\pi}, h, \partial_\mu \boldsymbol{\pi}, \partial_\mu h, \dots)\,,$$

with $f(0) = 1$, the Lagrangian is shifted

$$\mathcal{L} \to \mathcal{L}' = \mathcal{L} + \delta\mathcal{L}\,,$$

from the equivalence between $\mathcal{L}$ and $\mathcal{L}'$ it follows that $\delta\mathcal{L}$ must be unphysical. Thus, if an appropriate pion field redefinition is found which is able to absorb all NIDs, it automatically implies that NIDs do not contribute to the S-matrix, and therefore that chiral symmetry remains unbroken. *In other words, the non-invariant operators can be identified with quantities in the functional generator that vanish upon performing the path integral.*

Let us consider the following pion redefinition, in which we propose new terms not considered previously and which contain the h field:

$$\begin{aligned}\pi_i \to \pi_i \bigg(1 &+ \frac{\alpha_1}{2v^4}\boldsymbol{\pi}\Box\boldsymbol{\pi} + \frac{\alpha_2}{2v^4}\partial_\mu\boldsymbol{\pi}\partial^\mu\boldsymbol{\pi} + \frac{\beta}{2v^3}\Box h + \frac{\tilde{\gamma}_1}{2v^4}h\Box h + \frac{\gamma_2}{2v^4}\partial_\mu h \partial^\mu h\bigg) \\ &+ \frac{\alpha_3}{2v^4}\Box\pi_i(\boldsymbol{\pi}\boldsymbol{\pi}) + \frac{\alpha_4}{2v^4}\partial_\mu\pi_i(\boldsymbol{\pi}\partial^\mu\boldsymbol{\pi}).\end{aligned} \tag{3.12}$$

[7] This may be linked to the larger freedom of redefinition for fields not subject to chiral invariance.

[8] Notice that this field redefinition is by no means unique. A redefinition of the Higgs field involving the pion fields and derivatives could also be done together with the one proposed here, but this does not add anything relevant for this discussion.

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The application of this redefinition to $\mathcal{L}_4$ is immaterial, as it would only induce couplings of higher order. As all terms in the shift contain two derivatives, when applied to $\mathcal{L}_2$ contributions to $\mathcal{L}_4$ and NID operator coefficients do follow. Indeed, the action of eq. (3.12) on $\mathcal{L}_2$ reduces to that on the term

$$\frac{1}{4}\mathrm{Tr}(\partial_\mu \mathbf{U}\partial^\mu \mathbf{U})\mathcal{F}_C(h)\,, \tag{3.13}$$

which produces the additional contribution to NID vertices given by

$$\begin{aligned}\Delta\mathcal{L}^{\rm NID} =& -\boldsymbol{\pi}\Box\boldsymbol{\pi}\left(\frac{\alpha_1}{v^4}\boldsymbol{\pi}\Box\boldsymbol{\pi}+\frac{\alpha_2}{v^4}\partial_\mu\boldsymbol{\pi}\partial^\mu\boldsymbol{\pi}+\frac{\beta}{v^3}\Box h+\frac{\gamma_1}{v^4}h\Box h+\frac{\gamma_2}{v^4}\partial_\mu h\partial^\mu h\right)\\ & -\frac{\alpha_3}{v^4}(\Box\boldsymbol{\pi}\Box\boldsymbol{\pi})(\boldsymbol{\pi}\boldsymbol{\pi})-\frac{\alpha_4}{v^4}(\Box\boldsymbol{\pi}\partial_\mu\boldsymbol{\pi})(\boldsymbol{\pi}\partial^\mu\boldsymbol{\pi})-\frac{2a_C\beta}{v^4}\boldsymbol{\pi}\partial_\mu\boldsymbol{\pi}\partial^\mu h\Box h+\ldots\end{aligned} \tag{3.14}$$

where $\gamma_1 = 2a_C\beta+\tilde{\gamma}_1$, and where the dots indicate other operators with either six derivatives or that have more than four fields and are beyond the scope of this paper. Comparing the terms in $\Delta\mathcal{L}^{\rm NID}$ with the NID operators found, eqs. (3.8), (3.9) and (3.10), it follows that by choosing

$$\begin{aligned}\alpha_1 &= \left(9\eta^2+5\eta+\frac{3}{4}\right)\Delta_\varepsilon, & \beta &= -\left(\frac{3}{2}+5\eta\right)a_C\Delta_\varepsilon,\\ \alpha_2 &= \left[1+4\eta+\left(\frac{1}{2}+\eta\right)a_C^2\right]\Delta_\varepsilon, & \gamma_1 &= \left(\frac{3}{2}+5\eta\right)\left(2a_C^2-b_C\right)\Delta_\varepsilon,\\ \alpha_3 &= 2\eta^2\Delta_\varepsilon, & \gamma_2 &= \left(\frac{3}{2}+5\eta\right)\left(a_C^2-b_C\right)\Delta_\varepsilon.\\ \alpha_4 &= 2\eta\left(a_C^2-1\right)\Delta_\varepsilon, & &\end{aligned}$$

all 1-loop NIDs are removed away.

A few comments are in order. Because of chiral symmetry, the pure pionic or mixed pion-h operators do not encode all possible momentum structures, even after pion field redefinitions. Hence, the appearance of divergent structures that can be absorbed by $\delta\mathcal{L}_0$, $\delta\mathcal{L}_2$, $\delta\mathcal{L}_4$ and $\Delta\mathcal{L}^{\rm NID}$ is a manifestation of chiral invariance and of the field redefinition equivalence discussed above. We have shown consistently that NIDs appearing in the one-loop renormalization of the electroweak chiral Lagrangian do not contribute to on-shell quantities. In fact, a closer examination has revealed that the apparent chiral non-invariant divergences emerge from loop diagrams which have at least one four-pion vertex in it, and this is why all of them depend on η. We have also shown that the presence of a light Higgs boson modifies the coefficients of the unphysical counterterms made out purely of pions, but not their structure, neither -of course- breaks chiral symmetry.

The field redefinitions implemented above to reabsorb the scalar NIDs may indeed be equivalent to the application of the pion EOM [54], and contribute to other type of NIDs: for instance those involving simultaneously pions and fermions, not yet explored. Their exact computation is not called for when exploring the scalar sector and the set of purely scalar counterterms required at one loop by the theory, which is what is clarified here.

4 Renormalization Group Equations

It is straightforward to derive the RGE from the δc_i divergent contributions determined in the previous section. The complete RGE set can be found in appendix B. As illustration,

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the evolution of those Lagrangian coefficients which do not vanish in the SM limit is given by:

$$
\begin{aligned}
16\pi^2 \frac{d}{d\ln\mu} a_C =& \frac{1}{2} a_C \left[a_H \frac{\mu_3}{v} + \left(3b_C - 5a_H^2 + b_H\right) \frac{m_h^2}{v^2} \right] + a_C^2 \left(\frac{\mu_3}{2v} - 2a_H \frac{m_h^2}{v^2} \right) - \frac{3}{2} a_C^3 \frac{m_h^2}{v^2} \\
& - \frac{1}{2v} b_C \mu_3 + 2a_H b_C \frac{m_h^2}{v^2}\,, \qquad (4.1)\\
16\pi^2 \frac{d}{d\ln\mu} b_C =& b_C \left[2a_C \frac{\mu_3}{v} + 5a_H \frac{\mu_3}{v} - \frac{\lambda}{2} - \left(5a_C^2 + 8a_H a_C + 17a_H^2 - 3b_H\right) \frac{m_h^2}{v^2} \right] + b_C^2 \frac{m_h^2}{v^2} \\
& + \frac{1}{2} \left(-4a_C \frac{\mu_3}{v} - 8a_H \frac{\mu_3}{v} + \lambda \right) a_C^2 + 2 \left(2a_C^2 + 4a_H a_C + 6a_H^2 - b_H\right) a_C^2 \frac{m_h^2}{v^2} \\
16\pi^2 \frac{d}{d\ln\mu} m_h^2 =& -\frac{1}{2} \mu_3^2 + \left(5a_H \frac{\mu_3}{v} - \frac{\lambda}{2} \right) m_h^2 + \left(2b_H - 11a_H^2\right) \frac{m_h^4}{v^2}\,, \qquad (4.2)\\
16\pi^2 \frac{d}{d\ln\mu} \mu_3 =& \frac{1}{2} \mu_3 \left[\left(-a_C^2 + b_C - 87a_H^2 + 15b_H\right) \frac{m_h^2}{v^2} - 3\lambda \right] + \frac{15}{2v} \mu_3^2 a_H \\
& + 6a_H \lambda \frac{m_h^2}{v} + 6 \left(8a_H^3 - 3a_H b_H\right) \frac{m_h^4}{v^3}\,, \qquad (4.3)\\
16\pi^2 \frac{d}{d\ln\mu} \lambda =& \lambda \left[26a_H \frac{\mu_3}{v} + \left(14b_H - 82a_H^2\right) \frac{m_h^2}{v^2} \right] - \frac{3}{2} \lambda^2 + 12 \left(b_H - 6a_H^2\right) \frac{\mu_3^2}{v^2} \qquad (4.4)\\
& + 48a_H \left(8a_H^2 - 3b_H\right) \mu_3 \frac{m_h^2}{v^3} - 6 \left(80a_H^4 - 48b_H a_H^2 + 3b_H^2\right) \frac{m_h^4}{v^4}\,. \qquad (4.5)
\end{aligned}
$$

These and the rest of the RGE in appendix B show as well that the running of the parameters a_C, b_C, a_H, b_H, and v^2 is only induced by the couplings entering the Higgs potential, eq. (2.5).

Note that in the RGE for the Higgs quartic self-coupling λ, eq. (4.5), some terms are weighted by numerical factors of $\mathcal{O}(100)$. This suggests that if a BSM theory results in small couplings for a_H and b_H, those terms could still induce measurable phenomenological consequences. Nevertheless, physical amplitudes will depend on a large combination of parameters, which might yield cancellations or enhancements as pointed out earlier, and only a more thorough study can lead to firm conclusions. Such large coefficients turn out to be also present in the evolution of some BSM couplings, such as the four-Higgs coupling b_H for which

$$
\begin{aligned}
16\pi^2 \frac{d}{d\ln\mu} b_H =& b_H \left[20a_H \frac{\mu_3}{v} - \frac{3}{2}\lambda + \left(-a_C^2 + b_C - 87a_H^2\right) \frac{m_h^2}{v^2} \right] \\
& - 42 \frac{\mu_3}{v} a_H^3 + \frac{13}{2} \lambda a_H^2 + \left(7b_H^2 + 120a_H^4\right) \frac{m_h^2}{v^2}\,. \qquad (4.6)
\end{aligned}
$$

On general grounds a_H is expected to be small, and for instance the a_H^4 dependence in eq. (4.6) is not expected to be relevant in spite of the numerical prefactor. On the other side, present data set basically no bound on the couplings involving three or more external Higgs particles, and thus the future putative impact of this evolution should not be dismissed yet.

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5 Comparison with the literature

Previous works on the one-loop renormalization of the scalar sector of the non-linear Lagrangian with a light Higgs have used either the square root parametrization ($\eta = 0$ in our parametrization) or the exponential one ($\eta = -1/6$), producing very interesting results, and have

- concentrated on on-shell analyses,
- disregarded the impact of $\mathcal{F}_H(h)$,
- disregarded fermionic operators; in practice this means to neglect all fermion masses.

This last point is not uncorrelated with the fact that the basis of independent four-derivative operators determined here has a larger number of elements than previous works about the scalar sector. Those extra bosonic operators have been shown here to be required by the counterterm procedure. It is possible to demonstrate, though, that they can be traded via EOM by other type of operators including gauge corrections and Yukawa-like operators. In a complete basis of all possible operators it is up to the practitioner to decide which set is kept, as long as it is complete and independent. When restricting instead to a given subsector, the complete and consistent treatment requires to consider all independent operators of the kind selected (anyway the renormalization procedure will indicate their need), or to state explicitly any extra assumptions to eliminate them. Some further specific comments:

Ref. [28] considers, under the first two itemized conditions above plus disregarding the impact of $V(h)$ (and in particular neglecting the Higgs mass), the scattering processes $hh \to hh$, $\pi\pi \to hh$ and $\pi\pi \to \pi\pi$. With the off-shell treatment, five additional operators result in this case with respect to those obtained in that reference (assuming the rest of their assumptions), $\mathcal{P}_7$, $\mathcal{P}_9$, $\mathcal{P}_{10}$, $\mathcal{P}_{\Box H}$ and $\mathcal{P}_{\Delta H}$ in table 1. Note that all these operators contain either $\Box h$ or $\Box \pi$ inside; they may thus be implicitly traded by fermionic operators via EOM, and can only be disregarded if all fermion masses are neglected. Assuming this extra condition, we could reproduce their results using the EOMs. For instance, the RGEs derived here for c_6, c_8, c_{20}, a_C and b_C differ from the corresponding ones in that reference: an off-shell renormalization analysis entails the larger number of operators mentioned. In any case, we stress again that the results of both approaches coincide when calculating physical amplitudes. Another contrast appears in the running of a_C, b_C, a_H, b_H, as well as the mass, the triple, and the quartic coupling of the Higgs, for which the running is induced by the Higgs potential parameters, disregarded in that reference.

In ref. [29] the on-shell scattering process $W_L^+ W_L^- \to Z_L Z_L$ is considered (the L subscript refers to the longitudinal modes of the gauge bosons), disregarding $\mathcal{F}_H(h)$ but including the impact of $V(h)$. Our off-shell treatment results in this case in one additional pure-pion operator -assuming the rest of their assumptions- with respect to those in that reference: $\mathcal{P}_9$. This extra operator contains $\Box \pi$ in all its terms and it does not enter into physical amplitudes when all fermion masses are disregarded. In this case our results reduce to theirs.

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6 Conclusions

We have considered the one-loop off-shell renormalization of the effective non-linear Lagrangian in the presence of a light (Higgs) scalar particle, taking into account the finite Higgs mass and its potential. We have concentrated on its scalar sector: goldstone bosons (that is, the longitudinal components of the SM gauge bosons) and the light scalar h, choosing a leading order Lagrangian containing all possible scalar interactions up to two derivatives. No power counting has been assumed neither derived, as the renormalization procedure suffices to reveal the NLO Lagrangian.

Analyzing the custodial-preserving sector, we have determined the four-derivative counterterms required by the one-loop renormalization procedure, by considering the full set of 1-, 2-, 3- and 4-point functions involving pions and/or h. The off-shell treatment has allowed to determine all required counterterms, confirming for the sector analyzed that the generic low-energy effective non-linear Lagrangian with a light Higgs particle developed in refs. [11, 12] is complete: all four-derivative operators of that basis and nothing else is induced by the renormalization. Those operators are linearly independent and form thus a complete basis when that sector is taken by itself. It is shown that a larger number of operators than previously considered are then needed. They are independent and non redundant: the use of EOM would require to substitute some of them by other fermionic and/or gauge operators, maintaining the same total number of operators. As we do not analyze here the complete basis made out of all possible SM fields, we chose to maximize the number of independent operators in the scalar sector.

As a useful analysis tool, we have also proposed here a general parametrization for the Goldstone boson matrix, which at the order considered here depends on only one parameter η, and reduces to the popular parametrizations (square root, exponential etc.) for different values of η. All counterterms induced by the renormalization procedure are then easily seen to be parametrization independent, as it befits physical couplings.

Furthermore, new chiral non-invariant counterterms involving the Higgs particle and pions have been found in our perturbative analysis. These findings extend to the realm of the Higgs particle the apparent non chiral-invariant divergences identified decades ago for the non-linear sigma model [34]. Those apparent violations of chiral symmetry are an artifact of perturbative approaches, they vanish on-shell, and their origin had been tracked down to the insertion of the four-pion vertex in loops. In this paper, new non-invariant divergences are shown to appear in triple $h\pi\pi$ counterterms and in $hh\pi\pi$ ones, and shown to have the same origin. Interestingly: i) all apparently non-invariant divergences depend explicitly on η, consistent with their non-physical nature; ii) there is a value of the η parameter for which the non-invariant divergences involving the Higgs vanish, though, while no η value can cancel the ensemble of non-invariant divergences and in particular the pure pion ones.

Moreover, we have determined a pion-field redefinition which includes space-time derivatives and reabsorbs automatically *all* apparently chiral non-invariant counterterms. This field redefinition leaves invariant the S-matrix and thus the result shows automatically that chiral symmetry remains unbroken.

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For the physical counterterms induced, we observe a complete agreement with the naive dimensional analysis [15, 33] in the $h-\pi$ sector of the chiral Lagrangian. Finally, the RGEs for the scalar sector of the general non-linear effective Lagrangian for a Higgs particle have been also derived in this work. The complete set of equations can be found in appendix B. Factors of $\mathcal{O}(100)$ appear accompanying certain operator coefficients in the RGEs, and those terms may thus be specially relevant when comparing future Higgs and gauge boson data obtained at different energies. On more general grounds, although present data are completely consistent with the SM predictions, going for precision in constraining small parameters may be the best way to tackle BSM physics and we should not be deterred by the task: the dream of today may be the discovery of tomorrow and the background of the future.

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A The counterterms

Details about the computation of the counterterms and the renormalization of the chiral Lagrangian are given in this appendix, including the derivation of the RGEs.

The bare parameters (denoted by b) written in terms of the renormalized ones and the counterterms for the $\mathcal{L}_2$ and $\mathcal{L}_4$ Lagrangians are given by

$$\begin{aligned} h_b &= \sqrt{Z_h}h, \qquad \delta_h \equiv Z_h - 1, \\ \boldsymbol{\pi}_b &= \sqrt{Z_\pi}\boldsymbol{\pi}, \qquad \delta_\pi \equiv Z_\pi - 1, \end{aligned}$$

$$\begin{aligned} v_b^2 &= Z_\pi(v^2+\delta v^2)\mu^{-\varepsilon}, & a_C^b &= \frac{1}{Z_\pi^{1/2}Z_h^{1/2}}\left(a_C+\delta a_C+\frac{a_C}{2}\frac{\delta v^2}{v^2}\right), \\ (m_h^2)^b &= \frac{1}{Z_h}(m_h^2+\delta m_h^2), & b_C^b &= \frac{1}{Z_h}\left(b_C+\delta b_C+b_C\frac{\delta v^2}{v^2}\right), \\ (\mu_1^3)^b &= \frac{1}{Z_h^{1/2}}\left(\mu_1^3+\delta\mu_1^3\right)\mu^{3\varepsilon/2}, & a_H^b &= \frac{Z_\pi^{1/2}}{Z_h^{3/2}}\left(a_H+\delta a_H+\frac{a_H}{2}\frac{\delta v^2}{v^2}\right), \\ \mu_3^b &= \frac{1}{Z_h^{3/2}}\left(\mu_3+\delta\mu_3\right)\mu^{\varepsilon/2}, & b_H^b &= \frac{Z_\pi}{Z_h^2}\left(b_H+\delta b_H+b_H\frac{\delta v^2}{v^2}\right), \\ \lambda^b &= \frac{1}{Z_h^2}\left(\lambda+\delta\lambda\right)\mu^{\varepsilon}, \end{aligned} \tag{A.1}$$

where

$$
\begin{aligned}
X_i^b &= \left(X_i + \delta X_i + 2X_i \frac{\delta v^2}{v^2}\right)\mu^{-\varepsilon}, && X_i = c_6, c_9, c_{11},\\
X_i^b &= \frac{Z_\pi^{1/2}}{Z_h^{1/2}}\left(X_i + \delta X_i + \frac{3}{2}X_i \frac{\delta v^2}{v^2}\right)\mu^{-\varepsilon}, && X_i = c_7, a_9, c_{10},\\
X_i^b &= \frac{Z_\pi}{Z_h}\left(X_i + \delta X_i + 2X_i \frac{\delta v^2}{v^2}\right)\mu^{-\varepsilon}, && X_i = a_7, c_8, b_9, a_{10}, c_{20},\\
X_i^b &= \frac{Z_\pi}{Z_h}\left(X_i + \delta X_i + X_i \frac{\delta v^2}{v^2}\right)\mu^{-\varepsilon}, && X_i = c_{\Box H},\\
X_i^b &= \frac{Z_\pi^{3/2}}{Z_h^{3/2}}\left(X_i + \delta X_i + \frac{3}{2}X_i \frac{\delta v^2}{v^2}\right)\mu^{-\varepsilon}, && X_i = a_{\Box H}, c_{\Delta H},\\
X_i^b &= \frac{Z_\pi^2}{Z_h^2}\left(X_i + \delta X_i + 2X_i \frac{\delta v^2}{v^2}\right)\mu^{-\varepsilon}, && X_i = b_{\Box H}, a_{\Delta H}, c_{DH}.
\end{aligned}
\tag{A.2}
$$

The counterterms required to absorb the divergences of the hhh 3-point function are

$$
\begin{aligned}
\delta a_{\Box H} =& \frac{1}{2}\left(-\frac{3a_C b_C}{2} - \frac{a_H b_H}{2} + 3a_C^3 + a_H^3\right)\Delta_\varepsilon,\\
\delta c_{\Delta H} =& \frac{1}{2}\left(-3a_C b_C + 3a_C^3 - a_H^3\right)\Delta_\varepsilon,\\
\delta a_H =& \left[\frac{1}{2}\left(-9\frac{\mu_3}{v}a_H^2 + \lambda a_H + 2\frac{\mu_3}{v}b_H\right) + a_H\left(15a_H^2 - 7b_H\right)\frac{m_h^2}{v^2}\right]\Delta_\varepsilon,\\
\delta\mu_3 =& \left[\frac{3}{2}\mu_3\left(\lambda - 4\frac{\mu_3}{v}a_H\right) + 6\left(6\mu_3 a_H^2 - \lambda v a_H - \mu_3 b_H\right)\frac{m_h^2}{v^2}\right.\\
&\left. + 6a_H\left(3b_H - 8a_H^2\right)\frac{m_h^4}{v^3}\right]\Delta_\varepsilon,
\end{aligned}
\tag{A.3}
$$

while those for $\pi\pi \to h$ read

$$
\begin{aligned}
\delta a_C &= \frac{1}{2}\left(a_C^2 - b_C\right)\left[2(a_C + 2a_H)\frac{m_h^2}{v^2} - \frac{\mu_3}{v}\right]\Delta_\varepsilon,\\
\delta c_7 &= \frac{1}{4}\left(-a_H b_C + a_C^2 a_H - a_C^3 - 2a_C\right)\Delta_\varepsilon,\\
\delta a_9 &= -\frac{1}{8}a_C\left(a_C a_H + a_C^2 - b_C\right)\Delta_\varepsilon,\\
\delta c_{10} &= \frac{1}{2}a_C\left(-a_C a_H + a_C^2 + b_C\right)\Delta_\varepsilon.
\end{aligned}
\tag{A.4}
$$

In the case of the $\pi\pi \to hh$ amplitudes, the relevant diagrams are displayed in figure 6,

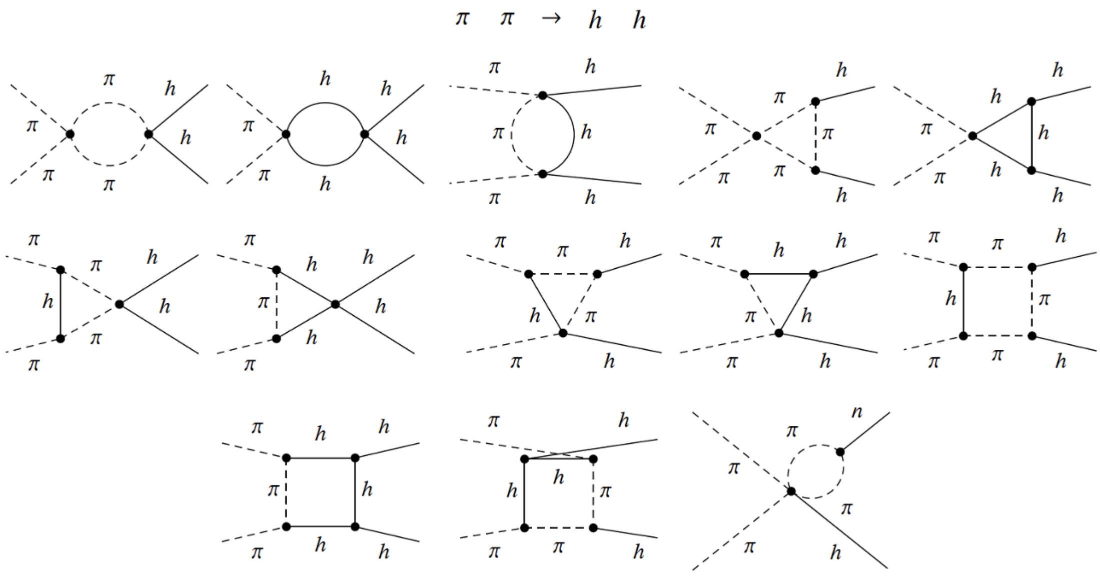

Figure 6. Diagrams contributing to the $\pi\pi \to hh$ amplitude, not including diagrams obtained by crossing.

and the counterterms correspond to

$$\begin{aligned}
\delta b_C =& \frac{1}{2}\left(a_C^2 - b_C\right)\Bigg[(4a_C + 8a_H)\frac{\mu_3}{v} - \lambda \\
& - 2\left(8a_C a_H + 4a_C^2 + 12a_H^2 - b_C - 2b_H\right)\frac{m_h^2}{v^2}\Bigg]\Delta_\varepsilon, \\
\delta a_7 =& \frac{1}{8}\Big[a_C^2\left(-4a_H^2 - 3b_C + b_H + 4\right) + 2a_C a_H b_C + b_C\left(4a_H^2 - b_H - 2\right) + 4a_C^4\Big]\Delta_\varepsilon\,, \\
\delta c_8 =& \frac{1}{3}\left[a_C^2\left(a_H^2 + b_C\right) - 2a_C a_H b_C - a_C^3 a_H + a_C^4 + b_C^2\right]\Delta_\varepsilon\,, \\
\delta b_9 =& \frac{1}{4}\left[-a_C^2\left(-4a_H^2 + 5b_C + b_H\right) - 4a_C a_H b_C + 4a_C^3 a_H + 4a_C^4 + b_C^2\right]\Delta_\varepsilon\,, \\
\delta a_{10} =& \frac{1}{4}\left[a_C^2\left(4a_H^2 + b_C - b_H\right) - 4a_C a_H b_C - 4a_C^4 + b_C^2\right]\Delta_\varepsilon\,, \\
\delta c_{20} =& \frac{1}{12}\Big[a_C^2\left(2a_H^2 - b_C + 6\right) + 2a_C a_H b_C \\
& - b_C\left(3a_H^2 + b_C + 6\right) - 2a_C^3 a_H + 2a_C^4\Big]\Delta_\varepsilon\,.
\end{aligned} \tag{A.5}$$

For $\pi\pi \to \pi\pi$ amplitudes, the relevant diagrams are displayed in figure 7, and the required counterterms are given by

$$\begin{aligned}
\delta c_6 &= \frac{1}{48}\left[a_C^2\left(6b_C - 8\right) - 2a_C^4 - 3b_C^2 - 2\right]\Delta_\varepsilon, \\
\delta c_{11} &= -\frac{1}{12}\left(a_C^2 - 1\right)^2\Delta_\varepsilon.
\end{aligned} \tag{A.6}$$

Finally, the relevant diagrams for $hh \to hh$ amplitudes are shown in figure 8, and the

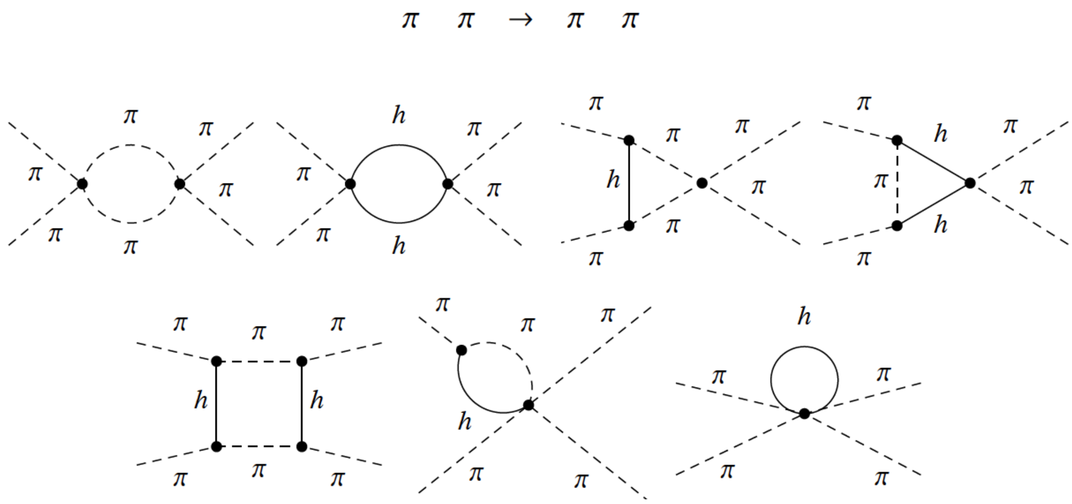

Figure 7. Diagrams contributing to the $\pi\pi \to \pi\pi$ amplitude, not including diagrams obtained by crossing.

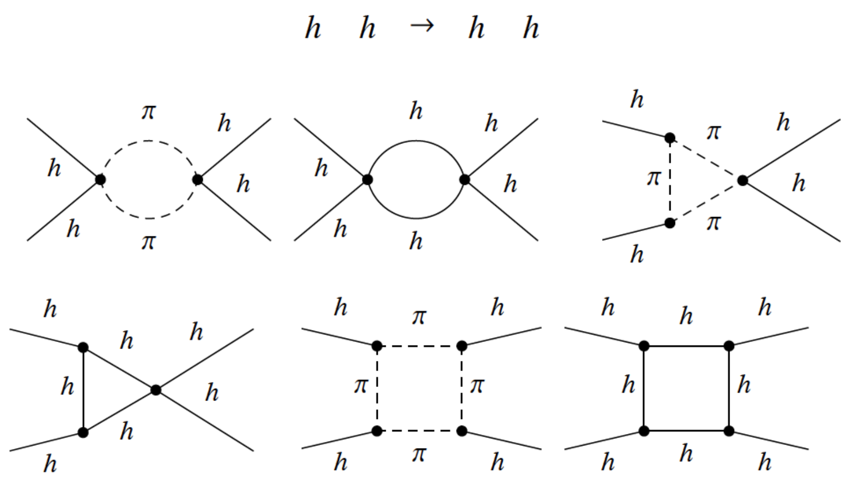

Figure 8. Diagrams contributing to the $hh \to hh$ amplitude, not including diagrams obtained by crossing.

renormalization conditions read

$$\begin{aligned}
\delta b_H =& \left[\frac{1}{2}\left(\frac{\mu_3}{v}\left(-40 a_H b_H + 84 a_H^3\right) - 13\lambda a_H^2 + 3\lambda b_H\right)\right. \\
&\left. + \left(87 a_H^2 b_H - 120 a_H^4 - 7 b_H^2\right)\frac{m_h^2}{v^2}\right]\Delta_\varepsilon, \\
\delta b_{\Box H} =& \frac{1}{4}\left[-3\left(4a_H^4 + b_C^2\right) + 30 a_C^2 b_C + 10 a_H^2 b_H - 36 a_C^4 - b_H^2\right]\Delta_\varepsilon, \\
\delta a_{\Delta H} =& -\frac{3}{4}\left(-7 a_C^2 b_C + a_H^2 b_H + 6 a_C^4 - 2 a_H^4 + b_C^2\right)\Delta_\varepsilon,
\end{aligned}$$

$$
\begin{aligned}
\delta c_{DH} =&\Big[-\frac{3}{4}\left(a_C^2 - b_C\right)^2 - \frac{a_H^4}{4}\Big]\Delta_\varepsilon, \\
\delta\lambda =&\Big\{ \frac{3}{2v^2}\left[8\mu_3^2\left(6a_H^2 - b_H\right) - 16\lambda\mu_3 v a_H + \lambda^2 v^2\right] \\
&- 12\left(-12\mu_3 a_H b_H + 32\mu_3 a_H^3 - 6\lambda v a_H^2 + \lambda v b_H\right)\frac{m_h^2}{v^3} \\
&+ 6\left(-48a_H^2 b_H + 80a_H^4 + 3b_H^2\right)\frac{m_h^4}{v^4}\Big\}\Delta_\varepsilon.
\end{aligned}
\tag{A.7}
$$

B The Renormalization Group Equations

This appendix provides the expressions for the RGE of all couplings discussed above, at the order considered in this paper:

$$
\begin{aligned}
16\pi^2\frac{d}{d\ln\mu}a_C =&a_C\left[\left(5a_H^2 - 3b_C - b_H\right)\frac{m_h^2}{v^2} - a_H\frac{\mu_3}{v}\right] \\
&+ a_C^2\left(4a_H\frac{m_h^2}{v^2} - \frac{\mu_3}{v}\right) + 3a_C^3\frac{m_h^2}{v^2} + b_C\frac{\mu_3}{v} - 4a_H b_C\frac{m_h^2}{v^2}\,,
\end{aligned}
\tag{B.1}
$$

$$
\begin{aligned}
16\pi^2\frac{d}{d\ln\mu}b_C =&b_C\left[2\left(5a_C^2 + 8a_C a_H + 17a_H^2 - 3b_H\right)\frac{m_h^2}{v^2} + \lambda - 2(2a_C + 5a_H)\frac{\mu_3}{v}\right] \\
&- 2b_C^2\frac{m_h^2}{v^2} + a_C^2\left(-\lambda + 4a_C\frac{\mu_3}{v} + 8a_H\frac{\mu_3}{v}\right) \\
&- 4a_C^2\left(2a_C^2 + 4a_C a_H + 6a_H^2 - b_H\right)\frac{m_h^2}{v^2}\,,
\end{aligned}
\tag{B.2}
$$

$$
16\pi^2\frac{d}{d\ln\mu}a_H =a_H\left[\lambda - \left(a_C^2 - b_C + 17b_H\right)\frac{m_h^2}{v^2}\right] - 12a_H^2\frac{\mu_3}{v} + 45a_H^3\frac{m_h^2}{v^2} + 2b_H\frac{\mu_3}{v}\,, \tag{B.3}
$$

$$
\begin{aligned}
16\pi^2\frac{d}{d\ln\mu}b_H =&b_H\left[2\left(-a_C^2 + 97a_H^2 + b_C\right)\frac{m_h^2}{v^2} - 44a_H\frac{\mu_3}{v} + 3\lambda\right] - 18b_H^2\frac{m_h^2}{v^2} \\
&+ a_H^2\left(-13\lambda + 84a_H\frac{\mu_3}{v}\right) - 240a_H^4\frac{m_h^2}{v^2}\,,
\end{aligned}
\tag{B.4}
$$

$$
16\pi^2\frac{d}{d\ln\mu}m_h^2 =m_h^2\left(\lambda - 10a_H\frac{\mu_3}{v}\right) + \left(22a_H^2 - 4b_H\right)\frac{m_h^4}{v^2} + \mu_3^2\,, \tag{B.5}
$$

$$
\begin{aligned}
16\pi^2\frac{d}{d\ln\mu}\mu_3 =&\mu_3\left[\left(87a_H^2 - 15b_H\right)\frac{m_h^2}{v^2} + 3\lambda\right] - 15a_H\frac{\mu_3^2}{v} \\
&- 12\lambda a_H\frac{m_h^2}{v} - \left(96a_H^3 - 36a_H b_H\right)\frac{m_h^4}{v^3}\,,
\end{aligned}
\tag{B.6}
$$

$$
\begin{aligned}
16\pi^2\frac{d}{d\ln\mu}\lambda =&\lambda\left[4\left(41a_H^2 - 7b_H\right)\frac{m_h^2}{v^2} - 52a_H\frac{\mu_3}{v}\right] + 3\lambda^2 + 24\left(6a_H^2 - b_H\right)\frac{\mu_3^2}{v^2} \\
&- 96a_H\left(8a_H^2 - 3b_H\right)\frac{\mu_3 m_h^2}{v^3} + 12\left(80a_H^4 - 48a_H^2 b_H + 3b_H^2\right)\frac{m_h^4}{v^4}\,,
\end{aligned}
\tag{B.7}
$$

$$
16\pi^2\frac{d}{d\ln\mu}v^2 = -2\left(a_C^2 - b_C\right)m_h^2\,, \tag{B.8}
$$

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$$16\pi^2 \frac{d}{d\ln\mu} c_6 = -\frac{1}{24}\left[2 + 2a_C^4 + 3b_C^2 - a_C^2\left(-8 + 6b_C\right)\right] , \tag{B.9}$$

$$\begin{aligned} 16\pi^2 \frac{d}{d\ln\mu} c_7 = & - c_7 \left[\left(a_C^2 - 5a_H^2 - b_C + b_H\right)\frac{m_h^2}{v^2} + a_H \frac{\mu_3}{v}\right] \\ & + \frac{1}{2}\left(-2a_C - a_C^3 + a_C^2 a_H - a_H b_C\right) , \end{aligned} \tag{B.10}$$

$$\begin{aligned} 16\pi^2 \frac{d}{d\ln\mu} a_7 = & - a_7 \left[2\left(a_C^2 - 5a_H^2 - b_C + b_H\right)\frac{m_h^2}{v^2} + 2a_H \frac{\mu_3}{v}\right] \\ + \frac{1}{4}\Big[4a_C^4 + 2a_C a_H b_C & + b_C\left(-2 + 4a_H^2 - b_H\right) + a_C^2\left(4 - 4a_H^2 - 3b_C + b_H\right)\Big] , \end{aligned} \tag{B.11}$$

$$\begin{aligned} 16\pi^2 \frac{d}{d\ln\mu} c_8 = & - c_8 \left[2\left(a_C^2 - 5a_H^2 - b_C + b_H\right)\frac{m_h^2}{v^2} + 2a_H \frac{\mu_3}{v}\right] \\ & + \frac{2}{3}\left[a_C^4 - a_C^3 a_H - 2a_C a_H b_C + b_C^2 + a_C^2\left(a_H^2 + b_C\right)\right] , \end{aligned} \tag{B.12}$$

$$16\pi^2 \frac{d}{d\ln\mu} c_9 = \frac{a_C^2}{2} , \tag{B.13}$$

$$\begin{aligned} 16\pi^2 \frac{d}{d\ln\mu} a_9 = & - a_9 \left[\left(a_C^2 - 5a_H^2 - b_C + b_H\right)\frac{m_h^2}{v^2} + a_H \frac{\mu_3}{v}\right] \\ & - \frac{1}{2} a_C \left(a_C^2 + a_C a_H - b_C\right) , \end{aligned} \tag{B.14}$$

$$\begin{aligned} 16\pi^2 \frac{d}{d\ln\mu} b_9 = & - b_9 \left[2\left(a_C^2 - 5a_H^2 - b_C + b_H\right)\frac{m_h^2}{v^2} + 2a_H \frac{\mu_3}{v}\right] \\ & + \frac{1}{2}\left[4a_C^4 + 4a_C^3 a_H - 4a_C a_H b_C + b_C^2 + a_C^2\left(4a_H^2 - 5b_C - b_H\right)\right] , \end{aligned} \tag{B.15}$$

$$16\pi^2 \frac{d}{d\ln\mu} c_{10} = - c_{10} \left[\left(a_C^2 - 5a_H^2 - b_C + b_H\right)\frac{m_h^2}{v^2} + a_H \frac{\mu_3}{v}\right] + a_C\left(a_C^2 - a_C a_H + b_C\right) , \tag{B.16}$$

$$\begin{aligned} 16\pi^2 \frac{d}{d\ln\mu} a_{10} = & - a_{10} \left[2\left(a_C^2 - 5a_H^2 - b_C + b_H\right)\frac{m_h^2}{v^2} + 2a_H \frac{\mu_3}{v}\right] \\ & + \frac{1}{2}\left(-4a_C^4 - 4a_C a_H b_C + b_C^2 + a_C^2\left(4a_H^2 + b_C - b_H\right)\right) , \end{aligned} \tag{B.17}$$

$$16\pi^2 \frac{d}{d\ln\mu} c_{11} = -\frac{1}{6}\left(a_C^2 - 1\right)^2 , \tag{B.18}$$

$$\begin{aligned} 16\pi^2 \frac{d}{d\ln\mu} c_{20} = & - c_{20} \left[2\left(a_C^2 - 5a_H^2 - b_C + b_H\right)\frac{m_h^2}{v^2} + 2a_H \frac{\mu_3}{v}\right] \\ + \frac{1}{6}\Big[2a_C^4 - 2a_C^3 a_H & + a_C^2\left(6 + 2a_H^2 - b_C\right) + 2a_C a_H b_C - b_C\left(6 + 3a_H^2 + b_C\right)\Big] , \end{aligned} \tag{B.19}$$

$$16\pi^2 \frac{d}{d\ln\mu} c_{\Box H} = - c_{\Box H} \left[2\left(a_C^2 - 5a_H^2 - b_C + b_H\right)\frac{m_h^2}{v^2} + 2a_H \frac{\mu_3}{v}\right] + \frac{1}{2}\left(-3a_C^2 - a_H^2\right) , \tag{B.20}$$

$$16\pi^2 \frac{d}{d\ln\mu} a_{\square H} = -\, a_{\square H} \left[3\left(a_C^2 - 5a_H^2 - b_C + b_H\right) \frac{m_h^2}{v^2} + 3a_H \frac{\mu_3}{v} \right] + 3a_C^3 + a_H^3 - \frac{3a_C b_C}{2} - \frac{a_H b_H}{2}\,, \tag{B.21}$$

$$16\pi^2 \frac{d}{d\ln\mu} b_{\square H} = -\, b_{\square H} \left[4\left(a_C^2 - 5a_H^2 - b_C + b_H\right) \frac{m_h^2}{v^2} + 4a_H \frac{\mu_3}{v} \right] - 18a_C^4 - 6a_H^4 + 15a_C^2 b_C - \frac{3b_C^2}{2} + 5a_H^2 b_H - \frac{b_H^2}{2}\,, \tag{B.22}$$

$$16\pi^2 \frac{d}{d\ln\mu} c_{\Delta H} = -\, c_{\Delta H} \left[3\left(a_C^2 - 5a_H^2 - b_C + b_H\right) \frac{m_h^2}{v^2} + 3a_H \frac{\mu_3}{v} \right] + 3a_C^3 - a_H^3 - 3a_C b_C\,, \tag{B.23}$$

$$16\pi^2 \frac{d}{d\ln\mu} a_{\Delta H} = -\, a_{\Delta H} \left[4\left(a_C^2 - 5a_H^2 - b_C + b_H\right) \frac{m_h^2}{v^2} + 4a_H \frac{\mu_3}{v} \right] - \frac{3}{2}\left(6a_C^4 - 2a_H^4 - 7a_C^2 b_C + b_C^2 + a_H^2 b_H\right)\,, \tag{B.24}$$

$$16\pi^2 \frac{d}{d\ln\mu} c_{DH} = -\, c_{DH} \left[4\left(a_C^2 - 5a_H^2 - b_C + b_H\right) \frac{m_h^2}{v^2} + 4a_H \frac{\mu_3}{v} \right] - \frac{1}{2}\left[a_H^4 + 3\left(a_C^2 - b_C\right)^2 \right]. \tag{B.25}$$

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