Effects of $K^0 - \bar{K}^0$ mixing on determining γ from $B^\pm \rightarrow DK^\pm$

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ABSTRACT: The decay $B^\pm \rightarrow DK^\pm$ followed by the subsequent decay of the D meson into final states involving a neutral kaon can be used to determine the CKM angle γ . We study CP violation effects due to mixing and decay of the final state kaon. We find that ignoring these effects produces a shift in γ of order ϵ_K/r_B , an enhancement of $1/r_B$ compared to the naive expectation. We then show how to take these effects into account such that, in principle, they will not introduce any theoretical error in the extraction of γ .

KEYWORDS: B-Physics, CP violation, Kaon Physics

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Contents

1	Introduction	1
2	GLW	2
3	The general case: Dalitz decays	3
3.1	General discussion	3
3.2	Including the kaon time dependence	5
3.3	Order $r_B \epsilon $ terms	6
4	Assuming Breit-Wigner dependence	7
5	Estimating the error in γ	9
6	Conclusion	10
A	Some explicit expressions	11
B	Other small weak phases	12
B.1	In $D \rightarrow K\pi^+\pi^-$	12
B.2	In $B^\pm \rightarrow DK^\pm$	13

1 Introduction

Interference between the $b \rightarrow c\bar{u}s$ and $b \rightarrow u\bar{c}s$ decay amplitudes can be used to determine the weak phase

$$\gamma \equiv \arg \left(-\frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*} \right). \quad (1.1)$$

There are many hadronic final states that can be used to add information about γ [1–7] and, what is relevant to our case, some of them have one (or more) neutral kaon in the final state. All methods of determining γ from $B^\pm \rightarrow DK^\pm$ involve deriving a system of equations for γ in terms of the decay widths and amplitudes for the various processes involved. If all other quantities can be determined experimentally this allows for a model-independent determination of γ .

Thus far CP violation associated with neutral kaons in the determination of γ has been neglected since the effect is expected to be much smaller than current experimental uncertainties. As more statistics become available new sources of CP violation in the overall process will become significant and will need to be taken into account. In this work we study the effects of CP violation in the kaon system on the determination of γ . Such effects were studied in other systems in [8–12]. As we will see, there are effects that are linear

in ϵ_K , and they are parametrically enhanced by $1/r_B$ (we use standard notations that are defined below). Once CP violation in the kaon system is included it is crucial that the time dependence of the kaon mixing be taken into account. This must be done in a way which considers time dependent detector efficiencies, and thus it can only be carried out by each experiment separately as part of their analysis.

2 GLW

To understand the main issues associated with including kaon mixing and CPV effects, we start with the theoretically simplest case, which is the GLW method [1, 2]. We define

$$A(B^- \rightarrow D^0 K^-) = A(B^+ \rightarrow \bar{D}^0 K^+) \equiv A_B, \quad (2.1)$$

$$A(B^- \rightarrow \bar{D}^0 K^-) \equiv A_B r_B e^{i(\delta_B - \gamma)}, \quad A(B^+ \rightarrow D^0 K^+) \equiv A_B r_B e^{i(\delta_B + \gamma)}, \quad (2.2)$$

where δ_B is a strong phase and r_B is a real parameter which accounts for color and CKM suppression, and is measured to be of order 10^{-1} . We consider $D \rightarrow K\pi^0$ decays, but the π^0 can be exchanged for another CP eigenstate such as ρ^0 and the same discussion would apply. We further assume

$$|A(D^0 \rightarrow \bar{K}^0 \pi^0)| = |A(\bar{D}^0 \rightarrow K^0 \pi^0)|, \quad |A(D^0 \rightarrow K^0 \pi^0)| = |A(\bar{D}^0 \rightarrow \bar{K}^0 \pi^0)| = 0. \quad (2.3)$$

The decay $A(D^0 \rightarrow K^0 \pi^0)$ and its CP conjugate are doubly Cabbibo suppressed, and there is no real complication in including them, they are unimportant for our current discussion, so for the moment we set them to zero (they are included in our final results). We also neglect terms of order $r_B |\epsilon_K|$ in this section for simplicity. For the kaons we use the standard notation

$$|K^0\rangle = \frac{1}{2p} (|K_L\rangle + |K_S\rangle), \quad |\bar{K}^0\rangle = \frac{1}{2q} (|K_L\rangle - |K_S\rangle), \quad \frac{A(K_L \rightarrow \pi\pi)}{A(K_S \rightarrow \pi\pi)} = \epsilon, \quad (2.4)$$

where in the last term we neglected direct CPV in kaon decays, that is, we set $\epsilon' = 0$. We define the time dependent asymmetry

$$a_{CP}(t) = \frac{\Gamma(B^+ \rightarrow (K_S \pi^0)_D K^+) - \Gamma(B^- \rightarrow (K_S \pi^0)_D K^-)}{\Gamma(B^+ \rightarrow (K_S \pi^0)_D K^+) + \Gamma(B^- \rightarrow (K_S \pi^0)_D K^-)}, \quad (2.5)$$

In the above t is the proper time of the kaon system, and by K_S we refer to a kaon that decays into two pions (see discussion in [12]). We work to first order in ϵ_K and we neglect terms of order $r_B |\epsilon_K|$ to find

$$a_{CP}(t) = \frac{2r_B \sin(\gamma) \sin(\delta_B) - 2\text{Re}(\epsilon) + 2e^{(\Gamma_S - \Gamma)t} \text{Re}(\epsilon^* e^{ix\Gamma t})}{1 + r_B^2 - 2r_B \cos(\gamma) \cos(\delta_B)}. \quad (2.6)$$

where as usual

$$\Gamma = \frac{\Gamma_S + \Gamma_L}{2}, \quad x = \frac{m_L - m_S}{\Gamma}. \quad (2.7)$$

It is useful to consider also the case where we integrate over the kaon lifetime. Following [12] we parametrize the experiment-dependent efficiency to detect the kaon by $F(t)$

with $0 \leq F(t) \leq 1$. We emphasize that F must be determined as part of the experimental analysis. We then define the time integrated asymmetry

$$A_{CP} = \frac{\int F(t) dt [\Gamma(B^+ \rightarrow (K_S \pi^0)_D K^+) - \Gamma(B^- \rightarrow (K_S \pi^0)_D K^-)]}{\int F(t) dt [\Gamma(B^+ \rightarrow (K_S \pi^0)_D K^+) + \Gamma(B^- \rightarrow (K_S \pi^0)_D K^-)]}, \quad (2.8)$$

where the integral is from zero to infinity. To demonstrate the effect we take a simple case where $F(t) = 1$ and we obtain

$$A_{CP} = \frac{2r_B \sin(\gamma) \sin(\delta_B) + 2\text{Re}(\epsilon)}{1 + r_B^2 - 2r_B \cos(\gamma) \cos(\delta_B)}. \quad (2.9)$$

We can check few limits of the above results:

1. For the case of no CPV in kaons, that is, $\epsilon \rightarrow 0$, and to first order in r_B we obtain

$$A_{CP} = 2r_B \sin(\gamma) \sin(\delta_B), \quad (2.10)$$

as we should.

2. The $r_B \rightarrow 0$ limit corresponds to the case where the only source of CP violation is in the kaon system. This case was studied for τ decays in [12]. Using eq. (2.9) for $r_B \rightarrow 0$ we see that

$$A_{CP} = 2\text{Re}(\epsilon), \quad (2.11)$$

in agreement with [12].

3. Last we work to first order in r_B and we get

$$A_{CP} = 2r_B \left[\sin(\gamma) \sin(\delta_B) + \frac{\text{Re}(\epsilon)}{r_B} \right]. \quad (2.12)$$

Therefore, when both effects are included, A_{CP} (and therefore the extracted value of γ) is shifted by a term of order $\text{Re}(\epsilon)/r_B$. It is the $1/r_B$ enhancement which makes the effect larger and somewhere at the level that is expected to be probed in the near future.

3 The general case: Dalitz decays

3.1 General discussion

Our goal is to obtain a system of equations for γ in terms of various experimentally determined quantities in the most general case of a multi-body D decay such as $D \rightarrow K_S \pi^+ \pi^-$. The quantities involved will be integrated over finite regions of the D decay phase space. Here we are using $D \rightarrow K_S \pi^+ \pi^-$ for concreteness, however our discussion will also apply to other decay modes such as $D \rightarrow K K^+ K^-$ and also to two-body decays, such as $D \rightarrow K_S \pi^0$, in which these amplitudes are not momentum dependent. We do not include other small effects like $D^0 - \bar{D}^0$ mixing which was discussed in [4, 13–16]. Note that, though we will

not discuss it further, it is imperative to include the effects of $D^0 - \bar{D}^0$ mixing since they will be competitive with those of CP violation in $K^0 - \bar{K}^0$ mixing.

With this in mind, we begin by defining quantities associated with the three-body D decay

$$\mathcal{A}(t) \equiv A(D^0 \rightarrow (\pi\pi)_K \pi^+ \pi^-) = A_{\pi\pi} \left(A_S e^{-im_S t - \Gamma_S t/2} + \epsilon A_L e^{-im_L t - \Gamma_L t/2} \right), \quad (3.1)$$

and its CP conjugate

$$\bar{\mathcal{A}}(t) \equiv A(\bar{D}^0 \rightarrow (\pi\pi)_K \pi^- \pi^+) = A_{\pi\pi} \left(\bar{A}_S e^{-im_S t - \Gamma_S t/2} + \epsilon \bar{A}_L e^{-im_L t - \Gamma_L t/2} \right), \quad (3.2)$$

where

$$A_{\pi\pi} \equiv A(K_S \rightarrow \pi\pi), \quad A_{S,L} \equiv A(D^0 \rightarrow K_{S,L} \pi^+ \pi^-), \quad \bar{A}_{S,L} \equiv A(\bar{D}^0 \rightarrow K_{S,L} \pi^- \pi^+), \quad (3.3)$$

and $\pi\pi$ is either of $\pi^+ \pi^-$ or $\pi^0 \pi^0$. Note that $\mathcal{A}$ and $\bar{\mathcal{A}}$ are analogous to $A_D(s_{12}, s_{13})$ and $A_D(s_{13}, s_{12})$ of [4] respectively. We are assuming that we use the $\pi\pi$ final state to tag the kaon as a K_S and we have used (2.4). From now on meson variables such as p, q, ϵ, x, y will be for the K unless stated otherwise.

Interference of the $\mathcal{A}$ and $\bar{\mathcal{A}}$ amplitudes will occur through the $B^\pm$ decay, where the relative phase of the interference involves γ . With the definitions (2.1) and (2.2) we can write down the amplitude for the overall process in terms of γ

$$A(B^- \rightarrow f^-) = A_B \left[\mathcal{A}(t) + r_B e^{i(\delta_B - \gamma)} \bar{\mathcal{A}}(t) \right], \quad f^\pm = [(\pi\pi)_K \pi^+ \pi^-]_D K^\pm. \quad (3.4)$$

One can obtain the amplitude for the $B^+ \rightarrow f^+$ by $\gamma \rightarrow -\gamma$ and $\mathcal{A} \leftrightarrow \bar{\mathcal{A}}$. Squaring this we can write the time-dependent differential normalized widths

$$\begin{aligned} d\hat{\Gamma}(B^- \rightarrow f^-) &= |\mathcal{A}|^2 + r_B^2 |\bar{\mathcal{A}}|^2 + 2r_B [\text{Re}(\mathcal{A}^* \bar{\mathcal{A}}) \cos(\delta_B - \gamma) - \text{Im}(\mathcal{A}^* \bar{\mathcal{A}}) \sin(\delta_B - \gamma)], \\ d\hat{\Gamma}(B^+ \rightarrow f^+) &= |\bar{\mathcal{A}}|^2 + r_B^2 |\mathcal{A}|^2 + 2r_B [\text{Re}(\mathcal{A}^* \bar{\mathcal{A}}) \cos(\delta_B + \gamma) + \text{Im}(\mathcal{A}^* \bar{\mathcal{A}}) \sin(\delta_B + \gamma)], \end{aligned} \quad (3.5)$$

where $\hat{\Gamma} \equiv \Gamma/|A_B|^2$.

Eqs. (3.5) are our system of equations which can be used to determine γ . Note that $|\mathcal{A}|^2$ and $|\bar{\mathcal{A}}|^2$ are directly measurable in D decays. In addition to these there is a phase, that of $\mathcal{A}^* \bar{\mathcal{A}}$, which is momentum dependent in the multi-body case and which must be obtained in addition to r_B and δ_B in order to determine γ . These equations are identical in form to those from [4], and similar to those in [2] except that we distinguish between $\mathcal{A}$ and $\bar{\mathcal{A}}$. The new complications are hidden in the time-dependence and CP violating parameters stored in $\mathcal{A}$ and $\bar{\mathcal{A}}$.

We should proceed by setting up equations for the Dalitz analysis. This is performed by integrating over bins in the $D \rightarrow K_S \pi\pi$ phase space. First, let us define the momenta of the decay products and the Mandelstam variables as in [4]

$$K(p_1), \quad \pi^-(p_2), \quad \pi^+(p_3), \quad s_{ij} = (p_i + p_j)^2. \quad (3.6)$$

In the $\epsilon \rightarrow 0$ limit one has $\mathcal{A}(s_{12}, s_{13}) = \bar{\mathcal{A}}(s_{13}, s_{12})$. One way to think of this is that if the K_S were a CP eigenstate then $K_S \pi^+ \pi^-$ is a CP eigenstate but for momentum

interchange. The effect due to the fact that the K_S is not a CP eigenstate is that $\mathcal{A}(s_{12}, s_{13}) = \bar{\mathcal{A}}(s_{13}, s_{12}) + O(|\epsilon|)$.

We would like to partition the (s_{12}, s_{13}) phase space into bins which are symmetric about the line $s_{12} = s_{13}$. With $\epsilon \rightarrow 0$ very simple relations exist between the various integrals above and below the $s_{12} = s_{13}$ line. The CP violation in the kaon system, however, complicates this. To this end, we define integrals over bins in phase space

$$T_i^-(t) \equiv \int_i d^2s |\mathcal{A}(t)|^2, \quad T_i^+(t) \equiv \int_i d^2s |\bar{\mathcal{A}}(t)|^2, \quad (3.7)$$

$$C_i(t) \equiv \int_i d^2s \operatorname{Re}(\mathcal{A}^* \bar{\mathcal{A}}), \quad S_i(t) \equiv \int_i d^2s \operatorname{Im}(\mathcal{A}^* \bar{\mathcal{A}}), \quad (3.8)$$

where we have emphasized that these quantities all depend on the proper time of the kaon. The index i labels a bin in (s_{12}, s_{13}) phase space, and we will let $\bar{i}$ denote the bin obtained by reflecting the bin i about $s_{12} = s_{13}$. We can then write a system of equations for integrals of the overall decay width in these bins. At this stage we assume that we integrate over time, and we obtain

$$\hat{\Gamma}_i^- = T_i^- + r_B^2 T_i^+ + 2r_B [C_i \cos(\delta_B - \gamma) - S_i \sin(\delta_B - \gamma)], \quad (3.9)$$

$$\hat{\Gamma}_i^+ = T_i^+ + r_B^2 T_i^- + 2r_B [C_i \cos(\delta_B + \gamma) + S_i \sin(\delta_B + \gamma)]. \quad (3.10)$$

Note that the quantities $T_i^\pm$ are the widths for $D \rightarrow K_S \pi^+ \pi^-$ with the K_S identified through its decay and thus can be determined from charm data, so we will treat these as known. In the $\epsilon \rightarrow 0$ limit we have $T_{\bar{i}}^- = T_i^+$. In contrast, the variables C_i and S_i arise from interference effects and therefore cannot be directly measured in non-interfering D decays.

Next we perform the counting of the number of observables and parameters to check if we can, in principle, obtain γ . We denote by k the number of bins above the $s_{12} = s_{13}$ line so that there are $2k$ bins in total. We consider n different B decay modes, such as $B \rightarrow D^{(*)} K^{(*)}$. We see that there are $2k$ C_i 's and $2k$ S_i 's, n of δ_B^f and r_B^f , and γ . We end up with $4k + 2n + 1$ unknowns and $4kn$ equations. We see that for $n \geq 2$ we can find k such that there are more observables than unknowns and thus γ can be determined without any approximations.

We can also determine γ in the $n = 1$ case using approximations. One approximation is to use a model for the Dalitz plot. Another approximation that can be made is to take advantage of the fact that the main correction to γ comes from the term that is not proportional to r_B [see, eq. (2.12)]. Thus, dropping terms of order $r_B |\epsilon|$ is equivalent to using $C_i = C_{\bar{i}}$ and $S_i = -S_{\bar{i}}$. In this case we have only $2k + 3$ unknowns with $4k$ equations which is solvable for $k \geq 2$.

Some other possibilities to reduce the number of unknowns were discussed in [4, 17–20]. They are applicable here except that one must be careful to distinguish between CP conjugate quantities.

3.2 Including the kaon time dependence

When we outlined how methods for determining γ from $B^\pm \rightarrow DK^\pm$ can be adapted to include CP violation in the $K^0 - \bar{K}^0$ system we glossed over the time dependence of the

kaon decay and oscillations by assuming we integrate over all of time. This is not realistic since time-dependent efficiencies must be taken into account. Below we discuss the time dependence issue in more detail.

We begin by writing the time dependence explicitly

$$|\mathcal{A}|^2 = |A_{\pi\pi}|^2 \left[|A_S|^2 e^{-\Gamma_S t} + 2e^{-\Gamma t} \text{Re}(\epsilon^* A_L^* A_S e^{ix\Gamma t}) \right], \quad (3.11)$$

$$|\bar{\mathcal{A}}|^2 = |A_{\pi\pi}|^2 \left[|\bar{A}_S|^2 e^{-\Gamma_S t} + 2e^{-\Gamma t} \text{Re}(\epsilon^* \bar{A}_L^* \bar{A}_S e^{ix\Gamma t}) \right], \quad (3.12)$$

$$\text{Re}(\mathcal{A}^* \bar{\mathcal{A}}) = \text{Re}(A_S^* \bar{A}_S) e^{-\Gamma_S t} + \text{Re} \left[\epsilon^* (A_S \bar{A}_L^* + A_L^* \bar{A}_S) e^{\Gamma t(ix-1)} \right], \quad (3.13)$$

$$\text{Im}(\mathcal{A}^* \bar{\mathcal{A}}) = \text{Im}(A_S^* \bar{A}_S) e^{-\Gamma_S t} + \text{Im} \left[\epsilon^* (A_L^* \bar{A}_S - A_S \bar{A}_L^*) e^{\Gamma t(ix-1)} \right], \quad (3.14)$$

where we have neglected the $|A_L|^2$ term as it is proportional to $|\epsilon|^2 \sim 10^{-6}$. In order to translate measurements performed in different timing windows, one must determine the various momentum dependent coefficients of the different time-dependent functions. For instance, we will need

$$\int dt F(t) |\mathcal{A}(t)|^2, \quad \int dt F(t) \text{Re}(\mathcal{A}^* \bar{\mathcal{A}}), \quad \int dt F(t) \text{Im}(\mathcal{A}^* \bar{\mathcal{A}}). \quad (3.15)$$

where $F(t)$ is the time-dependent detection efficiency defined above. To obtain such integrals we must determine both the real and imaginary parts of

$$|A_S|^2, \quad A_L^* A_S, \quad A_S^* \bar{A}_S, \quad A_S^* \bar{A}_L, \quad A_L^* \bar{A}_S \quad (3.16)$$

for each bin by distinguishing between the $\exp(-\Gamma_S t)$, $\exp(-\Gamma t) \cos(x\Gamma t)$, and $\exp(-\Gamma t) \sin(x\Gamma t)$ terms. When dropping order $r_B |\epsilon|$ terms only the first three of these need to be determined. (Some explicit expressions of time integrals can be found in appendix A.)

While in principle knowing $F(t)$ will enable us to determine γ , care must be taken, as the whole program involves data from different experiments. For example, one uses charm decay data as an input to the B decay analysis. In principle $F(t)$ can be very different in such experiments, so naive use of this data introduces error. In practice, we expect $F(t)$ to be similar for different experiments so that the induced error will be small. Ultimately estimating this effect depends on the details of particular experiments, so this cannot be done in a general way.

3.3 Order $r_B |\epsilon|$ terms

As we have already alluded to, there are complications that arise when terms of order $r_B |\epsilon|$ are included. This is because there is a new variable (roughly speaking it is the phase of $A_S^* \bar{A}_S$) which only appears in interference between D^0 and $\bar{D}^0$ amplitudes. To understand how these terms complicate the analysis, we should introduce new definitions. First, we define

$$A_D \equiv A(D^0 \rightarrow \bar{K}^0 \pi^+ \pi^-), \quad A_{DrDe^{i\delta_D}} \equiv A(D^0 \rightarrow K^0 \pi^+ \pi^-), \quad (3.17)$$

and the CP conjugate amplitudes

$$\bar{A}_D \equiv A(\bar{D}^0 \rightarrow K^0 \pi^- \pi^+), \quad \bar{A}_D \bar{r}_D e^{i\bar{\delta}_D} \equiv A(\bar{D}^0 \rightarrow \bar{K}^0 \pi^- \pi^+). \quad (3.18)$$

We emphasize that A_D , r_D , δ_D and their conjugates depend on (s_{12}, s_{13}) for multi-body decays. Here r_D is a CKM suppression factor, but we do not know how small it is in any particular region of phase space. From here on we neglect CP violation in the D decay, and then we have

$$A_D(s_{12}, s_{13}) = \bar{A}_D(s_{13}, s_{12}), \quad r_D(s_{12}, s_{13}) = \bar{r}_D(s_{13}, s_{12}), \quad \delta_D(s_{12}, s_{13}) = \bar{\delta}_D(s_{13}, s_{12}). \quad (3.19)$$

(In principle A_D, r_D, δ_D are each measurable through flavor specific decays of the kaon, for example, $D^0 \rightarrow (\pi \ell^+ \nu)_K \pi^+ \pi^-$ and its CP conjugates.) From these we find

$$A_S = \frac{A_D}{2pq} (qr_D e^{i\delta_D} - p) = \frac{A_D}{\sqrt{2}} \left((1 - \epsilon) r_D e^{i\delta_D} - 1 - \epsilon \right) + O(|\epsilon|^2), \quad (3.20)$$

$$A_L = \frac{A_D}{2pq} (qr_D e^{i\delta_D} + p) = \frac{A_D}{\sqrt{2}} \left((1 - \epsilon) r_D e^{i\delta_D} + 1 + \epsilon \right) + O(|\epsilon|^2), \quad (3.21)$$

The CP conjugate expressions for $\bar{A}_{S,L}$ are obtained by $A_D \rightarrow \bar{A}_D$, $\epsilon \rightarrow -\epsilon$, and $A_S \rightarrow -A_S$. The phase between p and q is unphysical but we adopt the convention $q = (1 - \epsilon)/\sqrt{2}$ and $p = (1 + \epsilon)/\sqrt{2}$. The new phase which occurs only in the r_B terms which cannot be obtained through measurements of non-interfering D decays alone is that of $A_D^* \bar{A}_D$ so we define

$$\theta_D(s_{12}, s_{13}) \equiv \arg(A_D^* \bar{A}_D). \quad (3.22)$$

Under an interchange of the pion momenta one has $\theta_D \rightarrow -\theta_D$. The value of this angle at a point in phase space is unphysical, but for a particular choice of δ_B it is fixed, and its momentum dependence is physical. It reduces to $\delta_{12,13} - \delta_{13,12}$ of [4] in the $\epsilon \rightarrow 0$ limit.

If one expands out the real and imaginary parts of $\mathcal{A}^* \bar{\mathcal{A}}$ one will obtain various combinations of $|A_D|^2, r_D, \delta_D$ and trig functions of $\theta_D, \delta_D, \bar{\delta}_D$ such as $r_D \cos(\theta - \delta_D)$, $\bar{r}_D \cos(\theta + \bar{\delta}_D)$, and $r_D \bar{r}_D \cos(\theta + \bar{\delta}_D - \delta_D)$. These are of course multiplied by $|A_D \bar{A}_D|$ and are ultimately integrated over bins of phase space. The full result can be found in appendix A.

4 Assuming Breit-Wigner dependence

As we alluded to previously, some unknowns can be eliminated by assuming a Breit-Wigner dependence for the D decays. The change one needs to keep in mind when generalizing the discussion of Breit-Wigner dependence from a case where kaon CP violation is neglected is that CP conjugate amplitudes are no longer related by only an exchange of pion momenta.

We substitute A_S above with a sum over Breit-Wigner functions

$$A_S(s_{12}, s_{13}) = a_0 e^{i\delta_0} + \sum_r a_r e^{i\delta_r} A_r(s_{12}, s_{13}), \quad (4.1)$$

where

$$A_r(s_{12}, s_{13}) = {}^J \mathcal{M}_r W^r, \quad (4.2)$$

and the index r labels the resonance. The factor $^J\mathcal{M}_r$ depends on the spin of the resonance, for example $^0\mathcal{M}_r = 1$, $^1\mathcal{M}_r = -2\vec{k}_1 \cdot \vec{k}_2$ where $\vec{k}_1, \vec{k}_2$ are the spatial momenta of the particles originating from the resonance. The Breit-Wigner function is

$$W^r = \frac{1}{s - M_r^2 + iM_r\Gamma_r(\sqrt{s})}, \quad (4.3)$$

where M_r is the mass of the resonance. The argument of W^r depends on the particles which participate in the resonance, for example for the ρ^0 the argument of W^r is s_{23} . Explicit expressions for the mass dependent width $\Gamma_r(\sqrt{s})$ and the other $^J\mathcal{M}_r$'s can be found in [21–24].

In order to account for CP violation in the kaon system one cannot assume that the amplitudes are related to their CP conjugates by a momentum exchange alone, for instance

$$a_\rho \propto A(D^0 \rightarrow \rho^0 K_S) \neq \bar{a}_\rho \propto A(\bar{D}^0 \rightarrow \rho^0 K_S). \quad (4.4)$$

The reason for this is that the K_S is not an exactly even superposition of K^0 and $\bar{K}^0$. Fortunately, these are related through simple momentum independent factors

$$A(D^0 \rightarrow \rho^0 K_S) = \frac{A_\rho}{2pq}(qR_\rho - p) = \frac{A_\rho}{\sqrt{2}}(R_\rho - \epsilon R_\rho - 1 - \epsilon) + O(|\epsilon|^2), \quad (4.5)$$

$$A(\bar{D}^0 \rightarrow \rho^0 K_S) = \frac{A_\rho}{2pq}(q - pR_\rho) = \frac{A_\rho}{\sqrt{2}}(1 - \epsilon - R_\rho - \epsilon R_\rho) + O(|\epsilon|^2), \quad (4.6)$$

where $A_\rho \equiv A(D^0 \rightarrow \rho^0 \bar{K}^0)$, $A_\rho R_\rho \equiv A(D^0 \rightarrow \rho^0 K^0)$ with R_ρ complex and we have neglected CP violation in the D decay so that $A(D^0 \rightarrow \rho^0 \bar{K}^0) = A(\bar{D}^0 \rightarrow \rho^0 K^0)$. Similar expressions apply for any resonance decay of the form $D^0 \rightarrow K_S X$.

The situation is slightly different for decays in which the K_S emerges from a resonance such as $D^0 \rightarrow K^{*-} \pi^+$. In this case

$$a_{K^*} \propto A(D^0 \rightarrow K^{*-} \pi^+) \neq \bar{a}_{K^*} \propto A(\bar{D}^0 \rightarrow K^{*+} \pi^-) \quad (4.7)$$

because $A(K^{*-} \rightarrow K_S \pi^-) \neq A(K^{*+} \rightarrow K_S \pi^+)$. Fortunately, these are again simply related by momentum-independent factors

$$A(K^{*-} \rightarrow K_S \pi^-) = -\frac{A_*}{2q} = -\frac{A_*}{\sqrt{2}}(1 + \epsilon), \quad (4.8)$$

$$A(K^{*+} \rightarrow K_S \pi^+) = \frac{A_*}{2p} = \frac{A_*}{\sqrt{2}}(1 - \epsilon), \quad (4.9)$$

where $A_* = A(K^{*-} \rightarrow \bar{K}^0 \pi^-) = A(K^{*+} \rightarrow K^0 \pi^+)$ and we have assumed $\Delta(\text{strangeness}) = 2$ decays to be forbidden entirely. In this way one can relate a_{K^*} to $\bar{a}_{K^*}$.

For $\epsilon \rightarrow 0$ one only has the amplitude A_S , but for finite ϵ , we also have A_L , which can be decomposed just like (4.1). The same discussion on how to account for CP violation applies, however the relation between some of the coefficients is different. A general procedure for relating amplitude coefficients is as follows

1. Express decay amplitudes in terms of $A(D \rightarrow KX)$ for resonances which do not decay to the final state neutral kaon, where K is one of $K^0, \bar{K}^0$.

2. Express decay amplitudes in terms of $A(X_s \rightarrow KX)$ where X_s is a resonance which decays to a neutral kaon.
3. Project out the $K_{S,L}$ component using the reciprocal basis

$$\langle K_{L,S} | = \frac{1}{p} \langle K^0 | \pm \frac{1}{q} \langle \bar{K}^0 | \quad (4.10)$$

to relate the various coefficients.

where the $+$ is for the K_L and the $-$ is for the K_S .

5 Estimating the error in γ

Next we estimate the error introduced on the extracted value of γ if one were to use the standard analysis and neglect CP violation in the kaon system. We do so by making the following simplifying assumptions: we neglect terms of order $r_B|\epsilon|$, we assume that we integrate over all of the $K_S\pi^+\pi^-$ phase space, and we assume that $F(t) = 1$. A simple expression for γ can be obtained in terms of the difference

$$D_{0\infty} = \int_0^\infty dt [\Gamma(B^+ \rightarrow (\pi^4)_D K^+) - \Gamma(B^- \rightarrow (\pi^4)_D K^-)] . \quad (5.1)$$

Inverting the result for $D_{0\infty}$ one finds

$$\sin(\gamma) = \frac{D_{0\infty}/N - 4\kappa'_D \text{Re}(\epsilon)}{4r_B F'_D \sin(\delta_B)} . \quad (5.2)$$

where $N \equiv |A(K_S \rightarrow \pi\pi)A_B|^2/2\Gamma_S$ and F'_D and κ'_D are functions of A_D, r_D, δ_D and $\bar{A}_D, \bar{r}_D, \bar{\delta}_D$ which are defined in appendix A. Next we define

$$\Delta\gamma \equiv \gamma - \gamma_0, \quad (5.3)$$

where γ_0 is γ as determined from A_{CP} with $\epsilon = 0$. We find

$$\Delta\gamma = -\frac{\kappa'_D \text{Re}(\epsilon)}{r_B F'_D \cos(\gamma_0) \sin(\delta_B)}, \quad (5.4)$$

and we expect $\kappa'_D/F'_D \sim 1$. To see this, note that in the limit where we neglect doubly Cabbibo suppressed D decays and let the D decay to a true CP eigenstate one obtains the same expression for $\Delta\gamma$ with $\kappa'_D/F'_D = 1$. Therefore we find $\Delta\gamma \sim |\epsilon|/r_B$.

There appear to be several limits in which $\Delta\gamma$ diverges. The first type is when any of r_B, F'_D , or δ_B vanish. None of these are problematic since they arise only when working to leading order, which is no longer justified in that case. Keeping the full expression we find

$$\frac{\sin(\gamma_0 + \Delta\gamma) - \sin(\gamma_0)}{\sin(\gamma_0)} = -\frac{4\kappa'_D \text{Re}(\epsilon)}{D_{0\infty}/N}, \quad (5.5)$$

which does not depend on r_B, F'_D , or δ_B . Next we see that $\Delta\gamma$ appears to diverge for $\gamma_0 \rightarrow \pi/2$. This divergence reflects the fact that $D_{0\infty}$ depends only very weakly on γ for

Figure 1. $\Delta\gamma$ (blue line) as a function of γ , as it appears in (5.4) for $\kappa'_D/F'_D = 1, r_B = 10^{-1}, \delta_B = \pi/2$. The shaded region represents an error due to $\delta(D_{0\infty}/N) = 0.01$. For kaon mixing to be relevant, uncertainty in the CP asymmetry $D_{0\infty}$ must be small enough that the blue line lies outside the shaded region.

$\gamma \approx \pi/2$. Therefore, any source of error in $D_{0\infty}/N$ would also cause a large shift in γ in this region. The uncertainty in γ using this method is then large for $\gamma \approx \pi/2$, so we expect that $\Delta\gamma$ is also large there. This uncertainty is only intrinsic to the Dalitz method where S_i is small. Where C_i is small there is a similar uncertainty near $\gamma \approx 0$. The effect is demonstrated in figure 1.

The best current determinations of γ from Belle and BaBar have uncertainties of roughly $\pm 10^\circ$ [22, 23]. We expect $|\Delta\gamma|$ to be of order $\text{Re}(\epsilon)/r_B \approx 1^\circ$ so it may be some time before the effect of CP violation in kaon mixing and decay becomes relevant. This correction is, however, very large compared to the largest irreducible theoretical error on the determination of γ from $B \rightarrow DK$ [25].

6 Conclusion

The $B \rightarrow DK$ program is known to have the smallest theoretical error in any determination of weak parameters [25]. As precision improves it becomes more important to look for sub-leading effects [26, 27]. We generalized the $B \rightarrow DK$ method for determining γ to account for additional sources of CP violation from mixing of final state neutral kaons. We found that γ is shifted by an amount of order $\epsilon/r_B \sim 10^{-2}$. While this effect is still below the current experimental sensitivity, it may be important in the near future. We have discussed how existing methods can be corrected to account for this effect.

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A Some explicit expressions

In this appendix we collect more general expressions that we omit in the main text. In the main text we usually kept the γ dependence explicit, while here we keep the time dependence explicit. Consider a D meson state which results from the decay of the B^- and which is a superposition of D^0 and $\bar{D}^0$ depending on $A_B, r_B, \delta_B, \gamma$. We denote that state D_B . It is useful to write quantities in terms of D_B decay amplitudes. We define

$$c_{K^0} \equiv A(D_B \rightarrow K^0 \pi^+ \pi^-), \quad c_{\bar{K}^0} \equiv A(D_B \rightarrow \bar{K}^0 \pi^+ \pi^-). \quad (\text{A.1})$$

We also define $c_{L,S}$ to be c_{K^0} with the K^0 replaced by $K_{L,S}$. These amplitudes are simply related

$$c_S \equiv \frac{1}{2pq} (qc_{K^0} - pc_{\bar{K}^0}), \quad c_L \equiv \frac{1}{2pq} (qc_{K^0} + pc_{\bar{K}^0}), \quad (\text{A.2})$$

and can be expressed in terms of γ

$$c_{K^0} = A_B \left(A_D r_D e^{i\delta_D} + \bar{A}_D r_B e^{i(\delta_B - \gamma)} \right), \quad (\text{A.3})$$

$$c_{\bar{K}^0} = A_B \left(A_D + \bar{A}_D r_B \bar{r}_D e^{i(\delta_B + \bar{\delta}_D - \gamma)} \right). \quad (\text{A.4})$$

These can be written for the B^+ by $\gamma \rightarrow -\gamma$ and $(A_D, r_D, \delta_D) \leftrightarrow (\bar{A}_D, \bar{r}_D, \bar{\delta}_D)$. The latter is equivalent to changing the sign of ϵ in the overall decay width. The kaon time dependent overall width of the $B \rightarrow DK$ followed by $D \rightarrow K\pi\pi$ and then $K \rightarrow \pi\pi$

$$\frac{d^2\Gamma}{ds^2}(B^- \rightarrow f^-) = |A_{\pi\pi}|^2 (|c_S|^2 e^{-\Gamma_S t} + |\epsilon c_L|^2 e^{-\Gamma_L t} + 2e^{-\Gamma t} \text{Re}(\epsilon^* c_L^* c_S e^{ix\Gamma t})), \quad (\text{A.5})$$

where f^- is defined in (3.4). When expanding $|c_S|^2$ and $c_L^* c_S$ there are a number of expressions which occur frequently, we define

$$\begin{aligned} F_D &\equiv |A_D|^2 (1 + r_D^2 - 2r_D \cos(\delta_D)), \\ F'_D &\equiv \text{Re}(A_D^* \bar{A}_D) - 2r_D \text{Re}(A_D^* \bar{A}_D e^{-i\delta_D}) - r_D \bar{r}_D \text{Re}(A_D^* \bar{A}_D e^{i(\bar{\delta}_D - \delta_D)}), \\ \kappa_D &\equiv |A_D|^2 (1 - r_D^2 - 2r_D \sin(\delta_D)), \\ \kappa'_D &\equiv |A_D|^2 (1 - r_D^2 + 2r_D \sin(\delta_D)), \end{aligned} \quad (\text{A.6})$$

with barred quantities can be obtained by $(A_D, r_D, \delta_D) \rightarrow (\bar{A}_D, \bar{r}_D, \bar{\delta}_D)$. Neglecting terms of order $r_B |\epsilon|$ we get, for example,

$$|c_S|^2 = \frac{|A_B|^2}{2} [F_D + r_B^2 \bar{F}_D + 2\kappa_D \text{Re}(\epsilon) - 2r_B F'_D \cos(\delta_B - \gamma)], \quad (\text{A.7})$$

$$\epsilon^* c_L^* c_S = -\frac{|A_B A_D|^2}{2} \epsilon^* (1 - r_D^2 - 2ir_D \sin(\delta_D)). \quad (\text{A.8})$$

When integrating over all of time, these combine giving expressions which include κ'_D , so that κ'_D/F'_D appears in the expression for $\Delta\gamma$.

Integrals over time involve a detection efficiency function $F(t)$ which must be determined for a particular experiment. As an illustrative example, consider $F(t) = 1$ for

$t_1 < t < t_2$ and $F(t) = 0$ otherwise. The following integrals involving an arbitrary complex number z may also be useful

$$\Gamma_S \int_{t_1}^{t_2} dt \operatorname{Re}(z^* e^{\Gamma t(ix-1)}) = (x \operatorname{Re}(z) - \operatorname{Im}(z)) Q_c(t_1, t_2) + (\operatorname{Im}(z) - x \operatorname{Re}(z)) Q_s(t_1, t_2), \quad (\text{A.9})$$

$$\Gamma_S \int_{t_1}^{t_2} dt \operatorname{Im}(z^* e^{\Gamma t(ix-1)}) = (x \operatorname{Re}(z) - \operatorname{Im}(z)) Q_c(t_1, t_2) + (\operatorname{Re}(z) + x \operatorname{Im}(z)) Q_s(t_1, t_2), \quad (\text{A.10})$$

where

$$Q_c(t_1, t_2) = -\frac{1-y}{1+x^2} [e^{-\Gamma t_2} \cos(x\Gamma t_2) - e^{-\Gamma t_1} \cos(x\Gamma t_1)], \quad (\text{A.11})$$

$$Q_s(t_1, t_2) = -\frac{1-y}{1+x^2} [e^{-\Gamma t_2} \sin(x\Gamma t_2) - e^{-\Gamma t_1} \sin(x\Gamma t_1)], \quad (\text{A.12})$$

where we have used $\Gamma_S = (1-y)\Gamma$. While we have not used any approximation involving the variables x, y , it greatly simplifies things to use $x \approx -y \approx 1$, which we use in the main body of the paper when estimating $\Delta\gamma$. When integrating over all time we have $Q_c(0, \infty) = (1-y)/(1+x^2) \approx 1$ and $Q_s(0, \infty) = 0$.

It is also useful to have an expansion of $\mathcal{A}^* \mathcal{A}$. Note that

$$\operatorname{Re}(\mathcal{A}^* \mathcal{A}) = \operatorname{Re}(A_S^* \bar{A}_S) e^{-\Gamma_S t} + |\epsilon|^2 \operatorname{Re}(A_L^* \bar{A}_L) e^{-\Gamma_L t} + \operatorname{Re} \left[\epsilon^* (A_S \bar{A}_L^* + A_L^* \bar{A}_S) e^{\Gamma t(ix-1)} \right], \quad (\text{A.13})$$

$$\operatorname{Im}(\mathcal{A}^* \mathcal{A}) = \operatorname{Im}(A_S^* \bar{A}_S) e^{-\Gamma_S t} + |\epsilon|^2 \operatorname{Im}(A_L^* \bar{A}_L) e^{-\Gamma_L t} + \operatorname{Im} \left[\epsilon^* (-A_S \bar{A}_L^* + A_L^* \bar{A}_S) e^{\Gamma t(ix-1)} \right]. \quad (\text{A.14})$$

Therefore, in terms of $\theta_D(s_{12}, s_{13}) \equiv \arg(A_D^* \bar{A}_D)$ to order $|\epsilon|$ we obtain

$$A_S^* \bar{A}_S = \frac{|A_D^* \bar{A}_D|}{2} e^{i\theta_D} \left[1 + (1 - 2\operatorname{Re}(\epsilon)) r_D \bar{r}_D e^{i(\bar{\delta}_D - \delta_D)} - r_D e^{-i\delta_D} - \bar{r}_D e^{i\bar{\delta}_D} - 2i r_D e^{-i\delta_D} \operatorname{Im}(\epsilon) + 2i \bar{r}_D e^{i\bar{\delta}_D} \operatorname{Im}(\epsilon) \right], \quad (\text{A.15})$$

$$\epsilon^* A_S \bar{A}_L^* = \epsilon^* \frac{|A_D^* \bar{A}_D|}{2} e^{-i\theta_D} \left(r_D e^{i\delta_D} - 1 - \bar{r}_D e^{-i\bar{\delta}_D} + r_D \bar{r}_D e^{i(\delta_D - \bar{\delta}_D)} \right). \quad (\text{A.16})$$

B Other small weak phases

There are two other sources of CP violation which we have not taken into account, but which may become important at the $r_B |\epsilon|$ level. Neither of these present any significant complication to the methods discussed in the body of the letter where they are largely ignored, and they are simply accounted for, at least to the extent to which they can be accurately measured in other experiments.

B.1 In $D \rightarrow K\pi^+\pi^-$

There is a small amount of CP violation involved in the decay of the D . This plays much the same role as the CP violation in the kaon mixing and decay, both are like CP violation in the D decay for the intents and purposes of the Dalitz analysis. Regardless of how small

the CP violation is in the D decay itself, this is already taken into account when neglecting $r_B|\epsilon|$ terms when we allow for $T_i^+ \neq T_i^-$. This CP violation makes its appearance in the variables δ_D and $\bar{\delta}_D$. This can be taken into account for all terms (including $r_B|\epsilon|$ terms) by allowing for $\delta_D(s_{12}, s_{13}) \neq \bar{\delta}_D(s_{13}, s_{12})$. In fact

$$\frac{\delta_D(s_{12}, s_{13}) - \bar{\delta}_D(s_{13}, s_{12})}{2} = \arg \left(\frac{V_{us}V_{cd}^*}{V_{ud}V_{cs}^*} \right). \quad (\text{B.1})$$

Note that $V_{us}V_{cd}^*/V_{ud}V_{cs}^* \sim \lambda^2$ with a non-zero phase coming in at $O(\lambda^6)$. This is of course extremely small, but it may be compensated for by kinematic effects in some regions of phase space (i.e. where r_D is not small). In that case the phase can be thought of as a λ^4 correction, which may be competitive with $O(|\epsilon|)$ effects. As noted however, this is most important in the $T_i^\pm$ terms where it is already taken into account, otherwise it is suppressed by r_B .

B.2 In $B^\pm \rightarrow DK^\pm$

The weak angle involved in the decay $B^\pm \rightarrow DK^\pm$ is not precisely γ but receives some small corrections from other CKM elements. The ratio of CKM matrix elements involved in the decay is $V_{cs}V_{ub}^*/V_{us}V_{cb}^*$. Relating this to γ involves

$$\frac{V_{cd}V_{cs}}{V_{ud}V_{us}} = -1 + \frac{A\lambda^4}{2} - A^2\lambda^5 \left(\rho + i\eta - \frac{1}{2} \right) + O(\lambda^6). \quad (\text{B.2})$$

From this we see that corrections to the phase $e^{-i\gamma}$ come in at order $\lambda^5 \sim 10^{-3}$. This is still unimportant at the $|\epsilon|/r_B$ level but is competitive with $O(|\epsilon|)$ corrections. In all previous sections where we have written γ , we in fact mean $\gamma + \phi_B$ where

$$\phi_B \equiv \arg \left(\frac{V_{cd}V_{cs}}{V_{ud}V_{us}} \right). \quad (\text{B.3})$$

This can be considered a fundamental limitation on $B^\pm \rightarrow DK^\pm$ methods, although it can be overcome to whatever accuracy ϕ_B can be measured by other means.

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