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# Effective field theories in $R_\xi$ gauges

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Abstract: In effective quantum field theories, higher dimensional operators can affect the canonical normalization of kinetic terms at tree level. These contributions for scalars and gauge bosons should be carefully included in the gauge fixing procedure, in order to end up with a convenient set of Feynman rules. We develop such a setup for the linear $R_\xi$-gauges. It involves a suitable reduction of the operator basis, a generalized gauge fixing term, and a corresponding ghost sector. Our approach extends previous results for the dimension-six Standard Model Effective Field Theory to a generic class of effective theories with operators of arbitrary dimension.

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## Contents



## 1 Introduction

The Higgs boson of the Standard Model (SM) remains the only particle discovered at the LHC so far, despite several years of searches at 13 TeV [1]. Thus, it becomes more and more likely that a sizeable energy gap between the new physics scale and the electroweak scale is present. In this region, the most convenient calculational framework is an Effective Field Theory (EFT) with only the SM degrees of freedom, the so-called SMEFT [2–4]. Higher-dimensional SMEFT operators can account for the neutrino masses and mixings, as well as for other indirect signals for beyond-SM physics that emerge with growing statistical significance in the magnetic moments of leptons [1, 5], and in several $B$-meson decay channels (see, e.g., ref. [6]).

Practical calculations within the SMEFT require introducing convenient gauge-fixing terms. In particular, it has been observed in refs. [7, 8] that effects of higher-dimensional operators should be taken into account in the definition of $R_\xi$ gauges to remove tree-level mixing between the gauge and would-be Goldstone (WBG) bosons, and to preserve simple relations among various masses. Explicit expressions for the dimension-six operator effects have been derived.

In the present paper, we extend the analysis of refs. [7, 8] to a wide class of EFTs with operators of arbitrary dimension. We consider a generic local EFT with linearly realized gauge symmetry, and matter fields in arbitrary representations of the gauge group $G$. The matter fields are assumed to contain spin-0 fields that develop Vacuum Expectation Values (VEVs), and the Higgs mechanism takes place, giving mass to some of the gauge bosons. In such a case, tree-level mixing between the gauge and WBG bosons arises not only from the dimension-four part of the Lagrangian, but also from operators of arbitrarily high dimension. To remove such a mixing with the help of $R_\xi$ gauge-fixing terms, we are going to arrange our operator basis in a particular manner, using the equations of Motion (EOM) to simplify the bilinear terms. Next, after introducing the $R_\xi$ gauge fixing in an appropriate manner, we shall verify that the standard relations between the gauge and WBG boson masses remain valid. Explicit expressions for the ghost terms and BRST transformations will be given.

Our article is organized as follows. In the next section, the operator basis simplification with the help of EOM is described. Section 3 is devoted to defining the gauge fixing and deriving the mass relations. In section 4, the ghost sector and the BRST transformations are specified. We conclude in section 5. In appendix A, we recall the arguments behind constructing the EFT operators from products of fields and their covariant derivatives. Appendix B is devoted to generalizing our results to the case of several distinct gauge-fixing parameters. Appendix C summarizes basic expressions for complex scalar field representations of $G$ in the real notation. The specific example of SMEFT is discussed in appendix D.

## 2 Operator basis reduction

Let us consider an EFT that arises after decoupling [9] of heavy particles whose masses are of the order of some scale $\Lambda$. We assume that the original theory at that scale is perturbative. The resulting EFT describes dynamics of light fields (with masses $\ll \Lambda$) at energy scales much lower than $\Lambda$. It is given by the following Lagrangian

$$\mathcal{L} = \mathcal{L}^{(4)} + \sum_{k=1}^{\infty} \frac{1}{\Lambda^k} \sum_i C_i^{(k+4)} Q_i^{(k+4)} , \tag{2.1}$$

where $\mathcal{L}^{(4)}$ is the dimension-four ("renormalizable") part of $\mathcal{L}$, while $Q_i^{(k+4)}$ stand for dimension-$(k+4)$ local operators built out of fields and their derivatives. They come with the Wilson coefficients $C_i^{(k+4)}$. Throughout the paper, we work at an arbitrary but *fixed* order $N$ in the $1/\Lambda^k$-expansion, i.e. we are going to neglect terms of order $1/\Lambda^{N+1}$ or higher.

Spin-0 degrees of freedom can always be described in terms of real scalar fields. They will be denoted collectively by $\Phi$. We are interested in situations when $\Phi$ acquires a non-vanishing VEV $\langle\Phi\rangle = v$ such that $|v| \ll \Lambda$. If $v$ is not a singlet under $G$, some of the gauge fields $A_\mu^a$ become massive via the Higgs mechanism. We assume absence of any other relevant contributions to the gauge boson masses (like, e.g., contributions from fermion-antifermion pair condensation).

To keep the notation compact, we absorb the gauge couplings into the structure constants $f^{abc}$ and the gauge group generators $T^a$. Then the field strength tensor for all the gauge fields

$$F_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a - f^{abc} A_\mu^b A_\nu^c , \tag{2.2}$$

as well as the covariant derivatives of $\Phi$ and $F_{\mu\nu}$

$$D_\mu \Phi = (\partial_\mu + i A_\mu^a T^a)\Phi, \qquad (D_\rho F_{\mu\nu})^a = \partial_\rho F_{\mu\nu}^a - f^{abc} A_\rho^b F_{\mu\nu}^c \tag{2.3}$$



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are given by the above short-hand expressions even if the gauge group G is not simple, and/or $\Phi$ resides in a reducible representation. The generators $T^a$ of this representation are both hermitian and antisymmetric, which means that all their components are imaginary (see appendix C).

Our goal in this section is selecting and simplifying all the terms in $\mathcal{L}$ (2.1) that matter for tree-level two-point Green's functions for the scalar and gauge fields. Thus, we shall consider such operators $\mathcal{L}$ that contain bilinear terms in $\varphi = \Phi - v$ and in the gauge fields $A^a_\mu$. Before gauge fixing, the part of $\mathcal{L}^{(4)}$ that matters for our considerations is the one containing solely $\Phi$ and $A^a_\mu$

$$\mathcal{L}^{(4)}_{\Phi,A} = \frac{1}{2}(D_\mu\Phi)^T(D^\mu\Phi) - \frac{1}{4}F^a_{\mu\nu}F^{a\,\mu\nu} - V(\Phi)\,, \tag{2.4}$$

where $V$ is the scalar potential. Fermionic matter fields with half-integer spins have no effect on the bilinear terms we are after. We assume that no other bosonic degrees of freedom but $\Phi$ and $A^a_\mu$ are present in our EFT.[1] Thus, all the terms in $\mathcal{L}$ (2.1) that contain any other field but $\Phi$ or $A^a_\mu$ are going to be ignored from now on.

In this section, we treat $\mathcal{L}$ (2.1) as the tree-level Lagrangian, before introducing gauge fixing and/or ultraviolet counterterms. It means that $\mathcal{L}$ is a linear combination of gauge-invariant operators that are built of gauge field strength tensors, matter fields, and their (multiple) covariant derivatives (see appendix A). It can be simplified with the help of EOM that take the following form

$$D^\mu D_\mu \Phi = \boxed{HL}\,, \qquad\qquad (D^\mu F_{\mu\nu})^a = \boxed{HL}\,, \tag{2.5}$$

where $\boxed{HL}$ stands for either higher-dimensional or lower-derivative terms. By lower-derivative terms we mean terms containing a lower number of covariant derivatives. If two terms contain the same number of covariant derivatives, then the one containing a lower number of field strength tensors is considered to be "lower".

Simplification with the help of EOM may be understood as writing as many interactions as possible in terms of expressions that vanish by EOM, i.e. EOM-vanishing operators. Green's functions with single insertions of such operators have no effect on on-shell amplitudes [10–17]. For this reason, such operators are often considered redundant, and are removed from the operator basis. Whenever multiple insertions of them matter, the right tool for the operator basis simplification are certain field redefinitions rather than the EOM themselves. However, the ultimate effect of such a procedure is that the simplified basis contains no EOM-vanishing operators (see, e.g., section 5 of ref. [17]).

One should not forget that EOM-vanishing operators may be relevant as ultraviolet counterterms in renormalizing off-shell Green's functions, along with certain gauge-variant operators [11]. Since our current discussion is restricted to the tree-level Lagrangian only, we can determine its structure by setting all the EOM-vanishing and gauge-variant terms to zero.

Any gauge-invariant operator containing $n$ scalar fields, $m$ field strength tensors, and $k$ covariant derivatives will be symbolically denoted by $\Phi^n F^m D^k$. The power $k$ must be even

[1] We neglect the gravitational interactions.



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due to Lorentz invariance. We shall show that the EOM allow to bring the Lagrangian into such a form that only $\Phi^n$, $\Phi^n D^2$ and $\Phi^n F^2$ operators matter for the scalar and gauge boson bilinear terms.

First, let us realize that the bilinear terms in question can only originate from such products where at most two objects have vanishing VEVs. By "objects" we mean tensors $F$, scalar fields $\Phi$ or (multiple) covariant derivatives of them written in any ordering. For instance, a contraction of four objects like $(D_\mu\Phi)^T(D^\mu\Phi)F^a_{\nu\rho}F^{a\,\nu\rho}$ does not affect the bilinear terms because all the four objects have vanishing VEVs. On the other hand, $(\Phi^T\Phi)(\Phi^T D^\mu D^\nu D_\mu D_\nu\Phi)$ can potentially affect the bilinear terms, because only one object there (the fourth covariant derivative of $\Phi$) has a vanishing VEV.

We are not going to consider contractions involving $\varepsilon_{\mu\nu\alpha\beta}$ in a separate manner. Since all our spin $\neq 0$ objects (tensors F and covariant derivatives of something) have vanishing VEVs, the $\varepsilon$ tensor can contract no more than two objects in the operators of interest. In such a case, it is easy to convince oneself that any contraction with $\varepsilon$ can be written in terms of the dual tensor $\widetilde{F}$ and no explicit $\varepsilon$. In our considerations below, whenever $F$ is being mentioned, in may sometimes mean also $\widetilde{F}$. If $F$ is inserted into the l.h.s. of its EOM (2.5), doing the same for $\widetilde{F}$ means using the Bianchi identity $(D^\mu\widetilde{F}_{\mu\nu})^a = 0$.

We shall proceed as follows. After picking up an operator of dimension $d$ that may matter for the bilinear terms in $\varphi$ or $A^a_\mu$, we are going to use the EOM to express it in terms ("reduce it to") either dimension $> d$ operators or lower-derivative ones. Such a procedure can be applied subsequently starting from the operators of lowest dimension, and then proceeding to higher dimensions. At a given dimension, we start from operators with the highest number of derivatives, and then proceed to lower-derivative ones. In this way, at each given dimension, all the operators with derivatives of $F$, and all the operators with multiple derivatives of $\Phi$ will turn out to be reducible. Higher-dimensional terms arising from the EOM at each step will eventually get shifted beyond the order $1/\Lambda^N$ that has been assumed to be the highest one in our treatment of the EFT.

Let us begin with considering an operator containing $D_{\mu_1}\ldots D_{\mu_k}\Phi$ where some of the Lorentz indices are contracted. We can permute the derivatives (at the cost of introducing lower-derivative terms via $[D_\mu, D_\nu] \sim F_{\mu\nu}$) to express the considered $k$-th derivative in terms of objects containing either more $F$ tensors or $D_\mu D^\mu\Phi$. In the latter term, we apply the EOM for $\Phi$ to reduce it to either higher-dimensional or lower-derivative terms. This way we can eliminate all the terms with contracted derivatives of a single $\Phi$, up to the highest dimension we would like to include.

Next, we consider a Lorentz-scalar operator containing $D_{\mu_1}\ldots D_{\mu_k}\Phi$ with $k \geq 2$, where none of the indices are contracted among themselves. They must be contracted with other objects carrying Lorentz indices to give an invariant operator. Since we allow only for two objects with non-vanishing VEVs, this other object must be either $D^{\mu_{\sigma(1)}}\ldots D^{\mu_{\sigma(k)}}\Phi$, where $\sigma$ is some permutation, or a (multiple) derivative of $F$.[2] Shifting one of the $D^{\mu_{\sigma(i)}}$ derivatives "by parts", we obtain either contracted derivatives acting on a single $\Phi$ (discussed above), or operators with three terms whose VEVs vanish.

[2]For $k = 2$, we skip the option with $F^{\mu_1\mu_2}$. In such a case, $D_{\mu_1}D_{\mu_2}\Phi$ can be replaced by $[D_{\mu_1}, D_{\mu_2}]\Phi \sim F^a_{\mu_1\mu_2}T^a\Phi$, and we obtain an operator with double $F$ and no derivatives of $\Phi$, to be considered below.



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Thus, what we have shown so far is that all the operators containing second and higher derivatives of $\Phi$ can be either removed via EOM or do not affect the bilinear terms.

Now, let us consider operators containing at least one gauge field strength tensor $F$. These tensors have vanishing VEVs, so only one or two such tensors are allowed in the operators that affect the bilinear terms. If there is only one such tensor, its indices can be contracted either with some of the derivatives acting on the very $F$, or with the first derivative of $\Phi$. Since only a single derivative of $\Phi$ is allowed at this point, one of the indices of $F$ must be contracted with one of the derivatives acting on the very $F$. After a permutation of derivatives (which brings to life more $F$'s), we find a contraction $[(\ldots)D^\mu F_{\mu\nu}]^a$ that reduces via EOM to higher-dimension or lower-derivative operators. Thus, all the operators with single $F$ can either be removed via EOM or do not affect the bilinear terms.

It remains to discuss operators with double $F$. In this case, no derivative of $\Phi$ allowed because three objects would have vanishing VEVs. Let us show that operators with derivatives acting on $F$ can be removed, too. None of the $F$'s can be fully contracted with derivatives acting on any single object because this would bring us to the case with at least three $F$'s via $F^{\mu\nu}D_\mu D_\nu = \frac{1}{2}F^{\mu\nu}[D_\mu, D_\nu] \sim F^{\mu\nu}F_{\mu\nu}$. On the other hand, if $F$ is contracted with at least one derivative acting on the very $F$, we proceed as in the previously discussed case with a single $F$, arrive at the EOM for $F$, and get moved to higher-dimension or lower-derivative operator classes. Thus, the only remaining options for contractions of Lorentz indices are:

$$Y^{ab}\;[(\ldots)(D_\mu F_{\nu\rho})]^a[(\ldots)(D^\mu F^{\nu\rho})]^b \quad \text{or} \quad Y^{ab}\;[(\ldots)(D_\mu F_{\nu\rho})]^a[(\ldots)(D^\nu F^{\mu\rho})]^b\,, \tag{2.6}$$

where $(\ldots)$ stand for possible extra derivatives, while $Y^{ab}$ is built out of the $\Phi$ fields only (with no derivatives). The first of the above options can be converted to the second one with the help of the Bianchi identity[3] $(D_{[\mu}F_{\nu\rho]})^a = 0$. In the second option, we shift $D_\mu$ from the middle term "by parts". After doing this, we ignore all the terms with derivatives of $\Phi$ or commutators of covariant derivatives because they contain more than two terms with vanishing VEV's. This way we arrive at

$$Y^{ab}\;[(\ldots)(F_{\nu\rho})]^a[(\ldots)(D^\nu D_\mu F^{\mu\rho})]^b\,, \tag{2.7}$$

where the EOM for $F$ can be applied, reducing the considered expression to higher-dimensional or lower-derivative terms.

At this point, our EFT Lagrangian (at the considered arbitrary but fixed order in $1/\Lambda$) already has the desired property, namely that only the $\Phi^n$, $\Phi^n D^2$ and $\Phi^n F^2$ operators matter for the scalar and gauge boson bilinear terms. As far as the $\Phi^n F^2$ operators are concerned, we can now remove the possibility $Y^{ab}F^a_{\mu\nu}\widetilde{F}^{b\,\mu\nu}$, in which case the only possible bilinear term is a total derivative.

[3] If the considered tensor is $\widetilde{F}$, we do the same via the EOM for $F$, up to higher-order and/or lower-derivative terms.

## 3 Gauge fixing

We have organized our Lagrangian in such a way that only operators with at most first derivatives of $\Phi$ and no derivatives of $F$ matter for the bilinear terms in $\varphi = \Phi - v$ and $A^a_\mu$. All such operators belonging to the classes $\Phi^n D^2$ and $\Phi^n F^2$ form a (gauge-invariant) part of the Lagrangian $\mathcal{L}$ (2.1) that can be written in the following form

$$\mathcal{L}_{J,K} = -\frac{1}{4} F^a_{\mu\nu}\, J^{ab}[\Phi]\, F^{b\,\mu\nu} + \frac{1}{2} (D_\mu\Phi)_i\, K_{ij}[\Phi]\, (D^\mu\Phi)_j\,, \tag{3.1}$$

where the $\Phi$-dependent matrices $J$ and $K$ are symmetric. They form a series in $1/\Lambda$ with the leading $(1/\Lambda^0)$ contributions coming from $\mathcal{L}^{(4)}_{\Phi,A}$ (2.4), and being equal to $\delta_{ij}$ and $\delta^{ab}$, respectively. Non-leading terms are polynomial in $\Phi/\Lambda$, and depend on the Wilson coefficients.

The form of eq. (3.1) has been used in ref. [8] to fix the gauge for the dimension-six SMEFT using the Background Field Method (BFM). In that paper, one can find explicit expressions for $J$ and $K$ at $\mathcal{O}(1/\Lambda^2)$, as they appear in this particular EFT. Note that the operator reduction presented in the previous section ensures that eq. (3.1) still holds at higher orders in the SMEFT expansion. This is required for extending the BFM beyond the dimension-six level, and it would not be guaranteed without the prior use of EOM. If the higher-derivative terms were not eliminated from the bilinear terms via the EOM, they would need to be treated as interactions affecting two-point functions at the tree level.

Let us note that a derivation of explicit expressions for $J[\Phi]$ and $K[\Phi]$ in cases when multiple insertions of EOM-vanishing operators might matter should be based on field redefinitions [16, 17] rather than simply setting the EOM-vanishing operators to zero.

We now return to our main task, which is to present a formalism for $R_\xi$ gauges in generic EFTs. We focus on the bilinear terms in eq. (3.1) that arise when $J$ and $K$ are set to their expectation values, i.e.

$$J^{ab}[\Phi] \to J^{ab}[v] \equiv J^{ab} \qquad \text{and} \qquad K_{ij}[\Phi] \to K_{ij}[v] \equiv K_{ij}\,. \tag{3.2}$$

Now $\mathcal{L}_{J,K}$ can be written as

$$\mathcal{L}_{J,K} = -\frac{1}{4} A^T_{\mu\nu}\, J\, A^{\mu\nu} + \frac{1}{2} (D_\mu\Phi)^T\, K\, (D^\mu\Phi) + \dots\,, \tag{3.3}$$

where $A^a_{\mu\nu} \equiv \partial_\mu A^a_\nu - \partial_\nu A^a_\mu$. The terms denoted with ellipses in the above equation describe interactions of three or more fields, and are irrelevant for our $R_\xi$ gauge fixing procedure. In specific models, the structure of the matrices $J$ and $K$ can be constrained by the remaining local or global symmetries of the theory. This happens, in particular, in the SMEFT — see appendix D.

Expanding the covariant derivatives in the second term of eq. (3.3), using integration by parts, and taking into account that $K$ is a symmetric matrix, one obtains the usual "unwanted" term

$$\mathcal{L}_{A\varphi} = -i \left(\partial^\mu A^a_\mu\right) \left[\varphi^T K T^a v\right]\,, \tag{3.4}$$

that describes the gauge and WBG boson mixing. In a convenient setup for perturbative calculations, the $R_\xi$ gauge fixing term should remove this unwanted mixing.



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The scalar fields in the square brackets in eq. (3.4) are identified as the WBG bosons. They correspond to excitations of the scalar fields $\varphi$ along orbits of the gauge group in directions of the broken generators. The remaining excitations of $\varphi$ (that correspond to physical scalars) span a space that is also determined by eq. (3.4). It is the space that is orthogonal to all the $T^a v$ vectors, with "orthogonality" defined by the scalar product $K$. Thus, operators suppressed by powers of $\Lambda$ that affect $K$ do have influence on our identification of the WBG and physical excitations of $\varphi$.

Before introducing the gauge fixing, the WBG excitations are massless. It is guaranteed by gauge invariance of the full scalar potential that includes both $V(\Phi)$ from $\mathcal{L}^{(4)}$ (2.4), and all the relevant contributions from higher-dimensional operators.

Let us now introduce the $R_\xi$ gauge fixing term

$$\mathcal{L}_{\rm GF} = -\frac{1}{2\xi}\,\mathcal{G}^a J^{ab}\mathcal{G}^b \qquad \text{with} \qquad \mathcal{G}^a = \partial^\mu A^a_\mu - i\xi(J^{-1})^{ac}\left[\varphi^T K T^c v\right]. \tag{3.5}$$

It is straightforward to check that the "unwanted" mixing of eq. (3.4) cancels in the sum $\mathcal{L}_{J,K} + \mathcal{L}_{\rm GF}$. The bilinear terms in this sum read

$$\begin{aligned}\mathcal{L}_{\rm kin,mass} = &-\frac{1}{4}A^T_{\mu\nu}JA^{\mu\nu} + \frac{1}{2}A^a_\mu\left[v^T T^a K T^b v\right]A^{b\,\mu} + \frac{1}{2}(\partial_\mu\varphi)^T K(\partial^\mu\varphi)\\ &-\frac{1}{2\xi}(\partial^\mu A_\mu)^T J(\partial^\nu A_\nu) - \frac{\xi}{2}\left[\varphi^T K T^a v\right](J^{-1})^{ab}\left[v^T T^b K\varphi\right].\end{aligned} \tag{3.6}$$

The last term is the WBG boson mass matrix that comes solely from $\mathcal{L}_{\rm GF}$. The physical scalar mass terms (coming from the full scalar potential) are not included in the above equation.

Let's diagonalize the above kinetic and mass terms. The matrices $J$ and $K$ are symmetric and strictly positive-definite because $|v| \ll \Lambda$. Thus, they are diagonalizable and invertible. Moreover, they possess positive-definite square roots that are also symmetric and invertible. We can use them to redefine the scalar and gauge boson fields as follows:

$$\tilde{\varphi}_i = \left(K^{\frac{1}{2}}\right)_{ij}\varphi_j, \qquad\qquad \tilde{A}^a_\mu = \left(J^{\frac{1}{2}}\right)^{ab}A^b_\mu. \tag{3.7}$$

After such a redefinition, we get

$$\begin{aligned}\mathcal{L}_{\rm kin,mass} = &-\frac{1}{4}\tilde{A}^T_{\mu\nu}\tilde{A}^{\mu\nu} + \frac{1}{2}\tilde{A}^T_\mu(M^T M)\tilde{A}^\mu + \frac{1}{2}(\partial_\mu\tilde{\varphi})^T(\partial^\mu\tilde{\varphi})\\ &-\frac{1}{2\xi}(\partial^\mu\tilde{A}_\mu)^T(\partial^\nu\tilde{A}_\nu) - \frac{\xi}{2}\tilde{\varphi}^T(MM^T)\tilde{\varphi}.\end{aligned} \tag{3.8}$$

The kinetic terms have already acquired the canonical form, while the mass matrices are given in terms of

$$M_j{}^b \equiv \left[K^{\frac{1}{2}}\left(iT^a\right)v\right]_j\left(J^{-\frac{1}{2}}\right)^{ab}. \tag{3.9}$$

The above matrix is not a square one (in general) because the scalars and gauge bosons usually reside in representations of different dimensionality. Below, we denote the number of real scalar fields by $m$, and the number of gauge bosons by $n$, which means that $M$ is a real $m \times n$ matrix.



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To diagonalize the mass matrices, one can apply the Singular Value Decomposition (SVD)

$$M = U^T \Sigma V \tag{3.10}$$

with certain orthogonal matrices $U_{m\times m}$ and $V_{n\times n}$, as well as a diagonal one $\Sigma_{m\times n}$ (i.e. a non-square matrix such that $\Sigma_j{}^b = 0$ when $j \neq b$). Consequently,

$$MM^T = U^T(\Sigma\Sigma^T)U \qquad \text{and} \qquad M^TM = V^T(\Sigma^T\Sigma)V\,. \tag{3.11}$$

Therefore, applying $U$ and $V$ respectively on the scalar and gauge boson multiplets

$$\phi_i = U_{ij}\tilde{\varphi}_j, \qquad\qquad W^a_\mu = V^{ab}\tilde{A}^b_\mu \tag{3.12}$$

gives the diagonal mass matrices

$$m^2_\phi = \Sigma\Sigma^T = \begin{bmatrix} D_p & \\ & 0 \end{bmatrix}_{m\times m} \qquad \text{and} \qquad m^2_W = \Sigma^T\Sigma = \begin{bmatrix} D_p & \\ & 0 \end{bmatrix}_{n\times n}\,. \tag{3.13}$$

Although $m^2_\phi$ and $m^2_W$ are in general of different dimension, this is only due to their null spaces. The diagonal blocks $D_p$ of dimension $p = \min(m,n)$ are identical, and include all the non-vanishing entries.

The Lagrangian including the gauge fixing term has now the desired form in the mass-eigenstate basis:

$$\begin{aligned}\mathcal{L}_{\text{kin,mass}} = &-\frac{1}{4}W^T_{\mu\nu}W^{\mu\nu} + \frac{1}{2}W^T_\mu m^2_W W^\mu + \frac{1}{2}(\partial_\mu\phi)^T(\partial^\mu\phi)\\ &-\frac{1}{2\xi}(\partial^\mu W_\mu)^T(\partial^\nu W_\nu) - \frac{\xi}{2}\phi^T m^2_\phi \phi\,.\end{aligned} \tag{3.14}$$

The WBG and gauge boson mass matrices are now diagonal. Non-vanishing squared masses are proportional to each other, with $\xi$ being the proportionality factor. The physical scalars are contained in $\phi$ but they do not receive any mass contribution from the gauge fixing, and therefore correspond to zero eigenvalues of $m^2_\phi$. As we have already mentioned (below eq. (3.6)), contributions to their mass matrix from the full scalar potential should be added to $\mathcal{L}_{\text{kin,mass}}$. Obviously, they can be diagonalized without affecting the r.h.s. of eq. (3.14).

## 4 Ghost sector and BRST

Our gauge-fixing functionals $\mathcal{G}^a$ in eq. (3.5) are linear in the fields. Consequently, the ghost Lagrangian $\mathcal{L}_{\text{FP}}$ can be derived from the Fadeev-Popov determinant (see, e.g., section 21.1 of ref. [18]). The kinetic terms and interactions for ghosts $N^a$ and antighosts $\bar{N}^a$ are then obtained from the variation of $\mathcal{G}^a$ under infinitesimal gauge transformations[4] $\delta\varphi = -i\alpha^a T^a(\varphi + v)$ and $\delta A^a_\mu = \partial_\mu\alpha^a - f^{abc}A^b_\mu\alpha^c$. Taking $\alpha^a(x) = \epsilon N^a(x)$ with an infinitesimal anticommuting constant $\epsilon$, one gets the BRST [19, 20] variations

$$\delta_{\text{BRST}}\varphi = -i\epsilon N^a T^a(\varphi + v) \qquad \text{and} \qquad \delta_{\text{BRST}}A^a_\mu = \epsilon\left(\partial_\mu N^a - f^{abc}A^b_\mu N^c\right)\,. \tag{4.1}$$

[4]The gauge couplings in our notation are absorbed into the generators and structure constants.

The BRST variation of $\mathcal{G}^a$ follows from the above equations, and can be expressed as

$$\delta_{\text{BRST}}\mathcal{G}^a = \epsilon M_F^{ab} N^b \,. \tag{4.2}$$

The ghost Lagrangian can now be written in a compact form

$$\mathcal{L}_{\text{FP}} = \bar{N}^a X^{ab} M_F^{bc} N^d \,, \tag{4.3}$$

where $X^{ab}$ is an arbitrary field-independent matrix, albeit with a non-vanishing determinant. A modification of $X^{ab}$ results in changing the Fadeev-Popov determinant by an irrelevant normalization constant. For future convenience, we set $X^{ab} = J^{ab}$. Then our explicit expression for $\mathcal{L}_{\text{FP}}$ becomes

$$\begin{aligned} \mathcal{L}_{\text{FP}} = {} & J^{ab}\bar{N}^a \Box N^b + \xi \bar{N}^a [v^T T^a K T^b v] N^b \\ & + \bar{N}^a \overleftarrow{\partial}^\mu J^{ab} f^{bcd} A_\mu^c N^d + \xi \bar{N}^a [v^T T^a K T^b \varphi] N^b \,, \end{aligned} \tag{4.4}$$

where the last two terms describe ghost interactions with the gauge bosons and scalars.

The BRST variations of ghost and antighosts take the standard form

$$\delta_{\text{BRST}} N^a = \frac{\epsilon}{2} f^{abc} N^b N^c \qquad \text{and} \qquad \delta_{\text{BRST}} \bar{N}^a = \frac{\epsilon}{\xi} \mathcal{G}^a \,. \tag{4.5}$$

The nilpotence of BRST on $\varphi$ and $A_\mu^a$ follows from eq. (4.1) and $\delta_{\text{BRST}} N^a$ (4.5). A short calculation to check this fact is exactly the same as in theories without higher-dimensional operators. Since $\mathcal{G}^a$ is linear in the fields, one concludes that BRST is nilpotent on $\mathcal{G}^a$, as well, which implies that $\delta_{\text{BRST}}\left(M_F^{ab} N^b\right) = 0$. The latter equality together with the expression for $\delta_{\text{BRST}}\bar{N}^a$ (4.5) and eq. (4.2) are sufficient to see that

$$\mathcal{L}_{\text{GF}} + \mathcal{L}_{\text{FP}} = -\frac{1}{2\xi} \mathcal{G}^a J^{ab} \mathcal{G}^b + \bar{N}^a J^{ab} M_F^{bc} N^c \tag{4.6}$$

is invariant under BRST, irrespectively of what the actual form of $\mathcal{G}^a$ is. The remaining parts of the Lagrangian are BRST-invariant thanks to their gauge invariance.

The matrices parameterizing the ghost kinetic and mass terms in eq. (4.4) are identical/proportional to those for the gauge bosons in eq. (3.6). Thus, the ghost mass eigenstates

$$\eta = V J^{\frac{1}{2}} N \qquad \text{and} \qquad \bar{\eta} = V J^{\frac{1}{2}} \bar{N} \,, \tag{4.7}$$

are obtained with precisely the same transformations as in eqs. (3.7) and (3.12), which leads to

$$\mathcal{L}_{\text{FP}} = \bar{\eta}^T \Box \eta + \xi\, \bar{\eta}^T m_W^2 \eta + (\text{interactions}) \,. \tag{4.8}$$

Ghost masses are thus proportional to the corresponding gauge boson ones, with $\xi$ being the proportionality factor, as in a theory with no higher-dimensional operators. However, the ghost interactions and the BRST variations of antighosts are, in general, affected by the presence of such operators.

## 5 Summary

We described a procedure for introducing the $R_\xi$ gauge fixing in effective theories that arise after heavy particle decoupling, taking into account operators of arbitrarily high dimension. The scalar field VEVs were assumed to be much smaller than the scale $\Lambda$ whose inverse powers multiply higher-dimensional terms in the Lagrangian. Treating all such terms as interactions allowed us to simplify their structure with the help of EOM. We showed that it is possible to perform this simplification in such a way that only operators with single derivatives of the scalar fields $\Phi$, and no derivatives of the gauge field strength tensor $F^a_{\mu\nu}$ matter for bilinear terms in $A^a_\mu$ and $\varphi = \Phi - \langle\Phi\rangle$. They were parameterized by two matrices depending on $\Phi$ alone, with no derivatives. Such matrices become constant when $\Phi$ is replaced by its VEV, and then all the bilinear terms can be resummed into the propagators.[5]

Further steps of our $R_\xi$ gauge-fixing procedure were technically similar to what one does in theories with initially non-diagonal kinetic terms and without higher-dimensional operators. Relations between masses of the gauge bosons, WBG bosons and ghosts remain the same as in the case with canonical kinetic terms. However, the BRST invariance is maintained only after taking into account the full dependence on $\Phi$ in the operators that contain the kinetic and mass terms. Diagonalization of these terms proceeds via field redefinitions that are not gauge-covariant, and depend on Wilson coefficients of higher-dimensional operators. For this reason, the ghost terms and BRST transformations are most conveniently specified before such a diagonalization is performed. The resulting interactions in the mass-eigenstate basis (including those of the ghosts) are affected by the presence of higher-dimensional operators.

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## A The EFT building blocks

In numerous approaches to EFTs with linearly realized gauge symmetries, higher-dimensional operators are constructed from products of gauge field strength tensors, matter fields,

[5] Beyond tree level, one renormalizes the two-point one-particle-irreducible Green's functions treating all the UV counterterms as interactions, including the EOM-vanishing and/or gauge-variant ones. Next, the renormalized bilinear terms are the basis for defining the propagators.



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and their covariant derivatives, as dictated by gauge invariance. However, a frequently asked question is whether any operator containing non-covariant objects like the usual partial derivatives could be gauge invariant and, at the same time, not expressible in terms of covariant derivatives. A very compact (negative) answer to this question was given in footnote 3 of ref. [3], while an extended version can be found in appendix A of ref. [21]. Here, we recall the relevant argument once again.

If the requirement of gauge invariance was not imposed, a local EFT Lagrangian density $\mathcal{L}$ at a spacetime point $x$ would be a polynomial in fields and their multiple partial derivatives at this point. For a scalar matter field $\Phi$, its partial derivative can be trivially re-written in terms of the covariant one as

$$\partial_\mu \Phi = \left(\partial_\mu + iA^a_\mu T^a - iA^a_\mu T^a\right)\Phi = D_\mu \Phi - iA^a_\mu T^a \Phi \,. \tag{A.1}$$

Another partial differentiation of this expression gives

$$\partial_\nu \partial_\mu \Phi = \left(D_\nu - iA^b_\nu T^b\right) D_\mu \Phi - i(\partial_\nu A^a_\mu) T^a \Phi - iA^a_\mu T^a \left(D_\nu - iA^b_\nu T^b\right)\Phi \,, \tag{A.2}$$

and so on. Thus, $\mathcal{L}$ can be re-written in terms of the matter fields, their covariant derivatives, as well as gauge fields and their multiple partial derivatives[6]

$$\begin{aligned}\partial_{\mu_1}\ldots\partial_{\mu_{k-1}} A^a_{\mu_k} &= \partial_{(\mu_1}\ldots\partial_{\mu_{k-1}} A^a_{\mu_k)} + \frac{1}{k!}\sum_\sigma \left(\partial_{\mu_1}\ldots\partial_{\mu_{k-1}} A^a_{\mu_k} - \partial_{\mu_{\sigma(1)}}\ldots\partial_{\mu_{\sigma(k-1)}} A^a_{\mu_{\sigma(k)}}\right)\\ &= \partial_{(\mu_1}\ldots\partial_{\mu_{k-1}} A^a_{\mu_k)} + \frac{1}{k}\sum_{j=1}^{k-1} \partial_{\mu_1}\ldots \not{\partial}_{\mu_j} \ldots\partial_{\mu_{k-1}}\left[\partial_{\mu_j} A^a_{\mu_k} - \partial_{\mu_k} A^a_{\mu_j}\right] .\end{aligned}$$

The last term in the square bracket equals to $F^a_{\mu_j\mu_k} + f^{abc} A^b_{\mu_j} A^c_{\mu_k}$ . Under further differentiation, the tensor $F$ can be treated in the same manner as $\Phi$ above, so only covariant derivatives of $F$ remain. Thus, further differentiation and subsequent symmetrization of partial derivatives of $A$ as above will eventually give us an expression containing $F$ and its covariant derivatives, as well as $A$ and its fully symmetrized partial derivatives only.

At this point, still before imposing gauge invariance on $\mathcal{L}$, all the EFT operators are expressed in terms of matter fields and gauge field strength tensors, covariant derivatives of them, as well as the fully symmetrized partial derivatives $\partial_{(\mu_1}\ldots\partial_{\mu_{k-1}} A^a_{\mu_k)}$, including the zeroth-order one ($k = 1$) being equal to the $A$ field itself.

It remains to be shown that no fully symmetrized derivatives of $A$ can survive once the gauge-invariance requirement is imposed. One can do this by considering a series of gauge transformations that sets all such derivatives ($k = 1, 2, 3, \ldots$) to zero at a single but arbitrary spacetime point $x_P$. We begin with a transformation whose infinitesimal form is

$$A^a_\nu(x) \to A^a_\nu(x) + \partial_\nu \alpha^a(x) - f^{abc} A^b_\nu(x) \alpha^c(x) \quad \text{with} \quad \alpha^a(x) = -(x - x_P)^\rho A^a_\rho(x_P) \,. \tag{A.3}$$

After such a transformation, we have $A^a_\nu(x_P) = 0$. Next, we perform another transformation choosing $\alpha^a(x) = -\frac{1}{2}(x - x_P)^\rho (x - x_P)^\sigma \partial_\rho A^a_\sigma(x_P)$. It preserves the condition

[6] Symmetrization and/or antisymmetrization of $k$ indices in our notation goes with a factor of $1/k!$ .

$A^a_\nu(x_P) = 0$ because $\alpha^a(x_P) = \partial_\mu \alpha^a(x_P) = 0$. Moreover, it nullifies the first symmetrized derivative of $A$ at $x_P$ because $\partial_\mu \partial_\nu \alpha^a = -\partial_{(\mu} A^a_{\nu)}(x_P)$. Further transformations proceed in an analogous manner. At the $k$-th step, we choose

$$\alpha^a(x) = -\frac{1}{k!}(x - x_P)^{\rho_1} \dots (x - x_P)^{\rho_k}\, \partial_{\rho_1} \dots \partial_{\rho_{k-1}} A^a_{\rho_k}(x_P)\,. \tag{A.4}$$

It preserves the conditions $A^a_\nu(x_P) = \partial_{(\mu} A^a_{\nu)}(x_P) = \dots = \partial_{(\mu_1} \dots \partial_{\mu_{k-2}} A^a_{\nu)}(x_P) = 0$ because $\alpha^a(x_P) = \partial_\mu \alpha^a(x_P) = \dots = \partial_{\mu_1} \dots \partial_{\mu_{k-1}} \alpha^a(x_P) = 0$. Moreover, it nullifies the $(k-1)$-th symmetrized derivative of $A$ at $x_P$ because $\partial_{\mu_1} \dots \partial_{\mu_k} \alpha^a = -\partial_{(\mu_1} \dots \partial_{\mu_{k-1}} A^a_{\mu_k)}(x_P)$. Working with a full (non-infinitesimal) form of the gauge transformations would not affect our arguments because higher-order terms in $\alpha^a$ go with higher powers of $(x - x_P)$.

We have thus shown that in a particular gauge, any local operator at $x_P$ (even a gauge-variant one) can be written in terms of matter fields, gauge field strength tensors and their covariant derivatives only. For a gauge invariant operator, this statement remains true at $x_P$ in any gauge, just because the operator is gauge invariant by definition. Since the point $x_P$ was arbitrary, we conclude that gauge invariant local Lagrangian densities at any point can be written in terms of matter fields, gauge field strength tensors and their covariant derivatives only.

## B Distinct gauge-fixing parameters

Our discussion in sections 3 and 4 was restricted to the case of a single gauge-fixing parameter $\xi$. Here, we generalize it to the case when distinct gauge-fixing parameters are used for each of the gauge-boson mass eigenstates. The last two terms of eq. (3.14) take then the form

$$-\frac{1}{2}(\partial^\mu W_\mu)^T\, \hat{\xi}_D^{-1}\, (\partial^\nu W_\nu) - \frac{1}{2}\phi^T (\Sigma\, \hat{\xi}_D\, \Sigma^T)\, \phi\,, \tag{B.1}$$

where $\hat{\xi}_D$ is a diagonal matrix with arbitrary but non-vanishing real entries. Since both $\Sigma_{m\times n}$ and $(\hat{\xi}_D)_{n\times n}$ are diagonal, it is evident that the scalar mass matrix $(\Sigma\, \hat{\xi}_D\, \Sigma^T)$ is diagonal. Moreover, all its non-vanishing entries are given by non-vanishing entries of the diagonal matrix $\Sigma^T \Sigma\, \hat{\xi}_D = m_W^2 \hat{\xi}_D$.

To achieve such a result, we start over with a differently defined $\mathcal{L}_{\rm GF}$, namely

$$\mathcal{L}_{\rm GF} = -\frac{1}{2}\mathcal{G}^a Z^{ab} \mathcal{G}^b\,, \tag{B.2}$$

with

$$Z = J^{\frac{1}{2}} V^T \hat{\xi}_D^{-1} V J^{\frac{1}{2}} \qquad \text{and} \qquad \mathcal{G}^a = \partial^\mu A^a_\mu - i(Z^{-1})^{ac}\left[\varphi^T K T^c v\right]. \tag{B.3}$$

The matrix $Z$ is specified in terms the same orthogonal matrix $V$ that appeared in eq. (3.10) for the $\hat{\xi}_D \sim \mathbf{1}$ case.

The "unwanted" mixing cancels out without making use of the explicit form of $Z$ in eq. (B.3), and we arrive at a new version of eq. (3.6), where the only modification is the replacement of $J/\xi$ by $Z$ in the last two terms. Next, the fields get redefined as in



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eq. (3.7), which gives us eq. (3.8) with the first three terms unaltered, and the last two taking the form

$$-\frac{1}{2}(\partial^\mu \tilde{A}_\mu)^T V^T \hat{\xi}_D^{-1} V (\partial^\nu \tilde{A}_\nu) - \frac{1}{2}\tilde{\varphi}^T \left( M V^T \hat{\xi}_D V M^T \right) \tilde{\varphi} \,. \tag{B.4}$$

Finally, we substitute $M = U^T \Sigma V$, and perform the final rotation of the fields as in eq. (3.12). This way we arrive at eq. (B.1).

As far as the ghost terms are concerned, the expression $\mathcal{L}_{\mathrm{FP}} = \bar{N}^a J^{ab} M_F^{bc} N^c$ remains valid. However, $M_F$ defined through eq. (4.2) now depends on $\hat{\xi}_D$ because $\mathcal{G}^a$ in eq. (B.3) does. Explicitly, one finds

$$\mathcal{L}_{\mathrm{FP}} = \bar{N}^T J \,\Box N + \bar{N}^T J^{\frac{1}{2}} V^T \hat{\xi}_D V M^T M J^{\frac{1}{2}} N + (\text{interactions}) \,. \tag{B.5}$$

Diagonalization of the ghost kinetic and mass terms proceeds as in eq. (4.7), which leads to

$$\mathcal{L}_{\mathrm{FP}} = \bar{\eta}^T \Box \eta + \bar{\eta}^T m_W^2 \hat{\xi}_D \eta + (\text{interactions}) \,. \tag{B.6}$$

The BRST variations of $\varphi$, $A_\mu^a$ and $N^a$ remain the same as in eqs. (4.1) and (4.5), while $\delta_{\mathrm{BRST}} \bar{N}^a = \epsilon \left(J^{-1} Z\right)^{ab} \mathcal{G}^b$.

## C Scalars in complex representations

When setting up our notation in section 2, all the spin-0 degrees of freedom were expressed in terms of real scalar fields. Such a notation is not common in the SM and/or SMEFT where scalars furnish complex representations of the gauge group. To facilitate re-expressing complex fields in terms of real ones, we recall a few useful identities below.

For $N$ complex scalar fields denoted collectively by $H$, the corresponding set of $2N$ real fields $\Phi$ is

$$\Phi = \mathcal{U}\Psi, \qquad \text{with} \qquad \Psi = \begin{pmatrix} H \\ H^\star \end{pmatrix} \qquad \text{and} \qquad \mathcal{U} = \frac{S}{\sqrt{2}} \begin{pmatrix} \mathbf{1}_{N\times N} & \mathbf{1}_{N\times N} \\ -i\mathbf{1}_{N\times N} & i\mathbf{1}_{N\times N} \end{pmatrix} , \tag{C.1}$$

where $S$ is an arbitrary orthogonal $2N \times 2N$ matrix. The matrix $\mathcal{U}$ is unitary. Denoting the gauge group representation generators for $H$ by $C^a$, we have $D_\mu H = \left(\partial_\mu + i A_\mu^a C^a\right) H$. Consequently, $D_\mu \Psi = \left(\partial_\mu + i A_\mu^a P^a\right) \Psi$ and $D_\mu \Phi = \mathcal{U} D_\mu \Psi = \mathcal{U}\left(\partial_\mu + i A_\mu^a P^a\right) \Psi = \mathcal{U}\left(\partial_\mu + i A_\mu^a P^a\right) \mathcal{U}^\dagger \Phi = \left(\partial_\mu + i A_\mu^a T^a\right) \Phi$, where

$$P^a = \begin{pmatrix} C^a & \mathbf{0}_{N\times N} \\ \mathbf{0}_{N\times N} & -C^{a\star} \end{pmatrix} \qquad \text{and} \qquad T^a = \mathcal{U} P^a \mathcal{U}^\dagger = iS \begin{pmatrix} \operatorname{Im} C^a & \operatorname{Re} C^a \\ -\operatorname{Re} C^a & \operatorname{Im} C^a \end{pmatrix} S^T . \tag{C.2}$$

Hermiticity of $C^a$ implies that $P^a = P^{a\,\dagger}$ and $T^a = T^{a\,\dagger}$. Moreover, $T^a$ are manifestly antisymmetric, and all their components are imaginary.

With the above expressions at hand, any operator containing $H$ and its covariant derivatives (or their complex conjugates) can easily be expressed in terms of $\Phi$ and its covariant derivatives. Returning to the notation in terms of $\Psi$ and then $H$ is also straightforward at any desired instance.



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## D Gauge fixing in the SMEFT

As an example, we apply our formalism to the electroweak sector of SMEFT, with the gauge group SU(2) × U(1), considered to any fixed order in the $1/\Lambda$ expansion. Following the notation of appendix C, the complex Higgs doublet and its covariant derivative can be written as

$$H = \begin{pmatrix} H^+ \\ H^0 \end{pmatrix} \equiv \frac{1}{\sqrt{2}} \begin{pmatrix} \phi_2 + i\phi_1 \\ \phi_4 - i\phi_3 \end{pmatrix}, \qquad D_\mu H = \left( \partial_\mu + \frac{ig}{2}\sigma^a W^a_\mu + \frac{ig'}{2} B_\mu \right) H\,. \tag{D.1}$$

For switching to the real notation, we choose

$$S = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 1 & 0 & 0 \end{pmatrix}, \tag{D.2}$$

which gives $\Phi = (\phi_1, \phi_2, \phi_3, \phi_4)$ and $D_\mu \Phi = \left(\partial_\mu + iT^a V^a_\mu\right)\Phi$, with $V^a_\mu = (W^1_\mu, W^2_\mu, W^3_\mu, B_\mu)$, and

$$\begin{aligned} T^1 &= \frac{ig}{2} S \begin{pmatrix} \mathbf{0}_{2\times2} & \sigma^1 \\ -\sigma^1 & \mathbf{0}_{2\times2} \end{pmatrix} S^T, \qquad & T^2 &= \frac{g}{2} S \begin{pmatrix} \sigma^2 & \mathbf{0}_{2\times2} \\ \mathbf{0}_{2\times2} & \sigma^2 \end{pmatrix} S^T, \\ T^3 &= \frac{ig}{2} S \begin{pmatrix} \mathbf{0}_{2\times2} & \sigma^3 \\ -\sigma^3 & \mathbf{0}_{2\times2} \end{pmatrix} S^T, \qquad & T^4 &= \frac{ig'}{2} S \begin{pmatrix} \mathbf{0}_{2\times2} & \mathbf{1}_{2\times2} \\ -\mathbf{1}_{2\times2} & \mathbf{0}_{2\times2} \end{pmatrix} S^T. \end{aligned} \tag{D.3}$$

The matrices $T^a$ are proportional to those in eq. (9) of ref. [8]. After the Higgs field takes its VEV $\langle\Phi\rangle = (0, 0, 0, v)$, the surviving electromagnetic $\mathrm{U}(1)_{\mathrm{em}}$ gauge transformations act on the charged gauge bosons as follows:

$$W^\pm_\mu = \frac{1}{\sqrt{2}}(W^1_\mu \mp iW^2_\mu) \to e^{\pm i\alpha} \frac{1}{\sqrt{2}}(W^1_\mu \mp iW^2_\mu)\,, \tag{D.4}$$

which is equivalent to

$$\begin{pmatrix} W^1_\mu \\ W^2_\mu \end{pmatrix} \to \begin{pmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{pmatrix} \begin{pmatrix} W^1_\mu \\ W^2_\mu \end{pmatrix} \equiv Q_\alpha \begin{pmatrix} W^1_\mu \\ W^2_\mu \end{pmatrix}. \tag{D.5}$$

Thus, the gauge boson kinetic matrix $J$ of eq. (3.2) must be invariant under the transformation

$$\begin{pmatrix} Q^T_\alpha & \mathbf{0}_{2\times2} \\ \mathbf{0}_{2\times2} & \mathbf{1}_{2\times2} \end{pmatrix} J \begin{pmatrix} Q_\alpha & \mathbf{0}_{2\times2} \\ \mathbf{0}_{2\times2} & \mathbf{1}_{2\times2} \end{pmatrix} = J\,, \tag{D.6}$$

which constrains it to the block-diagonal form

$$J = \begin{pmatrix} 1+J_+ & 0 & 0 & 0 \\ 0 & 1+J_+ & 0 & 0 \\ 0 & 0 & 1+J_1 & J_3 \\ 0 & 0 & J_3 & 1+J_2 \end{pmatrix} \equiv \begin{pmatrix} J_C & \mathbf{0}_{2\times2} \\ \mathbf{0}_{2\times2} & J_N \end{pmatrix}. \tag{D.7}$$



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The same argument ensures identical block-diagonal structure of the scalar kinetic matrix $K$ and, in consequence, of the matrices $M$, $U$, $V$ and $\Sigma$ in eqs. (3.9) and (3.10).

In the charged sector, one finds $\Sigma_C = M_W \, \mathbf{1}_{2\times2}$ and $U_C = V_C = \mathbf{1}_{2\times2}$, with the charged $W$-boson mass squared equal to

$$M_W^2 = \frac{g^2 v^2}{4} \frac{1+K_+}{1+J_+} \,. \tag{D.8}$$

In this sector, one should use a common gauge parameter $\xi_W$ to preserve the $\mathrm{U}(1)_{\mathrm{em}}$ gauge symmetry.

In the neutral sector, let us denote $J'_i = 1 + J_i + \sqrt{\det J_N}$, for $i = 1, 2$. Then one finds

$$\begin{aligned} J_N^{1/2} &= \frac{1}{\sqrt{J'_1 + J'_2}} \begin{pmatrix} J'_1 & J_3 \\ J_3 & J'_2 \end{pmatrix} , \\ J_N^{-1/2} &= \frac{1}{\sqrt{(J'_1 + J'_2)\det J_N}} \begin{pmatrix} J'_2 & -J_3 \\ -J_3 & J'_1 \end{pmatrix} , \end{aligned} \tag{D.9}$$

and similarly for the neutral scalar kinetic matrix $K_N$. The matrices appearing in the SVD decomposition (3.10) for the neutral sector are: $\Sigma_N = \mathrm{diag}(M_Z, 0)$,

$$U_N = \begin{pmatrix} \cos\omega & \sin\omega \\ -\sin\omega & \cos\omega \end{pmatrix} \qquad \text{and} \qquad V_N = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} , \tag{D.10}$$

where $\omega = \arctan(K_3/K'_1)$ and $\theta = \arctan[(g' J'_1 + g J_3)/(g J'_2 + g' J_3)]$. In the limit $\Lambda \to \infty$, we have $\omega \to 0$ and $\theta \to \theta_W \equiv \arctan(g'/g)$. Since $\mathcal{O}(v/\Lambda)$ effects are small by assumption, both angles should be close to these limiting values. The $Z$ boson mass squared equals to

$$M_Z^2 = \frac{v^2}{4} \left(g^2 + g'^2 + g'^2 J_1 + 2 g g' J_3 + g^2 J_2\right) \frac{1+K_1}{\det J_N} \,. \tag{D.11}$$

The above formulae for the SVD matrices and gauge boson masses are valid in the SMEFT including operators up to any (fixed) dimension. For consistency, they must always be expanded to the same order in $v/\Lambda$ to which the matrices $K$ and $J$ are known. Up to $\mathcal{O}(v^2/\Lambda^2)$, following ref. [7], one has:

$$J_+ = J_1 = -\frac{2v^2}{\Lambda^2} C^{\varphi W} \,, \qquad J_2 = -\frac{2v^2}{\Lambda^2} C^{\varphi B} \,, \qquad J_3 = \frac{v^2}{\Lambda^2} C^{\varphi WB} \,, \tag{D.12}$$

$$K_+ = K_3 = 0 \,, \qquad K_1 = \frac{v^2}{2\Lambda^2} C^{\varphi D} \,, \qquad K_2 = \frac{v^2}{2\Lambda^2} (C^{\varphi D} - 4 C^{\varphi\Box}) \,. \tag{D.13}$$

After introducing the effective gauge couplings $\bar{g} = g/\sqrt{1+J_1}$, $\bar{g}' = g'/\sqrt{1+J_2}$ and expanding in $v/\Lambda$, one recovers the gauge boson masses, gauge fixing terms and ghost interactions derived in ref. [7].

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