Radiative neutrino masses from order-4 CP symmetry

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ABSTRACT: Generalized CP symmetry of order 4 (CP_4) is surprisingly powerful in shaping scalar and quark sectors of multi-Higgs models. Here, we extend this framework to the neutrino sector. We build two simple Majorana neutrino mass models with unbroken CP_4 , which are analogous to Ma's scotogenic model. Both models use three Higgs doublets and two or three right-handed (RH) neutrinos. The minimal CP_4 symmetric scotogenic model uses only two RH neutrinos, leads to three non-zero light neutrino masses, and contains a built-in mechanism to further suppress them via phase alignment. With three RH neutrinos, one generates a type I seesaw mass matrix of rank 1, which is then corrected by the same scotogenic mechanism, naturally leading to two neutrino mass scales with mild hierarchy. These minimal CP_4 -based constructions emerge as a primer for introducing additional symmetry structures and exploring their phenomenological consequences.

KEYWORDS: Discrete Symmetries, Neutrino Physics, Beyond Standard Model

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1 Introduction

The tiny mass scale of neutrinos and their mixing patterns are considered by many a clear indication that a new mechanism beyond the Standard Model (bSM) is at work. Dozens of neutrino mass models with different levels of sophistication have been proposed, and many of them are based on symmetry arguments, see reviews [1–4] and references therein. Some models aim to quantitatively reproduce the mass and mixing parameters and employ for that purpose discrete or continuous symmetry groups and various new field multiplets transforming non-trivially under them. Others keep assumptions to the minimum and propose new qualitatively different mechanisms for neutrino mass generation.

One appealing example of the latter class of models is the scotogenic model suggested by Ma in 2006 [5]. It makes use of an additional “inert” Higgs doublet and the right-handed neutrinos, which are assumed to be odd under the new global $\mathbb{Z}_2$ -symmetry. If $\mathbb{Z}_2$ remains unbroken, the traditional tree-level seesaw mechanism is not at work. However, at one loop, the new inert scalars including the scalar dark matter (DM) candidate, generate the light neutrino mass matrix. Apart from rich DM consequences, this model may be testable at the LHC [6, 7] or in lepton-flavor violating (LFV) processes [8, 9].

In addition to proposing a radiative neutrino mass model, Ma’s 2006 paper [5] together with refs. [10, 11] boosted the exploration of its scalar sector known as the inert doublet model, see, for example, the recent review [12]. Various more elaborate scalar sectors with DM candidates have been studied later [13–15]. Such models can also be used to radiatively generate neutrino masses; the classification of one-loop [16] and two-loop [17] neutrino mass models generated by scalar DM candidates were established. In those models, one usually keeps $\mathbb{Z}_2$ as the symmetry that protects DM candidates although other family symmetries and their use for radiative neutrino mass generation have also been studied [18].

The intrinsic weakness of $\mathbb{Z}_2$ or $\mathbb{Z}_N$ family symmetries in multi-scalar models is that they still allow for many free parameters. Recently, a rather special multi-Higgs-doublet sector was proposed, in which the scalar DM candidates are protected by a CP symmetry [19]. Unlike all previously constructed models, this model used a generalized CP symmetry of order 4 (called CP4), which means that one must apply it four times to get the identity transformation. Imposing CP4 without producing additional accidental symmetry requires three Higgs doublets; this possibility was found in the course of systematic search for all symmetry groups available in three-Higgs-doublet models (3HDM) [20].

Imposing CP4 leads to a rather well shaped scalar sector [19]. If CP4 remains unbroken, it produces pairwise mass-degenerate scalar sector and DM candidates with peculiar properties. The same CP4 can also be extended to the quark sector [21, 22], also strongly shaping the Yukawa matrices. However, CP4 must be broken in this case to avoid mass-degenerate fermions.

These results naturally lead to the question of whether CP4 can be extended to the neutrino sector and, remaining unbroken, can produce a new, CP4-based version of the scotogenic model. In this paper we answer these questions in the affirmative. We first construct a CP4-symmetric neutrino sector and then build two minimalistic models with unbroken CP4, in which DM candidates play the key role in generating light neutrino masses. Despite the number of free parameters increases when we pass from two to three Higgs doublets, CP4 alone constrains these models *stronger* than $\mathbb{Z}_2$ in the original scotogenic model and generates features which were absent there.

The structure of the paper is the following. In the next section we give essential details of the two main ingredients: the original Ma’s scotogenic model and the CP4-symmetric 3HDM. Then, in section 3, we extend CP4 to the neutrino sector, and consider two minimal examples with two and with three RH neutrinos. We wrap up the paper with conclusions.

2 Scalar DM candidates and radiative neutrino masses

2.1 Ma’s scotogenic model

We begin with a recap of Ma’s scotogenic model proposed in [5]. The model postulates a new global symmetry $\mathbb{Z}_2$, under which the SM fields are even, and adds a second electroweak Higgs doublet Φ_2 and three RH neutrinos¹ N_i , all of which are odd. In this way the charged leptons are coupled only to the SM-like doublet Φ_1 , but the Yukawa neutrino interactions mediated by Φ_2 as well as the Majorana mass term for N ’s are allowed:

$$-\mathcal{L}_{\text{lept.}} = \Gamma_{\alpha\beta} \bar{L}_\alpha \Phi_1 \ell_{R\beta} + Y_{\alpha k} \bar{L}_\alpha \tilde{\Phi}_2 N_k + \frac{1}{2} M_{ij} \bar{N}_i^c N_j + \text{h.c.} \quad (2.1)$$

Here, the Greek letters $\alpha, \beta = 1, 2, 3$ denote charged leptons and the Roman letters $i, j, k = 1, 2, 3$ denote RH neutrinos. The $\mathbb{Z}_2$ symmetry stays unbroken upon minimization of the Higgs potential, $\langle \Phi_2 \rangle = 0$, which can be easily achieved in a significant part

¹Strictly speaking, since these RH singlets N_i do not mix, in the scotogenic model, with the LH neutrinos, naming them RH “neutrinos” is a slight abuse of notation. But since this is a widespread terminology, we will stick to it.

Figure 1. Left: the standard diagram representing the radiative neutrino mass generation in the scotogenic model. Right: the actual H and A -mediated loop diagrams one needs to calculate.

of the scalar sector parameter space. The second, inert doublet Φ_2 produces two neutral scalars denoted traditionally as H and A plus a pair of charged Higgses $H^\pm$, and the lightest among them, usually taken to be H , is the DM candidate (unless some of the new heavy neutrinos is even lighter). Due to $\langle \Phi_2 \rangle = 0$, no Dirac mass term appears, and the LH neutrinos remain massless at the tree level.

However, these masses are generated at the 1-loop level via loop diagrams mediated by the inert neutral scalars H and A , figure 1. Since the fermion line involves scalar interactions of the form

$$\cdots Y_{\alpha k} \frac{1}{\sqrt{2}} (H - iA) \cdots Y_{\beta k} \frac{1}{\sqrt{2}} (H - iA) \cdots, \quad (2.2)$$

one gets the difference between the H -loop and the A -loop with different masses $m_H \equiv m$ and $m_A \equiv M$. The scalar potential contains, among other, the interaction term $\lambda_5[(\Phi_1^\dagger \Phi_2)^2 + \text{h.c.}]/2$, which generates the H/A mass splitting:

$$m_H^2 - m_A^2 = \lambda_5 v^2. \quad (2.3)$$

Thus, the two 1-loop diagrams cancel only in their divergent parts and produce the finite piece proportional to the following scalar function:

$$J(m, M; M_k) = \frac{M_k}{16\pi^2} \left(\frac{m^2}{M_k^2 - m^2} \log \frac{M_k^2}{m^2} - \frac{M^2}{M_k^2 - M^2} \log \frac{M_k^2}{M^2} \right). \quad (2.4)$$

The resulting light neutrino mass matrix is written as

$$(\mathcal{M}_\nu)_{\alpha\beta} = \frac{1}{2} \sum_k Y_{\alpha k} Y_{\beta k} \cdot J(m, M; M_k). \quad (2.5)$$

In particular, for small mass splitting and heavy RH neutrinos, $\lambda_5 v^2 \ll m_0^2 \equiv (m^2 + M^2)/2 \ll M_k^2$, the scalar function J can be simplified as

$$J \approx \frac{\lambda_5 v^2}{16\pi^2 M_k} \left(\log \frac{M_k^2}{m_0^2} - 1 \right). \quad (2.6)$$

Thus, with respect to the classical seesaw mechanism, the scotogenic model generates an extra suppression factor $\lambda_5/(32\pi^2)$ potentially enhanced by the logarithm, and it can be rather small.

The minimum of assumptions is a very appealing feature of the model. One just adds an extra doublet, (usually) three RH neutrinos, and the smallest finite group $\mathbb{Z}_2$, and naturally derives several qualitative consequences. On the other hand, although it can provide tiny neutrino masses for reasonably large M_k , it cannot predict patterns in the mass matrix $\mathcal{M}_\nu$, since the Yukawa couplings $Y_{\alpha k}$ can be arbitrary.

2.2 CP4 3HDM

We aim to apply this scotogenic idea not to the inert doublet model but to the CP4 3HDM, three-Higgs-doublet model equipped with the generalized CP symmetry of order 4 (CP4) and no other accidental symmetries. It was very briefly mentioned in the last appendix of [20], then brought up in [19] as an example of a CP -conserving model without real basis, and later studied in finer detail in [21]. Without loss of generality, its scalar potential can be written as $V = V_0 + V_1$, where

$$\begin{aligned} V_0 = & -m_{11}^2(\Phi_1^\dagger\Phi_1) - m_{22}^2(\Phi_2^\dagger\Phi_2 + \Phi_3^\dagger\Phi_3) + \lambda_1(\Phi_1^\dagger\Phi_1)^2 + \lambda_2[(\Phi_2^\dagger\Phi_2)^2 + (\Phi_3^\dagger\Phi_3)^2] \\ & + \lambda_3(\Phi_1^\dagger\Phi_1)(\Phi_2^\dagger\Phi_2 + \Phi_3^\dagger\Phi_3) + \lambda'_3(\Phi_2^\dagger\Phi_2)(\Phi_3^\dagger\Phi_3) \\ & + \lambda_4[(\Phi_1^\dagger\Phi_2)(\Phi_2^\dagger\Phi_1) + (\Phi_1^\dagger\Phi_3)(\Phi_3^\dagger\Phi_1)] + \lambda'_4(\Phi_2^\dagger\Phi_3)(\Phi_3^\dagger\Phi_2), \end{aligned} \quad (2.7)$$

and

$$V_1 = \frac{\lambda_6}{2}[(\Phi_2^\dagger\Phi_1)^2 - (\Phi_1^\dagger\Phi_3)^2] + \lambda_8(\Phi_2^\dagger\Phi_3)^2 + \lambda_9(\Phi_2^\dagger\Phi_3)(\Phi_2^\dagger\Phi_2 - \Phi_3^\dagger\Phi_3) + \text{h.c.} \quad (2.8)$$

with real λ_6 but still complex $\lambda_{8,9}$. This potential is invariant under

$$J: \phi_i \mapsto X_{ij}\phi_j^*, \quad X = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & i \\ 0 & -i & 0 \end{pmatrix}. \quad (2.9)$$

For generic values of the parameters, there are no other global symmetries of this potential.

Notice that $J^2 = XX^* = \text{diag}(1, -1, -1)$, so that, as a byproduct, this model effectively incorporates a $\mathbb{Z}_2$ symmetry and uniquely assigns Φ_1 to be the $\mathbb{Z}_2$ -even and Φ_2 and Φ_3 to be the $\mathbb{Z}_2$ -odd doublets. If CP4 symmetry is to be conserved in vacuum, we must require that $\langle\Phi_2\rangle = \langle\Phi_3\rangle = 0$, which can be satisfied in a significant part of the scalar parameter space. In this way, Φ_2 and Φ_3 become inert: they do not contribute to the gauge boson or charged fermion masses. Expanding the potential near the vacuum as

$$\Phi_1 = \begin{pmatrix} G^+ \\ \frac{1}{\sqrt{2}}(v + h_{125} + iG^0) \end{pmatrix}, \quad \Phi_2 = \begin{pmatrix} H_2^+ \\ \frac{1}{\sqrt{2}}(H + ia) \end{pmatrix}, \quad \Phi_3 = \begin{pmatrix} H_3^+ \\ \frac{1}{\sqrt{2}}(h + iA) \end{pmatrix}, \quad (2.10)$$

we can find that all these scalars are already mass eigenstates with the following masses:

$$m^2 \equiv m_{h,a}^2 = \frac{1}{2}v^2(\lambda_3 + \lambda_4 - \lambda_6) - m_{22}^2, \quad (2.11)$$

$$M^2 \equiv m_{H,A}^2 = \frac{1}{2}v^2(\lambda_3 + \lambda_4 + \lambda_6) - m_{22}^2 = m^2 + \lambda_6 v^2, \quad (2.11)$$

$$m_{H^\pm}^2 \equiv m_{H_2^\pm, H_3^\pm}^2 = \frac{1}{2}v^2\lambda_3 - m_{22}^2 = m^2 + \frac{1}{2}v^2(\lambda_6 - \lambda_4). \quad (2.12)$$

The inert spectrum is pairwise degenerate, which is a rare instance of the state doubling beyond Kramers degeneracy [21]. Indeed, the conserved CP4 acts on the neutral scalars as

$$H \xrightarrow{\text{CP}} A, \quad A \xrightarrow{\text{CP}} -H, \quad h \xrightarrow{\text{CP}} -a, \quad a \xrightarrow{\text{CP}} h. \quad (2.13)$$

It is this symmetry which protects the lightest inert scalars h and a against decay. We also see that λ_6 in this model plays the role of λ_5 of the inert doublet model: it governs the mass splitting of the two pairs of inert neutral scalars.

CP4 symmetry can also be extended to the fermion sector [21, 22]. Requirement that the Yukawa interactions are CP4 invariant forces this transformation to mix fermion generations as well, in the way similar to (2.9). If CP4 remains unbroken, it leads to mass-degenerate quarks or leptons, which must be avoided. One can either assume that CP4 is spontaneously broken — this route was taken in [22]—or decouple fermions from the inert doublets altogether. In the latter path, we can nevertheless allow RH neutrinos to transform non-trivially under CP4, which will lead us to the desired scotogenic model.

3 Radiative neutrino masses from CP4

3.1 CP4 symmetric neutrino sector

We want to build a scotogenic model based on the generalized CP -symmetry CP4 rather than the family symmetry $\mathbb{Z}_2$. We work in the same CP4 3HDM scalars sector as before and build a 3HDM analog of eq. (2.1):

$$-\mathcal{L}_{\text{lept.}} = \Gamma_{\alpha\beta}^{(a)} \overline{L}_\alpha \Phi_a \ell_{R\beta} + Y_{\alpha k}^{(a)} \overline{L}_\alpha \tilde{\Phi}_a N_k + \frac{1}{2} M_{ij} \overline{N}_i^c N_j + \text{h.c.} \quad (3.1)$$

For the moment, we do not specify the total number of RH neutrinos N_k . CP4 acts on Higgs doublets according to eq. (2.9) and on fermions as

$$\Psi_i \xrightarrow{\text{CP}} U_{ij} \Psi_j^{cp}, \quad \text{where} \quad \Psi_j^{cp} = \gamma^0 C(\overline{\Psi}_j)^T, \quad (3.2)$$

where Ψ generically denotes any type of fermions present in (3.1). The matrices U accompanying a generalized CP transformation can be different for left doublets and right charged lepton and neutrino singlets, see [22] for the similar construction in the quark sector. To avoid massless or mass-degenerate charged leptons, we must assume that CP4 acts trivially on L_α and $\ell_{R\alpha}$. This leads to $\Gamma_{\alpha\beta}^{(2)} = \Gamma_{\alpha\beta}^{(3)} = 0$ and an arbitrary real $\Gamma_{\alpha\beta}^{(1)}$. In terminology of [22], this amounts to selecting case A for charged leptons among four possibilities.

The RH neutrinos N_k can transform non-trivially under CP4. Requiring that the lagrangian (3.1) stays invariant leads to the following set of conditions:

$$Y_{\alpha i}^{(a)} U_{ik} X_{ab}^* = (Y_{\alpha k}^{(b)})^*, \quad (3.3)$$

$$(U^T)_{ii'} M_{i'k'} U_{k'k} = M_{ik}^*. \quad (3.4)$$

Via an appropriate basis change in the RH neutrino space N_i , the matrix U can be brought to the block-diagonal form [23, 24], with the blocks being either 1×1 phases or 2×2 matrices of the following type:

$$\begin{pmatrix} c_\alpha & s_\alpha \\ -s_\alpha & c_\alpha \end{pmatrix} \quad \text{as in [23],} \quad \text{or} \quad \begin{pmatrix} 0 & e^{i\alpha} \\ e^{-i\alpha} & 0 \end{pmatrix} \quad \text{as in [24].} \quad (3.5)$$

Condition (3.3) is of the same type as derived for the quark sector [22] under the additional assumption that the left-handed fields are not mixed by CP transformation. The only non-trivial solution available corresponds to $\alpha = \pi/2$, which can also be checked by direct derivation. Thus, the minimal scotogenic model with CP4 requires *two RH neutrinos* N_i , which must be coupled only with the doublets Φ_2 and Φ_3 (thus, $Y^{(1)} = 0$).

To proceed further, we select the first form of matrix U :

$$U_{ij} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}, \quad (3.6)$$

and convert (3.3) to $-iY^{(2)}U = (Y^{(3)})^*$ and $iY^{(3)}U = (Y^{(2)})^*$. These conditions lead to the reciprocal dependence of elements of $Y^{(2)}$ and $Y^{(3)}$. For example, once all elements of $Y^{(2)}$ are chosen, $Y^{(3)}$ is fully reconstructed:

$$Y^{(2)} = \begin{pmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \\ y_{31} & y_{32} \end{pmatrix}, \quad Y^{(3)} = \begin{pmatrix} -iy_{12}^* & iy_{11}^* \\ -iy_{22}^* & iy_{21}^* \\ -iy_{32}^* & iy_{31}^* \end{pmatrix}. \quad (3.7)$$

Alternatively, one could take the first columns in both $Y^{(2)}$ and $Y^{(3)}$ as free parameters, and then the second columns would be fully determined. One can also verify by direct computation that

$$\sum_k Y_{\alpha k}^{(3)} Y_{\beta k}^{(3)} = - \left(\sum_k Y_{\alpha k}^{(2)} Y_{\beta k}^{(2)} \right)^*. \quad (3.8)$$

The same U given by eq. (3.6) constrains for the RH Majorana mass matrix M :

$$M = \begin{pmatrix} m_{11} & im_{12} \\ im_{12} & m_{11}^* \end{pmatrix}, \quad (3.9)$$

with complex m_{11} and real m_{12} . This complex symmetric matrix can be always brought, via a transformation $V \in \text{SU}(2)$ in the N_i space, to the diagonal form proportional to the identity matrix:

$$M = V^T D V, \quad \text{where} \quad D = \begin{pmatrix} M_0 & 0 \\ 0 & M_0 \end{pmatrix}. \quad (3.10)$$

Notice also that this transformation does not affect the CP4 matrix U : $V^T U V = U$. This is not surprising: the matrix U can be viewed as defining the skew-symmetric product, and the transformation group which leaves the skew-symmetric product invariant is known as the symplectic group $\text{Sp}(1)$ which is isomorphic to $\text{SU}(2)$.

Therefore, in the new basis, one can still parametrize Yukawa couplings $Y^{(2)}$ and $Y^{(3)}$ as in (3.7) and just replace M with its diagonal form D . In this way, the Majorana mass matrix for RH neutrinos N_i is diagonal and two real degenerate entries M_0 .

3.2 Two RH neutrinos: the minimal CP4 scotogenic model

We are now ready to write the light neutrino mass matrix in the minimal scotogenic model based on CP4. The expression resembles closely eq. (2.5):

$$\mathcal{M}_{\alpha\beta}^s = \frac{1}{2} \sum_k \left[Y_{\alpha k}^{(2)} Y_{\beta k}^{(2)} \cdot J(m, M; M_0) + Y_{\alpha k}^{(3)} Y_{\beta k}^{(3)} \cdot J(M, m; M_0) \right], \quad (3.11)$$

with the same loop function J as in eq. (2.4). Here we used the pairwise mass degeneracy of the inert neutral scalars (2.11) and the mass degeneracy between the two RH neutrinos N_i . Since $J(M, m; M_0) = -J(m, M; M_0)$ and using the property (3.8), we can further simplify it as

$$\mathcal{M}_{\alpha\beta}^s = \text{Re} \left[\sum_k Y_{\alpha k}^{(2)} Y_{\beta k}^{(2)} \right] \cdot J(m, M; M_0). \quad (3.12)$$

This is the final expression for the light neutrino mass matrix within the minimal CP4 scotogenic model. It closely resembles the original scotogenic model result (2.5) and supports the intuitive picture that, for unbroken CP4, the duplicated spectrum of inert scalars just add up in their loops.

However, there are several important differences with respect to Ma's scotogenic model. First, although we used only *two* RH neutrinos, the resulting expression does not force the lightest neutrino to be massless. Indeed, although the matrix $\sum_k Y_{\alpha k}^{(2)} Y_{\beta k}^{(2)}$ is of rank 2 and so is its complex conjugation, their *sum* in eq. (3.11) can be of rank 3 since they have distinct eigenvectors.²

Second, since $\mathcal{M}_{\alpha\beta}$ is manifestly real, it is diagonalized by an orthogonal rotation, which precludes CP -violation in the leptonic sector. This is, of course, to be expected: by construction, we work with an unbroken CP4, and therefore the model cannot display CP -violating effects.

Third, since eq. (3.12) explicitly uses taking the real part, one can envision another potential source of suppression or even cancellation of the neutrino masses: a common phase $\pi/4$ in all entries of $Y^{(2)}$. Whether such an alignment, exact or approximate, can be achieved by additional symmetry arguments is an open question. If it can, then we have extra suppression without fine-tuning among the values of the Yukawa parameters. It is remarkable that, in that case, neutrino masses will be vanishing at one loop, but the corresponding lepton flavor violating processes for charged leptons will persist. Indeed, they involve one-loop diagrams with charged inert scalars and are proportional to $\sum_k Y_{\alpha k}^{(a)} [Y_{\beta k}^{(a)}]^*$, see a detailed analysis [8, 9]. This expression is always non-vanishing, and the two mass-degenerate charged scalars from doublets Φ_2 and Φ_3 interfere constructively.

²As an elementary illustration, consider the following complex symmetric 2×2 matrix A :

$$A = \begin{pmatrix} 1 & i \\ i & -1 \end{pmatrix} = \begin{pmatrix} 1 \\ i \end{pmatrix} \cdot (1, i). \quad (3.13)$$

This matrix is of rank 1, but $\text{Re}A$ is a matrix of rank 2.

3.3 Three neutrino case: two mass scales with mild hierarchy

Now we turn to the case with three RH neutrinos N_k . The starting expressions (3.1) as well as the CP4 symmetry conditions (3.3) and (3.4) still hold, but all matrices including the Yukawa couplings $Y^{(a)}$ and the transformation matrix U are now 3×3 matrices. Again, using the basis change freedom in the N_k space, we make U block-diagonal

$$U = \begin{pmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad (3.14)$$

which forces the Yukawa matrices $Y^{(a)}$ to be of the following form:

$$Y^{(1)} = \begin{pmatrix} 0 & 0 & y_{13} \\ 0 & 0 & y_{23} \\ 0 & 0 & y_{33} \end{pmatrix}, \quad Y^{(2)} = \begin{pmatrix} y_{11} & y_{12} & 0 \\ y_{21} & y_{22} & 0 \\ y_{31} & y_{32} & 0 \end{pmatrix}, \quad Y^{(3)} = \begin{pmatrix} -iy_{12}^* & iy_{11}^* & 0 \\ -iy_{22}^* & iy_{21}^* & 0 \\ -iy_{32}^* & iy_{31}^* & 0 \end{pmatrix}. \quad (3.15)$$

Here, the entries of $Y^{(1)}$ are all real, while the entries of $Y^{(2)}$ and $Y^{(3)}$ can be complex. The Majorana mass matrix for the RH neutrinos becomes, after diagonalization,

$$M = \begin{pmatrix} M_0 & 0 & 0 \\ 0 & M_0 & 0 \\ 0 & 0 & M'_0 \end{pmatrix}, \quad (3.16)$$

where M'_0 does not have to be equal to M_0 .

The main effect of the third RH neutrino is that it can now couple to the SM-like Higgs doublet Φ_1 via $Y^{(1)}$. Upon the spontaneous symmetry breaking, it generate the Dirac mass term with the matrix $m_D = vY^{(1)}/\sqrt{2}$. Therefore, the standard Type I seesaw mechanism is at work and leads to the following tree-level light neutrino mass matrix:

$$\mathcal{M}_{\alpha\beta}^{\text{seesaw}} = -m_D M^{-1} m_D^T = -\frac{v^2}{2M'_0} y_{3\alpha} y_{3\beta}. \quad (3.17)$$

Once again, this matrix is purely real, so that no CP -violating phases emerge.

This tree-level mass matrix is not physically acceptable because it is of rank 1 and, therefore, shows insufficient lepton mixing. But now we recall that the scotogenic mechanism described above with the resulting matrix $\mathcal{M}^s$ given by eq. (3.12) is still at work and can be added to the seesaw term:

$$\mathcal{M}_\nu = \mathcal{M}^s + \mathcal{M}^{\text{seesaw}}. \quad (3.18)$$

The resulting mechanism and the roles of the three RH neutrinos N_i are schematically shown in figure 2. Although one cannot predict, on the basis of CP4 alone, the shape of the neutrino mass matrix and mixing patterns, there are several qualitative features which emerge from this construction.

First, the full matrix $\mathcal{M}_\nu$ is of rank 3. However it emerges as a loop-induced correction to the tree-level seesaw result, which is unavoidably of rank 1. If the loop-induced correction

Figure 2. In the CP4 symmetric model with three RH neutrinos, the first two $N_{1,2}$ generate the scotogenic mass terms, while the third one N_3 produces a rank-1 mass term via the usual type 1 seesaw.

is relatively small, it will naturally generate normally ordered neutrino masses with two mass scales:

$$m_1 \sim m_2 \sim \frac{\lambda_6}{16\pi^2} \frac{v^2}{M_0} [Y^{(2)}]^2 \log \left(\frac{M_0^2}{m^2} \right), \quad m_3 \sim \frac{v^2}{M'_0} [Y^{(1)}]^2, \quad (3.19)$$

where $[Y^{(a)}]^2$ schematically denote results of diagonalization of the $Y^{(a)}$ entry products which are present in (3.12) and (3.17).

Next, let us we assume, for a rough estimate, that $M'_0 = M_0$ and that there is no extra hierarchy among the entries of $Y^{(1)}$ and $Y^{(2,3)}$. Then the ratio of the two scales is naturally mild,

$$\frac{m_{1,2}}{m_3} \sim \frac{\lambda_6}{16\pi^2} \log \left(\frac{M_0^2}{m^2} \right). \quad (3.20)$$

If this ratio is compared with the experimental value of $(\Delta m_{21}^2/|\Delta m_{32}^2|)^{1/2} \approx 0.18$, one can easily match the two numbers, for any reasonable mass scale M_0 and the scalar DM candidate mass m , with $\lambda_6 \sim 1$. Although we consider this exercise just as a numerical example, there may exist a deeper reason for such relations. Namely, if CP4 arises at low-energy as the unique residual symmetry of a yet-unknown larger symmetry group with irreducible triplet, then $M'_0 = M_0$ as well as relations among Y 's follow automatically. It is intriguing to construct an explicit example of such a situation.

Of course, no quantitative conclusions can be drawn from this comparison, but at least the scales emerging in neutrino mass matrix in the CP4 hybrid seesaw-scotogenic model can easily incorporate the experimental data. The idea that loop contributions can correct the unsatisfactory tree-level neutrino mass matrix was exploited as early as in 1999 [25] and was recently embedded in the scotogenic framework in [26, 27], see also other illustrative examples [28–30]. Our model demonstrates that the same mechanism can be driven by a single albeit non-standard CP -symmetry.

4 Conclusions

In this work, we saw that the model-building strategy based on a single symmetry CP4, the generalized CP -symmetry of order 4, which was previously applied to 3HDM scalar [19] and quark sectors [21, 22], is equally well suited for building neutrino mass models.

To this end, we considered two versions of the model with unbroken CP4 and with two or three right-handed heavy neutrinos N_k . For two RH neutrinos — which is the minimal case with unbroken CP4 — the construction resembles Ma’s scotogenic model [5] but with additional features driven by CP4. This minimal model, naturally, does not contain CP violation, allows for three non-zero light neutrino masses despite using only two RH neutrinos, and has a built-in possibility to further suppress or even cancel this mass term via the phase alignment.

For the three RH neutrino case, one predicts the type I seesaw term, which produces the tree-level light neutrino mass matrix of rank 1. The scotogenic mechanism, which is in action here as well, brings the rank back to 3. In a way similar to refs. [25, 26, 29], the model naturally generates two mass scales for neutrino masses, with the relative magnitude which easily matches the experimentally observed pattern.

Certainly, these minimal models, as they stand, are not in the position to claim quantitative predictions. Just like the original scotogenic model, the power of these CP4-based models is the minimality of their assumptions and surprisingly far-fetched consequences. We show that using truly minimal models realizing unbroken CP4 without any freedom to assign representations, we get neutrino masses and potentially rich physics. Of course, one can elaborate on it by considering spontaneously broken CP4 [22] or even allowing for softly CP4-breaking terms. In short, the minimal CP4-based construction presented here can be used as a primer for introduction of additional features and deriving their phenomenological and astroparticle consequences.

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