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Butterfly effect and holographic mutual information under external field and spatial noncommutativity

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Abstract: We apply the transformation of mixing azimuthal and internal coordinate or mixing time and internal coordinate to a stack of N black M-branes to find the Melvin spacetime of a stack of N black D-branes with magnetic or electric flux in string theory, after the Kaluza-Klein reduction. We slightly extend previous formulas to investigate the external magnetic and electric effects on the butterfly effect and holographic mutual information. It shows that the Melvin fields do not modify the scrambling time and will enhance the mutual information. In addition, we also T-dualize and twist a stack of N black D-branes to find a Melvin Universe supported by the flux of the NSNS b-field, which describes a non-comutative spacetime. It also shows that the spatial noncommutativity does not modify the scrambling time and will enhance the mutual information. We also study the corrected mutual information in the backreaction geometry due to the shock wave in our three model spacetimes.

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JHEP02(2017)032

Contents

JHEP02(2017)032

1 Introduction

Butterfly effects in the holographic geometry have been extensively studied in recent, which led to interesting new insights into quantum chaos and the behaviours of entanglement in near-thermal systems [1–3]. These investigations use eternal AdS black hole [4] which has two asymptotically AdS to describe the thermofield double state. After adding a small perturbation to one of the two CFTs at early time t_w one then studies the effect on the structure of the state at t = 0. In this picture the perturbation can be modeled by a shock wave near the horizon of the black hole. In the dual holographic geometry the shock wave will travel across the horizon of the black hole and the mutual information of two space-like regions in each boundary given by

$$I(A,B) = S(A) + S(B) - S(A \cup B) \tag{1.1}$$

can be calculated by the Ryu-Takayanagi prescription for for entanglement entropy [5–7]. After an amount of time t_* the mutual information between the two sides is disrupted and it shows the butterfly effect familiar from chaotic dynamics.

In the initial investigation [1] the dual black hole geometry in is the non-rotating BTZ black hole. The later analysis has generalized to AdS_d [8], multiple shock waves [2], and localized shock waves [9, 10]. The string corrections of the scrambling time were presented in [11]. The butterfly effect for black holes with rotation or charge had also been studied in [8, 12, 13]. That on black D_p branes was investigated in [14].

In this article, our goals are to understand how the external field and spatial non-commutativity will affect the butterfly effect from the holographic dual geometry. The previous studies [8, 12, 13, 15, 16] which used the AdS-Reissner-Nordstrom geometry as the model spacetime is not the external field. The EM fields therein are used to form the black hole itself and dual to the field theory in the present of chemical potential. In our model the extra fields are the external electric field, magnetic field or NSNS b-field, which are called as Melvin fields [17]. The Melvin electric or magnetic field appears in this paper is the genuinely external field while the NSNS b-field describes the spatial non-commutativity.

The stringy effects in scrambling was studied by Shenker and Stanford [11]. They found that the stringy and Planckian correction will increase the scrambling time by

$$t_* = \frac{\beta}{2\pi}\left[1 + \frac{d(d-1)\ell_s^2}{4\ell_{\rm AdS}^2} + \dots\right]\log S \tag{1.2}$$

where d is the space-time dimension of the boundary theory, ℓ_s and $\ell_{\rm AdS}$ are the string and AdS space length respectively. S is the entropy. In our studies the three Melvin fields, however, do not modify the scrambling time. This property seems a little strange at first sight. Since the three spacetimes are constructed through the processes of dimensional reduction, twist and/or T-duality it is natural to conjecture that the scrambling time is invariant under any combination of the three processes. The general formula in [1] or (2.17) tells us that the scrambling time depends on entropy and temperature. While it has been known, for a long time, that certain duality transformations do not affect the temperature and entropy of various gravity solutions [18, 19] our three Melvin-field deformed spacetimes provide the interesting examples which have the same scrambling time.

The mutual information is an important concept in information theory and is a useful quantity to describe the chaos. Previous studies on AdS-Reissner-Nordstrom geometry had found that the chemical potential therein (will is proportion to the EM field) will decrease the mutual information [8, 12, 13, 17]. However, in our model spacetime the extra EM fields will increase the mutual information. We also find that the mutual information increases with the increase of the spacial noncommutativity. While the property is consistent to previous literature [20] we furthermore investigate the backreaction property by considering the mutual information on the shockwave Melvin field geometry.

This paper is organized as follows. In section 2 we first follow the method in [1] to derive the formula of scrambling time in the more general black branes background and apply it to our model spacetime. In section 3 we first apply the transformation of mixing azimuthal and internal coordinate [21–23] or mixing time and internal coordinate [24–30]

JHEP02(2017)032

to the 11D M-theory with a stack N black M5-branes [31, 32] to find the spacetime of a stack of N black D4-branes with magnetic or electric flux in 10 D IIA string theory, after the Kaluza-Klein reduction.[1] Next, we follow [34] to begin with a N black D_3 branes and by applying the three chains of operation to find a model spacetime: 1. T-dualize along z to obtain a N black D_2 branes [35–39]. 2. "Twist" the compactification. 3. T-dualize along z to obtain Melvin Universe supported by the flux of the NSNS b-field. After applying the mapping of Seiberg and Witten [40] we then find a new Melvin Universe which describe the non-comutative spacetime.[2] We calculate the scrambling time in the above three spacetimes to see how the background field will affect it.

In section 4 we calculate the mutual information in the above three spacetimes and determine the critial interval therein. In section 5 we follow the method in [1, 8] to derive the formula of extremal surface in the more general black branes background and apply it to study the corrected mutual information in the backreaction geometry due to the shock wave in our three model spacetimes in section 6. Last section is used to summarize our results and makes some discussions.

2 Scrambling time in general black branes background

2.1 Shockwave geometry

We first derive the formula of scrambling time in the following general black branes background

$$ds^2 = -a(r)f(r)dt^2 + \frac{dr^2}{b(r)f(r)} + d\Sigma_{d-1}^2 \tag{2.1}$$

in which the horizon locates at $r = r_H$ and $f(r_H) = 0$ while $a(r_H) \neq 0$, $b(r_H) \neq 0$. Contract to [1] above metric has two extra functions $a(r)$ and $b(r)$ which could help us to explicitly see how the external filed will affect the butterfly effect. The associated temperature is

$$T = \frac{f'(r_H)\sqrt{a(r_H)b(r_H)}}{4\pi} \tag{2.2}$$

The line element can be expressed in the Kruskal coordinate

$$ds^2 = \frac{4a(r)f(r)}{a(r_H)b(r_H)f'(r_H)^2}\, e^{-f'(r_H)r_*\sqrt{a(r_H)b(r_H)}} dUdV \tag{2.3}$$

$$U = e^{\frac{f'(r_H)}{2}(-t+r_*)\sqrt{a(r_H)b(r_H)}} \tag{2.4}$$

$$V = e^{\frac{f'(r_H)}{2}(t+r_*)\sqrt{a(r_H)b(r_H)}} \tag{2.5}$$

in which r_* is the tortoise coordinate defined by $dr_* = \frac{dr}{f(r)\sqrt{a(r)b(r)}}$.

We now follow [1] to add a small null perturbation of asymptotic energy $E \ll M$ (M is the ADM mass of black branes) at time t_w and radius $r = \Lambda$ in the left asymptotic region.

[1] The method had been applied by us in a previous paper to the zero temperature of M2-branes [33].

[2] The original method in [31] was applied to zero-temperature spacetime.

JHEP02(2017)032

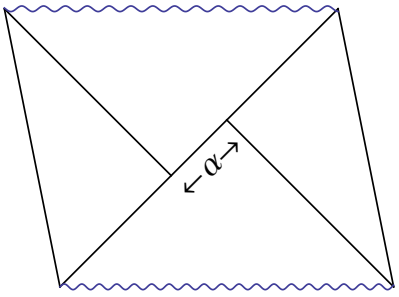

Figure 1. Penrose diagram of an eternal black hole perturbed by a shock wave.

We denote $\tilde{U},\ \tilde{V}$ as coordinates to the left of the perturbation and $U,\ V$ are those to the right. Then, the shell propagating on the constant U surface is described by

$$\tilde{U}_w = e^{\frac{f'(\tilde{r}_H)}{2}(-t_w+\tilde{r}_*(\Lambda))\sqrt{a(\tilde{r}_H)b(\tilde{r}_H)}} \tag{2.6}$$

$$U_w = e^{\frac{f'(r_H)}{2}(-t_w+r_*(\Lambda))\sqrt{a(r_H)b(r_H)}} \tag{2.7}$$

We can patch two side of geometry and the matching condition which relates $\tilde{V}$ to V is described by

$$\tilde{U}_w\tilde{V} = -e^{f'(\tilde{r}_H)\tilde{r}_*(r)\sqrt{a(\tilde{r}_H)b(\tilde{r}_H)}} \tag{2.8}$$

$$U_wV = -e^{f'(r_H)r_*(r)\sqrt{a(r_H)b(r_H)}} \tag{2.9}$$

In the case of linear order in small E and large t_w the matching condition can be used to find the shift

$$\tilde{V} = V + \alpha \tag{2.10}$$

which is shown in figure 1.

2.2 Scrambling time

Since the large value of t_w will lead $r \to r_H$ we can make a approximation $f(r) \approx f'(r_H)(r-r_H) + \cdots$ and tortoise coordinate becomes

$$r_*(r) \approx \int_0^r \frac{dr}{f'(r_H)(r-r_H)\sqrt{a(r_H)b(r_H)}} = \frac{1}{f'(r_H)\sqrt{a(r_H)b(r_H)}} \ln\Big((r-r_H)/r_H\Big) \tag{2.11}$$

Therefore

$$\tilde{U}_w\tilde{V} - UV \approx -\frac{dr_H}{r_H dM}E \tag{2.12}$$

In small E we can approx $\tilde{U}_w \approx U$ [1] and

$$\alpha = -\frac{1}{U_w}\frac{dr_H}{r_H dM}E \tag{2.13}$$

JHEP02(2017)032

To proceed we will express the shift value in terms of entropy. Using the black hole area law and thermodynamic first law we can find that

$$1 = \frac{TdS_{BH}}{dM} = \frac{1}{4}\frac{TdV_{d-1}(r_H)}{dM} = TS_{BH}\frac{V'_{d-1}(r_H)}{V_{d-1}(r_H)}\frac{dr_H}{dM} \tag{2.14}$$

and thus

$$\alpha = -\frac{1}{U_w}\frac{E}{TSr_H}\frac{V_{d-1}(r_H)}{V'_{d-1}(r_H)} \tag{2.15}$$

$$= -e^{-\frac{f'(r_H)}{2}(-t_w+r_*(\Lambda))\sqrt{a(r_H)b(r_H)}}\left[\frac{E}{T}\frac{V_{d-1}(r_H)}{r_H S_{BH}}\frac{V_{d-1}(r_H)}{V'_{d-1}(r_H)}\right] \tag{2.16}$$

The scrambling time t_* is defined to be the value of t_w when $\alpha = 1$ and $E \approx T$. Then we finally find the formula

$$t_* = r_*(\Lambda) + \frac{\beta}{2\pi}\ln\left[\frac{V'_{d-1}(r_H)}{V_{d-1}(r_H)}r_H S_{BH}\right] \tag{2.17}$$

It is interesting to see that while our general metric is different from that in [1, 14] (in which $a(r) = b(r) = 1$) the formula of scrambling time t_* has same form. Notice that, in the general metric the value of $V_{d-1}(r_H)$, T, S_{BH} may be different form that in the case of $a(r) = b(r) = 1$ and the scrambling time t_* therein could be different therefore.

3 Scrambling time in Melvin field deformed geometry

3.1 Scrambling time in electric field deformed geometry

We now apply the transformation of mixing time and internal coordinate to the 11D M-theory with a stack N black M5 branes to find the spacetime of a stack of N black D4 branes with electric flux after the Kaluza-Klein reduction.

The full N black M5-branes solution is given by

$$ds^2_{11} = H^{\frac{-1}{3}}\left(-f(r)dt^2 + dx_1^2 + dx_2^2 + dx_3^2 + dx_4^2 + dx_5^2\right) + H^{\frac{2}{3}}\left(\frac{dr^2}{f(r)} + r^2 d\Omega_4^2\right) \tag{3.1}$$

$$d\Omega_4^2 \equiv d\gamma^2 + cos^2\gamma d\varphi_1^2 + sin^2\gamma(d\psi^2 + cos^2\psi\ d\varphi_2^2) \tag{3.2}$$

where H is the harmonic function defined by

$$H = 1 + \frac{R^3}{r^3}, \quad R^3 \equiv \frac{16\pi G_{11}\, T_5\, N}{3} \tag{3.3}$$

in which G_{11} is the D-dimensional Newton's constant and T_5 is the M_5 brane tension. The function $f(r)$ specified by the horizon at r_H is

$$f(r) = 1 - \frac{r_H^3}{r^3} \tag{3.4}$$

JHEP02(2017)032

Now we transform the time t by mixing it with the compactified coordinate x_5 in the following substituting

$$t \to t - E\ x_5 \tag{3.5}$$

Using above substitution the line element (3.1) becomes

$$\begin{aligned} ds_{11}^2 = &-\frac{H^{\frac{-1}{3}} f(r)}{1-E^2 f(r)}\ dt^2 + H^{\frac{-1}{3}}\left(dx_1^2+dx_2^2+dx_3^2+dx_4^2\right) + H^{\frac{2}{3}}\left(\frac{dr^2}{f(r)}+r^2 d\Omega_4^2\right) \\ &+ H^{\frac{-1}{3}}\left(1-E^2 f(r)\right)\left(dx_5+\frac{Ef(r)}{1-E^2 f(r)}dt\right)^2 \end{aligned} \tag{3.6}$$

Using the relation between the 11D M-theory metric and string frame metric, dilaton field and 1-form potential

$$ds_{11}^2 = e^{-2\phi/3} ds_{10}^2 + e^{4\phi/3}(dx_{11} + 2A_\mu dx^\mu)^2 \tag{3.7}$$

the 10D IIA background is described by

$$ds_{10}^2 = -\frac{H^{\frac{-1}{2}} f(r)}{\sqrt{1-E^2 f(r)}}\, dt^2 + \sqrt{1-E^2 f(r)}\left[H^{\frac{-1}{2}}\left(dx_1^2+dx_2^2+dx_3^2+dx_4^2\right)+H^{\frac{1}{2}}\left(\frac{dr^2}{f(r)}+r^2 d\Omega_4^2\right)\right] \tag{3.8}$$

$$e^{\frac{4\Phi}{3}} = H^{\frac{-1}{3}}\left(1-E^2 f(r)\right), \qquad A_t = \frac{Ef(r)}{2(1-E^2 f(r))} \tag{3.9}$$

In this decomposition into ten-dimensional fields which do not depend on the x_5, the ten-dimensional Lagrangian density becomes

$$L = R - 2(\nabla\Phi)^2 - e^{2\sqrt{3}\Phi}\ F_{\mu\nu}F^{\mu\nu} \tag{3.10}$$

and from (3.10) we see that the parameter E represents the magnitude of the external electric field. In the case of $E = 0$ the above spacetime becomes the well-known geometry of a stack of N black D4-branes. Thus, the background describes the spacetime of a stack of N black D4-branes with electric flux.

To calculate the thermal quantities of the black branes system we have to change the metric in string frame into the Einstein frame by the relation

$$g_{\mu\nu}^E = e^{-\frac{\Phi}{2}} g_{\mu\nu}^S \tag{3.11}$$

Thus the line element in Einstein frame is

$$\begin{aligned} ds_E^2 = &-H^{\frac{-3}{8}} f(r)(1-E^2 f(r))^{-\frac{7}{8}}\, dt^2 + (1-E^2 f(r))^{\frac{1}{8}}\Big[H^{\frac{-3}{8}}\left(dx_1^2+dx_2^2+dx_3^2+dx_4^2\right) \\ &+ H^{\frac{5}{8}}\left(\frac{dr^2}{f(r)}+r^2 d\Omega_4^2\right)\Big] \end{aligned} \tag{3.12}$$

JHEP02(2017)032

In the "near horizon" limit we can approximate $H \sim \frac{R^3}{r^3}$ and metric become

$$ds_E^2 = -\left(\frac{R^3}{r^3}\right)^{\frac{-3}{8}} f(r)(1-E^2 f(r))^{-\frac{7}{8}}\, dt^2 + (1-E^2 f(r))^{\frac{1}{8}} \left[\left(\frac{R^3}{r^3}\right)^{\frac{-3}{8}} \left(dx_1^2+dx_2^2+dx_3^2+dx_4^2\right) + \left(\frac{R^3}{r^3}\right)^{\frac{5}{8}} \left(\frac{dr^2}{f(r)} + r^2 d\Omega_4^2\right)\right] \tag{3.13}$$

Use above metric we can calculate the black brane temperature

$$T(E) = T_0 \tag{3.14}$$

where T_0 is that without electirc field and scrambling time

$$t_*(E) = (t_*)_0 \tag{3.15}$$

in which $(t_*)_0$ is the scrambling time without electric field. The trivial property is the consequence of $E^2 f(r_H) = 0$.

3.2 Scrambling time in magnetic field deformed geometry

We now apply the transformation of mixing azimuthal and internal coordinate to the 11D M-theory with a stack N black M5-branes to find the spacetime of a stack of N black D4 branes with magnetic flux.

Using the full N black M5-branes metric described in (3.1) we can transform the angle φ_1 by mixing it with the compactified coordinate x_5 in the following substituting

$$\varphi_1 \to \varphi_1 + B x_5 \tag{3.16}$$

Using the above substitution the line element (3.1) becomes

$$\begin{aligned} ds_{11}^2 = {} & \frac{H^{\frac{2}{3}} r^2 \cos^2\gamma}{1+B^2 H r^2 \cos^2\gamma} d\varphi_1^2 + H^{-\frac{1}{3}} \left(-f\left(r\right) dt^2 + dx_1^2 + dx_2^2 + dx_3^2 + dx_4^2\right) \\ & + H^{\frac{2}{3}} \left[\frac{dr^2}{f\left(r\right)} + r^2 \left[d\gamma^2 + \sin^2\gamma \left(d\psi^2 + \cos^2\psi d\varphi_2^2\right)\right]\right] \\ & + H^{-\frac{1}{3}} \left(1 + B^2 H r^2 \cos^2\gamma\right) \left(dx_5 + \frac{B H r^2 \cos^2\gamma}{1+B^2 H r^2 \cos^2\gamma} d\varphi_1\right)^2 \end{aligned} \tag{3.17}$$

Using the relation between the 11D M-theory metric and string frame metric, dilaton field and 1-form potential, the 10D IIA background is then described by

$$\begin{aligned} ds_{10}^2 = {} & \frac{H^{\frac{1}{2}} r^2 \cos^2\gamma}{\sqrt{1+B^2 H r^2 \cos^2\gamma}} d\varphi_1^2 + \sqrt{1+B^2 H r^2 \cos^2\gamma} \\ & \times \left\{ H^{-\frac{1}{2}} \left(-f(r) dt^2 + dx_1^2 + dx_2^2 + dx_3^2 + dx_4^2\right) \right. \\ & \left. + H^{\frac{1}{2}} \left[\frac{dr^2}{f\left(r\right)} + r^2 (d\gamma^2 + \sin^2\gamma \left(d\psi^2 + \cos^2\psi d\varphi_2^2\right)\right] \right\} \end{aligned} \tag{3.18}$$

$$e^{\frac{4}{3}\Phi} = H^{-\frac{1}{3}} \left(1 + B^2 H r^2 \cos^2\gamma\right), \qquad A_{\varphi_1} = \frac{B H r^2 \cos^2\gamma}{2\left(1 + B^2 H r^2 \cos^2\gamma\right)} \tag{3.19}$$

JHEP02(2017)032

In this decomposition into ten-dimensional fields which do not depend on the x_5, the ten-dimensional Lagrangian density will be described by (3.10) and the parameter B is the magnetic field defined by $B^2 = \frac{1}{2}F_{\mu\nu}F^{\mu\nu}|_{\rho=0}$. In the case of $B = 0$ the above spacetime becomes the well-known geometry of a stack of N black D4-branes. Thus, the background describes the spacetime of a stack of N black D4-branes with magnetic flux.

The line element in Einstein frame is

$$\begin{aligned} ds_E^2 = & H^{\frac{5}{8}} r^2 \cos^2\gamma \left(1 + B^2 H r^2 \cos^2\gamma\right)^{-\frac{7}{8}} d\varphi_1^2 \\ & + \left(1 + B^2 H r^2 \cos^2\gamma\right)^{\frac{1}{8}} \left\{ H^{-\frac{3}{8}} \left(-f\left(r\right) dt^2 + dx_1^2 + dx_2^2 + dx_3^2 + dx_4^2\right) \right. \\ & \left. + H^{\frac{5}{8}} \left[\frac{dr^2}{f\left(r\right)} + r^2 \left(d\gamma^2 + \sin^2\gamma \left(d\psi^2 + \cos^2\psi d\varphi_2^2\right)\right) \right] \right\} \end{aligned} \tag{3.20}$$

In the "near horizon" the metric become

$$\begin{aligned} ds_E^2 = & \left(\frac{R^3}{r^3}\right)^{\frac{5}{8}} r^2 \cos^2\gamma \left(1 + B^2 \left(\frac{R^3}{r^3}\right) r^2 \cos^2\gamma\right)^{-\frac{7}{8}} d\varphi_1^2 \\ & + \left(1 + B^2 \left(\frac{R^3}{r^3}\right) r^2 \cos^2\gamma\right)^{\frac{1}{8}} \left\{ \left(\frac{R^3}{r^3}\right)^{-\frac{3}{8}} \left(-f\left(r\right) dt^2 + dx_1^2 + dx_2^2 + dx_3^2 + dx_4^2\right) \right. \\ & \left. + \left(\frac{R^3}{r^3}\right)^{\frac{5}{8}} \left[\frac{dr^2}{f\left(r\right)} + r^2 \left(d\gamma^2 + \sin^2\gamma \left(d\psi^2 + \cos^2\psi d\varphi_2^2\right)\right) \right] \right\} \end{aligned} \tag{3.21}$$

Use above metric we can calculate the black brane temperature

$$T(B) = T_0 \tag{3.22}$$

where T_0 is that without magnetic field and scrambling time

$$t_*(B) = (t_*)_0 \tag{3.23}$$

in which $(t_*)_0$ is the scrambling time without magnetic field. Therefore, likes as the Malvin electic field the Malvin magnetic field also does not modify the scrambling time.

3.3 Scrambling time in noncommutative geometry

To begin with, we quote the formula of T-duality [35–39]. After the T-duality on z coordinate the metric, dilaton field and B field become:

$$\tilde{g}_{zz} = 1/g_{zz}, \qquad \tilde{g}_{\mu z} = B_{\mu z}/g_{zz}, \quad \tilde{g}_{\mu\nu} = g_{\mu\nu} - (g_{\mu z}g_{\nu z} - B_{\mu z}B_{\nu z})/g_{zz} \tag{3.24}$$

$$e^{-2\tilde{\phi}} = g_{zz}\, e^{-2\phi}, \quad \tilde{B}_{z\mu} = g_{z\mu}/g_{zz}, \tag{3.25}$$

Now consider the metric of a stack of N black D_3 branes

$$ds_{10}^2 = H^{-1/2}\Big[- f(r)dt^2 + d\rho^2 + \rho^2 d\phi^2 + dz^2\Big] + H^{1/2}\Big[f(r)^{-1}dr^2 + r^2 d\Omega_5^2\Big] \tag{3.26}$$

JHEP02(2017)032

which has zero dilation field and $f(r) = 1 - r_h^4/r^4$. T-dualize along z gives a stack of N black D_2 branes which is described by

$$ds_{10}^2 = H^{-1/2}\Big[-f(r)dt^2 + d\rho^2 + \rho^2 d\phi^2\Big] + H^{1/2}\Big[d\tilde{z}^2 + f(r)^{-1}dt^2 + r^2 d\Omega_5^2\Big] \tag{3.27}$$

$$e^{-2\Phi} = H^{-1/2} \tag{3.28}$$

After performing the twist by replacing

$$\phi \to \phi + b\tilde{z} \tag{3.29}$$

and then T-dualize along $\tilde{z}$ will give a stack of N black D_3 branes described by

$$ds_{10}^2 = H^{-1/2}\left[-f(r)dt^2 + d\rho^2 + \frac{\rho^2 d\phi^2 + dz^2}{1 + H^{-1}b^2\rho^2}\right] + H^{1/2}\Big[f(r)^{-1}dt^2 + r^2 d\Omega_5^2\Big] \tag{3.30}$$

$$e^{-2\Phi} = 1 + H^{-1}b^2\rho^2 \tag{3.31}$$

$$B_{z\phi} = \frac{H^{-1}b\rho^2}{1 + H^{-1}b^2\rho^2} \tag{3.32}$$

in which $b_{z\phi}$ is the flux of the NSNS b-field. After applying the mapping of Seiberg and Witten [33]

$$(G + \theta)^{\mu\nu} = [(g + b)_{\mu\nu}]^{-1} \tag{3.33}$$

the metric G and non-commutativity parameter θ are

$$G_{\mu\nu}dx^\mu dx^\nu = H^{-1/2}\Big[-f(r)d\rho^2 + \rho^2 d\phi^2 + dz^2\Big] + H^{1/2}\Big[f(r)^{-1}dr^2 + r^2 d\Omega_5^2\Big] \tag{3.34}$$

$$\theta^{z\phi} = b \tag{3.35}$$

which describes the black branes geometry with non-commutativity $\theta^{z\phi} = b$ [31]. Note that in Cartesian coordinates the non-commutativity parameter is non-constant.

The line element in Einstein frame is

$$ds_{10}^2 = \left(1 + \frac{r^4}{R^4}\rho^2 B^2\right)^{\frac{1}{4}}\left[\left(\frac{R^4}{r^4}\right)^{\frac{-1}{2}}\left(-f(r)dt^2 + d\rho^2 + \frac{\rho^2 d\phi^2 + dz^2}{1 + \left(\frac{R^4}{r^4}\right)^{-1}B^2\rho^2}\right)\right.$$
$$\left. + \left(\frac{R^4}{r^4}\right)^{\frac{1}{2}}\Big(f(r)^{-1}dr^2 + r^2 d\Omega_5^2\Big)\right] \tag{3.36}$$

in which the "near horizon" lime $H \to \frac{R^4}{r^4}$ is used.

Use above metric we can calculate the black brane temperature

$$T(B) = T_0 \tag{3.37}$$

where T_0 is that without NSNS B-field and scrambling time

$$t_*(B) = (t_*)_0 \tag{3.38}$$

in which $(t_*)_0$ is the scrambling time without NSNS B-field. The trivial property is a consequence of the invariance of entropy under T duality [20].

JHEP02(2017)032

4 Mutual information in Melvin field deformed geometry

In this section we will compute the mutual information of a region A on the left asymptotic boundary and its partner B on the right asymptotic boundary. Note that A = B when the left and right boundaries are identified. For simplicity we consider the mutual information (1.1) between two strips contained in the left and right side of the geometry respectively. Therefore we can use the Ryu-Takayanagi prescription [5–7] to calculate the entangle entropy (EE) between each strip and the rest of the system ($S(A)$ and $S(B)$), and the EE of their union ($S(A\cup B)$).

To consider a strip we can pick up an interval L in a coordinate y with

$$-\frac{L}{2}<y<\frac{L}{2} \tag{4.1}$$

Since that in the black brane system there is a function $f(r)$ which is zero on horizon $r=r_H$ we can let $y=y(r)$ in the Ryu-Takayanagi prescription and, after integrating other coordinates besides r, the action can be described by

$$\mathcal{A}=2\int_{r_{\rm min}}^{\infty}dr\ C(r)\sqrt{f(r)^{-1}+D(r)y'(r)^2} \tag{4.2}$$

in which $r_{\rm min}$ is the turning point. This is the general action which includes those in [8, 14] and mode spacetimes studied in this paper. A minimal bulk surface is found by extremzing the area functional, which leads to

$$y'(r)=\frac{1}{\sqrt{f(r)D(r)\left(\frac{C^2(r)D(r)}{C^2_{\rm min}D_{\rm min}}-1\right)}} \tag{4.3}$$

in which $C_{\rm min}\equiv C(r_{\rm min})$ and $D_{\rm min}\equiv D(r_{\rm min})$. Note that the interval L can be calculated by

$$L=2\int_{r_{\rm min}}^{\infty}dr\ y'(r) \tag{4.4}$$

In this way the minimum surface becomes

$$\text{Area}=2\int_{r_{\rm min}}^{\infty}dr\ \frac{C(r)}{\sqrt{f(r)}}\frac{1}{\sqrt{1-\frac{C^2_{\rm min}D_{\rm min}}{C^2(r)D(r)}}} \tag{4.5}$$

and

$$I(L)=S(A)+S(B)-S(A\cup B)=\int_{r_{\rm min}}^{\infty}dr\frac{C(r)}{\sqrt{f(r)}}\frac{1}{\sqrt{1-\frac{C^2_{\rm min}D_{\rm min}}{C^2(r)D(r)}}}-\int_{r_H}^{\infty}dr\frac{C(r)}{\sqrt{f(r)}} \tag{4.6}$$

where the second term is $S(A\cup B)$ that from area of surface which passes through the horizon to connect to the other side [1].

JHEP02(2017)032

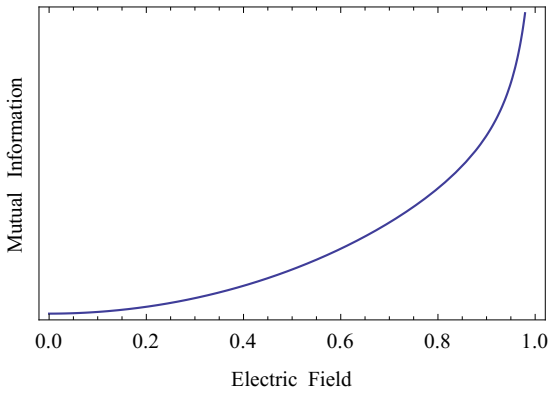

Figure 2. Mutual information under Melvin electric field for the case of $r_H = 1$ and $R = 10$.

We now want to find the critical interval L_c in which the mutual information becomes zero. To proceed, we first rewrite the mutual information function as

$$I(L) = \sqrt{C_{\min}^2 D_{\min}}\ \frac{L}{2} + \int_{r_{\min}}^{\infty} dr\ \frac{C(r)}{\sqrt{f(r)}} \sqrt{1 - \frac{C_{\min}^2 D_{\min}}{C^2(r) D(r)}} - \int_{r_H}^{\infty} dr\ \frac{C(r)}{\sqrt{f(r)}} \tag{4.7}$$

Notice that above result is an exact relation. Since the mutual information becomes zero when $r_{\min} \approx r_H$ above relation leads to

$$L_c = \frac{2}{\sqrt{C_H^2 D_H}} \int_{r_H}^{\infty} dr\ \frac{C(r)}{\sqrt{f(r)}} \left[1 - \sqrt{1 - \frac{C_H^2 D_H}{C^2(r) D(r)}}\right] \tag{4.8}$$

$$\approx C(r_H)\sqrt{D(r_H)} \int_{r_H}^{\infty} dr\ \frac{1}{\sqrt{f(r)}\ C(r) D(r)} + \cdots \tag{4.9}$$

Above formula gives same scaling relation between L_c and R as that in [8]. We now apply these formulas to the Melvin field deformed geometry.

4.1 Mutual information and critical interval in electric field deformed geometry

For the electric field deformed geometry the action becomes

$$\mathcal{A}(E) = \frac{16\pi^2}{3} \int_{r_{\min}}^{\infty} dr\ r R^3\ (1 - E^2 f(r))^{\frac{1}{2}}\ \sqrt{f(r)^{-1} + \frac{r^4}{R^4} y'(r)^2} \tag{4.10}$$

Using above formulation we present figure 2 to show the mutual information in Melvin electric field spactime.

The associated critical interval $L_c(E)$ calculated from above formula is plotted in figure 3.

Use the approximation relation

$$L_c(E) \approx \frac{1}{r_H^3} \int_{r_H}^{\infty} dr\ \frac{1}{r^5\ \sqrt{f(r)}\sqrt{1 - E^2 f(r)}} + \cdots \tag{4.11}$$

the increasing property of the function $L_c(E)$ can be easily read.

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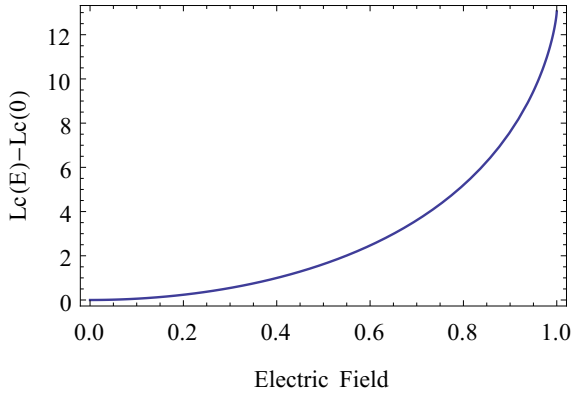

Figure 3. Critical interval $L_c(E)$ of the mutual information under Melvin electric field for the case of $r_H = 1$ and $R = 10$.

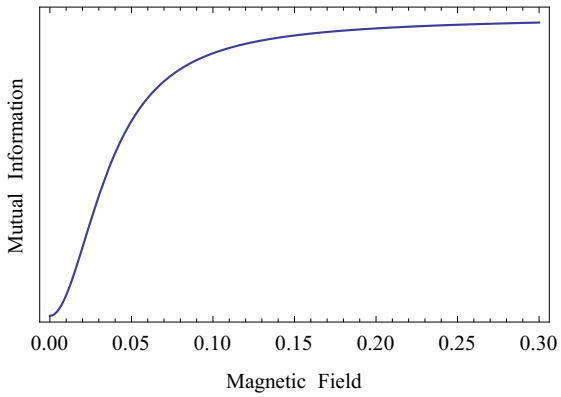

Figure 4. Mutual information under Melvin electric field for the case of $r_H = 1$ and $R = 10$.

4.2 Mutual information and critical interval in magnetic field deformed geometry

For the small-magnetic field deformed geometry the action becomes

$$\mathcal{A}(B) = \int_{r_{\min}}^{\infty} dr\ rR^3 \left(1 + \frac{2B^2R^3}{5r}\right)^{\frac{1}{2}} \sqrt{f(r)^{-1} + \frac{r^3}{R^3}\left(1 + \frac{8B^2R^3}{35r}\right)y'(r)^2} \tag{4.12}$$

Using above formulation we present figure 4 to show the mutual information in Melvin magnetic field spactime.

The associated critical interval $L_c(B)$ calculated from above formula is plotted in figure 5.

Use the approximation relation

$$L_c(B) \approx \frac{1}{r_H^3} \int_{r_H}^{\infty} dr\ \frac{1}{r^5\ \sqrt{f(r)}\sqrt{\left(1 + \frac{2B^2R^3}{5r}\right)}} + \cdots \tag{4.13}$$

the decreasing property of the function $L_c(B)$ can be easily read.

JHEP02(2017)032

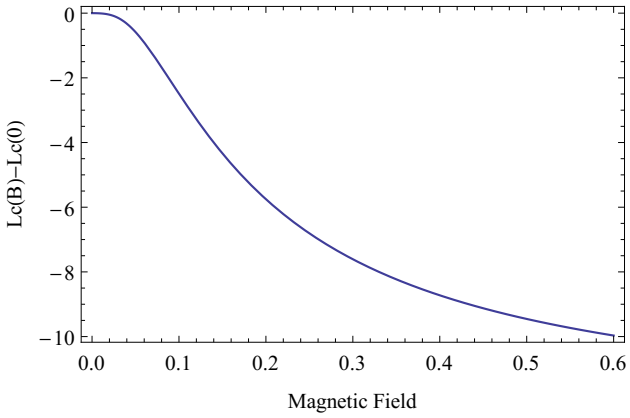

Figure 5. Critical interval $L_c(B)$ of the mutual information under Melvin magnetic field for the case of $r_H = 1$ and $R = 10$.

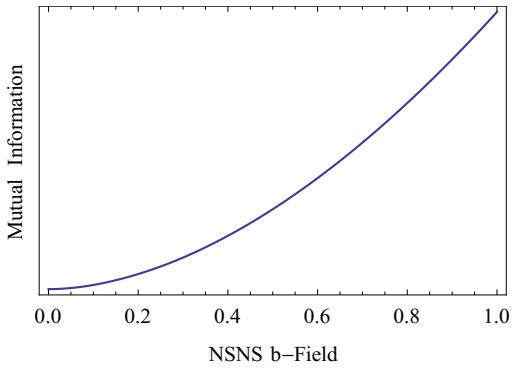

Figure 6. Mutual information under NSNS b-field for the case of $r_H = 1$ and $R = 10$.

4.3 Mutual information and critical interval in noncommutative geometry

For the NSNS b-field deformed geometry the action becomes

$$\mathcal{A}(b) = \int_{r_{\min}}^{\infty} dr\ r\rho R^4 \sqrt{1+\frac{r^4}{R^4}b^2}\ \sqrt{f(r)^{-1}+\frac{\frac{r^4}{R^4}}{1+\frac{r^4}{R^4}b^2}\ y'(r)^2} \tag{4.14}$$

Using above formulation we present figure 6 to show the mutual information in Melvin NSNS b-field spactime.

The associated critical interval $L_c(b)$ calculated from above formula is plotted in figure 7.

Use the approximation relation

$$L_c(b) \approx \int_{r_{\min}}^{\infty} dr\ \rho\ \frac{R^3 r_H^3\ \sqrt{1+\frac{r^4}{R^4}b^2}}{r^5\ \sqrt{f(r)}} + \cdots \tag{4.15}$$

the increasing property of the function $L_c(b)$ can be easily read.

Above results show that the Melvin electric field, Melvin magnetic field and NSNS b-field will increase the mutual information.

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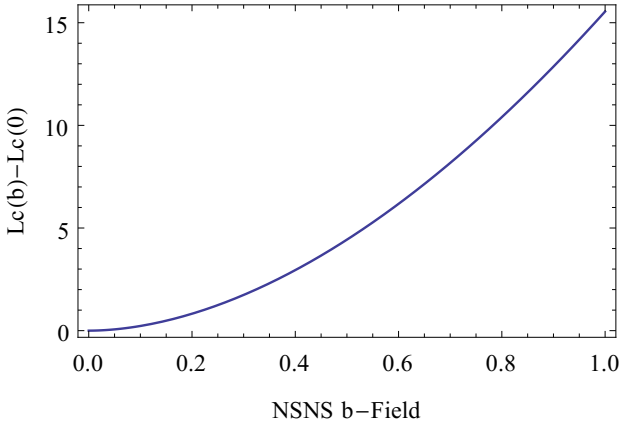

Figure 7. Critical interval $L_c(b)$ of the mutual information under NSNS b-field for the case of $r_H = 1$ and $R = 10$.

5 Extremal surface in shockwave Melvin field geometry

We now consider the mutual information in the backreaction geometry due to the present of shock wave.

5.1 Extremal surfaces

The area of minimal surface is reduced to two-dimensional problem and is dercribed by $r(t)$ [8]. It can be studied from the general form

$$\mathcal{A} = \int dt \sqrt{-a(r)f(r) + \frac{\dot{r}^2}{b(r)f(r)}} \tag{5.1}$$

Regarding $\mathcal{A}$ as a particle action we see that the t-translation symmetry in there gives a conserved quantity

$$\gamma \equiv \sqrt{-a(r_0)f(r_0)} = \frac{-a(r)f(r)}{\sqrt{-a(r)f(r) + \frac{\dot{r}^2}{b(r)f(r)}}} \tag{5.2}$$

where r_0 is define as the radial position where $\dot{r} = 0$. Since that r_0 is behind the horizon and thus $f(r_0)$ is negative as shown in figure 8. Above relation implies that

$$dt = \frac{dr}{\sqrt{a(r)b(r)f(r)^2\left(1 + \frac{a(r)f(r)}{\gamma^2}\right)}} \tag{5.3}$$

Therefore the entanglement entropy can be evaluated from the extremal surface [5–7]

$$S_{A\cup B}(r_0) = \frac{\mathcal{A}}{4} = \frac{1}{4}\int dr \, \frac{\sqrt{a(r)}}{\sqrt{b(r)\left(\gamma^2 + a(r)f(r)\right)}} \tag{5.4}$$

In order to examine how the EE depends on α we have to find the relation between α and r_0. In below, for self-consistent, we will slightly generalize [8] to derive this relation in our model spacetimes.

JHEP02(2017)032

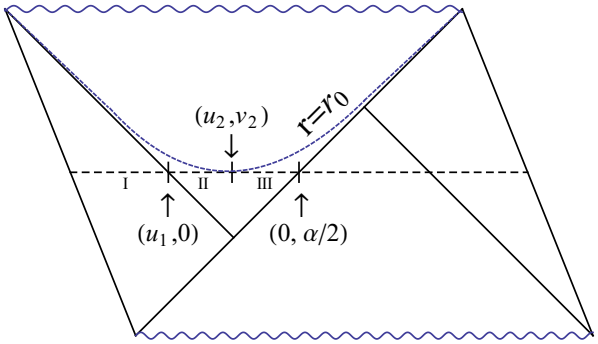

Figure 8. The Penrose diagram and minimal surface (dashing horizontal line) in the shockwave geometry. The left half of the surface is split into three segments, labeled I, II, and III. The smallest value of r attained by the surface is $r = r_0$, which marks the division between II and III and locates at (u_2, v_2).

5.2 Surface location

For the segment I, the minimal surface stretches from the boundary at $(u, v) = (1, 1)$ to $(u; v) = (u_1; 0)$. Using the definition of u we have

$$\begin{aligned} u_1 &\equiv \exp\left[\frac{f'(r_H)\sqrt{a(r_H)b(r_H)}}{2}\Big(\Delta r_* - \Delta t\Big)\right] \\ &= \exp\left[\frac{2\pi}{\beta}\int_\infty^R \frac{dr}{f(r)\sqrt{a(r)b(r)}}\left(1 - \frac{1}{\sqrt{1+\frac{a(r)f(r)}{\gamma^2}}}\right)\right] \end{aligned} \tag{5.5}$$

Since that $r = \infty$ at left boundary and $r = r_H$ at right boundary $(u_1, 0)$ as shown in figure 5.

For the segment II, the u_2 can be evaluated in a same way and

$$u_2 = \exp\left[\frac{2\pi}{\beta}\int_\infty^{r_0} \frac{dr}{f(r)\sqrt{a(r)b(r)}}\left(1 - \frac{1}{\sqrt{1+\frac{a(r)f(r)}{\gamma^2}}}\right)\right] \tag{5.6}$$

Since that $r = r_0$ at right boundary (u_2, v_2) as shown in figure 8. We now need to find the formula about v_2. In this case we can define a reference point $(\bar{u}, \bar{v})$ which is in the interior of the black brane and at some radial coordinate $r = \bar{r} < r_H$. Then

$$v_2 \equiv \frac{\bar{u}\bar{v}}{u_2}\exp\Big[f'(r_H)\Delta r_*(r)\sqrt{a(r_H)b(r_H)}\Big] = \frac{\bar{u}\bar{v}}{u_2}\exp\left[\frac{4\pi}{\beta}\int_{\bar{r}}^{r_0}\frac{dr}{f(r)\sqrt{a(r)b(r)}}\right] \tag{5.7}$$

Note that if we choose the point located at $r_*(\bar{r}) = 0$ then $\bar{u}\bar{v} = 1$ [8].

JHEP02(2017)032

For the segment III, the right boundary is at $(u, v) \equiv (u_3, v_3) = (0, \frac{\alpha}{2})$ as shown in figure 8. Using the definition of v we have

$$\begin{aligned}\frac{v_3}{v_2} \equiv \frac{\alpha}{2v_2} &= \exp\left[\frac{f'(r_H)\sqrt{a(r_H)b(r_H)}}{2}\Big(\Delta r^* + \Delta t\Big)\right] \\ &= \exp\left[\frac{2\pi}{\beta}\int_{r_0}^{r_H}\frac{dr}{f(r)\sqrt{a(r)b(r)}}\left(1 - \frac{1}{\sqrt{1+\frac{a(r)f(r)}{\gamma^2}}}\right)\right]\end{aligned} \tag{5.8}$$

Since that $r = r_0$ at left boundary (u_2, v_2) and $r = r_H$ at right boundary $(0, \frac{\alpha}{2})$ as shown in figure 8.

Combine above three relations we can find that

$$\alpha = 2\bar{u}\bar{v}\,\exp\left[K_1 + K_2 + K_3\right] \tag{5.9}$$

where

$$K_1 = \frac{4\pi}{\beta}\int_{\bar{r}}^{r_0}\frac{dr}{f(r)\sqrt{a(r)b(r)}} \tag{5.10}$$

$$K_2 = \frac{2\pi}{\beta}\int_{r_H}^{\infty}\frac{dr}{f(r)\sqrt{a(r)b(r)}}\left(1 - \frac{1}{\sqrt{1+\frac{a(r)f(r)}{\gamma^2}}}\right) \tag{5.11}$$

$$K_3 = \frac{4\pi}{\beta}\int_{r_0}^{r_H}\frac{dr}{f(r)\sqrt{a(r)b(r)}}\left(1 - \frac{1}{\sqrt{1+\frac{a(r)f(r)}{\gamma^2}}}\right) \tag{5.12}$$

Using above relation the entanglement entropy can be expressed as function α. Since it is divergent we can renormalized it by subtracting its value at $\alpha = 0$. Then

$$S^R_{A\cup B}(\alpha) = S_{A\cup B}(\alpha) - S_{A\cup B}(0) \tag{5.13}$$

In next section we will use above formulas to numerically study the mutual information in three kinds of shockwave Melvin field geometry.

6 Mutual information in shockwave Melvin field geometry

6.1 Mutual information in shockwave electric field deformed geometry

For the Melvin electric field deformed geometry the minimal surface is described by

$$\begin{aligned}\mathcal{A} = \frac{1}{4}\int dt\,\sqrt{\frac{R^3}{r^3}}\,r^4\sqrt{1-E^2f(r)} \\ \times\sqrt{-\left(\frac{R^3}{r^3}\right)^{\frac{-3}{8}}\Big(1-E^2f(r)\Big)^{\frac{-7}{8}}f(r) + \left(\frac{R^3}{r^3}\right)^{\frac{5}{8}}\Big(1-E^2f(r)\Big)^{\frac{1}{8}}\frac{\dot{r}^2}{f(r)}}\end{aligned} \tag{6.1}$$

The relation between α and r_0 is shown in figure 9.

The figure 10 describes the renormalized entanglement entropy "$-S^R_{A\cup B}$" as function of Malvin electric field E. Note that the total entanglement entropy is $S(A) + S(B) - S(A\cup B) - S^R_{A\cup B}$ and thus "$-S^R_{A\cup B}$" describes the corrected mutual information from backreaction geometry due to the shock wave.

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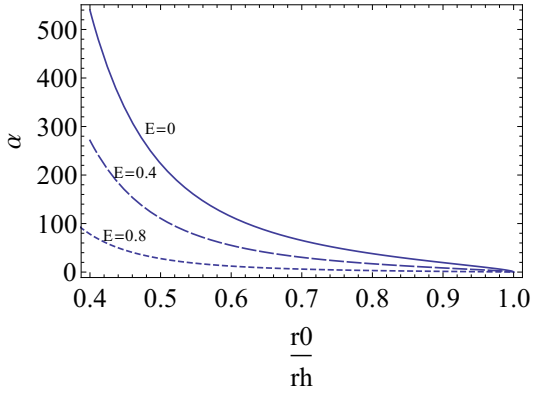

Figure 9. Relation between α and r_0 under Melvin electric field E.

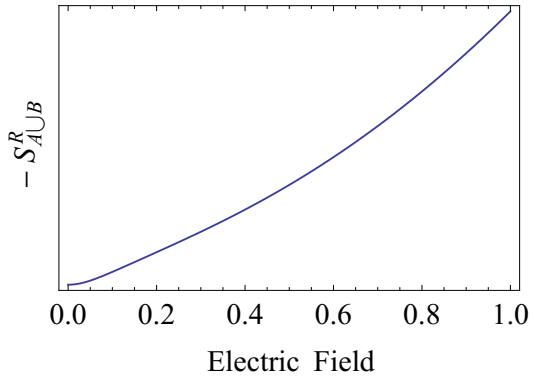

Figure 10. Renormalized entanglement entropy "$-S^R_{A\cup B}$" as function of Malvin electric field E.

6.2 Mutual information in shockwave magnetic field deformed geometry

For the Melvin magnetic field deformed geometry the minimal surface is described by

$$\mathcal{A} \approx \frac{1}{4}\int dt\,\sqrt{\frac{R^3}{r^3}}\,r^4\left(1+\frac{B^2R^3/r}{40}\right)\sqrt{-\left(\frac{R^3}{r^3}\right)^{\frac{-3}{8}}f(r)+\left(\frac{R^3}{r^3}\right)^{\frac{5}{8}}\frac{\dot{r}^2}{f(r)}} \tag{6.2}$$

in which we adopt the approximation for the case of small B field. The relation between α and B, and renormalized entanglement entropy $-S^R_{A\cup B}$ as function of Malvin magnetic field B are shown in the figures 11 and 12.

6.3 Mutual information in shockwave noncommutative geometry

For the NSNS b-field deformed geometry the minimal surface is described by

$$\mathcal{A} \approx \frac{1}{4}\int dt\,\sqrt{\frac{R^4}{r^4}}\,r^4\left(1+\frac{b^2r^4/R^4}{8}\right)\sqrt{-\left(\frac{R^4}{r^4}\right)^{\frac{-1}{2}}f(r)+\left(\frac{R^4}{r^4}\right)^{\frac{1}{2}}\frac{\dot{r}^2}{f(r)}} \tag{6.3}$$

in which we adopt the approximation for the case of small b field. The relation between α and r_0, and renormalized entanglement entropy $-S^R_{A\cup B}$ as function of non-commutativity b are shown in the figures 13 and 14.

JHEP02(2017)032

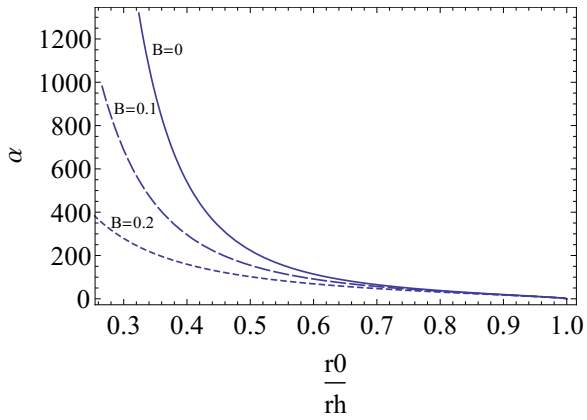

Figure 11. Relation between α and r_0 under Melvin magnetic field B.

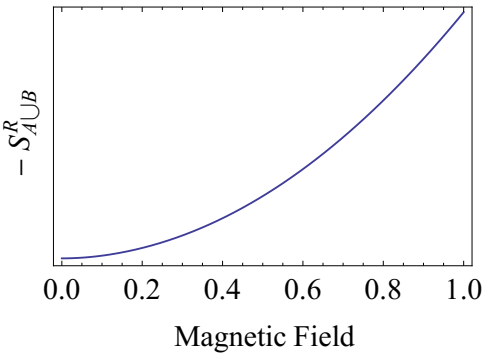

Figure 12. Renormalized entanglement entropy "$-S^R_{A\cup B}$" as function of Malvin magnetic field B.

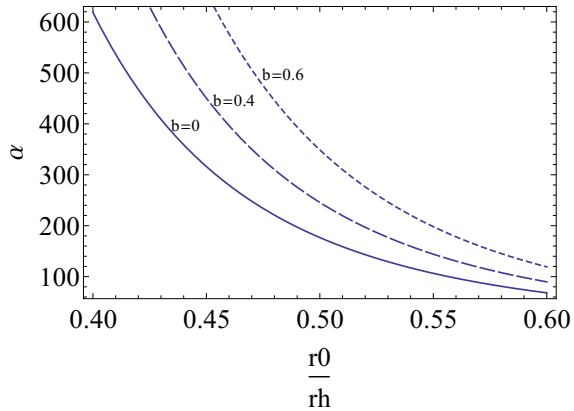

Figure 13. Relation between α and r_0 under Melvin NSNS b-field.

JHEP02(2017)032

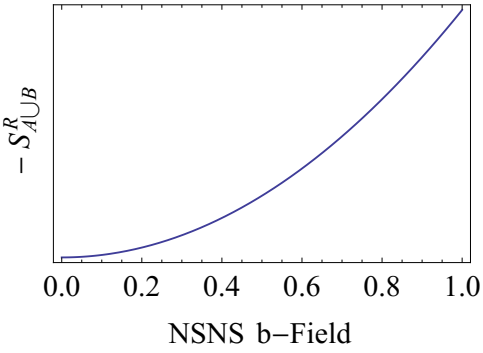

Figure 14. Renormalized entanglement entropy "$-S^R_{A\cup B}$" as function of non-commutativity b.

All above figures show that Melvin fields will enhance the mutual information in the backreaction geometry due to the shock wave.

7 Conclusion

The idea that entanglement entropy has some resemblance to thermodynamic entropy including a sort of first law-like relation has been vigorously investigated recently from the field theoretical and from the holographic sides [41, 42]. The study of mutual information might also serve to determine which set of quantities are expected to be different. In this paper we extend previous method [33, 34] to derive three kinds of Melvin geometry. First and second geometries describe the spacetime of a stack of N black D4-branes with magnetic or electric flux in 10 D IIA string theory. The third geometry describes a Melvin Universe supported by the flux of the NSNS b-field, which relates to the non-comutative spacetime. We then follow the method in [1, 8] to derive the new formula of scrambling time and holographic mutual information in more general spaceimes and use the new formulas to investigate the scrambling time and holographic mutual information in the three model spacetimes. We first find that while the three kinds of external field do no modify the scrambling time they can enhance the mutual information. We also study the mutual information corrected by the backreaction geometry due to the shock wave. It also shows that the three kinds of external field will enhance the mutual information.

Finally, we shall mention that the prescription of Melvin twist used in this paper is a powerful solution generating technique in string theories [21–30]. Applying it to the Dp-brane background and the subsequent near horizon limit gives rise to supergravity duals for a variety of decoupled field theories depending on the orientation of the brane and the Melvin twist. There are many geometries which can be generated from a slight variation of the Melvin twist, as shown in the table of reference [34]. One of our three geometries describes the supergravity background dual to the non-commutative gauge theory, in which the non-commutativity are not constant values [34, 43, 44]. The electric-field deformed and magnetic-field deformed spacetimes are related to the fluxbranes [45, 46]. The holographic duality for the fluxbranes remains an interesting and open problem [46]. Note that the procedure of Melvin twist relies on having a U(1) $\times$ U(1) compact isometry along which

JHEP02(2017)032

one performs a sequence of T-duality, twist, and a T-duality. For examples, if one of the U(1) is transverse to the brane, then one obtains a dipole field theory [47–49]. If taking both of the U(1) to be transverse to the brane gives rise to the construction of Lunin and Maldacena [50]. It is interesting to investigate the quantum chaos on these spacetimes. It is also interesting to see that whether the properties found in this paper could be shown in any simple field theory models. We are now searching the toy mode and investigating the properties from the field theoretic side.

Acknowledgments

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JHEP02(2017)032

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JHEP02(2017)032