

Model independent constraints on four-lepton operators

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ABSTRACT: We obtain constraints on 4-lepton interactions in the effective field theory with dimension-6 operators. To this end, we combine the experimental input from Z boson measurements in LEP-1, W boson mass and decays, muon and tau decays, lepton pair production in LEP-2, neutrino scattering on electrons, and parity violating electron scattering. The analysis does not rely on any assumptions about the flavor structure of the dimension-6 operators. Our main results are the confidence intervals for Wilson coefficients of 16 lepton-flavor conserving four-lepton operators, together with the full correlation matrix. Consequences for leptophilic models beyond the Standard Model are discussed.

KEYWORDS: Beyond Standard Model, Effective field theories

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1 Introduction

An effective field theory (EFT) provides a model-independent framework to characterize new physics beyond the Standard Model (SM). It gives an adequate description of physical processes in current and past experiments if the new particles are much heavier than the weak scale. Then new physics effects can be represented, without introducing new degrees of freedom, by operators with canonical dimensions $D > 4$ added to the SM Lagrangian. Assuming lepton number conservation, leading effects are expected to originate from $D=6$

operators [1]. It is important to understand and describe, with minimal theoretical assumptions, the existing experimental constraints on these operators. While many $D=6$ operators can be probed at the LHC collider, others can be better constrained by previous experiments at high and low energies.

The program of systematic characterization of experimental constraints on the Wilson coefficients $D=6$ operators was pioneered in ref. [2], and later continued e.g. in refs. [3–30]. Most often, due to a huge number of independent $D=6$ operators [31, 32], additional assumptions about the flavor structure of fermionic operators are made. The exception was ref. [21] where a completely generally flavor structure was allowed. That work derived electroweak precision constraints on the subset of $D=6$ operators that yields the so-called *vertex corrections* to Z and W boson interactions with the SM fermions. Only observables that are not affected at leading order by four-fermion operators were considered, so as to separate the analysis of vertex corrections from that of (more numerous) four-fermion operators. Assuming that $D=6$ operators give the dominant new physics contributions to observables, and that loop suppressed new physics contributions are sub-dominant, 20 flavor preserving vertex corrections can be simultaneously constrained. A Gaussian likelihood function in the space of the vertex corrections was given, which can be used to constrain any particular model beyond the SM predicting a more restricted pattern of vertex corrections in the low-energy EFT.

In this paper we extend the analysis of ref. [21] so as to also obtain constraints on some $D=6$ four-fermion operators. This step is important in order to probe a larger class of theories than what can be achieved using vertex corrections only. For example, if the UV theory contains vector boson coupled to the SM fermions but not mixing with the Z or W boson, then it gives rise *only* to 4-fermion operators in the low-energy EFT. Again, the main goal is to derive experimental constraints without assuming anything about the flavor structure of $D=6$ operators. We focus on *four-lepton* operators, leaving four-fermion operators with quarks for future publication. These operators were probed most precisely by lepton pair production in LEP-2 and W mass measurements. However, these observables leave several unconstrained directions in the space of four-lepton operators. Therefore, we also include in our analysis low-energy precision experiments, such as neutrino scattering on electrons, parity violating electron scattering, and leptonic decay of muons and taus. This way we are able to simultaneously constrain 16 linear combinations of 27 Wilson coefficients of lepton-flavor conserving four-lepton operators, in addition to the previous constraints on the vertex corrections. The correlation matrix is given, which allows one to reconstruct the full likelihood function. Given that, our results can be used to constrain large classes of models that give rise to an arbitrary pattern of vertex corrections and 4-lepton operators in the low-energy EFT.

The paper is organized as follows. In section 2 we lay out our formalism where the effects of $D=6$ operators are parametrized by vertex corrections and four-fermion operators. In section 3 we discuss how relevant experimental observables are affected by these EFT parameters. In section 4 we give the confidence intervals for the leptonic vertex corrections and a subset of lepton-flavor conserving four-lepton operators, together with the full correlation matrix (relegated to appendix C). In section 5 we discuss the consequences of our

general analysis for particular models with new leptophilic particles. In appendix A we discuss the relationship between our formalism and the more familiar oblique parameters, and in appendix B we review current constraints on lepton-flavor violating operators.

2 Formalism

We first summarize our conventions. The $SU(2) \times U(1)$ gauge couplings of the SM are denoted by g_L, g_Y . The photon coupling strength is $e = g_L s_\theta$, where $s_\theta = g_Y / \sqrt{g_L^2 + g_Y^2}$ is the weak mixing angle. The vacuum expectation value (VEV) of the SM Higgs field H is $\langle H^\dagger H \rangle = v^2/2$. The SM fermions are written using the two-component spinor notation, and we follow the conventions of ref. [33]. The left- and right-handed charged leptons are denoted by $e_I = (e, \mu, \tau)$, $e_I^c = (e^c, \mu^c, \tau^c)$, while the neutrinos are denoted by $\nu_I = (\nu_e, \nu_\mu, \nu_\tau)$, where $I = 1 \dots 3$ is the flavor index. We work in the basis where these fermions fields are mass eigenstates.

We consider an EFT with the Lagrangian

$$\mathcal{L}_{\text{eff}} = \mathcal{L}_{\text{SM}} + \frac{1}{v^2} \mathcal{L}^{D=6}, \quad \mathcal{L}^{D=6} = \sum_i c_i O_{D=6,i}. \quad (2.1)$$

Here \mathcal{L}_{SM} is the SM Lagrangian, and $O_{D=6,i}$ is a complete basis of $SU(3) \times SU(2) \times U(1)$ invariant operators of canonical dimension $D=6$ constructed out of the SM fields. The Wilson coefficients c_i are formally $\mathcal{O}(v^2/\Lambda^2)$ where Λ is the scale of new physics that sets the EFT expansion. We assume here that $O_{D=6,i}$ conserve the baryon and lepton number. Below we also assume that individual $U(1)_e \times U(1)_\mu \times U(1)_\tau$ lepton numbers are conserved; see appendix B for the discussion of lepton flavor violating operators. In eq. (2.1) we omit higher-dimensional operators with $D > 6$. While these are always present in an EFT derived as a low-energy description of specific UV models, throughout this paper we will assume that their contribution to the relevant observables is negligible. This assumption is generically true when the scale Λ of new physics is much larger than the electroweak scale v , since the Wilson coefficients of higher dimensional operators are formally $\mathcal{O}(v^4/\Lambda^4)$ or smaller. Consequently, our analysis will be performed at $\mathcal{O}(\Lambda^{-2})$, that is to say, we will take into account the corrections to observables that are linear in c_i , and ignore quadratic corrections which are formally $\mathcal{O}(\Lambda^{-4})$. We do not impose any constraints on the Wilson coefficients c_i . In particular, all $D=6$ operators can be simultaneously present in the Lagrangian, and the Wilson coefficients of fermionic operators can be flavor dependent.

Rather than parametrizing the theory space by the Wilson coefficients of $D=6$ operators, we find it more convenient to work directly with parameters describing interactions of mass eigenstates after electroweak symmetry breaking. Our formalism follows that in refs. [13, 34]. Without any loss of generality, \mathcal{L}_{eff} can be brought to a form where the kinetic terms of all mass eigenstates are diagonal and canonically normalized. Then the quadratic Lagrangian for the electroweak gauge boson and lepton mass eigenstates is given by

$$\begin{aligned} \mathcal{L}_{\text{eff}}^{\text{kin}} = & -\frac{1}{2} W_{\mu\nu}^+ W_{\mu\nu}^- - \frac{1}{4} Z_{\mu\nu} Z_{\mu\nu} - \frac{1}{4} A_{\mu\nu} A_{\mu\nu} + \frac{g_L^2 v^2}{4} (1 + \delta m)^2 W_\mu^+ W_\mu^- + \frac{(g_L^2 + g_Y^2) v^2}{8} Z_\mu Z_\mu \\ & + i \bar{e}_I \bar{\sigma}_\mu \partial_\mu e_I + i \bar{\nu}_I \bar{\sigma}_\mu \partial_\mu \nu_I + i e_I^c \sigma_\mu \partial_\mu \bar{e}_I^c. \end{aligned} \quad (2.2)$$

One flavor ($I = 1 \dots 3$)	Two flavors ($I < J = 1 \dots 3$)
$[O_{\ell\ell}]_{IIII} = \frac{1}{2}(\bar{\ell}_I \bar{\sigma}_\mu \ell_I)(\bar{\ell}_I \bar{\sigma}_\mu \ell_I)$	$[O_{\ell\ell}]_{IIJJ} = (\bar{\ell}_I \bar{\sigma}_\mu \ell_I)(\bar{\ell}_J \bar{\sigma}_\mu \ell_J)$
$[O_{\ell e}]_{IIII} = (\bar{\ell}_I \bar{\sigma}_\mu \ell_I)(e_I^c \sigma_\mu \bar{e}_I^c)$	$[O_{\ell e}]_{IJJI} = (\bar{\ell}_I \bar{\sigma}_\mu \ell_J)(\bar{\ell}_J \bar{\sigma}_\mu \ell_I)$
	$[O_{\ell e}]_{IIJJ} = (\bar{\ell}_I \bar{\sigma}_\mu \ell_I)(e_J^c \sigma_\mu \bar{e}_J^c)$
	$[O_{\ell e}]_{JJII} = (\bar{\ell}_J \bar{\sigma}_\mu \ell_J)(e_I^c \sigma_\mu \bar{e}_I^c)$
	$[O_{\ell e}]_{IJJI} = (\bar{\ell}_I \bar{\sigma}_\mu \ell_J)(e_J^c \sigma_\mu \bar{e}_I^c)$
$[O_{ee}]_{IIII} = \frac{1}{2}(e_I^c \sigma_\mu \bar{e}_I^c)(e_I^c \sigma_\mu \bar{e}_I^c)$	$[O_{ee}]_{IIJJ} = (e_I^c \sigma_\mu \bar{e}_I^c)(e_J^c \sigma_\mu \bar{e}_J^c)$

Table 1. The full set of lepton flavor conserving 4-lepton operators in the $D=6$ EFT Lagrangian.

Here, δm parametrizes the relative correction to the W boson mass that may arise in the presence of $D=6$ operators. By construction, there is no correction to the Z boson mass: a possible shift due to $D=6$ operators has been absorbed into the definition of the electroweak parameters g_L , g_Y and v . For the sake of our analysis we need to define the interactions of leptons with the SM gauge fields in the effective Lagrangian:

$$\begin{aligned} \mathcal{L}_{\text{eff}}^{v\ell\ell} = & -eA_\mu(\bar{e}_I \bar{\sigma}_\mu e_I + e_I^c \sigma_\mu \bar{e}_I^c) + \frac{g_L}{\sqrt{2}} \left[W_\mu^+ \bar{\nu}_I \bar{\sigma}_\mu (1 + \delta g_L^{WeI}) e_I + \text{h.c.} \right] + \sqrt{g_L^2 + g_Y^2} Z_\mu \\ & \times \left[\bar{\nu}_I \bar{\sigma}_\mu \left(\frac{1}{2} + \delta g_L^{ZeI} + \delta g_L^{W\ell I} \right) \nu_I + \bar{e}_I \bar{\sigma}_\mu \left(-\frac{1}{2} + s_\theta^2 + \delta g_L^{ZeI} \right) e_I + e_I^c \sigma_\mu \left(s_\theta^2 + \delta g_R^{ZeI} \right) \bar{e}_I^c \right], \end{aligned} \quad (2.3)$$

Here, the effects of $D = 6$ operators are parameterized by the vertex corrections δg . All δg 's in eq. (2.3) are independent parameters, which in general may depend on the lepton flavor. By construction, there is no vertex corrections to photon interactions. The parameters δg can be related by a linear transformation to Wilson coefficients of $D=6$ operators in any particular basis, see ref. [34] for a map to popular bases used in the literature. Therefore, δg 's are $\mathcal{O}(\Lambda^{-2})$ in the EFT expansion. Note that the vertex corrections to neutrino interactions with Z in eq. (2.3) are expressed by the other vertex corrections: $\delta g_L^{Z\nu I} = \delta g_L^{ZeI} + \delta g_L^{WeI}$. This relation is a consequence of the linearly realized SM gauge symmetry and the absence of operators with $D > 6$ in the Lagrangian, and holds independently of the basis of $D=6$ operators employed in eq. (2.1).

The main focus of this paper is on the lepton-flavor conserving 4-lepton operators in eq. (2.1) summarized in table 1. Overall, there is $3 \times 3 + 3 \times 6 = 27$ such operators. Three of those, denoted $[O_{\ell e}]_{IJJI}$, are complex, in which case the corresponding Wilson coefficient is complex, and the Hermitian conjugate operator is included in eq. (2.1). The goal of this paper is to derive simultaneous constraints on the Wilson coefficients of (as many as possible) 4-lepton operators and the leptonic vertex corrections in eq. (2.3). In our framework, the remaining parameter introduced above - the W mass correction δm in eq. (2.2) - is related to the leptonic vertex corrections and one 4-lepton operators [34]:

$$\delta m = \frac{\delta g_L^{We} + \delta g_L^{W\mu}}{2} - \frac{[c_{\ell\ell}]_{1221}}{4}. \quad (2.4)$$

Again, this relation is a consequence of the linearly realized SM gauge symmetry and the absence of operators with dimensions greater than 6. It also ensures that the Fermi

constant G_F measured in muon decays is given at tree-level by $G_F = 1/\sqrt{2}v^2$. This way, the tree-level relations between the electroweak parameters g_L , g_Y and v and the input observables α_{em} , m_Z and G_F are the same as in the SM.

3 Experimental input

In this section we discuss observables that will allow us to place constraints on 4-lepton operators and leptonic vertex corrections.

3.1 Z- and W-pole observables

The leptonic vertex corrections in eq. (2.3) can be probed by measurements of leptonic decays of on-shell Z and W bosons. Precise measurements of observables ultimately related to various Z and W partial decay widths were performed in LEP-1 (Z) [35] and LEP-2 (W) [36]. The dependence of these observables on leptonic and quark vertex corrections is correlated. On the other hand, the dependence on four-fermion operators is suppressed by Γ_V/m_V and can be neglected [2]. Simultaneous constraints on all flavor-preserving vertex corrections were derived recently in ref. [21], and we use directly these results. Marginalizing the likelihood over the quark vertex corrections, the constraints on the leptonic ones are given by:

$$\begin{pmatrix} \delta g_L^{We} \\ \delta g_L^{W\mu} \\ \delta g_L^{W\tau} \\ \delta g_L^{Ze} \\ \delta g_L^{Z\mu} \\ \delta g_L^{Z\tau} \\ \delta g_R^{Ze} \\ \delta g_R^{Z\mu} \\ \delta g_R^{Z\tau} \end{pmatrix} = \begin{pmatrix} -1.00 \pm 0.64 \\ -1.36 \pm 0.59 \\ 1.95 \pm 0.79 \\ -0.026 \pm 0.028 \\ 0.01 \pm 0.11 \\ 0.016 \pm 0.058 \\ -0.037 \pm 0.027 \\ 0.00 \pm 0.13 \\ 0.039 \pm 0.062 \end{pmatrix} \times 10^{-2}, \quad (3.1)$$

with the correlation matrix

$$\rho = \begin{pmatrix} 1. & -0.12 & -0.63 & -0.1 & -0.03 & 0.01 & 0.07 & -0.06 & -0.04 \\ . & 1. & -0.56 & -0.11 & -0.04 & 0.01 & 0.08 & -0.06 & -0.04 \\ . & . & 1. & -0.1 & -0.03 & 0.01 & 0.07 & -0.05 & -0.04 \\ . & . & . & 1. & -0.1 & -0.07 & 0.17 & -0.05 & 0.03 \\ . & . & . & . & 1. & 0.07 & -0.06 & 0.9 & -0.04 \\ . & . & . & . & . & 1. & 0.02 & -0.03 & 0.41 \\ . & . & . & . & . & . & 1. & -0.08 & -0.04 \\ . & . & . & . & . & . & . & 1. & 0.04 \\ . & . & . & . & . & . & . & . & 1. \end{pmatrix}. \quad (3.2)$$

Note that *all* leptonic vertex corrections are strongly constrained by the data in a model-independent way. In particular, the constraints on charged leptons couplings to Z (dominated by LEP-1) are at a per-mille level, while the constraints on lepton couplings to W (dominated by LEP-2) are at a percent level.

3.2 W mass

The W boson mass was measured very precisely at LEP-2 and the Tevatron. We use the result from ref. [37], $m_W = (80.385 \pm 0.015) \text{ GeV}$, where the SM prediction is $m_W = 80.364 \text{ GeV}$. This trivially translates into the constraint on the parameter δm in eq. (2.2),

$$\delta m = (2.6 \pm 1.9) \times 10^{-4}. \quad (3.3)$$

By virtue of eq. (2.4), this result constrains a combination of leptonic vertex corrections and one four-lepton operator.

3.3 Fermion pair production in LEP-2

The LEP-2 experiment measured differential cross sections for the processes $e^+e^- \rightarrow \ell^+\ell^-$, $\ell = e, \mu, \tau$ at energies above the Z boson resonance. Away from the Z-pole, these processes probe not only Z couplings to leptons but also 4-lepton operators, and the effect of the latter increases with increasing center-of-mass energy.

Let us first focus on the processes $e^-e^+ \rightarrow \mu^-\mu^+$ ($e^-e^+ \rightarrow \tau^-\tau^+$ is analogous). For the experimental input, we will use the total cross-sections and forward-backward asymmetries measured at 12 different center-of-mass energies between $\sqrt{s} \approx 130 \text{ GeV}$ and $\sqrt{s} \approx 209 \text{ GeV}$ [36]. We are interested in $\mathcal{O}(\Lambda^{-2})$ corrections to these observables from $D=6$ operators, which translates to linear corrections in the vertex corrections and Wilson coefficients of 4-fermion operators (i.e. the interference term between SM and new physics). At that order, the observables are affected by 5 four-leptons operators $[O_{\ell\ell}]_{1122}$, $[O_{\ell\ell}]_{1221}$, $[O_{ee}]_{1122}$, $[O_{\ell e}]_{1122}$, and $[O_{\ell e}]_{2211}$. In the limit of vanishing fermion masses, their effect on the forward (σ_F) and backward (σ_B) $e^-e^+ \rightarrow \mu^-\mu^+$ cross sections is given by

$$\begin{aligned} \delta(\sigma_F + \sigma_B) &= \frac{1}{24\pi v^2} \left\{ e^2 \left([c_{\ell\ell}]_{1122} + [c_{\ell\ell}]_{1221} + [c_{ee}]_{1122} + [c_{\ell e}]_{1122} + [c_{\ell e}]_{2211} \right) \right. \\ &\quad \left. + \frac{s(g_L^2 + g_Y^2)}{s - m_Z^2} \left[(g_{L,\text{SM}}^Z)^2 \left([c_{\ell\ell}]_{1122} + [c_{\ell\ell}]_{1221} \right) + (g_{R,\text{SM}}^Z)^2 [c_{ee}]_{1122} + g_{L,\text{SM}}^Z g_{R,\text{SM}}^Z \left([c_{\ell e}]_{1122} + [c_{\ell e}]_{2211} \right) \right] \right\}, \\ \delta(\sigma_F - \sigma_B) &= \frac{1}{32\pi v^2} \left\{ e^2 \left([c_{\ell\ell}]_{1122} + [c_{\ell\ell}]_{1221} + [c_{ee}]_{1122} - [c_{\ell e}]_{1122} - [c_{\ell e}]_{2211} \right) \right. \\ &\quad \left. + \frac{s(g_L^2 + g_Y^2)}{s - m_Z^2} \left[(g_{L,\text{SM}}^Z)^2 \left([c_{\ell\ell}]_{1122} + [c_{\ell\ell}]_{1221} \right) + (g_{R,\text{SM}}^Z)^2 [c_{ee}]_{1122} - g_{L,\text{SM}}^Z g_{R,\text{SM}}^Z \left([c_{\ell e}]_{1122} + [c_{\ell e}]_{2211} \right) \right] \right\}, \end{aligned} \quad (3.4)$$

where $g_{L,\text{SM}}^Z = -\frac{1}{2} + s_\theta^2$, $g_{R,\text{SM}}^Z = s_\theta^2$ are the couplings of the Z to left- and right-handed electrons. The effect of the vertex corrections δg_L^Z , δg_R^Z , $\delta g_L^{Z\mu}$, and $\delta g_R^{Z\mu}$ is also taken into account in the fit, but is not displayed here. The operator $[O_{\ell e}]_{1221}$ does not interfere with the SM due to the different helicity structure; thus it enters only at the quadratic ($\mathcal{O}(\Lambda^{-4})$) level and is neglected in this analysis.

One observes that measurements of the total cross section and asymmetry in $e^-e^+ \rightarrow \mu^-\mu^+$ in principle can constrain 3 linear combinations of the 5 four-lepton operators that

enter in eq. (3.4). $[O_{\ell\ell}]_{1122}$ and $[O_{\ell\ell}]_{1221}$ are indistinguishable for this process because their parts involving charged leptons are related by a Fierz transformation. $[O_{\ell e}]_{1122}$ and $[O_{\ell e}]_{2211}$ are also indistinguishable in this process, which can be traced to lepton flavor universality of the SM couplings. Accidentally, the LEP-2 observables depend very weakly on the combination $[O_{\ell\ell}]_{1122} + [O_{\ell\ell}]_{1221} - [O_{ee}]_{1122}$ due to the fact that, numerically, $(g_{L,SM}^{Z_e})^2 \approx (g_{R,SM}^{Z_e})^2$.

We move to the process $e^-e^+ \rightarrow e^-e^+$ (Bhabha scattering). In ref. [36], LEP-2 quotes the differential cross sections for the scattering angle $\cos\theta$ in the interval $[-0.9, 0.9]$, and the center-of-mass energies from 189 GeV to 207 GeV. Bhabha scattering is affected by the three four-leptons operators $[O_{\ell\ell}]_{1111}$, $[O_{ee}]_{1111}$ and $[O_{\ell e}]_{1111}$. In the limit of vanishing fermion masses their effect on the differential cross section is given by

$$\delta \frac{d\sigma}{d\cos\theta} = \frac{1}{8\pi s} \frac{1}{v^2} \left\{ u^2 \left[e^2 ([c_{\ell\ell}]_{1111} + [c_{ee}]_{1111}) \left(\frac{1}{s} + \frac{1}{t} \right) + (g_L^2 + g_Y^2) \left((g_{L,SM}^{Z_e})^2 [c_{\ell\ell}]_{1111} + (g_{R,SM}^{Z_e})^2 [c_{ee}]_{1111} \right) \left(\frac{1}{s-m_Z^2} + \frac{1}{t-m_Z^2} \right) \right] + t^2 \left[[c_{\ell e}]_{1111} \frac{e^2}{s} + [c_{\ell e}]_{1111} \frac{(g_L^2 + g_Y^2) g_{L,SM}^{Z_e} g_{R,SM}^{Z_e}}{s-m_Z^2} \right] + s^2 \left[[c_{\ell e}]_{1111} \frac{e^2}{t} + [c_{\ell e}]_{1111} \frac{(g_L^2 + g_Y^2) g_{L,SM}^{Z_e} g_{R,SM}^{Z_e}}{t-m_Z^2} \right] \right\}, \quad (3.5)$$

where $t = -\frac{s}{2}(1 - \cos\theta)$ and $u = -\frac{s}{2}(1 + \cos\theta)$. Again, the dependence on the vertex corrections $\delta g_L^{Z_e}$, $\delta g_R^{Z_e}$ is taken into account in our analysis but not displayed here. In principle, Bhabha scattering at LEP-2 constrains independently all 3 four-electron operators, but again an approximate flat direction along the direction $[O_{\ell\ell}]_{1111} - [O_{ee}]_{1111}$ arises due to the numerical accident $(g_{L,SM}^{Z_e})^2 \approx (g_{R,SM}^{Z_e})^2$.

3.4 Low-energy neutrino scattering

Interactions of SM leptons can be probed by neutrino scattering on electrons. We focus on processes with muon neutrinos: $\nu_\mu e^- \rightarrow \nu_\mu e^-$, and muon anti-neutrinos: $\bar{\nu}_\mu e^- \rightarrow \bar{\nu}_\mu e^-$, which were studied at center-of-mass energies far below the Z-pole by the CHARM [38], CHARM-II [39], and BNL-734 [40] experiments. The results are usually presented as constraints on the vector (g_V) and axial (g_A) coupling strength of the Z boson to electrons:

Experiment	Ref.	g_V	g_A
CHARM-II	[39]	-0.035 ± 0.017	-0.503 ± 0.0017
CHARM	[38]	-0.06 ± 0.07	-0.54 ± 0.07
BNL-E734	[40]	-0.107 ± 0.045	-0.514 ± 0.036

(3.6)

where the SM predicts $g_V = -0.0396$, $g_A = -0.5064$ [41].

In the presence of $D=6$ operators, the scattering cross sections measured in these experiments are sensitive not only to the Z boson couplings but also to the four-leptons operators involving the 2nd generation doublet: $[O_{\ell\ell}]_{1122}$ and $[O_{\ell e}]_{2211}$. Nevertheless, at energies below the Z-pole, the measurements of g_V and g_A can be easily recast as constraints

on the parameters in our framework. At the linear level, the vector and axial couplings are *effectively* modified as

$$\begin{aligned}\delta g_V &= \delta g_L^{Ze} + \delta g_R^{Ze} + \frac{3g_Y^2 - g_L^2}{g_L^2 + g_Y^2} \left(\delta g_L^{Z\mu} + \delta g_L^{W\mu} \right) - \frac{[c_{\ell\ell}]_{1122} + [c_{\ell e}]_{2211}}{2}, \\ \delta g_A &= \delta g_L^{Ze} - \delta g_R^{Ze} - \left(\delta g_L^{Z\mu} + \delta g_L^{W\mu} \right) - \frac{[c_{\ell\ell}]_{1122} - [c_{\ell e}]_{2211}}{2}.\end{aligned}\tag{3.7}$$

Notice that the dependence on the four-lepton operators is different than for the LEP-2 observables discussed in the previous subsection. Therefore, low-energy neutrino scattering provides us with complementary information that will allow us to constrain additional directions in the space of $D=6$ Wilson coefficients.

Experimental results on low-energy scattering of *electron* neutrinos [42] and anti-neutrinos [43] on electrons are also available. These probe the 4-electron operators $[O_{\ell\ell}]_{1111}$ and $[O_{\ell e}]_{1111}$. However, the current experimental accuracy is worse than for muon neutrinos scattering, and including this additional input would not affect the global fit in an appreciable way.

3.5 Parity violating electron scattering

The SLAC E158 experiment made a precise measurement of parity-violating asymmetry in Møller scattering $e^-e^- \rightarrow e^-e^-$ [44]. The asymmetry is defined as $A_{PV} = (\sigma_R - \sigma_L)/(\sigma_R + \sigma_L)$ where $\sigma_{L(R)}$ is the cross-section for incident left- (right-) handed electrons. The E158 experiment used a polarized electron beam of energy $E \approx 50$ GeV against an electron target at rest which corresponds to a center-of-mass energy of $\sqrt{s} \approx \sqrt{2m_e E} \approx 0.2$ GeV, far below the Z pole. The results are presented as a measurement of the weak mixing angle at low energies:

$$s_\theta^2(Q^2 = 0.026\text{GeV}^2) = 0.2397 \pm 0.0013,\tag{3.8}$$

where the SM predicts $s_\theta^2(Q^2 = 0.026\text{GeV}^2) = 0.2381 \pm 0.0006$ [45].

A_{PV} in Møller scattering is sensitive to the four-electron operators $[O_{ee}]_{1111}$ and $[O_{\ell\ell}]_{1111}$ ($[O_{\ell e}]_{1111}$ cancels out in $\sigma_R - \sigma_L$). At the linear order in the EFT parameters and leading order in s/m_Z^2 , the effect of these operators and the vertex corrections can be effectively represented as a shift of the measured weak mixing angle:

$$\delta s_\theta^2 = 2(g_{R,SM}^{Ze} \delta g_R^{Ze} - g_{L,SM}^{Ze} \delta g_L^{Ze}) - \frac{1}{4}([c_{ee}]_{1111} - [c_{\ell\ell}]_{1111})\tag{3.9}$$

Although Møller scattering probes the same 4-electron operators as LEP-2, cf. eq. (3.5), its importance rests in the sensitivity to the combination that is accidentally very weakly constrained by unpolarized electron scattering in LEP-2.

3.6 Tau and muon decays

The leptonic tau decays $\tau^- \rightarrow e^- \nu_\tau \bar{\nu}_e$, $\tau^- \rightarrow \mu^- \nu_\tau \bar{\nu}_\mu$, and the conjugates provide additional information on 4-lepton operators involving τ . In particular, they provide the only constraint

we are aware of on lepton-flavor conserving 4-lepton operators with muons and taus. The decays can be described by the following effective Lagrangian:

$$\mathcal{L} = -\frac{4G_{\tau f}}{\sqrt{2}}(\bar{\nu}_\tau \bar{\sigma}_\rho \tau)(\bar{f} \sigma_\rho \nu_f) + \text{h.c.}, \quad (3.10)$$

where $f = e, \mu$. At the linear level, the relative strength of the Fermi constant measured in the tau decays normalized to that measured in the muon decay is affected by the vertex corrections and four-lepton operators as

$$\begin{aligned} A_e &\equiv \frac{G_{\tau e}^2}{G_F^2} = 1 + 2\delta g_L^{W\tau} + 2\delta g_L^{We} - 4\delta m - [c_{\ell\ell}]_{1331}, \\ A_\mu &\equiv \frac{G_{\tau\mu}^2}{G_F^2} = 1 + 2\delta g_L^{W\tau} + 2\delta g_L^{W\mu} - 4\delta m - [c_{\ell\ell}]_{2332}, \end{aligned} \quad (3.11)$$

where the W mass corrections δm can be expressed by other EFT parameters, cf. eq. (3.3). The experimental values quoted by the PDG are [46]

$$\begin{aligned} A_e &= 1.0029 \pm 0.0046, \\ A_\mu &= 0.981 \pm 0.018, \end{aligned} \quad (3.12)$$

and the SM prediction is $A_f = 1$.

For the muon decay, $\mu^- \rightarrow e^- \nu_\mu \bar{\nu}_e$ and the conjugate, the total rate defines the SM input parameter v and by itself it does not probe new physics. However, additional information can be extracted from differential distributions in (polarized) muon decay. Customarily, these measurements are presented in the language of *Michel parameters* [47]. From the EFT perspective the most interesting are the so-called η and β'/A parameters, because they are the only ones that may receive contributions at $\mathcal{O}(1/\Lambda^2)$ [48, 49]:

$$\eta = \frac{\text{Re}[c_{\ell e}]_{1221}}{2}, \quad \beta'/A = -\frac{\text{Im}[c_{\ell e}]_{1221}}{4}. \quad (3.13)$$

These parameters have been measured in an experiment in the PSI [50]:

$$\eta = -0.0021 \pm 0.0071, \quad \beta'/A = -0.0013 \pm 0.0036. \quad (3.14)$$

Analogous limits from tau decays are much weaker.

4 General fit

We now do a global fit to all the data discussed above so as to simultaneously constrain $D=6$ operators in the EFT Lagrangian that give rise to leptonic vertex corrections and 4-lepton interactions. Previously, constraints on 4-lepton (and other 4-fermion) operators were obtained in refs. [2, 3] and recently updated in ref. [28], assuming the Wilson coefficients are the same for all 3 fermion generations. The novel aspect of our analysis is that we allow for a completely general flavor structure of the $D=6$ operators.

We combine the following experimental inputs discussed in section 3:

- Z boson production and decay in LEP-1 and leptonic W decays in LEP-2,
- W mass measurement,
- Two-lepton production in LEP-2,
- Muon-neutrino scattering on electrons,
- Parity violation in low-energy Møller scattering,
- G_F measurements in τ decays.

We consider the EFT Lagrangian with operators up to $D=6$, neglecting possible contributions of $D=8$ operators.¹ Consistently, in our analysis we only include corrections to observables that are linear in Wilson coefficients of $D=6$ operators. These are formally $\mathcal{O}(v^2/\Lambda^2)$ in the EFT counting, and come from interference between tree-level SM and $D=6$ contributions to the relevant amplitudes. We also ignore loop-suppressed effects proportional to $D=6$ Wilson coefficients. We use the experimental results, the SM predictions, and the analytic expression for $D=6$ contributions discussed in section 3 to construct a global Gaussian likelihood in the space of the relevant Wilson coefficients. With this procedure,

¹In the EFT expansion, Wilson coefficients of $D=8$ operators are suppressed by another factor of v^2/Λ^2 compared to those of $D=6$ operators. Thus, they are generically subleading when the EFT approach is valid, that is when the new physics scale Λ is greater than the electroweak scale. Exceptions to that rule could occur if symmetries or fine-tuning in the UV theory lead to a suppression of some $D=6$ (but not the corresponding $D=8$ or higher) Wilson coefficients in the low-energy EFT. Our results are not valid in such situations; see ref. [28] for a discussion relevant to these cases.

we get the following global constraints:

$$\begin{pmatrix}
 \delta g_L^{We} \\
 \delta g_L^{W\mu} \\
 \delta g_L^{W\tau} \\
 \delta g_L^{Ze} \\
 \delta g_L^{Z\mu} \\
 \delta g_L^{Z\tau} \\
 \delta g_R^{Ze} \\
 \delta g_R^{Z\mu} \\
 \delta g_R^{Z\tau} \\
 [c_{\ell\ell}]_{1111} \\
 [c_{\ell e}]_{1111} \\
 [c_{ee}]_{1111} \\
 [c_{\ell\ell}]_{1221} \\
 [c_{\ell\ell}]_{1122} \\
 [c_{\ell e}]_{1122} \\
 [c_{\ell e}]_{2211} \\
 [c_{ee}]_{1122} \\
 [c_{\ell\ell}]_{1331} \\
 [c_{\ell\ell}]_{1133} \\
 [c_{\ell e}]_{1133} + [c_{\ell e}]_{3311} \\
 [c_{ee}]_{1133} \\
 [c_{\ell\ell}]_{2332}
 \end{pmatrix}
 =
 \begin{pmatrix}
 -1.00 \pm 0.64 \\
 -1.36 \pm 0.59 \\
 1.95 \pm 0.79 \\
 -0.027 \pm 0.028 \\
 0.01 \pm 0.11 \\
 0.016 \pm 0.058 \\
 -0.037 \pm 0.027 \\
 0.00 \pm 0.13 \\
 0.039 \pm 0.062 \\
 0.99 \pm 0.39 \\
 -0.23 \pm 0.22 \\
 0.23 \pm 0.39 \\
 -4.8 \pm 1.6 \\
 2.0 \pm 2.3 \\
 0.9 \pm 2.3 \\
 -0.8 \pm 2.2 \\
 2.8 \pm 2.8 \\
 1.5 \pm 1.3 \\
 140 \pm 170 \\
 -0.55 \pm 0.64 \\
 -150 \pm 180 \\
 3.0 \pm 2.3
 \end{pmatrix}
 \times 10^{-2}, \tag{4.1}$$

with the correlation matrix written down in eq. (C.1).

A few general comments are in order:

- In the global fit, the constraints on the leptonic vertex corrections are the same as the ones in eq. (3.1) determined from on-shell Z and W data. The additional experimental input considered in this analysis constrains 4-lepton operators without affecting the limits on the vertex corrections in an appreciable way. Nevertheless, the correlations between vertex corrections and 4-lepton operators are non-negligible in some cases, as can be observed in eq. (C.1).
- Not all 4-lepton operators can be constrained by the current data. In particular, we are not aware of any experiments probing four-muon or four-tau interactions. On the other hand, most of the Wilson coefficients of 4-lepton operators involving electrons are constrained, in a model-independent way, at a percent level accuracy.

- In eq. (4.1), the limits on the electron-tau 4-fermion operators $[O_{\ell\ell}]_{1133}$ and $[O_{ee}]_{1133}$ are very weak. Actually, the combination $[O_{\ell\ell}]_{1133} + [O_{ee}]_{1133}$ is constrained at a percent level. However, τ -pair production in LEP-2 is accidentally insensitive to $[O_{\ell\ell}]_{1133} - [O_{ee}]_{1133}$, as discussed in section 3.3, and this is reflected in our fit by the -1 value of the corresponding correlation coefficient. Moreover, only the sum $[O_{\ell e}]_{1133} + [O_{\ell e}]_{3311}$ can be probed in LEP-2. We note that both flat directions would be absent if polarization of the colliding electrons was known. Measurements with polarized e^\pm beams in future linear colliders [51, 52] will provide additional information that will break these degeneracies and greatly improve model-independent constraints on electron-tau 4-fermion operators. For electron-muon operators the corresponding flat direction is absent in eq. (4.1) thanks to including the experimental input from muon neutrino scattering on electrons. From this point of view, it would be extremely interesting to study tau neutrino scattering on electrons, although we are not aware of any realistic experimental plans in this direction. For 4-electron operators, the direction $[O_{\ell\ell}]_{1111} - [O_{ee}]_{1111}$ is also practically unconstrained by LEP-2, but in this case the degeneracy is lifted thanks to parity violating Møller scattering.
- In eq. (4.1) we do not give any constraints on $[c_{\ell e}]_{IJJ I}$ with $I \neq J$. That is because, in the limit $m_{e_I} = 0$, the corresponding operators do not interfere with the SM, thus they contribute to the observables at $\mathcal{O}(\Lambda^{-4})$ and are neglected. However, as discussed in section 3.6, they contribute at $\mathcal{O}(\Lambda^{-2})$ to the Michel parameters η and β'/A in $e_I \rightarrow e_J \nu \nu$ decays (which are in fact defined only for $m_{e_I} > 0$). The experimental limits on the Michel parameters measured in muon decays translate to

$$\text{Re}([c_{\ell e}]_{1221}) = (-0.4 \pm 1.4) \times 10^{-2}, \quad \text{Im}([c_{\ell e}]_{1221}) = (0.5 \pm 1.4) \times 10^{-2}. \quad (4.2)$$

One can also constrain the analogous operators with tau leptons. Translating the constraints on the form factor g_{RR}^S in leptonic tau decays [46] one obtains: $\text{Re}([c_{\ell e}]_{2332}) = 0.19 \pm 0.15$, and $|[c_{\ell e}]_{1331}| < 0.70$, $|[c_{\ell e}]_{2332}| < 0.72$ at 95% confidence level (CL). Stronger limits may arise via 1-loop contributions of these operators to anomalous electric and magnetic moments [53], however in this case the limits concern in fact for a linear combination of $[c_{\ell e}]_{IJJ I}$ and the Wilson coefficients of $D=6$ dipole operators in the effective Lagrangian.

- Combining the results from this paper with the ones in ref. [21] one could also perform a global analysis of leptonic parameters together with quark vertex corrections. However, correlations between 4-lepton operators and quark vertex corrections are small: we find that the correlation coefficients are typically of order 0.01, and the largest is 0.07. All in all, the constraints on the quark vertex corrections and their correlations with the leptonic vertex corrections quoted in [21] are not affected by the combination.

Using the results in eq. (4.1) and the correlation matrix ρ one can reconstruct the complete Gaussian likelihood function in the space of leptonic vertex corrections and four-lepton operators:

$$\chi^2 = \sum_{ij} [x - x_0]_i \sigma_{ij}^{-2} [x - \delta x_0]_j, \quad (4.3)$$

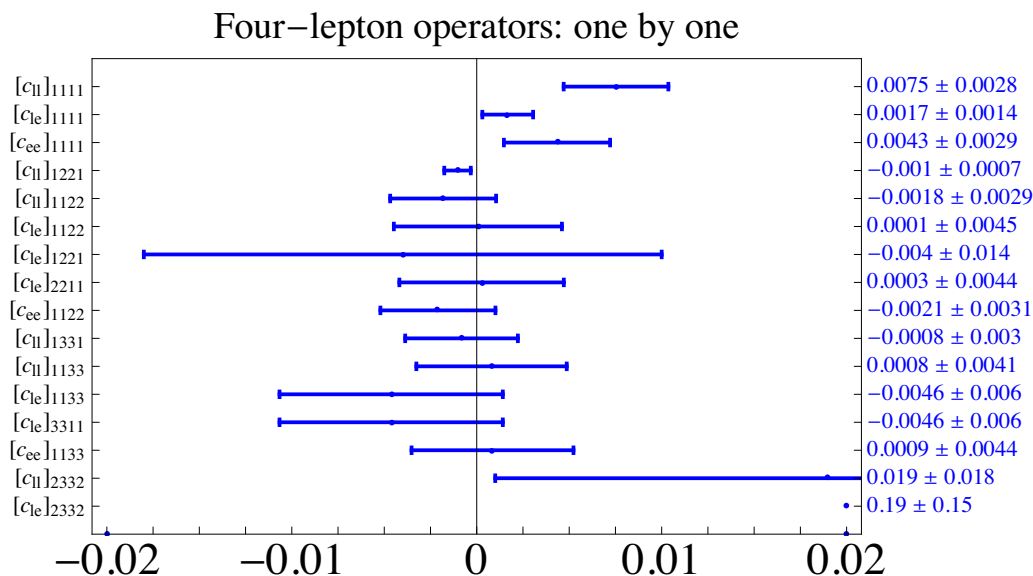


Figure 1. Summary of the 68% CL intervals for the Wilson coefficients of 4-lepton operators, assuming that only 1 such operator is present at a time and that leptonic vertex corrections are absent.

where $\sigma_{ij}^{-2} = [[\Delta x]_i \rho_{ij} [\Delta x]_j]^{-1}$, \vec{x} is a 14-dimensional vector collecting the δg 's and c 's as in eq. (4.1), and \vec{x}_0 , $\Delta \vec{x}$ are the corresponding central values and 1σ errors. In specific extensions of the SM only a subset of the general EFT parameters will be generated. In such a case, constraints on the model parameters can be obtained by restricting the full likelihood to the smaller subspace, and then minimizing the restricted likelihood. In the next section we perform this procedure for a handful of scenarios beyond the SM that affect only leptonic observables.

5 Leptophilic models

5.1 One by one

Before we attack specific models, we first discuss a general scenario where only one four-lepton operator and no vertex corrections is generated by new physics. Setting all but one Wilson coefficient to zero in the likelihood in eq. (4.3), and then minimizing the resulting function we obtain the constraints summarized in figure 1. The strongest constraint, at a per-mille level, is the one on $[c_{\ell\ell}]_{1221}$. The reason is that the corresponding operator affects the measurement of the Fermi constant G_F in muon decays, and this way, unlike other 4-fermion operators, it affects the electroweak precision observables very accurately measured in LEP-1. The constraints on the remaining operators containing electrons are dominated by lepton pair production in LEP-2 and are somewhat weaker. Finally, the muon-tau four-lepton operators are only weakly constrained by tau decays.

We also visualize these constraint in terms of the new physics scale probed by each operator. To this end, we write $c_i = \pm g_*^2 / \Lambda_i^2$, where Λ_i can be interpreted as the mass scale

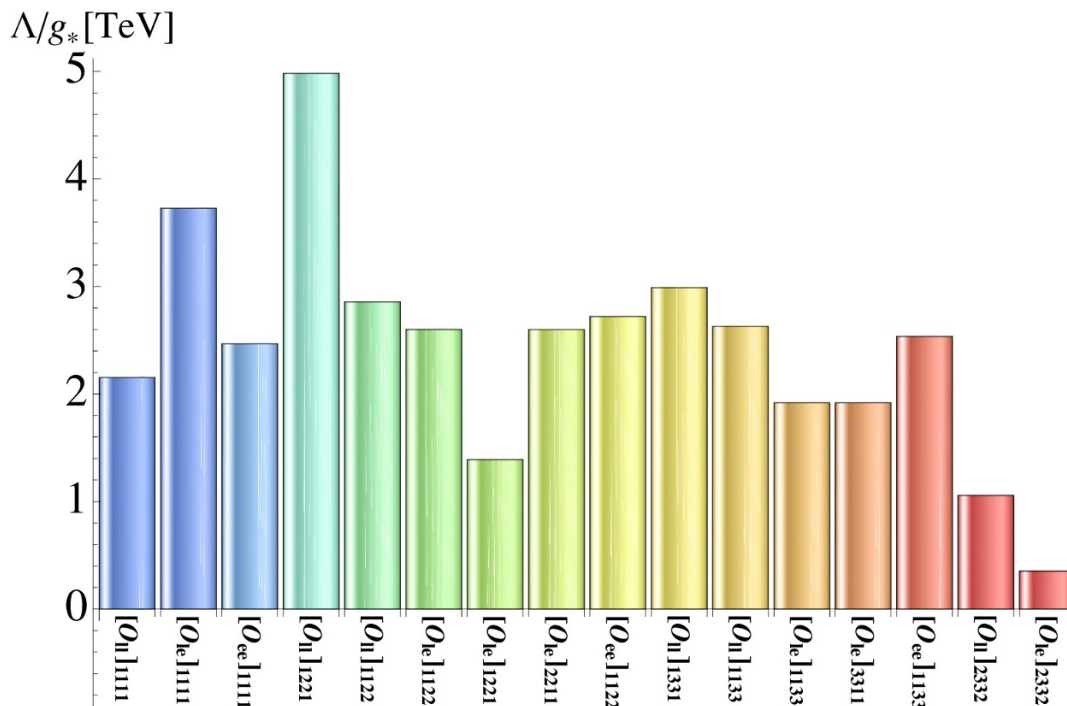


Figure 2. 95% CL lower limits on the scale suppressing 4-lepton operators in the EFT Lagrangian, assuming that only 1 such operator is present at a time and that leptonic vertex corrections are absent.

of new particles and g_* their coupling strength. Then we derive the 95% CL lower limit on the ratio Λ_i/g_* . In general, the limit depends on the sign of the Wilson coefficient, and for our presentation we always choose the lower one of the two possibilities. The results are shown in figure 2. Current data allow one to probe new particles with masses up to 5 TeV if they are coupled to the SM with order one strength.

5.2 Z prime

We now consider a model with a new neutral vector boson V_μ of mass m_V coupled to leptons as

$$\mathcal{L} \supset V_\mu \left(\kappa_{L,I} \bar{\ell}_I \bar{\sigma}_\mu \ell_I + \kappa_{R,I} e_I^c \sigma_\mu \bar{e}_I^c \right) \quad (5.1)$$

We assume the vector does not mix with the Z-boson, and does not couple to quarks. In such a case, it can be constrained neither by Z-pole observables nor by LHC dilepton resonance searches. However, it is constrained by the off-Z-pole observables discussed here. Integrating out the vector we get an effective theory with the Wilson coefficient of 4-lepton operators:

$$[c_{\ell\ell}]_{IIJJ} = -\kappa_{L,I} \kappa_{L,J} \frac{v^2}{m_V^2},$$

$$[c_{ee}]_{IIJJ} = -\kappa_{R,I} \kappa_{R,J} \frac{v^2}{m_V^2},$$

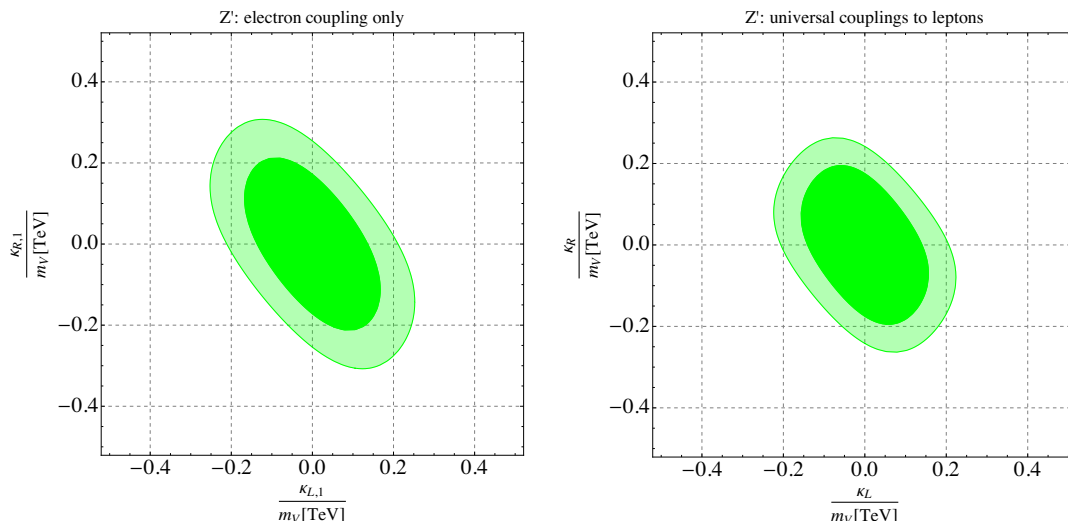


Figure 3. *Left:* 68 % CL (darker green) and 95 % CL (lighter green) regions for the ratio of the couplings over mass for the leptophilic Z' vector boson coupled to electrons only. *Right:* the same for Z' coupled universally to all leptons.

$$[c_{\ell e}]_{IIJJ} = -\kappa_{L,I}\kappa_{R,J}\frac{v^2}{m_V^2}. \quad (5.2)$$

Plugging these expressions in the general likelihood in eq. (4.3) we obtain the likelihood as a function of κ/m_V . In figure 3 we show examples of this likelihood for 2 scenarios: one where the vector couples to electrons only, and another one where the vector couples universally to all leptons, $\kappa_{L/R,I} = \kappa_{L/R}$. In both cases, we find that $\kappa/m_V \lesssim 0.1-0.3/\text{TeV}$, depending on the ratio of the left- and right-handed couplings. We can also observe that the vector-like couplings, $\kappa_L \approx \kappa_R$, are more strongly constrained than the axial ones, $\kappa_L \approx -\kappa_R$.

5.3 Vector triplet

Consider a model with a new triplet of vector bosons V_μ^i of mass m_V coupled to left-handed leptons as

$$\mathcal{L} \supset \kappa_I V_\mu^i \bar{\ell}_I \bar{\sigma}_\mu \frac{\sigma^i}{2} \ell_I. \quad (5.3)$$

Integrating out the triplet we get the following 4-lepton operators in the EFT:

$$\mathcal{L}_{\text{eff}} \supset -\frac{\kappa_I \kappa_J}{8m_V^2} (\bar{\ell}_I \bar{\sigma}_\mu \sigma^i \ell_I) (\bar{\ell}_J \bar{\sigma}_\mu \sigma^i \ell_J). \quad (5.4)$$

These operators were not introduced previously. The reason is that they are related to other 4-lepton operators in table 1 via Fierz transformations. Using, the identity

$$(\bar{\ell}_I \bar{\sigma}_\mu \sigma^i \ell_I) (\bar{\ell}_J \bar{\sigma}_\mu \sigma^i \ell_J) = 2(\bar{\ell}_I \bar{\sigma}_\mu \ell_J) (\bar{\ell}_J \bar{\sigma}_\mu \ell_I) - (\bar{\ell}_I \bar{\sigma}_\mu \ell_I) (\bar{\ell}_J \bar{\sigma}_\mu \ell_J), \quad (5.5)$$

one identifies the Wilson coefficients in the low-energy EFT as

$$[c_{\ell\ell}]_{IIII} = -\kappa_I^2 \frac{v^2}{4m_V^2},$$

$$\begin{aligned}
 [c_{\ell\ell}]_{IJJJ} &= -\kappa_I \kappa_J \frac{v^2}{2m_V^2}, & I < J, \\
 [c_{\ell\ell}]_{IIJJ} &= \kappa_I \kappa_J \frac{v^2}{4m_V^2}, & I < J.
 \end{aligned}
 \tag{5.6}$$

When the triplet couples to electrons only, $\kappa_1 \neq 0$ and $\kappa_{2,3} = 0$, only one four-lepton operator $[O_{\ell\ell}]_{1111}$ is generated. Plugging the above expression in the general likelihood in eq. (4.3) one obtains the following bound on the ratio of the vector mass and coupling:

$$\frac{m_V}{\kappa_1} \geq 2.9 \text{ TeV}, \quad @ 95\% \text{ CL.}
 \tag{5.7}$$

The limit is stronger than what might be inferred from the one-by-one limits plotted in figure 2, because the model predicts the negative sign of the Wilson coefficient $[c_{\ell\ell}]_{1111}$, for which the experimental constraints are stronger than for the positive one. On the other hand, when the triplet couples with the same strength to all leptons, $\kappa_I = \kappa$, we find a slightly stronger bound:

$$\frac{m_V}{\kappa} \geq 3.7 \text{ TeV}, \quad @ 95\% \text{ CL.}
 \tag{5.8}$$

5.4 Inert Higgs

The last example we study is a model with a scalar S of mass m_S transforming, much like the SM Higgs, as $2_{1/2}$ under $SU(2)_L \times U(1)_Y$, and coupled to leptons as

$$\mathcal{L} \supset -S^\dagger Y_I \ell_I e_I^c + \text{h.c.}
 \tag{5.9}$$

We assume that S does not get a VEV. Integrating out the scalar we get the following 4-lepton operators in the EFT:

$$\mathcal{L}_{\text{eff}} \supset \frac{Y_I^* Y_J}{m_S^2} (\bar{\ell}_I \bar{e}_I^c) (\ell_J e_J^c).
 \tag{5.10}$$

Once again these operators do not appear in table 1, but using the Fierz transformation, $\bar{\sigma}_\mu^{\dot{\alpha}\alpha} \sigma_\mu^{\beta\dot{\beta}} = 2\delta_\beta^\alpha \delta_{\dot{\beta}}^{\dot{\alpha}}$ we can rewrite them as

$$\mathcal{L}_{\text{eff}} \supset \frac{Y_I^* Y_J}{2m_S^2} (\bar{\ell}_I \bar{\sigma}_\mu \ell_J) (e_J^c \sigma_\mu \bar{e}_I^c).
 \tag{5.11}$$

This way, we identify the Wilson coefficients of the 4-lepton operators induced in the EFT by integrating out the scalar S :

$$[c_{\ell e}]_{IJJJ} = Y_I^* Y_J \frac{v^2}{2m_S^2}.
 \tag{5.12}$$

When the scalar couples to electrons only, $Y_1 \neq 0$ and $Y_2 = Y_3 = 0$, its mass over coupling is constrained as

$$\frac{m_S}{|Y_1|} \geq 2.6 \text{ TeV}, \quad @ 95\% \text{ CL.}
 \tag{5.13}$$

When the scalar couples to all 3 generations of leptons then the constraints on the Michel parameters discussed around eq. (4.2) can be relevant. If the couplings Y_I are the same

for all 3 generations, $Y_I = Y$, then these constraints have a small effect, and the bound $m_S/|Y|$ is the same as in eq. (5.13). However, if Y_I are proportional to fermion's mass, $Y_I = Y_3 m_{e_I}/m_\tau$, then the constraint on $[c_{\ell e}]_{2332}$ from tau decays is the dominant one, leading to $m_S/|Y_3| > 60 \text{ GeV}$ at 95% CL.

6 Conclusions

In this paper we discussed constraints on 4-lepton $D=6$ operators in the EFT beyond the SM. For the first time, the analysis was performed without any assumptions about the flavor structure of the $D=6$ operators. We presented our results such that they can readily be recast as constraints on specific models beyond the SM that, after integrating the new heavy particles, lead to leptonic vertex corrections and 4-lepton operators in the low-energy EFT. Our results are particularly relevant in relation to models where lepton flavor universality is not preserved.

We find that the typical current experimental sensitivity to the scale suppressing 4-lepton operators is of order a few TeV. In the best case, one can probe 50 TeV particles provided they interact strongly with the SM leptons ($g_* \sim 4\pi$), and they generate the best constrained operator $[O_{\ell\ell}]_{1221}$ (e.g. via exchange of an $SU(2)_L$ triplet of vector bosons). Our analysis also reveals several blind spots where the current sensitivity is weaker, which would be interesting targets for future experiments. In particular, certain linear combinations of 4-lepton operators involving electrons and taus are very weakly constrained. This can be cured by future e^+e^- colliders once polarization information about the initial state is available. Moreover, experimental information on 4-lepton operators involving muons and taus is currently very limited. Bounds on two such operators involving left-handed doublets could be improved by more precise measurements of the rate and differential distributions in $\tau \rightarrow \mu\nu\nu$ decays; probing the remaining operators would be one of the strong points of the physics program of a future $\mu^+\mu^-$ collider.

Including the available experimental information about $e^+e^- \rightarrow$ jets in LEP-2 and $pp \rightarrow$ leptons and jets at the LHC, as well as about low energy scattering on nuclei one can generalize this analysis so as to also constrain 4-fermion operators involving quarks. This is left for a future publication.

Acknowledgments

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A Relation to universal theories

In this paper we used the formalism where new physics effects in the $D=6$ EFT were represented by vertex corrections and 4-fermion operators. At the same time, the quadratic terms of electroweak gauge bosons in the Lagrangian were assumed to be the same as in the SM, except for a correction to the W boson mass. That can always be achieved without loss of generality, via field redefinitions and integration by parts. On the other hand, in

the literature, precision constraints on new physics are often expressed in the language of *oblique parameters*, such as the Peskin-Takeuchi S , T , U parameters [54]. These are, on the contrary, defined via corrections to kinetic terms of electroweak gauge bosons. In this appendix we discuss the relationship between the two formalisms.

Oblique parameters fully characterize the new physics effects for the so-called *universal* theories [4, 55]. The theory is universal if one can recast it to a form where new physics affects only propagators of the SM gauge bosons. Before introducing the oblique parameters, we first define the momentum expansion of the 2-point functions of electroweak gauge bosons:

$$\mathcal{M}(V_{1,\mu} \rightarrow V_{2,\nu}) = \eta_{\mu\nu} \left(\Pi_{V_1 V_2}^{(0)} + \Pi_{V_1 V_2}^{(2)} p^2 + \Pi_{V_1 V_2}^{(4)} p^4 + \dots \right) + p_\mu p_\nu (\dots), \quad (\text{A.1})$$

where p is the 4-momentum of the incoming gauge boson. We are interested in corrections $\delta\Pi_{V_1 V_2}^{(n)}$ with respect to the 2-point function in the SM. Two-point functions are not directly measurable, but certain combinations of $\delta\Pi_{V_1 V_2}$ affect measurable quantities. Up to order p^2 , the physical combinations are the 3 Peskin-Takeuchi oblique parameters:

$$\alpha S = -4 \frac{g_L g_Y}{g_L^2 + g_Y^2} \delta\Pi_{3B}^{(2)}, \quad \alpha T = \frac{\delta\Pi_{11}^{(0)} - \delta\Pi_{33}^{(0)}}{m_W^2}, \quad \alpha U = \frac{4g_Y^2}{g_L^2 + g_Y^2} \left(\delta\Pi_{11}^{(2)} - \delta\Pi_{33}^{(2)} \right). \quad (\text{A.2})$$

At order p^4 one can define [4] further oblique parameters:

$$\alpha V = m_W^2 \left(\delta\Pi_{11}^{(4)} - \delta\Pi_{33}^{(4)} \right), \quad \alpha W = -m_W^2 \delta\Pi_{33}^{(4)}, \quad \alpha X = -m_W^2 \delta\Pi_{3B}^{(4)}, \quad \alpha Y = -m_W^2 \delta\Pi_{BB}^{(4)}. \quad (\text{A.3})$$

Compared to ref. [4], we rescaled these parameters by $\alpha = e^2/4\pi$.

Now we want to relate the oblique parameters defined above to the vertex corrections and 4-fermion operators in the EFT Lagrangian of eq. (2.1). In universal theories, by definition, the new physics effects in the EFT Lagrangian can be represented by only bosonic operators with $D > 4$. These operators may lead to corrections to the gauge boson propagators, and one can relate their Wilson coefficients to the oblique parameters in eq. (A.2) and eq. (A.3). It turns out that, in an EFT with operators up to $D=6$, only the parameters S , T , W , Y can be generated at tree level [4]. Using field redefinitions and integration by parts we can get rid of the corrections to the gauge boson propagators, trading them for flavor diagonal vertex corrections and flavor conserving four-fermion operators as in our eq. (2.1). Completing this procedure, we obtain the following map:

$$\delta g^{Zf_I} = \alpha \left\{ T_f^3 \frac{T - W - \frac{g_Y^2}{g_L^2} Y}{2} + Q_f \frac{2g_Y^2 T - (g_L^2 + g_Y^2) S + 2g_Y^2 W + \frac{2g_Y^2 (2g_L^2 - g_Y^2) Y}{g_L^2}}{4(g_L^2 - g_Y^2)} \right\},$$

$$[c_{\ell\ell}]_{IIJJ} = \alpha \left[W - \frac{g_Y^2}{g_L^2} Y \right], \quad [c_{\ell\ell}]_{IJJ I} = -2\alpha W, \quad I < J,$$

$$[c_{\ell\ell}]_{IIII} = -\alpha \left[W + \frac{g_Y^2}{g_L^2} Y \right],$$

$$\begin{aligned}
 [c_{\ell e}]_{IIJJ} &= -\frac{2g_Y^2}{g_L^2}\alpha Y, & [c_{ee}]_{IIJJ} &= -\frac{4g_Y^2}{g_L^2}\alpha Y, \\
 [c'_{\ell q}]_{IIJJ} &= -\alpha W, \\
 [c_{\ell q}]_{IIJJ} &= \frac{g_Y^2}{3g_L^2}\alpha Y, & [c_{\ell u}]_{IIJJ} &= \frac{4g_Y^2}{3g_L^2}\alpha Y, & [c_{\ell d}]_{IIJJ} &= -\frac{2g_Y^2}{3g_L^2}\alpha Y, \\
 [c_{eq}]_{IIJJ} &= \frac{2g_Y^2}{3g_L^2}\alpha Y, & [c_{eu}]_{IIJJ} &= \frac{8g_Y^2}{3g_L^2}\alpha Y, & [c_{ed}]_{IIJJ} &= -\frac{4g_Y^2}{3g_L^2}\alpha Y. \quad (\text{A.4})
 \end{aligned}$$

Thus, the oblique parameters S , T , W , Y correspond to a special pattern of vertex corrections and 4-fermion operators [55]. In our language, the theory can be universal if the pattern of vertex corrections and 4-fermion operators can be matched to that in eq. (A.4). Note that both leptonic and quark operators are necessarily present in universal theories. Our formalism is more general and applies to a large class of models, as we don't need to make any assumptions about the pattern of vertex corrections or 4-fermion operators. None of the specific models discussed in section 5 is universal, and cannot be properly described by the oblique parameters.

The current constraints on the oblique parameters are

$$\begin{pmatrix} S \\ T \\ W \\ Y \end{pmatrix} = \begin{pmatrix} -0.04 \pm 0.11 \\ 0.05 \pm 0.08 \\ -0.09 \pm 0.08 \\ -0.04 \pm 0.09 \end{pmatrix}, \quad \rho = \begin{pmatrix} 1. & 0.88 & 0.57 & 0.08 \\ . & 1. & 0.3 & 0.04 \\ . & . & 1. & -0.4 \\ . & . & . & 1. \end{pmatrix}. \quad (\text{A.5})$$

These constraints are dominated by the leptonic observables discussed in this paper. To derive eq. (A.5), we also used the experimental input from the hadronic observables $e^+e^- \rightarrow q\bar{q}, b\bar{b}, c\bar{c}$ in LEP-1 and LEP-2 [35, 36] and atomic parity violation [56]. If, instead, we plugged in eq. (A.4) directly in the likelihood function of eq. (4.3), the result would be very similar as in eq. (A.5), up to a small $\mathcal{O}(0.05)$ shift of the central values. Setting $W = Y = 0$ one obtains the constraints on S and T alone: $S = 0.06 \pm 0.08$, $T = 0.09 \pm 0.07$ with the correlation coefficient $+0.92$, which is very close to the result in ref. [57] using the Z-pole and m_W measurements alone.

B Lepton flavor violating operators

Among four-lepton operators $[O_{\ell\ell}]_{IJKL}$, $[O_{ee}]_{IJKL}$, $[O_{\ell e}]_{IJKL}$ with general flavor indices there exist 66 complex ones that violate lepton flavor. Moreover, 9 complex lepton flavor violating vertex corrections $[\delta g_{L,R}^{Ze}]_{IJ}$, $[\delta g_L^{We}]_{IJ}$ with $I \neq J$ may arise from $D=6$ operators in the EFT. These operators do not interfere with the SM and thus, at the leading order, they do not affect the constraints on flavor conserving operators discussed in section 4. In this appendix, for completeness, we review experimental constraints on some lepton flavor violating vertex corrections and four-lepton operators. See also [53, 58, 59] for recent reviews.

B.1 From lepton flavor violating Z decays

Lepton flavor violating Z boson vertices can be probed by on-shell Z decays at LEP and the LHC, as recently discussed ref. [21]. The current experimental limits are:

Observable	95% CL limit	Ref.
$\text{Br}(Z \rightarrow e\mu)$	7.5×10^{-7}	[60]
$\text{Br}(Z \rightarrow e\tau)$	9.8×10^{-6}	[61]
$\text{Br}(Z \rightarrow \mu\tau)$	1.2×10^{-5}	[62]

At tree level, this translates to the following constraints on the vertex corrections:

$$\begin{aligned}
 \sqrt{|[\delta g_L^{Ze}]_{12}]^2 + |[\delta g_R^{Ze}]_{12}]^2} &< 1.2 \times 10^{-3}, \\
 \sqrt{|[\delta g_L^{Ze}]_{13}]^2 + |[\delta g_R^{Ze}]_{13}]^2} &< 4.3 \times 10^{-3}, \\
 \sqrt{|[\delta g_L^{Ze}]_{23}]^2 + |[\delta g_R^{Ze}]_{23}]^2} &< 4.8 \times 10^{-3}.
 \end{aligned}
 \tag{B.1}$$

B.2 From lepton flavor violating lepton decay

Searches for lepton flavor violating muon and tau decays have, so far, all given negative results and set tight constraints on lepton flavor violating operators. In what follows, we perform a tree-level computation, neglecting the masses of the daughter leptons.

The 90% CL constraints on the branching ratios given by PDG [46] are:

Decay mode	90% CL bound
$\mu^- \rightarrow e^- \bar{\nu}_\mu \nu_e$	1.2 %
$\mu^- \rightarrow e^- e^+ e^-$	1.0×10^{-12}
$\tau^- \rightarrow e^- e^+ e^-$	2.7×10^{-8}
$\tau^- \rightarrow \mu^- \mu^+ \mu^-$	2.1×10^{-8}
$\tau^- \rightarrow \mu^- e^+ e^-$	1.8×10^{-8}
$\tau^- \rightarrow e^- \mu^+ \mu^-$	2.7×10^{-8}
$\tau^- \rightarrow e^+ \mu^- \mu^-$	1.7×10^{-8}
$\tau^- \rightarrow \mu^+ e^- e^-$	1.5×10^{-8}

- $\mu^- \rightarrow e^- \bar{\nu}_\mu \nu_e$

This process can be induced by the operators $[O_{\ell\ell}]_{1212}$ and $[O_{\ell e}]_{1212}$. We get, at 90% CL:

$$\sqrt{4|[c_{\ell\ell}]_{1212}]^2 + |[c_{\ell e}]_{1212}]^2} < 0.219.
 \tag{B.3}$$

- $\mu^- \rightarrow e^- e^+ e^-$

This process can be induced by the operators $[O_{\ell\ell}]_{1112}, [O_{\ell e}]_{1112}, [O_{\ell e}]_{1211}$ and $[O_{ee}]_{1112}$ but also by vertex corrections. At 90% CL:

$$\begin{aligned}
 &\left\{ 2|[c_{\ell\ell}]_{1112} + 4g_{L,SM}^{Ze}[\delta g_L^{Ze}]_{12}]^2 + 2|[c_{ee}]_{1112} + 4g_{R,SM}^{Ze}[\delta g_R^{Ze}]_{12}]^2 \right. \\
 &\quad \left. + |[c_{\ell e}]_{1112} + [c_{\ell e}]_{1211} + 4g_{R,SM}^{Ze}[\delta g_L^{Ze}]_{12} + 4g_{L,SM}^{Ze}[\delta g_R^{Ze}]_{12}]^2 \right\}^{1/2} \\
 &\qquad\qquad\qquad < 2.0 \times 10^{-6}.
 \end{aligned}
 \tag{B.4}$$

- $\tau^- \rightarrow e^- e^+ e^-$

We get, at 90% CL:

$$\begin{aligned} & \left\{ 2 \left| [c_{\ell\ell}]_{1113} + 4g_{L,SM}^{Z_e} [\delta g_L^{Z_e}]_{13} \right|^2 + 2 \left| [c_{ee}]_{1113} + 4g_{R,SM}^{Z_e} [\delta g_R^{Z_e}]_{13} \right|^2 \right. \\ & \left. + \left| [c_{\ell e}]_{1113} + [c_{\ell e}]_{1311} + 4g_{R,SM}^{Z_e} [\delta g_L^{Z_e}]_{13} + 4g_{L,SM}^{Z_e} [\delta g_R^{Z_e}]_{13} \right|^2 \right\}^{1/2} \\ & < 7.8 \times 10^{-4}. \end{aligned} \quad (\text{B.5})$$

- $\tau^- \rightarrow \mu^- \mu^+ \mu^-$

We get, at 90% CL:

$$\begin{aligned} & \left\{ 2 \left| [c_{\ell\ell}]_{2223} + 4g_{L,SM}^{Z_e} [\delta g_L^{Z_e}]_{23} \right|^2 + 2 \left| [c_{ee}]_{2223} + 4g_{R,SM}^{Z_e} [\delta g_R^{Z_e}]_{23} \right|^2 \right. \\ & \left. + \left| [c_{\ell e}]_{2223} + [c_{\ell e}]_{2322} + 4g_{R,SM}^{Z_e} [\delta g_L^{Z_e}]_{23} + 4g_{L,SM}^{Z_e} [\delta g_R^{Z_e}]_{23} \right|^2 \right\}^{1/2} \\ & < 6.9 \times 10^{-4}. \end{aligned} \quad (\text{B.6})$$

- $\tau^- \rightarrow \mu^- e^+ e^-$

This process can be induced by the operators $[O_{\ell\ell}]_{1123}, [O_{\ell\ell}]_{1321}, [O_{\ell e}]_{1123}, [O_{\ell e}]_{1321}, [O_{\ell e}]_{2113}, [O_{\ell e}]_{2311}$ and $[O_{ee}]_{1123}$ but also by vertex corrections At 90% CL:

$$\begin{aligned} & \left\{ \left| [c_{\ell\ell}]_{1123} + [c_{\ell\ell}]_{1321} + 4g_{L,SM}^{Z_e} [\delta g_L^{Z_e}]_{23} \right|^2 + \left| [c_{ee}]_{1123} + 4g_{R,SM}^{Z_e} [\delta g_R^{Z_e}]_{23} \right|^2 + \right. \\ & \left. \left| [c_{\ell e}]_{1123} + [c_{\ell e}]_{1321} + 4g_{L,SM}^{Z_e} [\delta g_R^{Z_e}]_{23} \right|^2 + \left| [c_{\ell e}]_{2311} + [c_{\ell e}]_{2113} + 4g_{R,SM}^{Z_e} [\delta g_L^{Z_e}]_{23} \right|^2 \right\}^{1/2} \\ & < 6.4 \times 10^{-4}. \end{aligned} \quad (\text{B.7})$$

- $\tau^- \rightarrow e^- \mu^+ \mu^-$

We get, at 90% CL:

$$\begin{aligned} & \left\{ \left| [c_{\ell\ell}]_{1322} + [c_{\ell\ell}]_{1223} + 4g_{L,SM}^{Z_e} [\delta g_L^{Z_e}]_{13} \right|^2 + \left| [c_{ee}]_{1223} + 4g_{R,SM}^{Z_e} [\delta g_R^{Z_e}]_{13} \right|^2 + \right. \\ & \left. \left| [c_{\ell e}]_{2213} + [c_{\ell e}]_{2312} + 4g_{L,SM}^{Z_e} [\delta g_R^{Z_e}]_{13} \right|^2 + \left| [c_{\ell e}]_{1322} + [c_{\ell e}]_{1223} + 4g_{R,SM}^{Z_e} [\delta g_L^{Z_e}]_{13} \right|^2 \right\}^{1/2} \\ & < 7.8 \times 10^{-4}. \end{aligned} \quad (\text{B.8})$$

- $\tau^- \rightarrow e^+ \mu^- \mu^-$

This process can be induced by the operators $[O_{\ell\ell}]_{2123}, [O_{\ell e}]_{2123}, [O_{\ell e}]_{2321}$ and $[O_{ee}]_{2123}$ but is not affected at first order by vertex corrections. At 90% CL, we have:

$$\sqrt{2 \left| [c_{\ell\ell}]_{2123} \right|^2 + 2 \left| [c_{ee}]_{2123} \right|^2 + \left| [c_{\ell e}]_{2123} + [c_{\ell e}]_{2321} \right|^2} < 6.2 \times 10^{-4}. \quad (\text{B.9})$$

- $\tau^- \rightarrow \mu^+ e^- e^-$

We get, at 90% CL:

$$\sqrt{2 \left| [c_{\ell\ell}]_{1213} \right|^2 + 2 \left| [c_{ee}]_{1213} \right|^2 + \left| [c_{\ell e}]_{1213} + [c_{\ell e}]_{1312} \right|^2} < 5.8 \times 10^{-4}. \quad (\text{B.10})$$

B.3 From lepton decay parameters

In experiments studying lepton decays $\ell \rightarrow \ell_1 \nu \bar{\nu}$ such as ref. [63], the two emitted neutrinos are not detected and are assumed to conserve lepton flavor. A more general analysis of muon decay allowing lepton number violation was presented in ref. [64]. The authors show that there is a one-to-one correspondence between the form factors $g_{\ell\mu}^\gamma$ defined e.g. in ref. [63] in the lepton flavor conserving case and combinations of parameters in the lepton number violating case.

In our D=6 EFT framework, for the decay $\ell_I \rightarrow \ell_J \nu \bar{\nu}$, this correspondence is:

$$\begin{aligned}
 |g_{RR}^S|^2 &\rightarrow \sum_{k \geq l} |[c_{\ell e}]_{klJI}|^2, \\
 |g_{RR}^V|^2 &\rightarrow \sum_{k < l} |[c_{\ell e}]_{klJI}|^2, \\
 |g_{LL}^S|^2 &\rightarrow \sum_{k > l} |a_{lk}[c_{\ell\ell}]_{JIlk}|^2,
 \end{aligned}
 \tag{B.11}$$

where $a_{JI} = 2$ and $a_{kl} = 1$ in all other cases.

The limits given by PDG [46] are:

$$\begin{aligned}
 |g_{RR}^S| < 0.035, \quad |g_{RR}^V| < 0.017, \quad |g_{LL}^S| < 0.550, \quad \text{for } \mu^- \rightarrow e^- \bar{\nu} \nu \text{ at 90\% CL.} \\
 |g_{RR}^S| < 0.70, \quad |g_{RR}^V| < 0.17, \quad |g_{LL}^S| < 2.01, \quad \text{for } \tau^- \rightarrow e^- \bar{\nu} \nu \text{ at 95\% CL,} \\
 |g_{RR}^S| < 0.72, \quad |g_{RR}^V| < 0.18, \quad |g_{LL}^S| < 2.01, \quad \text{for } \tau^- \rightarrow \mu^- \bar{\nu} \nu \text{ at 95\% CL.}
 \end{aligned}
 \tag{B.12}$$

The constraints are quite weak for tau decays parameters, but give constraints better than 0.1 for Wilson coefficient of nine four-lepton operators. Explicitly they are:

$$\begin{aligned}
 \sqrt{|[c_{\ell e}]_{1112}|^2 + |[c_{\ell e}]_{2112}|^2 + |[c_{\ell e}]_{3112}|^2 + |[c_{\ell e}]_{2212}|^2 + |[c_{\ell e}]_{3212}|^2 + |[c_{\ell e}]_{3312}|^2} < 0.035, \\
 \sqrt{|[c_{\ell e}]_{1212}|^2 + |[c_{\ell e}]_{1312}|^2 + |[c_{\ell e}]_{2312}|^2} < 0.017,
 \end{aligned}
 \tag{B.13}$$

at 90% CL.

C Correlation matrix

$$\rho = \begin{pmatrix} 1. & -0.12 & -0.63 & -0.1 & -0.03 & 0.01 & 0.07 & -0.06 & -0.04 & 0. & -0.01 & 0. & 0.7 & 0.03 & 0.03 & -0.03 & -0.47 & 0.2 & 0. & 0. & -0.51 \\ . & 1. & -0.56 & -0.11 & -0.04 & 0.01 & 0.08 & -0.06 & -0.04 & 0. & -0.01 & 0. & 0.63 & -0.28 & -0.24 & 0.24 & -0.15 & -0.77 & 0.01 & 0. & 0.13 \\ . & . & 1. & -0.1 & -0.03 & 0.01 & 0.07 & -0.05 & -0.04 & 0. & -0.01 & 0. & -0.9 & 0.16 & 0.13 & -0.14 & 0.43 & 0.58 & 0. & 0. & 0.41 \\ . & . & . & 1. & -0.1 & -0.07 & 0.17 & -0.05 & 0.03 & -0.09 & 0.04 & 0.08 & -0.16 & 0.06 & 0.03 & -0.03 & 0.05 & -0.21 & 0. & -0.01 & 0. & -0.13 \\ . & . & . & . & 1. & 0.07 & -0.06 & 0.9 & -0.04 & 0.01 & 0. & -0.01 & -0.05 & -0.04 & -0.03 & 0.04 & 0.08 & -0.07 & 0. & 0. & 0. & -0.04 \\ . & . & . & . & . & 1. & 0.02 & -0.03 & 0.41 & 0.01 & 0. & 0. & 0.01 & -0.01 & -0.01 & 0.01 & 0. & 0.02 & 0.01 & -0.01 & -0.01 & 0.01 \\ . & . & . & . & . & . & 1. & -0.08 & -0.04 & -0.08 & -0.05 & 0.08 & 0.12 & -0.02 & -0.04 & 0.04 & -0.06 & 0.15 & 0. & 0.01 & 0. & 0.09 \\ . & . & . & . & . & . & . & 1. & 0.04 & 0.01 & 0. & -0.01 & -0.09 & -0.03 & -0.02 & 0.02 & 0.09 & -0.12 & 0. & 0. & 0. & -0.07 \\ . & . & . & . & . & . & . & . & 1. & 0. & 0. & 0. & -0.06 & 0.02 & 0.01 & -0.01 & 0.03 & -0.08 & 0. & 0.02 & 0. & -0.05 \\ . & . & . & . & . & . & . & . & . & 1. & -0.53 & -0.08 & 0.01 & 0. & 0. & 0. & 0.01 & 0. & 0. & 0. & 0. & 0.01 \\ . & . & . & . & . & . & . & . & . & . & 1. & -0.52 & -0.01 & 0. & 0. & 0.01 & -0.02 & 0. & 0. & 0. & -0.01 \\ . & . & . & . & . & . & . & . & . & . & . & 1. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ . & . & . & . & . & . & . & . & . & . & . & . & 1. & -0.18 & -0.15 & 0.15 & -0.48 & -0.39 & 0. & 0. & 0. & -0.3 \\ . & . & . & . & . & . & . & . & . & . & . & . & 1. & 0.04 & -0.04 & -0.77 & 0.21 & 0. & 0. & 0. & -0.04 \\ . & . & . & . & . & . & . & . & . & . & . & . & . & 1. & -0.98 & 0.05 & 0.18 & 0. & 0. & 0. & -0.03 \\ . & . & . & . & . & . & . & . & . & . & . & . & . & . & 1. & -0.06 & -0.19 & 0. & 0. & 0. & 0.03 \\ . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & 1. & 0.06 & 0. & 0. & 0. & 0.22 \\ . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & 1. & -0.01 & 0. & 0. & 0. & 0.01 \\ . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & 1. & 0.01 & -1. & 0. \\ . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & 1. & -0.01 & 0. \\ . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & 1. & 0. \\ . & 1. \end{pmatrix} \quad (C.1)$$

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References

- [1] W. Buchmüller and D. Wyler, *Effective Lagrangian analysis of new interactions and flavor conservation*, *Nucl. Phys. B* **268** (1986) 621 [[INSPIRE](#)].
- [2] Z. Han and W. Skiba, *Effective theory analysis of precision electroweak data*, *Phys. Rev. D* **71** (2005) 075009 [[hep-ph/0412166](#)] [[INSPIRE](#)].
- [3] Z. Han, *Electroweak constraints on effective theories with $U(2) \times U(1)$ flavor symmetry*, *Phys. Rev. D* **73** (2006) 015005 [[hep-ph/0510125](#)] [[INSPIRE](#)].
- [4] R. Barbieri, A. Pomarol, R. Rattazzi and A. Strumia, *Electroweak symmetry breaking after LEP-1 and LEP-2*, *Nucl. Phys. B* **703** (2004) 127 [[hep-ph/0405040](#)] [[INSPIRE](#)].
- [5] C. Grojean, W. Skiba and J. Terning, *Disguising the oblique parameters*, *Phys. Rev. D* **73** (2006) 075008 [[hep-ph/0602154](#)] [[INSPIRE](#)].
- [6] G. Cacciapaglia, C. Csáki, G. Marandella and A. Strumia, *The minimal set of electroweak precision parameters*, *Phys. Rev. D* **74** (2006) 033011 [[hep-ph/0604111](#)] [[INSPIRE](#)].
- [7] A. Pomarol and F. Riva, *Towards the ultimate SM fit to close in on Higgs physics*, *JHEP* **01** (2014) 151 [[arXiv:1308.2803](#)] [[INSPIRE](#)].
- [8] J. Elias-Miro, J.R. Espinosa, E. Masso and A. Pomarol, *Higgs windows to new physics through $D = 6$ operators: constraints and one-loop anomalous dimensions*, *JHEP* **11** (2013) 066 [[arXiv:1308.1879](#)] [[INSPIRE](#)].
- [9] B. Dumont, S. Fichet and G. von Gersdorff, *A Bayesian view of the Higgs sector with higher dimensional operators*, *JHEP* **07** (2013) 065 [[arXiv:1304.3369](#)] [[INSPIRE](#)].
- [10] C.-Y. Chen, S. Dawson and C. Zhang, *Electroweak effective operators and Higgs physics*, *Phys. Rev. D* **89** (2014) 015016 [[arXiv:1311.3107](#)] [[INSPIRE](#)].
- [11] J. de Blas, M. Chala and J. Santiago, *Global constraints on lepton-quark contact interactions*, *Phys. Rev. D* **88** (2013) 095011 [[arXiv:1307.5068](#)] [[INSPIRE](#)].
- [12] S. Willenbrock and C. Zhang, *Effective field theory beyond the Standard Model*, *Ann. Rev. Nucl. Part. Sci.* **64** (2014) 83 [[arXiv:1401.0470](#)] [[INSPIRE](#)].
- [13] R.S. Gupta, A. Pomarol and F. Riva, *BSM primary effects*, *Phys. Rev. D* **91** (2015) 035001 [[arXiv:1405.0181](#)] [[INSPIRE](#)].
- [14] E. Masso, *An effective guide to beyond the Standard Model physics*, *JHEP* **10** (2014) 128 [[arXiv:1406.6376](#)] [[INSPIRE](#)].
- [15] J. de Blas et al., *Global Bayesian analysis of the Higgs-boson couplings*, [arXiv:1410.4204](#) [[INSPIRE](#)].
- [16] M. Ciuchini, E. Franco, S. Mishima, M. Pierini, L. Reina and L. Silvestrini, *Update of the electroweak precision fit, interplay with Higgs-boson signal strengths and model-independent constraints on new physics*, [arXiv:1410.6940](#) [[INSPIRE](#)].
- [17] J. Ellis, V. Sanz and T. You, *The effective Standard Model after LHC Run I*, *JHEP* **03** (2015) 157 [[arXiv:1410.7703](#)] [[INSPIRE](#)].

- [18] A. Falkowski and F. Riva, *Model-independent precision constraints on dimension-6 operators*, *JHEP* **02** (2015) 039 [[arXiv:1411.0669](#)] [[INSPIRE](#)].
- [19] F. del Aguila, M. Chala, J. Santiago and Y. Yamamoto, *Collider limits on leptophilic interactions*, *JHEP* **03** (2015) 059 [[arXiv:1411.7394](#)] [[INSPIRE](#)].
- [20] T. Corbett, O.J.P. Eboli, D. Goncalves, J. Gonzalez-Fraile, T. Plehn and M. Rauch, *The Higgs legacy of the LHC Run I*, *JHEP* **08** (2015) 156 [[arXiv:1505.05516](#)] [[INSPIRE](#)].
- [21] A. Efrati, A. Falkowski and Y. Soreq, *Electroweak constraints on flavorful effective theories*, *JHEP* **07** (2015) 018 [[arXiv:1503.07872](#)] [[INSPIRE](#)].
- [22] M. Gonzalez-Alonso, A. Greljo, G. Isidori and D. Marzocca, *Electroweak bounds on Higgs pseudo-observables and $h \rightarrow 4\ell$ decays*, *Eur. Phys. J. C* **75** (2015) 341 [[arXiv:1504.04018](#)] [[INSPIRE](#)].
- [23] A. Buckley et al., *Global fit of top quark effective theory to data*, *Phys. Rev. D* **92** (2015) 091501 [[arXiv:1506.08845](#)] [[INSPIRE](#)].
- [24] J. de Blas, M. Chala and J. Santiago, *Renormalization group constraints on new top interactions from electroweak precision data*, *JHEP* **09** (2015) 189 [[arXiv:1507.00757](#)] [[INSPIRE](#)].
- [25] A. Falkowski, *Effective field theory approach to LHC Higgs data*, [arXiv:1505.00046](#) [[INSPIRE](#)].
- [26] F. del Aguila, M. Chala, J. Santiago and Y. Yamamoto, *Four and two-lepton signals of leptophilic gauge interactions at large colliders*, *PoS(CORFU2014)109* [[arXiv:1505.00799](#)] [[INSPIRE](#)].
- [27] J.D. Wells and Z. Zhang, *Status and prospects of precision analyses with $e^+e^- \rightarrow W^+W^-$* , *Phys. Rev. D* **93** (2016) 034001 [[arXiv:1507.01594](#)] [[INSPIRE](#)].
- [28] L. Berthier and M. Trott, *Consistent constraints on the Standard Model effective field theory*, [arXiv:1508.05060](#) [[INSPIRE](#)].
- [29] J. Ellis and T. You, *Sensitivities of prospective future e^+e^- colliders to decoupled new physics*, [arXiv:1510.04561](#) [[INSPIRE](#)].
- [30] C. Englert, R. Kogler, H. Schulz and M. Spannowsky, *Higgs coupling measurements at the LHC*, [arXiv:1511.05170](#) [[INSPIRE](#)].
- [31] B. Grzadkowski, M. Iskrzynski, M. Misiak and J. Rosiek, *Dimension-six terms in the Standard Model Lagrangian*, *JHEP* **10** (2010) 085 [[arXiv:1008.4884](#)] [[INSPIRE](#)].
- [32] R. Alonso, E.E. Jenkins, A.V. Manohar and M. Trott, *Renormalization group evolution of the Standard Model dimension six operators III: gauge coupling dependence and phenomenology*, *JHEP* **04** (2014) 159 [[arXiv:1312.2014](#)] [[INSPIRE](#)].
- [33] H.K. Dreiner, H.E. Haber and S.P. Martin, *Two-component spinor techniques and Feynman rules for quantum field theory and supersymmetry*, *Phys. Rept.* **494** (2010) 1 [[arXiv:0812.1594](#)] [[INSPIRE](#)].
- [34] LHC HIGGS CROSS SECTION WORKING GROUP 2 collaboration, *Higgs basis: proposal for an EFT basis choice for LHC HXSWG*, [LHCHXSWG-INT-2015-001](#), CERN, Geneva Switzerland (2015).
- [35] SLD ELECTROWEAK GROUP, DELPHI, ALEPH, SLD, SLD HEAVY FLAVOUR GROUP, OPAL, LEP ELECTROWEAK WORKING GROUP and L3 collaborations, S. Schael et al., *Precision electroweak measurements on the Z resonance*, *Phys. Rept.* **427** (2006) 257 [[hep-ex/0509008](#)] [[INSPIRE](#)].

- [36] DELPHI, OPAL, LEP ELECTROWEAK, ALEPH and L3 collaborations, S. Schael et al., *Electroweak measurements in electron-positron collisions at W-boson-pair energies at LEP*, *Phys. Rept.* **532** (2013) 119 [[arXiv:1302.3415](#)] [[INSPIRE](#)].
- [37] TEVATRON ELECTROWEAK WORKING GROUP, CDF and D0 collaborations, 2012 *update of the combination of CDF and D0 results for the mass of the W boson*, [arXiv:1204.0042](#) [[INSPIRE](#)].
- [38] CHARM collaboration, J. Dorenbosch et al., *Experimental results on neutrino-electron scattering*, *Z. Phys.* **C 41** (1989) 567 [*Erratum ibid.* **C 51** (1991) 142] [[INSPIRE](#)].
- [39] CHARM-II collaboration, P. Vilain et al., *Precision measurement of electroweak parameters from the scattering of muon-neutrinos on electrons*, *Phys. Lett.* **B 335** (1994) 246 [[INSPIRE](#)].
- [40] L.A. Ahrens et al., *Determination of electroweak parameters from the elastic scattering of muon-neutrinos and anti-neutrinos on electrons*, *Phys. Rev.* **D 41** (1990) 3297 [[INSPIRE](#)].
- [41] J. Erler and S. Su, *The weak neutral current*, *Prog. Part. Nucl. Phys.* **71** (2013) 119 [[arXiv:1303.5522](#)] [[INSPIRE](#)].
- [42] LSND collaboration, L.B. Auerbach et al., *Measurement of electron-neutrino-electron elastic scattering*, *Phys. Rev.* **D 63** (2001) 112001 [[hep-ex/0101039](#)] [[INSPIRE](#)].
- [43] T2K collaboration, K. Abe et al., *First muon-neutrino disappearance study with an off-axis beam*, *Phys. Rev.* **D 85** (2012) 031103 [[arXiv:1201.1386](#)] [[INSPIRE](#)].
- [44] SLAC E158 collaboration, P.L. Anthony et al., *Precision measurement of the weak mixing angle in Møller scattering*, *Phys. Rev. Lett.* **95** (2005) 081601 [[hep-ex/0504049](#)] [[INSPIRE](#)].
- [45] A. Czarnecki and W.J. Marciano, *Electroweak radiative corrections to polarized Møller scattering asymmetries*, *Phys. Rev.* **D 53** (1996) 1066 [[hep-ph/9507420](#)] [[INSPIRE](#)].
- [46] PARTICLE DATA GROUP collaboration, K.A. Olive et al., *Review of particle physics*, *Chin. Phys.* **C 38** (2014) 090001 [[INSPIRE](#)].
- [47] C. Bouchiat and L. Michel, *Theory of μ -meson decay with the hypothesis of nonconservation of parity*, *Phys. Rev.* **106** (1957) 170 [[INSPIRE](#)].
- [48] M. González-Alonso, *TAU2014 opening talk*, *Nucl. Part. Phys. Proc.* **260** (2015) 3 [[arXiv:1411.4529](#)] [[INSPIRE](#)].
- [49] M. González-Alonso, *Test del Modelo Estándar a energías bajas* (in Spanish), Ph.D. thesis, Universitat de València, Valencia Spain (2010).
- [50] N. Danneberg et al., *Muon decay: measurement of the transverse polarization of the decay positrons and its implications for the Fermi coupling constant and time reversal invariance*, *Phys. Rev. Lett.* **94** (2005) 021802 [[INSPIRE](#)].
- [51] H. Baer et al., *The International Linear Collider technical design report — volume 2: physics*, [arXiv:1306.6352](#) [[INSPIRE](#)].
- [52] M. Koratzinos et al., *TLEP: a high-performance circular e^+e^- collider to study the Higgs boson*, [arXiv:1305.6498](#) [[INSPIRE](#)].
- [53] A. Crivellin, S. Najjari and J. Rosiek, *Lepton flavor violation in the Standard Model with general dimension-six operators*, *JHEP* **04** (2014) 167 [[arXiv:1312.0634](#)] [[INSPIRE](#)].
- [54] M.E. Peskin and T. Takeuchi, *Estimation of oblique electroweak corrections*, *Phys. Rev.* **D 46** (1992) 381 [[INSPIRE](#)].

- [55] J.D. Wells and Z. Zhang, *Effective theories of universal theories*, *JHEP* **01** (2016) 123 [[arXiv:1510.08462](#)] [[INSPIRE](#)].
- [56] C.S. Wood et al., *Measurement of parity nonconservation and an anapole moment in cesium*, *Science* **275** (1997) 1759 [[INSPIRE](#)].
- [57] GFITTER GROUP collaboration, M. Baak et al., *The global electroweak fit at NNLO and prospects for the LHC and ILC*, *Eur. Phys. J. C* **74** (2014) 3046 [[arXiv:1407.3792](#)] [[INSPIRE](#)].
- [58] F. Feruglio, *Theoretical aspects of flavour and CP-violation in the lepton sector*, in 27th *Rencontres de Blois on Particle Physics and Cosmology Blois*, France May 31–June 5 2015 [[arXiv:1509.08428](#)] [[INSPIRE](#)].
- [59] G.M. Pruna and A. Signer, *Lepton-flavour violating decays in theories with dimension-6 operators*, [arXiv:1511.04421](#) [[INSPIRE](#)].
- [60] ATLAS collaboration, *Search for the lepton flavor violating decay $Z \rightarrow e\mu$ in pp collisions at $\sqrt{s} = 8$ TeV with the ATLAS detector*, *Phys. Rev. D* **90** (2014) 072010 [[arXiv:1408.5774](#)] [[INSPIRE](#)].
- [61] OPAL collaboration, R. Akers et al., *A search for lepton flavor violating Z^0 decays*, *Z. Phys. C* **67** (1995) 555 [[INSPIRE](#)].
- [62] DELPHI collaboration, P. Abreu et al., *Search for lepton flavor number violating Z^0 decays*, *Z. Phys. C* **73** (1997) 243 [[INSPIRE](#)].
- [63] A. Stahl, *Michel parameters: averages and interpretation*, *Nucl. Phys. Proc. Suppl.* **76** (1999) 173 [[INSPIRE](#)].
- [64] P. Langacker and D. London, *Analysis of muon decay with lepton number nonconserving interactions*, *Phys. Rev. D* **39** (1989) 266 [[INSPIRE](#)].