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Noncompact gauging of N=2 7D supergravity and AdS/CFT holography

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Abstract: Half-maximal gauged supergravity in seven dimensions coupled to n vector multiplets contains $n+3$ vectors and $3n+1$ scalars parametrized by $\mathbb{R}^+ \times \mathrm{SO}(3,\mathrm{n})/\mathrm{SO}(3) \times \mathrm{SO}(\mathrm{n})$ coset manifold. The two-form field in the gravity multiplet can be dualized to a three-form field which admits a topological mass term. Possible non-compact gauge groups take the form of $G_0 \times H \subset \mathrm{SO}(3,\mathrm{n})$ with a compact group H. G_0 is one of the five possibilities; $\mathrm{SO}(3,1)$, $\mathrm{SL}(3,\mathbb{R})$, $\mathrm{SO}(2,2)$, $\mathrm{SO}(2,1)$ and $\mathrm{SO}(2,2) \times \mathrm{SO}(2,1)$. We investigate all of these possible non-compact gauge groups and classify their vacua. Unlike the gauged supergravity without a topological mass term, there are new supersymmetric AdS_7 vacua in the $\mathrm{SO}(3,1)$ and $\mathrm{SL}(3,\mathbb{R})$ gaugings. These correspond to new $N=(1,0)$ superconformal field theories (SCFT) in six dimensions. Additionally, we find a class of $AdS_5 \times S^2$ and $AdS_5 \times H^2$ backgrounds with $\mathrm{SO}(2)$ and $\mathrm{SO}(2)\times\mathrm{SO}(2)$ symmetries. These should correspond to $N=1$ SCFTs in four dimensions obtained from twisted compactifications of six-dimensional field theories on S^2 or H^2. We also study RG flows from six-dimensional $N=(1,0)$ SCFT to $N=1$ SCFT in four dimensions and RG flows from a four-dimensional $N=1$ SCFT to a six-dimensional SYM in the IR. The former are driven by a vacuum expectation value of a dimension-four operator dual to the supergravity dilaton while the latter are driven by vacuum expectation values of marginal operators.

Keywords: Gauge-gravity correspondence, AdS-CFT Correspondence, Supergravity Models

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Contents

1 Introduction

Gauged supergravities play an important role in string/M theory compactification and gauge/gravity correspondence. Generally, a gauge supergravity theory admits many types of gauge groups namely compact, non-compact and non-semisimple groups, and different types of gauge groups give rise to different vacuum structures. Gauged supergravity theories may be accordingly classified into two categories by the vacua they admit. AdS supergravities are theories admitting a maximally supersymmetric AdS space as a vacuum solution while those with a half-maximally supersymmetric domain wall vacuum are called domain-wall supergravities. The former is useful in the context of the AdS/CFT correspondence [1], and the latter is relevant in the DW/QFT correspondence [2, 3].

The study of $N = (1, 0)$ superconformal field theories (SCFT) in the context of $\mathrm{AdS}_7/\mathrm{CFT}_6$ correspondence has originally done by orbifolding the $AdS_7 \times S^4$ geometry of M-theory giving rise to the gravity dual of $N = (2, 0)$ SCFT [4–6]. And, recently, many AdS_7 solutions to type IIA string theory have been identified in [7]. These backgrounds are dual to $N = (1, 0)$ SCFTs in six dimensions, and the holographic study of these SCFTs has been given in [8]. Furthermore, a number of $N = (1, 0)$ SCFTs in six dimensions have been found and classified in the context of F-theory in [9]. It would be desirable to have a description of these SCFT in terms of the gravity solutions to seven-dimensional gauged supergravity. However, it has been pointed out in [10] that AdS_7 solutions found in [7] cannot be obtained from seven-dimensional gauged supergravity.

In the framework of seven-dimensional gauged supergravity, there are only a few results in the holography of $N = (1, 0)$ SCFTs. It has been proposed in [11] that the $N = (1, 0)$ SCFTs arising in the M5-brane world-volume theories should be described by $N = 2$ seven-dimensional gauged supergravity and its matter-coupled version. A non-supersymmetric holographic RG flow within pure $N = 2$ gauged supergravity has been studied in [12], and recently, new supersymmetric AdS_7 critical points and holographic RG flows between these critical points have been explored in [13]. The gauged supergravity considered in [13] is the $N = 2$ gauged supergravity coupled to three vector multiplets resulting in $SO(4) \sim SU(2) \times SU(2)$ gauge group with two coupling constants for the two SU(2)'s. When these couplings are equal, the theory can be embedded in eleven dimensions by using the reduction ansatz recently obtained in [14].

To find more supersymmetric AdS_7 backgrounds, in this paper, we will consider the $N = 2$ gauged supergravity in seven dimensions coupled to a number of vector multiplets with non-compact gauge groups. The gauged supergravity is obtained from coupling pure $N = 2$ supergravity constructed in [15] to vector multiplets [16]. Furthermore, the two-form field in the supergravity multiplet can be dualized to a three-form field [17]. It turns out to be possible to add a topological mass term to this three-form field resulting in a gauged supergravity with a massive three-form field [18]. The latter differs considerably from the theory without topological mass in the sense that it is possible to have maximally supersymmetric AdS_7 backgrounds.

We will see that there are new AdS_7 critical points for non-compact gauging of the $N = 2$ supergravity with topological mass term. These provide more examples of AdS_7 solutions with sixteen supercharges. We will also find that some non-compact gauge groups admit $AdS_5 \times S^2$ and $AdS_5 \times H^2$ geometries as a background solution. In the context of twisted field theories, these solutions should describe a six-dimensional SCFT wrapped on a two-dimensional Riemann surface. In the IR, the six-dimensional SCFT would flow to another SCFT in four dimensions. These results give new AdS_5 backgrounds dual to $N = 1$ four-dimensional SCFTs.

The holographic study of twisted field theories has originally been applied to $N = 4$ SYM [19]. Until now, the method has been applied to other dimensions, see for example [20–23]. In [23], AdS_5 solutions from a truncation of the maximal $N = 4$ gauged supergravity in seven dimensions have been found. These AdS_5 geometries correspond to a class of $N = 1$ SCFTs in four dimensions obtained from M5-branes wrapped on complex curves. In this paper, we will give more examples of these $N = 1$ SCFTs by finding new AdS_5 geometries with eight supercharges in the half-maximal $N = 2$ gauged supergravity. We

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also give some examples of RG flows from six-dimensional SCFTs to these four-dimensional SCFTs. Furthermore, we find an RG flow from a four-dimensional $N = 1$ SCFT in the UV to a six-dimensional $N = (1, 0)$ SYM in the IR. This flow gives another example of the flows considered in [24] in which the flows from $N = 4$ SYM to six-dimensional $N = (2, 0)$ SCFT and $N = 2^*$ theory to five dimensional $N = 2$ SCFT have been studied.

The paper is organized as follow. In section 2, we describe $N = 2$ gauged supergravity in seven dimensions to set up the notation and discuss all possible non-compact gauge groups. These gauge groups will be studied in detail in section 3, 4, 5 and 6 in which possible vacua and RG flow solutions will be given. In section 7, we give a summary of the results and some conclusions.

2 Seven-dimensional $N = 2$ gauged supergravity coupled to n vector multiplets

In this section, we give a description of the matter-coupled minimal $N = 2$ gauged supergravity in seven dimensions with topological mass term. All of the notations are the same as those in [18] to which the reader is referred to for further details.

A general matter-coupled theory is constructed by coupling n vector multiplets to pure $N = 2$ supergravity constructed in [15]. The supergravity multiplet $(e^m_\mu, \psi^A_\mu, A^i_\mu, \chi^A, B_{\mu\nu}, \sigma)$ consists of the graviton, two gravitini, three vectors, two spin-$\frac{1}{2}$ fields, a two-form field and a real scalar, the dilaton. The only matter mutiplet is the vector multiplet $(A_\mu, \lambda^A, \phi^i)$ consisting of a vector field, two gauginos and three scalars. We use the convention that curved and flat space-time indices are denoted by $\mu, \nu, \ldots$ and $m, n, \ldots$, respectively. Spinor fields, ψ^A_μ, χ^A, λ^A, and the supersymmetry parameter ϵ^A are symplectic-Majorana spinors transforming as doublets of the R-symmetry $\mathrm{USp}(2)_\mathrm{R} \sim \mathrm{SU}(2)_\mathrm{R}$. From now on, the $\mathrm{SU}(2)_\mathrm{R}$ doublet indices $A, B = 1, 2$ will be dropped. Indices $i, j = 1, 2, 3$ label triplets of $\mathrm{SU}(2)_\mathrm{R}$.

The supergravity theory coupled to n vector multiplets has $\mathrm{SO}(3, \mathrm{n})$ global symmetry. The n vector multiplets will be labelled by an index $r = 1, \ldots n$. There are then $n + 3$ vector fields in total. Accordingly, only a subgroup G of the global symmetry $\mathrm{SO}(3, \mathrm{n})$ of dimension $\dim G \leq n + 3$ can be gauged. Possible gauge groups with structure constants ${f_{IJ}}^K$ and gauge algebra

$$[T_I, T_J] = {f_{IJ}}^K T_K \tag{2.1}$$

can be gauged provided that the $\mathrm{SO}(3, \mathrm{n})$ Killing form η_{IJ}, $I, J = 1, \ldots n + 3$, is invariant under G

$${f_{IK}}^L \eta_{LJ} + {f_{JK}}^L \eta_{LI} = 0\,. \tag{2.2}$$

Since η_{IJ} has only three negative eigenvalues, any gauge group can have three or less compact generators or three or less non-compact generators. It follows from (2.2) that the part of η_{IJ} corresponding to each simple subgroup G_α of G must be a multiple of the G_α Killing form. Therefore, possible non-compact gauge groups take the form of $G_0 \times H$ with a compact group $H \subset \mathrm{SO}(3, \mathrm{n})$ of dimension $\dim H \leq (n + 3 - \dim G_0)$ [18]. The G_0 factor can only be one of the five possibilities: $\mathrm{SO}(3, 1)$, $\mathrm{SL}(3, \mathbb{R})$, $\mathrm{SO}(2, 1)$, $\mathrm{SO}(2, 2) \sim \mathrm{SO}(2, 1) \times \mathrm{SO}(2, 1)$ and $\mathrm{SO}(2, 2) \times \mathrm{SO}(2, 1)$.

Apart from the dilaton σ which is a singlet under the gauge group, there are $3n$ scalar fields ϕ^{ir} parametrized by $\mathrm{SO(3,n)/SO(3) \times SO(n)}$ coset manifold. The associated coset representative $L = (L_I{}^i, L_I{}^r)$ transforms under the global $\mathrm{SO(3,n)}$ and the local $\mathrm{SO(3) \times SO(n)}$ by left and right multiplications, respectively. Its inverse is denoted by $L^{-1} = (L^I{}_i, L^I{}_r)$ with the relations $L^I{}_i = \eta^{IJ} L_{Ji}$ and $L^I{}_r = \eta^{IJ} L_{Jr}$.

The two-form field $B_{\mu\nu}$ can be dualized to a three-form field $C_{\mu\nu\rho}$ which admits a topological mass term

$$\frac{h}{36}\epsilon^{\mu_1\ldots\mu_7} H_{\mu_1\ldots\mu_4} C_{\mu_5\ldots\mu_7} \tag{2.3}$$

where the four-form field strength is defined by $H_{\mu\nu\rho\sigma} = 4\partial_{[\mu} C_{\nu\rho\sigma]}$.

The bosonic Lagrangian of the $N = 2$ massive-gauged supergravity is then given by

$$\begin{aligned} e^{-1}\mathcal{L} = & \frac{1}{2}R - \frac{1}{4}e^{\sigma} a_{IJ} F^I_{\mu\nu} F^{J\mu\nu} - \frac{1}{48}e^{-2\sigma} H_{\mu\nu\rho\sigma} H^{\mu\nu\rho\sigma} - \frac{5}{8}\partial_\mu \sigma \partial^\mu \sigma - \frac{1}{2} P^{ir}_\mu P^\mu_{ir} \\ & - \frac{1}{144\sqrt{2}} e^{-1} \epsilon^{\mu_1\ldots\mu_7} H_{\mu_1\ldots\mu_4} \omega_{\mu_5\ldots\mu_7} + \frac{1}{36} h e^{-1} \epsilon^{\mu_1\ldots\mu_7} H_{\mu_1\ldots\mu_4} C_{\mu_5\ldots\mu_7} - V \end{aligned} \tag{2.4}$$

where the scalar potential is given by

$$V = \frac{1}{4}e^{-\sigma}\left(C^{ir}C_{ir} - \frac{1}{9}C^2\right) + 16h^2 e^{4\sigma} - \frac{4\sqrt{2}}{3} h e^{\frac{3\sigma}{2}} C\,. \tag{2.5}$$

The Chern-Simons term is defined by

$$\omega_{\mu\nu\rho} = 3\eta_{IJ} F^I_{[\mu\nu} A^J_{\rho]} - f_{IJ}{}^K A^I_\mu \wedge A^J_\nu \wedge A_{\rho K} \tag{2.6}$$

with $F^I_{\mu\nu} = 2\partial_{[\mu} A^I_{\nu]} + f_{JK}{}^I A^J_\mu A^K_\nu$.

We are going to find supersymmetric bosonic background solutions, so the supersymmetry transformations of fermions are needed. Since, in the following analysis, we will set $C_{\mu\nu\rho} = 0$, we will accordingly give the supersymmetry transformations with all fermions and the three-form field vanishing. These are given by

$$\delta\psi_\mu = 2D_\mu\epsilon - \frac{\sqrt{2}}{30}e^{-\frac{\sigma}{2}} C\gamma_\mu\epsilon - \frac{i}{20}e^{\frac{\sigma}{2}} F^i_{\rho\sigma}\sigma^i\left(3\gamma_\mu\gamma^{\rho\sigma} - 5\gamma^{\rho\sigma}\gamma_\mu\right)\epsilon - \frac{4}{5}he^{2\sigma}\gamma_\mu\epsilon, \tag{2.7}$$

$$\delta\chi = -\frac{1}{2}\gamma^\mu\partial_\mu\sigma\epsilon - \frac{i}{10}e^{\frac{\sigma}{2}} F^i_{\mu\nu}\sigma^i\gamma^{\mu\nu}\epsilon + \frac{\sqrt{2}}{30}e^{-\frac{\sigma}{2}} C\epsilon - \frac{16}{5}e^{2\sigma}h\epsilon, \tag{2.8}$$

$$\delta\lambda^r = -i\gamma^\mu P^{ir}_\mu\sigma^i\epsilon - \frac{1}{2}e^{\frac{\sigma}{2}} F^r_{\mu\nu}\gamma^{\mu\nu}\epsilon - \frac{i}{\sqrt{2}}e^{-\frac{\sigma}{2}} C^{ir}\sigma^i\epsilon\,. \tag{2.9}$$

The covariant derivative of ϵ is defined by

$$D_\mu\epsilon = \partial_\mu\epsilon + \frac{1}{4}\omega^{ab}_\mu\gamma_{ab} + \frac{i}{4}\sigma^i\epsilon^{ijk} Q_{\mu jk} \tag{2.10}$$

where γ^a are space-time gamma matrices.

The quantities appearing in the Lagrangian and the supersymmetry transformations are defined by

$$\begin{aligned}
P^{ir}_\mu &= L^{Ir}\left(\delta^K_I\partial_\mu + f_{IJ}{}^K A^J_\mu\right)L^i{}_K, \qquad Q^{ij}_\mu = L^{Ij}\left(\delta^K_I\partial_\mu + f_{IJ}{}^K A^J_\mu\right)L^i{}_K, \\
C_{ir} &= \frac{1}{\sqrt{2}} f_{IJ}{}^K L^I{}_j L^J{}_k L_{Kr}\epsilon^{ijk}, \qquad C = -\frac{1}{\sqrt{2}} f_{IJ}{}^K L^I{}_i L^J{}_j L_{Kk}\epsilon^{ijk}, \\
C_{rsi} &= f_{IJ}{}^K L^I{}_r L^J{}_s L_{Ki}, \qquad a_{IJ} = L^i{}_I L_{iJ} + L^r{}_I L_{rJ}, \\
F^i_{\mu\nu} &= L_I{}^i F^I, \qquad F^r_{\mu\nu} = L_I{}^r F^I\,.
\end{aligned} \tag{2.11}$$

In the following sections, we will study all possible non-compact gauge groups G_0 without the compact H factor. This is a consistent truncation since all scalar fields we retain are H singlets. All of the solutions found here are automatically solutions of the gauged supergravity with $G_0 \times H$ gauge group according to the result of Schur's lemma as originally discussed in [25].

Before going to the computation, we will give a general parametrization of the $\mathrm{SO}(3,\mathrm{n})/\mathrm{SO}(3)\times\mathrm{SO}(\mathrm{n})$ coset. We first introduce $(n+3)^2$ basis elements of a general $(n+3)\times(n+3)$ matrix as follow

$$(e_{IJ})_{KL} = \delta_{IK}\delta_{JL}\,. \tag{2.12}$$

The composite $\mathrm{SO}(3)\times\mathrm{SO}(\mathrm{n})$ generators are given by

$$\begin{aligned}
\mathrm{SO}(3):&\qquad J^{(1)}_{ij} = e_{ji} - e_{ij}, \qquad i,j = 1,2,3, \\
\mathrm{SO}(\mathrm{n}):&\qquad J^{(2)}_{rs} = e_{s+3,r+3} - e_{r+3,s+3}, \qquad r,s = 1,\ldots,n\,.
\end{aligned} \tag{2.13}$$

The non-compact generators corresponding to the $3n$ scalars are given by

$$Y^{ir} = e_{i,r+3} + e_{r+3,i}\,. \tag{2.14}$$

The coset representative in each case will be given by an exponential of the relevant Y^{ir} generators.

3 SO(3, 1) gauge group

The minimal scalar coset for embedding $\mathrm{SO}(3,1)$ gauge group is $\mathrm{SO}(3,3)/\mathrm{SO}(3)\times\mathrm{SO}(3)$. We will choose the gauge structure constants to be

$$f_{IJK} = -g(\epsilon_{ijk}, \epsilon_{rsi}), \qquad i,j,r,s = 1,2,3 \tag{3.1}$$

from which we find $f_{IJ}{}^K = \eta^{KL} f_{IJL}$ with $\eta^{IJ} = (-1,-1,-1,1,1,1)$. Together with the dilaton σ, there are ten scalars in this case. At the vacuum, the full $\mathrm{SO}(3,1)$ gauge symmetry is broken down to its the maximal compact subgroup $\mathrm{SO}(3)$. The ten scalars transform as $\mathbf{1}+\mathbf{1}+\mathbf{3}+\mathbf{5}$ with the first singlet being the dilaton.

Critical point	σ	V_0	L
I	0	$-240h^2$	$\frac{1}{4h}$
II	$\frac{2}{5}\ln 2$	$-160(2^{\frac{3}{5}})h^2$	$\frac{\sqrt{3}}{2(2^{\frac{4}{5}})h}$

Table 1. Supersymmetric and non-supersymmetric AdS_7 critical points in SO(3,1) gauging.

$\mathrm{SO}(3)_{\mathrm{diag}}$	m^2L^2	Δ
1	-8	4
1	40	10
3	0	6
5	16	8

Table 2. Scalar masses at the supersymmetric AdS_7 critical point in SO(3,1) gauging.

3.1 AdS_7 critical points

We now investigate the vacuum structure of the $N = 2$ gauged supergravity with SO(3,1) gauge group. We simplify the task by restricting the potential to the two SO(3) $\subset$ SO(3,1) singlet scalars. This truncation is consistent in the sense that all critical points found on this restricted scalar manifold are automatically critical points of the potential computed on the full scalar manifold as pointed out in [25].

The scalar potential on these SO(3) singlets is given by

$$V = \frac{1}{16}e^{-\sigma-6\phi}\left[\left(1+8e^{2\phi}+3e^{4\phi}-32e^{6\phi}+3e^{8\phi}+8e^{10\phi}+e^{12\phi}\right)g^2 \right. \\ \left. -32e^{\frac{5}{2}\sigma+3\phi}\left(1+e^{2\phi}+e^{4\phi}+e^{6\phi}\right)gh+256h^2e^{5\sigma+6\phi}\right]. \tag{3.2}$$

The scalar ϕ is an SO(3) singlet coming from SO(3,3)/SO(3) $\times$ SO(3). It can be easily checked that this potential admits two critical points at $\phi = 0$ and

$$\sigma = \frac{2}{5}\ln\frac{g}{16h}, \qquad \text{and} \qquad \sigma = \frac{2}{5}\ln\frac{g}{8h}. \tag{3.3}$$

As in the SO(4) gauging studied in [13], the second critical point is non-supersymmetric as can be checked by computing the supersymmetry transformations of fermions. We will shift the dilaton field so that the supersymmetric AdS_7 occurs at $\sigma = 0$. This is effectively achieved by setting $g = 16h$. The gauge group SO(3,1) is broken down to its maximal compact subgroup SO(3), so the two critical points have SO(3) symmetry. At these critical points, the values of the cosmological constant (V_0) and the AdS_7 radius (L) are given in table 1.

In our convention, the relation between V_0 and L is given by $L = \sqrt{-\frac{15}{V_0}}$. We can compute scalar masses at the trivial critical point, $\sigma = 0$, as shown in the table 2.

In the table, we have given the representations under the unbroken SO(3) $\subset$ SO(3,1) symmetry. The conformal dimension Δ of the dual operators in the six-dimensional SCFT is also given. The three scalars in the **3** representation correspondence to the Goldstone bosons in the symmetry breaking SO(3,1) to SO(3). These scalars correspond to marginal

SO(3)	m^2L^2	Δ
1	12	$3+\sqrt{21}$
1	36	$3(1+\sqrt{5})$
3	0	6
5	0	6

Table 3. Scalar masses at the non-supersymmetric AdS_7 critical point in $SO(3,1)$ gauging.

operators of dimension six. From the table, we see that only the operator dual to the dilaton is relevant. The other are either marginal or irrelevant.

Unlike in the SO(4) gauging in which the non-supersymmetric AdS_7 is unstable, we find that, in $SO(3,1)$ gauging, it is indeed stable as can be seen from the scalar masses given in table 3. From the table, we see that the operator dual to σ becomes irrelevant at this critical point. We then expect that there should be an RG flow driven by this operator from the $N = 2$ supersymmetric fixed point to this CFT. The gravity solution would involve the metric $g_{\mu\nu}$ and σ. Since the flow is non-supersymmetric, the flow solution has to be found by solving the full second-order field equations. In general, these equations do not admit an analytic solution. We will not go into the detail of this flow here and will not give the corresponding numerical flow solution. A similar study in the case of pure $N = 2$ SU(2) gauged supergravity can be found in [12].

3.2 AdS_5 critical points

We now look for a vacuum solution of the form $AdS_5 \times S^2$. In this case, an abelian gauge field is turned on. There are six gauge fields A^I, $I = 1, \ldots, 6$, of $SO(3,1)$ in which the first three gauge fields are those of the compact subgroup SO(3). We will choose the non-zero gauge field to be A^3. The seven-dimensional metric is given by

$$ds^2 = e^{2f(r)}dx^2_{1,3} + dr^2 + e^{2g(r)}(d\theta^2 + \sin^2 d\phi^2) \tag{3.4}$$

where $dx^2_{1,3}$ is the flat metric on the four-dimensional Minkowski space. The ansatz for the gauge field is given by

$$A^3 = a\cos\theta d\phi, \qquad F^3 = -a\sin\theta d\theta \wedge d\phi\,. \tag{3.5}$$

From the metric, we can compute the following spin connections

$$\begin{aligned} &\omega^{\hat{\phi}}{}_{\hat{\theta}} = e^{-g(r)}\cot\theta e^{\hat{\phi}}, && \omega^{\hat{\phi}}{}_{\hat{r}} = g(r)'e^{\hat{\phi}}, \\ &\omega^{\hat{\theta}}{}_{\hat{r}} = g(r)'e^{\hat{\theta}}, && \omega^{\hat{\mu}}{}_{\hat{r}} = f'e^{\hat{\mu}}\,. \end{aligned} \tag{3.6}$$

From $SO(3,3)/SO(3)\times SO(3)$ coset, there are three singlets under this $SO(2) \subset SO(3)$. One of them is the SO(3) singlet mentioned before. The other two come from **3** and **5** representations of SO(3) with the former being one of the three Goldstone bosons. We can then set up relevant BPS equations by computing the supersymmetry transformations of ψ_μ, χ and λ^r. We will not give $\delta\psi_r = 0$ equation here. This will give rise to the equation for the Killing spinors as a function of r.

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We then impose the projections

$$\gamma_r \epsilon = \epsilon \qquad \text{and} \qquad i\gamma^{\hat{\theta}\hat{\phi}}\sigma^3\epsilon = \epsilon \tag{3.7}$$

where hatted indices are tangent space indices. By imposing the twist condition

$$ag = 1, \tag{3.8}$$

we find that equation $\delta\psi_\theta = 0$ is the same as $\delta\psi_\phi = 0$. The Killing spinors are then given by constant spinors on S^2. Equations $\delta\psi_\mu$, $\mu = 0, 1, 2, 3$ lead to a single equation for $f(r)$. With all these, we find the following set of the BPS equations

$$\phi_1' = \frac{e^{-\frac{\sigma}{2}-2\phi_1+2\phi_2-\phi_3}\left(1+e^{2\phi_3}\right)\left(e^{2\phi_3}-1\right)g}{2\left(1+e^{4\phi_2}\right)}, \tag{3.9}$$

$$\phi_2' = 0, \tag{3.10}$$

$$\begin{aligned}\phi_3' = &-\frac{1}{4}e^{-\frac{\sigma}{2}-2\phi_1-\phi_3-2g(r)}\Big[2ae^{\sigma+2\phi_1}\left(e^{2\phi_3}-1\right) \\ &-e^{2g(r)}\left(2e^{2\phi_1}+e^{4\phi_1}-e^{2\phi_3}-2e^{2(\phi_1+\phi_3)}+e^{4\phi_1+2\phi_3}-1\right)g\Big],\end{aligned} \tag{3.11}$$

$$\begin{aligned}\sigma' = &\frac{1}{10}e^{-\frac{\sigma}{2}-2\phi_1-\phi_3-2g(r)}\Big[2ae^{\sigma+2\phi_1}\left(1+e^{2\phi_3}\right)+64he^{\frac{5}{2}\sigma+2\phi_1+\phi_3+2g(r)} \\ &-e^{2g(r)}\left(1-2e^{2\phi_1}-e^{4\phi_1}-e^{2\phi_3}-2e^{2(\phi_1+\phi_3)}+e^{4\phi_1+2\phi_3}\right)g\Big],\end{aligned} \tag{3.12}$$

$$\begin{aligned}g(r)' = &-\frac{2}{5}ae^{\frac{\sigma}{2}-\phi_3-2g(r)}\left(1+e^{2\phi_3}\right)+\frac{4}{5}he^{2\sigma} \\ &+\frac{1}{20}e^{-\frac{\sigma}{2}-2\phi_1-\phi_3}\left(1-2e^{2\phi_1}-e^{4\phi_1}-e^{2\phi_3}-2e^{2(\phi_1+\phi_3)}+e^{4\phi_1+2\phi_3}\right)g,\end{aligned} \tag{3.13}$$

$$\begin{aligned}f' = &\frac{1}{10}ae^{\frac{\sigma}{2}-\phi_3-2g(r)}\left(1+e^{2\phi_3}\right)+\frac{4}{5}he^{2\sigma} \\ &+\frac{1}{20}e^{-\frac{\sigma}{2}-2\phi_1-\phi_3}\left(1-2e^{2\phi_1}-e^{4\phi_1}-e^{2\phi_3}-2e^{2(\phi_1+\phi_3)}+e^{4\phi_1+2\phi_3}\right)g\end{aligned} \tag{3.14}$$

where ϕ_i, $i = 1, 2, 3$ are the three singlets from $SO(3,3)/SO(3) \times SO(3)$. The $'$ denotes $\frac{d}{dr}$. To avoid the confusion with the gauge coupling g, we have explicitly written the S^2 warp factor as $g(r)$.

ϕ_2, being one of the Goldstone bosons, disappears entirely from the scalar potential which, for these SO(2) singlets, is given by

$$\begin{aligned}V = &\frac{1}{16}e^{-\sigma-4\phi_1-2\phi_3}\Big[\left(1+2e^{4\phi_1}+e^{4\phi_3}+2e^{4(\phi_1+\phi_3)}-16e^{4\phi_1+2\phi_3}+e^{8\phi_1+4\phi_3}\right)g^2 \\ &+32ghe^{\frac{5\sigma}{2}+2\phi_1+\phi_3}\left(1-2e^{2\phi_1}-e^{4\phi_1}-e^{2\phi_3}-2e^{2(\phi_1+\phi_3)}+e^{4\phi_1+2\phi_3}\right) \\ &+256h^2e^{5\sigma+4\phi_1+2\phi_3}\Big].\end{aligned} \tag{3.15}$$

When $\phi_3 = \phi_1$, this reduces to the SO(3) invariant potential (3.2). Equation (3.10) implies that ϕ_2 is a constant. We will choose $\phi_2 = 0$ from now on in order to be consistent with the supersymmetric AdS_7 critical point.

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The $AdS_5 \times S^2$ geometry is characterized by the fixed point solution of $g(r)' = \phi_i' = \sigma' = 0$. From the above equations, there is a solution only for $\phi_i = 0$ and

$$\sigma = \frac{2}{5}\ln\frac{g}{12h}, \qquad g(r) = -\frac{1}{2}\ln\frac{g}{3a} + \frac{1}{5}\ln\frac{g}{12h}\,. \tag{3.16}$$

Near this fixed point with $g = 16h$, we find $f \sim \left(\frac{512}{9}\right)^{\frac{2}{5}} hr$. Therefore, the AdS_5 radius is given by $L_{AdS_5} = \frac{1}{h}\left(\frac{9}{512}\right)^{\frac{2}{5}}$. At this fixed point, the projection $\gamma_r\epsilon = \epsilon$ is not needed, so the number of unbroken supercharges is eight. According to the AdS/CFT correspondence, we will identify this AdS_5 solution with an $N = 1$ SCFT in four dimensions.

3.3 RG flows from 6D $N = (1, 0)$ SCFT to 4D $N = 1$ SCFT

The existence of $AdS_5 \times S^2$ geometry indicates that the $N = (1, 0)$ SCFT in six dimensions corresponding to AdS_7 critical point can undergo an RG flow to a four-dimensional $N = 1$ SCFT. We begin the study of this RG flow solution by rewriting the BPS equations for $\phi_i = 0$

$$\sigma' = \frac{2}{5}e^{-\frac{\sigma}{2}}\left(ae^{\sigma-2g(r)} + g - 16he^{\frac{5\sigma}{2}}\right), \tag{3.17}$$

$$g(r)' = \frac{1}{5}e^{-\frac{\sigma}{2}}\left(g - 4ae^{\sigma-2g(r)} + 4he^{\frac{5\sigma}{2}}\right), \tag{3.18}$$

$$f' = \frac{1}{5}e^{-\frac{\sigma}{2}}\left(g + ae^{\sigma-2g(r)} + 4he^{\frac{5\sigma}{2}}\right). \tag{3.19}$$

Near the IR AdS_5 fixed point, we find

$$\begin{aligned} \sigma \sim g(r) &\sim e^{(\sqrt{7}-1)\frac{r}{L_{AdS_5}}}, \\ f &\sim \frac{r}{L_{AdS_5}}\,. \end{aligned} \tag{3.20}$$

We then conclude that the operators dual to σ and $g(r)$ become irrelevant in four dimensions with dimension $\Delta = 3 + \sqrt{7}$. We are not able to find an analytic solution to the above equations. We therefore give an example of numerical solutions in figure 1.

At the IR fixed point, the value of σ does not depend on a, but different values of a give rise to different solutions for $g(r)$. In figure 1, we have given some examples of the $g(r)$ solutions with three different values of a, $a = 1, 2, 3$ with $g = 16h$ and $h = 1$. From the solutions, we see that, at large r, $g(r) \sim r$ and $\sigma \sim 0$. Furthermore, as $g(r) \sim r \to \infty$, we find $f(r) \sim g(r) \sim r$. The UV geometry is AdS_7 corresponding to the six-dimensional $N = (1, 0)$ SCFT. The behavior of σ near the UV point is given by

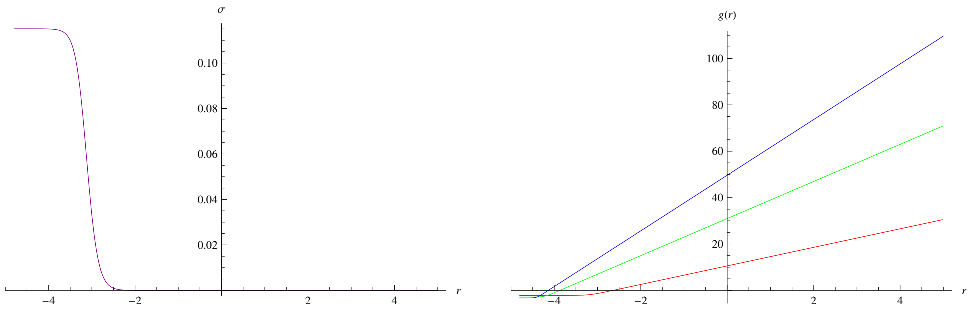
$$\sigma \sim e^{-\frac{4r}{L_{AdS_7}}} \tag{3.21}$$

which indicates that the flow is driven by a VEV of a dimension-four operator.

3.4 $AdS_5 \times H^2$ geometry

We now consider a fixed point of the form $AdS_5 \times H^2$ with H^2 being a genus $g > 1$ Riemann surface. In this case, we take the metric ansatz to be

$$ds^2 = e^{2f(r)}dx_{1,3}^2 + dr^2 + \frac{e^{2g(r)}}{y^2}(dx^2 + dy^2). \tag{3.22}$$



(a) A solution for σ. **(b)** Solutions for $g(r)$.

Figure 1. RG flow solutions from $N = (1, 0)$ SCFT in six dimensions to four-dimensional $N = 1$ SCFT with the $g(r)$ solution given for three different values of a; $a = 1$ (red),$a = 2$ (green), $a = 3$ (blue).

The SO(2) gauge field is then given by

$$A = \frac{a}{y} dx, \qquad F = \frac{a}{y^2} dx \wedge dy\,. \tag{3.23}$$

The spin connections computed from the above metric are given by

$$\omega^{\hat{x}}{}_{\hat{r}} = g(r)' e^{\hat{x}}, \qquad \omega^{\hat{y}}{}_{\hat{r}} = g(r)' e^{\hat{y}}, \qquad \omega^{\hat{x}}{}_{\hat{y}} = -e^{-g(r)} e^{\hat{x}}\,. \tag{3.24}$$

The twisted condition is still given by $ga = 1$. The BPS equations change by some signs, and it is still true that the AdS_5 is possible only for $\phi_i = 0$. The BPS equations, for $\phi_i = 0$, are then given by

$$\sigma' = \frac{2}{5} e^{-\frac{\sigma}{2}} \left(-a e^{\sigma - 2g(r)} + g - 16 h e^{\frac{5\sigma}{2}} \right), \tag{3.25}$$

$$g(r)' = \frac{1}{5} e^{-\frac{\sigma}{2}} \left(g + 4a e^{\sigma - 2g(r)} + 4 h e^{\frac{5\sigma}{2}} \right), \tag{3.26}$$

$$f' = \frac{1}{5} e^{-\frac{\sigma}{2}} \left(g - a e^{\sigma - 2g(r)} + 4 h e^{\frac{5\sigma}{2}} \right). \tag{3.27}$$

The fixed point conditions $\sigma' = g(r)' = 0$ have the solution

$$\sigma = \frac{2}{5} \ln \frac{g}{12h}, \qquad g(r) = -\frac{1}{2} \ln \left[-\frac{g}{3a} \right] + \frac{1}{5} \ln \frac{g}{12h}\,. \tag{3.28}$$

In this case, there is no real solution for $g(r)$ since the twisted condition requires that g must have the same sign as a. Therefore, we conclude that there is no supersymmetric $AdS_5 \times H^2$ solution for SO(3, 1) gauging.

4 SL(3, ℝ) gauge group

In this section, we consider the SL(3, ℝ) gauge group. The minimal scalar manifold to accommodate this eight-dimensional gauge group is SO(3, 5)/SO(3) × SO(5). The structure constants can be obtained from the generators $T_I = (i\lambda_2, i\lambda_5, i\lambda_7, \lambda_1, \lambda_3, \lambda_4, \lambda_6, \lambda_8)$ with $I = 1, \ldots, 8$. λ_i are the usual Gell-mann matrices.

SO(3)	m^2L^2	Δ
1	-8	4
3	112	14
5	0	6
7	72	12

Table 4. Scalar masses at the supersymmetric AdS_7 critical point in $SL(3,\mathbb{R})$ gauging.

SO(3)	m^2L^2	Δ
1	12	$3+\sqrt{21}$
3	96	$3+\sqrt{105}$
5	0	6
7	36	$3(1+\sqrt{5})$

Table 5. Scalar masses at the non-supersymmetric AdS_7 critical point in $SL(3,\mathbb{R})$ gauging.

Under $SL(3,\mathbb{R})$, the adjoint representation of $SO(3,5)$ decomposes as

$$\mathbf{28} \to \mathbf{8} + \mathbf{10} + \mathbf{10}'\,.$$

At the vacuum, the $SL(3,\mathbb{R})$ symmetry is broken down to SO(3) with the embedding $\mathbf{3} \to \mathbf{3}$. Therefore, under SO(3), the $\mathbf{28}$ of $SO(3,5)$ further decomposes as

$$\mathbf{28} \to \mathbf{3} + \mathbf{5} + \mathbf{3} + \mathbf{7} + \mathbf{3} + \mathbf{7}\,.$$

The fifteen scalars transform under SO(3) as $\mathbf{3}+\mathbf{5}+\mathbf{7}$. The other representations $\mathbf{3}+\mathbf{3}+\mathbf{7}$ combine into the adjoint representation of the composite local $SO(3) \times SO(5)$ symmetry.

4.1 AdS_7 critical points

By computing the scalar potential, we find that there are two AdS_7 critical points with SO(3) symmetry as in the $SO(3,1)$ gauging for vanishing vector multiplet scalars. One of them is supersymmetric, and the other one is non-supersymmetric. We will similarly set $g = 16h$ to bring the supersymmetric AdS_7 to $\sigma = 0$. The characteristics of these two critical points are the same as in $SO(3,1)$ gauging, so we will not repeat them here. However, scalar masses at these two critical point are different and are given in table 4 and 5.

As in the previous case, the SO(3) singlet is the dilaton. In this case, there are five Goldstone bosons from the $SL(3,\mathbb{R}) \to SO(3)$ symmetry breaking. The non-supersymmetric AdS_7 is stable as in the $SO(3,1)$ gauging and can be interpreted as a unitary six-dimensional CFT. We then expect that there should be an RG flow from the supersymmetric AdS_7 to the non-supersymmetric one. As in the previous case, the flow is driven by a VEV of the operator dual to the dilaton σ. In the IR, the operator becomes irrelevant with dimension $\Delta = 3 + \sqrt{21}$.

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4.2 AdS_5 critical points

We now study possible AdS_5 fixed points. We will turn on a gauge field of SO(2) which is a subgroup of the compact subgroup $SO(3) \subset SL(3, \mathbb{R})$. Among the fifteen scalars, there are three singlets under this SO(2), and we will denote them by ϕ_i, $i = 1, 2, 3$. Each of the three SO(3) representations, $\mathbf{3} + \mathbf{5} + \mathbf{7}$, gives one SO(2) singlet.

We again use the metric ansatz (3.4) and the gauge field $A^3 = a \cos\theta d\phi$. With the twisted condition $ga = 1$ and the projectors $\gamma_r \epsilon = \epsilon$ and $i\gamma^{\hat{\theta}\hat{\phi}}\sigma^3\epsilon = \epsilon$, we obtain a system of complicated BPS equations. Since these equations might be useful for other applications, we explicitly give them here

$$\phi_1' = \frac{\sqrt{3}ge^{-\frac{\sigma}{2}-2\phi_1-\frac{2}{\sqrt{3}\phi_3}}\left(e^{4\phi_1}-1\right)\left(e^{4\phi_2}-1\right)\left(e^{\frac{4\phi_3}{\sqrt{3}}}-1\right)}{4\left(1+e^{4\phi_2}\right)}, \tag{4.1}$$

$$\phi_2' = \frac{\sqrt{3}}{4}ge^{-\frac{\sigma}{2}-2\phi_2-\frac{2\phi_3}{\sqrt{3}}}\left(1+e^{4\phi_2}\right)\left(e^{\frac{4\phi_3}{\sqrt{3}}}-1\right), \tag{4.2}$$

$$\begin{aligned}\phi_3' = \frac{1}{16}e^{-\frac{\sigma}{2}-2\phi_1-2\phi_2-\frac{2\phi_3}{\sqrt{3}}-2g(r)}\bigg[&4\sqrt{3}ae^{\sigma+2\phi_1+2\phi_2}\left(1-e^{\frac{4\phi_3}{\sqrt{3}}}\right)\\ &+ge^{g(r)}\bigg(3e^{4\phi_1+4\phi_2+\frac{4\phi_3}{\sqrt{3}}}+3e^{4\phi_2+\frac{4\phi_3}{\sqrt{3}}}-4\sqrt{3}e^{2\phi_1+2\phi_2+\frac{4\phi_3}{\sqrt{3}}}-3e^{4\phi_1+\frac{4\phi_3}{\sqrt{3}}}-3e^{\frac{4\phi_3}{\sqrt{3}}}\\ &+3e^{4(\phi_1+\phi_2)}+4\sqrt{3}e^{2(\phi_1+\phi_2)}+3e^{4\phi_2}-3e^{4\phi_1}-3\bigg)\bigg],\end{aligned} \tag{4.3}$$

$$\begin{aligned}\sigma' = \frac{1}{20}e^{-\frac{\sigma}{2}-2\phi_1-2\phi_2-\frac{2\phi_3}{\sqrt{3}}-2g(r)}\bigg[&4ae^{\sigma+2(\phi_1+\phi_2)}\left(1+e^{\frac{4\phi_3}{\sqrt{3}}}+128he^{\frac{5\sigma}{2}+2\phi_1+2\phi_2+\frac{2\phi_3}{\sqrt{3}}+2g(r)}\right)\\ &ge^{2g(r)}\bigg(\sqrt{3}\left(1+e^{4\phi_1}\right)-\sqrt{3}e^{4\phi_2}-4e^{2(\phi_1+\phi_2)}-\sqrt{3}e^{4(\phi_1+\phi_2)}-\sqrt{3}e^{\frac{4\phi_3}{\sqrt{3}}}\\ &-\sqrt{3}e^{4\phi_1+\frac{4\phi_3}{\sqrt{3}}}-4e^{2\phi_1+2\phi_2+\frac{4\phi_3}{\sqrt{3}}}+\sqrt{3}e^{4\phi_2+\frac{4\phi_3}{\sqrt{3}}}+\sqrt{3}e^{4\phi_1+4\phi_2+\frac{4\phi_3}{\sqrt{3}}}\bigg)\bigg],\end{aligned} \tag{4.4}$$

$$\begin{aligned}g(r)' = &-\frac{2}{5}ae^{\frac{\sigma}{2}-\frac{2\phi_3}{\sqrt{3}}-2g(r)}\left(1+e^{\frac{4\phi_3}{\sqrt{3}}}\right)+\frac{4}{5}he^{2\sigma}\\ &-\frac{1}{40}ge^{-\frac{\sigma}{2}-2\phi_1-2\phi_2-\frac{2\phi_3}{\sqrt{3}}}\bigg[\sqrt{3}\left(1+e^{4\phi_1}\right)-\sqrt{3}e^{4\phi_2}-4e^{2(\phi_1+\phi_2)}-\sqrt{3}e^{4(\phi_1+\phi_2)}\\ &-\sqrt{3}e^{\frac{4\phi_3}{\sqrt{3}}}\left(1+e^{4\phi_1}\right)-4e^{2\phi_1+2\phi_2+\frac{4\phi_3}{\sqrt{3}}}+\sqrt{3}e^{4\phi_2+\frac{4\phi_3}{\sqrt{3}}}+\sqrt{3}e^{4\phi_1+4\phi_2+\frac{4\phi_3}{\sqrt{3}}}\bigg],\end{aligned} \tag{4.5}$$

$$\begin{aligned}f' = &\frac{1}{10}ae^{\frac{\sigma}{2}-\frac{2\phi_3}{\sqrt{3}}-2g(r)}\left(1+e^{\frac{4\phi_3}{\sqrt{3}}}\right)+\frac{4}{5}he^{2\sigma}\\ &-\frac{1}{40}ge^{-\frac{\sigma}{2}-2\phi_1-2\phi_2-\frac{2\phi_3}{\sqrt{3}}}\bigg[\sqrt{3}\left(1+e^{4\phi_1}\right)-\sqrt{3}e^{4\phi_2}-4e^{2(\phi_1+\phi_2)}-\sqrt{3}e^{4(\phi_1+\phi_2)}\\ &-\sqrt{3}e^{\frac{4\phi_3}{\sqrt{3}}}\left(1+e^{4\phi_1}\right)-4e^{2\phi_1+2\phi_2+\frac{4\phi_3}{\sqrt{3}}}+\sqrt{3}e^{4\phi_2+\frac{4\phi_3}{\sqrt{3}}}+\sqrt{3}e^{4\phi_1+4\phi_2+\frac{4\phi_3}{\sqrt{3}}}\bigg].\end{aligned} \tag{4.6}$$

It can be easily verified that the first three equations have a fixed point solution only when $\phi_i = 0$ for all $i = 1, 2, 3$. The remaining equations then reduce to the same form as in the SO(3, 1) case. The RG flow solutions can also be studied in a similar manner, and we will not repeat it here.

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As a final remark, we note here that similar to the previous case, it is not possible to have an $AdS_5 \times H^2$ solution.

5 SO(2, 2) gauge group

Unlike the previous two cases, this gauging does not admit a maximally supersymmetric AdS_7. The vacuum is rather a half-supersymmetric domain wall. This is not unexpected since the minimal superconformal algebra in six dimensions has $SU(2)_R$ R-symmetry, but the vacuum of this gauging has only SO(2)×SO(2) symmetry. The minimal scalar manifold for embedding this gauge group is SO(3, 3)/SO(3) × SO(3). The embedding of SO(2, 2) in SO(3, 3) is given by the following structure constants

$$f_{IJ}{}^{K} = (g_1\epsilon_{\bar{i}\bar{j}\bar{l}}\eta^{\bar{k}\bar{l}}, g_2\epsilon_{\bar{r}\bar{s}\bar{t}}\eta^{\bar{q}\bar{t}}) \tag{5.1}$$

with $\bar{i} = 1, 2, 6$, $\bar{r} = 3, 4, 5$, $\eta_{\bar{i}\bar{j}} = (-1, -1, 1)$ and $\eta_{\bar{r}\bar{s}} = (-1, 1, 1)$.

5.1 Domain wall solutions

The vacuum of this gauging will have SO(2) × SO(2) symmetry. Among the nine scalars from SO(3, 3)/SO(3) × SO(3), there is one SO(2) × SO(2) singlet which will be denoted by ϕ. The scalar potential for SO(2) × SO(2) singlet scalars is given by

$$V = \frac{1}{2}g_1 e^{-\sigma} + 4g_1 h e^{\frac{3\sigma}{2}}\left(e^{-\phi} - e^{\phi}\right) + 16h^2 e^{4\sigma}\,. \tag{5.2}$$

It can be checked that this potential does not admit any critical points unless $h = g_1 = 0$. The vacuum is then a domain wall.

To study the domain wall solution, we write down the associated BPS equations by setting all the fields but the metric and scalars to zero. The metric is given by the domain wall ansatz

$$ds^2 = e^{2A(r)}dx^2_{1,5} + dr^2\,. \tag{5.3}$$

With the projection $\gamma_r\epsilon = \epsilon$, the relevant BPS equations read

$$\phi' = -\frac{1}{2}g_1 e^{-\frac{\sigma}{2}-\phi}\left(1 + e^{2\phi}\right), \tag{5.4}$$

$$\sigma' = \frac{1}{5}e^{-\frac{\sigma}{2}-\phi}\left[g_1\left(e^{2\phi} - 1\right) - 32he^{\frac{5\sigma}{2}+\phi}\right], \tag{5.5}$$

$$A' = \frac{1}{10}e^{-\frac{\sigma}{2}-\phi}\left[g_1\left(e^{2\phi} - 1\right) + 8he^{\frac{5\sigma}{2}+\phi}\right]. \tag{5.6}$$

By changing the radial coordinate from r to $\tilde{r}$ with the relation $\frac{d\tilde{r}}{dr} = e^{-\frac{\sigma}{2}}$, it is not difficult to find the solutions for ϕ, σ and A. These are given by

$$\phi = \ln\left[\tan\frac{C_1 - g_1\tilde{r}}{2}\right], \tag{5.7}$$

$$\sigma = \frac{2}{5}\phi - \frac{2}{5}\ln\left[\frac{16h}{g_1}\left(4C_2(1 + e^{2\phi}) - 1\right)\right], \tag{5.8}$$

$$A = \frac{1}{5}\phi - \frac{1}{4}\ln(1 + e^{2\phi}) + \frac{1}{20}\ln\left[1 - 4C_2\left(1 + e^{2\phi}\right)\right] \tag{5.9}$$

where C_1 and C_2 are integration constants. We have omitted the additive constant to A since this can be removed by rescaling $dx^2_{1,5}$ coordinates. According to the general DW/QFT correspondence, this solution should be dual to a non-conformal $N = (1,0)$ gauge theory in six dimensions. As $\tilde{r} \to \frac{C_1}{g_1}$, the two scalars are logarithmically divergent. After changing the coordinate from $\tilde{r}$ back to r, we find the behavior of ϕ and σ as $\tilde{r} \sim \frac{C_1}{g_1}$, which is equivalent to $r \sim \frac{C}{g_1}$,

$$\phi \sim \frac{5}{6} \ln \left[\frac{C - g_1 r}{2}\right], \qquad \sigma \sim \frac{1}{3} \ln \left[\frac{C - g_1 r}{2}\right] \tag{5.10}$$

where C is a new integration constant coming from solving for $\tilde{r}$ in term of r. After rescaling $dx^2_{1,5}$ coordinates, the metric in this limit is given by

$$ds^2 = (C - g_1 r)^{\frac{1}{3}} dx^2_{1,5} + dr^2 \,. \tag{5.11}$$

5.2 AdS_5 critical points

We now look for a vacuum solution of the form $AdS_5 \times S^2$. In this case, there are two abelian SO(2) gauge groups. The corresponding gauge fields are denoted by

$$A^3 = a \sin\theta d\phi, \qquad A^6 = b \sin\theta d\phi \,. \tag{5.12}$$

The metric is still given by (3.4). In order to find the BPS equations, we impose the projectors $\gamma_r \epsilon = \epsilon$ and $i\gamma^{\hat{\theta}\hat{\phi}}\sigma^3\epsilon = \epsilon$. The twisted condition is now given by

$$g_1 b = 1 \,. \tag{5.13}$$

Proceed as in the previous cases but with one more gauge field, we find the following BPS equations

$$\begin{aligned}
\phi' &= \frac{1}{2} e^{-\frac{\sigma}{2} - \phi - 2g(r)} \left[a e^{\sigma} \left(1 - e^{2\phi}\right) - \left(1 + e^{2\phi}\right) \left(b e^{\sigma} + e^{2g(r)} g_1\right)\right], && (5.14) \\
\sigma' &= \frac{1}{5} e^{-\frac{\sigma}{2} - \phi - 2g(r)} \left[(a - b) e^{\sigma} + (a + b) e^{\sigma + 2\phi} \right. \\
&\quad \left. + e^{2g(r)} \left[\left(e^{2\phi} - 1\right) g_1 - 32 h e^{\frac{5\sigma}{2} + \phi} \right] \right], && (5.15) \\
g(r)' &= \frac{1}{10} e^{-\frac{\sigma}{2} - \phi - 2g(r)} \left[e^{2g(r)} \left[\left(e^{2\phi} - 1\right) g_1 + 8 h e^{\frac{5\sigma}{2} + \phi} \right] \right. \\
&\quad \left. + 4(b - a) e^{\sigma} - 4(a + b) e^{\sigma + 2\phi} \right], && (5.16) \\
f' &= \frac{1}{10} e^{-\frac{\sigma}{2} - \phi - 2g(r)} \left[e^{2g(r)} \left[\left(e^{2\phi} - 1\right) g_1 + 8 h e^{\frac{5\sigma}{2} + \phi} \right] \right. \\
&\quad \left. + (a - b) e^{\sigma} + (a + b) e^{\sigma + 2\phi} \right] && (5.17)
\end{aligned}$$

where ϕ is the $SO(2) \times SO(2)$ singlet scalar from $SO(3,3)/SO(3) \times SO(3)$.

The equations $\phi' = \sigma' = g(r)' = 0$ admit a fixed point solution given by

$$\begin{aligned}
\phi &= \frac{1}{2}\ln\left[\frac{\sqrt{4b^2-3a^2}-a}{2(a+b)}\right],\\
\sigma &= \frac{1}{5}\ln\left[\frac{a^2g_1^2\left(\sqrt{4b^2-3a^2}-a\right)}{32(a+b)h^2\left(2b-3a+\sqrt{4b^2-3a^2}\right)}\right],\\
g(r) &= \frac{1}{10}\ln\left[\frac{(a+b)^4\left(a-2b+\sqrt{4b^2-3a^2}\right)^5\left(3a-2b-\sqrt{4b^2-3a^2}\right)^3}{1024a^3g_1^3h^2\left(a-\sqrt{4b^2-3a^2}\right)^4}\right].
\end{aligned}\tag{5.18}$$

It can be checked that the solution exists for $g_1 < 0$ and $a < 0$ with $b > -a$ or $g_1 < 0$ with $a > 0$ and $b > a$. This in turn implies that g_1 and b always have opposite sign in contradiction with the twisted condition $g_1 b = 1$. Therefore, the SO(2, 2) gauging does not admit $AdS_5 \times S^2$ geometry.

However, there exists an $AdS_5 \times H^2$ geometry. In this case, we have the metric (3.22) with the gauge fields given by

$$A^3 = \frac{a}{y}dx, \qquad A^6 = \frac{b}{y}dx\,. \tag{5.19}$$

The twisted condition is still given by $g_1 b = 1$. The BPS equations are given by (5.14), (5.15), (5.16) and (5.17) but with (a, b) replaced by $(-a, -b)$. The values of scalar fields at the AdS_5 fixed point solution are real for $g_1 < 0$ and $a < 0$ with $b < a$ in compatible with the twisted condition. Furthermore, it is not possible to have an AdS_5 fixed point with $a = \pm b$. This rules out the possibility of AdS_5 fixed point with $\mathrm{SO}(2)_{\mathrm{diag}} \subset \mathrm{SO}(2) \times \mathrm{SO}(2)$ symmetry. For $a = 0$, only one SO(2) gauge field turned on, it can also be checked that the AdS_5 fixed point does not exist. The $b = 0$ case is not possible since this is not consistent with the twisted condition with finite g_1.

5.3 RG flows from $N = 1$ 4D SCFT to 6D $N = (1, 0)$ SYM

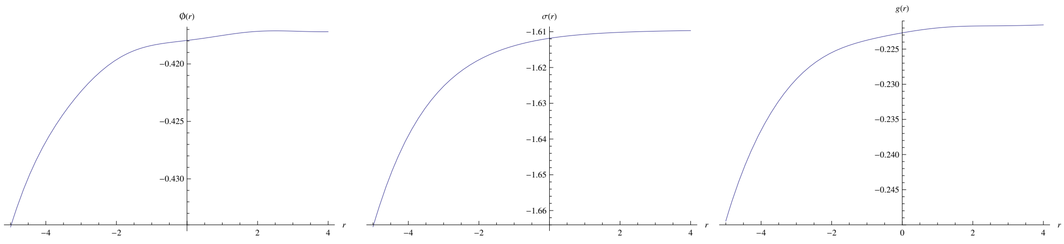
According to the AdS/CFT correspondence, the existence of AdS_5 fixed point implies a dual $N = 1$ SCFT in four dimensions. Near this AdS_5 critical point, the linearized BPS equations give

$$\phi \sim \sigma \sim g(r) \sim e^{-\frac{4r}{L}} \tag{5.20}$$

where L is the AdS_5 radius. We see that the AdS_5 should appear in the UV identified with $r \to \infty$. This UV SCFT in four dimensions undergoes an RG flow to a six-dimensional $N = (1, 0)$ SYM corresponding to the domain wall solution given by equations (5.7), (5.8) and (5.9). In the IR, the warped factors behave as $f(r) \sim g(r) \sim \ln(C - g_1 r)^{\frac{1}{3}}$ while the behavior of the scalars σ and ϕ is given in (5.10). The flow is then driven by vacuum expectations value of marginal operators dual to ϕ, σ and $g(r)$. We give an example of numerical flow solutions to the BPS equations in figure 2. This solution is found for particular values of $a = -1$, $b = -2$, $g = -\frac{1}{2}$ and $h = 1$ which give

$$\phi = -0.4171, \qquad \sigma = -1.6095, \qquad g(r) = -0.2214 \tag{5.21}$$

at the AdS_5 fixed point.



(a) A solution for ϕ. **(b)** A solution for σ. **(c)** A solution for $g(r)$.

Figure 2. An RG flow solution from $N = 1$ SCFT in four dimensions to six-dimensional $N = (1, 0)$ SYM.

As usual in flows to non-conformal field theories, the domain wall geometry in the IR is singular. We have checked that the domain wall solution given in equation (5.10) gives rise to a good singularity according to the criterion of [26]. Given the behavior of σ and ϕ in (5.10), we find that the scalar potential is bounded above $V \to -\infty$. Therefore, the IR domain wall corresponds to a physical gauge theory in six dimensions.

6 $\mathrm{SO}(2,1)$ and $\mathrm{SO}(2,2) \times \mathrm{SO}(2,1)$ gauge groups

In this section, we consider the last two possible non-compact gauge groups $\mathrm{SO}(2,1)$ and $\mathrm{SO}(2,2) \times \mathrm{SO}(2,1)$. We will see that both of them admit a vacuum solution in the form of a domain wall.

6.1 Vacua of $\mathrm{SO}(2,1)$ gauging

In this case, the minimal scalar manifold is given by $\mathrm{SO}(3,1)/\mathrm{SO}(3)$. There are three scalars in this manifold. The structure constants of the $\mathrm{SO}(2,1)$ gauge group can be chosen to be

$$f_{IJK} = (g\epsilon_{\bar{i}\bar{j}\bar{k}}, 0), \qquad \bar{i} = 1, 2, 4\,. \tag{6.1}$$

This corresponds to choosing the $\mathrm{SO}(2,1)$ generators to be (T_{41}, T_{42}, T_{12}) from the $\mathrm{SO}(3,1)$ generators (T_{ij}, T_{4i}), $i, j = 1, 2, 3$.

The scalar potential does not have any critical points. Therefore, we expect that the vacuum is a domain wall. Using the domain wall ansatz for the metric and the projector $\gamma_r \epsilon = \epsilon$, we find the BPS equations for all of the four scalars

$$\phi_1' = -\frac{e^{-\frac{\sigma}{2}-\phi_1}\left(e^{2\phi_1}-1\right)\left(e^{2\phi_3}-1\right)g}{2\left(1+e^{2\phi_3}\right)}, \tag{6.2}$$

$$\phi_2' = -\frac{e^{-\frac{\sigma}{2}-\phi_2}\left(e^{2\phi_2}-1\right)\left(e^{2\phi_3}-1\right)g}{2\left(1+e^{2\phi_3}\right)}, \tag{6.3}$$

$$\phi_3' = -\frac{1}{2}e^{-\frac{\sigma}{2}-\phi_3}\left(1+e^{2\phi_3}\right)g, \tag{6.4}$$

$$\sigma' = \frac{1}{20}e^{-\frac{\sigma}{2}-\phi_1-\phi_2-\phi_3}\left(1+e^{2\phi_1}\right)\left(1+e^{2\phi_2}\right)\left(e^{2\phi_3-1}\right)g - \frac{32}{5}he^{2\sigma}, \tag{6.5}$$

$$A' = \frac{1}{40}e^{-\frac{\sigma}{2}-\phi_1-\phi_2-\phi_3}\left(1+e^{2\phi_1}\right)\left(1+e^{2\phi_2}\right)\left(e^{2\phi_3-1}\right)g + \frac{4}{5}he^{2\sigma}\,. \tag{6.6}$$

In these equations, ϕ_i, $i = 1, 2, 3$ are scalars in $\mathrm{SO}(3,1)/\mathrm{SO}(3)$.

It is difficult to find an exact solution with all scalars non-vanishing. On the other hand, a numerical solution could be obtained by the same procedure as in the previous sections. Since analytic solutions might be more interesting, we consider only a domain wall solution preserving $SO(2) \subset SO(2,1)$ symmetry. Among these ϕ_i's, ϕ_3 is an SO(2) singlet. It turns out that on this scalar submanifold the solution is the same as that given in (5.7), (5.8) and (5.9) with ϕ replaced by ϕ_3.

6.2 Vacua of $SO(2,2) \times SO(2,1)$ gauging

The last gauge group to be considered is $SO(2,2)\times SO(2,1) \sim SO(2,1)\times SO(2,1)\times SO(2,1)$. The minimal scalar manifold in this case is $SO(3,6)/SO(3) \times SO(6)$ with the embedding of $SO(2,2) \times SO(2,1)$ in $SO(3,6)$ given by the following structure constants

$$f_{IJ}{}^{K} = (g_1\epsilon_{\bar{i}\bar{j}\bar{k}}\eta^{\bar{k}\bar{l}}, g_2\epsilon_{\bar{r}\bar{s}\bar{t}}\eta^{\bar{t}\bar{q}}, g_3\epsilon_{\tilde{i}\tilde{j}\tilde{k}}\eta^{\tilde{k}\tilde{l}}), \qquad \bar{i} = 1,4,5, \quad \bar{r} = 2,6,7, \quad \tilde{i} = 3,8,9\,. \tag{6.7}$$

The Killing metrics are given by $\eta_{\bar{i}\bar{j}} = (-1,1,1)$, $\eta_{\bar{r}\bar{s}} = (-1,1,1)$ and $\eta_{\tilde{i}\tilde{j}} = (-1,1,1)$, and g_1, g_2 and g_3 are gauge couplings of the three SO(2,1) factors.

Apart from the dilaton, there are no scalars which are singlet under the maximal compact subgroup $SO(2) \times SO(2) \times SO(2)$. However, it can be shown that the potential does not have any critical points for $g_i, h \neq 0$. A simple domain wall solution can be obtained by solving the BPS equations for σ and the metric. There might be other solutions with non-vanishing scalars from $SO(3,6)/SO(3) \times SO(6)$, but we have not found any of them. Therefore, we will restrict ourselves to the domain wall with only σ and the metric non-vanishing. Using the projector $\gamma_r\epsilon = \epsilon$ as usual, we find the following BPS equations

$$\sigma' = -\frac{32}{5}e^{2\sigma}h, \tag{6.8}$$

$$A' = = \frac{4}{5}e^{2\sigma}h\,. \tag{6.9}$$

These equations can be readily solved for the solution

$$\sigma = -\frac{1}{2}\ln\left[\frac{64hr}{5}+C\right], \tag{6.10}$$

$$A = \frac{1}{16}\ln\left[\frac{64hr}{5}+C\right] \tag{6.11}$$

where C is an integration constant. The seven-dimensional metric is given by

$$ds^2 = (64hr+5C)^{\frac{1}{8}}dx_{1,5}^2 + dr^2 \tag{6.12}$$

where we have rescaled the $dx_{1,5}^2$ coordinates by $\frac{1}{5}$.

For $h = 0$, there is a Minkowski vacuum with $V_0 = 0$. All scalar masses at this critical point are given in table 6. The $SO(2)^3$ singlet is the dilaton which is massless while the other six massless scalars are Goldstone bosons of the symmetry breaking $SO(2,1)^3 \to SO(2)^3$.

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m^2	$\mathrm{SO}(2) \times \mathrm{SO}(2) \times \mathrm{SO}(2)$ representation
0	$(\mathbf{1},\mathbf{1},\mathbf{1})$
0	$(\mathbf{1},\mathbf{1},\mathbf{2})+(\mathbf{1},\mathbf{2},\mathbf{1})+(\mathbf{2},\mathbf{1},\mathbf{1})$
g_1^2	$2\times(\mathbf{2},\mathbf{1},\mathbf{1})$
g_2^2	$2\times(\mathbf{1},\mathbf{2},\mathbf{1})$
g_3^2	$2\times(\mathbf{1},\mathbf{1},\mathbf{2})$

Table 6. Scalar masses at the supersymmetric Minkowski vacuum in $\mathrm{SO}(2,2)\times\mathrm{SO}(2,1)$ gauging.

7 Conclusions

We have studied $N=2$ gauged supergravity in seven dimensions with non-compact gauge groups. In $\mathrm{SO}(3,1)$ and $\mathrm{SL}(3,\mathbb{R})$ gaugings, we have found new supersymmetric AdS_7 critical points. These should correspond to new $N=(1,0)$ SCFTs in six dimensions. We have also found that there exist $AdS_5\times S^2$ solutions to these gaugings. The solutions preserve eight supercharges and should be dual to some $N=1$ four-dimensional SCFT with $\mathrm{SO}(2)\sim\mathrm{U}(1)$ global symmetry identified with the R-symmetry. We have then studied RG flows from the six-dimensional $N=(1,0)$ SCFT to the $N=1$ SCFT in four dimensions and argued that the flow is driven by a vacuum expectation value of a dimension-four operator dual to the supergravity dilaton. A numerical solution for an example of these flows has also been given. In addition, we have shown that both of the gauge groups admit a stable non-supersymmetric AdS_7 solution which should be interpreted as a unitary CFT. This is not the case for the compact $\mathrm{SO}(4)$ gauging studied in [13] in which the non-supersymmetric critical point has been shown to be unstable.

In the $\mathrm{SO}(2,2)$ gauging, we have given a domain wall vacuum solution preserving half of the supersymmetry. According to the DW/QFT correspondence, this is expected to be dual to a non-conformal SYM in six dimensions. This $\mathrm{SO}(2,2)$ gauging does not admit an $AdS_5\times S^2$ solution but an $AdS_5\times H^2$ geometry with eight supercharges. The latter corresponds to an $N=1$ SCFT in four dimensions with $\mathrm{SO}(2)\times\mathrm{SO}(2)$ global symmetry. It is likely that the a-maximization [27–29] is needed in order to identify the correct $\mathrm{U}(1)_\mathrm{R}$ symmetry out of the $\mathrm{SO}(2)\times\mathrm{SO}(2)$ symmetry. We have studied an RG flow from this SCFT to a non-conformal SYM in six dimensions, dual to the seven-dimensional domain wall, and argued that the flow is driven by vacuum expectation values of marginal operators. We have also investigated $\mathrm{SO}(2,1)$ and $\mathrm{SO}(2,2)\times\mathrm{SO}(2,1)$ gaugings. Both of them admit a half-supersymmetric domain wall as a vacuum solution. For vanishing topological mass, the $\mathrm{SO}(2,2)\times\mathrm{SO}(2,1)$ gauging admits a seven-dimensional Minkowski vacuum preserving all of the supersymmetry and $\mathrm{SO}(2)\times\mathrm{SO}(2)\times\mathrm{SO}(2)$ symmetry.

Due to the existence of new supersymmetric AdS_7 critical points, the results of this paper might be useful in $\mathrm{AdS}_7/\mathrm{CFT}_6$ correspondence within the framework of seven-dimensional gauged supergravity. The new AdS_5 backgrounds could be of interest in the context of $\mathrm{AdS}_5/\mathrm{CFT}_4$ correspondence. RG flows across dimensions described by gravity solutions connecting these geometries would provide additional examples of flows in twisted field theories. It is also interesting, if possible, to identify these AdS_5 critical points with the known four-dimensional SCFTs.

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Until now, only the embedding of the SO(4) gauging of $N = 2$ supergravity coupled to three vector multiplets in eleven-dimensional supergravity has been given [14]. The embedding of non-compact gauge groups in ten or eleven dimensions in the presence of topological mass term is presently not known. It would be of particular interest to find such an embedding so that the results reported here would be given an interpretation in terms of brane configurations in string/M theory.

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References

[1] J.M. Maldacena, *The Large-N limit of superconformal field theories and supergravity*, Int. J. Theor. Phys. **38** (1999) 1113 [hep-th/9711200] [INSPIRE].

[2] H.J. Boonstra, K. Skenderis and P.K. Townsend, *The domain wall/QFT correspondence*, JHEP **01** (1999) 003 [hep-th/9807137] [INSPIRE].

[3] T. Gherghetta and Y. Oz, *Supergravity, nonconformal field theories and brane worlds*, Phys. Rev. **D 65** (2002) 046001 [hep-th/0106255] [INSPIRE].

[4] M. Berkooz, *A Supergravity dual of a (1,0) field theory in six-dimensions*, Phys. Lett. **B 437** (1998) 315 [hep-th/9802195] [INSPIRE].

[5] C.-h. Ahn, K. Oh and R. Tatar, *Orbifolds of $AdS_7 \times S^4$ and six-dimensional (0,1) SCFT*, Phys. Lett. **B 442** (1998) 109 [hep-th/9804093] [INSPIRE].

[6] E.G. Gimon and C. Popescu, *The Operator spectrum of the six-dimensional (1,0) theory*, JHEP **04** (1999) 018 [hep-th/9901048] [INSPIRE].

[7] F. Apruzzi, M. Fazzi, D. Rosa and A. Tomasiello, *All AdS_7 solutions of type-II supergravity*, JHEP **04** (2014) 064 [arXiv:1309.2949] [INSPIRE].

[8] D. Gaiotto and A. Tomasiello, *Holography for (1,0) theories in six dimensions*, JHEP **12** (2014) 003 [arXiv:1404.0711] [INSPIRE].

[9] J.J. Heckman, D.R. Morrison and C. Vafa, *On the Classification of 6D SCFTs and Generalized ADE Orbifolds*, JHEP **05** (2014) 028 [arXiv:1312.5746] [INSPIRE].

[10] U.H. Danielsson, G. Dibitetto, M. Fazzi and T. Van Riet, *A note on smeared branes in flux vacua and gauged supergravity*, JHEP **04** (2014) 025 [arXiv:1311.6470] [INSPIRE].

[11] S. Ferrara, A. Kehagias, H. Partouche and A. Zaffaroni, *Membranes and five-branes with lower supersymmetry and their AdS supergravity duals*, Phys. Lett. **B 431** (1998) 42 [hep-th/9803109] [INSPIRE].

[12] V.L. Campos, G. Ferretti, H. Larsson, D. Martelli and B.E.W. Nilsson, *A Study of holographic renormalization group flows in $D = 6$ and $D = 3$*, *JHEP* **06** (2000) 023 [hep-th/0003151] [INSPIRE].

[13] P. Karndumri, *RG flows in* 6*D* N *= (1,0) SCFT from* SO(4) *half-maximal 7D gauged supergravity*, *JHEP* **06** (2014) 101 [arXiv:1404.0183] [INSPIRE].

[14] P. Karndumri, $N = 2\mathrm{SO}(4) 7D$ *gauged supergravity with topological mass term from 11 dimensions*, *JHEP* **11** (2014) 063 [arXiv:1407.2762] [INSPIRE].

[15] P.K. Townsend and P. van Nieuwenhuizen, *Gauged seven-dimensional supergravity*, *Phys. Lett.* **B 125** (1983) 41 [INSPIRE].

[16] E. Bergshoeff, I.G. Koh and E. Sezgin, *Yang-Mills/Einstein Supergravity in Seven-dimensions*, *Phys. Rev.* **D 32** (1985) 1353 [INSPIRE].

[17] Y.-J. Park, *Gauged Yang-Mills-Einstein Supergravity With Three Index Field in Seven-dimensions*, *Phys. Rev.* **D 38** (1988) 1087 [INSPIRE].

[18] E. Bergshoeff, D.C. Jong and E. Sezgin, *Noncompact gaugings, chiral reduction and dual σ-models in supergravity*, *Class. Quant. Grav.* **23** (2006) 2803 [hep-th/0509203] [INSPIRE].

[19] J.M. Maldacena and C. Núñez, *Supergravity description of field theories on curved manifolds and a no go theorem*, *Int. J. Mod. Phys.* **A 16** (2001) 822 [hep-th/0007018] [INSPIRE].

[20] C. Núñez, I.Y. Park, M. Schvellinger and T.A. Tran, *Supergravity duals of gauge theories from F(4) gauged supergravity in six-dimensions*, *JHEP* **04** (2001) 025 [hep-th/0103080] [INSPIRE].

[21] P. Karndumri and E.O. Colgáin, 3*D Supergravity from wrapped D*3*-branes*, *JHEP* **10** (2013) 094 [arXiv:1307.2086] [INSPIRE].

[22] F. Benini and N. Bobev, *Two-dimensional SCFTs from wrapped branes and c-extremization*, *JHEP* **06** (2013) 005 [arXiv:1302.4451] [INSPIRE].

[23] I. Bah, C. Beem, N. Bobev and B. Wecht, *Four-Dimensional SCFTs from M5-Branes*, *JHEP* **06** (2012) 005 [arXiv:1203.0303] [INSPIRE].

[24] C. Hoyos, *Higher dimensional conformal field theories in the Coulomb branch*, *Phys. Lett.* **B 696** (2011) 145 [arXiv:1010.4438] [INSPIRE].

[25] N.P. Warner, *Some New Extrema of the Scalar Potential of Gauged $N = 8$ Supergravity*, *Phys. Lett.* **B 128** (1983) 169 [INSPIRE].

[26] S.S. Gubser, *Curvature singularities: The Good, the bad and the naked*, *Adv. Theor. Math. Phys.* **4** (2000) 679 [hep-th/0002160] [INSPIRE].

[27] K.A. Intriligator and B. Wecht, *The Exact superconformal R symmetry maximizes a*, *Nucl. Phys.* **B 667** (2003) 183 [hep-th/0304128] [INSPIRE].

[28] Y. Tachikawa, *Five-dimensional supergravity dual of a-maximization*, *Nucl. Phys.* **B 733** (2006) 188 [hep-th/0507057] [INSPIRE].

[29] P. Szepietowski, *Comments on a-maximization from gauged supergravity*, *JHEP* **12** (2012) 018 [arXiv:1209.3025] [INSPIRE].

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