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Quantum corrections to extremal black brane solutions

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Abstract: We discuss quantum corrections to extremal black brane solutions in $N = 2$ U(1) gauged supergravity in four dimensions. We consider modifications due to a certain class of higher-derivative terms as well as perturbative corrections to the prepotential. We use the entropy function formalism to assess the impact of these corrections on singular brane solutions and we give a few examples. We then use first-order flow equations to construct solutions that interpolate between quantum corrected fixed points of the associated potentials.

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1 Introduction

Gauged supergravity in four dimensions allows for extremal solutions that have Killing horizons with vanishing entropy density [1]. These solutions do, however, generically suffer from singularities due to the presence of tidal forces in the near-horizon region [2–6].[1] The associated near-horizon geometry is not described by an $AdS_2 \times \mathbb{R}^2$ line element, but instead takes a different form. It may, for instance, be of Lifshitz type [2], of hyperscaling violating type [10] or it may describe an η-geometry [11]. There turn out to be various ways to regularize these geometries, for instance by adding electric/magnetic charges [12, 13] or by taking quantum corrections into account [14–16]. In both cases the near-horizon geometry of the solution gets modified to an $AdS_2 \times \mathbb{R}^2$ geometry. This in turn implies that the regularized solution will have non-vanishing entropy density.[2]

In this paper we consider quantum corrections to extremal solutions in $N = 2$ gauged supergravity with U(1) Fayet-Iliopoulos gauging in four dimensions ($N = 2$ U(1) gauged supergravity for short). This is a step towards a string theory embedding of the proposals to regularize the singular brane solutions by quantum corrections. The corrections we consider are of two different types. They either represent quantum corrections to the prepotential of $N = 2$ supergravity, or they represent higher-derivative corrections proportional to the

[1] There are exceptions to this, though, cf. [7–9].

[2] It is well known that the resulting infrared $AdS_2 \times \mathbb{R}^2$ geometries are often unstable as well, suffering from spatially modulated instabilities, cf. [17–22]. We will not analyze this kind of instability in the following.

square of the Weyl tensor.[3] Analogous effects have been explored in [14–16] in the context of Einstein-Maxwell-dilaton systems.

One way to study the impact of quantum corrections on extremal brane solutions is to study the interpolating solution that is obtained by solving the associated first-order flow equations. First-order flow equations for extremal solutions to $N = 2$ U(1) gauged supergravity in four dimensions were first studied in [23–25] and reformulated in terms of homogeneous coordinates in [26]. We will use the latter approach to study the effect of quantum corrections that are encoded in the prepotential. On the other hand, if we choose to focus on the near-horizon geometry of the regularized solutions, the impact of the quantum corrections may also be assessed by using Sen's entropy function formalism [27]. This formalism is amenable to the inclusion of corrections due to higher-derivative terms, and was explored in the context of extremal black holes in $N = 2$ ungauged supergravity in [28, 29]. Here, we will apply it to $N = 2$ U(1) gauged supergravity in the presence of higher-derivative corrections proportional to the square of the Weyl tensor.

We will begin by deriving the entropy function for extremal black branes in $N = 2$ U(1) gauged supergravity in the presence of the aforementioned higher-derivative terms. To this end, we adapt the results of [28, 29] to the case at hand. Extremizing the entropy function with respect to the various fields yields a set of attractor equations whose solution describes the near-horizon solution of an extremal, not necessarily supersymmetric black brane. They take a complicated form that simplifies substantially when restricting to supersymmetric black branes. We give the form of these attractor equations with and without higher-derivative terms, and we discuss a few examples, which includes a non-supersymmetric one. The examples we give describe AdS_2 solutions that only exist because of the presence of quantum corrections.

Having constructed AdS_2 solutions, we turn to interpolating solutions that interpolate between AdS_2 and AdS_4 solutions. We switch off higher-derivative terms and use the formalism of first-order flow equations to construct these interpolating solutions. We discuss examples where both end points of the flow only exist due to quantum corrections to the prepotential.

2 The entropy function for extremal black branes

Extremal black brane solutions with non-vanishing entropy density are solutions which are supported by scalar fields that are subjected to the attractor mechanism. When focussing on the near-horizon region, the associated attractor equations can be efficiently derived by extremizing Sen's entropy function [27]. The entropy function framework offers the additional advantage that higher-derivative corrections to the entropy density can be dealt with in an efficient manner.

[3]We sometimes refer to both of these kinds of corrections as *quantum corrections*, even though the higher derivative corrections can arise at tree level in the genus expansion of string theory. However, they do correspond to quantum corrections in the world-sheet theory and it is in this sense that we also refer to them as quantum corrections.

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The entropy function formalism relies on the existence of an AdS_2 factor in the near-horizon geometry, but not on supersymmetry. Thus, the attractor equations derived from the entropy function formalism will be more general than those derived in the supersymmetric context. In the following, we derive the attractor equations for extremal black branes in $N = 2$ U(1) gauged supergravity in the presence of a certain class of higher-derivative interactions. In the absence of the latter, we obtain attractor equations that encompass those derived in the supersymmetric context [23–26]. We give an example of a solution that is not supersymmetric. In the presence of higher-curvature interactions, the resulting attractor equations are more complicated than their counterparts of the ungauged case. In order to display the differences between the attractor equations in the gauged and in the ungauged case, we introduce a parameter k, related to the curvature of the spatial cross section of the Killing horizon, that takes the value $k = 0$ in the black brane case, and the value $k = 1$ in the black hole case.

2.1 Entropy function

In the following, we compute the entropy function for extremal black brane solutions in $N = 2$ gauged supergravity with U(1) Fayet-Iliopoulos gauging. The associated supergravity Lagrangian contains complex scalar fields X^I (with $I = 0, \ldots, n$) that reside in $N = 2$ vector multiplets. We allow for the presence of a class of higher-derivative terms, namely terms that are proportional to the square of the Weyl tensor. These so-called F-terms play an important role in $N = 2$ string compactifications, and they can be dealt with in a systematic fashion by using the superconformal approach to supergravity [30–33]. In ungauged supergravity, the coupling of the vector multiplets to the Weyl multiplet is encoded in a holomorphic function $F(X, \hat{A})$ that is homogeneous of degree two, i.e. $F(\lambda X, \lambda^2 \hat{A}) = \lambda^2 F(X, \hat{A})$. Here $\hat{A}$ denotes the lowest component of the square of the Weyl superfield. We will assume that in $N = 2$ U(1) gauged supergravity these higher-derivative terms are encoded in the Lagrangian through the function $F(X, \hat{A})$, as in the ungauged case. Thus, the Lagrangian we will consider is

$$L = L_u - g^2 \, e^{-2\mathcal{K}} \, V \, , \tag{2.1}$$

where L_u denotes the bosonic part of the Lagrangian of $N = 2$ ungauged supergravity with higher-derivative terms [34], and V denotes the flux potential

$$V = N^{IJ} \hat{h}_I \bar{\hat{h}}_J - 2 e^{\mathcal{K}} \, |W|^2 \, , \tag{2.2}$$

where

$$\begin{aligned} N_{IJ} &= -i \left(F_{IJ} - \bar{F}_{IJ} \right) \, , \\ e^{-\mathcal{K}} &= i \left(\bar{X}^I F_I - X^I \bar{F}_I \right) \, , \\ \hat{h}_I &= h_I - F_{IK} h^K \, , \\ W &= h^I F_I - h_I X^I \, . \end{aligned} \tag{2.3}$$

Here, $F_I = \partial F(X, \hat{A}) / \partial X^I$ and $F_{IJ} = \partial^2 F(X, \hat{A}) / \partial X^I \partial X^J$. The (h_I, h^I) denote electric/magnetic fluxes. Observe that V is defined in terms of $F(X, \hat{A})$, and that it constitutes a symplectic function. The presence of the factor $e^{\mathcal{K}}$ ensures that V is invariant under scalings $(X^I, \hat{A}) \to (\lambda X^I, \lambda^2 \hat{A})$. In the absence of higher-derivative terms (in which case $F = F(X)$) and in the Poincaré frame (where $e^{-\mathcal{K}} = 1$), V reduces to the standard form of the flux potential in $N = 2$ U(1) gauged supergravity.

The Lagrangian L_u consists of various parts. One part describes the couplings of $N = 2$ vector multiplets to supergravity and to the square of the Weyl multiplet, as mentioned above. Another part describes a hyper multiplet that acts as a compensating supermultiplet. Additional hyper multiplets may be coupled as well, but they will only play a passive role in the following. The hyper multiplets give rise to the hyper-Kähler potential χ. This field couples to a real scalar field D that belongs to the Weyl multiplet.

Let us evaluate the Lagrangian (2.1) in an AdS_2 background,

$$ds^2 = v_1 \left(-r^2 dt^2 + \frac{dr^2}{r^2} \right) + v_2 \, d\Omega_k^2 \,, \tag{2.4}$$

where $d\Omega_k^2$ denotes the line element of a two-dimensional space of constant curvature, either flat space ($k = 0$) or a unit two-sphere S^2 ($k = 1$). Even though we will be interested in extremal black branes, and hence in the case $k = 0$, we will carry k along so as to be able to compare with the attractor equations for extremal black holes in ungauged supergravity, which necessarily have $k = 1$. The background (2.4) will be supported by electric fields ${F_{rt}}^I = e^I$, magnetic charges p^I as well as electric and magnetic fluxes (h_I, h^I). We will consider solutions that have the symmetries of the line element (2.4). We follow the exposition of [29] and adapt the steps given there to the background (2.4).

In this background, the fields $e^I, X^I, \hat{A}, D, \chi$ take constant values, and the Lagrangian L_u will depend on constant parameters $v_1, v_2, e^I, p^I, X^I, w, D, \chi$, where $\hat{A} = -4w^2$ [28, 29]. Since $\sqrt{-g}\, L_u$ is derived in the superconformal framework, it is invariant under rescalings with a complex parameter Λ, namely [29],

$$v_{1,2} \to |\Lambda|^{-2} v_{1,2} \,, \quad w \to \bar{\Lambda} w \,, \quad D \to |\Lambda|^2 D \,, \quad X^I \to \bar{\Lambda} X^I \,, \quad \chi \to |\Lambda|^2 \chi \,, \tag{2.5}$$

while e^I and p^I are invariant under this scale transformation (and so are the fluxes). The presence of the factor $e^{-2\mathcal{K}}$ in (2.1) ensures that the reduced Lagrangian $\sqrt{-g}\, L$ will be invariant under this transformation, and therefore it is natural to express it in terms of scale invariant variables, which may be chosen as follows [29],

$$\begin{aligned} Y^I &= \frac{1}{4} v_2 \, \bar{w} \, X^I \,, & \Upsilon &= \frac{1}{16} v_2^2 \, \bar{w}^2 \, \hat{A} = -\frac{1}{4} v_2^2 \, |w|^4 \,, & \Xi &= \frac{v_1}{v_2} \,, \\ \tilde{D} &= v_2 \left(D + \frac{1}{3} R \right) , & \tilde{\chi} &= v_2 \, \chi \,. \end{aligned} \tag{2.6}$$

Here R denotes the curvature scalar computed in the background (2.4) (see appendix B).

Observe that Υ is real and negative, and that $\sqrt{-\Upsilon}$ and Ξ are real and positive. The potential V, when expressed in terms of the rescaled variables (2.6), reads

$$V(Y, \bar{Y}) = N^{IJ} \hat{h}_I \bar{\hat{h}}_J - 2 \, \frac{|W(Y)|^2}{K(Y, \bar{Y})} \,, \tag{2.7}$$

where

$$
\begin{aligned}
K(Y,\bar Y) &= i\left(\bar Y^I\, F_I(Y,\Upsilon) - Y^I\,\bar F_I(\bar Y,\bar\Upsilon)\right)\,,\\
W(Y) &= h^I F_I(Y,\Upsilon) - h_I Y^I\,.
\end{aligned}
\tag{2.8}
$$

Here $F(Y,\Upsilon)$ denotes the rescaled function $F(X,\hat A)$, and homogeneity of the function $F(Y,\Upsilon)$ implies

$$
F(Y,\Upsilon) = \frac{1}{2}Y^I F_I(Y,\Upsilon) + \Upsilon F_\Upsilon(Y,\Upsilon)\,,
\tag{2.9}
$$

where $F_I(Y,\Upsilon) = \partial F(Y,\Upsilon)/\partial Y^I$ and $F_\Upsilon = \partial F(Y,\Upsilon)/\partial\Upsilon$. Further homogeneity relations are listed in appendix A.

When imposing the equations of motion for the redefined fields $\tilde D$ and $\tilde\chi$, one finds $\tilde D = 0$, while $\tilde\chi$ gets expressed in terms of the remaining fields (see appendix B). Inserting $\tilde D = 0$ back into L_u removes the dependence on $\tilde\chi$, since the latter couples to $\tilde D$. Then, the reduced Lagrangian $\sqrt{-g}\,L$ is expressed in terms of the rescaled parameters Y^I,Υ,Ξ,e^I,p^I and the fluxes (h_I,h^I).

The free energy $\mathcal F$ is defined to equal the integral of $\sqrt{-g}\,L$ over a unit cell of the spatial cross section of the Killing horizon. Thus, for black branes ($k=0$), $\mathcal F$ equals $\sqrt{-g}\,L$, while for black holes ($k=1$) $\mathcal F$ equals the integral of $\sqrt{-g}\,L$ over a unit two-sphere. The entropy function $\mathcal E$ is defined by the Legendre transform of the free energy $\mathcal F$ with respect to the electric fields e^I, so that $\mathcal E = -\mathcal F - e^I\,q_I$. Adapting the results of [29] to the case at hand (see appendix B), we obtain for $\mathcal F$,

$$
\begin{aligned}
\frac{1}{2}\mathcal F = {}& \frac{1}{8}N_{IJ}\Big[\Xi^{-1}e^Ie^J - \Xi p^Ip^J\Big] - \frac{1}{4}(F_{IJ}+\bar F_{IJ})e^Ip^J\\
&+\frac{1}{2}ie^I\Big[F_I + F_{IJ}\bar Y^J - \text{h.c.}\Big] - \frac{1}{2}\Xi\,p^I\Big[F_I - F_{IJ}\bar Y^J + \text{h.c.}\Big]\\
&+\frac{4}{\sqrt{-\Upsilon}}\,K(Y,\bar Y)\,(k\,\Xi-1)\\
&+i\,\Xi\Big[F - Y^IF_I - 2\Upsilon F_\Upsilon + \frac{1}{2}\bar F_{IJ}Y^IY^J - \text{h.c.}\Big]\\
&+i(F_\Upsilon - \bar F_\Upsilon)\Big[32(k^2\,\Xi + \Xi^{-1} - 2k) - 8(1+k\,\Xi)\sqrt{-\Upsilon}\Big]\\
&+32g^2\,\Xi\,\Upsilon^{-1}\,\big[K(Y,\bar Y)\big]^2\,V(Y,\bar Y)\,,
\end{aligned}
\tag{2.10}
$$

while $\mathcal E$ is given by

$$
\begin{aligned}
\frac{1}{2}\mathcal E = {}& \frac{1}{2}\Xi\,\Sigma + \frac{1}{2}\Xi\,N^{IJ}(\mathcal Q_I - F_{IK}\mathcal P^K)\,(\mathcal Q_J - \bar F_{JL}\mathcal P^L)\\
&-\frac{4}{\sqrt{-\Upsilon}}\,K(Y,\bar Y)\,(k\,\Xi-1)\\
&-i(F_\Upsilon - \bar F_\Upsilon)\Big[-2\Xi\,\Upsilon + 32(k^2\,\Xi + \Xi^{-1} - 2k) - 8(1+k\Xi)\sqrt{-\Upsilon}\Big]\\
&-32g^2\,\Xi\,\Upsilon^{-1}\,\big[K(Y,\bar Y)\big]^2\,V(Y,\bar Y)\,.
\end{aligned}
\tag{2.11}
$$

To arrive at (2.11), we used the homogeneity (2.9) of the function $F(Y,\Upsilon)$. The expressions (2.10) and (2.11) depend on k, which denotes the curvature of the two-dimensional space with line element $d\Omega_k^2$. The quantities $\mathcal{Q}_I$, $\mathcal{P}^I$, and Σ are defined by [29]

$$\begin{aligned}
\mathcal{Q}_I &= q_I + i\left(F_I - \bar{F}_I\right)\,, \\
\mathcal{P}^I &= p^I + i\left(Y^I - \bar{Y}^I\right)\,, \\
\Sigma &= -i\left(\bar{Y}^I F_I - Y^I \bar{F}_I\right) - 2i\left(\Upsilon F_\Upsilon - \bar{\Upsilon}\bar{F}_\Upsilon\right) - q_I(Y^I + \bar{Y}^I) + p^I(F_I + \bar{F}_I)\,.
\end{aligned} \tag{2.12}$$

The entropy function (2.11) depends on the variables Ξ, Υ and Y^I, whose values in the near-horizon geometry (2.4) are determined by extremizing $\mathcal{E}$. The resulting equations are called attractor equations. In the following, we will discuss the extremization of $\mathcal{E}$ with respect to these variables, first in the absence of higher-derivative terms, and then with higher-derivative terms. The extremization equations depend on k, and this implies that the attractor equations for black branes in gauged supergravity (which corresponds to the case $k = 0$ and $V \neq 0$) are markedly different from those for black holes in ungauged supergravity (which corresponds to $k = 1$ and $V = 0$). When evaluated at the extremum, the entropy function yields the value of the entropy of the extremal black hole when $k = 1$, and yields the entropy density of the extremal black brane when $k = 0$.

2.2 Variational equations without higher-derivative terms

In this subsection, we derive the attractor equations in the absence of Weyl interactions. The attractor equations we obtain apply to extremal, not necessarily supersymmetric black configurations, and they simplify considerably when restricting to supersymmetric configurations.

When switching off higher-derivative terms, the function F does not any longer depend on Υ, i.e. $F = F(Y)$, and the entropy function (2.11) reduces to

$$\begin{aligned}
\frac{1}{2}\mathcal{E}(Y,\bar{Y},\Upsilon,\Xi) = &\ \frac{1}{2}\Xi\,\Sigma + \frac{1}{2}\Xi\, N^{IJ}(\mathcal{Q}_I - F_{IK}\mathcal{P}^K)\,(\mathcal{Q}_J - \bar{F}_{JL}\mathcal{P}^L) \\
&- \frac{4}{\sqrt{-\Upsilon}}\,K(Y,\bar{Y})\,(k\,\Xi - 1) \\
&- 32 g^2\,\Xi\,\Upsilon^{-1}\,\left[K(Y,\bar{Y})\right]^2\,V(Y,\bar{Y})\,,
\end{aligned} \tag{2.13}$$

where now

$$\Sigma = -i\left(\bar{Y}^I F_I - Y^I \bar{F}_I\right) - q_I(Y^I + \bar{Y}^I) + p^I(F_I + \bar{F}_I)\,. \tag{2.14}$$

Varying $\mathcal{E}$ with respect to Y^I, Υ, Ξ and demanding the vanishing of these variations results in the following equations. Varying with respect to Υ gives

$$\frac{1 - k\,\Xi}{\Xi} = -16 g^2\,\frac{K(Y,\bar{Y})}{\sqrt{-\Upsilon}}\,V(Y,\bar{Y})\,, \tag{2.15}$$

where we assumed that $K(Y,\bar{Y})$ is non-vanishing. In the ungauged case ($k = 1, V = 0$) we obtain $\Xi = 1$, which implies $v_1 = v_2$, whereas in the gauged case ($k = 0, V \neq 0$) Ξ becomes a non-trivial function of Υ and Y^I, namely

$$\frac{1}{\Xi} = -16 g^2\,\frac{K(Y,\bar{Y})}{\sqrt{-\Upsilon}}V(Y,\bar{Y})\,, \tag{2.16}$$

where consistency requires the right hand side of (2.16) to be positive.

Varying (2.13) with respect to Ξ yields,

$$\begin{aligned}\Sigma + \left(\mathcal{Q}_I - F_{IK}\,\mathcal{P}^K\right) N^{IJ} \left(\mathcal{Q}_J - \bar F_{JL}\,\mathcal{P}^L\right) - \frac{8k}{\sqrt{-\Upsilon}}\, K(Y,\bar Y)& \\ -64 g^2\, \Upsilon^{-1} \left[K(Y,\bar Y)\right]^2 V(Y,\bar Y) = 0\,,&\end{aligned} \tag{2.17}$$

which determines the value of Υ in terms of the Y^I. This reflects the fact that in the absence of R^2 -interactions, the quantity Υ is related to an auxiliary field in the original Lagrangian whose field equation is algebraic. Again, depending on the case ($k = 1, V = 0$ or $k = 0, V \neq 0$) the relation is markedly different. When $k = 0$, we get

$$\frac{64 g^2}{\Upsilon} = \frac{\Sigma + \left(\mathcal{Q}_I - F_{IK}\,\mathcal{P}^K\right) N^{IJ} \left(\mathcal{Q}_J - \bar F_{JL}\,\mathcal{P}^L\right)}{\left[K(Y,\bar Y)\right]^2 V(Y,\bar Y)}\,. \tag{2.18}$$

Let us explore some of the consequences of (2.16) and (2.18) in the Einstein frame, where $e^{\mathcal{K}} = 1$. Using the scaling relations (2.6), we infer

$$|w|^2 = -\frac{\Upsilon}{4K(Y,\bar Y)}\,, \quad v_2 = \frac{8K(Y,\bar Y)}{\sqrt{-\Upsilon}}\,, \tag{2.19}$$

and from (2.16) we obtain

$$v_1 = -\frac{1}{2g^2\, V(Y,\bar Y)}\,. \tag{2.20}$$

This expresses the scale factors v_1, v_2 in terms of Y^I and Υ. Inserting (2.18) into (2.13) yields

$$\mathcal{E} = \frac{8K(Y,\bar Y)}{\sqrt{-\Upsilon}}\,, \tag{2.21}$$

which, according to (2.19), equals v_2, as expected for the black brane entropy density. Now consider a uniform rescaling of the charges (q_I, p^I), of the fluxes (h_I, h^I) and of the variables Y^I. Then, we infer from (2.18) that $[K(Y,\bar Y)]^2/\Upsilon$ is inert under such a rescaling. It follows that (2.21) scales with weight zero, and this implies that when expressing $\mathcal{E}$ in terms of charges and fluxes, it will be of weight zero in the charges and fluxes. This differs markedly from the case of big black holes in ungauged supergravity, where $\mathcal{E}$ scales quadratically in the charges.

Next, consider varying the entropy function (2.13) with respect to Y^I. We focus on the black brane case $k = 0$, and obtain

$$\mathcal{P}^J F_{JI} - \mathcal{Q}_I + \frac{1}{2} i \left(\mathcal{Q}_K - \bar F_{KM}\,\mathcal{P}^M\right) N^{KP}\, F_{PIJ}\, N^{JL} \left(\mathcal{Q}_L - \bar F_{LN} \mathcal{P}^N\right) + \frac{1}{2} v_2^2\, g^2 V_I = 0\,, \tag{2.22}$$

where $V_I = \partial V(Y,\bar Y)/\partial Y^I$, and we used (2.19) and (2.16). Writing out (2.22) gives

$$q_I - F_{IJ} p^J - N_{IJ} \bar Y^J - \frac{1}{2} i \left(q_K - \bar F_{KM} p^M\right) N^{KP}\, F_{PIJ}\, N^{JL} \left(q_L - \bar F_{LN}\, p^N\right) = \frac{1}{2} v_2^2\, g^2 V_I\,, \tag{2.23}$$

where we made use of the special geometry relation $F_{IJK} Y^K = 0$. Next we compute V_I,

$$V_I = i N^{KP} F_{PQI} N^{QL} \hat h_K \bar{\hat h}_L - \frac{2}{K^2}\left(\bar Y^M N_{MI}\, Y^K \hat h_K + K\, \hat h_I\right) \bar Y^N \bar{\hat h}_N - N^{KL} F_{IKP} h^P \bar{\hat h}_L\,, \tag{2.24}$$

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which satisfies $V_I Y^I = 0$. Using the expression for V_I as well as the relation $\hat{h}_I = \bar{\hat{h}}_I - iN_{IL}h^L$, we obtain for (2.23),

$$\hat{Q}_I - N_{IJ}\bar{Y}^J - \frac{1}{2}iF_{IJK}N^{JL}N^{KM}\bar{\hat{Q}}_L\bar{\hat{Q}}_M = \frac{1}{2}v_2^2\,g^2\left[iN^{PQ}F_{QMI}N^{MN}\bar{\hat{h}}_P\bar{\hat{h}}_N \right. \\ \left. -\frac{2}{K^2}\left(\bar{Y}^M N_{MI}\,Y^K\hat{h}_K + K\,\hat{h}_I\right)\bar{Y}^N\bar{\hat{h}}_N\right], \tag{2.25}$$

where we introduced the combination

$$\hat{Q}_I = q_I - F_{IJ}\,p^J\ . \tag{2.26}$$

Finally, we rewrite (2.25) as

$$\hat{Q}_I + g^2\,\frac{v_2^2}{K}\,\bar{Y}^N\bar{\hat{h}}_N\,\hat{h}_I - \frac{1}{2}iF_{IJK}N^{JL}N^{KM}\left(\bar{\hat{Q}}_L\bar{\hat{Q}}_M + g^2\,v_2^2\,\bar{\hat{h}}_L\bar{\hat{h}}_M\right) \\ = \bar{Y}^J N_{JI}\left(1 - g^2\,\frac{v_2^2}{K^2}\,Y^K\hat{h}_K\,\bar{Y}^N\bar{\hat{h}}_N\right)\ . \tag{2.27}$$

These are the black brane attractor equations for the Y^I. Contracting them with Y^I yields the constraint

$$K(Y,\bar{Y}) = -\hat{Q}_I Y^I \equiv Z(Y) = \bar{Z}(\bar{Y})\ , \tag{2.28}$$

and hence, on the attractor, we obtain from (2.14),

$$\Sigma = K(Y,\bar{Y})\ . \tag{2.29}$$

Next, we relate the entropy function (2.21) to the black hole potential which, in the Poincaré frame ($e^{\mathcal{K}} = 1$), takes the form

$$V_{\rm BH} = \left[N^{IJ} + 2\,X^I\,\bar{X}^J\right]\hat{Q}_I\bar{\hat{Q}}_J\ , \tag{2.30}$$

as follows. First we observe that (2.21) can be written as

$$\mathcal{E} = 2\,\Xi\left[2Z(Y) + N^{IJ}\hat{Q}_I\bar{\hat{Q}}_J\right] \tag{2.31}$$

by making use of (2.16), (2.18) and (2.28). Next, we express Y^I as

$$Y^I = \bar{Z}(\bar{X})\,X^I\ , \tag{2.32}$$

where

$$Z(X) = -\hat{Q}_I\,X^I\ , \tag{2.33}$$

which is consistent with (2.6) by virtue of (2.19) and (2.28). This yields $Z(Y) = |Z(X)|^2$, which results in

$$\mathcal{E} = 2\,\Xi\,V_{\rm BH}\ . \tag{2.34}$$

Note that the entropy function approach needs to be supplemented by the Hamiltonian constraint, which imposes the following restriction on the charges and the fluxes [24],

$$q_I\,h^I = p^I\,h_I\ . \tag{2.35}$$

Now let us return to the black brane attractor equations (2.27). They take a form that is very different from their black hole counterpart in ungauged supergravity [28, 29]. The latter simplify substantially when restricting to supersymmetric solutions, in which case they are given by $\mathcal{Q}_I = \mathcal{P}^I = 0$. In the case of gauged supergravity, an analogous simplification occurs when considering supersymmetric solutions, as follows. To solve (2.27), we make the ansatz

$$\hat{Q}_I = g\, e^{i\delta} v_2\, \hat{h}_I \; , \tag{2.36}$$

which, upon contraction with Y^I, results in

$$Z(Y) = g\, e^{i\delta} v_2\, W(Y) = g\, e^{-i\delta} v_2\, \bar{W}(\bar{Y}) = K(Y, \bar{Y}) \; . \tag{2.37}$$

Inserting these relations into (2.27) gives

$$\hat{Q}_I - g\, e^{i\delta} v_2\, \hat{h}_I - \frac{1}{2} i F_{IJK} N^{JL} N^{KM} \bar{\hat{h}}_L \bar{\hat{h}}_M\, g^2\, v_2^2 \left(1 + e^{-2i\delta}\right) = 0 \; . \tag{2.38}$$

This vanishes provided that

$$e^{-2i\delta} = -1 \; . \tag{2.39}$$

Inserting (2.39) into (2.36) yields the attractor values derived in [24, 26]. They apply to supersymmetric solutions [24] as well as to solutions derived from supersymmetric ones by applying a transformation $\mathcal{S}$ [35] to the charges and to the fluxes. We will refer to (2.36) and (2.39) as supersymmetric attractor equations, for simplicity. They constitute a simplification compared to the non-supersymmetric ones based on (2.27).

Since in (2.36) the dependence on Y^I only enters through F_{IJ}, which is homogeneous of degree zero, the Y^I only appear as ratios, i.e. as projective coordinates $z^i = Y^i/Y^0$ with $i = 1, \ldots, n$. The equations (2.36) can thus be viewed as equations that determine the values of the n parameters z^i and v_2 in terms of charges and fluxes. Using (2.37), (2.39) and (2.19), we find that the supersymmetric attractor equations can be recast in the form

$$\begin{aligned} \hat{Q}_I + 64 g^2 \Upsilon^{-1} K \bar{W} \hat{h}_I &= 0 \; , \\ -\Upsilon &= 64 g^2\, |W|^2 \\ \sqrt{-\Upsilon}\, \Xi^{-1} &= -16 g^2\, K\, V \; , \\ W &= -\bar{W} \; . \end{aligned} \tag{2.40}$$

Note that combining (2.35) with (2.37) results in $v_2\, \hat{h}_I N^{IJ} \bar{\hat{h}}_J = 0$ [24, 26]. Taking $v_2 \neq 0$, and inserting $\hat{h}_I N^{IJ} \bar{\hat{h}}_J = 0$ into (2.7) yields $V = -2|W|^2/K$ on a supersymmetric attractor. Combining this with (2.40) gives

$$\Xi^{-1} = 4g\, |W(Y)| \; . \tag{2.41}$$

The attractor equations for the scalars, (2.36) together with (2.39), can be obtained by extremizing the effective potential of the associated one-dimensional effective Lagrangian [24, 26].[4] The latter is obtained by evaluating the Lagrangian in a static black brane background with line element

$$ds^2 = -e^{2U} dt^2 + e^{-2U} dr^2 + e^{2(\psi - U)} (dx^2 + dy^2) \tag{2.42}$$

[4] We thank the referee for raising this question.

which, at the horizon, reduces to the line element (2.4) (with $k = 0$). The resulting effective potential takes the form

$$V_{\rm tot} = g^2 \big[N^{IJ} \partial_I \tilde{W} \partial_{\bar{J}} \bar{\tilde{W}} - 2|\tilde{W}|^2 \big] + e^{-4(\psi - U)} \big[N^{IJ} \partial_I \tilde{Z} \partial_{\bar{J}} \bar{\tilde{Z}} + 2 |\tilde{Z}|^2 \big] \,, \tag{2.43}$$

where Z and W are given by (2.33) and (2.3), respectively, with X^I replaced by the rescaled, U(1) invariant field $\tilde{X}^I = \bar{\varphi} X^I$ [26]. Here $\partial_I = \partial / \partial \tilde{X}^I$. This effective potential can be expressed in terms of a quantity Δ given by

$$\Delta = e^{2U} Z(\tilde{X}) - ig\, e^{2\psi}\, W(\tilde{X}) \,, \tag{2.44}$$

which depends holomorphically on $\tilde{X}^I$. We then obtain

$$V_{\rm tot} = e^{-4\psi} \left[N^{IJ} \partial_I \Delta \partial_{\bar{J}} \bar{\Delta} + \frac{1}{2} \partial_U \Delta \partial_U \bar{\Delta} - \frac{1}{2} \partial_\psi \Delta \partial_\psi \bar{\Delta} \right] + g e^{2(U-\psi)} \left(q_I h^I - p^I h_I \right) . \tag{2.45}$$

Now consider varying $V_{\rm tot}$ with respect to the scalar fields $\tilde{X}^I$. The variation of the first term in the bracket can be set to zero by demanding $\partial_I \Delta = 0$, which in turn implies $\Delta = 0$, since $\Delta = \tilde{X}^I \, \partial_I \Delta$ by virtue of special geometry. The variation of the sum of the second and third terms in the bracket also vanishes when imposing $\partial_I \Delta = 0$. Thus, we obtain an extremum of the potential by demanding $\partial_I \Delta = 0$. If we now take U and ψ to have the form of an AdS_2 background (2.4), i.e. $e^{2U} = r^2/v_1, e^{2\psi} = v_2\, e^{2U}$ (with a subsequent rescaling $t \to v_1 t$), we obtain from $\partial_I \Delta = 0$,

$$\hat{Q}_I - i g v_2 \hat{h}_I = 0 \,, \tag{2.46}$$

in agreement with (2.36) and (2.39).

In the ungauged case, it is known that extrema of the effective potential correspond to minima, as long as the metric on the moduli space of physical scalars is positive definite, cf. [36]. In the case at hand, it is not obvious that an extremum is a minimum of the effective potential, as we proceed to analyze. To do so, we have to take into account that the scalar fields X^I are constrained to satisfy $N_{IJ} X^I \bar{X}^J = -1$. Expressing the X^I in terms of the physical scalar fields $z^i = X^i / X^0$ $(i = 1, \ldots, n)$, we obtain

$$\begin{aligned} D_i \left(|Z|^2 + D_k Z g^{k\bar{j}} \bar{D}_{\bar{j}} \bar{Z} \right) &= 2 (D_i Z) \bar{Z} + i C_i{}^{\bar{k}\bar{l}} \bar{D}_{\bar{k}} \bar{Z} \bar{D}_{\bar{l}} \bar{Z} \,, \\ D_i \left(-3 |W|^2 + D_k W g^{k\bar{j}} \bar{D}_{\bar{j}} \bar{W} \right) &= -2 (D_i W) \bar{W} + i C_i{}^{\bar{k}\bar{l}} \bar{D}_{\bar{k}} \bar{W} \bar{D}_{\bar{l}} \bar{W} \,, \end{aligned} \tag{2.47}$$

where $g^{k\bar{j}}$ is the inverse of the metric on the moduli space of the physical scalars, C_{ijk} is a covariantly holomorphic symmetric tensor and D_i is the covariant derivative with respect to the usual Levi-Civita connection and the Kähler connection. In deriving (2.47), we used the following identities from special geometry (see for instance (3.2) in [37])

$$\begin{aligned} D_i D_j X^I &= i C_{ijk} g^{k\bar{l}} \bar{D}_{\bar{l}} \bar{X}^I \,, \\ D_i \bar{D}_{\bar{j}} \bar{X}^I &= g_{i\bar{j}} \bar{X}^I \,, \\ D_i \bar{X}^I &= 0 \,. \end{aligned} \tag{2.48}$$

Using the identity (see (23) of [38])

$$N^{IJ} = g^{i\bar{\jmath}}\, D_i X^I\, \bar{D}_{\bar{\jmath}} \bar{X}^J - X^I\, \bar{X}^J\,, \tag{2.49}$$

one sees that (2.47) and $\Delta = 0$ indeed imply $\partial_i V_{\rm tot} = 0$, i.e. an extremum. Using the identities (2.48), one also shows that at the extremum,

$$\begin{aligned} D_j D_i V_{\rm tot} &= D_j \partial_i V_{\rm tot} = \partial_j \partial_i V_{\rm tot} = e^{-4(\psi-U)} 4i C_{ij}{}^{\bar{k}}\, \bar{Z} \bar{D}_{\bar{k}} \bar{Z}\,, \\ \bar{D}_{\bar{\jmath}} D_i V_{\rm tot} &= \partial_{\bar{\jmath}} \partial_i V_{\rm tot} = e^{-4(\psi-U)} 4 C_i{}^{\bar{k}\bar{l}} \bar{C}_{\bar{\jmath}\bar{l}}{}^{p}\, D_p Z \bar{D}_{\bar{k}} \bar{Z}\,. \end{aligned} \tag{2.50}$$

This is markedly different from the ungauged case, where $\partial_j \partial_i V$ vanishes at the extremum and $\partial_{\bar{\jmath}} \partial_i V$ is positive definite there [36]. Thus, in the presence of fluxes, a more detailed analysis is required to decide whether an extremum of $V_{\rm tot}$ is actually a minimum. This we leave for future work.

Summarizing, for extremal black brane solutions ($k = 0$) the attractor equations for Ξ, Υ and Y^I are given by (2.16), (2.18) and (2.27). In the supersymmetric case, these become (2.40). The entropy density is related to the black hole potential by (2.34). When expressed in terms of charges and fluxes, it has weight zero under uniform scalings of the charges and of the fluxes.

Finally, let us consider the free energy (2.10). Using (2.15) and introducing the combination $\mathcal{Y}^I = \frac{1}{2}\left(\Xi^{-1}\, e^I + i p^I\right)$, we obtain

$$\begin{aligned} \frac{1}{2}\mathcal{F} = \; & \frac{1}{4}\Xi\Big[N_{IJ}\left(\mathcal{Y}^I\mathcal{Y}^J + \bar{\mathcal{Y}}^I\bar{\mathcal{Y}}^J\right) + i\left(F_{IJ} + \bar{F}_{IJ}\right)\left(\mathcal{Y}^I\mathcal{Y}^J - \bar{\mathcal{Y}}^I\bar{\mathcal{Y}}^J\right)\Big] \\ & -\frac{1}{2}\Xi\, N_{IJ}\Big[\left(\mathcal{Y}^I + \bar{\mathcal{Y}}^I\right)\left(Y^J + \bar{Y}^J\right) + \left(\mathcal{Y}^I - \bar{\mathcal{Y}}^I\right)\left(Y^J - \bar{Y}^J\right)\Big] \\ & +\frac{1}{2}\Xi\, N_{IJ}\left(Y^I Y^J + \bar{Y}^I \bar{Y}^J\right) \\ & -32 g^2\, \Xi\, \Upsilon^{-1}\, [K(Y,\bar{Y})]^2\, V(Y,\bar{Y})\,. \end{aligned} \tag{2.51}$$

In the absence of fluxes we have $k = 1$ and $\Xi = 1$, as can be seen from (2.15). In this case, BPS solutions satisfy $Y^I = \mathcal{Y}^I$, and the free energy evaluated on these solutions equals [39],

$$\mathcal{F} = -4\, {\rm Im}\, F(Y)\,. \tag{2.52}$$

In the presence of fluxes, no analogous simplification occurs.

2.2.1 Examples

The attractor equations (2.27) allow for supersymmetric solutions as well as for non-supersymmetric solutions. In the following, we give two examples of solutions to the attractor equations. The first example is non-supersymmetric, while the second example is supersymmetric, and therefore satisfies (2.36). The first example is based on the prepotential

$$F(Y) = -\left(Y^1\right)^3/Y^0 + ic\,(Y^0)^2 = i\left(Y^0\right)^2\left(t^3 + c\right)\,, \tag{2.53}$$

where $t = -iY^1/Y^0$ and $c < 0$. We take the solution to be supported by a non-vanishing electric charge q_0 and a non-vanishing electric flux h_1 satisfying $q_0\, h_1 < 0$. Then, we find that the attractor equations (2.16), (2.18) and (2.27) can be solved *exactly*, with the solution given by

$$t = \beta_1\,|c|^{1/3}\ ,\quad \Xi = \beta_2\,\frac{|c|^{2/3}}{|q_0 h_1|}\ ,\quad Y^0 = \beta_3\,\frac{q_0}{c}\ ,\quad \sqrt{-\Upsilon} = \beta_4\,\frac{|q_0 h_1|}{|c|^{2/3}}\ , \tag{2.54}$$

where the β_i denote fixed real constants given by

$$\beta_1 = 0.323\ ,\quad \beta_2 = 1.971\ ,\quad \beta_3 = 0.234\ ,\quad \beta_4 = 2.267\ . \tag{2.55}$$

Observe that the solution only exists because of the presence of the c-term in $F(Y)$, and that the modulus t takes a real and positive value that is independent of (q_0, h_1). On the solution, $F(Y) \neq 0$ since $\beta_1^3 + 1 \neq 0$. Using (2.19) we obtain

$$v_2 = -32\,\frac{(2\beta_1^3+1)\beta_3^2}{\beta_4}\,\frac{q_0}{h_1}\,\frac{1}{|c|^{1/3}}\ , \tag{2.56}$$

which is positive. In the limit of large c, t becomes large, while v_2 shrinks to zero. When embedding a supergravity model of the form (2.53) into type II string theory, requiring a large value of $\mathrm{Re}\,t$ is necessary in order to neglect worldsheet instanton contributions to $F(Y)$. However, in type II string theory the term c constitutes an α' correction, and hence a subleading term, while for the above solution both terms in $F(Y)$, t^3 and c, are of similar order (even though there is indeed a small hierarchy as $\beta_1^3 \approx 1/30$). Thus, while the above solution constitutes a solution to the supergravity toy model (2.53), for it to also constitute a solution to a string model would require taking worldsheet instanton effects into account.

The second example is based on the prepotential

$$F(Y) = -\left(Y^1Y^2Y^3 + a\,(Y^3)^3\right)/Y^0 = i\left(Y^0\right)^2\left(STU + a\,U^3\right)\ , \tag{2.57}$$

where $S = -iY^1/Y^0, T = -iY^2/Y^0, U = -iY^3/Y^0$ and $a > 0$. When embedded into heterotic string theory, the U^3-term constitutes a perturbative (one-loop) correction, and the prepotential (2.57) describes the perturbative chamber $S \gg T > U$ [40]. We consider solutions that are supported by charges (q_0, p^3) and fluxes (h_1, h_2, h_3, h^0). We demand that these satisfy the Hamiltonian constraint (2.35), $q_0 h^0 = p^3 h_3$, and we take the fluxes to be all positive. We seek a supersymmetric solution, and hence we proceed to solve (2.36), where we set $e^{i\delta} = i$, for concreteness. These equations constitute equations for S, T, U and v_2 and they take the form (we set $g = 1$)

$$\begin{aligned}
q_0 + p^3\left(ST + 3a\,U^2\right) &= 2v_2\,h^0\left(STU + a\,U^3\right)\ ,\\
p^3 T &= v_2\,\left(h_1 + h^0\,TU\right)\ ,\\
p^3 S &= v_2\,\left(h_2 + h^0 SU\right)\ ,\\
6a\,p^3 U &= v_2\,\left(h_3 + 3ah^0U^2 + h^0 ST\right)\ .
\end{aligned} \tag{2.58}$$

We focus on a solution satisfying

$$h_2\,T = h_1 S\ , \tag{2.59}$$

which is consistent with the second and third equations. We find that we can numerically construct an exact solution to (2.58) satisfying (2.59) that has the feature that it only exists for non-vanishing a. In particular, the field U blows up as $a \to 0$, with v_2 shrinking to zero in this limit. This is shown in figure 1. Thus, this solution only exists due to quantum corrections: they turn a non-AdS_2 geometry into an AdS_2 geometry. The solution can also be constructed iteratively, as follows. We expand S, T, U, v_2 as follows,

$$\begin{aligned} S &= s_0 + s_1\sqrt{a} + s_2 a + \dots \,, \\ T &= t_0 + t_1\sqrt{a} + t_2 a + \dots \,, \\ U &= \frac{u_0}{\sqrt{a}} + u_1 + u_2\sqrt{a} + u_3 a + \dots \,, \\ v_2 &= \alpha_0\sqrt{a} + \alpha_1 a + \dots \,. \end{aligned} \tag{2.60}$$

Inserting this ansatz into the attractor combination $\Delta_I = \hat{Q}_I - i\, v_2 \hat{h}_I$ yields the expansion

$$\begin{aligned} \Delta_i &= \Delta_i^{(0)} + \Delta_i^{(1)}\sqrt{a} + \dots \,, \quad i = 0,1,2, \\ \Delta_3 &= \Delta_3^{(1)}\sqrt{a} + \dots \,. \end{aligned} \tag{2.61}$$

This system can be solved iteratively, order by order in the expansion parameter a. When doing so, we find $u_1 = 0$. To simplify the expressions below, we set $u_1 = 0$ in (2.60) from the start. Then, to lowest order, the system $\Delta_i^{(0)} = 0$ yields

$$s_0\, t_0 = \frac{h_3 + h^0 u_0^2}{h^0} \,, \quad \alpha_0\, u_0 = \frac{p_3}{h^0} \,, \quad h_2\, t_0 = h_1\, s_0 \,. \tag{2.62}$$

This determines s_0, t_0 and α_0 in terms of u_0 which, at this order, remains undetermined, but gets determined recursively by going to the next order. At the next order, the system $\Delta_I^{(1)} = 0$ determines the values of the parameters s_0, t_0, u_0, α_0 to be

$$s_0 = \sqrt{\frac{2h_2h_3}{h^0 h_1}} \,, \quad t_0 = \sqrt{\frac{2h_1h_3}{h^0 h_2}} \,, \quad u_0 = \sqrt{\frac{h_3}{h^0}} \,, \quad \alpha_0 = \frac{p^3}{\sqrt{h^0 h_3}} \,. \tag{2.63}$$

In addition, using $h_2\, t_1 = h_1\, s_1$ (which follows from (2.59)), we obtain

$$t_1 = \frac{3}{2}\frac{h_1}{\sqrt{h^0 h_3}} \,, \quad \alpha_1 = -\sqrt{\frac{h_1 h_2}{2h^0 h_3^3}}\, p^3 \,. \tag{2.64}$$

The value of u_2 is again determined recursively by going to the next order. The approximate solution, obtained by solving (2.61), can be compared with the exact solution, see figures 1 and 2. The values (2.60) are invariant under uniform scalings of the charges and the fluxes, since the attractor equations (2.36) scale uniformly.

In addition, to determining the values of S, T, U, v_2, we also need to determine the values of Υ, Y^0 and v_1 (or equivalently of Ξ). We expand these fields as

$$\begin{aligned} \sqrt{-\Upsilon} &= \lambda_0 + \lambda_1\sqrt{a} + \dots \,, \\ Y^0 &= y_0\sqrt{a} + y_1 a + \dots \,, \\ v_1 &= \beta_0\,\sqrt{a} + \beta_1\, a + \dots \,. \end{aligned} \tag{2.65}$$

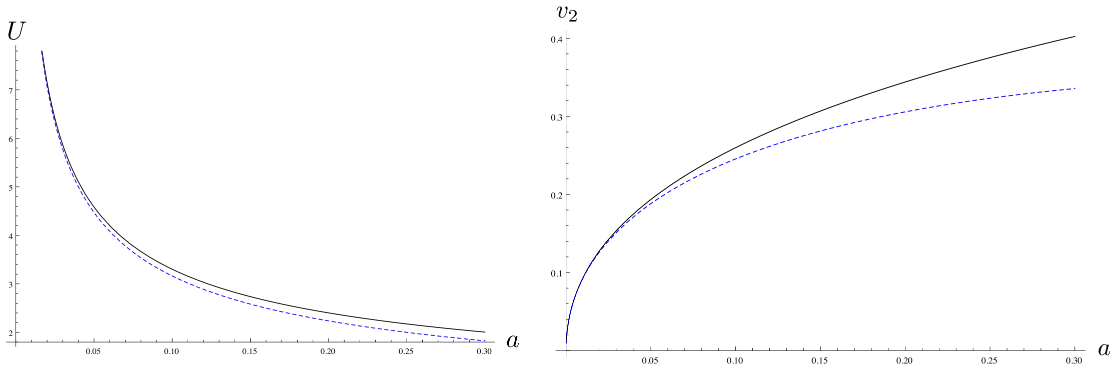

Figure 1. Dashed (leading approximation), solid (exact) ($h^0 = h_1 = h_2 = h_3 = p^3 = 1$).

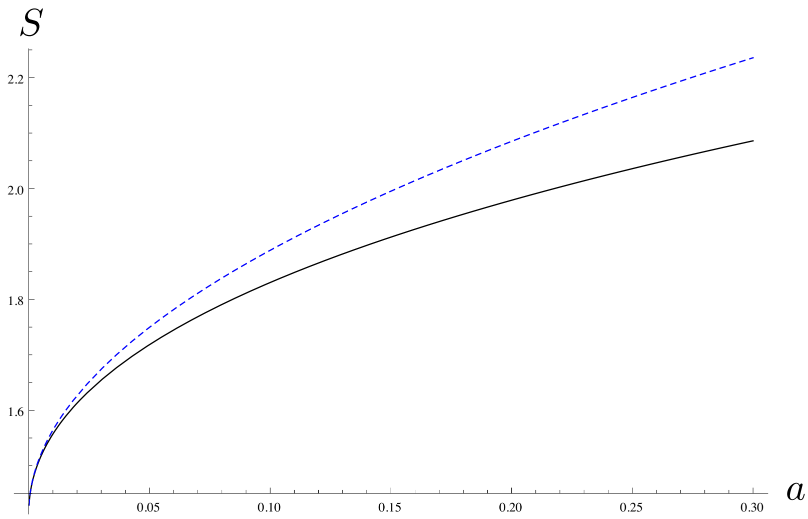

Figure 2. Dashed (leading approximation), solid (exact) ($h^0 = h_1 = h_2 = h_3 = p^3 = 1$).

Inserting this into (2.20) yields the following value for β_0,

$$\beta_0 = \frac{3}{8\sqrt{h^0\, h_3^3}}\ . \tag{2.66}$$

On the other hand, inserting the ansatz (2.65) into (2.16) and (2.18) leads to a determination of the lowest order coefficients λ_0 and y_0. We find (for h_3, h^0 and p^3 positive, for concreteness)

$$\begin{aligned} \lambda_0 &= \frac{16(3289 + 592\sqrt{30})}{1083} h_3 p^3\ , \\ y_0 &= \frac{37 + 8\sqrt{30}}{114} \frac{\sqrt{h^0}p^3}{\sqrt{h_3}}\ . \end{aligned} \tag{2.67}$$

To ensure that the exact solution to (2.58) is in the perturbative chamber $S \gg T > U \gg 1$ (we set $a = \frac{1}{3}$) , we have to choose the fluxes appropriately. By choosing h^0 to

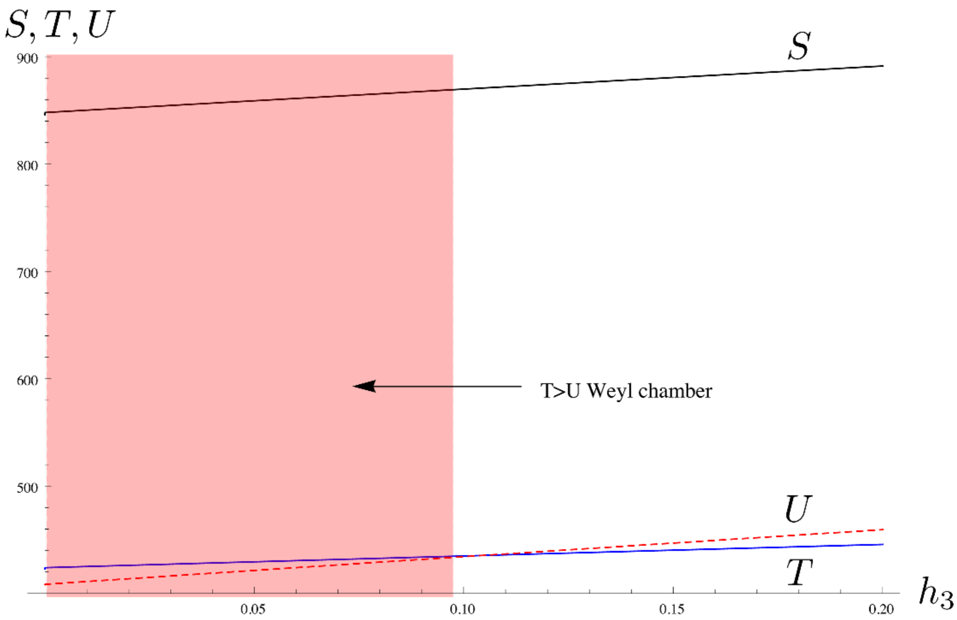

Figure 3. Ensuring $S \gg T > U \gg 1$ ($h^0 = 0.00001, h_1 = 1, h_2 = 2, a = 1/3$).

be small and taking $h_2/h_1 > 1$ we can ensure $S \gg T \gg 1$. Picking h_3 accordingly, we can then also enforce $T > U \gg 1$, as depicted in figure 3. In addition, the values of the fluxes may be chosen in such a way to ensure that there exists an interpolating solution that connects the $AdS_2 \times \mathbb{R}^2$ background discussed here to a solution that asymptotes to AdS_4. This will be discussed in section 3. Interestingly, we will obtain a flow to AdS_4 that only exists when $a \neq 0$.

Finally, we note that we could add a term proportional to $ic\,(Y^{(0)})^2$ to the prepotential (2.57) and repeat the analysis given above. Such a term also represents a perturbative correction in heterotic string theory. Its presence would lead to a modification of the solution given above. We have chosen not to include such a term in our analysis, for simplicity.

2.3 Variational equations with higher-derivative terms

Next, we turn to the entropy function (2.11) in the presence of higher-derivative terms, and we compute the associated extremization equations for the fields Ξ, Υ and Y^I. The quantities F, K, W are now given by (2.9) and (2.8). Although we will be interested in the black brane case ($k = 0$), we keep k as a bookkeeping device. We follow the exposition given in [29].

Varying with respect to Ξ gives

$$\Sigma + \left(\mathcal{Q}_I - F_{IK}\,\mathcal{P}^K\right) N^{IJ} \left(\mathcal{Q}_J - \bar{F}_{JL}\,\mathcal{P}^L\right) - \frac{8k}{\sqrt{-\Upsilon}}\, K(Y,\bar{Y}) \tag{2.68}$$
$$-i(F_\Upsilon - \bar{F}_\Upsilon)\Big[-4\Upsilon + 64(k^2 - \Xi^{-2}) - 16k\sqrt{-\Upsilon}\Big] - 64g^2\,\Upsilon^{-1}\,\left[K(Y,\bar{Y})\right]^2\,V(Y,\bar{Y}) = 0\,.$$

Expressing the combination $\mathcal{Q}_I - F_{IK}\,\mathcal{P}^K$ in terms of the combination (2.26) gives

$$\mathcal{Q}_I - F_{IK}\,\mathcal{P}^K = -\Sigma_I \;, \tag{2.69}$$

where

$$\Sigma_I = -\left(\hat{Q}_I + K_I + 2i\Upsilon F_{\Upsilon I}\right) \;. \tag{2.70}$$

Here $\Sigma_I = \partial_I \Sigma$, where Σ is given in (2.12). For $k = 0$, (2.68) becomes

$$\Sigma + \Sigma_I N^{IJ} \Sigma_{\bar{J}} - i(F_\Upsilon - \bar{F}_\Upsilon)\Big[-4\Upsilon - 64\Xi^{-2}\Big] - 64 g^2\,\Upsilon^{-1}\,\left[K(Y,\bar{Y})\right]^2\,V(Y,\bar{Y}) = 0 \;. \tag{2.71}$$

Next, we consider the variation of the entropy function (2.11) with respect to arbitrary variations of the fields Y^I and Υ and their complex conjugates. Denoting this variation by $\delta = \delta Y^I \partial/\partial Y^I + \delta\bar{Y}^I \partial/\partial \bar{Y}^I + \delta\Upsilon \partial/\partial\Upsilon + \delta\bar{\Upsilon}\partial/\partial\bar{\Upsilon}$, we obtain

$$\begin{aligned}
\frac{1}{2}\delta\mathcal{E} \;=\; & \Xi\left[\mathcal{P}^I\,\delta(F_I + \bar{F}_I) - \mathcal{Q}_I\,\delta(Y^I + \bar{Y}^I)\right] \\
& + \frac{1}{2} i\,\Xi\left[(\mathcal{Q}_K - \bar{F}_{KM}\,\mathcal{P}^M) N^{KI}\,\delta F_{IJ}\,N^{JL}(\mathcal{Q}_L - \bar{F}_{LN}\,\mathcal{P}^N) - \text{h.c.}\right] \\
& - 4i(-\Upsilon)^{-1/2}\,(k\Xi - 1)\,\left[(F_I - \bar{F}_I)\,\delta(Y^I + \bar{Y}^I) - (Y^I - \bar{Y}^I)\,\delta(F_I + \bar{F}_I)\right] \\
& + i\left[2\Xi\,\Upsilon - 32(k^2\Xi + \Xi^{-1} - 2k) + 16\sqrt{-\Upsilon}\right]\delta(F_\Upsilon - \bar{F}_\Upsilon) \\
& + i\,\Xi\left[\delta\Upsilon\,F_{\Upsilon I} N^{IJ}(\mathcal{Q}_J - \bar{F}_{JL}\,\mathcal{P}^L) - \text{h.c.}\right] \\
& - 2i(-\Upsilon)^{-3/2}\,(k\Xi - 1)\,(\bar{Y}^I F_I - Y^I \bar{F}_I)\,\delta\Upsilon \\
& + i(F_\Upsilon - \bar{F}_\Upsilon)\left[\Xi - 4(-\Upsilon)^{-1/2}\,(1 + k\Xi)\right]\,\delta\Upsilon \\
& - 32 g^2 \Xi\,\Upsilon^{-1}\left[2K\,V\,\delta K - \Upsilon^{-1}\,K^2\,V\,\delta\Upsilon + K^2\,\delta V\right] \;,
\end{aligned} \tag{2.72}$$

where we took into account that the variable Υ is real. For $k = 1$ this reduces to the expression derived in [29].

Restricting to variations δY^I gives

$$\begin{aligned}
& \Xi\left(\mathcal{Q}_I - F_{IJ}\,\mathcal{P}^J\right) - \frac{1}{2} i\,\Xi\left(\mathcal{Q}_K - \bar{F}_{KM}\,\mathcal{P}^M\right) N^{KP}\,F_{PIQ}\,N^{QL}\left(\mathcal{Q}_L - \bar{F}_{LN}\,\mathcal{P}^N\right) \\
& + 4i(-\Upsilon)^{-1/2}(k\Xi - 1)\left[F_I - \bar{F}_I - F_{IJ}(Y^J - \bar{Y}^J)\right] \\
& - i\left[2\Xi\,\Upsilon - 32(k^2\Xi + \Xi^{-1} - 2k) + 16\sqrt{-\Upsilon}\right]F_{\Upsilon I} + 32 g^2 \Xi\,\Upsilon^{-1}\left[2K\,K_I\,V + K^2\;V_I\right] = 0 \;.
\end{aligned} \tag{2.73}$$

Using $\hat{h}_I = \bar{\hat{h}}_I - iN_{IJ}\,h^J$, we obtain for V_I,

$$V_I = iN^{KP}\,F_{PQI}\,N^{QL}\,\bar{\hat{h}}_K\,\bar{\hat{h}}_L + 2K^{-2}\left[|W|^2\,K_I + \bar{W}\,K\,\hat{h}_I\right] \;. \tag{2.74}$$

Focussing on the black brane case ($k = 0$), we obtain from (2.73),

$$\begin{aligned}
& -\Xi\,\Sigma_I - \frac{1}{2} i\,\Xi\,\bar{\Sigma}_{\bar{K}} N^{KP}\,F_{PIQ}\,N^{QL}\,\bar{\Sigma}_{\bar{L}} - 4(-\Upsilon)^{-1/2}\left[K_I + 2i\Upsilon F_{\Upsilon I}\right] \\
& - i\left[2\Xi\,\Upsilon - 32\Xi^{-1} + 16\sqrt{-\Upsilon}\right]F_{\Upsilon I} + 32 g^2 \Xi\,\Upsilon^{-1}\left[2K\,K_I\,V + K^2\;V_I\right] = 0 \;,
\end{aligned} \tag{2.75}$$

and, using (2.70), we get

$$
\begin{aligned}
&\Xi\left(\hat{Q}_I+64g^2\Upsilon^{-1}K\bar{W}\hat{h}_I\right)-\frac{1}{2}i\,\Xi\,N^{KP}\,F_{PIQ}\,N^{QL}\left(\Sigma_{\bar{K}}\Sigma_{\bar{L}}-64g^2\,\Upsilon^{-1}\,K^2\,\bar{\hat{h}}_K\,\bar{\hat{h}}_L\right)\\
&+\Xi K_I\left(1-4(-\Upsilon)^{-1/2}\,\Xi^{-1}+64g^2\,\Upsilon^{-1}K\,V+64g^2\,\Upsilon^{-1}\,|W|^2\right)\\
&+i\left[32\,\Xi^{-1}-8\sqrt{-\Upsilon}\right]F_{\Upsilon I}=0\,.
\end{aligned}
\tag{2.76}
$$

Observe that all the terms transform as vectors under symplectic transformations.

Next, let us restrict (2.72) to variations $\delta\Upsilon$ (recall that Υ is a real variable). Setting $k=0$, and using

$$
\begin{aligned}
K_\Upsilon &= i\left(\bar{Y}^I F_{\Upsilon I}-Y^I\bar{F}_{\Upsilon\bar{I}}\right)\,, \qquad (2.77)\\
V_\Upsilon &= i\left(N^{KP}\,F_{PQ\Upsilon}\,N^{QL}\,\bar{\hat{h}}_K\,\bar{\hat{h}}_L-\text{h.c.}\right)+2K^{-2}\left[|W|^2\,K_\Upsilon-K\left(\bar{W}\,F_{\Upsilon I}+W\,\bar{F}_{\Upsilon\bar{I}}\right)h^I\right]\,,
\end{aligned}
$$

we obtain

$$
\begin{aligned}
&\Xi\,\left[iF_{\Upsilon I}N^{IJ}\left(-\Sigma_J-64ig^2\Upsilon^{-1}K\bar{W}\,N_{JK}h^K\right)+\text{h.c.}\right]\\
&+\frac{1}{2}\,i\,\Xi\left[N^{KP}\,F_{\Upsilon PQ}\,N^{QL}\left(\Sigma_{\bar{K}}\Sigma_{\bar{L}}-64g^2\,\Upsilon^{-1}\,K^2\,\bar{\hat{h}}_K\,\bar{\hat{h}}_L\right)-\text{h.c.}\right]\\
&+2\,\Xi\left(16g^2\Upsilon^{-2}K^2V+(-\Upsilon)^{-3/2}\Xi^{-1}K\right)\\
&-\Xi K_\Upsilon\left(-4(-\Upsilon)^{-1/2}\,\Xi^{-1}+64g^2\,\Upsilon^{-1}K\,V+64g^2\,\Upsilon^{-1}\,|W|^2\right)\\
&+i\left(\Xi-4(-\Upsilon)^{-1/2}\right)\left(F_\Upsilon-\bar{F}_\Upsilon\right)\\
&+i\left[2\Xi\,\Upsilon-32\Xi^{-1}+8\sqrt{-\Upsilon}\right]\left(F_{\Upsilon\Upsilon}-\bar{F}_{\Upsilon\Upsilon}\right)=0\,.
\end{aligned}
\tag{2.78}
$$

Observe that not all combinations are symplectic functions. This is so, because the derivative $\partial/\partial\Upsilon$, when acting on a symplectic function, does not yield a symplectic function [41]. To obtain combinations that are symplectic functions, we may use the mixed derivative $Y^I\partial/\partial Y^I+\bar{Y}^I\partial/\partial\bar{Y}^I+2\Upsilon\partial/\partial\Upsilon$, where Υ is real so that $\partial/\partial\Upsilon$ acts on both Υ and $\bar{\Upsilon}$ [29]. Then, using the homogeneity relation

$$
Y^IK_I+\bar{Y}^IK_{\bar{I}}+2\Upsilon K_\Upsilon=2K\,,
\tag{2.79}
$$

we obtain

$$
\begin{aligned}
&-2\Xi K\left(1-4(-\Upsilon)^{-1/2}\,\Xi^{-1}+64g^2\,\Upsilon^{-1}K\,V+64g^2\,\Upsilon^{-1}\,|W|^2\right)\\
&+2\Xi\left(16g^2\Upsilon^{-2}K^2V+(-\Upsilon)^{-3/2}\Xi^{-1}K\right)\\
&+2i\left(\Xi-4(-\Upsilon)^{-1/2}\right)\left(F_\Upsilon-\bar{F}_\Upsilon\right)\\
&+\Xi\left[Z(Y)+64g^2\,\Upsilon^{-1}\,K\,|W|^2-2i\Upsilon F_{\Upsilon I}N^{IJ}\Sigma_{\bar{J}}+\text{h.c.}\right]=0\,,
\end{aligned}
\tag{2.80}
$$

where $Z(Y)$ denotes the extension of (2.28) given by

$$
Z(Y)=p^IF_I(Y,\Upsilon)-q_IY^I\,.
\tag{2.81}
$$

Observe that each line of (2.80) constitutes a symplectic function.

Inserting (2.71) into the entropy function (2.11) with $k = 0$ gives

$$\mathcal{E} = \frac{8K}{\sqrt{-\Upsilon}} + 16i(F_\Upsilon - \bar{F}_{\bar{\Upsilon}})(\sqrt{-\Upsilon} - 8\Xi^{-1}) \ . \tag{2.82}$$

The black brane entropy density is given by (2.82), with Y^I, Υ and Ξ expressed in terms of charges and fluxes by solving the extremization equations (2.76), (2.80) and (2.71). To solve these equations, one may proceed iteratively by power expanding in Υ.

2.3.1 An example

In the presence of higher-derivative interactions, the extremization equations (2.76), (2.80) and (2.71) take a rather complicated form. One way to deal with these complications is to expand $F(Y,\Upsilon)$ in powers of Υ,

$$F(Y,\Upsilon) = \sum_{g=0}^{\infty} \Upsilon^g \, F^{(g)}(Y) \ , \tag{2.83}$$

and to solve the extremization equations order by order in Υ.

In the following, we will focus on a particular model with an $F^{(0)}$ and $F^{(1)}$ only, namely

$$F(Y,\Upsilon) = -\frac{Y^1 Y^2 Y^3}{Y^0} - \Upsilon \, c_1 \, \frac{Y^1}{Y^0} \ , \tag{2.84}$$

with $c_1 > 0$. In ungauged supergravity, this model captures features of $N = 4$ models in the presence of R^2 interactions. In particular, it allows for supersymmetric small black holes, which are solutions that only exist due to the presence of the term proportional to c_1 in (2.84) [42, 43]. Consider a small black hole that carries charges (q_0, p^1) with $q_0 p^1 < 0$. The supersymmetric attractor equations for the Y^I are $Y^I - \bar{Y}^I = ip^I$ and $F_I - \bar{F}_I = iq_I$. They can be readily solved for the model (2.84) [44]. The attractor values for $T = -iY^2/Y^0$ and $U = -iY^3/Y^0$ are zero, while $S = -iY^1/Y^0$ and Y^0 take non-vanishing values that exhibit the following scaling behavior with c_1 [45],

$$\begin{aligned} S + \bar{S} &= \frac{s_0}{\sqrt{c_1}} \ , \\ Y^0 = \bar{Y}^0 &= y_0 \sqrt{c_1} \ , \end{aligned} \tag{2.85}$$

where s_0 and y_0 are given by $s_0 = \sqrt{|q_0 p^1/\Upsilon|}$, $y^0 = \sqrt{|\Upsilon \, p^1/q_0|}$, and the field Υ takes the value $\Upsilon = -64$ at the horizon [46]. The entropy $\mathcal{E}$, which is non-vanishing, is determined in terms of $S + \bar{S}$ as $\mathcal{E} = 32\pi\sqrt{|q_0 p^1|}\,\sqrt{c_1}$.

Now consider turning on fluxes (h^0, h_1, h_2, h_3). For large charges, and for a certain range of fluxes, we expect that there exist black brane solutions whose near-horizon geometry can be approximated by the geometry of a small black hole. Thus, we expect to be able to construct black brane solutions to the extremization equations (2.76), (2.80) and (2.71) that are supported by charges (q_0, p^1) and have the scaling behavior

$$\begin{aligned}
S+\bar{S} &= \frac{s_0}{\sqrt{c_1}} + s_1 + \dots\ , \\
T+\bar{T} &= t_0 + t_1\sqrt{c_1} + \dots\ , \\
U+\bar{U} &= u_0 + u_1\sqrt{c_1} + \dots\ , \\
Y^0 &= y_0\sqrt{c_1} + y_1 c_1 + \dots\ , \\
\sqrt{-\Upsilon} &= \lambda_0 + \lambda_1\sqrt{c_1} + \dots\ , \\
\Xi &= \xi_0 + \xi_1\sqrt{c_1} + \dots\ .
\end{aligned} \tag{2.86}$$

We note the ansatz (2.86) for the scaling behavior is analogous to the one discussed in (2.60) for the model (2.57).

We proceed to solve the attractor equations (2.76), (2.80) and (2.71) iteratively by power expanding in c_1, in a manner similar to what we did for the model (2.57). Concretely we found a solution for $h^0 = 0 = h_1$ and with $q_0 < 0$ and $p^1 > 0$.[5] Plugging the ansatz (2.86) into the five attractor equations (2.76), (2.80) and (2.71) and expanding each one in a power series in c_1, the five leading order equations are solved by

$$s_0 = 0.441\frac{\sqrt{|q_0|p_1}}{\lambda_0}\ , \quad t_0 = u_0 = 1.306\sqrt{\frac{|q_0|}{p_1}}\ , \quad y_0 = 1.190\frac{\sqrt{p_1}\lambda_0}{\sqrt{|q_0|}}\ , \quad \xi_0 = \frac{4.898}{\lambda_0}\ , \tag{2.87}$$

whereas λ_0 is not constrained at this order. Inserting (2.87) into the equations obtained by expanding each of the attractor equations to their next order in c_1 gives five constraints involving λ_0 and the subleading coefficients of the expansion (2.86). However, λ_0 and λ_1 are again not constrained and we find

$$\begin{aligned}
s_1 &= -0.441\frac{\sqrt{|q_0|p_1}\lambda_1}{\lambda_0^2} = -s_0\frac{\lambda_1}{\lambda_0}\ , \\
t_1 &= u_1 = 0\ , \\
y_1 &= 1.190\frac{\sqrt{p_1}\lambda_1}{\sqrt{|q_0|}} = y_0\frac{\lambda_1}{\lambda_0}\ , \\
\xi_1 &= -\frac{4.898\lambda_1}{\lambda_0^2} = -\xi_0\frac{\lambda_1}{\lambda_0}\ .
\end{aligned} \tag{2.88}$$

Observe that whereas the small black hole solution (2.85) represents an exact solution to the supersymmetric attractor equations of ungauged supergravity, the black brane solution discussed here will receive corrections order by order in c_1. This is due to the complicated form of the attractor equations (2.76), (2.80) and (2.71). Moreover, naively it appears as if the solution we found does not depend on the values of the fluxes h_2 and h_3. However, this is an artefact of our truncation to the lowest orders in the c_1-expansion. The full attractor equations do depend on h_2 and h_3 and we expect the more subleading coefficients in the expansion (2.86) to depend on them as well. It would be interesting to pursue this point further.

[5]Given that we turned off the fluxes h^0 and h_1 it is likely impossible to extend this case to an asymptotic AdS_4 solution. Finding such an interpolating solution with higher derivative corrections is clearly outside the scope of this paper.

3 Interpolating solutions

In the following, we will switch off higher-derivative interactions and consider interpolating extremal black brane solutions in the presence of quantum corrections to the prepotential. We will focus on solutions that only exist due to the presence of these quantum corrections. For concreteness, we pick the STU-model described by (2.57), and we construct solutions that interpolate between a near-horizon geometry $AdS_2 \times \mathbb{R}^2$ and an AdS_4 geometry. These solutions will be supported by fluxes (h^0, h_1, h_2, h_3) as well as by charges (q_0, p^1, p^2, p^3). More precisely, we will consider what happens when one or two of the magnetic charges p^A $(A = 1, 2, 3)$ are turned off. The fluxes and the charges are subjected to the Hamiltonian constraint (2.35).

The choice of the fluxes ensures that the flux potential (2.7) has AdS_4 extrema. For the prepotential (2.57) we find various such extrema. We will focus on two of them, as follows. We take the fluxes (h^0, h_1, h_2, h_3) to be all positive. The first extremum is of standard type, i.e. it occurs for the uncorrected prepotential $(a = 0)$. The values of the scalar fields S, T, U at this extremum are given by

$$S = \sqrt{\frac{h_2 h_3}{h^0 h_1}} \,, \quad T = \sqrt{\frac{h_1 h_3}{h^0 h_2}} \,, \quad U = \sqrt{\frac{h_1 h_2}{h^0 h_3}} \,. \tag{3.1}$$

These values will get corrected when switching on a. At the extremum (3.1) the flux potential takes the value

$$V_F = -6\sqrt{h^0 h_1 h_2 h_3} \,. \tag{3.2}$$

In the figures given below, this extremum is denoted by type 1 AdS_4 fixed point and is represented by a blue dot.

The second extremum is not of standard type, and only exists in the presence of quantum corrections, i.e. when $a \neq 0$. The values of the scalar fields S, T, U at this extremum are, to leading order in a, given by

$$S = h_2 \left(\frac{3}{h^0 h_3}\right)^{1/2} \sqrt{a} \,, \quad T = h_1 \left(\frac{3}{h^0 h_3}\right)^{1/2} \sqrt{a} \,, \quad U = \left(\frac{h_3}{3h^0}\right)^{1/2} \frac{1}{\sqrt{a}} \,, \tag{3.3}$$

and the value of the flux potential at this extremum is, to leading order in a,

$$V_F = -2\left(\frac{h^0 h_3^3}{3}\right)^{1/2} \frac{1}{\sqrt{a}} \,. \tag{3.4}$$

In the figures given below, this extremum is denoted by type 2 AdS_4 fixed point and is represented by a red dot.

Next, we construct interpolating black brane solutions that asymptotically flow to one of these two AdS_4 extrema. These interpolating solutions are obtained as solutions to first-order flow equations [23–26].[6] They are described by a static line element of the form

$$ds^2 = -e^{2U} dt^2 + e^{-2U} dr^2 + e^{2A}\left(dx^2 + dy^2\right) \,, \tag{3.5}$$

[6] See [47] for a discussion of first-order flow equations for extremal black branes in five dimensions.

where $U = U(r)$ and $A = A(r)$. The solutions are supported by scalar fields X^I. It is convenient to introduce rescaled scalar fields Y^I given by[7] [26]

$$Y^I = e^A \bar{\varphi} X^I \, , \tag{3.6}$$

where the field φ denotes a U(1) compensator. The first-order flow equations can then be expressed in terms of the scalars $Y^I = Y^I(r)$ as follows,

$$\begin{aligned} \left(Y^I\right)' &= e^{-\psi - i\gamma} N^{IK} \left(\bar{\hat{Q}}_K + i g e^{2A} \bar{\hat{h}}_K \right) \, , \\ \psi' &= 2 g e^{-\psi} \, \mathrm{Im} \left[e^{i\gamma} W(Y) \right] \, , \end{aligned} \tag{3.7}$$

where $\psi = A + U$, and with $\hat{Q}, \hat{h}$ and $W(Y)$ as defined in (2.26), (2.3) and (2.8) (with F restricted to $F(Y)$). The quantity e^{2A} is determined in terms of the Y^I by

$$e^{2A} = K(Y, \bar{Y}) \, , \tag{3.8}$$

whereas the phase γ satisfies

$$e^{-2i\gamma} = \frac{Z(Y) - i g e^{2A} W(Y)}{\bar{Z}(\bar{Y}) + i g e^{2A} \bar{W}(\bar{Y})} \, , \tag{3.9}$$

with $Z(Y)$ given in (2.28).

The first-order flow equations (3.7) may have fixed points determined by (2.36), where $e^{2A} = v_2$ and $e^{i\delta} = i$. One such fixed point was already obtained in (2.63) and (2.66), and it arises when the two magnetic charges p^1, p^2 are switched off. Another fixed point occurs when switching off the magnetic charge p^3. In this case the attractor values for S, T, U and e^{2A} are, to leading order in a, given by (we set $g = 1$ in the following)

$$S = \sqrt{\frac{h_3 p^1}{h^0 p^2}} \, , \quad T = \sqrt{\frac{h_3 p^2}{h^0 p^1}} \quad U = \left(\frac{h_3}{h^0 p^1 p^2} \right)^{1/6} \left(\frac{q_0}{a} \right)^{1/3} \, , \quad e^{2A} = \sqrt{\frac{p^1 p^2}{h^0 h_3}} \, , \tag{3.10}$$

while the value of the flux potential at the attractor is, to leading order in a, given by

$$V_F \sim -h^0 h_3 \left(\frac{h_3}{h^0 p^1 p^2} \right)^{1/6} \left(\frac{q_0}{a} \right)^{1/3} \, . \tag{3.11}$$

Yet another fixed point is obtained when p^2 is switched off. In this case, and taking into account that the fluxes (h^0, h_1, h_2, h_3) are all positive, we find the following attractor values at leading order in a,

$$S = \frac{s_0}{\sqrt{a}} \, , \quad T = t_0 \sqrt{a} \, , \quad U = \frac{u_0}{\sqrt{a}} \, , \quad e^{2A} = \alpha_0 \sqrt{a} \, , \tag{3.12}$$

where the values s_0, t_0, u_0, α_0 are rather complicated expressions in terms of charges and fluxes, which we do not give here. We only note the relations

$$p^1 = -h_1 \left(\frac{s_0 \alpha_0}{t_0 u_0} \right) < 0 \, , \quad p^3 = \alpha_0 \left(\frac{h_1 + h^0 t_0 u_0}{t_0} \right) > 0 \, , \tag{3.13}$$

[7] Note that the field Y^I differs from the one introduced in (2.6).

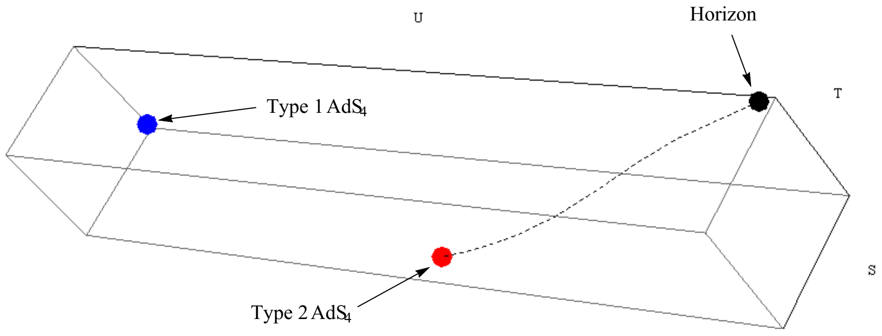

Figure 4. Flow from AdS_2 (black dot) to the type 2 AdS_4 fixed point (3.3) (red dot) ($q_0 = 10, p^1 = p^2 = 0, p^3 = 10; h^0 = h_1 = h_2 = h_3 = 1$).

which constrain the signs of the magnetic charges p^1, p^3. The value of the flux potential at the attractor is, to leading order, given by

$$V_F = -\frac{v(h,p,q)}{\sqrt{a}} \, , \tag{3.14}$$

with $v > 0$.

The three fixed points discussed above give rise to $AdS^2 \times \mathbb{R}^2$ geometries that only exist due to the presence of the a-term in the prepotential (2.57). The fixed points with either non-vanishing p^3 or non-vanishing p^1 and p^3 give rise to $AdS_2 \times \mathbb{R}^2$ geometries (2.4) with $v_1, v_2 \sim a^{1/2}$, as can be seen using (2.20). The fixed point with non-vanishing charges p^1 and p^2 has $v_1 \sim a^{1/3}$ and $v^2 = \mathcal{O}(a^0)$. All these geometries have in common that in the limit $a \to 0$, the AdS_2 factor v_1 shrinks to zero. The two fixed points for which, in addition, also $v_2 \to 0$, have entropy densities that exhibit Nernst behavior in the limit $a = 0$. Note that the ratio v_1/v_2 remains finite in this limit.

Next, we would like to check whether the three AdS_2 fixed points can be connected to the two AdS_4 fixed points discussed earlier. This can be done by numerically solving the first-order flow equations (3.7), as explained in appendix C. Our findings are summarized in figures 4–6. They represent the flows in the three-dimensional S-T-U moduli space for different charge configurations. In these figures, the black dot represents the AdS_2 fixed point, while the blue and red dots represent the AdS_4 fixed points (3.1) and (3.3), which we denote by type 1 and type 2 AdS_4 fixed points, respectively. We also took $a = 0.01$ in these plots.

First consider the case when $p^1 = p^2 = 0$. Then, we find a flow connecting the associated AdS_2 fixed point to the AdS_4 fixed point (3.3), as depicted in figure 4. When $p^3 = 0$, we find a flow connecting the associated AdS_2 fixed point to the AdS_4 fixed point (3.1), as depicted in figure 5. And finally, when $p^2 = 0$, we find a flow connecting the associated AdS_2 fixed point to the AdS_4 fixed point (3.3), as depicted in figure 6.

When $p^3 = 0$, we can show that the flow of the scalar fields remains in the perturbative chamber $S \gg T > U$ when suitably choosing the charges and the fluxes. This is depicted in

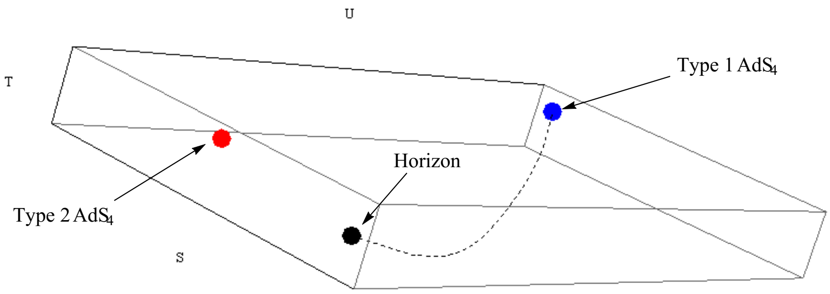

Figure 5. Flow from AdS_2 (black dot) to the type 1 AdS_4 fixed point (3.1) (blue dot) ($q_0 = 14, p^1 = 12, p^2 = 2, p^3 = 0; h^0 = h_1 = h_2 = h_3 = 1$).

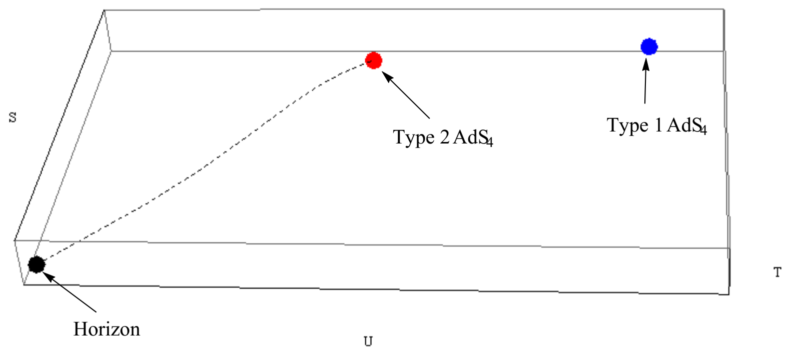

Figure 6. Flow from AdS_2 (black dot) to the type 2 AdS_4 fixed point (3.3) (red dot) ($q_0 = 9, p^1 = -1, p^2 = 0, p^3 = 10; h^0 = h_1 = h_2 = h_3 = 1$).

figure 7. The behavior of the metric factors e^{2U} and e^{2A} is depicted in figure 8. Note that, in a regime where e^{2U} or e^{2A} simply scale as powers, the quantities $\frac{d(\ln e^{2U})}{d\ln r}$ and $\frac{d(\ln e^{2A})}{d\ln r}$ give these powers, i.e. $\frac{d(\ln r^\rho)}{d\ln r} = \rho$. From figure 8 one reads off an intermediate scaling regime where both e^{2U} and e^{2A} roughly scale linearly in r. One might wonder whether this can be interpreted as a scaling regime with non-trivial dynamical critical exponent z and hyperscaling violation parameter θ. To answer his question we need to know the relation between the scalings of e^{2U} and e^{2A} on the one hand and θ and z on the other hand. For

$$e^{2U} \sim r^{2\alpha}, \qquad e^{2A} \sim r^{2\beta} \tag{3.15}$$

one finds [15]

$$\theta = \frac{2(\alpha - 1)}{\alpha + \beta - 1}, \qquad z = \frac{2\alpha - 1}{\alpha + \beta - 1}. \tag{3.16}$$

To answer the question whether there is an intermediate scaling regime we focus on

$$\eta = -\frac{\theta}{z} = -\frac{2(\alpha - 1)}{2\alpha - 1}. \tag{3.17}$$

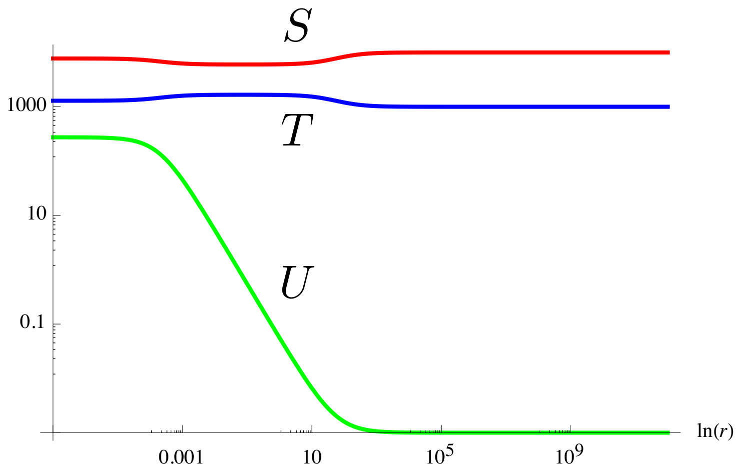

Figure 7. Flow in the perturbative chamber $S \gg T > U$ ($q_0 = 32, p^1 = 12, p^2 = 2, p^3 = 0; h^0 = h_1 = 1, h_2 = 10, h_3 = 10^7$), using $a = 0.001$.

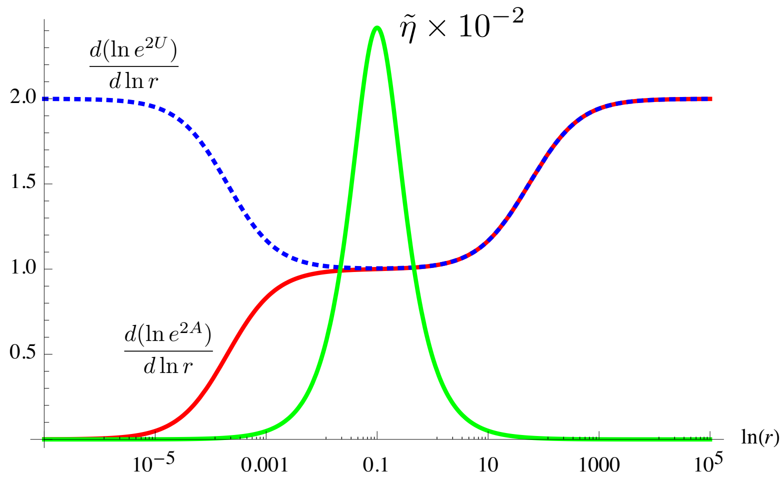

Figure 8. Metric behavior ($q_0 = 32, p^1 = 12, p^2 = 2, p^3 = 0; h^0 = h_1 = 1, h_2 = 10, h_3 = 10^7$), using $a = 0.001$. See (3.18) for a definition of $\tilde{\eta}$.

If there was a scaling regime with particular values of θ and z, one would have to see a plateau when plotting

$$\tilde{\eta} = -\frac{2\left(\frac{d(\ln e^U)}{d\ln r} - 1\right)}{2\frac{d(\ln e^U)}{d\ln r} - 1} \,. \tag{3.18}$$

This, however, is not the case, as depicted in figure 8.

Let us next come to an analysis of the near-horizon geometry for the (q_0, p^1, p^2) configuration when a is completely switched off. It is straightforward to check that the following ansatz solves the flow equations (3.7) in the limit $r \to 0$,

$$Y^0 \sim \sqrt{r}\,, \quad Y^1, Y^2 \sim r^0\,, \quad Y^3 \sim r\,, \quad e^{\psi} = r\,, \tag{3.19}$$

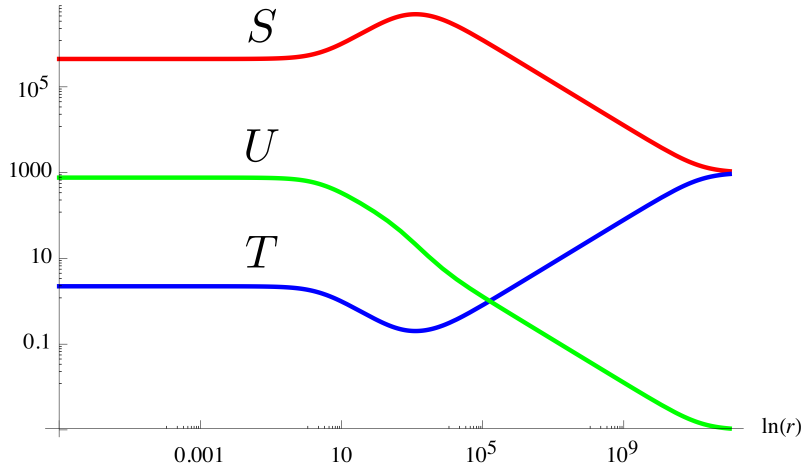

Figure 9. Flow in the purely magnetic case ($q_0 = 0, p^1 = 1.999.990, p^2 = 10, p^3 = -2; h^0 = h_1 = h_2 = 1, h_3 = 10^6$), using $a = 0.001$.

with $\gamma = 0$. This results in $e^{2A} \sim r^{1/2}$ and $e^{2U} \sim r^{3/2}$. The associated line element

$$ds^2 = r^{1/2} \left[-r dt^2 + r^{-2} dr^2 + dx^2 + dy^2\right] \ , \tag{3.20}$$

describes a so-called η-geometry [13], namely

$$ds^2 = \tilde{r}^{-\eta} \left[-\tilde{r}^{-2} dt^2 + l^2\, \tilde{r}^{-2} d\tilde{r}^2 + dx^2 + dy^2\right] \ , \tag{3.21}$$

where $r = \tilde{r}^2$, $l^2 = 4$ and $\eta = 1$. When turning on the regulator a, this η-geometry gets modified into the $AdS_2 \times \mathbb{R}^2$-geometry discussed above for the (q_0, p^1, p^2)-system.

Finally we would like to give an example of a flow which is purely magnetic, as this is the case that much of the earlier literature focussed on, cf. [14, 15]. In this case it is possible to find a scaling regime with $\eta \approx 1$. However, it does not stay all the way in the perturbative chamber, cf. figures 9 and 10. One way to enforce staying in the perturbative chamber in the purely magnetic case would be to choose p_1 and p_2 of equal order (i.e. both of order 10^6 in the example of figures 9 and 10). However, in that case the η-scaling regime disappears and the plots look very similar to figures 7 and 8.

We would like to end with a comment on the $a \to 0$ limit. One might expect that in this limit the scaling regime of figure 10 extends more and more into the $AdS_2 \times \mathbb{R}^2$-region. To a small extend this indeed happens and the scaling regime also gets more extended to larger values of r when decreasing the value of a. However, the effect of decreasing a is actually surprisingly small. Changing a from 10^{-1} to 10^{-7} cuts the $AdS_2 \times \mathbb{R}^2$-region only by a factor of about 10. The reason for this small effect seems to be that the attractor values of the scalars S, T and U are getting larger for smaller values of a, counterbalancing the decrease of a in the quantum correction aU^3 to the prepotential and, thus, preventing it from becoming negligible.

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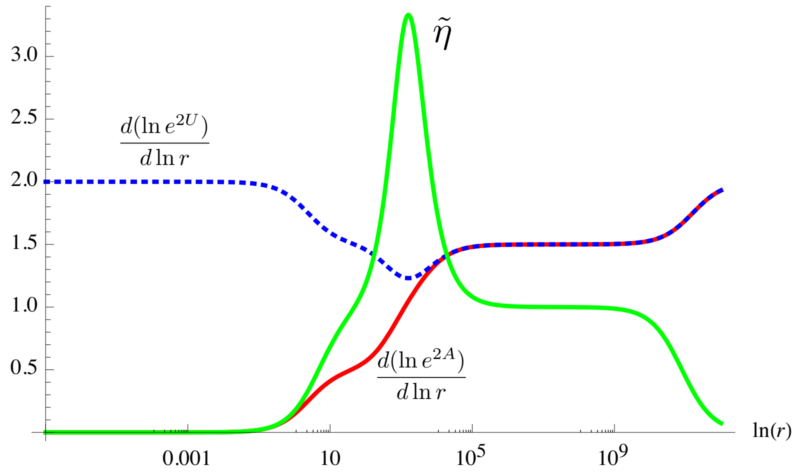

Figure 10. Metric behavior ($q_0 = 0, p^1 = 1.999.990, p^2 = 10, p^3 = -2; h^0 = h_1 = h_2 = 1, h_3 = 10^6$), using $a = 0.001$. See (3.18) for a definition of $\tilde{\eta}$.

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A Homogeneity relations

The homogeneity relation $F(\lambda\, Y, \lambda^2\, \Upsilon^2) = \lambda^2\, F(Y, \Upsilon)$ implies

$$\begin{aligned} 2F &= Y^I F_I + 2\Upsilon F_\Upsilon \,, \\ Y^I F_{IJK} &= -2\Upsilon F_{\Upsilon JK} \,, \\ Y^I F_{I\Upsilon} &= -2\Upsilon F_{\Upsilon\Upsilon} \,, \\ F_{\Upsilon I} + Y^J F_{\Upsilon JI} &= -2\Upsilon F_{\Upsilon\Upsilon I} \,. \end{aligned} \tag{A.1}$$

B Field combinations

The computation of the free energy (2.10) makes use of various field combinations that in [29] were computed for an $AdS_2 \times S^2$ geometry. Here we adapt these results to a

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background of the form (2.4). Indices i, j refer to the AdS_2 coordinates r, t, whereas indices α, β refer to coordinates of $d\Omega_k^2$. We obtain for the field combinations considered in [29],

$$\begin{aligned}
R &= 2\left(v_1^{-1} - k\, v_2^{-1}\right) \,, \\
f_i{}^j &= \left[\frac{1}{2}v_1^{-1} - \frac{1}{4}(D + \frac{1}{3}R) - \frac{1}{32}|w|^2\right]\delta_i{}^j \,, \\
f_\alpha{}^\beta &= \left[-\frac{k}{2}v_2^{-1} - \frac{1}{4}(D + \frac{1}{3}R) + \frac{1}{32}|w|^2\right]\delta_\alpha{}^\beta \,, \\
\mathcal{R}(M)_{ij}{}^{kl} &= \left(D + \frac{1}{3}R\right)\,\delta_{ij}{}^{kl} \,, \\
\mathcal{R}(M)_{\alpha\beta}{}^{\gamma\delta} &= \left(D + \frac{1}{3}R\right)\,\delta_{\alpha\beta}{}^{\gamma\delta} \,, \\
\mathcal{R}(M)_{i\alpha}{}^{j\beta} &= \frac{1}{2}\left(D - \frac{1}{6}R\right)\,\delta_i^j\,\delta_\alpha^\beta \,, \\
\hat{A} &= -4w^2 \,, \\
\hat{F}^-_{\underline{rt}} &= -16w\left(D + \frac{1}{3}R\right) \,, \\
\hat{C} &= 192D^2 + \frac{32}{3}R^2 - 16|w|^2(v_1^{-1} + k\, v_2^{-1}) + 2|w|^4 \,,
\end{aligned} \tag{B.1}$$

where we recall that k denotes the curvature of the two-dimensional space with line element $d\Omega_k^2$.

The resulting field equations for $\tilde{D}$ and $\tilde{\chi}$ become

$$\begin{aligned}
\tilde{D} &= 0\,, \\
\tilde{\chi} &= -\frac{16i}{\sqrt{-\Upsilon}}(\bar{Y}^I F_I - Y^I \bar{F}_I) - 256i(F_\Upsilon - \bar{F}_\Upsilon)(k - \Xi^{-1}) \\
&\quad +32i\sqrt{-\Upsilon}\Big[F_{I\Upsilon}N^{IJ}(\mathcal{Q}_J - \bar{F}_{JK}\mathcal{P}^K) - \text{h.c.}\Big] \,.
\end{aligned} \tag{B.2}$$

C Numerical interpolation

Here we outline a procedure to perform the numerical interpolation of the flow equations (3.7). It is convenient to recast these equations in terms of a radial coordinate τ defined by

$$e^\psi \frac{\partial}{\partial r} = -\frac{\partial}{\partial \tau}. \tag{C.1}$$

For the following it is important that the flow equations are autonomous (i.e. the right hand sides do not depend explicitly on the independent variable τ) and they read

$$\dot{Y}^I = -N^{IK}\left(\bar{\hat{Q}}_K + ige^{2A}\bar{\hat{h}}_K\right), \tag{C.2}$$

$$\dot{\psi} = -2g\,\mathrm{Im}\left[W(Y)\right]. \tag{C.3}$$

Here we set $\gamma = 0$. In terms of τ, the attractor nature of the horizon becomes manifest since, as $\tau \to \infty$, the moduli flow towards an equilibrium state, i.e. they tend towards

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constant values $\tilde{Y}^I$. Since e^ψ tends to zero when approaching an $AdS_2 \times \mathbb{R}^2$ geometry, it follows from (C.3) that

$$2g\,\mathrm{Im}\left[W(\tilde{Y})\right] < 0. \tag{C.4}$$

In order to find interpolating solutions we must figure out how to move away from this equilibrium configuration. To understand the possible deviations we need to linearize the system (C.2) around the attractor point and study the eigenvalues of the Jacobian of the system. This is a rather difficult task, but it is possible to show that there always exists at least one stable direction, i.e. a class of deformations that eventually evolves back towards equilibrium. This can be shown as follows.

Consider a deviation of the form

$$Y^I(\tau) = \tilde{Y}^I + \epsilon v^I e^{\lambda\tau}\ , \tag{C.5}$$

where $\epsilon \ll 1$. To linear order in ϵ, eq. (C.2) becomes

$$\lambda v^I = (\tilde{\mathcal{J}}^{(1)})^I{}_J v^J + (\tilde{\mathcal{J}}^{(2)})^I{}_J \bar{v}^J\ , \tag{C.6}$$

where

$$(\tilde{\mathcal{J}}^{(1)})^I{}_J = N^{IK}\left(\bar{F}_{JKL}\left(p^L + ige^{2A}h^L\right) + ig\left(N\bar{Y}\right)_J \bar{\hat{h}}_K\right)\Big|_{Y^I(\tau)=\tilde{Y}^I} \tag{C.7}$$

and

$$(\tilde{\mathcal{J}}^{(2)})^I{}_J = ig\left(NY\right)_J N^{IK}\bar{\hat{h}}_K\Big|_{Y^I(\tau)=\tilde{Y}^I} \tag{C.8}$$

are the two contributions from the Jacobian of (C.2). Then we can show that

$$\tilde{K}_{\bar{I}}(\tilde{\mathcal{J}}^{(1)})^I{}_J = -igW(\tilde{Y})\tilde{K}_{\bar{J}}\quad ,\quad \tilde{K}_{\bar{I}}(\tilde{\mathcal{J}}^{(2)})^I{}_J = -igW(\tilde{Y})\tilde{K}_J\ , \tag{C.9}$$

where we used $K_{\bar{I}} = \partial_{\bar{Y}^I}K = -(NY)_I$ and $K_J = \partial_{Y^J}K = -(N\bar{Y})_J$ and the tilde over $K_{\bar{I}}$ indicates that it is evaluated at the equilibrium values $\tilde{Y}$. The last equation, when combined with (C.6), implies that

$$\lambda = 2g\,\mathrm{Im}\left[W(\tilde{Y})\right]\ , \tag{C.10}$$

which is negative due to (C.4). Hence $Y(\tau)$ given in (C.5) approaches equilibrium as $\tau \to \infty$. The next step is to find the components of v^I. As a matter of fact we can only find the direction in moduli space in which v^I points, but this is all we need, since ϵ can be used to tune the size of the vector. Having determined the aforementioned direction, we can perform a numerical integration in order to find the flow line passing through

$$Y(0) = \tilde{Y}^I + \epsilon v^I. \tag{C.11}$$

on its way towards the horizon. Effectively, in the examples we looked at, the resulting flow coincides with the one that follows from the procedure outlined in section 3.1.2 of [26].[8]

[8]We would be happy to make our mathematica code available upon request.

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