$\mathcal{N} = 1$ SUSY AdS_4 vacua in IIB SUGRA on group manifolds

G. Solard

*Sorbonne Universités, UPMC Univ Paris 06, UMR 7589, LPTHE,
F-75005, Paris, France
CNRS, UMR 7589, LPTHE,
F-75005, Paris, France*

E-mail: solard@lpthe.jussieu.fr

ABSTRACT: We study $\mathcal{N} = 1$ compactification of IIB supergravity to AdS_4 . The internal manifold must have $\text{SU}(2)$ -structure. By putting some restrictions on the $\text{SU}(2)$ torsion classes, we can perform an exhaustive scan of all possible solutions on group manifolds (a priori non compact). We show that sourceless solutions don't exist in our class and that the presence of O5 and O7 orientifolds is mandatory. We also find a new solution and relate it by T-duality to a known type IIA solution with O6 planes.

KEYWORDS: Flux compactifications, Superstring Vacua

ARXIV EPRINT: [1310.4836](https://arxiv.org/abs/1310.4836)

Contents

1	Introduction	1
2	The supersymmetry conditions	3
3	Looking for vacua	7
3.1	SUSY constraints	7
3.2	The quadratic equations for group manifolds	8
3.2.1	Solving the Jacobi Identities on group manifold	8
3.3	Bianchi identities	9
3.3.1	Sourceless solutions	9
3.3.2	Sign of charges	10
4	Scanning group manifolds	11
4.1	A new solution	13
5	Intermediate $SU(2)$ structures on group manifolds	14
6	Conclusion	16
A	Structure constants	17
B	Solving the NS Bianchi identity	18
C	T-duality of $(0,0,25,-26,-23,24)$	18
D	Name of the new solution	20

1 Introduction

Even if of no direct phenomenological interest, compactifications to four-dimensional Anti de Sitter space are worth studying for several reasons. For instance, they are relevant for the CFT_3/AdS_4 correspondence and they might represent the first step in the construction of de Sitter vacua in string theory. In type IIA the literature on SUSY AdS_4 flux vacua is plentiful: examples have been found both with [1–5] or without sources [6, 7]. Among the vacua with sources some (see for instance [8, 9] and their T-duals [10–12]) contain fully localized sources. However most examples involve intersecting sources, D-branes or O-planes, which are smeared in the transverse directions. This raises the question of what is the meaning of a smeared orientifold plane and how such solutions can lift to full string theory [13, 14]¹

¹Some partial result about O6 planes can be found in [15].

The aim of this paper is to study AdS_4 vacua in type IIB theory. These are less studied than their IIA counterparts. Some work in this direction can be found in [16, 17] and more recently in [18]. A first question we aim at answering is the existence of AdS_4 SUSY vacua without sources. These clearly avoid all the aforementioned problems about the possible validity of the solutions and are clearly important in the context of AdS/CFT. While supersymmetric AdS_4 vacua without sources are known in type IIA, there is only one known example in type IIB [19]. Using $\text{SU}(2)$ structure techniques,² the authors of [16] showed that, for constant warp factor and a specific choice of $\text{SU}(2)$ torsions, only non supersymmetric sourceless vacua can be found. One goal of this paper is to extend the analysis to a larger class of manifolds. The most general form of the supersymmetry equations is too complicated to give general results. For this reason we will focus on group manifolds admitting an $\text{SU}(2)$ structure and look for solutions with constant warp factor. From simple manipulations of the traced Einstein equations it is easy to see that in absence of sources the internal manifold must have positive curvature [18]. This condition already rules out all nilmanifolds as candidates for sourceless vacua in type IIB. In this paper we will prove a stronger result: under some restrictions on the $\text{SU}(2)$ torsions (namely the vectors in the torsion classes are set to zero), it is not possible to have sourceless solutions. We will leave the analysis of the warped case for future work.

The same approach used to look for sourceless solutions, allows to scan all possible AdS_4 $\mathcal{N} = 1$ SUSY vacua in type IIB on group manifolds. As before we will consider constant warp factor and no torsions in the vector representation of $\text{SU}(2)$. We use the pure spinors formalism developed in [10, 20, 21]. This permits to solve the SUSY equations using algebraic equations on the components of the fields on the internal manifold. In this way we have been able to put constraints on the possible group manifolds and obtain an exhaustive list of which manifolds admit an AdS_4 vacuum. An important side result of this analysis is the fact that O5 and O7 planes are necessary in order to have such vacua. Unfortunately, the presence of intersecting orientifolds forces us to use smeared sources.

This paper is organized as follows. In section 2 we give the general conditions that the fields on the internal manifold have to satisfy in order to have $\mathcal{N} = 1$ AdS_4 . We also give the general form of $\text{SU}(2)$ -structure and rewrite the SUSY equations in this case. In section 3, we specialize to group manifolds and solve the SUSY equations in this case. We also discuss the charges coming from the RR Bianchi identities and we prove that no sourceless solution exist. In section 4, we give the exhaustive list of group manifolds admitting an AdS_4 solution in our framework and we discuss a new solution on a solvmanifold. There we also prove that O5 and O7 orientifolds are necessarily present. Finally in section 5, we prove that there can't be any AdS_4 vacua in our framework on manifolds with intermediate $\text{SU}(2)$ -structure.

Appendix A gives the structure constants of the group manifold after having solved the SUSY equations. Appendix B gives the solutions to the NS Bianchi identity. Finally, in appendix C we discuss how our new solution is related by T-duality to the IIA solutions given in [10].

²As shown in [4, 18] $\mathcal{N} = 1$ SUSY vacua in type IIB supergravity only exist on manifolds with $\text{SU}(2)$ structure.

2 The supersymmetry conditions

We are interested in $\mathcal{N} = 1$ SUSY AdS₄ vacua in type IIB theories. As discussed in [16, 22] such solutions are only possible when the compactification manifolds have SU(2) structure group. Let us recall that a manifold is said to be of SU(2) structure if it admits a complex one form z , a real and a holomorphic two-form, j and ω , that are globally defined and satisfy

$$z_{\perp} \bar{z} = 2, \quad z_{\perp} z = \bar{z}_{\perp} \bar{z} = 0, \quad (2.1)$$

$$j \wedge \omega = 0, \quad (2.2)$$

$$z_{\perp} j = z_{\perp} \omega = 0, \quad (2.3)$$

$$j \wedge j = \frac{1}{2} \omega \wedge \bar{\omega}. \quad (2.4)$$

Such forms can be constructed as bilinears of two globally defined spinors, η_+ and $\chi_+ = \frac{1}{2} z \eta_-$

$$z_m = -\chi_-^{\dagger} \gamma_m \eta_+, \quad (2.5)$$

$$j_{mn} = -i \eta_+^{\dagger} \gamma_{mn} \eta_+ + i \chi_+^{\dagger} \gamma_{mn} \chi_+, \quad (2.6)$$

$$\omega_{mn} = -i \chi_+^{\dagger} \gamma_{mn} \eta_+. \quad (2.7)$$

In order to study $\mathcal{N} = 1$ vacua with non trivial fluxes, it is convenient to use the language of Generalized Complex Geometry [23, 24]. The idea is to express the ten-dimensional supersymmetry variations as differential equations on a pair of polyforms defined on the internal manifold. In type IIB, one can decompose the ten-dimensional supersymmetry parameters as

$$\epsilon_i = \zeta_+ \otimes \eta_+^i + \zeta_- \otimes \eta_-^i \quad i = 1, 2, \quad (2.8)$$

where ζ_+ and η_+^i are Weyl spinors in four and six dimensions respectively³ and

$$\begin{aligned} \eta_+^1 &= a \eta_+, \\ \eta_+^2 &= b \left(k_{\parallel} \eta_+ + \frac{1}{2} k_{\perp} z_m \gamma^m \eta_- \right), \end{aligned} \quad (2.9)$$

where $z = z_m \gamma^m$ is the SU(2) structure one-form and $\eta_{\pm}$ are as in (2.5). The complex functions a and b give the norms of η^1 and η^2

$$\|\eta_+^1\|^2 = |a|^2, \quad \|\eta_+^2\|^2 = |b|^2, \quad (2.10)$$

while the parameters $k_{\parallel}$ and $k_{\perp}$ ($k_{\parallel}^2 + k_{\perp}^2 = 1$) are related to the choice of structure on the internal manifold. When $k_{\parallel} = 0$ and $k_{\perp} = 1$ the structure is SU(2), while the general case where both $k_{\parallel}$ and $k_{\perp}$ are non-zero is often referred to as dynamical SU(2) structure.⁴ When $k_{\parallel}$ and $k_{\perp}$ are non zero and constant, we speak of intermediate SU(2) structure rather than dynamical SU(2) structure [25].

³We choose the gamma matrices in such a way that $\eta_-^i = (\eta_+^i)^*$ and $\zeta_- = (\zeta_+)^*$.

⁴When $k_{\parallel} = 1$ and $k_{\perp} = 0$ the internal manifold is said to be of SU(3) structure. We will not consider this case here.

Then, one can use the spinors η_i to define two polyforms on the internal manifold M

$$\Phi_{\pm} = \eta_{+}^1 \otimes \eta_{\pm}^{2\dagger}, \quad (2.11)$$

whose explicit form depend on the choice of structure on M . In the most general case of intermediate $SU(2)$ structure they are

$$\Phi_{-} = -\frac{ab}{8} z \wedge (k_{\perp} e^{-ij} + i k_{\parallel} \omega), \quad (2.12)$$

$$\Phi_{+} = \frac{a\bar{b}}{8} e^{z\bar{z}/2} (k_{\parallel} e^{-ij} - i k_{\perp} \omega), \quad (2.13)$$

where z, j and ω are the forms defining the $SU(2)$ structure. The norm of the pure spinors $\Phi_{\pm}$ is related to the norm of the spinors η^i by

$$\langle \Phi_{\pm}, \bar{\Phi}_{\pm} \rangle = -i \|\Phi_{\pm}\|^2 \text{vol}_6 = -\frac{i}{8} |a|^2 |b|^2 \text{vol}_6, \quad (2.14)$$

where vol_6 is the volume of the internal manifold and the product

$$\langle A, B \rangle = (A \wedge \lambda(B))|_{\text{top}} \quad (2.15)$$

is the Mukai pairing among forms, where λ acts on a form as the transposition of all indices

$$\lambda(\omega_p) = (-)^{\lfloor p/2 \rfloor} \omega_p. \quad (2.16)$$

As shown in [20], for type IIB compactifications to AdS_4 the ten-dimensional supersymmetry variations are equivalent to the following set of equations on the pure spinors $\Phi_{\pm}$

$$(d - H \wedge)(e^{2A-\phi} \Phi_{-}) = -2\mu e^{A-\phi} \text{Re} \Phi_{+}, \quad (2.17)$$

$$(d - H \wedge)(e^{A-\phi} \text{Re} \Phi_{+}) = 0, \quad (2.18)$$

$$(d - H \wedge)(e^{3A-\phi} \text{Im} \Phi_{+}) = -3e^{A-\phi} \text{Im}(\bar{\mu} \Phi_{-}) - \frac{1}{8} e^{4A} * \lambda(F), \quad (2.19)$$

where ϕ is the dilaton, A the warp factor

$$ds^2 = e^{2A} ds_{(4)}^2 + ds_{(6)}^2, \quad (2.20)$$

and F is the sum of the RR field strength on M , $F = F_1 + F_3 + F_5$. The ten-dimensional fluxes are defined in terms of F by

$$F_{(10)} = \text{vol}_4 \wedge \lambda(*F) + F. \quad (2.21)$$

The complex number μ determines the size of the AdS_4 cosmological constant

$$\Lambda = -|\mu|^2. \quad (2.22)$$

Notice also that, for AdS vacua, supersymmetry constraints the norms of the two six-dimensional spinors to be equal [10]

$$|a|^2 = |b|^2 = e^A. \quad (2.23)$$

Only the relative scale between the spinor being relevant, we can always rescale η_+ in such a way that

$$\bar{b} = a, \quad \frac{b}{a} = e^{-i\theta}. \quad (2.24)$$

It is convenient to introduce the rescaled forms

$$\hat{\omega} = e^{i\theta} \omega, \quad (2.25)$$

$$\hat{z} = \frac{\bar{\mu}}{|\mu|} z, \quad (2.26)$$

but for simplicity of notation, we will drop the $\hat{}$ symbols in the rest of the paper.

Plugging the explicit form of (2.12) and (2.13), into the SUSY variations (2.17)–(2.19), one can deduce the general conditions for $\text{AdS}_4 \mathcal{N} = 1$ SUSY vacua in terms of the forms z, ω, j and the fluxes. As discussed in [18], (2.17) implies

$$k_{\parallel} = 0 \quad \text{or} \quad \cos \theta = 0. \quad (2.27)$$

The first choice corresponds to a rigid $\text{SU}(2)$ structure, while the second fixes the relative phase of a and b .

The supersymmetry conditions for rigid $\text{SU}(2)$ structure were derived in this language in [18] and amount to a set of differential conditions on the $\text{SU}(2)$ forms

$$d(e^{3A-\phi} z) = 2|\mu| e^{2A-\phi} \omega_I, \quad (2.28)$$

$$z \wedge (dj - iH + |\mu| e^{-A} \bar{z} \wedge \omega_R) = 0, \quad (2.29)$$

$$d(e^{2A-\phi} \omega_I) = 0, \quad (2.30)$$

$$d(e^{2A-\phi} z \wedge \bar{z} \wedge \omega_R) = 2ie^{2A-\phi} H \wedge \omega_I, \quad (2.31)$$

plus equations for the RR fluxes

$$*F_5 = 3e^{-A-\phi} |\mu| z_I, \quad (2.32)$$

$$*F_3 = -e^{-4A} d(e^{4A-\phi} \omega_R) + 3e^{-A-\phi} |\mu| z_R \wedge j, \quad (2.33)$$

$$\begin{aligned} *F_1 = & -i d(2A - \phi) z \wedge \bar{z} \wedge \omega_I - e^{-\phi} H \wedge \omega_R \\ & + \frac{1}{2} e^{-A-\phi} |\mu| z_I \wedge j \wedge j. \end{aligned} \quad (2.34)$$

With non trivial fluxes, the $\text{SU}(2)$ -structure forms are in general not closed. Their differentials can be expanded into representations of $\text{SU}(2)$, the $\text{SU}(2)$ torsion classes (see for instance [4, 18, 26]). There are 20 $\text{SU}(2)$ torsion classes: 8 complex singlets, 8 complex doublets and 4 complex triplets. In view of the application to group manifolds, we choose to parametrize the torsions in a slightly different way than what was used in [4, 18, 26]. The form z defines an almost product structure on M , which induces a (global) decomposition of the tangent space

$$TM = T_2 M \oplus T_4 M, \quad (2.35)$$

where the subbundle T_2M is spanned by the real and imaginary parts of the form z . On T_4M we define the following basis of two-forms

$$T^a = \{j, \omega_R, \omega_I, \tilde{j}_1, \tilde{j}_2, \tilde{j}_3\} \quad a = 1, \dots, 6, \quad (2.36)$$

where j and ω define the $SU(2)$ structure and the $\tilde{j}_a$ form a triplet of anti self-dual two-forms such that⁵

$$\tilde{j}_i \wedge \tilde{j}_k = -\delta_{ik} j \wedge j, \quad \tilde{j}_i \wedge j = \tilde{j}_i \wedge \omega = 0. \quad (2.39)$$

In what follows, in order to simplify the computations, we will set to zero the torsion classes in the **2** of $SU(2)$ ⁶ and we will parametrize the exterior derivatives of the two-forms T^a as follows

$$dz = S_0 z_R \wedge z_I + S_a T^a, \quad (2.40)$$

$$dT^a = (M_1)^a_b z_R \wedge T^b + (M_2)^a_b z_I \wedge T^b, \quad (2.41)$$

where S_a are seven complex scalars and M_i are two real torsion matrices of the form

$$M_i = \begin{pmatrix} t_{i1}^1 & t_{i2}^1 & t_{i3}^1 & t_{i4}^1 & t_{i5}^1 & t_{i6}^1 \\ -t_{i2}^1 & t_{i1}^1 & t_{i3}^2 & t_{i4}^2 & t_{i5}^2 & t_{i6}^2 \\ -t_{i3}^1 & -t_{i3}^2 & t_{i1}^1 & t_{i4}^3 & t_{i5}^3 & t_{i6}^3 \\ t_{i4}^1 & t_{i4}^2 & t_{i4}^3 & t_{i1}^1 & t_{i5}^4 & t_{i6}^4 \\ t_{i5}^1 & t_{i5}^2 & t_{i5}^3 & -t_{i5}^4 & t_{i1}^1 & t_{i6}^5 \\ t_{i6}^1 & t_{i6}^2 & t_{i6}^3 & -t_{i6}^4 & -t_{i6}^5 & t_{i1}^1 \end{pmatrix}. \quad (2.42)$$

With an abuse of notation, we will call torsions the singlets S_a and the coefficient of the matrices M_i . The redundancy in the elements t_{ib}^a is fixed by imposing on the two-forms T^a the constraints

$$\begin{aligned} d(T^a \wedge T^b) &= 0, \\ d^2 T^a &= 0. \end{aligned} \quad (2.43)$$

We have already enforced the first set of the above conditions. We still have to impose equations of the type $d(dT^a) = 0$, which are much harder to solve since they are quadratic in the coefficients t_{aj}^i . We will come back to this problem in section 3.2 where we found a way to linearly solve them.

⁵Given the decomposition (2.35), we can write the 6-dimensional Hodge star as $*_6(A \wedge B) = *_2 A \wedge *_4 B$, where $A \in T_2M$ and $B \in T_4M$. One can derive

$$*_2 1 = -z_R \wedge z_I \quad *_2 z_R = z_I \quad *_2 z_I = -z_R \quad *_2(z_R \wedge z_I) = -1 \quad (2.37)$$

$$*_4 1 = \frac{1}{2} j \wedge j \quad *_4 j, \omega_R, \omega_I = j, \omega_R, \omega_I \quad *_4 \tilde{j}_i = -\tilde{j}_i \quad *_4 j \wedge j = 2. \quad (2.38)$$

⁶Notice that in presence of orientifolds, these torsion classes are automatically put to zero.

3 Looking for vacua

In order to find $\mathcal{N} = 1$ supersymmetric vacua one has to solve the supersymmetry variations and then the Bianchi identities for the fluxes. In this formalism, the supersymmetry conditions are easy to solve, since they reduce to a set of linear algebraic constraints for the elements of the matrices M_i . The Bianchi identities are harder to handle since they require solving the quadratic constraints $d^2 T^a$ we introduced above. In this section we will solve the supersymmetry equations (2.28)–(2.31). Then we will solve the constraints $d^2 T^a$ in the case of group manifolds. Finally we will solve the Bianchi identities: we will show that there exist no sourceless solutions and give the list of all group manifolds admitting a vacuum in our class.

3.1 SUSY constraints

In order to solve the SUSY variations (2.28)–(2.31), besides (2.40) and (2.41) for the $SU(2)$ structure forms, we need a similar decomposition of the NS H flux

$$H = h_{1a} z_R \wedge T^a + h_{2a} z_I \wedge T^a. \quad (3.1)$$

Then we use the SUSY equations to put constraints on the torsions classes and on the coefficients of H . Indeed, by plugging (2.40), (2.41) and (3.1) into (2.28)–(2.31) (we take ϕ and A constant), one can show that

$$S_0 = S_1 = S_2 = S_4 = S_5 = S_6 = 0, \quad (3.2)$$

$$S_3 = 2e^{-A}|\mu|, \quad (3.3)$$

$$h_{11} = h_{21} = h_{13} = h_{13} = 0, \quad (3.4)$$

and that the torsion matrices take the following forms:

$$M_1 = \begin{pmatrix} 0 & -(2e^{-A}|\mu| + h_{22}) & 0 & -h_{24} & -h_{25} & -h_{26} \\ 2e^{-A}|\mu| + h_{22} & 0 & 0 & t_{14}^2 & t_{15}^2 & t_{16}^2 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ -h_{24} & t_{14}^2 & 0 & 0 & t_{15}^4 & t_{16}^4 \\ -h_{25} & t_{15}^2 & 0 & -t_{15}^4 & 0 & t_{16}^5 \\ -h_{26} & t_{16}^2 & 0 & -t_{16}^4 & -t_{16}^5 & 0 \end{pmatrix}, \quad (3.5)$$

$$M_2 = \begin{pmatrix} 0 & h_{12} & 0 & h_{14} & h_{15} & h_{16} \\ -h_{12} & 0 & 0 & t_{24}^2 & t_{25}^2 & t_{26}^2 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ h_{14} & t_{24}^2 & 0 & 0 & t_{25}^4 & t_{26}^4 \\ h_{15} & t_{25}^2 & 0 & -t_{25}^4 & 0 & t_{26}^5 \\ h_{16} & t_{26}^2 & 0 & -t_{26}^4 & -t_{26}^5 & 0 \end{pmatrix}. \quad (3.6)$$

The RR fluxes are then determined from (2.32)–(2.34).

3.2 The quadratic equations for group manifolds

Before looking at the Bianchi identities, we will now solve the quadratic equations of the form $d(dT^a) = 0$. Unfortunately, plugging (3.4)–(3.6) into (2.41), the equations $d(dT^a) = 0$ are quadratic in the torsion coefficients t_{ib}^a and the flux parameters h_{ia} , so that it is not an easy task to solve them. We will limit ourselves to group manifolds and enforce these quadratic conditions directly on the basis of one-forms e^i , which will simplify the problem.

As already said, we are interested in six-dimensional group manifolds. A group manifold is specified by a basis of globally defined one forms, e^i , satisfying the Maurer-Cartan equations

$$de^i = -\frac{1}{2}f_{jk}^i e^j \wedge e^k, \quad (3.7)$$

with f_{jk}^i constant. In this case imposing $d^2 e^i = 0$ gives the Jacobi identities

$$f_{[jk}^i f_{l]i}^k = 0. \quad (3.8)$$

Note that this group manifold is not required a priori to be compact. However, in some cases, the manifold we consider can be quotiented by a discrete subgroup, called a lattice, in order to make it compact. We refer the interested reader to section 2 of [10] which give a review of this matter. We won't make the distinction in this paper and call group manifold a manifold defined by a given set of structure constants.

By an appropriate choice of basis e^i , the $SU(2)$ -structure can always be put in the form

$$\begin{aligned} z &= z_1 e^1 + iz_2 e^2, \\ j &= j_1 e^{36} + j_2 e^{45}, \\ \omega_R &= \frac{j_1 j_2}{\omega_1} e^{34} + \omega_1 e^{56}, \\ \omega_I &= -\frac{j_1 j_2}{\omega_2} e^{35} + \omega_2 e^{46}, \end{aligned} \quad (3.9)$$

where all the coefficient are real. Similarly, the $SU(2)$ triplet of anti-self dual two forms $\tilde{j}_i$ can be written as

$$\begin{aligned} \tilde{j}_1 &= j_1 e^{36} - j_2 e^{45}, \\ \tilde{j}_2 &= -\frac{j_1 j_2}{\omega_1} e^{34} + \omega_1 e^{56}, \\ \tilde{j}_3 &= -\frac{j_1 j_2}{\omega_2} e^{35} - \omega_2 e^{46}. \end{aligned} \quad (3.10)$$

3.2.1 Solving the Jacobi Identities on group manifold

Once we use the expressions (3.9) and (3.10) for z and T^a , the quadratic constraints $d^2 T^a = 0$ is clearly satisfied since we know that the one-forms e^i must satisfy the Jacobi identity $d^2 e^i = 0$. The idea is then to express the structure constants in terms of the torsion parameters and the fluxes h_{ia} and then solve explicitly the Jacobi identities in terms of such parameters.

Plugging (3.9) and (3.10) in the torsion equations (2.40) and (2.41) gives the structure constant in terms of the torsion t_{ib}^a and the coefficient h_{ia} . Since they are not particularly enlightening we give the explicit expressions in appendix A. Then we can impose the Jacobi identities. Fortunately some of these equations are linear in t_{ib}^a and in h_{ia} after we impose the SUSY constraints and one can solve the full set of the Jacobi identities. The result is quite simple. It puts all coefficients of the matrix (3.5) to zero whereas (3.6) is unchanged. Concerning the structure constants, here it is also very simple, f_{j1}^3 , f_{j1}^4 , f_{j1}^5 , f_{j1}^6 are put to zero and the other ones are left unchanged.

3.3 Bianchi identities

In order to find a supersymmetric vacuum we still have to solve the Bianchi identities for the NS and RR fluxes. These are the equations that tell us whether we can have sourceless solutions and, if sources are needed, of which type they must be. In this work we assume that there are no NS5 branes, meaning that the NS Bianchi identity, $dH = 0$, simply puts more constraints on the parameters t_{ib}^a and h_{ia} . Such constraints are in general quadratic in t_{ib}^a and h_{ia} and admit different possible solutions. We give the explicit form of the NS Bianchi identities as well as its various classes of solutions in appendix B.

Since one of the aim of this paper is to see whether sourceless solutions are possible, we discuss in detail the RR Bianchi identities

$$dF_1 = \delta(D7/O7), \quad (3.11)$$

$$dF_3 - H \wedge F_1 = \delta(D5/O5), \quad (3.12)$$

$$dF_5 - H \wedge F_3 = \delta(D3/O3), \quad (3.13)$$

where $\delta(Dp/Op)$ is the charge densities of the space-filling sources. It is easy to see that for our solutions

$$\delta(D3/O3) = 0. \quad (3.14)$$

The other two equations give

$$\delta(D7/O7) = -10e^{-\phi}|\mu e^{-A}|^2\omega_I, \quad (3.15)$$

$$\begin{aligned} \delta(D5/O5) = e^{-\phi}z_R \wedge z_I \wedge [& (10|\mu e^{-A}|^2 + 3h_{12}^2 + (t_{24}^2)^2 + (t_{24}^2)^2 + (t_{25}^2)^2 + (t_{26}^2)^2)\omega_R \\ & + (h_{14}t_{24}^2 + h_{15}t_{25}^2 + h_{16}t_{26}^2)j + (3h_{12}h_{14} - t_{25}^2t_{25}^4 - t_{26}^2t_{26}^4)\tilde{j}_1 \\ & + (3h_{12}h_{15} + t_{24}^2t_{25}^4 - t_{26}^2t_{26}^5)\tilde{j}_2 + (3h_{12}h_{16} + t_{24}^2t_{26}^4 + t_{25}^2t_{26}^5)\tilde{j}_3] . \end{aligned} \quad (3.16)$$

Note that we haven't enforced the condition $dH = 0$ in above expressions above. A common feature of the solutions of appendix B of the NS Bianchi identities is that the coefficient in front of $z_R \wedge z_I \wedge j$ in $\delta(D5/O5)$ vanishes.

3.3.1 Sourceless solutions

For constant warp factor, there are some general arguments allowing to characterize the manifolds that could give sourceless solutions. In particular, by taking appropriate linear

combinations of the 10-dimensional equations of motion in absence of sources, we arrive at

$$R_4 = 2V = -2 \sum_p |F_p|^2 < 0, \quad (3.17)$$

$$R_6 = \sum_p \frac{9-p}{2} |F_p|^2 > 0, \quad (3.18)$$

which imply that in order to have sourceless solutions the internal manifold must have positive curvature. This condition rules out all nilmanifolds as possible candidates for sourceless solutions, but still leaves open the possibility of having other group manifolds.

However, from the Bianchi identities in the previous section, we immediately see that it is not possible to avoid O7 or D7 sources (see (3.15)), thus ruling out the possibility of having sourceless AdS SUSY vacua in type IIB in the class of group manifold we consider. This is very unlike type IIA where many source free solutions were found [6, 7].

Since T-duality does not create sources out of nothing, one might wonder whether it is possible to generate sourceless solutions in IIB T-dualizing one of the known sourceless solutions of IIA, for instance those in [6]. One can check that in all known IIA solutions the isometries on which one could T-dualize correspond to cycles that shrink to zero at some points, thus giving rise to non compact solutions, which are outside the classification of this paper.

3.3.2 Sign of charges

In this section we want to look at the sign of the charges in the Bianchi identities in order to see if D-branes or O-planes are involved. We will follow the approach of [10]. The source terms (3.15) and (3.16) can be written as sum of decomposable forms (this is necessary if they are to describe a plane)

$$\begin{aligned} \delta(D7/O7) &= \sum N_{(D7/O7)ij} e^i \wedge e^j, \\ \delta(D5/O5) &= \sum N_{(D5/O5)ijkl} e^i \wedge e^j \wedge e^k \wedge e^l. \end{aligned}$$

We want to rewrite them in terms of the volume forms orthogonal to the cycles wrapped by the source. We define vol^{ij} as the decomposable form proportional to $e^i \wedge e^j$ and normalized such that

$$\langle \text{vol}^{ij}, \text{Im } \Phi_+ \rangle = -\text{vol}_6 = -i \langle \Phi_+, \bar{\Phi}_+ \rangle. \quad (3.19)$$

Similarly, one can introduce vol^{ijkl} . With these definitions, the sources can be written as

$$\delta(D7/O7) = \sum n_{(D7/O7)ij} \text{vol}^{ij}, \quad (3.20)$$

$$\delta(D5/O5) = \sum n_{(D5/O5)ijkl} \text{vol}^{ijkl}, \quad (3.21)$$

where $n_{(D7/O7)ij}$ and $n_{(D5/O5)ijkl}$ can be seen as densities of charges on the cycle wrapped by the sources, they give the sign of the charges. On the other hand, $N_{(D7/O7)ij}$ and $N_{(D5/O5)ijkl}$ are more similar to total charges and these should be generally of order one on a solution since they are directly related to the number of D-branes or orientifolds.

In order to fix our conventions for the signs of $n_{(D7/O7)ij}$ and $n_{(D5/O5)ijkl}$ we recall briefly the case of Minkowski compactifications. In this case, one can show that the tadpole condition is equivalent to [10]

$$\begin{aligned} \sum_p |F_p|^2 \text{vol}_6 &= \langle F, * \lambda(F) \rangle \\ &= \sum_i n_i \langle \text{vol}^i, \text{Im } \Phi_+ \rangle = - \sum_i n_i \text{vol}_6, \end{aligned} \quad (3.22)$$

where, for simplicity, we denoted by n_i the charge density of a generic source and by vol^i the corresponding transverse volume. In order to solve the tadpole condition, we need a net orientifold charge. This means that we can associate orientifolds to a negative charge density n_i and branes to a positive one. In this paper we will use the same conventions: if $n_{(D7/O7)ij}$ and/or $n_{(D5/O5)ijkl}$ are negative we have an overall O-plane charge and viceversa.

Using the definition above, one finds that

$$n_{(D7/O7)35} = n_{(D7/O7)46} = -\frac{5e^{-3A-\phi}}{4} |\mu|^2 < 0, \quad (3.23)$$

implying that, in order to have an AdS SUSY solution on a group manifold, one must always have two intersecting O7-planes with our assumptions. Since we have intersecting orientifolds, we will only consider smeared sources. The D5/O5 case does not give any general condition, and we analyze it case by case in the next section.

4 Scanning group manifolds

We can use the formalism developed above to perform a complete search of group manifolds admitting $\mathcal{N} = 1$ AdS₄ solutions in type IIB with our assumptions. This amounts to determining the set of structure constants that solve both the supersymmetry conditions and the Bianchi identities.

The logic is the following. We first impose the supersymmetry constraints (2.28)–(2.34) and the Jacobi identities for the SU(2) structure on the generic group manifold.

We then look at the Bianchi identities: first the RR 1-form, then the NS 3-form and finally the RR 3-form. The BI for the RR 1-form implies that the solutions necessarily have intersecting O7-planes. Then the compatibility of the algebras with the orientifold involution forces to set

$$h_{16} = t_{26}^4 = t_{26}^2 = t_{26}^5 = 0. \quad (4.1)$$

As given in appendix A, the expressions for f_{jk}^i are not very manageable. Moreover, in looking for solutions, one usually proceed in the opposite way: given an internal manifold, one would like to find the coefficients of the torsions and H flux in terms of the structure constants. By inverting these relation we find the following expressions for the parameters in the SU(2)

$$\omega_2 = -\frac{f_{46}^1 z_1}{2e^{-A} |\mu|} \quad j_2 = -\frac{D}{4j_1}, \quad (4.2)$$

algebra	O5	type of manifold	name
$(35 + \epsilon 46, 0, 0, 0, 23, 24)$	✓	nilmanifold	$n3.13, n3.14$
$(35 + 46, 0, 0, 0, 23, 0)$	✓	nilmanifold	$n4.1$
$(35 + 46, 0, 0, 0, 0, 0)$	✓	nilmanifold	$n5.1$
$(35 + \epsilon 46, 0, 25, -\epsilon 26, -\epsilon 23, 24)$	✓	solvmanifold	$\mathfrak{g}6.88$ and solv_1

Table 1. List of group manifolds admitting an AdS SUSY solution ($\epsilon = \pm 1$).

with $D = \frac{f_{35}^1 f_{46}^1 z_1^2}{4|e^{-A}\mu|^2}$, the torsions

$$\begin{aligned}
 t_{25}^4 &= -\frac{1}{8j_1 z_2 \omega_1} \left(-f_{26}^4 D - 4f_{23}^5 \omega_1^2 + 4j_1^2 \left(f_{25}^3 - \frac{4f_{24}^6 \omega_1^2}{D} \right) \right), \\
 t_{25}^2 &= \frac{f_{23}^3 + f_{24}^4}{z_2}, \\
 t_{24}^2 &= -\frac{1}{8j_1 z_2 \omega_1} \left(-f_{26}^4 D + 4f_{23}^5 \omega_1^2 + 4j_1^2 \left(f_{25}^3 - \frac{4f_{24}^6 \omega_1^2}{D} \right) \right),
 \end{aligned} \tag{4.3}$$

and the H flux

$$\begin{aligned}
 h_{15} &= -\frac{1}{8j_1 z_2 \omega_1} \left(f_{26}^4 D + 4f_{23}^5 \omega_1^2 + 4j_1^2 \left(f_{25}^3 + \frac{4f_{24}^6 \omega_1^2}{D} \right) \right), \\
 h_{12} &= -\frac{1}{8j_1 z_2 \omega_1} \left(f_{26}^4 D - 4f_{23}^5 \omega_1^2 + 4j_1^2 \left(f_{25}^3 - \frac{4f_{24}^6 \omega_1^2}{D} \right) \right), \\
 h_{14} &= \frac{-f_{23}^3 + f_{24}^4}{z_2}.
 \end{aligned} \tag{4.4}$$

We also find that

$$f_{25}^5 = -f_{23}^3 \quad f_{26}^6 = -f_{24}^4. \tag{4.5}$$

The non-zero free parameters are now: $j_1, z_1, z_2, \omega_1, f_{35}^1, f_{46}^1$ and $f_{23}^3, f_{25}^3, f_{24}^4, f_{26}^4, f_{23}^5, f_{24}^6$.

As already mentioned, the NS Bianchi identity is quadratic in the free parameters. As a result we find three different solutions, which we give in detail in appendix B.

We then study the O5 source equations in each of the three solutions of the NS Bianchi identity. If the charge densities $n_{(D5/O5)}$ are negative, there are additional constraints on the structure constants due to the presence of an O5-plane in the directions 34 and 56

$$f_{23}^3 = f_{24}^4 = 0. \tag{4.6}$$

As a result of this procedure we find that $\mathcal{N} = 1$ AdS₄ solutions are very rare and all have intersecting O5-planes besides the O7 we already mentioned. We give the list of solutions in table 1 where the O5 column tells us if there is presence or absence of O5 plane.

The solutions on nilmanifolds have already been studied in this framework in [18], and we refer to that reference for details. In the next section we describe the last solution in the table, which, to our knowledge, is new. Concerning the names of the new manifold, look at appendix D for a detailed explanation.⁷

⁷The author would like to thank David Andriot for a useful discussion about solvmanifold algebras.

4.1 A new solution

As far as the author knows, $(35 + \epsilon 46, 0, 25, -\epsilon 26, -\epsilon 23, 24)$ is a new solution in the literature on the subject. But as we will see in appendix C, it can be obtained as a T-dual of a Lust-Tsimpis type solution in IIA given in [10].

Before giving the explicit vacua, let's give some properties of the algebra. It is easy to see that it is a solvable algebra. From its Killing form

$$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -4\epsilon & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}, \quad (4.7)$$

one can see that this algebra is compact if and only if $\epsilon = -1$ (the caveat of section 3.2 on the compactness of the manifold still holds).

We give here all the quantities of interest on the vacua. The SU(2) structure forms are⁸

$$\begin{aligned} z &= \epsilon \epsilon_1 2 |e^{-A} \mu| j_1 e^1 + z_2 e^2, \\ j &= j_1 (e^3 \wedge e^6 - \epsilon e^4 \wedge e^5), \\ \omega_R &= -\epsilon \frac{j_1^2}{\omega_1} e^3 \wedge e^4 + \omega_1 e^5 \wedge e^6, \\ \omega_I &= \epsilon_1 j_1 (\epsilon e^3 \wedge e^5 + e^4 \wedge e^6). \end{aligned} \quad (4.9)$$

We can easily compute the Ricci scalar

$$R_6 = -4 |\mu e^{-A}|^2 - \left(\frac{j_1^2 - \epsilon \omega_1^2}{j_1 z_2 \omega_1} \right)^2. \quad (4.10)$$

The solution has non trivial NS and RR fluxes

$$H = \frac{1}{\omega_1} 2 |\mu e^{-A}| z_2 (\epsilon j_1^2 e^2 \wedge e^3 \wedge e^4 - \omega_1 e^2 \wedge e^5 \wedge e^6), \quad (4.11)$$

$$F_1 = 10 e^{-\phi} \epsilon_1 |e^{-A} \mu|^2 j_1 e^1, \quad (4.12)$$

$$\begin{aligned} F_3 &= e^{-\phi} \frac{|e^{-A} \mu| j_1}{z_2 \omega_1} [3 z_2^2 \omega_1^2 (\epsilon e^2 \wedge e^4 \wedge e^5 - e^2 \wedge e^3 \wedge e^6) \\ &\quad + 2 \epsilon_1 (j_1^2 - \epsilon \omega_1^2) (e^1 \wedge e^3 \wedge e^6 + \epsilon_1 e^1 \wedge e^4 \wedge e^5)], \end{aligned} \quad (4.13)$$

$$F_5 = -6 e^{-\phi} \epsilon \epsilon_1 |e^{-A} \mu|^2 j_1^3 e^1 \wedge e^3 \wedge e^4 \wedge e^5 \wedge e^6, \quad (4.14)$$

⁸Similarly the triplet of anti-self dual two-forms are given by

$$\begin{aligned} \tilde{j}_1 &= j_1 (e^3 \wedge e^6 + \epsilon e^4 \wedge e^5), \\ \tilde{j}_2 &= \epsilon \frac{j_1^2}{\omega_1} e^3 \wedge e^4 + \omega_1 e^5 \wedge e^6, \\ \tilde{j}_3 &= \epsilon_1 j_1 (\epsilon e^3 \wedge e^5 - e^4 \wedge e^6). \end{aligned} \quad (4.8)$$

weak coupling	Large volume	scale separation (1)	scale separation (2)
✓	×	✓	✓

Table 2. Scale separation study for the new solution.

giving rise to the the charges

$$n_{(D5/O5)1234} = \frac{e^{-\phi}}{4} \left(-5|e^{-A}\mu|^2 + \frac{\epsilon - \frac{\omega_1^2}{j_1^2}}{z_2^2} \right), \quad (4.15)$$

$$n_{(D5/O5)1256} = \frac{e^{-\phi}}{4} \left(-5|e^{-A}\mu|^2 + \frac{\epsilon - \frac{j_1^2}{\omega_1^2}}{z_2^2} \right). \quad (4.16)$$

with $\epsilon_1 = \pm 1$. Notice that we still have three free parameters (j_1 , z_2 and ω_1). Note also that

$$n_{(D5/O5)1234} + n_{(D5/O5)1256} = -\frac{5e^{-\phi}}{2}|e^{-A}\mu|^2 - e^{-\phi} \left(\frac{j_1}{2z_2\omega_1} - \frac{\epsilon\omega_1}{2j_1z_2} \right)^2 < 0, \quad (4.17)$$

which means that at least one of them is negative and so there is, as we said, at least one O5-plane.

A natural question to ask is whether this solution has a good classical supergravity limit, it allows for separation of scales and large volume limit. Using the same approach of [18] we obtain the results in table 2.

5 Intermediate SU(2) structures on group manifolds

Up to now we restricted our analysis to the case of rigid SU(2) structure $k_{\parallel} = 0$. In this section we will briefly discuss the case of intermediate SU(2) structure when restricted on group manifolds and only give the main results. The analysis is parallel to that of the strict SU(2) case. As we saw in section 2, having both $k_{\parallel} \neq 0$ and $k_{\perp} \neq 0$ forces to set $\theta = -\frac{\pi}{2}$. Then, for constant dilaton and warp factor, (2.17) and (2.18) give

$$dz_R = -\frac{2e^{-A}|\mu|}{k_{\perp}} [k_{\perp}\omega_R + k_{\parallel}(j + z_R \wedge z_I)], \quad (5.1)$$

$$dz_I = 0, \quad (5.2)$$

$$d\omega_R = \frac{k_{\parallel}}{k_{\perp}^2} [-k_{\perp}dj + 2|\mu|e^{-A}z_I \wedge (k_{\parallel}j + k_{\perp}\omega_R)] \quad (5.3)$$

$$z \wedge \left[dj - ik_{\perp}^2 H - ik_{\perp}k_{\parallel}d\omega_I - |\mu|e^{-A}\bar{z} \wedge \left(\frac{k_{\parallel}}{k_{\perp}}(1 - 2k_{\perp}^2)2ij - \omega_I + 2ik_{\parallel}^2\omega_R \right) \right] = 0, \quad (5.4)$$

$$ik_{\perp}z \wedge \bar{z} \wedge d\omega_I - 2k_{\parallel}j \wedge H - 2k_{\perp}\omega_R \wedge H - ik_{\parallel}z \wedge \bar{z} \wedge H = 0, \quad (5.5)$$

while (2.19) can be used, as always, to determine the RR fluxes

$$e^\phi * F_5 = 3z_I |\mu e^{-A}| k_\perp, \quad (5.6)$$

$$e^\phi * F_3 = k_\parallel H - k_\perp d\omega_I + 3|\mu e^{-A}| (k_\perp z_R \wedge j + k_\parallel (z_I \wedge \omega_I - z_R \wedge \omega_R)), \quad (5.7)$$

$$e^\phi * F_1 = \frac{-1}{2k_\perp} \left(2k_\parallel k_\perp j \wedge dj + 2k_\perp^2 \omega_I \wedge H - |\mu e^{-A}| z_I \wedge j \wedge j(1 + 3k_\parallel^2) \right). \quad (5.8)$$

Repeating the analysis of the SUSY conditions, we can show that:

$$S_0 = S_1 = \frac{k_\parallel}{k_\perp} S_2 = -\frac{2e^{-A} |\mu| k_\parallel}{k_\perp} \quad S_3 = S_4 = S_5 = S_6 = 0, \quad (5.9)$$

$$h_{i1} = -\frac{k_\perp h_{i2}}{k_\parallel} \quad h_{13} = -\frac{h_{22}}{k_\parallel^2} \quad h_{23} = 2e^{-A} |\mu| + \frac{h_{12}}{k_\parallel^2} \quad (5.10)$$

and that the torsion matrices take the following forms:

$$M_1 = \begin{pmatrix} 0 & 0 & -4e^{-A} |\mu| - \frac{k_\perp^2 h_{12}}{k_\parallel^2} - \frac{k_\perp t_{14}^2}{k_\parallel} - \frac{k_\perp t_{15}^2}{k_\parallel} - \frac{k_\perp t_{16}^2}{k_\parallel} \\ 0 & 0 & \frac{4e^{-A} k_\parallel}{k_\perp} |\mu| + \frac{k_\perp h_{12}}{k_\parallel} & t_{14}^2 & t_{15}^2 & t_{16}^2 \\ 4e^{-A} |\mu| + \frac{k_\perp^2 h_{12}}{k_\parallel^2} - \frac{4e^{-A} k_\parallel}{k_\perp} |\mu| - \frac{k_\perp h_{12}}{k_\parallel} & 0 & \tilde{t}_4 & \tilde{t}_5 & \tilde{t}_6 \\ -\frac{k_\perp t_{14}^2}{k_\parallel} & t_{14}^2 & \tilde{t}_4 & 0 & t_{15}^4 & t_{16}^4 \\ -\frac{k_\perp}{k_\parallel} t_{15}^2 & t_{15}^2 & \tilde{t}_5 & -t_{15}^4 & 0 & t_{16}^5 \\ -\frac{k_\perp}{k_\parallel} t_{16}^2 & t_{16}^2 & \tilde{t}_6 & -t_{16}^4 & -t_{16}^5 & 0 \end{pmatrix}, \quad (5.11)$$

$$M_2 = \begin{pmatrix} \frac{2e^{-A} |\mu| k_\parallel}{k_\perp} & 0 & -\frac{k_\perp^2 h_{22}}{k_\parallel^2} & -\frac{k_\perp t_{24}^2}{k_\parallel} & -\frac{k_\perp t_{25}^2}{k_\parallel} & -\frac{k_\perp t_{26}^2}{k_\parallel} \\ 0 & \frac{2e^{-A} |\mu| k_\parallel}{k_\perp} & \frac{k_\perp h_{22}}{k_\parallel} & t_{24}^2 & t_{25}^2 & t_{26}^2 \\ \frac{k_\perp^2 h_{22}}{k_\parallel^2} & -\frac{k_\perp h_{22}}{k_\parallel} & \frac{2e^{-A} |\mu| k_\parallel}{k_\perp} & \hat{t}_4 & \hat{t}_5 & \hat{t}_6 \\ -\frac{k_\perp t_{24}^2}{k_\parallel} & t_{24}^2 & \hat{t}_4 & \frac{2e^{-A} |\mu| k_\parallel}{k_\perp} & t_{25}^4 & t_{26}^4 \\ -\frac{k_\perp t_{25}^2}{k_\parallel} & t_{25}^2 & \hat{t}_5 & -t_{25}^4 & \frac{2e^{-A} |\mu| k_\parallel}{k_\perp} & t_{26}^5 \\ -\frac{k_\perp t_{26}^2}{k_\parallel} & t_{26}^2 & \hat{t}_6 & -t_{26}^4 & -t_{26}^5 & \frac{2e^{-A} |\mu| k_\parallel}{k_\perp} \end{pmatrix}, \quad (5.12)$$

with $\tilde{t}_i = -\frac{k_\perp k_\parallel h_{1i} + t_{2i}^2}{k_\parallel^2}$ and $\hat{t}_i = -\frac{k_\perp k_\parallel h_{2i} - t_{1i}^2}{k_\parallel^2}$. The RR fluxes are then determined from (5.6)–(5.8). Next, we impose the Jacobi identities as was done in section 3.2.1 for the strict SU(2)-structure case. This gives following relations:

$$\begin{aligned} h_{12} &= -\frac{4e^{-A} |\mu| k_\parallel^2}{k_\perp}, & h_{14} &= -\frac{t_{24}^2}{k_\perp k_\parallel}, \\ t_{14}^2 &= t_{15}^2 = t_{16}^2 = t_{15}^4 = t_{16}^4 = t_{16}^5 = 0, & t_{25}^2 &= -k_\perp k_\parallel h_{15}, \\ t_{26}^2 &= -k_\perp k_\parallel h_{16}. \end{aligned} \quad (5.13)$$

Then we look at the Bianchi identities as in section 3.3. Here, we find in particular that:

$$n_{(D7/O7)36} = n_{(D7/O7)12} = -\frac{e^{-3A-\phi}|\mu|^2 k_{\parallel}^2(-8+3k_{\perp}^2)}{4k_{\perp}^2} < 0 \quad (5.14)$$

This means that we must have at least an orientifold in each of the orthogonal direction. Then the orientifold involution on z (namely $\sigma(z) = z$) can't be satisfied. It means that in our framework, there can't be any AdS_4 vacua on manifolds with intermediate $\text{SU}(2)$ -structure.

6 Conclusion

In this paper, we studied AdS_4 flux vacua with $\mathcal{N} = 1$ SUSY in type IIB supergravity with and without sources. It is well known that such vacua are only possible on manifold with $\text{SU}(2)$ structure. The presence of fluxes implies that the intrinsic torsions are non trivial. For constant dilaton and warp factor, it was shown in [4] that, if only the singlets in the torsions are non-zero, no supersymmetric sourceless solutions exist. In this paper we extend the analysis to a more general class of $\text{SU}(2)$ torsions where only the vector representations are set to zero, but we keep the warp factor and dilaton constant. In order to be able to solve the Bianchi identities, we have also restrict our analysis to group manifold. Under this restriction we show that, contrary to type IIA, no sourceless supersymmetric solutions are allowed. In particular we show that O7 and O5/D5 sources are necessarily present.

We also performed an exhaustive scan of possible $\mathcal{N} = 1$ AdS_4 vacua on group manifolds. Such solutions are quite rare. The presence of the orientifold planes and supersymmetry severely constrain the form of the structure constant and we find that very few solutions are possible. Namely some nilmanifolds that had already been studied in [18] and one new solution on a solvmanifold.

Our analysis assumes constant dilaton and warp factor and also does not cover the most general $\text{SU}(2)$ structure manifold, since we set to zero all the torsions in vector representations. Notice that such torsions are put automatically to zero in presence of orientifold, so one can hope that by turning them on, one can evade the necessity of O-planes. One can also devise a method to solve the quadratic equations in another way than the one used here in order to free oneself from the use of group manifold only. The general method would still work and one could discover new vacua.

There are also more general developments one can do that are not directly related to the method used. For example, one should understand how one could have fully localized sources and the use of the pure spinors formalism could help [15]. As we said we can try to look to non SUSY vacua close to the SUSY one and hope to find stable dS vacua (unstable ones were found for example in [27–32]). But with more vacua available, there is hope to find a stable dS vacuum.

Acknowledgments

I would like to thank Michela Petrini, Alessandro Tomasiello and Thomas Van Riet for helpful discussions and advice.

A Structure constants

Here we give the expression of the structure constants in term of torsion classes and coefficients of the NS three form H after having solved the SUSY equations. We only list the non-zero structure constants

$$\begin{aligned}
 f_{jk}^1 &= \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{|\mu|e^{-A}j_1j_2}{z_1\omega_2} & 0 \\ 0 & 0 & 0 & 0 & -\frac{|\mu|e^{-A}\omega_2}{z_1} \\ 0 & 0 & -\frac{|\mu|e^{-A}j_1j_2}{z_1\omega_2} & 0 & 0 \\ 0 & 0 & 0 & \frac{|\mu|e^{-A}\omega_2}{z_1} & 0 \end{pmatrix} & f_{jk}^2 &= \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} \\
 f_{j1}^3 &= \begin{pmatrix} 0 \\ 0 \\ -\frac{1}{4}(h_{24} + t_{15}^2)z_1 \\ \frac{(h_{26}-t_{16}^4)z_1\omega_2}{4j_1} \\ -\frac{(h_{22}+h_{25}-t_{14}^2-t_{15}^4+2|\mu|e^{-A})z_1\omega_1}{4j_1} \\ \frac{(t_{16}^2-t_{16}^5)z_1\omega_2\omega_1}{4j_1j_2} \end{pmatrix} & f_{j2}^3 &= \begin{pmatrix} 0 \\ 0 \\ \frac{1}{4}(h_{14} - t_{25}^2)z_2 \\ -\frac{(h_{16}+t_{26}^4)z_2\omega_2}{4j_1} \\ \frac{(h_{12}+h_{15}+t_{24}^2+t_{25}^4)z_2\omega_1}{4j_1} \\ \frac{(t_{26}^2-t_{26}^5)z_2\omega_2\omega_1}{4j_1j_2} \end{pmatrix} \\
 f_{j1}^4 &= \begin{pmatrix} 0 \\ 0 \\ \frac{(h_{26}+t_{16}^4)j_1z_1}{4\omega_2} \\ \frac{1}{4}(h_{24} - t_{15}^2)z_1 \\ \frac{(t_{16}^5-t_{16}^2)z_1\omega_1}{4\omega_2} \\ \frac{(h_{22}+h_{25}+t_{14}^2+t_{15}^4+2|\mu|e^{-A})z_1\omega_1}{4j_2} \end{pmatrix} & f_{j2}^4 &= \begin{pmatrix} 0 \\ 0 \\ \frac{(t_{26}^4-h_{16})j_1z_2}{4\omega_2} \\ -\frac{1}{4}(h_{14} + t_{25}^2)z_2 \\ \frac{(t_{26}^5-t_{26}^2)z_2\omega_1}{4\omega_2} \\ \frac{(-h_{12}-h_{15}+t_{24}^2+t_{25}^4)z_2\omega_1}{4j_2} \end{pmatrix} \\
 f_{j1}^5 &= \begin{pmatrix} 0 \\ 0 \\ \frac{(h_{22}-h_{25}+t_{14}^2-t_{15}^4+2|\mu|e^{-A})j_1z_1}{4\omega_1} \\ -\frac{(t_{16}^2+t_{16}^5)z_1\omega_2}{4\omega_1} \\ \frac{1}{4}(h_{24} + t_{15}^2)z_1 \\ \frac{(h_{26}+t_{16}^4)z_1\omega_2}{4j_2} \end{pmatrix} & f_{j2}^5 &= \begin{pmatrix} 0 \\ 0 \\ \frac{(-h_{12}+h_{15}+t_{24}^2-t_{25}^4)j_1z_2}{4\omega_1} \\ -\frac{(t_{26}^2+t_{26}^5)z_2\omega_2}{4\omega_1} \\ \frac{1}{4}(t_{25}^2 - h_{14})z_2 \\ \frac{(t_{26}^4-h_{16})z_2\omega_2}{4j_2} \end{pmatrix} \\
 f_{j1}^6 &= \begin{pmatrix} 0 \\ 0 \\ \frac{(t_{16}^2+t_{16}^5)j_1j_2z_1}{4\omega_2\omega_1} \\ -\frac{(h_{22}-h_{25}-t_{14}^2+t_{15}^4+2|\mu|e^{-A})j_2z_1}{4\omega_1} \\ \frac{(h_{26}-t_{16}^4)j_2z_1}{4\omega_2} \\ \frac{1}{4}(t_{15}^2 - h_{24})z_1 \end{pmatrix} & f_{j2}^6 &= \begin{pmatrix} 0 \\ 0 \\ \frac{(t_{26}^2+t_{26}^5)j_1j_2z_2}{4\omega_2\omega_1} \\ \frac{(h_{12}-h_{15}+t_{24}^2-t_{25}^4)j_2z_2}{4\omega_1} \\ -\frac{(h_{16}+t_{26}^4)j_2z_2}{4\omega_2} \\ \frac{1}{4}(h_{14} + t_{25}^2)z_2 \end{pmatrix}
 \end{aligned}$$

B Solving the NS Bianchi identity

In this appendix we list the explicit solutions of the NS Bianchi identity

$$\begin{aligned} dH = & (h_{12}^2 - h_{14}^2 - h_{15}^2)z_R \wedge z_I \wedge j - (h_{14}t_{24}^2 + h_{15}t_{25}^2)z_R \wedge z_I \wedge \omega_R \\ & + (h_{15}t_{25}^4 - h_{12}t_{24}^2)z_R \wedge z_I \wedge \tilde{j}_1 - (h_{12}t_{25}^2 + h_{14}t_{25}^4)z_R \wedge z_I \wedge \tilde{j}_2 \\ = & 0 \end{aligned} \quad (\text{B.1})$$

We found three different types of solutions (here $\epsilon = \pm 1$)

Type 1

$$h_{12} = \epsilon \sqrt{h_{14}^2 + h_{15}^2} \quad t_{25}^2 = -\frac{h_{14}t_{24}^2}{h_{15}} \quad t_{25}^4 = \epsilon \frac{\sqrt{h_{14}^2 + h_{15}^2}t_{24}^2}{h_{15}} \quad (\text{B.2})$$

Type 2

$$h_{12} = \epsilon h_{14} \quad h_{15} = 0 \quad t_{24}^2 = 0 \quad t_{25}^4 = -\epsilon t_{25}^2 \quad (\text{B.3})$$

Type 3

$$h_{12} = 0 \quad h_{14} = 0 \quad h_{15} = 0 \quad (\text{B.4})$$

C T-duality of (0,0,25,-26,-23,24)

In type IIA theories, AdS_4 vacua with $\mathcal{N} = 1$ supersymmetry are only possible on $\text{SU}(3)$ structure manifolds. The general class of solutions was derived in [4], and later in the language of Generalized Geometry in [10]. In [10], the authors also give an explicit solution of this type on a solvmanifold with intersecting O6 planes. We will first rewrite it in our notation and then do a T-duality in order to recover the solution of section 4.1.

The solution is found on the solvmanifold defined by the algebra (0,0,25,-26,-23,24). The $\text{SU}(3)$ structure has the form

$$\begin{aligned} J &= e^1 \wedge e^2 - e^4 \wedge e^5 + e^3 \wedge e^6, \\ \Omega &= -i(e^1 - ie^2) \wedge (e^3 + ie^6) \wedge (e^4 - ie^5), \end{aligned} \quad (\text{C.1})$$

and the non-zero fluxes are

$$\begin{aligned} H &= 2|\mu|\Omega_R, \\ F_0 &= 5|\mu|, \\ F_4 &= 3/2|\mu|J \wedge J. \end{aligned} \quad (\text{C.2})$$

We can also define the flux part of H with:

$$B = -2|\mu|(-1 + \alpha)e^4 \wedge e^5, \quad (\text{C.3})$$

$$H_{fl} = 2|\mu|(-e^1 \wedge e^3 \wedge e^5 - e^1 \wedge e^4 \wedge e^6 + 2\alpha e^2 \wedge e^3 \wedge e^4 - (2 - \alpha)e^2 \wedge e^5 \wedge e^6), \quad (\text{C.4})$$

with α an arbitrary real parameter. The manifold is $S^1_{\{1\}} \times M_5$, where M^5 is a $T^2_{\{3,5\}} \times T^2_{\{4,6\}}$ -fibration over $S^1_{\{2\}}$. Thus we can perform a T-duality along the direction 1. We will follow the rules for T-duality given in [10]. We will note all the T-dual quantities with tilde.

Since the metric is the identity and the B field is only along the base, these two quantities don't change and we have:

$$d\tilde{s}^2 = \mathbb{I}_6, \quad \tilde{B} = -2|\mu|(-1 + \alpha)\tilde{e}^4 \wedge \tilde{e}^5. \quad (\text{C.5})$$

Next we will do the T-duality on the pure spinors. We will first define

$$\Phi_{+B} = e^B \Phi_+, \quad \Phi_{-B} = e^B \Phi_-. \quad (\text{C.6})$$

Then, the T-duality rules are

$$\tilde{\Phi}_{+B} = -(\iota_1 \Phi_{-B} - e^1 \wedge \Phi_{-B}), \quad (\text{C.7})$$

$$\tilde{\Phi}_{-B} = (\iota_1 \Phi_{+B} - e^1 \wedge \Phi_{+B}), \quad (\text{C.8})$$

with e replaced by $\tilde{e}$ in the above expressions. Similarly, the T-dual of the RR fluxes is

$$\tilde{F} = e^{-2\tilde{B}} \wedge (\iota_1 - e^1 \wedge)(e^{2B} \wedge F), \quad (\text{C.9})$$

with, again, e replaced by $\tilde{e}$.

According to the T-duality rules, the component of the H -flux with one leg along the duality direction is exchanged with some structure constants

$$f_{ab}^1 \leftrightarrow H_{fl\ 1ab}, \quad (\text{C.10})$$

giving the new algebra: $(2|\mu|35 + 2|\mu|46, 0, 25, -26, -23, 24)$ and a new $\tilde{H}_{fl}$:

$$\tilde{H}_{fl} = 2|\mu|(\alpha \tilde{e}^2 \wedge \tilde{e}^3 \wedge \tilde{e}^4 - (2 - \alpha) \tilde{e}^2 \wedge \tilde{e}^5 \wedge \tilde{e}^6). \quad (\text{C.11})$$

We do the transformation $\tilde{e}^1 \rightarrow 2|\mu|\tilde{e}^1$ in order to find the solvalgebra we considered in section 4.1 (with $\epsilon = 1$): $(35 + 46, 0, 25, -26, -23, 24)$. We can now give explicitly the T-dual solution. From the transformation of the pure spinors, (C.7), we can read out the new SU(2) structure

$$\begin{aligned} z &= 2|\mu|\tilde{e}^1 + i\tilde{e}^2, \\ j &= \tilde{e}^3 \wedge \tilde{e}^6 - \tilde{e}^4 \wedge \tilde{e}^5, \\ \omega_R &= -\tilde{e}^3 \wedge \tilde{e}^4 + \tilde{e}^5 \wedge \tilde{e}^6, \\ \omega_I &= \tilde{e}^3 \wedge \tilde{e}^5 + \tilde{e}^4 \wedge \tilde{e}^6, \end{aligned} \quad (\text{C.12})$$

and H flux

$$H = 2|\mu|(\tilde{e}^2 \wedge \tilde{e}^3 \wedge \tilde{e}^4 - \tilde{e}^2 \wedge \tilde{e}^5 \wedge \tilde{e}^6). \quad (\text{C.13})$$

Similarly, from (C.9) we have

$$\begin{aligned} F_1 &= 10|\mu|^2 \tilde{e}^1, \\ F_3 &= 3|\mu|(\tilde{e}^2 \wedge \tilde{e}^4 \wedge \tilde{e}^5 - \tilde{e}^2 \wedge \tilde{e}^3 \wedge \tilde{e}^6), \\ F_5 &= -6|\mu|^2 \tilde{e}^1 \wedge \tilde{e}^3 \wedge \tilde{e}^4 \wedge \tilde{e}^5 \wedge \tilde{e}^6. \end{aligned} \quad (\text{C.14})$$

This is the solution we had in section 4.1 with $j_1 = 1$, $\epsilon_1 = 1$, $\omega_1 = 1$.

$(\epsilon_3, \epsilon_4, \epsilon_5)$	Name
$(1, 1, 1)$	$\mathfrak{g}_{6.88}$
$(-1, -1, 1)$	$\times$
$(-1, 1, -1)$	$\times$
$(1, -1, -1)$	$\mathfrak{g}_{6.92}^*$
$(1, 1, -1)$	$\mathfrak{g}_{6.89}^{0,-1,1} = \mathfrak{g}_{6.91}$
$(-1, 1, 1)$	$\mathfrak{g}_{6.89}^{0,1,1} = \mathfrak{g}_{6.90}^{0,-1}$

Table 3. The different names of algebra $(24+35, \epsilon_4 \ 46, \epsilon_5 \ 56, \epsilon_2 \ 26, \epsilon_3 \ 36, 0)$.

D Name of the new solution

We looked at the classification of six dimensional manifold in [33]. One can easily see that our manifold $(35 + \epsilon 46, 0, 25, -\epsilon 26, -\epsilon 23, 24)$ has nilradical $\mathfrak{g}_{5.4}$. We rewrite our solution in Bock's notation in order to make the comparisons easier:

$$\begin{aligned}
 [X_2, X_4] &= X_1 & [X_3, X_5] &= X_1 \\
 [X_2, X_6] &= X_4 & [X_3, X_6] &= -\epsilon X_5 \\
 [X_4, X_6] &= -\epsilon X_2 & [X_5, X_6] &= X_3
 \end{aligned} \tag{D.1}$$

We consider manifolds that one can put under the following form:

$$\begin{aligned}
 [X_2, X_4] &= X_1 & [X_3, X_5] &= X_1 \\
 [X_2, X_6] &= \epsilon_2 X_4 & [X_3, X_6] &= \epsilon_3 X_5 \\
 [X_4, X_6] &= \epsilon_4 X_2 & [X_5, X_6] &= \epsilon_5 X_3
 \end{aligned} \tag{D.2}$$

with $\epsilon_i = \pm 1$. One can see that by redefining X_6 in $-X_6$, one can change all the signs of the ϵ_i . So without loss of generality, one can assume that $\epsilon_2 = 1$. These manifolds are all unimodular (ie $\forall X, \text{Tr}(\text{Ad}_X) = 0$) solvmanifolds of dimension six with nilradical $\mathfrak{g}_{5.4}$. The different cases, up to real redefinitions of the X 's, are listed in table 3. The first row of the table corresponds to our solution with $\epsilon = -1$ whereas the second row corresponds to $\epsilon = 1$. As one can see, we weren't able to find it in the classification and called it solv_1 in the main text. Note that if one permits complex redefinitions of the X 's, the first four rows are equivalent and the last two are also equivalent.

Open Access. This article is distributed under the terms of the Creative Commons Attribution License ([CC-BY 4.0](https://creativecommons.org/licenses/by/4.0/)), which permits any use, distribution and reproduction in any medium, provided the original author(s) and source are credited.

References

- [1] J.-P. Derendinger, C. Kounnas, P.M. Petropoulos and F. Zwirner, *Superpotentials in IIA compactifications with general fluxes*, *Nucl. Phys. B* **715** (2005) 211 [[hep-th/0411276](https://arxiv.org/abs/hep-th/0411276)] [[INSPIRE](https://arxiv.org/abs/hep-th/0411276)].

- [2] K. Behrndt and M. Cvetič, *General $N = 1$ supersymmetric flux vacua of (massive) type IIA string theory*, *Phys. Rev. Lett.* **95** (2005) 021601 [[hep-th/0403049](#)] [[INSPIRE](#)].
- [3] O. DeWolfe, A. Giryavets, S. Kachru and W. Taylor, *Type IIA moduli stabilization*, *JHEP* **07** (2005) 066 [[hep-th/0505160](#)] [[INSPIRE](#)].
- [4] D. Lüst and D. Tsimpis, *Supersymmetric AdS_4 compactifications of IIA supergravity*, *JHEP* **02** (2005) 027 [[hep-th/0412250](#)] [[INSPIRE](#)].
- [5] C. Caviezel et al., *The Effective theory of type IIA AdS_4 compactifications on nilmanifolds and cosets*, *Class. Quant. Grav.* **26** (2009) 025014 [[arXiv:0806.3458](#)] [[INSPIRE](#)].
- [6] A. Tomasiello, *New string vacua from twistor spaces*, *Phys. Rev. D* **78** (2008) 046007 [[arXiv:0712.1396](#)] [[INSPIRE](#)].
- [7] P. Koerber and S. Körs, *A landscape of non-supersymmetric AdS vacua on coset manifolds*, *Phys. Rev. D* **81** (2010) 105006 [[arXiv:1001.0003](#)] [[INSPIRE](#)].
- [8] K. Dasgupta, G. Rajesh and S. Sethi, *M theory, orientifolds and G-flux*, *JHEP* **08** (1999) 023 [[hep-th/9908088](#)] [[INSPIRE](#)].
- [9] S.B. Giddings, S. Kachru and J. Polchinski, *Hierarchies from fluxes in string compactifications*, *Phys. Rev. D* **66** (2002) 106006 [[hep-th/0105097](#)] [[INSPIRE](#)].
- [10] M. Graña, R. Minasian, M. Petrini and A. Tomasiello, *A scan for new $N = 1$ vacua on twisted tori*, *JHEP* **05** (2007) 031 [[hep-th/0609124](#)] [[INSPIRE](#)].
- [11] J. Blaback et al., *Smeared versus localised sources in flux compactifications*, *JHEP* **12** (2010) 043 [[arXiv:1009.1877](#)] [[INSPIRE](#)].
- [12] J. Blaback, B. Janssen, T. Van Riet and B. Vercnocke, *Fractional branes, warped compactifications and backreacted orientifold planes*, *JHEP* **10** (2012) 139 [[arXiv:1207.0814](#)] [[INSPIRE](#)].
- [13] J. McOrist and S. Sethi, *M-theory and Type IIA flux compactifications*, *JHEP* **12** (2012) 122 [[arXiv:1208.0261](#)] [[INSPIRE](#)].
- [14] M.R. Douglas and R. Kallosh, *Compactification on negatively curved manifolds*, *JHEP* **06** (2010) 004 [[arXiv:1001.4008](#)] [[INSPIRE](#)].
- [15] F. Saracco and A. Tomasiello, *Localized $O6$ -plane solutions with Romans mass*, *JHEP* **07** (2012) 077 [[arXiv:1201.5378](#)] [[INSPIRE](#)].
- [16] D. Lüst and D. Tsimpis, *Classes of AdS_4 type IIA/ IIB compactifications with $SU(3) \times SU(3)$ structure*, *JHEP* **04** (2009) 111 [[arXiv:0901.4474](#)] [[INSPIRE](#)].
- [17] C. Caviezel, T. Wrase and M. Zagermann, *Moduli Stabilization and Cosmology of Type IIB on $SU(2)$ -Structure Orientifolds*, *JHEP* **04** (2010) 011 [[arXiv:0912.3287](#)] [[INSPIRE](#)].
- [18] M. Petrini, G. Solard and T. Van Riet, *AdS vacua with scale separation from IIB supergravity*, *JHEP* **11** (2013) 010 [[arXiv:1308.1265](#)] [[INSPIRE](#)].
- [19] D. Lüst and D. Tsimpis, *New supersymmetric AdS_4 type-II vacua*, *JHEP* **09** (2009) 098 [[arXiv:0906.2561](#)] [[INSPIRE](#)].
- [20] M. Graña, R. Minasian, M. Petrini and A. Tomasiello, *Supersymmetric backgrounds from generalized Calabi-Yau manifolds*, *JHEP* **08** (2004) 046 [[hep-th/0406137](#)] [[INSPIRE](#)].
- [21] M. Graña, R. Minasian, M. Petrini and A. Tomasiello, *Generalized structures of $N = 1$ vacua*, *JHEP* **11** (2005) 020 [[hep-th/0505212](#)] [[INSPIRE](#)].

- [22] K. Behrndt, M. Cvetič and P. Gao, *General type IIB fluxes with SU(3) structures*, *Nucl. Phys. B* **721** (2005) 287 [[hep-th/0502154](#)] [[INSPIRE](#)].
- [23] N. Hitchin, *Generalized Calabi-Yau manifolds*, *Quart. J. Math. Oxford Ser.* **54** (2003) 281 [[math/0209099](#)] [[INSPIRE](#)].
- [24] M. Gualtieri, *Generalized complex geometry*, [math/0401221](#) [[INSPIRE](#)].
- [25] D. Andriot, *New supersymmetric flux vacua with intermediate SU(2) structure*, *JHEP* **08** (2008) 096 [[arXiv:0804.1769](#)] [[INSPIRE](#)].
- [26] G. Dall’Agata and N. Prezas, *$N = 1$ geometries for M-theory and type IIA strings with fluxes*, *Phys. Rev. D* **69** (2004) 066004 [[hep-th/0311146](#)] [[INSPIRE](#)].
- [27] U.H. Danielsson, S.S. Haque, G. Shiu and T. Van Riet, *Towards classical de Sitter solutions in string theory*, *JHEP* **09** (2009) 114 [[arXiv:0907.2041](#)] [[INSPIRE](#)].
- [28] U.H. Danielsson, P. Koerber and T. Van Riet, *Universal de Sitter solutions at tree-level*, *JHEP* **05** (2010) 090 [[arXiv:1003.3590](#)] [[INSPIRE](#)].
- [29] D. Andriot, E. Goi, R. Minasian and M. Petrini, *Supersymmetry breaking branes on solvmanifolds and de Sitter vacua in string theory*, *JHEP* **05** (2011) 028 [[arXiv:1003.3774](#)] [[INSPIRE](#)].
- [30] U.H. Danielsson et al., *de Sitter hunting in a classical landscape*, *Fortsch. Phys.* **59** (2011) 897 [[arXiv:1103.4858](#)] [[INSPIRE](#)].
- [31] C. Caviezel et al., *On the cosmology of Type IIA compactifications on SU(3)-structure manifolds*, *JHEP* **04** (2009) 010 [[arXiv:0812.3551](#)] [[INSPIRE](#)].
- [32] R. Flauger, S. Paban, D. Robbins and T. Wrase, *Searching for slow-roll moduli inflation in massive type IIA supergravity with metric fluxes*, *Phys. Rev. D* **79** (2009) 086011 [[arXiv:0812.3886](#)] [[INSPIRE](#)].
- [33] C. Bock, *On low-dimensional solvmanifolds*, [arXiv:0903.2926](#).