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The super-Higgs mechanism in fluids

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ABSTRACT: Supersymmetry is spontaneously broken when the field theory stress-energy tensor has a non-zero vacuum expectation value. In local supersymmetric field theories the massless gravitino and goldstino combine via the super-Higgs mechanism to a massive gravitino. We study this mechanism in four-dimensional fluids, where the vacuum expectation value of the stress-energy tensor breaks spontaneously both supersymmetry and Lorentz symmetry. We consider both constant as well as space-time dependent ideal fluids. We derive a formula for the gravitino mass in terms of the fluid velocity, energy density and pressure. We discuss some of the phenomenological implications.

KEYWORDS: Supersymmetry Breaking, Supersymmetric Effective Theories, Supersymmetric gauge theory

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1 Introduction

A spontaneous breaking of supersymmetry is manifested by the generation of a massless fermionic Goldstone mode, the *goldstino* [1]. At non-zero temperature the vacuum expectation value of the stress-energy tensor breaks spontaneously supersymmetry as well as Lorentz symmetry, and the goldstino mode is called *phonino* (see e.g. [8–10]). We may view the phonino as a (supersymmetric) sound mode. However, unlike the ordinary bosonic sound mode that can be treated as a classical field, the phonino is fermionic and is therefore inherently a quantum field [11]. Its dispersion relation at leading order in momenta is fixed by the supersymmetry algebra. It is linear with velocity $v = \left| \frac{p}{\epsilon} \right|$, where p and ϵ are the fluid pressure and energy density, respectively. When the spontaneous supersymmetry breaking is due to a cosmological constant with the equation of state $\epsilon = -p$, the phonino is the ordinary goldstino, whose velocity is the speed of light.

In local supersymmetric field theories the goldstino combines with the massless gravitino via the super-Higgs mechanism to form a massive gravitino [2–7]. The aim of this paper is to study the super-Higgs mechanism in four-dimensional ideal fluids. Supersymmetry breaking is parametrized by the vacuum expectation value (vev) of the ideal fluid stress-energy tensor. We construct the effective field theory of the fermionic low-energy modes, the phonino and the gravitino, in the background of the fluid stress-energy tensor. We work up to quadratic order in the fields and to first order in derivatives. In the following we outline the results. Consider first the case, where the energy density ϵ and the pressure

p are constant, and the fluid is in the rest frame. Diagonalizing the field equations, we show that the longitudinal mode of the gravitino mixes with the phonino and acquires a mass

$$m_{\text{gravitino}} = \frac{\sqrt{3}}{4M_p} \left| \frac{p - \frac{\epsilon}{3}}{\sqrt{\epsilon}} \right|, \quad (1.1)$$

where M_p is the Planck mass. The dispersion relation of this mode is inherited from that of the phonino and is non-relativistic. The transverse part of the gravitino acquires the same mass (1.1), however, its dispersion relation is relativistic. When the spontaneous supersymmetry breaking is due to a cosmological constant $T^{\mu\nu} = -F^2\eta^{\mu\nu}$, one gets from (1.1) the well known formula for the gravitino mass $\frac{F}{\sqrt{3}M_p}$. Interestingly, the gravitino mass vanishes for a conformal fluid, where the stress-energy tensor is traceless and the equation of state is $\epsilon - 3p = 0$.

We study next the super-Higgs mechanism in the background of a non-constant slowly varying stress-energy tensor. We derive a general constraint on the gravitino field and analyze in detail the case of time-dependent energy density and pressure. The mass terms in this case include contributions from derivatives of the energy density.

The paper is organised as follows. In section 2 we briefly review various aspects of the goldstino, gravitino and the standard super-Higgs mechanism. In section 3 we consider the spontaneous breaking of supersymmetry and Lorentz invariance due to a non-zero vacuum expectation value of the fluid stress-energy tensor. We introduce the phonino field and its couplings to the gravitino, and study the super-Higgs mechanism. We consider first the case of a constant stress-energy tensor, and then extend the analysis to the case to space-time dependence, working to first order in derivatives. In section 4 we study in detail the field equations and find the propagating modes. For a constant stress-energy tensor we show that the goldstino is eaten by the gravitino, but retains its identity as it survives as the longitudinal mode of the gravitino with its own dispersion relation. The transverse and longitudinal component become massive, with the same mass (1.1). We generalize the discussion and study the field equations and the mass terms in the background of time-dependent stress-energy tensor. The last section is devoted to a discussion of some phenomenological implications.

2 F-term susy breaking and the super-Higgs mechanism

2.1 Goldstino and gravitino

In a global supersymmetric theory in flat space time, supersymmetry is broken spontaneously when the vacuum has non-zero energy. Preserving Lorentz invariance, this is typically accomplished for $N = 1$ susy in 4 dimensions by giving a vev to an auxiliary field in a chiral multiplet (F-term) or in a vector multiplet (D-terms). As a consequence of Goldstone theorem, the low energy spectrum contains a fermionic massless mode, known as the *goldstino*.

The goldstino is a spin $\frac{1}{2}$ field $(G_\alpha, \bar{G}_{\dot{\alpha}})$ in the $(\frac{1}{2}, 0) \oplus (0, \frac{1}{2})$ representation of the Lorentz group.¹ Its mass dimension is $\frac{3}{2}$. At quadratic order, the Lagrangian that describes its dynamics is only a kinetic term

$$\mathcal{L}_G = -i\bar{G}\bar{\sigma}^\mu\partial_\mu G, \quad (2.1)$$

and the field satisfies the Dirac equation

$$\bar{\sigma}^\mu\partial_\mu G = 0, \quad \sigma^\mu\partial_\mu \bar{G} = 0. \quad (2.2)$$

Theories with $N = 1$ local supersymmetry contain a gravitino field $(\psi_{\mu\alpha}, \bar{\psi}_{\mu\dot{\alpha}})$ of spin $\frac{3}{2}$ and mass dimension $\frac{3}{2}$. Following Fierz and Pauli, the irreducible spin $\frac{3}{2}$ representation is obtained from $\psi_{\mu\alpha}$ in the $(\frac{1}{2}, \frac{1}{2}) \otimes (\frac{1}{2}, 0) = (1, \frac{1}{2}) \oplus (0, \frac{1}{2})$ representation, and $\bar{\psi}_{\mu\dot{\alpha}}$ in the $(\frac{1}{2}, \frac{1}{2}) \otimes (0, \frac{1}{2}) = (\frac{1}{2}, 1) \oplus (\frac{1}{2}, 0)$ representation by imposing constraints that project out the additional spin $\frac{1}{2}$ components. The $(0, \frac{1}{2})$ and $(\frac{1}{2}, 0)$ parts in the decomposition of $(\psi_{\mu\alpha}, \bar{\psi}_{\mu\dot{\alpha}})$ are removed by imposing

$$\bar{\sigma}^\mu\psi_\mu = 0, \quad \sigma^\mu\bar{\psi}_\mu = 0. \quad (2.3)$$

The representations $(1, \frac{1}{2})$ and $(\frac{1}{2}, 1)$ have dimension six each. In order to reduce the number of degrees of freedom to four we impose

$$\partial^\mu\psi_{\mu\alpha} = 0, \quad \partial^\mu\bar{\psi}_{\mu\dot{\alpha}} = 0. \quad (2.4)$$

One can get this structure of equations and constraints from a Lagrangian. The massless gravitino Rarita-Schwinger Lagrangian is:

$$\mathcal{L}_\psi = \epsilon^{\mu\nu\rho\sigma}\bar{\psi}_\mu\bar{\sigma}_\nu\partial_\rho\psi_\sigma. \quad (2.5)$$

The field equations are

$$\epsilon^{\mu\nu\rho\sigma}\bar{\sigma}_\nu\partial_\rho\psi_\sigma = 0, \quad \epsilon^{\mu\nu\rho\sigma}\sigma_\nu\partial_\rho\bar{\psi}_\sigma = 0. \quad (2.6)$$

By imposing on this equation the condition (2.3) we get

$$\bar{\sigma}^\rho\partial_\rho\psi_\sigma = 0, \quad \sigma^\rho\partial_\rho\bar{\psi}_\sigma = 0. \quad (2.7)$$

It is easy to see that (2.7) and (2.3) imply (2.4).

2.2 The super-Higgs mechanism

Consider a spontaneous F-term supersymmetry breaking in a theory with local supersymmetry. The stress-energy tensor in this case has a vev $T^{\mu\nu} = -F^2\eta^{\mu\nu}$, where F is the

¹We will use Wess and Bagger notations. $\eta_{\mu\nu} = \text{diag}(-, +, +, +)$, $\epsilon^{12} = -\epsilon^{21} = 1$. ζ_α is a left Weyl spinor in the $(\frac{1}{2}, 0)$ representation. $\bar{\zeta}_{\dot{\alpha}}$ is a right Weyl spinor in the $(0, \frac{1}{2})$ representation. Complex conjugation exchanges $SU(2)_L$ and $SU(2)_R$. The complex conjugate of a left Weyl spinor is a right Weyl spinor.

vev of the auxiliary field. We take F real with mass dimension two. The supersymmetry transformations (we suppress here the spinor index) are:

$$\begin{aligned}\delta\psi_\mu &= -M_p(2\partial_\mu\epsilon + n\sigma_\mu\bar{\epsilon}), & \delta G &= \sqrt{2}F\epsilon, \\ \delta\bar{\psi}_\mu &= -M_p(2\partial_\mu\bar{\epsilon} + \bar{n}\bar{\sigma}_\mu\epsilon), & \delta\bar{G} &= \sqrt{2}F\bar{\epsilon}.\end{aligned}\quad (2.8)$$

In order to have a Lagrangian for the gravitino and goldstino invariant under (2.8) we need to add mass terms:

$$\mathcal{L}_{\text{mass}} = M^{\mu\nu}\psi_\mu\psi_\nu + \bar{M}^{\mu\nu}\bar{\psi}_\mu\bar{\psi}_\nu + (C^{\mu\nu}\psi_\mu\sigma_\nu\bar{G} - \bar{C}^{\mu\nu}\bar{\psi}_\mu\bar{\sigma}_\nu G) + \frac{1}{2}(mGG + \bar{m}\bar{G}\bar{G}). \quad (2.9)$$

$\bar{M}^{\mu\nu}$, $\bar{C}^{\mu\nu}$ and $\bar{m}$ are the complex conjugates of $M^{\mu\nu}$, $C^{\mu\nu}$ and m , respectively. We require that the total action

$$\mathcal{L} = \mathcal{L}_\psi + \mathcal{L}_G + \mathcal{L}_{\text{mass}}, \quad (2.10)$$

be invariant under the supersymmetry transformations (2.8).

The supersymmetry invariance of the action implies fixes the action uniquely. From terms of the form $\partial\epsilon\psi$ we get

$$M^{\mu\nu} = i\bar{n}\sigma^{\mu\nu}. \quad (2.11)$$

We set $-i\bar{n} = m_{\frac{3}{2}}$, thus, the gravitino mass matrix reads

$$M^{\mu\nu}\psi_\mu\psi_\nu = -m_{\frac{3}{2}}\psi_\mu\sigma^{\mu\nu}\psi_\nu. \quad (2.12)$$

From the $\bar{\epsilon}\psi$ terms we get

$$C^{\mu\nu} = \frac{i\sqrt{3}}{\sqrt{2}}m_{\frac{3}{2}}\eta^{\mu\nu}. \quad (2.13)$$

The mass of the gravitino is determined from the $\partial\epsilon\bar{G}$ terms

$$m_{\frac{3}{2}} = \frac{F}{\sqrt{3}M_p}. \quad (2.14)$$

From the ϵG terms we get

$$m = -m_{\frac{3}{2}}. \quad (2.15)$$

We can read the propagating degrees of freedom most easily by going to the unitary gauge, where we use the susy transformations to set $G = \bar{G} = 0$; then we find the Lagrangian for a massive gravitino

$$\mathcal{L}_g = \epsilon^{\mu\nu\rho\sigma}\bar{\psi}_\mu\bar{\sigma}_\nu\partial_\rho\psi_\sigma - m_{\frac{3}{2}}\psi_\mu\sigma^{\mu\nu}\psi_\nu - m_{\frac{3}{2}}^*\bar{\psi}_\mu\sigma^{\mu\nu}\bar{\psi}_\nu. \quad (2.16)$$

From the lagrangian we can also read the form of the supercurrents $(S_{\mu\alpha}, \bar{S}_{\mu\dot{\alpha}})$, $\bar{S}_{\mu\dot{\alpha}} = (S_{\mu}^{\alpha})^\dagger$. They couple to the gravitino as

$$\frac{1}{M_p} \int d^4x (S_{\mu\alpha}\psi^{\mu\alpha} + \bar{S}_{\mu\dot{\alpha}}\bar{\psi}^{\mu\dot{\alpha}}). \quad (2.17)$$

Comparing to (2.9) we see that

$$S^{\mu\alpha} = i\frac{F}{\sqrt{2}}\sigma^\mu\bar{G}, \quad \bar{S}^{\mu\dot{\alpha}} = -i\frac{F}{\sqrt{2}}\bar{\sigma}^\mu G. \quad (2.18)$$

These expressions are the leading non-derivative terms in the derivative expansion of the supercurrents. The conservation laws

$$\partial_\mu S^{\mu\alpha} = 0, \quad \partial_\mu \bar{S}^{\mu\dot{\alpha}} = 0 \quad (2.19)$$

are the field equations of the goldstino (2.2).

3 The super-Higgs mechanism in fluids

In this section we will study the super-Higgs mechanism in fluids, where the vacuum expectation value of the stress-energy tensor $T_{\mu\nu}$ breaks spontaneously both supersymmetry and Lorentz symmetry. One of the motivations for our study is to understand the fate of the *phonino* in supergravity theories.

3.1 Supersymmetric fluids

Consider a supersymmetric field theory in thermal equilibrium described by a background stress-energy tensor

$$T^{\mu\nu} = \text{diag}(\varepsilon, p, p, p) . \quad (3.1)$$

p is the pressure and ε is the energy density, and the two are related by an equation of state $p(\varepsilon)$. The expectation value of the stress-energy tensor (3.1) breaks spontaneously supersymmetry and Lorentz symmetry but keeps rotational invariance. Two special cases of (3.1) are: $-p = \varepsilon = F^2$ corresponding to the F-term breaking, and $p = \varepsilon/3$ that describes a conformal fluid.

The spontaneous breaking of supersymmetry implies in general a massless fermionic field in the spectrum called phonino. The existence of this mode can be understood as a consequence of a supersymmetric Ward-Takahashi identity for the supercurrent two-point function:

$$\partial_\mu \langle T \{ S^\mu(x) \bar{S}^\nu(y) \} \rangle \sim \delta^{(4)}(x-y) \langle T^{\nu\rho} \rangle \sigma_\rho . \quad (3.2)$$

Going to momentum space and assuming a constant energy-momentum tensor the correlator has to have a singularity when $k \rightarrow 0$. With Lorentz invariance one concludes that there must be a massless fermionic mode. Without Lorentz invariance it is possible to have a singularity without having a massless particle. This happens for instance in a free theory. In a generic interacting system it is expected that the massless mode is present (see e.g. [12] for a discussion of these issues), and we will consider these cases.

The field equations of the phonino take the form

$$T^{\mu\nu} \bar{\sigma}_\mu \partial_\nu G = 0, \quad T^{\mu\nu} \sigma_\mu \partial_\nu \bar{G} = 0 . \quad (3.3)$$

These equations arise from the Lagrangian

$$\mathcal{L}_G = -\frac{i}{\mathcal{T}_4} T^{\mu\nu} \bar{G} \bar{\sigma}_\mu \partial_\nu G , \quad (3.4)$$

where² $\mathcal{T} = |\det \langle T^{\mu\nu} \rangle|^{\frac{1}{16}}$. When $T^{\mu\nu} = -F^2 \eta^{\mu\nu}$ the Lagrangian (3.4) reduces to (2.1) and the propagator of the phonino becomes that of the usual goldstino.

3.2 Generalized super-Higgs mechanism

In the following we will be working with an expansion in powers of the dimensionless parameter $\frac{\mathcal{T}}{M_P}$. The effective Lagrangian for the gravitino and phonino at leading order in this expansion reads

$$\begin{aligned} \mathcal{L} = & \epsilon^{\mu\nu\rho\sigma} \bar{\psi}_\mu \bar{\sigma}_\nu \partial_\rho \psi_\sigma + i D^{\mu\nu} \bar{G} \bar{\sigma}_\mu \partial_\nu G + i C^{\mu\nu} (\bar{\psi}_\mu \bar{\sigma}_\nu G + \psi_\mu \sigma_\nu \bar{G}) \\ & + \frac{1}{2} G m G + \frac{1}{2} \bar{G} m^* \bar{G} + M_{\rho\tau}^{\mu\nu} \psi_\mu \sigma^{\rho\tau} \psi_\nu + M_{\rho\tau}^{\mu\nu*} \bar{\psi}_\mu \bar{\sigma}^{\rho\tau} \bar{\psi}_\nu. \end{aligned} \quad (3.5)$$

The mass matrices m and $M_{\rho\tau}^{\mu\nu}$ have suppressed spinor indices. We could have added also a term $M^{\mu\nu} \psi_\mu \psi_\nu$, however it turns out that it is not allowed by supersymmetry and we omit it. Note also, that at leading order in $\frac{\mathcal{T}}{M_P}$ the gravitino has the standard kinetic term.

The supersymmetry transformations need to be modified to allow for Lorentz violating coefficients:

$$\begin{aligned} \delta G^\alpha &= \sqrt{2} \mathcal{T}^2 \varepsilon^\alpha, \\ \delta \psi_{\mu\alpha} &= -M_P \left(2\partial_\mu \varepsilon_\alpha + i n_{\mu\nu} \sigma_{\alpha\dot{\alpha}}^\nu \bar{\varepsilon}^{\dot{\alpha}} \right), \\ \delta \bar{\psi}_{\mu\dot{\alpha}} &= -M_P \left(2\partial_\mu \bar{\varepsilon}_{\dot{\alpha}} - i n_{\mu\nu}^* \varepsilon^\alpha \sigma_{\alpha\dot{\alpha}}^\nu \right). \end{aligned} \quad (3.6)$$

The requirement that the Goldstino equation of motion reproduces, at the lowest order, the phonino dispersion relation fixes:

$$D^{\mu\nu} = \frac{T^{\mu\nu}}{\mathcal{T}^4}. \quad (3.7)$$

Note, that this is the Volkov-Akulov standard leading term describing the coupling between matter and Goldstinos, where the stress-energy tensor appears explicitly with its non-vanishing vacuum expectation value. We also assumed that the supersymmetric vacuum is obtained in flat space when the stress-energy tensor vanishes.

Performing the supersymmetry variation, terms of the form $\bar{G} \partial_\nu \varepsilon_\alpha$ coming from $\bar{G} \partial G$ and $\bar{\psi} G$ fix:

$$C^{\mu\nu} = -\frac{1}{\sqrt{2}} \frac{\mathcal{T}^2}{M_P} \frac{T^{\mu\nu}}{\mathcal{T}^4}. \quad (3.8)$$

This is consistent with a gravitino-phonino coupling of the form (2.17) if the supercurrent has the form

$$S^{\mu\alpha} \sim \frac{T^{\mu\nu}}{\mathcal{T}^2} \sigma_\nu^{\alpha\dot{\alpha}} \bar{G}_{\dot{\alpha}}. \quad (3.9)$$

As in the previous section, the conservation equation for the supercurrent is equivalent to the propagation equation for the phonino.

²This quantity is introduced for the sake of defining dimensionless factors involving $T^{\mu\nu}$. A slight modification would be needed in the case of a degenerate stress-energy tensor with zero determinant, for instance if $p = 0$, we should take $\mathcal{T} = \varepsilon^{1/4}$.

Terms of the form $\psi\partial\epsilon$ coming from $\bar{\psi}\partial\psi$ and $\psi\psi$ give

$$M_{\lambda\kappa}^{\mu\nu}\sigma^\lambda\bar{\sigma}^\kappa = -\frac{i}{2}\epsilon^{\mu\nu\rho\sigma}\sigma_\rho\bar{\sigma}^\gamma n_{\sigma\gamma}^* , \quad (3.10)$$

and terms $\psi\epsilon$ from $\bar{\psi}G$ and $\psi\psi$

$$M_{\rho\tau}^{\mu\nu}n_{\nu\lambda}\sigma^{\rho\tau}\sigma^\lambda = -\frac{T^{\mu\nu}}{2M_P^2}\sigma_\nu . \quad (3.11)$$

The last two equations lead to:

$$\frac{i}{2}\epsilon^{\mu\nu\rho\sigma}n_{\nu\lambda}^*n_{\sigma\gamma}\sigma_\rho\bar{\sigma}^\gamma\sigma^\lambda = \frac{T^{\mu\nu}}{M_P^2}\sigma_\nu . \quad (3.12)$$

The last equation can be put in a simpler form when n is real, which we will assume from now on. We antisymmetrize in $\rho\gamma\lambda$ and get

$$-\frac{1}{2}\epsilon^{\mu\nu\sigma\rho}\epsilon_\rho^{\lambda\gamma\kappa}n_{\nu\lambda}n_{\sigma\gamma} = \frac{T^{\mu\kappa}}{M_P^2} . \quad (3.13)$$

This equation determines $n_{\mu\nu}$ in terms of $T_{\mu\nu}$. Finally, from the terms ϵG we have:

$$m_\alpha{}^\beta = \frac{1}{2}\frac{T^{\mu\nu}n_{\mu\nu}}{\mathcal{T}^4}\delta_\alpha{}^\beta . \quad (3.14)$$

To arrive at this form we used the fact that $T^{\mu\rho}n_\rho^\nu$ is symmetric in $\mu\nu$. Putting all the results together gives the Lagrangian

$$\begin{aligned} \mathcal{L} = & \epsilon^{\mu\nu\rho\sigma}\bar{\psi}_\mu\bar{\sigma}_\nu\partial_\rho\psi_\sigma + \frac{i}{4}\epsilon^{\mu\nu\rho\sigma}n_{\sigma\gamma}\bar{\psi}_\mu\bar{\sigma}_\rho\sigma^\gamma\bar{\psi}_\nu - \frac{i}{4}\epsilon^{\mu\nu\rho\sigma}n_{\sigma\gamma}\psi_\mu\sigma_\rho\bar{\sigma}^\gamma\psi_\nu \\ & - \frac{i}{\sqrt{2}}\frac{\mathcal{T}^2}{M_P}\frac{T^{\mu\nu}}{\mathcal{T}^4}(\bar{\psi}_\mu\bar{\sigma}_\nu G + \psi_\mu\sigma_\nu\bar{G}) \\ & + i\frac{T^{\mu\nu}}{\mathcal{T}^4}\bar{G}\bar{\sigma}_\mu\partial_\nu G + \frac{1}{4}\frac{T^{\mu\nu}n_{\mu\nu}}{\mathcal{T}^4}GG + \frac{1}{4}\frac{T^{\mu\nu}n_{\mu\nu}}{\mathcal{T}^4}\bar{G}\bar{G} . \end{aligned}$$

The unitary gauge is obtained by making a supersymmetry transformation to set $G = 0$:

$$\psi_{\mu\alpha} \rightarrow \psi_{\mu\alpha} + \frac{\sqrt{2}M_P}{\mathcal{T}^2}\partial_\mu G_\alpha + i\frac{M_P}{\sqrt{2}\mathcal{T}^2}n_{\mu\nu}\sigma_{\alpha\dot{\alpha}}^\nu\bar{G}^{\dot{\alpha}} . \quad (3.15)$$

The resulting Lagrangian reads

$$\mathcal{L} = \epsilon^{\mu\nu\rho\sigma}\bar{\psi}_\mu\bar{\sigma}_\nu\partial_\rho\psi_\sigma - \frac{i}{2}\epsilon^{\mu\nu\rho\sigma}n_{\sigma\gamma}\bar{\psi}_\mu\bar{\sigma}_\rho\sigma^\gamma\bar{\psi}_\nu + \frac{i}{2}\epsilon^{\mu\nu\rho\sigma}n_{\sigma\gamma}\psi_\mu\sigma_\rho\bar{\sigma}^\gamma\psi_\nu . \quad (3.16)$$

The equation of motion is

$$\epsilon^{\mu\nu\rho\sigma}\bar{\sigma}_\nu\partial_\rho\psi_\sigma - \frac{i}{2}\epsilon^{\mu\nu\rho\sigma}n_{\sigma\gamma}\bar{\sigma}_\rho\sigma^\gamma\bar{\psi}_\nu = 0 . \quad (3.17)$$

Consider now the constraints that are necessary in order to reduce the number of degrees of freedom of ψ_μ to the four that describe a massive gravitino. Acting on the equation of motion by $n_{\mu\lambda}\sigma^\lambda$ gives

$$-\frac{i}{2}\epsilon^{\mu\nu\rho\sigma}n_{\mu\lambda}n_{\sigma\gamma}\sigma^\lambda\bar{\sigma}_\rho\sigma^\gamma\bar{\psi}_\nu = 0 . \quad (3.18)$$

Using the symmetry of $n_{\mu\lambda}$, this can be put in the form:

$$T^{\mu\nu}\sigma_\mu\bar{\psi}_\nu = 0, \quad (3.19)$$

which replaces the standard F-term breaking constraint $\bar{\sigma}^\mu\psi_\mu = 0$ of the gravitino. As in the case of curved space-time [13], a second constraint is obtained by taking the component $\mu = 0$ of (3.17). We analyze the consequences of the constraint in the next section.

Consider next the general case of a space-time dependent stress-energy tensor. In the hydrodynamic regime the fluid is in local thermal equilibrium. One can use a derivative expansion since the charge densities are slowly varying functions of the space-time coordinates. At leading order the gravitino Lagrangian in the unitary gauge takes the form (3.16) with $n_\mu^\nu(x^\alpha)$. As a consistency check we take the susy variation of the Lagrangian (3.16). It yields

$$\delta\mathcal{L} = -iM_P \epsilon^{\rho\mu\nu\sigma} \partial_\rho n_\mu^\tau \varepsilon_{\sigma\tau} \bar{\sigma}_\nu \psi_\sigma + \frac{i}{\sqrt{2}\mathcal{T}^2} \partial_\nu (\mathcal{T}^4 D^{\mu\nu}) \bar{G} \bar{\sigma}_\mu \varepsilon, \quad (3.20)$$

the second term vanishes by the stress-energy tensor conservation. The variation (3.20) can be compensated by adding new terms:

$$\begin{aligned} \mathcal{L}^{(1)} = & i \frac{M_P}{\sqrt{2}\mathcal{T}^2} \epsilon^{\rho\mu\nu\sigma} \partial_\rho n_\mu^\tau \left[G \sigma_\tau \bar{\sigma}_\nu \psi_\sigma + h.c. \right. \\ & + \frac{M_P}{\sqrt{2}\mathcal{T}^2} G \sigma_\tau \bar{\sigma}_\nu \partial_\sigma G + \frac{M_P}{\sqrt{2}\mathcal{T}^2} \bar{G} \sigma_\tau \bar{\sigma}_\nu \partial_\sigma \bar{G} \\ & \left. + i \frac{M_P}{\sqrt{2}\mathcal{T}^2} n_\sigma^\lambda (G \sigma_\tau \bar{\sigma}_\nu \sigma_\lambda \bar{G} + i G \epsilon_{\nu\lambda\tau\gamma} \sigma^\gamma \bar{G}) \right]. \end{aligned} \quad (3.21)$$

Using the equation (3.13), the last term can put in the form $-\frac{i}{2\mathcal{T}^4} \partial_\nu T^{\mu\nu} \bar{G} \bar{\sigma}_\mu G$, so it vanishes when $T^{\mu\nu}$ is conserved.³ All terms in (3.21) contain G , so they vanish in the unitary gauge, giving back, at this leading order in the varying mass term, the lagrangian (3.16) and field equations (3.17). However, there is a new constraint that replaces (3.19) and takes the form

$$\frac{T^{\mu\nu}}{M_P^2} \sigma_\mu \bar{\psi}_\nu - \epsilon^{\mu\nu\rho\sigma} \partial_\mu n_\sigma^\gamma \sigma_\rho \bar{\sigma}_\gamma \psi_\nu = 0. \quad (3.22)$$

4 Ideal fluid

We will consider now relativistic ideal fluids with stress-energy tensor

$$T^{\mu\nu} = (\epsilon + p) u^\mu u^\nu + p \eta^{\mu\nu}, \quad (4.1)$$

where u^μ is the fluid four-velocity $u^\mu u_\mu = -1$. We will derive the gravitino mass as a function of the fluid variables.

In order to solve (3.13) we parametrize the solution $n_{\mu\nu}$ as

$$n_{\mu\nu} = (n_T - n_L) u_\mu u_\nu + n_T \eta_{\mu\nu}. \quad (4.2)$$

³The stress-energy tensor is conserved when studying a closed system, but we could also consider non-conserved stress-energy tensors, for instance if we apply our formalism to systems subject to an external force.

Plugging $n_{\mu\nu}$ and $T_{\mu\nu}$ and solving for n_T and n_L we get

$$n_T^2 = \frac{\varepsilon}{3M_P^2}, \quad -n_T(n_T + 2n_L) = \frac{p}{M_P^2}, \quad (4.3)$$

hence

$$n_L = -n_T \left(\frac{\varepsilon + 3p}{2\varepsilon} \right). \quad (4.4)$$

For F -term breaking, $\varepsilon = -p$, $n_L = n_T$, and for a CFT, $\varepsilon = 3p$, $n_L = -n_T$.

4.1 Constant stress-energy tensor

In the following we study in detail the gravitino equations and constraints when the energy density and pressure are constant and the 4-velocity is $u^\mu = (1, 0, 0, 0)$. In this case $n_\mu^\nu = \text{diag}(n_L, n_T, n_T, n_T)$. We introduce the notation

$$\not{D} = \sigma^\mu \partial_\mu, \quad \not{\partial} = \sigma^i \partial_i, \quad (4.5)$$

and

$$\Psi = \bar{\sigma}^\mu \psi_\mu, \quad \psi_{\frac{1}{2}} = \bar{\sigma}^i \psi_i, \quad \bar{\psi}_{\frac{1}{2}} = \sigma^i \bar{\psi}_i. \quad (4.6)$$

One has

$$\epsilon^{ijk} \bar{\sigma}_i \partial_j \psi_k = i \bar{\sigma}^0 \left(\not{\partial} \bar{\psi}_{\frac{1}{2}} + \partial \cdot \psi \right) \quad (4.7a)$$

$$\epsilon^{0\nu\rho\sigma} n_\sigma^\gamma \bar{\sigma}_{\rho\gamma} \bar{\psi}_\nu = i n_T \bar{\sigma}^0 \bar{\psi}_{\frac{1}{2}}. \quad (4.7b)$$

We rewrite the gravitino equation in the following form

$$\not{D} \psi_\mu - \bar{\sigma}_\mu \partial_\nu \psi^\nu - \partial_\mu \Psi - \bar{\sigma}_\mu \not{D} \Psi + \epsilon_\mu^{\nu\rho\sigma} n_\sigma^\gamma \bar{\sigma}_{\rho\gamma} \bar{\psi}_\nu = 0. \quad (4.8)$$

The constraint (3.19) can be used to solve for one of the components

$$\psi_0 = -v \sigma_0 \bar{\psi}_{\frac{1}{2}}, \quad (4.9)$$

where $v = \left| \frac{p}{\varepsilon} \right|$ is the phonino velocity. The component $\mu = 0$ of equation (4.8) gives the constraint

$$\not{\partial} \psi_{\frac{1}{2}} - i n_T \bar{\psi}_{\frac{1}{2}} + \partial \cdot \psi = 0. \quad (4.10)$$

Putting all the constraints together leads to

$$(\bar{\sigma}^0 \partial_0 + v \not{\partial}) \bar{\psi}_{\frac{1}{2}} - i \hat{m} \psi_{\frac{1}{2}} = 0. \quad (4.11)$$

This is the Dirac equation satisfied by the longitudinal spin-1/2 mode with mass

$$\hat{m} = \frac{n_L + n_T}{2} = \frac{n_T}{4} |(1 - 3v)| = \frac{\sqrt{3}}{4M_P} \left| \frac{p - \frac{\varepsilon}{3}}{\sqrt{\varepsilon}} \right|. \quad (4.12)$$

Notice that the eqs. (4.3) determine n_L, n_T only up to a sign; we have used this freedom in the last equation to have a positive mass $\hat{m}$.

Using (4.9) and (4.10) one finds

$$\Psi = (1+v)\psi_{\frac{1}{2}}, \quad \partial_\mu \psi^\mu = (v^2 - 1) \not{\partial} \psi_{\frac{1}{2}} + i(n_T + \hat{m}v)\bar{\psi}_{\frac{1}{2}}. \quad (4.13)$$

Finally, using the last relations in the equation of motion with $\mu = j$ gives

$$(\bar{\sigma}^0 \partial_0 + \bar{\not{\partial}}) \psi_j + i\hat{m}\bar{\psi}_j - (1+v) \left(\partial_j \bar{\psi}_{\frac{1}{2}} + i \frac{n_T}{2} \bar{\sigma}_j \psi_{\frac{1}{2}} \right) = 0. \quad (4.14)$$

One can verify that contracting this equation with σ^j gives back the equation for $\psi_{\frac{1}{2}}$. The projector on the transverse part of the spinor is

$$\psi_j = \psi_j^T - \left(\frac{1}{2} \sigma_j - \frac{k_j \not{k}}{2k^2} \right) \bar{\psi}_{\frac{1}{2}} + \left(\frac{3k_j}{2k^2} + \frac{1}{2} \frac{\sigma_j \not{k}}{k^2} \right) k \cdot \psi. \quad (4.15)$$

Replacing $k \cdot \psi$ using (4.10) we have

$$\begin{aligned} \psi_j &= \psi_j^T - \frac{k_j \not{k}}{k^2} \bar{\psi}_{\frac{1}{2}} + \frac{n_T}{2k^2} (k_j - \not{k} \sigma_j) \psi_{\frac{1}{2}} \\ \bar{\psi}_j &= \bar{\psi}_j^T - \frac{k_j \not{\bar{k}}}{k^2} \psi_{\frac{1}{2}} - \frac{n_T}{2k^2} (k_j - \not{\bar{k}} \sigma_j) \bar{\psi}_{\frac{1}{2}}. \end{aligned} \quad (4.16)$$

One can check plugging it in (4.14) that the transverse part satisfies the decoupled equation

$$(\bar{\sigma}^0 \partial_0 + \bar{\not{\partial}}) \psi_j^T + i\hat{m} \bar{\psi}_j^T = 0. \quad (4.17)$$

Eqs. (4.11), (4.12) and (4.17) are our main results. In the fluid, the goldstino is eaten by the gravitino. The gravitino has two distinct propagating modes, the longitudinal and the transversal, with the same mass but different dispersion relations. It is interesting to note that the gravitino and the goldstino remain massless in a CFT fluid.

4.2 Slowly varying stress-energy tensor

Consider an ideal fluid with time dependent stress-energy, $\epsilon = \epsilon(t), p = p(t)$. The field equation (4.8) is still valid, however the constraint (4.10) now reads

$$\frac{\epsilon}{M_P} \sigma_0 \bar{\psi}_0 + \frac{p}{M_P} \sigma_k \bar{\psi}_k - i \frac{\dot{\epsilon}}{\sqrt{3}\epsilon} \sigma_0 \bar{\sigma}_k \psi_k = 0. \quad (4.18)$$

It is straightforward to solve the constraints and derive the mass for the longitudinal and transverse components. First notice that (4.7a), (4.7b), (4.8), (4.10) are unchanged, while for ψ_0 we get

$$\bar{\psi}_0 = -v \bar{\sigma}_0 \bar{\psi}_{\frac{1}{2}} + iM_P \frac{\dot{\epsilon}}{\sqrt{3}\epsilon^{3/2}} \psi_{\frac{1}{2}}, \quad (4.19)$$

and the equation for the longitudinal mode becomes

$$(\bar{\sigma}^0 \partial_0 + v \bar{\not{\partial}}) \bar{\psi}_{\frac{1}{2}} - iM_P \frac{\dot{\epsilon}}{\sqrt{3}\epsilon^{3/2}} \bar{\sigma}^0 \not{\partial} \psi_{\frac{1}{2}} - i\hat{m} \bar{\psi}_{\frac{1}{2}} + \frac{\dot{\epsilon}}{2\epsilon} \bar{\sigma}^0 \bar{\psi}_{\frac{1}{2}} = 0. \quad (4.20)$$

In comparison to the time-independent case, we see that both the dispersion relation and the mass are modified by the time-dependent terms, and couple the different chiralities.

These general features are in agreement with the results of [15, 17] though the details differ. We comment on these differences in the final section.

Equation (4.13) takes the form

$$\begin{aligned}\Psi &= (1+v)\psi_{\frac{1}{2}} - iM_P \frac{\dot{\epsilon}}{\sqrt{3}\epsilon^{3/2}} \bar{\sigma}^0 \bar{\psi}_{\frac{1}{2}}, \\ \partial_\mu \psi^\mu &= (v^2 - 1) \not{\partial} \psi_{\frac{1}{2}} + i(n_T + \hat{m}v) \bar{\psi}_{\frac{1}{2}} - \hat{m}M_P \frac{\dot{\epsilon}}{\sqrt{3}\epsilon^{3/2}} \sigma^0 \psi_{\frac{1}{2}}.\end{aligned}\tag{4.21}$$

Considering (4.8) for $\mu = j$, one sees that equation of motion for the transverse part will remain unchanged.

5 Discussion

In this work we considered the spontaneous supersymmetry breaking triggered by the expectation value of a (non-degenerate) fluid stress-energy tensor $T^{\mu\nu}$. The mass of the gravitino is $m_{\text{gravitino}} = \frac{\sqrt{3}}{4M_p} \left| \frac{p-\frac{5}{3}}{\sqrt{\epsilon}} \right|$. The super-Higgs mechanism analysis that we performed assumed a flat space-time. One may question the validity of this assumption, since the stress-energy tensor will in general curve space-time, with a curvature scale of the same order of magnitude as the gravitino mass. The contribution of this stress-energy tensor to the curvature scale of space-time is assumed to be negligible as it is done in the more familiar F -term breaking case.

In order to justify this assumption, we should notice first that the quantity $T^{\mu\nu}$ considered here is not the total stress-energy tensor appearing on the right hand side of Einstein equations. For instance, in the case of a stress-energy tensor with the equation of state of a cosmological constant $T^{\mu\nu} = -F^2 \eta^{\mu\nu}$, the Volkov-Akulov result shows that the Goldstino variation contains a non-linear term proportional to $1/F$ and the Lagrangian contains a cosmological constant given by F^2 . Supersymmetry enforces the relation between the gravitino mass and the cosmological constant $3M_p^2 m_{\text{gravitino}}^2 = F^2$, as the vacuum energy is cancelled. In a similar way, the assumption of a flat space-time enforces the exact numerical coefficients in (1.1). While we have performed the analysis at the quadratic level, we expect that our assumption of a flat space time will also hold consistently at higher orders but leave this issue for future study.

The case a constant stress-energy tensor with a general equation of state, that contributes to the vacuum energy and whose back-reaction generates an FRW space-time, has been studied in [15–17]. It is interesting to note that their results agree with our computation in flat space time (1.1) after proper rescalings by the expansion scale to take into account the covariant derivatives.

Our results can find diverse phenomenological applications. One can consider Standard Model particles as part of the fluid under consideration. This would be the situation during early universe evolution, and it has been studied in the paper mentioned above [14–17].

Another phenomenological application is to identify the fluid as a hidden sector. The supersymmetry breaking is mediated through very weak interactions not sufficient to thermalize the whole system. For instance gravitational interactions will lead to soft terms of the order of $m_{\text{soft}} \sim \frac{\mathcal{T}^2}{M_p}$. The mediation also induces a Lorentz violation in the visible sector, therefore implying a bound on $1 - \frac{v}{c}$ for the viability of this scenario [18].

Beyond the original motivation of studying supersymmetry breaking, the super-Higgs mechanism allows to engineer Lagrangian for massive Rarita-Schwinger fields that do not exhibit pathologies such as breakdown of causality [19]. In this way, our Lagrangian, and the corresponding equations of motion, can be thought of as describing the propagation of a spin $\frac{3}{2}$ state, e.g. a hadronic resonance, in a non-Lorentz invariant background.

Finally, it is of interest to continue the study of the hydrodynamics of supersymmetric field theories [11, 20] beyond the ideal order (see [21–24] for a computation of transport coefficients at strong coupling using AdS/CFT). The study of couplings of the supersymmetry hydrodynamic modes to the gravitino is a useful framework to pursue, at least for the analysis of non-dissipative transports.

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