RECEIVED: October 17, 2013

REVISED: December 4, 2013

ACCEPTED: January 15, 2014

PUBLISHED: February 4, 2014

Asymmetric Gepner models in type II

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ABSTRACT: We describe new four-dimensional type II compactifications with $\mathcal{N} = 2$ supersymmetry, based on asymmetric Gepner models for $K3 \times T^2$. In more than half of these models, all the $K3$ moduli are lifted, giving at low energies $\mathcal{N} = 2$ supergravity with the STU vector multiplets and no hypermultiplets.

KEYWORDS: Conformal Field Models in String Theory, Superstring Vacua

ARXIV EPRINT: [1310.4116](https://arxiv.org/abs/1310.4116)

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1 Introduction

Understanding compactifications with fewer moduli and fewer supersymmetries is still a major goal of string theory. Besides compactifications with Ramond-Ramond fluxes, that are quite successful in this respect but lack a usable worldsheet formulation, it is desirable to find models with a better grip on α' corrections beyond the supergravity regime.

Unlike in heterotic strings, it is not possible to consider type II compactifications with NSNS-fluxes only, since the three-form H is closed. It leaves us the possibility of using non-geometric fluxes, described either as asymmetric orbifolds of rational tori [1–3], using free-fermion constructions [4, 5] or as (generalized) T-duals of so-called T-folds that are locally geometric [6–8] (see [9] and references therein for a recent review). Studying such (geometric or non-geometric) fluxes in *interacting* rather than free worldsheet conformal field theories would allow to understand how these twists can be defined in non-trivial backgrounds, beyond the twisted tori or free-fermions examples.

A large class of supersymmetric compactifications on Calabi-Yau manifolds, in their stringy regime of negative Kähler moduli, can be described by non-trivial superconformal field theories constructed by Gepner using $\mathcal{N} = (2, 2)$ minimal models as building

blocks [10, 11]. The structure of Gepner model is very rigid, being tightly constrained by modular invariance of the one-loop partition function. Nevertheless some asymmetric heterotic Gepner models with $(2, 0)$ superconformal symmetry have been considered in the past, corresponding to (the small volume limit of) compactifications with non-standard embedding gauge bundles [12–15]. To our knowledge, analogous supersymmetric constructions in type IIA/IIB superstrings have not been considered yet.¹

In this work we describe a simple method to construct asymmetric Gepner models in type IIA/B superstrings, starting with an $\mathcal{N} = 4$ compactification on $K3 \times T^2$ where the $K3$ surface is described by a Gepner model; they can be thought as some sort of non-geometric T^2 fibration. These asymmetric Gepner models have several interesting and unusual properties. They provide $\mathcal{N} = 2$ compactifications to four dimensions such that all the eight space-time supercharges come from the left, as in [18], unlike compactifications on CY threefolds for which four supercharges comes from the left and four from the right. Since the right Ramond ground states are massive, there are also no massless states from the whole RR sector.

Having studied the massless spectra for all the 62 asymmetric models of this sort, we have found that they all have at most few remaining massless scalars in the spectrum, and that 33 of them are actually devoid of any remaining massless modulus from the $K3$ two-fold. Their field content corresponds at low energies to four-dimensional $\mathcal{N} = 2$ supergravity with the STU vector multiplets and no hypermultiplets. Some supersymmetric compactifications with few moduli were constructed using free-fermionic constructions [19, 20] or freely acting toroidal orbifolds [2, 3]. Our models provide a broad generalization of these models to interacting superconformal field theory compactifications.²

This note is organized as follows. In section 2 we review the construction of $K3$ Gepner models. In section 3 we explain how to build the asymmetric Gepner models, provide their partition function and give some of their generic properties. In section 4 we give an overview of the massless spectra for all models. Finally we summarize our findings and give some future directions of research in section 5. We give a detailed list of all massless spectra in appendix A and recall some basic facts about $\mathcal{N} = 2$ characters in appendix B.

2 Gepner models for $K3$

In this section we review the construction of Gepner models with a $K3$ target-space, using a slightly different method compared to the original work of Gepner, that is more convenient for our purposes. We also explain how to derive their massless spectra.

Our construction starts with $K3 \times T^2$ compactifications in type IIA/B superstrings, preserving $\mathcal{N} = 4$ supersymmetry in four dimensions, where the $K3$ surface is chosen at a Gepner point in its moduli space [11]. Gepner models for $K3$, except for two out of the sixteen available, are made out of four $\mathcal{N} = (2, 2)$ minimal models, whose *supersymmetric* levels will be denoted by $\{k_i, i = 1, \dots, 4\}$. In these conventions (see appendix B), the left

¹Different constructions of non-supersymmetric asymmetric models were considered in [16, 17].

²As Gepner models with small levels for all minimal models are free actually theories the aforementioned examples should be given by specific cases of our construction.

and right central charges of a minimal model are equal to $3 - 6/k$. In particular a model with $k = 2$ is trivial, being a SCFT with $(c, \bar{c}) = (0, 0)$.

The central charges of an $\mathcal{N} = (2, 2)$ SCFT with a $K3$ target space should be $(c, \bar{c}) = (6, 6)$, which translates into

$$\sum_{i=1}^4 \frac{1}{k_i} = 1. \quad (2.1)$$

For simplicity of notation we consider only models with four minimal model factors in the following. These models are entirely specified by a quadruplet $(k_1, \dots, k_4)$, where $k_i \geq 2$, if a diagonal modular-invariant is chosen for each affine $SU(2)$ spin as we shall assume for definiteness.

2.1 Partition function

The main achievement of Gepner's work is to find a modular-invariant combination of minimal models with a generalized GSO projection onto integer left and right R-charges, allowing to build a supersymmetric string compactification out of the model.

In Gepner models, the GSO projection can be decomposed in two steps. The first step is a *diagonal* $\mathbb{Z}_K$ orbifold of the minimal models, with

$$K = \text{lcm}(k_1, \dots, k_4), \quad (2.2)$$

giving integer left and right R-charges, while the second step involves two *chiral* $\mathbb{Z}_2$ orbifolds, enforcing that the former are both odd integers.

A type IIB modular invariant partition function for such a $K3 \times T^2$ compactification is given by the following expression:

$$Z = \frac{\Gamma_{2,2}(T, U)}{\tau_2^2 \eta^4 \bar{\eta}^4} \frac{1}{K} \sum_{\gamma, \delta \in \mathbb{Z}_K} \frac{1}{2} \sum_{a, b=0}^1 (-)^{a+b} \frac{1}{2} \sum_{\bar{a}, \bar{b}=0}^1 (-)^{\bar{a}+\bar{b}} \frac{\eta^2 \left[\frac{a}{b} \right]}{\eta^2} \frac{\bar{\eta}^2 \left[\frac{\bar{a}}{\bar{b}} \right]}{\bar{\eta}^2} \prod_{i=1}^4 \sum_{2j_i=0}^{k_i-2} \sum_{m_i \in \mathbb{Z}_{2k_i}} e^{i\pi(2\delta-b+\bar{b}) \frac{m_i}{k_i}} C_{m_i+a}^{j_i} \left[\frac{a}{b} \right] \bar{C}_{m_i+\bar{a}+2\gamma}^{j_i} \left[\frac{\bar{a}}{\bar{b}} \right], \quad (2.3)$$

in terms of the minimal models characters $C_m^j \left[\frac{a}{b} \right]$, see appendix B, and of the T^2 lattice $\Gamma_{2,2}(T, U)$. The sectors of the $\mathbb{Z}_K$ orbifold are labeled by (γ, δ) , while those of the chiral $\mathbb{Z}_2$ projections are (a, b) and $(\bar{a}, \bar{b})$ respectively. An explicit check of modular invariance is presented in appendix B.³

2.2 Massless spectrum

Massless scalars in space-time are obtained by combining states of the left and right (anti-) chiral rings of the $\mathcal{N} = 2$ superconformal algebra. A very detailed study of Gepner models spectra can be found e.g. in [21]; a more specific analysis of $K3$ Gepner models with an emphasis on the $(4, 4)$ structure can be found in [22].

³We do not use the ‘beta-method’ introduced by Gepner in the original article [10], as we find that our alternative formulation makes the computation of the massless spectrum easier for asymmetric models.

In the case at hand, the S , T and U moduli of $K3 \times T^2$ compactifications are built using the identity operator in the Gepner model. Second, the $K3$ moduli are given by chiral/antichiral operators on the left and on the right with $|Q_R| = |\bar{Q}_R| = 1$. In the case of (generic) CY threefolds, one has to consider four different rings, (a, a) , (c, c) , (a, c) and (c, a) , depending on the choice of chiral or antichiral operators on both sides. In the case of K3 models, thanks to $\mathcal{N} = (4, 4)$ superconformal symmetry, these four rings are related by (inner) automorphisms of $SU(2)_R \times \overline{SU(2)}_R$.

Massless states in the (a, a) ring. Since the four rings are isomorphic to each other, it is enough to consider one of them to get the full spectrum of massless scalars. We can choose for instance to study the (a, a) spectrum.

Anti-chiral operators in each of the minimal models have even fermion number, $m = 2j$ and conformal dimension⁴

$$\Delta = -\frac{Q_{R,i}}{2} = \frac{m_i}{2k_i} = \frac{j_i}{k_i}, \quad i = 1, \dots, 4, \quad (2.4)$$

see appendix B. One gets then a left antichiral state with $\bar{Q}_R = -1$ provided that

$$\sum_{i=1}^4 \frac{m_i}{k_i} = \sum_{i=1}^4 \frac{2j_i}{k_i} = 1, \quad (2.5)$$

which, naturally, satisfies the $\mathbb{Z}_K$ -orbifold invariance.

On the right-moving side, once the $\{j_i\}$'s have been chosen in order to satisfy (2.5), states in the untwisted sector ($\gamma = 0$) are automatically anti-chiral states with $\bar{Q}_R = -1$, hence giving at the end massless scalars in space-time.

In the twisted sectors ($\gamma \neq 0$), anti-chiral states occur on the right if one of the two following conditions is satisfied *for every minimal model*:⁵

1. If $\gamma \equiv 0 \pmod{k_i}$, due to the periodicity of minimal model characters, see eq. (B.4).
2. If $2j_i + \gamma + 1 \equiv 0 \pmod{k_i}$, the state equivalence $(j, 2j + 2\gamma, 2) \sim (k/2 - j - 1, 2j + 2\gamma - k, 0)$, see again eq. (B.4), gives an anti-chiral state, as $2j + 2\gamma - k_i \equiv k - 2j - 2 \pmod{2k_i}$. However this map flips the GSO parity, see eq. (B.10), hence it should be used in an *even* number of minimal models only.

One easily checks that solutions of these constraints automatically give right anti-chiral states with $\bar{Q}_R = -1$, hence massless scalars again.

Once the massless states in the (a, a) ring have been determined, massless states in the three other rings follow from $\mathcal{N} = (4, 4)$ superconformal symmetry.

⁴We have chosen the representation of chiral and antichiral states in relation with our choice of domain $m_i \in \{0, 1, \dots, 2k_i - 1\}$.

⁵Likewise, in the $(\star, c)$ rings one has either $\gamma \equiv 1 \pmod{k_i}$ or $2j_i + \gamma \equiv 0 \pmod{k_i}$ for each minimal model.

An example. As an example we consider the (3, 3, 4, 12) Gepner model. Marginal operators in any of the four chiral rings satisfy the constraint

$$4j_1 + 4j_2 + 3j_3 + j_4 = 6, \quad (2.6)$$

with the restrictions $0 \leq 2j_{1,2} \leq 1$, $0 \leq 2j_3 \leq 2$ and $0 \leq 2j_4 \leq 10$.

We use the notation $[2j_1, 2j_2, 2j_3, 2j_4]_\gamma$, identifying massless states by their $SU(2)^4$ spins and their twisted sector γ . One finds first the following ten (a, a) operators in the untwisted sector:

$$\begin{aligned} & [0, 0, 1, 9]_0 \\ & [0, 1, 0, 8]_0, [1, 0, 0, 8]_0, \\ & [0, 0, 2, 6]_0 \\ & [0, 1, 1, 5]_0, [1, 0, 1, 5]_0, \\ & [1, 1, 0, 4]_0, \\ & [0, 1, 2, 2]_0, [1, 0, 2, 2]_0, \\ & [1, 1, 1, 1]_0. \end{aligned} \quad (2.7)$$

For each of these operators, there is one and precisely one twisted sector such that the right operator is also anti-chiral and of right R-charge minus one:

$$\begin{aligned} \gamma = 2 & \longleftrightarrow [0, 0, 1, 9]_2 \\ \gamma = 3 & \longleftrightarrow [0, 1, 0, 8]_3, [1, 0, 0, 8]_3 \\ \gamma = 5 & \longleftrightarrow [0, 0, 2, 6]_5 \\ \gamma = 6 & \longleftrightarrow [0, 1, 1, 5]_6, [1, 0, 1, 5]_6 \\ \gamma = 7 & \longleftrightarrow [1, 1, 0, 4]_7 \\ \gamma = 9 & \longleftrightarrow [0, 1, 2, 2]_9, [1, 0, 2, 2]_9 \\ \gamma = 10 & \longleftrightarrow [1, 1, 1, 1]_{10} \end{aligned} \quad (2.8)$$

hence giving ten additional massless states in the (a, a) ring. Notice that this one-to-one correspondence between the untwisted and twisted sector antichiral operators is accidental. It is not true for every $K3$ Gepner model.

The same story holds for the (c, c) and (a, c) and (c, a) rings. Altogether one finds $20 + 20 + 20 + 20 = 80$ operators. One has in addition the identity operator, that gives the universal S modulus containing the dilaton and NSNS axion. One gets then, as expected, 81 massless scalars spanning the moduli of $K3$ compactifications. On top of this, the two-torus provides the usual T and U moduli.

3 Asymmetric Gepner models

We now present a class of asymmetric Gepner models that provide new types of supersymmetric type II compactifications.⁶

3.1 The idea

The basic idea behind these constructions is very simple. Consider the combination of characters

$$\frac{\Theta_{m,k}}{\eta} \bar{\chi}^j \frac{\bar{\vartheta} \left[\frac{\bar{a}}{\bar{b}} \right]}{\bar{\eta}}, \quad (3.1)$$

⁶Lowest levels models at small radii may likely be similar to some free-fermion constructions as [20].

where χ^j is an affine $SU(2)$ character at level $k-2$ and $\Theta_{m,k}$ is a theta-function at level k , which gives the lattice of a holomorphic compact boson at radius $\sqrt{\alpha'k}$.

The modular properties of (3.1) are the same as those of the anti-holomorphic minimal model character $\bar{C}_m^j[\frac{\bar{a}}{\bar{b}}]$, thus we can trade the latter for the former in a Gepner model partition function without spoiling modular invariance. Crucially, as far as the $\mathbb{Z}_{2k}$ charge is concerned, an $\mathcal{N} = 2$ minimal model character with $c = 3 - 6/k$ transforms as the *conjugate* of a $U(1)$ character at level k .

A careful reader may be worried about right superconformal invariance, as one considers type II models which should have at least $\mathcal{N} = (1, 1)$ superconformal symmetry. Let us consider the product of two terms like (3.1), together with the corresponding left minimal model characters. The two right bosonic $SU(2)_{k_i-2}$ characters can be combined with free-fermion characters in order to make explicit an $\mathcal{N} = 2$ superconformal symmetry on the right:

$$\begin{aligned} & \frac{\Theta_{m_3,k_3}}{\eta} \frac{\Theta_{m_4,k_4}}{\eta} \frac{\vartheta[\frac{a}{b}]}{\eta} \bar{\chi}^{j_3} \bar{\chi}^{j_4} \frac{\bar{\vartheta}^3[\frac{\bar{a}}{\bar{b}}]}{\bar{\eta}^3} \\ &= \sum_{\bar{m}_3 \in \mathbb{Z}_{2k_3}} \sum_{\bar{m}_4 \in \mathbb{Z}_{2k_4}} \bar{C}_{\bar{m}_3}^{j_3} \left[\frac{\bar{a}}{\bar{b}} \right] \bar{C}_{\bar{m}_4}^{j_4} \left[\frac{\bar{a}}{\bar{b}} \right] \left(\frac{\Theta_{m_3,k_3}}{\eta} \frac{\Theta_{m_4,k_4}}{\eta} \frac{\vartheta[\frac{a}{b}]}{\eta} \right) \left(\frac{\bar{\Theta}_{\bar{m}_3,k_3}}{\bar{\eta}} \frac{\bar{\Theta}_{\bar{m}_4,k_4}}{\bar{\eta}} \frac{\bar{\vartheta}[\frac{\bar{a}}{\bar{b}}]}{\bar{\eta}} \right). \quad (3.2) \end{aligned}$$

The decomposition (3.2) shows first that this construction uses perfectly well-defined $\mathcal{N} = (2, 2)$ superconformal field theories, as we get a product of ordinary holomorphic and anti-holomorphic characters for a couple of minimal models and for a couple of free $c = 2$ theories. Second, as the $\mathbb{Z}_{2k_i}$ charges of the minimal models are mixed with the lattice of the free bosons, it corresponds to ‘fibering’ S^1 ’s over minimal models. In order to get a four-dimensional theory at the end, one considers fibering at most a T^2 over the $K3$ Gepner model. For definiteness, one considers below the generic case with a non-degenerate T^2 fiber.

3.2 Partition function

Following the rules defined in the previous subsection, one obtains a modular-invariant partition function for an asymmetric Gepner model in type II by replacing in a $K3 \times T^2$ ordinary Gepner model the right-moving characters of the last two minimal models and the T^2 contribution by a combination of characters of the type (3.2). Explicitly, one gets:

$$\begin{aligned} Z &= \frac{1}{\tau_2^2 \eta^2 \bar{\eta}^2} \frac{1}{K} \sum_{\gamma, \delta \in \mathbb{Z}_K} \frac{1}{2} \sum_{a,b=0}^1 (-)^{a+b} \frac{1}{2} \sum_{\bar{a}, \bar{b}=0}^1 (-)^{\bar{a}+\bar{b}} \frac{\vartheta^2[\frac{a}{b}]}{\eta^2} \frac{\bar{\vartheta}^2[\frac{\bar{a}}{\bar{b}}]}{\bar{\eta}^2} \\ & \quad \prod_{i=1}^2 \sum_{2j_i=0}^{k_i-2} \sum_{m_i \in \mathbb{Z}_{2k_i}} e^{i\pi(2\delta-b+\bar{b})\frac{m_i}{k_i}} C_{m_i+a}^{j_i} \left[\frac{a}{b} \right] \bar{C}_{m_i+\bar{a}+2\gamma}^{j_i} \left[\frac{\bar{a}}{\bar{b}} \right] \\ & \quad \prod_{i=3}^4 \sum_{2j_i=0}^{k_i-2} \sum_{m_i, \bar{m}_i \in \mathbb{Z}_{2k_i}} e^{i\pi(2\delta-b+\bar{b})\frac{m_i}{k_i}} \left(\frac{\Theta_{m_i+\bar{a}+2\gamma, k_i}}{\eta} \frac{\bar{\Theta}_{\bar{m}_i, k_i}}{\bar{\eta}} \right) C_{m_i+a}^{j_i} \left[\frac{a}{b} \right] \bar{C}_{\bar{m}_i}^{j_i} \left[\frac{\bar{a}}{\bar{b}} \right]. \quad (3.3) \end{aligned}$$

This modular-invariant partition function corresponds to a well-defined type IIB string background, given that the underlying conformal field theory is unitary, has $(2, 2)$ superconformal symmetry, left and right central charges $(c, \bar{c}) = (12, 12)$ and satisfies the requested spin-statistics connection in space-time thanks to the GSO projection.

As we have noticed before, this model can be thought as some sort of freely-acting asymmetric $\mathbb{Z}_{k_3} \times \mathbb{Z}_{k_4}$ orbifold of $K3 \times T^2$ that acts as a shift in the lattice of the two-torus. This shift is asymmetric since the term in parenthesis in eq. (3.3) contains left and right $U(1)_{k_i}$ characters with different charges.

3.3 Space-time supersymmetry and Ramond-Ramond ground states

The left-moving sector of the SCFT defined by eq. (3.3) provides eight space-time supercharges, as the holomorphic part of its partition function is the same as in an ordinary symmetric Gepner model, see eq. (2.3); it guarantees a left $(c = 6, \mathcal{N} = 4) \times (c = 3, \mathcal{N} = 2)$ superconformal symmetry.

In contrast, there are no space-time supercharges coming from the right-moving sector. As follows from (3.3), the right R-charge of the $\mathcal{N} = 2$ superconformal algebra is of the form (with $\bar{Q}_{fer}$ the right fermion number)

$$\bar{Q}_R = \bar{Q}_{fer} - \frac{m_1 + 2\gamma}{k_1} - \frac{m_2 + 2\gamma}{k_2} - \frac{\bar{m}_3}{k_3} - \frac{\bar{m}_4}{k_4} \mod 2, \quad (3.4)$$

where $\bar{m}_3$ and $\bar{m}_4$ are left unconstrained by the GSO projection. This charge being generically fractional, it is not possible to achieve space-time supersymmetry using spectral flow of the right $\mathcal{N} = 2$ superconformal algebra (see [23] for a detailed account on this mechanism).

To be more explicit, Ramond ground states in minimal models correspond to the quantum numbers $(j, m, a) = (0, \pm 1, 1)$. In the right-moving sector, since $\bar{m}_3$ and $\bar{m}_4$ appear *both* as minimal model $\mathbb{Z}_{k_i}$ charges and in the T^2 lattice, two out of the four gravitini get a mass.⁷

$$M_{\psi_\mu} = \sqrt{\frac{1}{\alpha' k_3} + \frac{1}{\alpha' k_4}}. \quad (3.5)$$

Therefore, space-time four-dimensional $\mathcal{N} = 4$ supersymmetry is broken to $\mathcal{N} = 2$. Naturally the same holds for the RR ground states, which have the same mass shift (3.5) thanks to space-time supersymmetry.

These $\mathcal{N} = 2$ four-dimensional compactifications of type II superstrings are markedly distinct from usual compactifications on Calabi-Yau three-folds, as one has eight space-time supercharges from the left-movers, instead of having four supercharges from each side.

This demonstrates the non-geometric nature of these compactifications for the following reason.⁸ One can construct in principle non-linear sigma-models giving a different number of space-time supercharges from the left-movers and from the right-movers by adding H-flux (torsion) to the background, as the left- and right-handed worldsheet fermions couple to different torsionfull spin connections. It is not possible however to obtain type II

⁷The same holds for model with a single S^1 fiber, however there is only one term in the mass formula.

⁸Another more trivial indication of their non-geometric nature is simply that their partition function is asymmetric as we have already emphasized.

compactifications with a standard supergravity limit having only NSNS-flux.⁹ Typically non-geometric compactifications have a reduced number of moduli; we shall now study in detail the massless spectra of our models.

4 Massless spectra of asymmetric models

The asymmetric Gepner models given by the partition function (3.3) have different numbers of massless moduli, depending on the values of $(k_1, \dots, k_4)$, unlike the case of ordinary Gepner models for $K3$. We shall now study these massless spectra in detail, finding in particular that more than half of them have no massless moduli besides S , T and U .

4.1 General rules

On the left, the analysis of massless states starts similar to the case of ordinary minimal models that was considered in section 2, namely one has to look for elements of the chiral and antichiral rings with $|Q_R| = 1$.

There is however one important new ingredient. States in the T^2 theory have to be chiral or anti-chiral as well, which sets their left-moving momentum to zero. This gives a pair of non-trivial constraints, since the minimal model and $U(1)^2$ quantum numbers are mixed with each other, see (3.3). Explicitly one gets the conditions

$$j_\ell + \gamma \equiv 0 \pmod{k_\ell} \quad , \quad \ell = 3, 4, \quad (4.1)$$

which already cut out a large part of the chiral and antichiral rings of the original Gepner model.

For each of the surviving left chiral/antichiral states one needs to check whether it is possible to obtain a massless state in space-time, i.e. whether there exists a right primary state of dimension one-half associated with its quantum numbers. As the right R-charges are generically fractional rather than integer-valued, massless states are not necessarily associated with right chiral or antichiral states; the usual argument relating massless states with BPS states of smallest R-charge in absolute value does not hold any more.

In full generality, the right conformal dimension of conformal primaries that we need to consider, in the γ -th twisted sector, reads:

$$\bar{\Delta} = \bar{\Delta}_1 + \bar{\Delta}_2 + \frac{j_3(j_3 + 1)}{k_3} + \frac{j_4(j_4 + 1)}{k_4} \quad (4.2)$$

where $\bar{\Delta}_\ell$ is the conformal dimension of the ℓ th minimal model primary such that $m_\ell = 2j_\ell + 2\gamma \pmod{2k_\ell}$, for $\ell = 1, 2$, see eq. (B.5).

For every asymmetric Gepner model, we first list the elements of the left (anti)chiral ring that satisfy the extra conditions (4.1) in some twisted sectors γ . For the states that remain, we look for the minimal right conformal dimensions with the given quantum numbers $[2j_1, 2j_2, 2j_3, 2j_4]_\gamma$. There are *a priori* three possibilities:

⁹In the non-compact case, no such obstruction exist, see e.g. [24].

(2, 3, 10, 15), (2, 10, 3, 15), (2, 15, 3, 10), (3, 10, 2, 15), (3, 15, 2, 10), (10, 15, 2, 3)
(2, 3, 8, 24), (2, 8, 3, 24), (2, 24, 3, 8), (3, 8, 2, 24), (3, 24, 2, 8), (8, 24, 2, 3)
(2, 3, 9, 18), (2, 9, 3, 18), (2, 18, 3, 9), (3, 9, 2, 18), (3, 18, 2, 9), (9, 18, 2, 3)
(2, 3, 7, 42), (2, 7, 3, 42), (2, 42, 3, 7), (3, 42, 2, 7), (7, 42, 2, 3), (3, 7, 2, 42)
(2, 4, 6, 12), (2, 6, 4, 12), (2, 12, 4, 6), (4, 12, 2, 6), (6, 12, 2, 4), (4, 6, 2, 12)
(2, 4, 5, 20), (2, 5, 4, 20), (2, 20, 4, 5), (5, 20, 2, 4), (4, 5, 2, 20), (4, 20, 2, 5)
(2, 3, 12, 12), (2, 12, 3, 12), (12, 12, 2, 3), (3, 12, 2, 12)
(3, 3, 4, 12), (3, 4, 3, 12), (3, 12, 3, 4), (4, 12, 3, 3)
(2, 5, 5, 10), (2, 10, 5, 5), (5, 5, 2, 10), (5, 10, 2, 5)
(2, 4, 8, 8), (2, 8, 4, 8), (8, 8, 2, 4), (4, 8, 2, 8)
(3, 4, 4, 6), (3, 6, 4, 4), (4, 4, 3, 6), (4, 6, 3, 4)
(3, 3, 6, 6), (3, 6, 3, 6), (6, 6, 3, 3)
(2, 6, 6, 6), (6, 6, 2, 6)
(4, 4, 4, 4)

Table 1. List of all inequivalent asymmetric K3 Gepner models, the last two minimal models being the asymmetric ones in each case.

1. The contribution of the WZW models $\bar{\Delta}_w = j_3(j_3 + 1)/k_3 + j_4(j_4 + 1)/k_4$ is already too large whatever the contribution of the first two minimal models is.
2. If there are candidates with $j_3 = j_4 = 0$, the contribution of the WZW models vanishes hence we get a subset of the chiral rings of the original symmetric Gepner model.
3. If there exists states with $0 < \bar{\Delta}_w \leq 1/2$ one needs to check explicitly the overall smallest conformal dimension with the given quantum numbers.

4.2 Massless spectra for all asymmetric Gepner models

Setting aside for convenience the two Gepner models constructed with six minimal models – that are actually free theories so can be studied using free-fermion constructions – we have to consider, for each of the 14 remaining models, all inequivalent ways of choosing the two minimal models that are ‘twisted’ in our construction. Overall, one gets a list of 62 models which are given in table 1. In our conventions, a model (k_1, k_2, k_3, k_4) has its last two minimal models, at levels k_3 and k_4 , asymmetrized.

STU models. Among those listed in table 1, one finds that 33 models are actually free of any massless modulus from the asymmetrized K3 Gepner model; these are the models that do not appear in appendix A.

In all cases, the T and U moduli of the two-torus and the axio-dilaton modulus S are still part of the spectrum; they are obtained by taking the identity operator in the asymmetric Gepner model, and vanishing left and right momenta along the twisted T^2 . The massless spectrum contains also four U(1) gauge fields coming from the Kaluza-Klein reduction on the two-torus (being states with zero momentum along the T^2 they also survive

the twist). In order to organize these degrees of freedom and their fermionic partners in $\mathcal{N} = 2$ multiplets, we have no choice but to consider that the three complex scalars S , T and U are part of vector multiplets. Hence unlike for type II compactifications on Calabi-Yau three-folds the dilaton belongs to a vector multiplet.

Therefore these models flow at low energies to the so-called STU model of $\mathcal{N} = 2$ supergravity [25, 26], which contains besides the supergravity multiplet only three Abelian vector multiplets, whose scalar components are denoted S , T and U .

Models with surviving moduli. In the other 29 models, that are given in appendix A, some massless hypermultiplets remain in the spectrum. The massless states of any such asymmetric Gepner model are given by the subset of massless states in the associated symmetric Gepner model that satisfy the extra conditions (4.1).

As was argued before it may have been possible to find some massless states that do not belong to the right (anti-)chiral ring in the asymmetric models, as the right R-charge is not integer-valued. The analysis done for all models, whose results are summarized in appendix A, shows that there aren't any other massless states besides the truncated chiral rings. There is no particular relation between the right chiral and antichiral rings as the right $\mathcal{N} = 4$ superconformal symmetry of the Gepner model factor is broken. We indeed find that the dimension of these two rings do not match generically.

Interestingly, since the dilaton belongs to a vector multiplet, the hypermultiplet moduli space receives no quantum corrections, unlike in CY_3 compactifications. This is related to the underlying $\mathcal{N} = 4$ supergravity theory corresponding to the symmetric Gepner model at low energies, even though the gravitini masses, set by the T^2 moduli, are not necessarily small hence the breaking not necessarily of the 'spontaneous' type.

4.3 Moduli spaces

To summarize, the moduli space of a given asymmetric Gepner model splits into two subspaces. The first one (hypermultiplets moduli space) is spanned by the leftover moduli of $K3$ that are not lifted by the fibration, if there are any. As we have noticed before, all the RR ground states are lifted, so there are no moduli coming from RR p -forms integrated over cycles of $K3$. The second subspace (vector multiplets moduli space) is spanned by the torus moduli T and U , which survive in all models, and by the axio-dilaton S .

The asymmetric models were built by choosing particular values for the torus moduli, using an orthogonal two-torus with $R_x = \sqrt{\alpha' k_3}$ and $R_y = \sqrt{\alpha' k_4}$ and no B-field, see eq. (3.2). As the T and U moduli are not lifted, it is possible to reach any value for them by exact marginal deformations built out of the $U(1)^2$ left- and right-moving currents. The masses of the lifted $K3$ moduli and of the massive gravitini are therefore generically functions of the complex and Kähler structure of the two-torus. In particular in the decompactification limit $U/\alpha' \rightarrow \infty$ one finds a $K3 \times \mathbb{R}^2$ with all the moduli of the original $K3$ Gepner model restored. In general the masses of the two massive gravitini are given by

$$M_{\psi_\mu}(T, U) = \sqrt{\frac{T_2}{U_2} + \frac{(T_1 \pm 1)^2}{U_2 T_2}}. \quad (4.3)$$

At the point in the moduli space where we originally defined the models, namely $U = i\alpha'\sqrt{k_3k_4}$ and $T = i\sqrt{k_4/k_3}$, there is a non-Abelian symmetry enhancement. The two asymmetric minimal models used in the construction have indeed an unbroken affine right-moving $SU(2)_{k_3} \times SU(2)_{k_4}$ symmetry, as can be seen from equation (3.1).¹⁰ Moving away from these values by marginal deformations will naturally break this $SU(2)^2$ symmetry.

In our scan of all asymmetric models, we have found that all space-time massless scalars transform in the trivial representation of both $SU(2)$'s. Hence this symmetry, while not visible in the supergravity limit, is an organizing principle for the massive states. Likewise there are no massless space-time gauge fields that would make this symmetry local.

5 Conclusions and perspectives

In this work we have constructed a large class of type II compactifications with $\mathcal{N} = 2$ supersymmetry, using asymmetric Gepner models. Interestingly, the space-time supercharges come only from the left-movers, indicating that these constructions should be somehow related to asymmetric freely-acting orbifolds of $K3 \times T^2$ at Gepner points.

More than half of the models have no massless hypermultiplets in their spectrum, reproducing at low energies $\mathcal{N} = 2$ supergravity with STU vector multiplets (having as scalar components the axio-dilaton and the torus moduli), and no other massless fields. The remaining models have a hypermultiplets moduli space which receives no quantum corrections as the dilaton sits in a vector multiplet, and whose dimension is model-dependent.

A very interesting generalization of this work would be to consider orientifolds of these models, for example analogues of the well-studied type IIB $K3 \times T^2$ compactifications with $O7$ -planes (using the involution $\Omega(-)^{F_L} \mathcal{I}_{T^2}$ where $\mathcal{I}_{T^2}$ is the inversion along the two-torus), $D7$ -branes, $D3$ -branes and/or fluxes [27]. In the case of symmetric Gepner models, orientifolds have been constructed e.g. in [28]; similar techniques can be used here. In our asymmetric Gepner models space-time supersymmetry comes only from the left-movers, and there are no RR fluxes, hence one can wonder whether such models are actually supersymmetric. As a preliminary step, we have considered boundary states for $D7$ -branes, generalizing the results of [29], and obtained that their open string spectrum is not supersymmetric. Instead of having a generalized GSO-projection like (2.5), giving a spectrum of integer R-charges, one gets the weaker condition

$$\sum_{i=1}^4 \frac{m_i}{k_i} + \frac{M_3}{k_3} + \frac{M_4}{k_4} \in \mathbb{Z}, \quad (5.1)$$

where $M_{3,4}$ are the $U(1)_{k_3} \times U(1)_{k_4}$ charges along the T^2 . The presence of fractional R-charges pinpoints the absence of supersymmetry. It would be very interesting to study these orientifold compactifications in more detail, in particular to check whether the open string spectrum on branes contains tachyonic states. It is worthwhile to notice finally that $D7$ -branes are not strictly necessary as there is no RR tadpole to cancel. Hence unoriented

¹⁰This enhancement is distinct from what happens while compactifying at the self-dual radius.

models without open strings are possible, the price to pay being that the closed vacuum is corrected by the dilaton tadpole.

Another important problem is the understanding of non-perturbative dualities using these compactifications as starting points, more precisely to find dual descriptions under STU triality [26]. In particular, heterotic duals should be given by some twisted tori compactifications with nonperturbative duality twists (as U and S are exchanged).

Finally, the (non-)geometric interpretation of the asymmetric Gepner models is also quite interesting to consider, if one is able to find suitable ‘R-fluxes’ and ‘Q-fluxes’ added to $K3 \times T^2$ that could reproduce the spectra that we obtained, at the effective action level, using for instance the ten-dimensional formulation of non-geometric backgrounds developed in [30, 31] (see in particular [32] for a study of free-fermions models). In our case, having a clear understanding of the original symmetric Gepner model and of the potentially surviving moduli in terms of the $K3$ geometry would give some interesting insights on string (non-)geometry.

Acknowledgments

We would like to thank Luca Carlevaro, Nick Halmagyi, Josh Lapan, Michela Petrini and Jan Troost for enlightening and entertaining discussions. This work was supported by French state funds managed by the ANR within the Investissements d’Avenir programme under reference ANR-11-IDEX-0004-02.

A Massless moduli for all models

We provide below the complete list of massless moduli in all the models that actually admit massless scalars in their spectra besides S , T and U , among those given in table 1. All others models are free of hypermultiplets.

As in the text we use the notation (k_1, k_2, k_3, k_4) for the models themselves, the last two minimal models being the asymmetric ones, and $[2j_1, 2j_2, 2j_3, 2j_4]_\gamma$ to label the massless states by their $SU(2)$ spins and twisted sector γ . In all cases one finds that massless states in the untwisted sector ($\gamma = 0$) belong to the (a, a) and (c, a) rings while states in the twisted sectors ($\gamma \neq 0$) belong to the (a, c) and (c, c) rings. In all models, we did not find any non-chiral massless state.

(2,3,10,15) family

- $(3,15,2,10) : [1, 10, 0, 0]_0$ and $[1, 10, 0, 0]_{20}$
- $(10,15,2,3) : [2, 12, 0, 0]_0, [2, 12, 0, 0]_{18}, [4, 9, 0, 0]_0, [4, 9, 0, 0]_6, [6, 6, 0, 0]_0, [6, 6, 0, 0]_{24}, [8, 3, 0, 0]_0$ and $[8, 3, 0, 0]_{12}$

(2,3,8,24) family

- $(3, 24, 2, 8) : [1, 16, 0, 0]_0$ and $[1, 16, 0, 0]_8$
- $(8, 24, 2, 3) : [1, 21, 0, 0]_0$

(2,3,9,18) family

- $(3, 9, 2, 18) : [1, 6, 0, 0]_0$
- $(3, 18, 2, 9) : [1, 12, 0, 0]_0$
- $(9, 18, 2, 3) : [1, 16, 0, 0]_0, [2, 14, 0, 0]_0$ and $[3, 12, 0, 0]_0, [3, 12, 0, 0]_6, [4, 10, 0, 0]_0, [5, 8, 0, 0]_0, [6, 6, 0, 0]_0$ and $[6, 6, 0, 0]_{12}$

(2, 4, 6, 12) family

- $(4, 12, 2, 6) : [1, 9, 0, 0]_0, [2, 6, 0, 0]_0$ and $[2, 6, 0, 0]_6$
- $(6, 12, 2, 4) : [1, 10, 0, 0]_0, [2, 8, 0, 0]_0, [2, 8, 0, 0]_4, [3, 6, 0, 0]_0, [4, 4, 0, 0]_0$ and $[4, 4, 0, 0]_8$
- $(4, 6, 2, 12) : [2, 3, 0, 0]_0$

(2, 4,5, 20) family

- $(5, 20, 2, 4) : [1, 16, 0, 0]_0, [1, 16, 0, 0]_4, [2, 12, 0, 0]_0, [2, 12, 0, 0]_8, [3, 8, 0, 0]_0$ and $[3, 8, 0, 0]_{12}$
- $(4, 20, 2, 5) : [1, 15, 0, 0]_0, [2, 10, 0, 0]_0$ and $[2, 10, 0, 0]_{10}$

(2, 3, 7, 42) family

- $(3, 42, 2, 7) : [1, 28, 0, 0]_0$ and $[1, 28, 0, 0]_{14}$
- $(7, 42, 2, 3) : [1, 36, 0, 0]_0, [1, 36, 0, 0]_6, [2, 30, 0, 0]_0, [2, 30, 0, 0]_{12}, [3, 24, 0, 0]_0, [3, 24, 0, 0]_{18}, [4, 18, 0, 0]_0, [4, 18, 0, 0]_{24}, [5, 12, 0, 0]_0$ and $[5, 12, 0, 0]_{30}$

(2, 3, 12, 12) family

- $(12, 12, 2, 3) : [2, 10, 0, 0]_0, [3, 9, 0, 0]_0, [4, 8, 0, 0]_0, [5, 7, 0, 0]_0, [6, 6, 0, 0]_0, [6, 6, 0, 0]_6, [7, 5, 0, 0]_0, [8, 4, 0, 0]_0, [8, 4, 0, 0]_6, [9, 3, 0, 0]_0$ and $[10, 2, 0, 0]_0$
- $(3, 12, 2, 12) : [1, 8, 0, 0]_0$

(3, 3, 4, 12) family

- $(3, 12, 3, 4) : [1, 8, 0, 0]_0$
- $(4, 12, 3, 3) : [1, 9, 0, 0]_0, [1, 9, 0, 0]_3, [2, 6, 0, 0]_0$ and $[2, 6, 0, 0]_6$

(2, 5, 5, 10) family

- $(5, 5, 2, 10) : [2, 3, 0, 0]_0$ and $[3, 2, 0, 0]_0$
- $(5, 10, 2, 5) : [1, 8, 0, 0]_0, [2, 6, 0, 0]_0$ and $[3, 4, 0, 0]_0$

(2, 4, 8, 8) family

- $(8, 8, 2, 4) : [2, 6, 0, 0]_0, [3, 5, 0, 0]_0, [4, 4, 0, 0]_0, [4, 4, 0, 0]_4, [5, 3, 0, 0]_0$ and $[6, 2, 0, 0]_0$
- $(4, 8, 2, 8) : [1, 6, 0, 0]_0$ and $[2, 4, 0, 0]_0$

(3, 4, 4, 6) family

- (3, 6, 4, 4) : $[1, 4, 0, 0]_0$ and $[1, 4, 0, 0]_8$
- (4, 4, 3, 6) : $[2, 2, 0, 0]_0$ and $[2, 2, 0, 0]_6$
- (4, 6, 3, 4) : $[2, 3, 0, 0]_0$

(3, 3, 6, 6) family

- (3, 6, 3, 6) : $[1, 4, 0, 0]_0$
- (6, 6, 3, 3) : $[2, 4, 0, 0]_0$ and $[3, 3, 0, 0]_0$, $[3, 3, 0, 0]_3$, and $[4, 2, 0, 0]_0$

(2, 6, 6, 6) family

- (6, 6, 2, 6) : $[2, 4, 0, 0]_0$, $[3, 3, 0, 0]_0$ and $[4, 2, 0, 0]_0$

(4, 4, 4, 4) family

- (4, 4, 4, 4) : $[2, 2, 0, 0]_0$

B $\mathcal{N} = 2$ characters

The characters of the $\mathcal{N} = 2$ minimal models with $c = 3 - 6/k$, i.e. the supersymmetric $SU(2)_k/U(1)$ gauged WZW model, are conveniently defined through the characters $C_m^{j(s)}$ of the $[SU(2)_{k-2} \times U(1)_2]/U(1)_k$ bosonic coset, obtained by splitting the Ramond and Neveu–Schwarz sectors according to the fermion number mod 2 [10]. Defining $q = e^{2\pi i\tau}$ and $z = e^{2\pi i\nu}$, these characters are determined implicitly through the identity:

$$\chi_{k-2}^j(\nu|\tau)\Theta_{s,2}(\nu - \nu'|\tau) = \sum_{m \in \mathbb{Z}_{2k}} C_m^{j(s)}(\nu'|\tau)\Theta_{m,k}(\nu - \frac{2\nu'}{k}|\tau), \quad (\text{B.1})$$

in terms of the theta functions of $\widehat{\mathfrak{su}(2)}_k$:

$$\Theta_{m,k}(\tau, \nu) = \sum_n q^{k(n + \frac{m}{2k})^2} z^{k(n + \frac{m}{2k})}, \quad m \in \mathbb{Z}_{2k} \quad (\text{B.2})$$

and χ_{k-2}^j the characters of the affine algebra $\widehat{\mathfrak{su}(2)}_{k-2}$:

$$\chi_{k-2}^j(\nu|\tau) = \frac{\Theta_{2j+1,k}(\nu|\tau) - \Theta_{-(2j+1),k}(\nu|\tau)}{i\vartheta_1(\nu|\tau)}. \quad (\text{B.3})$$

Highest-weight representations are labeled by (j, m, s) , corresponding to primaries of $SU(2)_{k-2} \times U(1)_k \times U(1)_2$. The following identifications apply:

$$(j, m, s) \sim (j, m + 2k, s) \sim (j, m, s + 4) \sim \left(\frac{k}{2} - j - 1, m + k, s + 2\right) \quad (\text{B.4})$$

as the selection rule $2j + m + s = 0 \pmod{2}$. The spin j is restricted to $0 \leq j \leq \frac{k}{2} - 1$. The conformal weights of the superconformal primary states are:

$$\Delta = \frac{j(j+1)}{k} - \frac{n^2}{4k} + \frac{s^2}{8} \quad \text{for } -2j \leq n - s \leq 2j \quad (\text{B.5a})$$

$$\Delta = \frac{j(j+1)}{k} - \frac{n^2}{4k} + \frac{s^2}{8} + \frac{n-s-2j}{2} \quad \text{for } 2j \leq n - s \leq 2k - 2j - 4 \quad (\text{B.5b})$$

and their R -charge reads:

$$Q_R = \frac{s}{2} - \frac{m}{k} \pmod{2}. \quad (\text{B.6})$$

Chiral primary states are obtained for $m = 2(j+1)$ and $s = 2$ (thus odd fermion number). Their conformal dimension reads:

$$\Delta = \frac{Q_R}{2} = \frac{1}{2} - \frac{j+1}{k}. \quad (\text{B.7})$$

Anti-chiral primary states are obtained for $m = 2j$ and $s = 0$ (thus even fermion number). Their conformal dimension reads:

$$\Delta = -\frac{Q_R}{2} = \frac{j}{k}. \quad (\text{B.8})$$

The usual Ramond and Neveu-Schwarz characters are obtained as:

$$C_m^j \begin{bmatrix} a \\ b \end{bmatrix} (\nu|\tau) = e^{\frac{i\pi ab}{2}} \left[C_m^{j(a)}(\nu|\tau) + (-)^b C_m^{j(a+2)}(\nu|\tau) \right], \quad (\text{B.9})$$

where $a = 0$ (resp. $a = 1$) denote the NS (resp. R) sector, and characters with $b = 1$ are twisted by $(-)^F$. In terms of these characters one has the reflexion symmetry:

$$C_m^j \begin{bmatrix} a \\ b \end{bmatrix} (\nu|\tau) = (-)^b C_{m+k}^{\frac{k}{2}-j-1} \begin{bmatrix} a \\ b \end{bmatrix} (\nu|\tau). \quad (\text{B.10})$$

Modular transformations. The S and T transformations give

$$C_m^j \begin{bmatrix} a \\ b \end{bmatrix} (-1/\tau) = e^{\frac{i\pi ab}{2}} \frac{1}{\sqrt{2k}} \sum_{n \in \mathbb{Z}_{2k}} e^{\frac{i\pi mn}{k}} \sum_{j'} S_{j'}^j C_n^{j'} \begin{bmatrix} b \\ -a \end{bmatrix} (\tau) \quad (\text{B.11a})$$

$$C_m^j \begin{bmatrix} a \\ b \end{bmatrix} (\tau+1) = e^{2i\pi(\frac{j(j+1)}{k} - \frac{m^2}{4k} - \frac{a(a-2)}{8})} C_m^j \begin{bmatrix} a \\ a+b-1 \end{bmatrix} (\tau) \quad (\text{B.11b})$$

with $S_{j'}^j = \sqrt{2/k} \sin \pi \frac{(1+2j)(1+2j')}{k}$. Let us consider now the full partition function for type IIB on $K3 \times T^2$, the $K3$ being a Gepner model. Under an S transformation one gets

$$Z = \frac{1}{\tau_2^2 \eta^4 \bar{\eta}^4} \frac{1}{K} \sum_{\gamma, \delta \in \mathbb{Z}_K} \frac{1}{2} \sum_{a, b=0}^1 (-)^{a+b} \frac{1}{2} \sum_{\bar{a}, \bar{b}=0}^1 (-)^{\bar{a}+\bar{b}} \frac{\vartheta^2 \left[\begin{smallmatrix} b \\ -a \end{smallmatrix} \right]}{\eta^2} \frac{\bar{\vartheta}^2 \left[\begin{smallmatrix} \bar{b} \\ -\bar{a} \end{smallmatrix} \right]}{\bar{\eta}^2} \\ \prod_{i=1}^4 \sum_{j_i, n_i} e^{i\pi\gamma(-2\gamma+a-\bar{a})\frac{n_i}{k_i}} (k_i-2) S_{j_i'}^{j_i} C_{n_i+b}^{j_i'} \begin{bmatrix} b \\ -a \end{bmatrix} (k_i-2) S_{\bar{j}_i'}^{\bar{j}_i} \bar{C}_{n_i+\bar{b}+2\delta}^{\bar{j}_i'} \begin{bmatrix} \bar{b} \\ -\bar{a} \end{bmatrix}. \quad (\text{B.12})$$

Therefore it is invariant as is seen after the obvious redefinitions $(a', b') = (b, -a)$ and $(\gamma', \delta') = (\delta, -\gamma)$, using that $\sum_j S_{j'}^j S_{\bar{j}'}^j = \delta_{j', \bar{j}'}$. Let us now consider a T transformation. One gets

$$Z = \frac{1}{\tau_2^2 \eta^4 \bar{\eta}^4} \frac{1}{K} \sum_{\gamma, \delta \in \mathbb{Z}_K} \frac{1}{2} \sum_{a, b=0}^1 (-)^b \frac{1}{2} \sum_{\bar{a}, \bar{b}=0}^1 (-)^{\bar{b}} \frac{\vartheta^2 \left[\begin{smallmatrix} a \\ a+b-1 \end{smallmatrix} \right]}{\eta^2} \frac{\bar{\vartheta}^2 \left[\begin{smallmatrix} \bar{a} \\ \bar{a}+\bar{b}-1 \end{smallmatrix} \right]}{\bar{\eta}^2} \\ \prod_{i=1}^4 \sum_{j_i, m_i} e^{i\pi(2(\delta+\gamma)-(b+a)+\bar{b}+\bar{a})\frac{m_i}{k_i}} C_{m_i+a}^{j_i} \left[\begin{smallmatrix} a \\ a+b-1 \end{smallmatrix} \right] \bar{C}_{m_i+\bar{a}+2\gamma}^{\bar{j}_i} \left[\begin{smallmatrix} \bar{a} \\ \bar{a}+\bar{b}-1 \end{smallmatrix} \right]. \quad (\text{B.13})$$

After redefining $\delta' = \delta + \gamma$ and $b' = b + a - 1$ it is also invariant under T .

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