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Predictions for $b \rightarrow ss\bar{d}$ and $b \rightarrow dd\bar{s}$ decays in the SM and with new physics

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ABSTRACT: The $b \rightarrow ss\bar{d}$ and $b \rightarrow dd\bar{s}$ decays are highly suppressed in the SM, and are thus good probes of new physics (NP) effects. We discuss in detail the structure of the relevant SM effective Hamiltonian pointing out the presence of nonlocal contributions which can be about $\lambda^{-4}(m_c^2/m_t^2) \sim 30\%$ of the local operators ($\lambda = 0.21$ is the Cabibbo angle). The matrix elements of the local operators are computed with little hadronic uncertainty by relating them through flavor SU(3) to the observed $\Delta S = 0$ decays. We identify a general NP mechanism which can lead to the branching fractions of the $b \rightarrow ss\bar{d}$ modes in the B^+, B^0 or B_s meson decays at or just below the present experimental bounds, while satisfying the bounds from $K - \bar{K}$ and $B_{(s)} - \bar{B}_{(s)}$ mixing. It involves the exchange of a NP field carrying a conserved charge, broken only by its flavor couplings. The size of branching fractions within MFV, NMFV and general flavor violating NP are also predicted. We show that in the future energy scales higher than 10^3 TeV could be probed without hadronic uncertainties even for $b \rightarrow s$ and $b \rightarrow d$ transitions, if enough statistics becomes available.

KEYWORDS: Rare Decays, Beyond Standard Model, B-Physics

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1 Introduction

The decays $b \rightarrow ss\bar{d}$ and $b \rightarrow dd\bar{s}$ are highly suppressed in the SM: they are both loop and CKM suppressed (by six powers of small CKM elements V_{ts} and/or V_{td}). As such they can be used for searches of New Physics (NP) signals [1–8]. The types of NP that would generate $b \rightarrow ss\bar{d}$ and $b \rightarrow dd\bar{s}$ transitions will commonly also give contributions to $K - \bar{K}$, $B - \bar{B}$ and $B_s - \bar{B}_s$ mixing. Since no clear deviations from the SM predictions are seen in the meson mixing, is it possible to have deviations in $b \rightarrow ss\bar{d}$ and $b \rightarrow dd\bar{s}$ transition observable at Belle II and at LHCb? A related question is: with improved statistics, can the experiments using $b \rightarrow ss\bar{d}$ and $b \rightarrow dd\bar{s}$ decays push the bounds on flavor violation scale beyond what can be achieved from the mixing observables?

We address the second question first. For simplicity let us assume that NP contributions can be matched onto the SM operator basis, so that $H^{\Delta S} = C_1^{sd}(\bar{d}_L\gamma^\mu s_L)(\bar{d}_L\gamma_\mu s_L)$,

$H^{\Delta B} = C_1^{bs}(\bar{s}_L\gamma^\mu b_L)(\bar{s}_L\gamma_\mu b_L)$ and $H^{b\rightarrow ss\bar{d}} = C_1^{b\rightarrow ss\bar{d}}(\bar{s}_L\gamma^\mu b_L)(\bar{s}_L\gamma_\mu d_L)$. Using $|C_1^i| = 1/(\Lambda^i)^2$ one finds [9]

$$\begin{aligned} K - \bar{K} \text{ mixing : } & \Lambda^{sd} > 1.0 \cdot 10^3 \text{ TeV}, \\ B_d - \bar{B}_d \text{ mixing : } & \Lambda^{bd} > 210 \text{ TeV}, \\ B_s - \bar{B}_s \text{ mixing : } & \Lambda^{bs} > 30 \text{ TeV}, \end{aligned} \quad (1.1)$$

with $\text{Im}(C_1^{sd})$ additionally constrained from ε_K . The above bounds should be compared with the following prediction for the $b \rightarrow ss\bar{d}$ transition in the presence of NP with scale $\Lambda^{b\rightarrow ss\bar{d}}$ (see section 5 for derivation)

$$\mathcal{B}(\bar{B}^0 \rightarrow \bar{K}^{0*} \bar{K}^{0*}) = 0.3 \times 10^{-6} \left(\frac{10 \text{ TeV}}{\Lambda^{b\rightarrow ss\bar{d}}} \right)^4, \quad (1.2)$$

while the SM prediction for this branching ratio is of $\mathcal{O}(10^{-15})$. Let us take as an estimate $\Lambda^{b\rightarrow ss\bar{d}} \sim \sqrt{\Lambda^{bs}\Lambda^{sd}}$, a relation that holds in a wide set of NP models including the Minimal Flavor Violation (MFV) and Next-to-Minimal Flavor Violation (NMFV) frameworks. With enough statistics the bound on Λ^{bs} can then be pushed up to 10^3 TeV and higher without running into SM background. The $b \rightarrow ss\bar{d}$ decay modes could thus be used to constrain the NP flavor structure for $b \rightarrow s$ transitions as precisely as it is possible for $s \rightarrow d$ transitions from kaon physics. However, the statistics needed is very large. For instance, even to probe this type of flavor violating NP beyond the mixing bounds, the LHCb and Belle II luminosities will not be enough. In this scenario the $K - \bar{K}$ and $B_d - \bar{B}_d$ mixing bounds translate to $\mathcal{B}(b \rightarrow dd\bar{s}) \lesssim 10^{-13}$ and the bounds from $K - \bar{K}$ and $B_s - \bar{B}_s$ mixing translate to $\mathcal{B}(b \rightarrow ss\bar{d}) \lesssim 10^{-11}$.

Does this mean that any NP discoveries using $b \rightarrow dd\bar{s}$ and $b \rightarrow ss\bar{d}$ transitions are excluded at Belle II and LHCb? Certainly not. It is possible to have significant effects in $b \rightarrow dd\bar{s}$ and $b \rightarrow ss\bar{d}$ while obeying the bounds from the meson mixing, if (i) the exchanged particle (or a set of particles) X carries an approximately conserved global charge and, if (ii) additionally there is some hierarchy in the couplings (or alternatively some cancellations in $K - \bar{K}$ mixing). Consider the NP Lagrangian of a generic form

$$\mathcal{L}_{\text{flavor}} = g_{b\rightarrow s}(\bar{s}\Gamma b)X + g_{s\rightarrow b}(\bar{b}\Gamma s)X + g_{d\rightarrow s}(\bar{s}\Gamma d)X + g_{s\rightarrow d}(\bar{d}\Gamma s)X + \text{h.c.}, \quad (1.3)$$

and assume that X carries a conserved quantum number broken only by the above terms. We also assume for simplicity that the field X couples to a fixed Dirac structure Γ . Integrating out the field X produces flavor-changing operators

$$\begin{aligned} \mathcal{L}_{\text{eff}} = \frac{1}{M_X^2} [& g_{d\rightarrow s}g_{s\rightarrow d}^*(\bar{s}\Gamma d)(\bar{s}\bar{\Gamma}d) + g_{b\rightarrow s}g_{s\rightarrow b}^*(\bar{s}\Gamma b)(\bar{s}\bar{\Gamma}b) \\ & + g_{b\rightarrow s}g_{s\rightarrow d}^*(\bar{s}\Gamma b)(\bar{s}\bar{\Gamma}d) + g_{d\rightarrow s}g_{s\rightarrow b}^*(\bar{s}\bar{\Gamma}b)(\bar{s}\Gamma d)], \end{aligned} \quad (1.4)$$

with the terms in the first line contributing to $K - \bar{K}$ mixing and $B_s - \bar{B}_s$ mixing, and in the second line to $b \rightarrow ss\bar{d}$ decays (we also introduced $\bar{\Gamma} = \gamma^0\Gamma^\dagger\gamma^0$). It is now possible to set contributions to meson mixing to zero, while keeping $b \rightarrow ss\bar{d}$ unbounded. This happens for instance, if

$$g_{b\rightarrow s} \ll g_{s\rightarrow b}, \quad g_{s\rightarrow d} \ll g_{d\rightarrow s}, \quad \text{or} \quad g_{b\rightarrow s} \gg g_{s\rightarrow b}, \quad g_{s\rightarrow d} \gg g_{d\rightarrow s}. \quad (1.5)$$

In this way all the present experimental bounds can be satisfied, while branching ratios for $b \rightarrow ss\bar{d}$ and $b \rightarrow dd\bar{s}$ induced decays are $\mathcal{O}(10^{-6})$ (see section 5 for details).

The important ingredient in the above argument was that X carried a conserved quantum number, so that there were no terms in $\mathcal{L}_{\text{eff}}$ of the form

$$g_{b \rightarrow s}^2(\bar{s}\Gamma b)(\bar{s}\Gamma b) + g_{d \rightarrow s}^2(\bar{s}\Gamma d)(\bar{s}\Gamma d) + g_{s \rightarrow b}^{*2}(\bar{s}\bar{\Gamma} b)(\bar{s}\bar{\Gamma} b) + g_{s \rightarrow d}^{*2}(\bar{s}\bar{\Gamma} d)(\bar{s}\bar{\Gamma} d) \dots, \quad (1.6)$$

These would be generated for $X = X^\dagger$, which is impossible, if X carries a conserved charge. If terms (1.6) are present, then $B_s - \bar{B}_s$ mixing forces both $g_{b \rightarrow s}$ and $g_{s \rightarrow b}$ to be small, and the hierarchy in (1.5) is not possible (similarly $K - \bar{K}$ mixing bounds $g_{d \rightarrow s}$ and $g_{s \rightarrow d}$ to both be small). An explicit example of a NP scenario where only terms of the form (1.4) are generated is R -parity violating MSSM [4]. The R -parity violating term in the superpotential, $W = \lambda'_{ijk} L_i Q_j \bar{d}_k$, leads to $\tilde{\nu}_i \bar{q}_{Lj} d_{kR}$ flavor violating coupling. Sneutrino exchange generates operators of the form (1.4), while operators of the form (1.6) are not generated, since the sneutrino carries lepton charge broken only by R -parity violating terms.

A hierarchy of couplings in (1.5) is also present in (N)MFV models, if left-right terms give dominant contributions [10]. Both terms in (1.4) and (1.6) are generated, on the other hand, for FCNCs induced by Z' exchange, since Z' does not carry any conserved charge.

In this paper we will not confine ourselves to a particular model but keep the analysis completely general using effective field theory. We will improve on the existing SM predictions, and also give predictions for general NP contributions. The most general local NP hamiltonian for $b \rightarrow ss\bar{d}$ transition is [2]

$$H^{\text{NP}} = \frac{1}{\Lambda_{\text{NP}}^2} \left(\sum_{j=1}^5 c_j Q_j + \sum_{j=1}^5 \tilde{c}_j \tilde{Q}_j \right), \quad (1.7)$$

where c_j are dimensionless Wilson coefficients, Λ_{NP} the NP scale, and the operators are

$$\begin{aligned} Q_1 &= (\bar{s}_L \gamma_\mu b_L)(\bar{s}_L \gamma^\mu d_L), \\ Q_2 &= (\bar{s}_R b_L)(\bar{s}_R d_L), \quad Q_3 = (\bar{s}_R^\alpha b_L^\beta)(\bar{s}_R^\beta d_L^\alpha), \\ Q_4 &= (\bar{s}_R b_L)(\bar{s}_L d_R), \quad Q_5 = (\bar{s}_R^\alpha b_L^\beta)(\bar{s}_L^\beta d_R^\alpha). \end{aligned} \quad (1.8)$$

The $\tilde{Q}_j$ operators are obtained from Q_j by $L \leftrightarrow R$ exchange. In SM only Q_1 is present. The $b \rightarrow dd\bar{s}$ effective Hamiltonian is obtained by exchanging $s \leftrightarrow d$ in the above equations, while the $K - \bar{K}$ and $B_s - \bar{B}_s$ mixing Hamiltonians follow from $b \rightarrow d$ and $d \rightarrow s$ replacements.

The predictions following from the NP hamiltonian (1.7) require calculating the QCD matrix elements of the four-quark operators. In this paper we show that the Q_1 matrix elements can be related by SU(3) flavor symmetry to linear combinations of observable $\Delta S = 0$ decay amplitudes. This gives clean predictions for the branching fractions of the exclusive $b \rightarrow ss\bar{d}$ and $b \rightarrow dd\bar{s}$ modes in the SM and the NP models where Q_1 dominate. This happens in a large class of NP models, including the two-Higgs doublet model with small $\tan\beta$, and the MSSM with conserved R parity [4]. The effects of the operators with non-standard chirality can be estimated using factorization.

Figure 1. Matching the box diagrams contributing to $b \rightarrow ss\bar{d}$ decays onto an effective theory with $m_W \geq \mu \geq m_b$. The top quark box diagram (above) is matched onto a local four-quark operator, while the box diagrams with u, c internal quarks (below) are matched onto local and nonlocal operators. The mixed top-charm and top-up box diagrams are power suppressed by $m_{u,c}^2/m_W^2$ and do not contribute at leading order, see appendix A.

The outline of the paper is as follows. In section 2 we review the structure of the effective Hamiltonian mediating the $b \rightarrow ss\bar{d}, dd\bar{s}$ decays in the Standard Model. We point out that in addition to the local operators, the effective Hamiltonian contains also nonlocal operators which have not been included in the previous literature. In section 3 we derive the flavor SU(3) relations for the matrix elements of the Q_1 operator. The resulting numerical predictions for $b \rightarrow ss\bar{d}, dd\bar{s}$ decays in the SM are given in section 4. NP predictions in the case of Q_1 operator dominance are discussed in section 5, while in section 6 the modifications needed for a general chiral structure are given. Three appendices contain further technical details.

2 SM Effective Hamiltonian for $b \rightarrow ss\bar{d}$ and $b \rightarrow dd\bar{s}$ decays

In the SM the $b \rightarrow ss\bar{d}, dd\bar{s}$ decays are mediated by the box diagram with internal u, c, t quarks, figure 1. For notational simplicity let us focus on the case of $b \rightarrow ss\bar{d}$, while the results for $b \rightarrow dd\bar{s}$ can be obtained through a replacement $s \leftrightarrow d$. The effective weak Hamiltonian for $b \rightarrow ss\bar{d}$ is obtained in analogy to the one for $K^0 - \bar{K}^0$ mixing [11–14], but with several important differences. First, the CKM structure is more involved. Second, the presence of the massive b quark in the initial state introduces a correction, which is however suppressed by m_b^2/m_W^2 , and is thus numerically negligible. Finally, in applications to $K^0 - \bar{K}^0$ mixing the charm quark can be integrated out of the theory, while this cannot be done for exclusive B decays, where there is no clear separation between the charm mass m_c and the energy scales relevant in nonleptonic exclusive B decays into two pseudoscalars.

At scales $m_b \leq \mu \leq m_W$, the effective weak Hamiltonian mediating $b \rightarrow ss\bar{d}$ decays contains both local $\Delta S = 2$ terms as well as nonlocal terms arising from T-products of

$\Delta S = 1$ effective weak Hamiltonians

$$\mathcal{H}_{ss\bar{d}} = \mathcal{H}^{\Delta S=2} + \int d^d x T \{ \mathcal{H}_d^{\Delta S=1}(x), \mathcal{H}_b^{\Delta S=1}(0) \}. \quad (2.1)$$

The local part is

$$\mathcal{H}^{\Delta S=2} = \frac{G_F^2 m_W^2}{16\pi^2} (\lambda_t^d \lambda_t^b C_{tt} + \lambda_c^d \lambda_t^b C_{ct} + \lambda_t^d \lambda_c^b C_{tc}) [(\bar{s}d)_{V-A}(\bar{s}b)_{V-A}], \quad (2.2)$$

where the CKM structures are defined as $\lambda_q^{d'} = V_{qq'} V_{qs}^*$. The Wilson coefficient coming from the top box loop is $C_{tt} \sim O(x_t)$ and from the top-charm box loop $C_{ct} = C_{tc} \sim O(x_c)$, where $x_i = m_i^2/m_W^2$. The scaling of the three contributions in the local Hamiltonian (2.2) in terms of Cabibbo angle $\lambda = 0.22$ and quark masses is then: $\sim \lambda^7 x_t$, $\sim \lambda^3 x_c$ and $\sim \lambda^7 x_c$ (for $b \rightarrow dd\bar{s}$ all three terms are suppressed by another factor of λ). The third term in (2.2) can thus easily be neglected. Note also that there is no $\lambda_c^d \lambda_c^b$ term. The resulting absence of large $\log x_c$ from the charm box contribution is sometimes called the *super-hard* GIM mechanism [14], and follows from the chiral structure of the weak interaction in the SM, as explained in appendix A. The precise values of the Wilson coefficients in (2.2) can be read off from the expressions for $K^0 - \bar{K}^0$ mixing [11], where the RG running is performed only down to scale $\mu \sim m_b$. For $\bar{m}_t(\bar{m}_t) = 160.9 \text{ GeV}$, $\bar{m}_c(\bar{m}_c) = 1.27 \text{ GeV}$, $\alpha_s(m_Z) = 0.118$ they are at $\mu = m_b = 4.2 \text{ GeV}$: $C_{tt}(m_b) = 1.92$, $C_{tc}(m_b) = 3.75 x_c = 9.35 \cdot 10^{-4}$ at leading order (LO) (see appendix B for the derivation).

The nonlocal contributions in $b \rightarrow ss\bar{d}$ transition, eq. (2.1), come from insertions of $\Delta S = 1$ effective weak Hamiltonians $\mathcal{H}_b^{\Delta S=1}$ and $\mathcal{H}_d^{\Delta S=1}$ (see also figure 1). The $\Delta S = 1$ effective weak Hamiltonian $\mathcal{H}_b^{\Delta S=1}$ is the same weak Hamiltonian relevant for hadronic B decays

$$\mathcal{H}_b^{\Delta S=1} = \frac{G_F}{\sqrt{2}} \left(\sum_{q,q'=u,c} V_{qs}^* V_{q'b} \sum_{i=1,2} C_i Q_{i,b}^{qq'} - V_{ts}^* V_{tb} \sum_{j=3}^6 C_j Q_j^b \right), \quad (2.3)$$

with the tree operators $Q_{1,b}^{qq'} = (\bar{q}b)_{V-A}(\bar{s}q')_{V-A}$, $Q_{2,b}^{qq'} = (\bar{q}_\beta b_\alpha)_{V-A}(\bar{s}_\alpha q'_\beta)_{V-A}$, and penguin operators $Q_{3,5}^b = (\bar{s}b)_{V-A}(\bar{q}q)_{V \mp A}$, $Q_{4,6}^b = (\bar{s}_\alpha b_\beta)_{V-A}(\bar{q}_\beta q_\alpha)_{V \mp A}$, where the color indices α, β are displayed only when the sum is over the fields in different brackets. In the definition of the penguin operators Q_{3-6}^b a sum over $q = \{u, d, s, c, b\}$ is implied. The weak Hamiltonian $\mathcal{H}_d^{\Delta S=1}$ follows from (2.3) by making the replacement $b \rightarrow d$.

Using CKM unitarity we can rewrite the CKM factors as $V_{us}^* V_{ub} = -V_{ts}^* V_{tb} - V_{cs}^* V_{cb}$. The insertions of tree operators with u and c quarks will generate contributions with CKM structure $\lambda_c^d \lambda_c^b$, that are not present in the local $\Delta S = 2$ Hamiltonian (2.2),

$$\begin{aligned} \mathcal{H}_{cc} = \frac{G_F^2}{2} \lambda_c^d \lambda_c^b \int d^d x \sum_{i,j=1,2} C_i C_j T \{ & Q_{i,d}^{cc}(x) Q_{j,b}^{cc}(0) + Q_{i,d}^{uu}(x) Q_{j,b}^{uu}(0) - \\ & - Q_{i,d}^{cu}(x) Q_{j,b}^{uc}(0) - Q_{i,d}^{uc}(x) Q_{j,b}^{cu}(0) \}. \end{aligned} \quad (2.4)$$

From dimensional analysis, the size of this contribution is roughly

$$\mathcal{H}_{cc} \sim \frac{G_F^2}{16\pi^2} m_c^2 \lambda_c^d \lambda_c^b (\bar{s}d)_{V-A}(\bar{s}b)_{V-A}, \quad (2.5)$$

which is comparable to (2.2) and needs to be kept. Another set of contributions of comparable size coming from double $\Delta S = 1$ weak Hamiltonian insertions has CKM structure $\lambda_c^d \lambda_t^b$. The nonlocal contributions proportional to $\lambda_t^d \lambda_t^b$, on the other hand, are power suppressed, scaling as m_c^2 , compared to the corresponding ones in (2.2), which scale as m_t^2 . These contributions can be safely neglected.

The appearance of nonlocal contributions is similar to the situation for $K^0 - \bar{K}^0$ mixing, where the effective Hamiltonian below the charm scale contains the T-product of two $\Delta S = 1$ operators mediating $s \rightarrow du\bar{u}$ transitions, in addition to the local operator $(\bar{s}d)_{V-A}(\bar{s}d)_{V-A}$. The only difference is that in exclusive $b \rightarrow ss\bar{d}$ decays the charm quark can not be integrated out because of the large momenta of the light mesons in the final state.

The dominant nonlocal operators have CKM structure $\lambda_c^d \lambda_t^b$ eq. (B.8) and $\lambda_c^d \lambda_c^b$ eq. (2.4). These operators contribute to the physical decay amplitude through rescattering effects with $D\bar{D}, D\pi, \bar{D}\pi, \dots$ intermediate states. Their matrix elements are suppressed relative to those of the top box contribution $\sim C_{tt}$ by $\lambda^{-4}(m_c^2/m_t^2) \simeq 30\%$, which suggests that the approximation of neglecting m_c^2 suppressed (but CKM enhanced) nonlocal terms may be a reasonable first attempt.

We leave a complete calculation of the nonlocal contributions for the future and present only a partial evaluation of $b \rightarrow ss\bar{d}$ branching ratios by relating the matrix elements of the local contributions (2.2) to the already measured charmless two body decays using flavor SU(3). We note that the nonlocal contributions were estimated in ref. [15] using a hadronic saturation model, and were found to be suppressed relative to the local contributions.

For the purpose of the SU(3) relations to be discussed below, it is useful to rewrite the effective Hamiltonian (2.2) as

$$\mathcal{H}_i = \frac{G_F}{\sqrt{2}} V_{ub} V_{ud}^* \kappa_i O_i, \quad i = ss\bar{d}, dd\bar{s}. \quad (2.6)$$

The operators O_i are

$$O_{ss\bar{d}} = (\bar{s}b)_{V-A}(\bar{s}d)_{V-A}, \quad O_{dd\bar{s}} = (\bar{d}b)_{V-A}(\bar{d}s)_{V-A}, \quad (2.7)$$

and the dimensionless coefficients κ_i depend only on the CKM factors and calculable hard QCD coefficients. We have

$$\kappa_{ss\bar{d}} = \frac{\sqrt{2}G_F m_W^2}{(4\pi)^2} \frac{V_{tb} V_{ts}^*}{V_{ub} V_{ud}^*} (V_{td} V_{ts}^* C_{tt} + V_{cd} V_{cs}^* C_{ct}), \quad (2.8)$$

and similarly for $\kappa_{dd\bar{s}}$. Numerically, the coefficients are (at $\mu = m_b = 4.2 \text{ GeV}$, with CKM elements from [16])

$$\kappa_{ss\bar{d}} = (6.9 \cdot 10^{-6}) e^{i51^\circ}, \quad \kappa_{dd\bar{s}} = (1.5 \cdot 10^{-6}) e^{-i74^\circ}. \quad (2.9)$$

The SU(3) symmetry relations derived below require also the $C_1 + C_2$ combination of Wilson coefficients, evaluated at the same scale $\mu = m_b$. At leading log order this is given by

$$(C_1 + C_2)(m_b) = \left(\frac{\alpha_s(m_W)}{\alpha_s(m_b)} \right)^{6/23} = \eta_2(m_b) = 0.85. \quad (2.10)$$

3 SU(3) predictions

We next show how two body B decay widths for $b \rightarrow ss\bar{d}$ and $b \rightarrow dd\bar{s}$ transitions can be calculated using flavor SU(3). As a first approximation we neglect the nonlocal charm-top contributions, as justified in the previous section. Then the processes $b \rightarrow ss\bar{d}$ and $b \rightarrow dd\bar{s}$ are mediated in the SM only by the local operators O_i in eq. (2.7). Under flavor SU(3) these operators transform as $\overline{\mathbf{15}}$

$$O_{ss\bar{d}} = \overline{\mathbf{15}}_{1/2}, \quad O_{dd\bar{s}} = \overline{\mathbf{15}}_1, \quad (3.1)$$

where the subscripts denote the isospin. They belong to the same SU(3) multiplet as the $\overline{\mathbf{15}}$ in the decomposition of the $b \rightarrow du\bar{u}$ tree operators [17]

$$C_1(\bar{u}b)_{V-A}(\bar{d}u)_{V-A} + C_2(\bar{d}b)_{V-A}(\bar{u}u)_{V-A} = \frac{1}{2}(C_1 + C_2) \left(-\frac{2}{\sqrt{3}}\overline{\mathbf{15}}_{3/2} - \frac{1}{\sqrt{6}}\overline{\mathbf{15}}_{1/2} + \frac{1}{\sqrt{2}}\overline{\mathbf{3}}_{1/2}^{(s)} \right) + \frac{1}{2}(C_1 - C_2)(\mathbf{6}_{1/2} - \overline{\mathbf{3}}_{1/2}^{(a)}). \quad (3.2)$$

These operators contribute to $\Delta S = 0$ decays such as $B \rightarrow \pi\pi$. The explicit expressions for $\overline{\mathbf{15}}$ operators in (3.2) are

$$\overline{\mathbf{15}}_{1/2} = -\frac{1}{2\sqrt{6}}[(\bar{b}u)(\bar{u}d) + (\bar{b}d)(\bar{u}u)] + \frac{1}{2}\sqrt{\frac{3}{2}}[(\bar{b}s)(\bar{s}d) + (\bar{b}d)(\bar{s}s)] - \frac{1}{\sqrt{6}}(\bar{b}d)(\bar{d}d), \quad (3.3)$$

$$\overline{\mathbf{15}}_{3/2} = -\frac{1}{\sqrt{3}}[(\bar{b}u)(\bar{u}d) + (\bar{b}d)(\bar{u}u)] + \frac{1}{\sqrt{3}}(\bar{b}d)(\bar{d}d). \quad (3.4)$$

We list the $b \rightarrow dd\bar{s}, ss\bar{d}$ exclusive decays in table 1 for $B \rightarrow PP$ and in table 2 for $B \rightarrow PV$. The PP final states transform under SU(3) as $\mathbf{1}, \mathbf{8}, \mathbf{27}$, the operators $O_{ss\bar{d}}$ and $O_{dd\bar{s}}$ are in $\overline{\mathbf{15}}$, and thus there are only two reduced matrix elements, $\langle \mathbf{8} | \overline{\mathbf{15}} | \mathbf{3} \rangle, \langle \mathbf{27} | \overline{\mathbf{15}} | \mathbf{3} \rangle$. These two reduced matrix elements also appear in the predictions for measured $\Delta S = 0$ decays mediated by the operators in eq. (3.2). This means that the $B \rightarrow PP$ matrix elements of the operators $O_{ss\bar{d}}$ and $O_{dd\bar{s}}$ can be expressed in terms of $\Delta S = 0$ decay amplitudes such as $A(B^0 \rightarrow \pi^+\pi^-)$ and others. A similar analysis applies to $B \rightarrow PV$ decays, where there are four independent reduced matrix elements of the $\overline{\mathbf{15}}$ operators: $\langle \mathbf{8}_S | \overline{\mathbf{15}} | \mathbf{3} \rangle, \langle \mathbf{8}_A | \overline{\mathbf{15}} | \mathbf{3} \rangle, \langle \overline{\mathbf{10}} | \overline{\mathbf{15}} | \mathbf{3} \rangle, \langle \mathbf{27} | \overline{\mathbf{15}} | \mathbf{3} \rangle$. These can again be expressed in terms of physical $B \rightarrow PV$ $\Delta S = 0$ amplitudes. We now derive these relations separately for the $B \rightarrow PP$ and $B \rightarrow PV$ final states.

3.1 $B \rightarrow PP$ decays

We use the formalism of the graphical amplitudes [18], which makes the derivation of SU(3) decompositions quite intuitive. The two independent reduced matrix elements of the $\overline{\mathbf{15}}$ operator are given in terms of graphical amplitudes [17, 19] as

$$-\frac{\sqrt{10}}{6}\langle \mathbf{27} | \overline{\mathbf{15}} | \mathbf{3} \rangle = -\frac{1}{C_1 + C_2}(T + C) = -\frac{1}{\kappa_{ss\bar{d}}}(t + c), \quad (3.5)$$

$$\langle \mathbf{8} | \overline{\mathbf{15}} | \mathbf{3} \rangle = -\frac{1}{C_1 + C_2} \left(\frac{1}{5}(T + C) + A + E \right) = -\frac{1}{\kappa_{ss\bar{d}}} \left(\frac{1}{5}(t + c) + a + e \right). \quad (3.6)$$

Transition	Mode	Amplitude
$b \rightarrow ss\bar{d}$	$B^+ \rightarrow K^+ K^0$	$t + c$
	$B^0 \rightarrow K^0 K^0$	$t + c$
	$B_s \rightarrow K^0 \pi^0$	$\frac{1}{\sqrt{2}}(a + e)$
	$B_s \rightarrow K^+ \pi^-$	$-(a + e)$
	$B_s \rightarrow K^0 \eta_8$	$\sqrt{\frac{2}{3}}(t + c + a + e)$
$b \rightarrow dd\bar{s}$	$B^+ \rightarrow \bar{K}^0 \pi^+$	$t + c$
	$B^0 \rightarrow \bar{K}^0 \pi^0$	$\frac{1}{\sqrt{2}}(t + c + a + e)$
	$B^0 \rightarrow K^- \pi^+$	$-(a + e)$
	$B_s \rightarrow \bar{K}^0 \bar{K}^0$	$t + c$

Table 1. $B \rightarrow PP$ exclusive decays mediated by the $b \rightarrow ss\bar{d}$ and $b \rightarrow dd\bar{s}$ transitions.

This gives two relations between the graphical amplitudes T (tree), C (color-suppressed tree), A (annihilation), E (exchange) in the $\Delta S = 0$ modes (the expression for $B \rightarrow PP$ decays can be found in [18]) and the corresponding graphical amplitudes t, c, a, e in $b \rightarrow ss\bar{d}$ transitions (the decay amplitudes for $B \rightarrow PP$ modes in terms of these are collected in table 1). Equivalent relations apply between $\Delta S = 0$ and $b \rightarrow dd\bar{s}$ decay amplitudes.

The most useful for our purposes is the relation (3.5). This gives the following prediction for the exclusive $b \rightarrow ss\bar{d}$ decays

$$A(B^+ \rightarrow K^+ K^0) = A(B^0 \rightarrow K^0 K^0) = \frac{\kappa_{ss\bar{d}}}{C_1 + C_2} \sqrt{2} A(B^+ \rightarrow \pi^+ \pi^0), \quad (3.7)$$

and similarly for the $b \rightarrow dd\bar{s}$ decay

$$A(B^+ \rightarrow \bar{K}^0 \pi^+) = A(B_s \rightarrow \bar{K}^0 \bar{K}^0) = \frac{\kappa_{dd\bar{s}}}{C_1 + C_2} \sqrt{2} A(B^+ \rightarrow \pi^+ \pi^0). \quad (3.8)$$

Neglecting the $1/m_b$ suppressed amplitudes e, a one also has

$$\frac{\sqrt{3}}{2} A(B_s \rightarrow K^0 \eta_8) = \frac{\kappa_{ss\bar{d}}}{\kappa_{dd\bar{s}}} A(B^0 \rightarrow \bar{K}^0 \pi^0) \simeq \frac{\kappa_{ss\bar{d}}}{C_1 + C_2} A(B^+ \rightarrow \pi^+ \pi^0). \quad (3.9)$$

The remaining amplitudes in table 1 are proportional to e, a . They are $1/m_b$ suppressed, therefore we do not consider them further.

The same SU(3) relations hold also for the decays into two vector mesons, $B \rightarrow V_\lambda V_\lambda$, separately for each helicity amplitude $\lambda = 0, \pm$. For example, the analog of eq. (3.7) is

$$A(B^+ \rightarrow K_\lambda^{*+} K_\lambda^{*0}) = A(B^0 \rightarrow K_\lambda^{*0} K_\lambda^{*0}) = \frac{\kappa_{ss\bar{d}}}{C_1 + C_2} \sqrt{2} A(B^+ \rightarrow \rho_\lambda^+ \rho_\lambda^0). \quad (3.10)$$

As a consequence the $b \rightarrow ss\bar{d}$ and $b \rightarrow dd\bar{s}$ $B \rightarrow VV$ decays are longitudinally polarized in the same way as the $B^+ \rightarrow \rho^+ \rho^0$ decay.

3.2 $B \rightarrow PV$ decays

Table 2 lists the decomposition of $B \rightarrow PV$ decays in terms of graphical amplitudes. The subscripts P, V on t, c identify the final state meson that contains the spectator quark,

Transition	Mode	Amplitude
$b \rightarrow ss\bar{d}$	$B^+ \rightarrow K^{*+}K^0$	$t_V + c_V$
	$B^+ \rightarrow K^+K^{*0}$	$t_P + c_P$
	$B^0 \rightarrow K^{*0}K^0$	$t_P + t_V + c_P + c_V$
	$B_s \rightarrow K^{*0}\pi^0$	$\frac{1}{\sqrt{2}}(a_P + e_P)$
	$B_s \rightarrow K^0\rho^0$	$\frac{1}{\sqrt{2}}(a_V + e_V)$
	$B_s \rightarrow K^{*0}\eta_8$	$\sqrt{\frac{2}{3}}(t_P + c_P + a_V + e_V)$
	$B_s \rightarrow K^0\phi_8$	$\sqrt{\frac{2}{3}}(t_V + c_V + a_P + e_P)$
	$B_s \rightarrow K^{*+}\pi^-$	$-(a_P + e_P)$
	$B_s \rightarrow K^+\rho^-$	$-(a_V + e_V)$
$b \rightarrow dd\bar{s}$	$B^+ \rightarrow \bar{K}^{*0}\pi^+$	$t_P + c_P$
	$B^+ \rightarrow \bar{K}^0\rho^+$	$t_V + c_V$
	$B^0 \rightarrow \bar{K}^{*0}\pi^0$	$\frac{1}{\sqrt{2}}(t_P + c_P + a_V + e_V)$
	$B^0 \rightarrow \bar{K}^0\rho^0$	$\frac{1}{\sqrt{2}}(t_V + c_V + a_P + e_P)$
	$B^0 \rightarrow K^{*-}\pi^+$	$-(a_V + e_V)$
	$B^0 \rightarrow K^-\rho^+$	$-(a_P + e_P)$
	$B_s \rightarrow \bar{K}^{*0}\bar{K}^0$	$t_V + t_P + c_V + c_P$

Table 2. $B \rightarrow PV$ exclusive decays mediated by the $b \rightarrow ss\bar{d}$ and $b \rightarrow dd\bar{s}$ transitions.

while the subscripts on a, e denote the final state meson containing the q_3 quark from $\bar{b} \rightarrow \bar{q}_1\bar{q}_2q_3$ (here the spectator participates in the weak interaction) [20, 21].

We have $T_{P,V} + C_{P,V} \propto \langle \overline{10} | \overline{15} | 3 \rangle \pm \langle 27 | \overline{15} | 3 \rangle$. The analogs of the relation (3.5) are then

$$t_P + c_P = \frac{\kappa_{ss\bar{d}}}{C_1 + C_2}(T_P + C_P), \quad t_V + c_V = \frac{\kappa_{ss\bar{d}}}{C_1 + C_2}(T_V + C_V), \quad (3.11)$$

where the graphical amplitudes on the right-hand side are for $\Delta S = 0$ decays. The expansion of the corresponding decay amplitudes in terms of graphical amplitudes can be found in refs. [20, 21]. Combining them with expansions in table 2 gives the SU(3) relations for the $t_i + c_i$ exclusive $b \rightarrow ss\bar{d}$ decay amplitudes (for $\Delta S = 0$ amplitude we only denote the final state)

$$A(B^+ \rightarrow K^{*+}K^0) = \frac{\kappa_{ss\bar{d}}}{C_1 + C_2} \left[-(A_{\rho^+\pi^-} - A_{\rho^-\pi^+}) - \sqrt{2}A_{\rho^0\pi^+} + (A_{K^{*0}\bar{K}^0} - A_{\bar{K}^{*0}K^0}) - (A_{K^{*-}K^+} - A_{K^{*+}K^-}) + (A_{\bar{K}^{*0}K^+} - A_{K^{*+}\bar{K}^0}) \right], \quad (3.12)$$

$$A(B^+ \rightarrow K^+K^{*0}) = \frac{\kappa_{ss\bar{d}}}{C_1 + C_2} \left[(A_{\rho^+\pi^-} - A_{\rho^-\pi^+}) - \sqrt{2}A_{\rho^+\pi^0} - (A_{K^{*0}\bar{K}^0} - A_{\bar{K}^{*0}K^0}) + (A_{K^{*-}K^+} - A_{K^{*+}K^-}) - (A_{\bar{K}^{*0}K^+} - A_{K^{*+}\bar{K}^0}) \right], \quad (3.13)$$

$$A(B^0 \rightarrow K^{*0}K^0) = -\frac{\kappa_{ss\bar{d}}}{C_1 + C_2} \sqrt{2}(A_{\rho^+\pi^0} + A_{\rho^0\pi^+}). \quad (3.14)$$

The B_s decay amplitudes containing $t_i + c_i$ are given in terms of the above $b \rightarrow ss\bar{d}$

amplitudes

$$A(B_s \rightarrow K^{*0} \eta_8) = \sqrt{\frac{2}{3}} [A(B^+ \rightarrow K^+ \bar{K}^{*0}) - A(B_s \rightarrow K^+ \rho^-)], \quad (3.15)$$

$$A(B_s \rightarrow K^0 \phi_8) = \sqrt{\frac{2}{3}} [A(B^+ \rightarrow K^{*+} K^0) - A(B_s \rightarrow K^{*+} \pi^-)], \quad (3.16)$$

where the $1/m_b$ suppressed pure annihilation and exchange decay amplitudes are

$$A(B_s \rightarrow K^{*+} \pi^-) = -\sqrt{2} A(B_s \rightarrow K^{*0} \pi^0) = -\frac{\kappa_{ss\bar{d}}}{C_1 + C_2} [A_{\bar{K}^{*0} K^+} - A_{K^{*-} K^+} - A_{\bar{K}^{*0} K^0}], \quad (3.17)$$

$$A(B_s \rightarrow K^+ \rho^-) = -\sqrt{2} A(B_s \rightarrow K^0 \rho^0) = -\frac{\kappa_{ssd}}{C_1 + C_2} [A_{K^{*+} \bar{K}^0} - A_{K^{*+} K^-} - A_{K^{*0} \bar{K}^0}]. \quad (3.18)$$

The relations for the $b \rightarrow dd\bar{s}$ transitions are derived in an analogous way, giving for the $t_i + c_i$ amplitudes

$$A(B^+ \rightarrow \bar{K}^{*0} \pi^+) = \frac{\kappa_{dd\bar{s}}}{C_1 + C_2} \left[(A_{\rho^+ \pi^-} - A_{\rho^- \pi^+}) - \sqrt{2} A_{\rho^+ \pi^0} - (A_{K^{*0} \bar{K}^0} - A_{\bar{K}^{*0} K^0}) + (A_{K^{*-} K^+} - A_{K^{*+} K^-}) - (A_{\bar{K}^{*0} K^+} - A_{K^{*+} \bar{K}^0}) \right], \quad (3.19)$$

$$A(B^+ \rightarrow \bar{K}^0 \rho^+) = \frac{\kappa_{dd\bar{s}}}{C_1 + C_2} \left[- (A_{\rho^+ \pi^-} - A_{\rho^- \pi^+}) - \sqrt{2} A_{\rho^0 \pi^+} + (A_{K^{*0} \bar{K}^0} - A_{\bar{K}^{*0} K^0}) - (A_{K^{*-} K^+} - A_{K^{*+} K^-}) + (A_{\bar{K}^{*0} K^+} - A_{K^{*+} \bar{K}^0}) \right], \quad (3.20)$$

and

$$\sqrt{2} A(B^0 \rightarrow \bar{K}^{*0} \pi^0) = A(B^+ \rightarrow \bar{K}^{*0} \pi^+) - A(B^0 \rightarrow K^{*-} \pi^+), \quad (3.21)$$

$$\sqrt{2} A(B^0 \rightarrow \bar{K}^0 \rho^0) = A(B^+ \rightarrow \bar{K}^0 \rho^+) - A(B^0 \rightarrow K^- \rho^+). \quad (3.22)$$

The $1/m_b$ suppressed pure annihilation and exchange amplitudes are

$$A(B^0 \rightarrow K^{*-} \pi^+) = \frac{\kappa_{dd\bar{s}}}{C_1 + C_2} [-A_{K^{*+} \bar{K}^0} + A_{K^{*+} K^-} + A_{K^{*0} \bar{K}^0}], \quad (3.23)$$

$$A(B^0 \rightarrow K^- \rho^+) = \frac{\kappa_{dd\bar{s}}}{C_1 + C_2} [-A_{\bar{K}^{*0} K^+} + A_{K^{*-} K^+} + A_{\bar{K}^{*0} K^0}]. \quad (3.24)$$

The remaining B_s mode is given by

$$A(B_s \rightarrow \bar{K}^{*0} \bar{K}^0) = -\frac{\kappa_{dd\bar{s}}}{C_1 + C_2} \sqrt{2} [A_{\rho^+ \pi^0} + A_{\rho^0 \pi^+}]. \quad (3.25)$$

These SU(3) relations will be used in the next section to predict branching fractions of exclusive $b \rightarrow ss\bar{d}$ and $b \rightarrow dd\bar{s}$ decays in the SM. The measured branching fractions of $\Delta S = 0$ modes are then the inputs in the predictions and are collected in table 5. We only quote results for those decays that are not $1/m_b$ suppressed.

4 SM predictions from the SU(3) relations

Experimentally one will be able to search for NP effects in the following $b \rightarrow ss\bar{d}$ decays $\bar{B}^0 \rightarrow \bar{K}^{0*}\bar{K}^{0*}$, $B^- \rightarrow K^-\bar{K}^{0*}$, $\bar{B}_s^0 \rightarrow \phi\bar{K}^{0*}$. The flavor of $\bar{K}^{0*}$ is tagged using the decay $K^{0(*)} \rightarrow K^+\pi^-$. The same decays with $\bar{K}^0$ instead of $\bar{K}^{0*}$, on the other hand, cannot be used to probe $b \rightarrow ss\bar{d}$ transitions. The K^0 mixes with $\bar{K}^0$ so that mass eigenstates $K_{S,L}$ are observed in the experiment. The "wrong kaon" decays listed above are thus only a subleading contribution in the SM rate. For easier comparison with previous calculations in the literature we will still quote results for $\bar{B}^0 \rightarrow \bar{K}^0\bar{K}^0$, $\dots$, "branching ratios", knowing that these are unobservable in practice. Similar comments apply to $b \rightarrow dd\bar{s}$ transitions, where NP effects can be probed in $\bar{B}^0 \rightarrow \pi^0 K^{0*}$, $\rho^0 K^{0*}$, $B^- \rightarrow \pi^- K^{0*}$, $\rho^- K^{0*}$ and $\bar{B}_s^0 \rightarrow K^{0*}K^{0*}$ decays, again using flavor tagged K^{0*} decays.

We derive next numerical predictions for the branching fractions of the exclusive $b \rightarrow ss\bar{d}, dd\bar{s}$ modes. The branching fraction of a given mode $B_q \rightarrow M_1 M_2$ is given by

$$\mathcal{B}(B_q \rightarrow M_1 M_2) = \tau_{B_q} |A(B_q \rightarrow M_1 M_2)|^2 \frac{|\vec{p}|}{8\pi m_{B_q}^2} \quad (4.1)$$

To predict $b \rightarrow ss\bar{d}, dd\bar{s}$ decay amplitudes, $A(B_q \rightarrow M_1 M_2)$, we use the SU(3) relations derived in section 3 which relate them to the amplitudes of the already measured $B^+ \rightarrow \pi^+\pi^0, \rho^+\rho^0$, and $B \rightarrow \rho\pi$ decays. The results are collected in tables 3 and 4. As mentioned, we do not present results for the branching ratios of the $1/m_b$ suppressed annihilation modes.

In the calculation of $B \rightarrow PV$ branching ratios we neglect the contributions of the small penguin dominated $B \rightarrow \bar{K}^*K, K^*\bar{K}$ decays in the SU(3) relations (with experimental upper bounds supporting this approximation). Furthermore, the application of the SU(3) relations requires that we know also the relative phases of the $B \rightarrow \rho\pi$ amplitudes. These phases are small, and can be neglected to a good approximation. This can be verified using the isospin pentagon relation

$$A(\rho^+\pi^0) + A(\rho^0\pi^+) = \frac{1}{\sqrt{2}}(A(\rho^+\pi^-) + A(\rho^-\pi^+)) + \sqrt{2}A(\rho^0\pi^0). \quad (4.2)$$

Neglecting the relative phases, and using data from table 5, the left-hand side of this equality is 6.25 ± 0.29 (in units of $\sqrt{\mathcal{B} \cdot 10^6}$), which compares well with the right-hand side of 6.91 ± 0.34 . This justifies the assumption made of neglecting the relative phases of the $B \rightarrow \rho\pi$ amplitudes.

To factor out the dependence on CKM elements, we also quote the predictions for $B \rightarrow PP, PV, VV$ modes in a common form as

$$\mathcal{B}(B \rightarrow X_i) = \frac{|\kappa_{ssd}|^2}{(C_1 + C_2)^2} c_i, \quad (4.3)$$

where c_i are coefficients specific to each final state calculated using the SU(3) relations and measured $\Delta S = 0$ branching fractions. In the predictions we used the branching fractions for the $\Delta S = 0$ modes listed in table 5. We use $\tau(B^+)/\tau(B^0) = 1.071 \pm 0.009$ and $\tau(B_s^0)/\tau(B^0) = 0.965 \pm 0.017$ [22].

Mode	$c_i[\times 10^{-6}]$	$\mathcal{B}_{\text{SM}}$	Literature
$B^+ \rightarrow K^+ K^0$	11.0 ± 0.8	$(0.7 \pm 0.1) \cdot 10^{-15}$	2.5×10^{-14}
$B^0 \rightarrow K^0 K^0$	10.2 ± 0.7	$(0.7 \pm 0.1) \cdot 10^{-15}$	—
$B^+ \rightarrow K^{*+} K^0$	29.3 ± 4.3	$(1.9 \pm 0.3) \cdot 10^{-15}$	1.7×10^{-14}
$B^+ \rightarrow K^+ K^{*0}$	11.3 ± 3.0	$(0.7 \pm 0.2) \cdot 10^{-15}$	6.5×10^{-14}
$B^0 \rightarrow K^{*0} K^0$	71.5 ± 6.2	$(4.7 \pm 0.4) \cdot 10^{-15}$	—
$B^+ \rightarrow K^{*-+} K^{*0}$	47.2 ± 3.7	$(3.1 \pm 0.2) \cdot 10^{-15}$	6.8×10^{-14}
$B^0 \rightarrow K^{*0} K^{*0}$	43.9 ± 3.5	$(2.9 \pm 0.2) \cdot 10^{-15}$	—

Table 3. SU(3) predictions for the branching fractions of the $b \rightarrow ss\bar{d}$ modes in the SM. Errors shown are due to experimental inputs. In addition there is an estimated $O(50\%)$ and $O(30\%)$ error on the predictions due to neglected nonlocal contributions and due to SU(3) breaking. The last column shows the predictions from a previous calculation [3].

Mode	$c_i[\times 10^{-6}]$	$\mathcal{B}_{\text{SM}}$
$B^+ \rightarrow \bar{K}^0 \pi^+$	11.1 ± 0.8	$(35 \pm 2) \times 10^{-18}$
$B_s \rightarrow \bar{K}^0 \bar{K}^0$	9.7 ± 0.7	$(30 \pm 2) \times 10^{-18}$
$B^+ \rightarrow \bar{K}^{*0} \pi^+$	11.4 ± 2.9	$(36 \pm 9) \times 10^{-18}$
$B^+ \rightarrow \bar{K}^0 \rho^+$	29.5 ± 4.3	$(92 \pm 13) \times 10^{-18}$
$B^0 \rightarrow \bar{K}^{*0} \pi^0$	5.3 ± 1.4	$(17 \pm 4) \times 10^{-18}$
$B^0 \rightarrow \bar{K}^0 \rho^0$	13.7 ± 2.0	$(43 \pm 6) \times 10^{-18}$
$B_s \rightarrow \bar{K}^{*0} \bar{K}^0$	69.1 ± 6.0	$(215 \pm 19) \times 10^{-18}$
$B^+ \rightarrow \bar{K}^{*0} \rho^+$	47.6 ± 3.8	$(148 \pm 12) \times 10^{-18}$
$B_s \rightarrow \bar{K}^{*0} \bar{K}^{*0}$	41.7 ± 3.3	$(130 \pm 10) \times 10^{-18}$

Table 4. SU(3) predictions for the branching fractions of the $b \rightarrow dd\bar{s}$ modes in the SM. Regarding errors the same comments as in table 3 apply.

Both Belle [23] and BABAR [24] collaborations presented the results of a search for these modes and report the 90% C.L. upper bounds (BABAR bounds are in square brackets)

$$b \rightarrow dd\bar{s} : \mathcal{B}(B^+ \rightarrow K^- \pi^+ \pi^+) < 45.0 [9.5] \times 10^{-7}; \quad \text{BELLE [BABAR]}, \quad (4.4)$$

$$b \rightarrow ss\bar{d} : \mathcal{B}(B^+ \rightarrow K^+ K^+ \pi^-) < 24.0 [1.6] \times 10^{-7}; \quad \text{BELLE [BABAR]}, \quad (4.5)$$

$$\mathcal{B}(B^0 \rightarrow K^0 K^+ \pi^-) < 180 \times 10^{-7}; \quad \text{BELLE}. \quad (4.6)$$

The quasi two-body decay $B^+ \rightarrow \bar{K}^{*0} \pi^+$ is part of the $B^+ \rightarrow K^- \pi^+ \pi^+$ three body decay, $B^+ \rightarrow K^+ K^{*0}$ is part of $B^+ \rightarrow K^+ K^+ \pi^-$, while $B^0 \rightarrow K^0 K^{*0}$ is part of $B^0 \rightarrow K^0 K^+ \pi^-$. The bounds on three body decays thus imply bound on two-body decays. These are 8 orders of magnitude or more above the estimates for the SM signal, but the situation could improve at a future super-B factory [25] or at LHCb. Note that $B^0 \rightarrow K^0 K^+ \pi^-$ is observed in $K_S K^+ \pi^-$ final states which also receives contributions from $b \rightarrow d$ penguin decay $B^0 \rightarrow \bar{K}^0 K^{*0}$ and from annihilation decay $B^0 \rightarrow K^+ K^-$. It thus cannot be used as a null probe of NP.

Mode	$\mathcal{B}(\times 10^{-6})$	Mode	$\mathcal{B}(\times 10^{-6})$
$B^+ \rightarrow \pi^0 \pi^+$	$5.59^{+0.41}_{-0.40}$	$B^0 \rightarrow \rho^\pm \pi^\mp$	23.0 ± 2.3
$\rho^0 \pi^+$	$8.7^{+1.0}_{-1.1}$	$\rho^+ \pi^-$	15.4 ± 1.8^a
$\rho^+ \pi^0$	$10.9^{+1.4}_{-1.5}$	$\rho^- \pi^+$	7.2 ± 1.1^b
$\rho^+ \rho^0$	2.40 ± 0.19	$\rho^0 \pi^0$	2.0 ± 0.5

Table 5. Branching ratios for $B \rightarrow \pi\pi, \rho\pi, \rho\rho$ decays, from ref. [22] apart from: a) the average of 15.5 ± 3.4 [26] and 15.3 ± 2.2 [27], and b) the average of 7.1 ± 1.9 [26] and 7.3 ± 1.4 [27].

5 $b \rightarrow ss\bar{d}$ and $b \rightarrow dd\bar{s}$ transitions in the presence of NP

Next we consider the $b \rightarrow ss\bar{d}$ and $b \rightarrow dd\bar{s}$ decays in the presence of generic NP. The most general local NP hamiltonian mediating the $b \rightarrow ss\bar{d}$ and $b \rightarrow dd\bar{s}$ transitions was given in eq. (1.7). In this section we will assume that NP matches onto the local operator Q_1 in eq. (1.7) with SM chirality $(V - A) \times (V - A)$. This is true for a large class of NP models, such as the two-Higgs doublet model with small $\tan\beta$, or the constrained MSSM [4]. Effects of NP that matches to other chiral structures will be given in the next section.¹ To simplify the notation we focus on $b \rightarrow ss\bar{d}$ transitions — the expressions for $b \rightarrow dd\bar{s}$ can be obtained through a simple $s \leftrightarrow d$ exchange — but show numerical results for both types of decays.

We consider three representative cases of NP: i) the exchange of NP fields that carry a conserved charge, where large effects are possible as explained in the Introduction, ii) minimally flavor violating (MFV) new physics with small $\tan\beta$ [28], NMFV [29] and iii) general flavor violation with a $\sim 10^3$ TeV scale suppression. For this analysis it is useful to rewrite the $b \rightarrow ss\bar{d}$ SM effective Hamiltonian (2.6) as

$$\mathcal{H}_{ss\bar{d}} = \frac{G_F}{\sqrt{2}} V_{ub} V_{ud}^* \kappa_{ss\bar{d}} O_{ss\bar{d}} = \frac{1}{\Lambda_0^2} e^{-i\gamma} \kappa_{ss\bar{d}} Q_1, \quad (5.1)$$

where $\Lambda_0 = 2^{1/4}/(2\sqrt{G_F|V_{ub}V_{ud}|}) = 2.98$ TeV and Q_1 is defined in (1.8) (the flavor dependence of Q_1 is not shown). The NP Hamiltonian for $b \rightarrow ss\bar{d}$ is

$$\mathcal{H}_{ss\bar{d}}^{\text{NP}} = \frac{c_1}{\Lambda_{\text{NP}}^2} \eta_2 Q_1, \quad (5.2)$$

where the Wilson coefficient c_1 contains possible extra flavor hierarchy in the new physics transitions, and Λ_{NP} is the scale of NP. The hard QCD correction to the c_1 coefficient describing the RG running from the weak scale m_W to m_b has been explicitly factored out, $\eta_2(m_b) = 0.85$. It is now very easy to obtain the branching ratio in the presence of NP from the SM predictions,

$$\mathcal{B}_{\text{NP}}(B \rightarrow f) = \left| \frac{\Lambda_0^2}{\kappa_{ss\bar{d}}} \frac{c_1 \eta_2}{\Lambda_{\text{NP}}^2} \right|^2 \mathcal{B}_{\text{SM}}(B \rightarrow f), \quad (5.3)$$

and similarly for $b \rightarrow dd\bar{s}$ decays.

¹There is also the possibility that NP contributes through the $\Delta S = 1$ hamiltonians appearing in the nonlocal term in eq. (2.1), for instance through a $(\bar{s}b)(\bar{c}c)$ term. We do not pursue this possibility further.

5.1 NP with conserved charge

As discussed in the introduction it is possible to have large NP effects in $b \rightarrow ss\bar{d}$ decays, if the transition is mediated by NP fields that carry a total conserved charge, and if in addition there exists a hierarchy in the couplings. In this case we have for the Wilson coefficient in the NP Hamiltonian (5.2) (cf. eq. (1.4))

$$\frac{c_1}{\Lambda_{\text{NP}}^2} = \frac{1}{M_X^2} (g_{b \rightarrow s} g_{s \rightarrow d}^* + g_{d \rightarrow s} g_{s \rightarrow b}^*). \quad (5.4)$$

From $K - \bar{K}$ and $B_s - \bar{B}_s$ mixing we have the bounds, eq. (1.1),

$$\frac{|g_{d \rightarrow s} g_{s \rightarrow d}^*|^{1/2}}{M_X} < \frac{1}{10^3 \text{ TeV}}, \quad \frac{|g_{b \rightarrow s} g_{s \rightarrow b}^*|^{1/2}}{M_X} < \frac{1}{30 \text{ TeV}}. \quad (5.5)$$

These bounds are trivially satisfied, if for instance $g_{s \rightarrow d} = g_{b \rightarrow s} = 0$, since then no mixing contributions are induced. The $b \rightarrow ss\bar{d}$ transitions, on the other hand, can still be large, if $g_{d \rightarrow s}$ and $g_{s \rightarrow b}$ are nonzero. Taking $g_{d \rightarrow s} = g_{s \rightarrow b} = 1$, the BABAR experimental bound on $\mathcal{B}(B^+ \rightarrow K^+ K^{*0})$, eq. (4.5), gives $M_X > 5.0 \text{ TeV}$. A similar bound $M_X > 5.0 \text{ TeV}$ is found for $b \rightarrow dd\bar{s}$ from the BABAR bound on $B^+ \rightarrow \bar{K}^{*0} \pi^+$ branching ratio. The resulting predictions for $b \rightarrow ss\bar{d}$ and $b \rightarrow dd\bar{s}$ branching ratios are of $\mathcal{O}(10^{-6})$ as shown in table 6 and may well be probed at Belle II and LHCb.

A more generic situation may be that only one of the g_i couplings is accidentally small. Unlike in the previous example, we choose M_X such that we do not saturate the present experimental bounds on $b \rightarrow ss\bar{d}$. As an illustration let us take $g_{s \rightarrow d} = 0$ and all the other couplings to be equal to 1. In this case the $K - \bar{K}$ mixing bound in (5.5) is trivially satisfied, while $B_s - \bar{B}_s$ mixing implies that $M_X > 30 \text{ TeV}$. The $b \rightarrow ss\bar{d}$ branching ratios are

$$\mathcal{B}(\{B^+ \rightarrow K^+ K^{*0}, B^+ \rightarrow K^{*+} K^{*0}, B^0 \rightarrow K^{*0} K^{*0}\}) = \{1.1, 4.6, 4.3\} \times 10^{-9} \left(\frac{30 \text{ TeV}}{M_X} \right)^4. \quad (5.6)$$

For $b \rightarrow dd\bar{s}$ transitions the same choice for the values of coupling, $g_{s \rightarrow d} = 0$ and all the other $g_i = 1$, sets $M_X > 210 \text{ TeV}$ due to the present absence of NP effects in $B_d - \bar{B}_d$ mixing. This gives

$$\mathcal{B}(\{B^+ \rightarrow \bar{K}^{*0} \pi^+, B^+ \rightarrow \bar{K}^{*0} \rho^+, B_s^0 \rightarrow \bar{K}^{*0} \bar{K}^{*0}\}) = \{0.5, 1.9, 1.7\} \times 10^{-12} \left(\frac{210 \text{ TeV}}{M_X} \right)^4. \quad (5.7)$$

Finally, we mention that, if $b \rightarrow ss\bar{d}$ or $b \rightarrow dd\bar{s}$ modes are observed in the near future, this would imply nontrivial exclusions on the parameter space of the models. In particular models with $g_{s \rightarrow b} \sim g_{s \rightarrow d}$ and/or $g_{b \rightarrow s} \sim g_{d \rightarrow s}$ would be excluded as discussed in appendix C.

5.2 NP with MFV and NMFV structures

Both MFV [28] and NMFV [29] fall in the class of new physics models where the $b \rightarrow ss\bar{d}$ suppression scale $\Lambda_{ss\bar{d}}$ is the geometric average of the NP scales in $K - \bar{K}$ and $B_s - \bar{B}_s$ mixing (1.1), $\Lambda_{ss\bar{d}} \sim \sqrt{\Lambda_{sd} \Lambda_{bs}} \gtrsim 173 \text{ TeV}$. In this paper we will restrict ourselves to MFV with small $\tan \beta$, where the $\Delta F = 2$ processes are mediated by a single operator with

Mode ($ss\bar{d}$)	$\mathcal{B}_X[\times(\frac{5.0 \text{ TeV}}{M_X})^4]$	$\mathcal{B}_{\text{NMFV}}[\times(\frac{173 \text{ TeV}}{\Lambda_{ss\bar{d}}})^4]$	$\mathcal{B}_{\text{gen.}}$	$\mathcal{B}_{\text{SM}}$
$B^+ \rightarrow K^+ K^{*0}$	1.4×10^{-6}	1.0×10^{-12}	0.3×10^{-14}	$(0.7 \pm 0.2) \times 10^{-15}$
$B^+ \rightarrow K^{*+} K^{*0}$	6.0×10^{-6}	4.2×10^{-12}	1.4×10^{-14}	$(3.1 \pm 0.2) \times 10^{-15}$
$B^0 \rightarrow K^{*0} K^{*0}$	5.5×10^{-6}	3.9×10^{-12}	1.3×10^{-14}	$(2.9 \pm 0.2) \times 10^{-15}$
Mode ($dd\bar{s}$)	$\mathcal{B}_X[\times(\frac{5.0 \text{ TeV}}{M_X})^4]$	$\mathcal{B}_{\text{NMFV}}[\times(\frac{458 \text{ TeV}}{\Lambda_{dd\bar{s}}})^4]$	$\mathcal{B}_{\text{gen.}}$	$\mathcal{B}_{\text{SM}}$
$B^+ \rightarrow \bar{K}^{*0} \pi^+$	1.4×10^{-6}	2.0×10^{-14}	1.3×10^{-15}	$(3.6 \pm 0.9) \times 10^{-17}$
$B^+ \rightarrow \bar{K}^{*0} \rho^+$	6.0×10^{-6}	8.5×10^{-14}	5.4×10^{-15}	$(14.8 \pm 1.2) \times 10^{-17}$
$B_s \rightarrow \bar{K}^{*0} \bar{K}^{*0}$	5.3×10^{-6}	7.5×10^{-14}	4.7×10^{-15}	$(13 \pm 1) \times 10^{-17}$

Table 6. Predictions for the branching fractions of the $b \rightarrow ss\bar{d}, dd\bar{s}$ modes in the presence of NP carrying conserved charge ($\mathcal{B}_X$), with NMFV flavor structure ($\mathcal{B}_{\text{NMFV}}$), and general flavor violation at high scale ($\mathcal{B}_{\text{gen.}}$), in all cases assuming dominance of the SM operator $(\bar{s}d)_{V-A}(\bar{s}b)_{V-A}$ (see also text for details). The branching fractions $\mathcal{B}_{\text{gen}}$ include (maximally constructive) interference with the SM amplitude, and were obtained using $\Lambda_{ss\bar{d}, dd\bar{s}} = 10^3 \text{ TeV}$. Only modes which do not contain $K_{S,L}$ are shown.

$(V - A) \times (V - A)$ structure [28]. This implies that the $K - \bar{K}$ and $B_s - \bar{B}_s$ mixing operators are

$$\frac{V_{ts}^2 V_{td}^{*2}}{\Lambda_{\text{MFV}}^2} (\bar{s}_L \gamma_\mu d_L)^2 \equiv \frac{1}{\Lambda_{sd}^2} (\bar{s}_L \gamma_\mu d_L)^2, \quad \frac{V_{tb}^2 V_{ts}^{*2}}{\Lambda_{\text{MFV}}^2} (\bar{b}_L \gamma_\mu s_L)^2 \equiv \frac{1}{\Lambda_{bs}^2} (\bar{b}_L \gamma_\mu s_L)^2, \quad (5.8)$$

and the $b \rightarrow ss\bar{d}$ local operator is

$$\frac{1}{\Lambda_{\text{MFV}}^2} V_{tb} V_{ts}^* V_{td} V_{ts}^* (\bar{b}_L \gamma_\mu s_L) (\bar{d}_L \gamma^\mu s_L) \equiv \frac{1}{\Lambda_{ss\bar{d}}^2} (\bar{b}_L \gamma_\mu s_L) (\bar{d}_L \gamma^\mu s_L), \quad (5.9)$$

all of which depend only on one unknown parameter, the MFV scale Λ_{MFV} . From a global fit the UTfit collaboration finds $\Lambda_{\text{MFV}} > 5.5 \text{ TeV}$ [9]. We have also defined the suppression scales $\Lambda_{sd}, \Lambda_{bs}, \Lambda_{ss\bar{d}}$ that include the hierarchy of the NP induced flavor changing couplings, which in the MFV case are just the appropriate CKM matrix elements. They are related as stated above $\Lambda_{ss\bar{d}} = \sqrt{\Lambda_{sd} \Lambda_{bs}}$.

In NMFV the operators in (5.8) and (5.9) are still parameterically suppressed by the CKM matrix elements, but the strict correlation between the Wilson coefficients is lost — they are multiplied by $O(1)$ complex coefficients. We then have approximately $\Lambda_{ss\bar{d}} \sim \sqrt{\Lambda_{sd} \Lambda_{bs}}$. Using the bounds from (1.1), $\Lambda_{ss\bar{d}} \gtrsim 173 \text{ TeV}$, which gives $b \rightarrow ss\bar{d}$ branching ratios of $\mathcal{O}(10^{-12})$, table 6. Similarly we have $\Lambda_{dd\bar{s}} \sim \sqrt{\Lambda_{sd} \Lambda_{bd}} \gtrsim 458 \text{ TeV}$, giving $b \rightarrow dd\bar{s}$ branching ratios of $\mathcal{O}(10^{-13})$. In MFV the predicted branching ratios are much smaller, of the order of the SM branching ratios in table 6. The reason for the difference between NMFV and MFV is that in MFV NP the Wilson coefficient generating $K - \bar{K}$ mixing carries a weak phase (the same phase as it does in the SM), while in NMFV the NP contribution can be real.

5.3 General flavor violation with a high scale

As a final example we consider the case where NP is at the mass scale probed by $K - \bar{K}$ mixing, $\Lambda \sim 10^3 \text{ TeV}$, and assume that flavor violating couplings are all of $\mathcal{O}(1)$. The resulting branching fractions for $b \rightarrow ss\bar{d}$ and $b \rightarrow dd\bar{s}$ decays, assuming positive interference

between SM and NP contributions, are collected in table 6. For $b \rightarrow ss\bar{d}$ decays the NP and SM contributions are roughly of the same size, while for $b \rightarrow dd\bar{s}$ the NP induced branching ratios are more than two orders of magnitude larger than the SM ones. This means that with enough statistics one could probe flavor violation without theoretical uncertainty to scales $\Lambda \sim 10^3$ TeV both in $3 \rightarrow 2$ and $3 \rightarrow 1$ transitions and not just in $2 \rightarrow 1$ transitions as is possible now from $K - \bar{K}$ mixing. Of course, the statistics needed to achieve such an ambitious goal is well beyond the reach of present and planned flavor factories.

6 NP leading to non-SM chiralities

We now turn to the description of effects induced by the local operators with non-standard chiralities $Q_{2-5}, \tilde{Q}_{1-5}$. It is convenient to normalize the matrix elements of these operators to the ones of the SM operator Q_1

$$r_j(B \rightarrow M_1 M_2) \equiv \frac{\langle M_1 M_2 | Q_j | B \rangle}{\langle M_1 M_2 | Q_1 | B \rangle}, \quad (6.1)$$

and similarly for $\tilde{Q}_{1-5}$, where the ratio is denoted as $\tilde{r}_j$. To obtain predictions for a $b \rightarrow ss\bar{d}$ decay branching ratio due to a particular NP chiral structure, one only needs to multiply the results in table 6 with appropriate r_j^2 or $\tilde{r}_j^2$.

Using parity one can relate r_j and $\tilde{r}_j$, since $P^\dagger Q_j P = \tilde{Q}_j$. For $B \rightarrow PP$ ($B \rightarrow VP$) decays one then has

$$\tilde{r}_1 = \mp 1, \quad \text{and} \quad \tilde{r}_j = \mp r_j, \quad j = 2, \dots, 5. \quad (6.2)$$

For $B \rightarrow VV$ decays it is convenient to define ratios $r_{\lambda,j}, \tilde{r}_{\lambda,j}$ for final states with definite helicities, $|V_{1,\lambda} V_{2,\lambda}\rangle$, where $\lambda = 0, \pm$. We then have $\tilde{r}_{1,\pm} = \tilde{r}_{1,0} = -1$ and

$$\tilde{r}_{j,\pm} = -r_{j,\mp}, \quad \tilde{r}_{j,0} = -r_{j,0}, \quad j = 2, \dots, 5 \quad (6.3)$$

We only need to compute the ratios r_j , $j = 2, \dots, 5$. The ratios $\tilde{r}_j$ are then already given by the above relations. To compute r_j we use naive factorization [30], which suffices for the accuracy required here. Strictly speaking, naive factorization is not valid at leading order in the heavy quark expansion, but corresponds to assuming dominance of the soft-overlap contributions in the complete SCET factorization formula [31–33], and keeping only terms of leading order in $\alpha_s(m_b)$. In the QCDF approach, this corresponds to neglecting hard spectator scattering contributions [35, 36]. If needed, these assumptions can be relaxed.

Naive factorization, or the vacuum insertion approximation, is also justified in the $1/N_c$ expansion for the matrix elements of the operators $Q_{1,2,4}$, but not for $Q_{3,5}$. To see this, one can rewrite Q_3 as a sum of color singlet and color octet terms using the color Fierz identity,

$$Q_3 = (\bar{s}_R^\alpha b_L^\beta)(\bar{s}_R^\beta d_L^\alpha) = \frac{1}{N_c}(\bar{s}_R b_L)(\bar{s}_R d_L) + 2(\bar{s}_R t^a b_L)(\bar{s}_R t^a d_L), \quad (6.4)$$

and analogously for Q_5 . The matrix element of the color-singlet operator scales as $N_c^{1/2}$, while that of the color-octet scales as $N_c^{-1/2}$. The two term in the above decomposition thus contribute at the same order in $1/N_c$ expansion, and both should in principle be kept.

For the experimentally interesting $B \rightarrow PV$ and $B \rightarrow VV$ decay modes all the ratios can be expressed in terms of $r_{2,4}$. One has $r_{3,5} = 3r_{2,4}$, and $r_4 = -1/[2(N_c + 1)]$.

The ratio r_2 is common to all the PV modes which depend only on the graphical amplitudes $t_P + c_P$ (for which the spectator quark ends up in the pseudoscalar meson) and is given by

$$r_2 = \frac{1}{8(N_c + 1)} \frac{f_V^\perp f_T(m_V^2)}{f_V f_+(m_V^2)} \frac{2(m_B^2 - m_P^2 - m_V^2)}{m_V(m_B + m_P)}. \quad (6.5)$$

Using $f_{K^*} = 218 \text{ MeV}$, $f_{\bar{K}^*} = 175 \text{ MeV}$ and the form factors from ref. [37] we find $r_2(K^+ K^{*0}) = 0.28$ and $r_2(\pi^+ \bar{K}^{*0}) = 0.27$.

For the VV modes we quote only the ratios corresponding to longitudinally polarized vector mesons, which dominate the total rate. We find $r_4^\parallel = -1/[2(N_c + 1)]$ and

$$r_2^\parallel = -\frac{1}{8(N_c + 1)} \frac{3f_{V1}^\perp}{f_{V1} m_{V1}} \frac{4m_B^2 \bar{p}^2 T_1(m_{V1}^2) - (m_B^2 - m_{V2}^2)(m_B^2 - m_{V1}^2 - m_{V2}^2) T_2(m_{V1}^2)}{(m_B + m_{V2})(m_B^2 - m_{V1}^2 - m_{V2}^2) A_1(m_{V1}^2) - \frac{4m_B^2 \bar{p}^2}{m_B + m_{V2}} A_2(m_{V1}^2)}. \quad (6.6)$$

Here V_1 denotes the neutral K^* meson (K^{*0} for $b \rightarrow ss\bar{d}$ transitions, and $\bar{K}^{*0}$ for the $b \rightarrow dd\bar{s}$ transitions), if V_1, V_2 are different vector mesons. Numerically we find

$$\begin{aligned} r_2^\parallel(B^+ \rightarrow K^{*+} K^{*0}, K^{*0} K^{*0}) &= 0.002, \\ r_2^\parallel(B^+ \rightarrow \bar{K}^{*0} \rho^+) &= 10^{-5}, \quad r_2^\parallel(B_s \rightarrow \bar{K}^{*0} \bar{K}^{*0}) = 0.002, \end{aligned} \quad (6.7)$$

where we used the $B \rightarrow V$ form factors from ref. [38].

7 Conclusions

The exclusive rare B decays $b \rightarrow ss\bar{d}$ and $b \rightarrow dd\bar{s}$ analyzed in this paper appear in the SM only at second order in the weak interactions and have thus very small branching fractions, but in NP models they can be greatly enhanced. We construct the complete effective Hamiltonian contributing to these modes in the SM, and point out the presence of nonlocal contributions, not included in previous work, which can contribute about 30% of the local term.

We show that the hadronic matrix elements of the local operators contributing to these exclusive decays in the SM can be determined using SU(3) flavor symmetry in terms of measured $\Delta S = 0$ decay amplitudes. Detailed numerical predictions are given for all $B \rightarrow PP, VP, VV$ modes of experimental interest, both in the SM and for several examples of NP models: NP with conserved global charge, (N)MFV models and general flavor violating models.

A general NP mechanism was identified which can enhance the branching fractions of these modes, while obeying existing constraints on NP in $\Delta S = 2$ mixing processes. This mechanism represents a generalization of the sneutrino exchange in R parity violating

SUSY. Any observation of such a decay mode gives a constraint on the ratio of flavor couplings to the NP, and can exclude regions in the parameter space of the NP theory for branching fractions observable at LHC-b and super-B factories.

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A The structure of the effective Hamiltonian

Consider for definiteness the $b \rightarrow ss\bar{d}$ transitions. At scales $m_W > \mu > m_b$, these transitions are described by an effective Hamiltonian with propagating u, c quarks. Writing explicitly the quarks propagating in the box diagram, and not assuming the unitarity of the CKM matrix, the effective Hamiltonian is given by

$$\mathcal{H}_{ss\bar{d}} = \lambda_t^d \lambda_t^b \mathcal{H}(t, t) + \sum_{q_1, q_2=u, c} \lambda_{q_1}^d \lambda_{q_2}^b \mathcal{H}(q_1, q_2), \quad (\text{A.1})$$

with $\lambda_q^{q'} = V_{qq'} V_{qs}^*$ (so that for instance $\lambda_t^d = V_{td} V_{ts}^*$). The top term in the Hamiltonian is a local operator

$$\mathcal{H}(t, t) = \frac{G_F^2 m_W^2}{2} C(m_t^2/m_W^2, \mu/m_W) [(\bar{s}d)_{V-A} (\bar{s}b)_{V-A}], \quad (\text{A.2})$$

where $C(m_t^2/m_W^2, \mu/m_W)$ is a Wilson coefficient. The box diagram with internal quarks $q_1, q_2 = u, c$, on the other hand, is matched onto an effective Hamiltonian containing both local and nonlocal terms [12]

$$\begin{aligned} \mathcal{H}(q_1, q_2) = & \frac{G_F^2}{2} \left\{ A \left(\frac{\mu}{m_W} \right) m_W^2 [(\bar{s}d)_{V-A} (\bar{s}b)_{V-A}] + B \left(\frac{\mu}{m_W} \right) (m_1^2 + m_2^2) [(\bar{s}d)_{V-A} (\bar{s}b)_{V-A}] \right. \\ & \left. + D \left(\frac{\mu}{m_W} \right) m_b^2 [(\bar{s}d)_{V-A} (\bar{s}b)_{V-A}] \right\} + \int d^d x T \{ \mathcal{H}_d(q_2, q_1)(x), \mathcal{H}_b(q_1, q_2)(0) \}. \end{aligned} \quad (\text{A.3})$$

The effective Hamiltonian $\mathcal{H}_b(q_1, q_2)$ mediates $b \rightarrow sq_1 \bar{q}_2$ transitions, and is given by

$$\mathcal{H}_b(q_1, q_2) = \frac{G_F}{\sqrt{2}} \left(\sum_{i=1,2} C_i Q_{i,b}^{q_1 q_2} + \delta_{q_1 q_2} \sum_{j=3}^6 C_j Q_j^b \right). \quad (\text{A.4})$$

$\mathcal{H}_d(q_1, q_2)$ mediates $d \rightarrow s \bar{q}_1 q_2$ and is given by a similar expression, with the replacement $b \rightarrow d$. Note that there is no top-charm contribution at leading order in the m_i^2/m_W^2 expansion. The top-charm box is matched in the effective theory onto six-quark operators of the form $(\bar{c}b)(\bar{s}c)(\bar{s}d)$, which are power suppressed by $1/m_W^2$ relative to the 4-quark operators shown. Terms proportional to $\lambda_c^d \lambda_t^b, \lambda_t^d \lambda_c^b$ appear only after using the unitarity of the CKM matrix.

The dependence on the light quark masses $m_{1,2}$ in the effective theory expression eq. (A.3) can be obtained in the mass insertion approximation. The $W^\pm$ coupling

$W_\mu^+(\bar{u}_L\gamma_\mu d_L)$ conserves chirality, which implies that only m_1^2, m_2^2 terms are allowed, but not $m_1 m_2$, which would require one mass insertion on each propagating line. The m_b^2 term arises from two mass insertions on the incoming b quark line. This term is not present in $K^0 - \bar{K}^0$ mixing. On the other hand, in a theory with chiral-odd quark couplings, such as e.g. the charged Higgs couplings $H^+(\bar{u}_L d_R)$ in the 2HDM, another term can appear in eq. (A.3), proportional to $m_1 m_2$. Chirality prevents also the appearance of terms of the form $m_b m_1, m_b m_2$.

Under renormalization, the local operator with Wilson coefficient $A(\mu/m_W)$ renormalizes multiplicatively, while the nonlocal operators mix into the local operators with coefficients $B(\mu/m_W), D(\mu/m_W)$.

Making use of the unitarity of the CKM matrix, it is possible to eliminate λ_u^b, λ_u^d as $\lambda_u^i = -\lambda_c^i - \lambda_t^i$, $i = b, d$. This reproduces the effective Hamiltonian quoted in text eq. (2.2). The terms proportional to A, C and D are combined into $C_{tt} = 8\pi^2(A + C + (m_b^2/m_W^2)D)$, while the B term in eq. (A.4) reproduces the C_{tc} and C_{ct} coefficients. The total contribution of the local terms proportional to the Wilson coefficient $B(\mu/m_W)$ is equal to

$$\begin{aligned} & \lambda_u^d \lambda_u^b \cdot 0 + \lambda_c^d \lambda_u^b \cdot m_c^2 + \lambda_u^d \lambda_c^b \cdot m_c^2 + \lambda_c^d \lambda_c^b \cdot 2m_c^2 \\ &= -\lambda_c^d(\lambda_c^b + \lambda_t^b) \cdot m_c^2 - \lambda_c^b(\lambda_c^d + \lambda_t^d) \cdot m_c^2 + \lambda_c^d \lambda_c^b \cdot 2m_c^2 = -(\lambda_c^d \lambda_t^b + \lambda_c^d \lambda_t^b) m_c^2. \end{aligned} \quad (\text{A.5})$$

This proves the two properties of the local effective Hamiltonian $\mathcal{H}^{\Delta S=2}$ stated in the text: i) the equality $C_{tc} = C_{ct}$, and ii) the absence of a $\lambda_c^d \lambda_c^b$ local term. The latter property does not hold in the presence of chiral-odd quark couplings, as for example in the 2HDM as discussed above.

B $\Delta S = 2$ Wilson coefficients

In this appendix we show the translation of results obtained for $\bar{K}^0 - K^0$ mixing to the case of $b \rightarrow ss\bar{d}$ decays (the results for $b \rightarrow dd\bar{s}$ decays are equivalent). The results for $\bar{K}^0 - K^0$ mixing were derived in [13] in the leading-log approximation, and in [14] in the next-to-leading log approximation.

We start with the Wilson coefficient C_{tt} , which is obtained by matching the u, c, t loops at the weak scale $\mu = m_W$ onto the local operator $(\bar{s}b)_{V-A}(\bar{s}d)_{V-A}$. Below this scale, QCD radiative corrections introduce a correction $\eta_2(\mu)$, and one has

$$C_{tt}(\mu) = \eta_2(\mu) S_0(x_t) + 8\pi^2 \frac{m_b^2}{m_W^2} D(\mu/m_W). \quad (\text{B.1})$$

The box function $S_0(x_t) = 8\pi^2(A+C)(\mu = m_W)$ with $x_t = m_t^2/m_W^2$ is the same as obtained in the one-loop matching at the m_W scale for $\bar{K}^0 - K^0$ mixing (external b quark leg can be considered as massless for the purpose of this calculation). It is given by [11]

$$S_0(x_t) = \frac{4x_t - 11x_t^2 + x_t^3}{4(1-x_t)^2} - \frac{3x_t^3 \log x_t}{2(1-x_t)^3} = 2.26. \quad (\text{B.2})$$

with the numerical value given for $\bar{m}_t(\bar{m}_t) = 160.9 \text{ GeV}$. The QCD correction $\eta_2(\mu)$ is obtained by solving the renormalization group equation

$$\mu \frac{d\eta_2(\mu)}{d\mu} = \gamma_+ \eta_2(\mu) \quad (\text{B.3})$$

At one-loop order, the anomalous dimension is $\gamma_+ = \alpha_s/\pi$, which gives using $\alpha_s(m_Z) = 0.118$ (from which $\Lambda_{\overline{\text{MS}}}^{n_f=5} = 226 \text{ MeV}$)

$$\eta_2(\mu_b) = \left(\frac{\alpha_s(m_W)}{\alpha_s(m_b)} \right)^{6/23} = 0.85, \quad m_b = 4.2 \text{ GeV}, \quad (\text{B.4})$$

so that

$$C_{tt}(\mu_b) = 1.92. \quad (\text{B.5})$$

The coefficient $D(\mu)$ parameterizes the b quark mass effects, and is introduced by mixing from the nonlocal operators into the local operator $m_b^2(\bar{s}b)_{V-A}(\bar{s}d)_{V-A}$. This mixing has not been computed yet. We will neglect this contribution since it is suppressed by the small ratio $m_b^2/m_W^2 \sim 0.2\%$.

The $\lambda_t^b \lambda_t^d$ nonlocal contributions due to insertions of two four-quark operators are power suppressed and can be neglected as discussed in appendix A. This is no longer true for top-charm contributions, where both local and nonlocal contributions are power suppressed by m_i^2/m_W^2 , and mix under renormalization.

We use the derivation of [14], which we adapt to the $b \rightarrow ss\bar{d}$ process at hand. The local part of the $\bar{K}^0 - K^0$ mixing weak Hamiltonian for μ above the charm quark mass (i.e. before charm quark is integrated out) is given by [14]

$$H_{\text{eff}}^{\bar{K}-K} = \frac{G_F^2}{2} \lambda_c^d \lambda_t^d \tilde{C}_7 \tilde{Q}_7, \quad \tilde{Q}_7 = \frac{m_c^2}{g^2} [(\bar{s}d)_{V-A}(\bar{s}d)_{V-A}]. \quad (\text{B.6})$$

The corresponding local part of the $b \rightarrow ss\bar{d}$ effective Hamiltonian on the other hand is

$$\frac{G_F^2 m_W^2}{16\pi^2} (\lambda_c^d \lambda_t^b C_{ct} + \lambda_t^d \lambda_c^b C_{tc}) [(\bar{s}d)_{V-A}(\bar{s}b)_{V-A}], \quad (\text{B.7})$$

The RG evolution calculation for $b \rightarrow ss\bar{d}$ process is the same as for $\bar{K}^0 - K^0$ mixing, except that the total contribution is split into two because of two different CKM element structures in (B.7). As shown in appendix A, these structures have identical coefficients in the SM $C_{ct} = C_{tc}$.

The same equality can be seen also in the anomalous dimension matrices for the running of these coefficients. Consider the nonlocal contribution to $b \rightarrow ss\bar{d}$ with insertions of the tree operators $T\{Q_{1,2}Q_{1,2}\}$, which is given by

$$\sum_{i,j=1,2} C_i C_j \left\{ \lambda_c^d \lambda_t^b (Q_{i,d}^{uu} Q_{j,b}^{uu} - Q_{i,d}^{cu} Q_{j,b}^{uc}) + \lambda_t^d \lambda_c^b (Q_{i,d}^{uu} Q_{j,b}^{uu} - Q_{i,d}^{uc} Q_{j,b}^{cu}) \right\}. \quad (\text{B.8})$$

When computing the mixing into the local operator $\tilde{Q}_7$, the terms in the first and the second brackets give the same contributions, since the quark masses are not relevant for

the calculation of the anomalous dimensions (it does not matter whether c quark or u quark runs in the lower leg of the loop in Fig 1). This shows that the RG running for C_{ct}, C_{tc} is the same. Furthermore, this running is the same as that of $\tilde{C}_7$ in $K^0 - \bar{K}^0$ mixing. This can be seen by comparing (B.8) with the nonlocal operator contributing to $\bar{K}^0 - K^0$ mixing

$$\sum_{i,j=1,2} C_i C_j \lambda_c^d \lambda_t^d (2Q_{i,d}^{uu} Q_{j,d}^{uu} - Q_{i,d}^{cu} Q_{j,d}^{uc} - Q_{i,d}^{cu} Q_{j,d}^{uc}), \quad (\text{B.9})$$

The two operators are identical, provided that one sets $b \rightarrow d$ in (B.8). The same correspondence between $K^0 - \bar{K}^0$ mixing and $b \rightarrow ss\bar{d}$ applies also for the nonlocal contributions involving penguin operators.

In conclusion, comparing the eqs. (B.6) and (B.7) we find that for $\mu > m_c$, we have

$$C_{ct}(\mu) = C_{tc}(\mu) = \tilde{C}_7(\mu) x_c \pi / \alpha_s, \quad (\text{B.10})$$

where $\tilde{C}_7(\mu)$ is obtained from RG evolution in the same way as for $\bar{K}^0 - K^0$ mixing. A very compact form of RG equations was presented in [14]

$$\mu \frac{d}{d\mu} \vec{D} = \hat{\gamma}^T \cdot \vec{D}, \quad (\text{B.11})$$

with $\hat{\gamma}$ the 8×8 anomalous dimension, given in eqs. (6.23)–(6.26) and (12.50)–(12.56) of [11] and

$$\vec{D}^T = (\vec{C}^T, C_{7+}/C_+, C_{7-}/C_-). \quad (\text{B.12})$$

Here $\vec{C}$ is a vector of C_i , $i = 1, \dots, 6$, $C_{\pm} = C_1 \pm C_2$,² and $\tilde{C}_7$ was split to $\tilde{C}_7 = C_{7+} + C_{7-}$, where the distribution between C_{7+} and C_{7-} is arbitrary. At LO we have for the matching at weak scale $\vec{D}^T(\mu_W) = (1, 0, 0, 0, 0, 0, 0, 0)$, so that the nonzero value of $\tilde{C}_7(\mu)$ comes entirely from the running, from mixing with C_1 . At μ_b the solution of RG running at LO is

$$\vec{D}(\mu) = V \left(\left[\frac{\alpha_S(m_W)}{\alpha_S(\mu)} \right]^{\tilde{\gamma}^{(0)}/2\beta_0} \right)_D V^{-1}, \quad (\text{B.13})$$

with $\hat{\gamma} = \frac{\alpha_S}{4\pi} \gamma^{(0)}$ and V a matrix that diagonalizes the LO anomalous dimension matrix, $\gamma_D^{(0)} = V^{-1} \gamma^{(0)T} V$. This gives

$$\tilde{C}_7(m_b) = 0.268, \quad m_b = 4.2 \text{ GeV}, \quad (\text{B.14})$$

and finally

$$C_{tc}(m_b) = C_{ct}(m_b) = 3.75 x_c = 9.35 \cdot 10^{-4}, \quad (\text{B.15})$$

where in the last equality we used $m_c = 1.27 \text{ GeV}$.

²Here we caution about the definition of $C_{1,2}$, which differs from the one in [11]. We use the definition, where $C_1(\mu_W) \sim 1$, $C_2(\mu_W) \sim 0$.

C Bounds on the flavor-changing couplings

We have showed in the introduction that $b \rightarrow ss\bar{d}$ branching ratios can be large, if NP effects are due to exchange of particle(s) with conserved charge. The resulting effective weak Hamiltonian, eq. (1.4), depends on four couplings, $g_{s \rightarrow d}, g_{d \rightarrow s}, g_{b \rightarrow s}, g_{s \rightarrow b}$ and an overall mass scale M_X , that in this appendix we set to $M_X = 10 \text{ TeV}$ (this then fixes the overall normalization of g_i). In order to have large $b \rightarrow ss\bar{d}$ branching ratios and simultaneously avoid bounds from $K - \bar{K}$ mixing and $B_s - \bar{B}_s$ mixing a hierarchy between couplings is required. Another way of looking at this is that, if a large $b \rightarrow ss\bar{d}$ decay branching ratio (we will quantify what "large" means below) is found by Belle II and/or LHCb this would imply that a region of parameter space with $g_{s \rightarrow b} \sim g_{s \rightarrow d}$ and/or $g_{b \rightarrow s} \sim g_{d \rightarrow s}$ would be excluded. We show this below.

The experimental constraints from $K - \bar{K}$ mixing and $B_s - \bar{B}_s$ mixing give the following upper bounds (fixing $M_X = 10 \text{ TeV}$ and using bounds from eq. (1.1))

$$\varepsilon_{sd} \equiv |g_{d \rightarrow s} g_{s \rightarrow d}^*| \leq \frac{M_X^2}{\Lambda_{sd}^2} = 10^{-4}, \quad \varepsilon_{bs} \equiv |g_{b \rightarrow s} g_{s \rightarrow b}^*| \leq \frac{M_X^2}{\Lambda_{bs}^2} = 0.11. \quad (\text{C.1})$$

We also define the following two ratios of coupling constants

$$R = \frac{g_{s \rightarrow b}}{g_{s \rightarrow d}}, \quad \bar{R} = \frac{g_{b \rightarrow s}}{g_{d \rightarrow s}}. \quad (\text{C.2})$$

We now show that a measured *lower* bound on the $b \rightarrow ss\bar{d}$ branching fraction excludes values of $R, \bar{R}$ that are close to 1. For definiteness, we assume that the NP field X couples to the quarks with the Dirac structure $\Gamma = P_R$, as in RPV SUSY. Similar bounds can be derived for any other Dirac structure Γ .

The amplitude for the $\bar{B} \rightarrow f$ transition mediated by the operator $(\bar{s}b)(\bar{s}d)$, eq. (1.4), is

$$A(\bar{B} \rightarrow f) = \frac{1}{M_X^2} \langle f | g_{d \rightarrow s} g_{s \rightarrow b}^* Q_4 + g_{b \rightarrow s} g_{s \rightarrow d}^* \tilde{Q}_4 | \bar{B} \rangle = \frac{r_4}{M_X^2} \langle f | Q_1 | \bar{B} \rangle (g_{d \rightarrow s} g_{s \rightarrow b}^* \mp g_{b \rightarrow s} g_{s \rightarrow d}^*), \quad (\text{C.3})$$

where the upper (lower) sign is for a $PP(PV)$ final state. The combination of couplings g_i can be written in terms of the ratios $R, \bar{R}$ defined in (C.2)

$$g_{b \rightarrow s} g_{s \rightarrow d}^* \mp g_{d \rightarrow s} g_{s \rightarrow b}^* = g_{b \rightarrow s} g_{s \rightarrow b}^* \frac{1}{R^*} \mp g_{d \rightarrow s} g_{s \rightarrow d}^* R^* = (g_{d \rightarrow s} g_{s \rightarrow d}^*) \bar{R} \mp (g_{b \rightarrow s} g_{s \rightarrow b}^*) \frac{1}{\bar{R}}. \quad (\text{C.4})$$

The products of coefficients on the r.h.s are now exactly the ones bounded from the meson mixing, eq. (C.1). The absolute value of the l.h.s on the other hand is assumed to be bounded from below from the measurement of $b \rightarrow ss\bar{d}$ branching ratio, cf. eq. (C.3). We then have

$$B^2 < |g_{b \rightarrow s} g_{s \rightarrow d}^* \mp g_{d \rightarrow s} g_{s \rightarrow b}^*|^2 \leq \varepsilon_{sd}^2 |R|^2 + \varepsilon_{bs}^2 \frac{1}{|R|^2} + 2\varepsilon_{sd}\varepsilon_{bs}. \quad (\text{C.5})$$

If $B \geq 2\sqrt{\varepsilon_{sd}\varepsilon_{bs}}$, then the above inequality rules out a range of values for $|R|$,

$$\frac{1}{2\varepsilon_{sd}^2} [B - 2\varepsilon_{sd}\varepsilon_{bs} - \sqrt{B^2 - 4\varepsilon_{sd}\varepsilon_{bs}}] \leq |R|^2 \leq \frac{1}{2\varepsilon_{sd}^2} [B - 2\varepsilon_{sd}\varepsilon_{bs} + \sqrt{B^2 - 4\varepsilon_{sd}\varepsilon_{bs}}]. \quad (\text{C.6})$$

The same bound with $\varepsilon_{sd} \leftrightarrow \varepsilon_{bs}$ holds also for $|\bar{R}|$. The requirement $B \geq 2\sqrt{\varepsilon_{sd}\varepsilon_{bs}}$ corresponds to the requirement that $\mathcal{B}(B \rightarrow f) > 4\mathcal{B}(B \rightarrow f)_{\text{NMFV}}$, with the NMFV predictions for branching ratios given in table 6.

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