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The $B \rightarrow \pi K$ puzzle revisited

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ABSTRACT: For a number of years, there has been a certain inconsistency among the measurements of the branching ratios and CP asymmetries of the four $B \rightarrow \pi K$ decays ($B^+ \rightarrow \pi^+ K^0$, $B^+ \rightarrow \pi^0 K^+$, $B_d^0 \rightarrow \pi^- K^+$, $B_d^0 \rightarrow \pi^0 K^0$). In this paper, we re-examine this $B \rightarrow \pi K$ puzzle. We find that the key unknown parameter is $|C'/T'|$, the ratio of color-suppressed and color-allowed tree amplitudes. If this ratio is large, $|C'/T'| = 0.5$, the SM can explain the data. But if it is small, $|C'/T'| = 0.2$, the SM cannot explain the $B \rightarrow \pi K$ puzzle — new physics (NP) is needed. The two types of NP that can contribute to $B \rightarrow \pi K$ at tree level are Z' bosons and diquarks. Z' models can explain the puzzle if the Z' couples to right-handed $u\bar{u}$ and/or $d\bar{d}$, with $g_R^{dd} \neq g_R^{uu}$. Interestingly, half of the many Z' models proposed to explain the present anomalies in $b \rightarrow s\mu^+\mu^-$ decays have the required Z' couplings to $u\bar{u}$ and/or $d\bar{d}$. Such models could potentially explain both the $b \rightarrow s\mu^+\mu^-$ anomalies and the $B \rightarrow \pi K$ puzzle. The addition of a color sextet diquark that couples to ud can also explain the puzzle.

KEYWORDS: Beyond Standard Model, Heavy Quark Physics, CP violation

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1 Introduction

Over the past 15 years there has been a great deal of interest in $B \rightarrow \pi K$ decays. There are four such processes: $B^+ \rightarrow \pi^+ K^0$ (designated as $+0$ below), $B^+ \rightarrow \pi^0 K^+$ ($0+$), $B_d^0 \rightarrow \pi^- K^+$ ($-+$) and $B_d^0 \rightarrow \pi^0 K^0$ (00). Their amplitudes are not independent, but obey a quadrilateral isospin relation:

$$\sqrt{2}A^{00} + A^{-+} = \sqrt{2}A^{0+} + A^{+0}. \quad (1.1)$$

Using these decays, nine observables have been measured: the four branching ratios, the four direct CP asymmetries A_{CP} , and the mixing-induced indirect CP asymmetry S_{CP} in $B_d^0 \rightarrow \pi^0 K^0$. Shortly after these measurements were first made (in the early 2000s), it was noted that there was an inconsistency among them. This was referred to as the “ $B \rightarrow \pi K$ puzzle” [1–3].

In ref. [4], it was pointed out that, by performing a full fit to the data, one can quantify the discrepancy with the standard model (SM) implied by the $B \rightarrow \pi K$ puzzle. Ref. [4] (2004) contains the results of the first such fit, an update was done in ref. [5] (2007), and the last fit was performed in ref. [6] (2009). This latter study finds that “if one adds a constraint on the weak phase γ coming from independent measurements — the SM fit — one finds that the fit is poor. On the other hand, it is not terrible. If one is willing to accept some deficiencies in the fit, it can be argued that the SM can explain the $B \rightarrow \pi K$ data.” In other words, one cannot say that the $B \rightarrow \pi K$ puzzle suggests the presence of new physics (NP). A more correct statement would be that the measurements of $B \rightarrow \pi K$ decays *allow* for NP.

In the present paper, we update the fit using the latest experimental results. As the data has not changed enormously since 2007, we do not expect to find a different conclusion. However, the first purpose of our study is to be more precise in this conclusion. In what regions of parameter space does the SM yield a reasonable explanation of the $B \rightarrow \pi K$ data? And in what regions is the SM fit to the data terrible, so that NP is definitely required?

There are two types of NP that can contribute at tree level to $B \rightarrow \pi K$. One is a Z' boson that has a flavor-changing coupling to $\bar{s}b$ and also couples to $\bar{u}u$ and/or $\bar{d}d$. The other is a diquark that has db and ds couplings or ub and us couplings. The second goal of our work is to determine what precise couplings the Z' or diquark must have in order to improve the fit with the $B \rightarrow \pi K$ data.

Finally, we make one other observation. Over the past few years, several measurements have been made that disagree with the predictions of the SM. These include (i) R_K (LHCb [7]) and (ii) R_{K^*} (LHCb [8]), where $R_{K,K^*} \equiv \mathcal{B}(B^{+,0} \rightarrow K^{+,*0} \mu^+ \mu^-) / \mathcal{B}(B^{+,0} \rightarrow K^{+,*0} e^+ e^-)$, (iii) the angular distribution of $B \rightarrow K^* \mu^+ \mu^-$ (LHCb [9, 10], Belle [11], ATLAS [12] and CMS [13]), and (iv) the branching fraction and angular distribution of $B_s^0 \rightarrow \phi \mu^+ \mu^-$ (LHCb [14, 15]). Recent analyses of these discrepancies [16–26] combine constraints from all measurements and come to the following conclusions: (i) there is indeed a significant disagreement with the SM, somewhere in the range of $4\text{--}6\sigma$, and (ii) the most probable explanation is that the NP primarily affects $b \rightarrow s \mu^+ \mu^-$ transitions. Arguably the simplest NP explanation is that its contribution to $b \rightarrow s \mu^+ \mu^-$ comes from the tree-level exchange of a Z' boson that has a flavor-changing coupling to $\bar{s}b$, and also couples to $\mu^+ \mu^-$. Many models of this type have been proposed to explain the data. In some of these models, the Z' also has couplings to $\bar{u}u$ and/or $\bar{d}d$. In this case, it will also contribute at tree level to $B \rightarrow \pi K$ decays¹ and could potentially furnish an explanation of the $B \rightarrow \pi K$ puzzle. If so, in such models there would be a connection between the anomalies in $b \rightarrow s \mu^+ \mu^-$ and $B \rightarrow \pi K$ decays, which is quite intriguing.

¹Another class of models involves the tree-level exchange of a leptoquark (LQ). However, LQs cannot contribute to $B \rightarrow \pi K$ at tree level.

In section 2 we review the $B \rightarrow \pi K$ puzzle and update its status. In section 3 we examine whether the SM can explain the puzzle. We find that it can if $|C'/T'| = 0.5$, but not if $|C'/T'| = 0.2$ (both values are allowed theoretically). For the case of $|C'/T'| = 0.2$, in section 4 we examine the puzzle in the presence of NP. We find that the $B \rightarrow \pi K$ puzzle can be explained if the NP is a Z' boson or a diquark. We make the connection with Z' models that explain the $b \rightarrow s\mu^+\mu^-$ anomalies. We conclude in section 5.

2 The $B \rightarrow \pi K$ puzzle

We begin by reviewing the $B \rightarrow \pi K$ puzzle. Within the diagrammatic approach [27, 28], B -decay amplitudes are expressed in terms of six diagrams:² the color-favored and color-suppressed tree amplitudes T' and C' , the gluonic penguin amplitudes P'_{tc} and P'_{uc} , and the color-favored and color-suppressed electroweak penguin amplitudes P'_{EW} and P'_{EW}^C . (The primes on the amplitudes indicate $\bar{b} \rightarrow \bar{s}$ transitions.) The $B \rightarrow \pi K$ decay amplitudes are given in terms of diagrams by

$$\begin{aligned} A^{+0} &= -P'_{tc} + P'_{uc}e^{i\gamma} - \frac{1}{3}P'_{EW}^C, \\ \sqrt{2}A^{0+} &= -T'e^{i\gamma} - C'e^{i\gamma} + P'_{tc} - P'_{uc}e^{i\gamma} - P'_{EW} - \frac{2}{3}P'_{EW}^C, \\ A^{-+} &= -T'e^{i\gamma} + P'_{tc} - P'_{uc}e^{i\gamma} - \frac{2}{3}P'_{EW}^C, \\ \sqrt{2}A^{00} &= -C'e^{i\gamma} - P'_{tc} + P'_{uc}e^{i\gamma} - P'_{EW} - \frac{1}{3}P'_{EW}^C. \end{aligned} \quad (2.1)$$

We have explicitly written the weak-phase dependence (including the minus sign from $V_{tb}^*V_{ts}$ [in P'_{tc}]), so that the diagrams contain both strong phases and the magnitudes of the Cabibbo-Kobayashi-Maskawa (CKM) matrix elements. The amplitudes for the CP-conjugate processes can be obtained from the above by changing the sign of the weak phase γ .

P'_{tc} and P'_{uc} are defined as follows. There are three gluonic penguin contributions P_i , where $i = u, c, t$ indicates the identity of the quark in the loop. The full penguin amplitude is then

$$\begin{aligned} P &= V_{ub}^*V_{us}P'_u + V_{cb}^*V_{cs}P'_c + V_{tb}^*V_{ts}P'_t \\ &= V_{ub}^*V_{us}(P'_u - P'_c) + V_{tb}^*V_{ts}(P'_t - P'_c). \end{aligned} \quad (2.2)$$

The second line arises due to the unitarity of the CKM matrix ($V_{ub}^*V_{us} + V_{cb}^*V_{cs} + V_{tb}^*V_{ts} = 0$), and we define $P'_{tc} \equiv |V_{tb}^*V_{ts}|(P'_t - P'_c)$ and $P'_{uc} \equiv |V_{ub}^*V_{us}|(P'_u - P'_c)$. Since $|V_{ub}^*V_{us}| = O(\lambda^4)$ and $|V_{tb}^*V_{ts}| = O(\lambda^2)$, where $\lambda = 0.22$ is the sine of the Cabibbo angle, $|P'_{uc}| \ll |P'_{tc}|$.

²We neglect the annihilation, exchange and penguin-annihilation diagrams, which are expected to be very small in the SM.

It has been shown [29–31] that, to a good approximation, the diagrams P'_{EW} and P'^C_{EW} can be related to T' and C' within the SM using flavor SU(3) symmetry:

$$\begin{aligned} P'_{EW} &= \frac{3}{4} \frac{c_9 + c_{10}}{c_1 + c_2} R(T' + C') + \frac{3}{4} \frac{c_9 - c_{10}}{c_1 - c_2} R(T' - C'), \\ P'^C_{EW} &= \frac{3}{4} \frac{c_9 + c_{10}}{c_1 + c_2} R(T' + C') - \frac{3}{4} \frac{c_9 - c_{10}}{c_1 - c_2} R(T' - C'). \end{aligned} \quad (2.3)$$

We refer to these as EWP-tree relations. Here, the c_i are Wilson coefficients [32] and $R \equiv |(V_{tb}^* V_{ts})/(V_{ub}^* V_{us})| = 49.1 \pm 1.0$ [33]. Now, $c_{10}/c_9 = c_2/c_1$ holds to a few percent. In this limit, the above EWP-tree relations become

$$P'_{EW} = \frac{3}{2} \frac{c_9}{c_1} R T' \quad , \quad P'^C_{EW} = \frac{3}{2} \frac{c_9}{c_1} R C'. \quad (2.4)$$

Thus, P'_{EW} and T' are roughly the same size, as are P'^C_{EW} and C' .

Taking the above information into account, the relative sizes of all the $B \rightarrow \pi K$ diagrams can be roughly estimated as³

$$1 : |P'_{tc}| \quad , \quad \mathcal{O}(\bar{\lambda}) : |T'|, |P'_{EW}| \quad , \quad \mathcal{O}(\bar{\lambda}^2) : |C'|, |P'_{uc}|, |P'^C_{EW}|, \quad (2.5)$$

where $\bar{\lambda} \sim 0.2$.

2.1 Naive $B \rightarrow \pi K$ puzzle

Neglecting the diagrams of $\mathcal{O}(\bar{\lambda}^2)$ in eq. (2.5), the $B \rightarrow \pi K$ amplitudes become

$$\begin{aligned} A^{+0} &= -P'_{tc}, \\ \sqrt{2}A^{0+} &= -T' e^{i\gamma} + P'_{tc} - P'_{EW}, \\ A^{-+} &= -T' e^{i\gamma} + P'_{tc}, \\ \sqrt{2}A^{00} &= -P'_{tc} - P'_{EW}. \end{aligned} \quad (2.6)$$

With these amplitudes, consider the direct CP asymmetries of $B^+ \rightarrow \pi^0 K^+$ and $B_d^0 \rightarrow \pi^- K^+$. Such CP asymmetries are generated by the interference of two amplitudes with nonzero relative weak and strong phases. In both A^{0+} and A^{-+} , T' - P'_{tc} interference leads to a direct CP asymmetry. On the other hand, in A^{0+} , P'_{EW} and T' have the same strong phase ($P'_{EW} \propto T'$ [eq. (2.4)]), while P'_{EW} and P'_{tc} have the same weak phase ($= 0$), so that P'_{EW} does not contribute to the direct CP asymmetry. This means that we expect $A_{CP}(B^+ \rightarrow \pi^0 K^+) = A_{CP}(B_d^0 \rightarrow \pi^- K^+)$.

The latest $B \rightarrow \pi K$ measurements are shown in table 1. Not only are $A_{CP}(B^+ \rightarrow \pi^0 K^+)$ and $A_{CP}(B_d^0 \rightarrow \pi^- K^+)$ not equal, they are of opposite sign! Experimentally, we have $(\Delta A_{CP})_{\text{exp}} = (12.2 \pm 2.2)\%$. This differs from 0 by 5.5σ . This is the naive $B \rightarrow \pi K$ puzzle.

³These estimates were first given in ref. [28], which predates the derivation of the EWP-tree relations.

Mode	$BR[10^{-6}]$	A_{CP}	S_{CP}
$B^+ \rightarrow \pi^+ K^0$	23.79 ± 0.75	-0.017 ± 0.016	
$B^+ \rightarrow \pi^0 K^+$	12.94 ± 0.52	0.040 ± 0.021	
$B_d^0 \rightarrow \pi^- K^+$	19.57 ± 0.53	-0.082 ± 0.006	
$B_d^0 \rightarrow \pi^0 K^0$	9.93 ± 0.49	-0.01 ± 0.10	0.57 ± 0.17

Table 1. Branching ratios, direct CP asymmetries A_{CP} , and mixing-induced CP asymmetry S_{CP} (if applicable) for the four $B \rightarrow \pi K$ decay modes. The data are taken from ref. [34].

2.2 Statistics

As noted in the Introduction, a more accurate measure of the (dis)agreement with the SM can be obtained by performing a fit to the data. From here on, in looking for SM or NP explanations of the $B \rightarrow \pi K$ puzzle, we will use only fits. In such fits, the $B \rightarrow \pi K$ amplitudes are expressed in terms of a certain number of unknown theoretical parameters. In order to perform a fit, there must be fewer theoretical unknowns than observables. We define χ^2 as

$$\chi^2 = \sum_i \frac{(\mathcal{O}_i^{th} - \mathcal{O}_i^{exp})^2}{(\Delta \mathcal{O}_i)^2}, \quad (2.7)$$

where $\mathcal{O}_i$ are the various observables used as constraints. $\mathcal{O}_i^{exp}$ and $\Delta \mathcal{O}_i$ are, respectively, the experimentally-measured central values and errors. $\mathcal{O}_i^{th}$ are the theoretical predictions for the observables, and are functions of the unknown theoretical parameters. We use the program MINUIT [35–37] to find the values of the unknowns that minimize the χ^2 .

At this point, it is useful to review some basic properties of χ^2 distributions in order to establish what constitutes a good fit. The χ^2 probability distribution depends on a single parameter, n , which is the number of degrees of freedom (d.o.f.). It is given by

$$P(\chi^2) = \frac{1}{2^{n/2} \Gamma(n/2)} (\chi^2)^{(n/2-1)} e^{-\chi^2/2}. \quad (2.8)$$

For n large, this becomes a normal distribution with central value n and standard deviation $\sqrt{2n}$. That is, in this limit the preferred value of $\chi^2/\text{d.o.f.}$ is 1, with an error $\sqrt{2/n}$. This says that, even if we have the correct underlying theory, we still expect $\chi^2/\text{d.o.f.} \simeq 1$, just due to statistical fluctuations. For this reason, it is common to say that, if we find $\chi_{\min}^2/\text{d.o.f.} \simeq 1$ in a fit, it is acceptable.

We stress that this only holds for n large — it is not justified to apply the same criterion for small values of n . One way to see this is to compute the p-value. For n large, the p-value corresponding to $\chi^2/\text{d.o.f.} = 1$ is 50%. That is, a p-value of $\simeq 0.5$ constitutes an acceptable fit. However, here are the p-values corresponding to $\chi^2/\text{d.o.f.} = 1$ for smaller

$\chi^2_{\min}/\text{d.o.f.} = 30.9/6,$ $\text{p-value} = 3.0 \times 10^{-5}$	
Parameter	Best-fit value
γ	$(67.2 \pm 4.7)^\circ$
β	$(21.80 \pm 0.68)^\circ$
$ T' $	7.0 ± 1.4
$ P'_{tc} $	50.5 ± 0.6
$\delta_{P'_{tc}} - \delta_{T'}$	$(-15.6 \pm 3.4)^\circ$

Table 2. $\chi^2_{\min}/\text{d.o.f.}$ and best-fit values of unknown parameters in amplitudes of eq. (2.6). Constraints: $B \rightarrow \pi K$ data, measurements of β and γ .

values of n :

$$\begin{aligned}
n = 1 : \quad & \text{p-value} = 0.32, \\
n = 2 : \quad & \text{p-value} = 0.37, \\
n = 3 : \quad & \text{p-value} = 0.39, \\
n = 5 : \quad & \text{p-value} = 0.42, \\
n = 10 : \quad & \text{p-value} = 0.44.
\end{aligned} \tag{2.9}$$

These reflect the fact that, for n small, the central value of the distribution is not at $\chi^2/\text{d.o.f.} = 1$ — it is at smaller values. This shows that, if $n = 1$, though $\chi^2/\text{d.o.f.} = 1$ is not a bad fit, it is still somewhat less than acceptable (which corresponds to a p-value of $\simeq 0.5$). It also suggests that p-values are easier than $\chi^2/\text{d.o.f.}$ for judging the goodness-of-fit when n is small.

2.3 True $B \rightarrow \pi K$ puzzle

Taking into account the first EWP-tree relation of eq. (2.4), the amplitudes of eq. (2.6) depend on four unknown parameters: the magnitudes $|T'|$ and $|P'_{tc}|$, one relative strong phase, and the weak phase γ . In addition, the indirect CP asymmetry in $B_d^0 \rightarrow \pi^0 K^0$, S_{CP} , depends on the weak phase β . These parameters are constrained by the $B \rightarrow \pi K$ data of table 1, as well as by the independent measurements of the weak phases [33]:

$$\beta = (21.85 \pm 0.68)^\circ, \quad \gamma = (72.1 \pm 5.8)^\circ. \tag{2.10}$$

With more observables (11) than theoretical unknowns (5), a fit can be performed. The results are shown in table 2. Unsurprisingly, we find a terrible fit: $\chi^2_{\min}/\text{d.o.f.} = 30.9/6$, corresponding to a p-value of 3.0×10^{-5} . This can be considered the true $B \rightarrow \pi K$ puzzle.

3 SM fits

In the previous section, we saw that the SM cannot account for the $B \rightarrow \pi K$ data if the small diagrams C' , P'_{uc} and P'_{EW} are neglected. Thus, in order to test whether the data can

be explained by the SM, the small diagrams must be included in the fit. With the EWP-tree relations of eq. (2.3), there are only four independent diagrams in the $B \rightarrow \pi K$ amplitudes:⁴ T' , C' , P'_{tc} and P'_{uc} . This corresponds to 8 unknown parameters: four magnitudes of diagrams, three relative strong phases, and the weak phase γ . (As before, the value of β , which is required for S_{CP} , can be constrained from independent measurements. This is done in all fits.) We therefore have more observables (9) than theoretical unknowns (8), so that a fit can be done. Additional constraints can come from the independent measurement of γ and/or theoretical input. In this section we perform various SM fits in order to determine under what circumstances the SM can explain the $B \rightarrow \pi K$ puzzle.

Regarding the strong phases, these are mainly generated by QCD rescattering from another diagram with the same CKM matrix elements. As an example, take P'_c in eq. (2.2). It can arise directly via a gluonic penguin amplitude with a c quark in the loop, or it can be generated by rescattering from the tree operator $\bar{b} \rightarrow \bar{s}c\bar{c}$:

$$(P'_c)_{\text{tot}} = (P'_c)_{\text{dir}} + (P'_c)_{\text{rescatt}}. \quad (3.1)$$

Only the rescattered contribution has a substantial strong phase. Now, the tree diagram is much larger than the penguin diagram. But rescattering comes with a cost: the rescattered penguin is only about 5-10% as large as the tree. Indeed it is of the same order as $(P'_c)_{\text{dir}}$. The net effect is that $(P'_c)_{\text{tot}}$ can have a sizeable strong phase. On the other hand, the strong phase of T' can only arise due to self-rescattering. Since this self-rescattered amplitude is only about 5-10% as large as the original amplitude, the strong phase of T' is expected to be small. The bottom line is that $\delta_{T'}$ should be small, while the other strong phases can be large. In the fits we adopt the convention that $\delta_{T'} = 0$. (In any case, only relative strong phases are measurable.)

3.1 All diagrams free, constraint on γ added

In this fit, we keep all diagrams and allow their values to vary, but constrain γ by including the independent measurement of eq. (2.10) (β is always constrained in this way.) The results of the fit are given in table 3. We see that the p-value is only 17%, which is below the 50% required for an acceptable fit. More importantly, the best fit has $|C'/T'| = 0.75 \pm 0.32$, which is considerably larger than the estimate in eq. (2.5).

Even though the fit is only fair, it is clear that the SM fit prefers a large value of $|C'/T'|$. But this raises the question: what does theory predict for $|C'/T'|$?

1. In ref. [39], $B \rightarrow \pi K$ decays were analyzed in the context of QCD factorization (QCdf). The various NLO contributions were computed for three different values of the renormalization scale, $\mu = m_b/2$, m_b and $2m_b$. In all three cases it was found that $|C'/T'| \simeq 0.2$.
2. The NNLO corrections within QCdf have been considered in refs. [40–43]. Including these corrections, it is found [44] that $0.13 \leq |C'/T'| \leq 0.43$, with a central value of $|C'/T'| = 0.23$, very near its NLO value.

⁴The EWP-tree relations were first applied to $B \rightarrow \pi K$ decays in ref. [38], which predates discussions of the $B \rightarrow \pi K$ puzzle.

SM fit (2): $\chi^2/\text{d.o.f.} = 3.5/2$, p-value = 0.17	
Parameter	Best-fit value
γ	$(72.0 \pm 5.8)^\circ$
β	$(21.85 \pm 0.68)^\circ$
$ T' $	5.2 ± 1.5
$ C' $	3.9 ± 1.2
$ P'_{tc} $	50.7 ± 0.9
$ P'_{uc} $	1.1 ± 2.4
$\delta_{C'}$	$(209.8 \pm 21.3)^\circ$
$\delta_{P'_{tc}}$	$(-16.2 \pm 7.3)^\circ$
$\delta_{P'_{uc}}$	$(4.9 \pm 51.3)^\circ$

Table 3. $\chi^2_{\min}/\text{d.o.f.}$ and best-fit values of unknown parameters in amplitudes of eq. (2.1). Constraints: $B \rightarrow \pi K$ data, measurements of β and γ .

- Ref. [45] does NNLO calculations within perturbative QCD (pQCD), and finds $|C'/T'| = 0.53$. We note that a range of values is not given, which suggests that this result is for a specific choice of the theoretical parameters. It is not clear what the smallest allowed value of $|C'/T'|$ is within pQCD.

It therefore appears that $|C'/T'|$ can be as large as ~ 0.5 . But it may also be the case that $|C'/T'|$ is quite a bit smaller. In light of this, below we repeat the SM fit, taking as theoretical input $|C'/T'| = 0.2$ or 0.5 .

There is one more thing. In eq. (2.5), it is estimated that $|P'_{uc}/P'_{tc}| = \mathcal{O}(\bar{\lambda}^2)$. However, it has been argued that $|P'_{uc}|$ is actually even smaller: $|P'_{uc}/P'_{tc}| = \mathcal{O}(\bar{\lambda}^3)$ [46], so that it can be neglected, to a good approximation. Indeed, in table 3 P'_{uc} is the smallest diagram. Therefore, from now on we will also add the theoretical input $P'_{uc} = 0$. Note that this is the most favorable assumption for the SM: it increases the d.o.f., and hence the p-value, of a fit. If we find that a particular SM fit is poor, it would be even worse had we allowed P'_{uc} to vary.

3.2 $|C'/T'| = 0.2$, $P'_{uc} = 0$, constraint on γ added

We now perform the same fit as above, but add the theoretical constraints $|C'/T'| = 0.2$ and $P'_{uc} = 0$. The results of the fit are given in table 4. The situation is better than in table 2, but we still have a poor fit: $\chi^2_{\min}/\text{d.o.f.} = 12.1/5$, corresponding to a p-value of 3%. This demonstrates conclusively that, if $|C'/T'| = 0.2$, the $B \rightarrow \pi K$ puzzle cannot be explained by the SM.

$\chi^2_{\min}/\text{d.o.f.} = 12.1/5,$ p-value = 0.03	
Parameter	Best-fit value
γ	$(67.2 \pm 4.6)^\circ$
β	$(21.80 \pm 0.68)^\circ$
$ T' $	7.9 ± 1.2
$ P'_{tc} $	50.7 ± 0.6
$\delta_{P'_{tc}}$	$(346.5 \pm 2.6)^\circ$
$\delta_{C'}$	$(253.1 \pm 23.5)^\circ$

Table 4. $\chi^2_{\min}/\text{d.o.f.}$ and best-fit values of unknown parameters in amplitudes of eq. (2.1). Constraints: $B \rightarrow \pi K$ data, measurements of β and γ , theoretical inputs $|C'/T'| = 0.2$, $P'_{uc} = 0$.

$\chi^2_{\min}/\text{d.o.f.} = 4.9/5,$ p-value = 0.43	
Parameter	Best-fit value
γ	$(70.6 \pm 5.3)^\circ$
β	$(21.82 \pm 0.68)^\circ$
$ T' $	6.2 ± 0.9
$ P'_{tc} $	50.5 ± 0.5
$\delta_{P'_{tc}}$	$(162.4 \pm 3.5)^\circ$
$\delta_{C'}$	$(42.8 \pm 18.1)^\circ$

Table 5. $\chi^2_{\min}/\text{d.o.f.}$ and best-fit values of unknown parameters in amplitudes of eq. (2.1). Constraints: $B \rightarrow \pi K$ data, measurements of β and γ , theoretical inputs $|C'/T'| = 0.5$, $P'_{uc} = 0$.

3.3 $|C'/T'| = 0.5$, $P'_{uc} = 0$, constraints on γ added

We now perform the SM fit, but with the theoretical constraints $|C'/T'| = 0.5$ and $P'_{uc} = 0$. The results of the fit are given in table 5. We find $\chi^2_{\min}/\text{d.o.f.} = 4.9/5$, for a p-value of 43%, which is an acceptable fit. We therefore conclude that, if $|C'/T'| = 0.5$, there is no $B \rightarrow \pi K$ puzzle — the data can be explained by the SM.

It is interesting to compare the results of this fit with those in table 3. By $\chi^2_{\min}$, this fit is worse, since $(\chi^2_{\min})_{\text{table 5}} > (\chi^2_{\min})_{\text{table 3}}$. On the other hand, by p-value it is better. The reason is that the number of d.o.f. has changed, so that $(\chi^2_{\min}/\text{d.o.f.})_{\text{table 5}} < (\chi^2_{\min}/\text{d.o.f.})_{\text{table 3}}$. This type of behavior often occurs when the number of d.o.f. is small, making it difficult to judge which of two fits is truly better.

Note that the strong phase $\delta_{C'}$ is produced mainly by rescattering from T' . Thus, for a large value of $|C'/T'|$, $\delta_{C'}$ should be on the small side. And indeed, from table 5, we see that $\delta_{C'}$ is not that large.

$\chi^2_{\min}/\text{d.o.f.} = 4.3/4,$ $\text{p-value} = 0.36$	
Parameter	Best-fit value
γ	$(51.2 \pm 5.1)^\circ$
β	$(21.78 \pm 0.68)^\circ$
$ T' $	10.1 ± 3.4
$ P'_{tc} $	51.8 ± 1.0
$\delta_{P'_{tc}}$	$(168.6 \pm 4.6)^\circ$
$\delta_{C'}$	$(131.2 \pm 24.7)^\circ$

Table 6. $\chi^2_{\min}/\text{d.o.f.}$ and best-fit values of unknown parameters in amplitudes of eq. (2.1). Constraints: $B \rightarrow \pi K$ data, measurement of β , theoretical inputs $|C'/T'| = 0.5$, $P'_{uc} = 0$.

Now, in this fit γ is constrained by its independently-measured value. However, this is not necessary — γ can be treated as an unknown parameter and extracted from the measurements of $B \rightarrow \pi K$ decays. Indeed, if the SM explains the $B \rightarrow \pi K$ data, we would expect the extracted value of γ to be the same as that measured in tree-level decays. We investigate this in the following subsection.

3.4 $|C'/T'| = 0.5$, $P'_{uc} = 0$, γ free

Here we repeat the fit of the previous subsection, but remove the constraint from the independent measurement of γ . The results are given in table 6. By $\chi^2_{\min}$, this fit is better than that of table 5. However, by p-value it is worse. This is a result of the different d.o.f. in the two fits. More importantly, it is found that $\gamma = (51.2 \pm 5.1)^\circ$, which deviates from its measured value of $(72.1 \pm 5.8)^\circ$ by 2.7σ . So this is a reason not to be entirely satisfied that the SM explains the $B \rightarrow \pi K$ puzzle, even if $|C'/T'| = 0.5$.

3.5 Summary

If $|C'/T'| = 0.2$, we have found that the SM cannot explain the $B \rightarrow \pi K$ puzzle. Even in the most optimistic scenario, where P'_{uc} is neglected, the p-value is only 3%, constituting a poor fit. On the other hand, if $|C'/T'| = 0.5$ and $P'_{uc} = 0$, the p-value is 43%, which is acceptable. There is still an aspect of this fit that is not entirely satisfactory, but on the whole we find that, in this case, the SM can explain the $B \rightarrow \pi K$ data.

We therefore conclude that it is the size of $|C'/T'|$ that determines whether or not there is truly a $B \rightarrow \pi K$ puzzle. If $|C'/T'| = 0.5$, which is theoretically its maximally-allowed value, the SM can explain the $B \rightarrow \pi K$ data. However, if $|C'/T'| = 0.2$, which is towards the lower end of its theoretically-allowed range, then the SM cannot explain the data, and NP is required. In this case, it is natural to investigate what type of NP is required. We do this in the next section.

4 NP fits

In this section we assume $|C'/T'| = 0.2$ and examine whether the $B \rightarrow \pi K$ puzzle can be explained with the addition of new physics.

4.1 Model-independent formalism

In the general approach of ref. [47], the NP operators that contribute to the $B \rightarrow \pi K$ amplitudes take the form $\mathcal{O}_{NP}^{ij,q} \sim \bar{s}\Gamma_i b \bar{q}\Gamma_j q$ ($q = u, d$), where $\Gamma_{i,j}$ represent Lorentz structures, and color indices are suppressed. The NP contributions to $B \rightarrow \pi K$ are encoded in the matrix elements $\langle \pi K | \mathcal{O}_{NP}^{ij,q} | B \rangle$. In general, each matrix element has its own NP weak and strong phases.

Now, above we noted that strong phases are basically generated by QCD rescattering from diagrams with the same CKM matrix elements. We then argued that the strong phase of T' is expected to be very small since it is due to self-rescattering. For the same reason, all NP strong phases are also small, and can be neglected. In this case, many NP matrix elements can be combined into a single NP amplitude, with a single weak phase:

$$\sum \langle \pi K | \mathcal{O}_{NP}^{ij,q} | B \rangle = \mathcal{A}^q e^{i\Phi_q}. \quad (4.1)$$

Here the strong phase is zero. There are two classes of such NP amplitudes, differing only in their color structure: $\bar{s}_\alpha \Gamma_i b_\alpha \bar{q}_\beta \Gamma_j q_\beta$ and $\bar{s}_\alpha \Gamma_i b_\beta \bar{q}_\beta \Gamma_j q_\alpha$ ($q = u, d$). They are denoted $\mathcal{A}'^q e^{i\Phi'_q}$ and $\mathcal{A}'^{C,q} e^{i\Phi_q'^C}$, respectively [48]. Here, Φ'_q and $\Phi_q'^C$ are the NP weak phases. In general, $\mathcal{A}'^q \neq \mathcal{A}'^{C,q}$ and $\Phi'_q \neq \Phi_q'^C$. Note that, despite the “color-suppressed” index C , the matrix elements $\mathcal{A}'^{C,q} e^{i\Phi_q'^C}$ are not necessarily smaller than $\mathcal{A}'^q e^{i\Phi'_q}$.

There are therefore four NP matrix elements that contribute to $B \rightarrow \pi K$ decays. However, only three combinations appear in the amplitudes: $\mathcal{A}'^{\text{comb}} e^{i\Phi'} \equiv -\mathcal{A}'^u e^{i\Phi'_u} + \mathcal{A}'^d e^{i\Phi'_d}$, $\mathcal{A}'^{C,u} e^{i\Phi_u'^C}$, and $\mathcal{A}'^{C,d} e^{i\Phi_d'^C}$ [48]. The $B \rightarrow \pi K$ amplitudes can now be written in terms of the SM diagrams and these NP matrix elements. Here we neglect the small SM diagram P'_{uc} :

$$\begin{aligned} A^{+0} &= -P'_{tc} - \frac{1}{3}P'_{EW}^C + \mathcal{A}'^{C,d} e^{i\Phi_d'^C}, \\ \sqrt{2}A^{0+} &= P'_{tc} - T' e^{i\gamma} - P'_{EW} - C' e^{i\gamma} - \frac{2}{3}P'_{EW}^C + \mathcal{A}'^{\text{comb}} e^{i\Phi'} - \mathcal{A}'^{C,u} e^{i\Phi_u'^C}, \\ A^{-+} &= P'_{tc} - T' e^{i\gamma} - \frac{2}{3}P'_{EW}^C - \mathcal{A}'^{C,u} e^{i\Phi_u'^C}, \\ \sqrt{2}A^{00} &= -P'_{tc} - P'_{EW} - C' e^{i\gamma} - \frac{1}{3}P'_{EW}^C + \mathcal{A}'^{\text{comb}} e^{i\Phi'} + \mathcal{A}'^{C,d} e^{i\Phi_d'^C}. \end{aligned} \quad (4.2)$$

In ref. [6], a different set of NP operators is defined:

$$\begin{aligned} P'_{EW,NP} e^{i\Phi'_{EW}} &\equiv \mathcal{A}'^u e^{i\Phi'_u} - \mathcal{A}'^d e^{i\Phi'_d}, \\ P'_{NP} e^{i\Phi'_P} &\equiv \frac{1}{3}\mathcal{A}'^{C,u} e^{i\Phi_u'^C} + \frac{2}{3}\mathcal{A}'^{C,d} e^{i\Phi_d'^C}, \\ P'_{EW,NP}^C e^{i\Phi_{EW}'^C} &\equiv \mathcal{A}'^{C,u} e^{i\Phi_u'^C} - \mathcal{A}'^{C,d} e^{i\Phi_d'^C}. \end{aligned} \quad (4.3)$$

In order, these imply the inclusion of NP in the color-allowed electroweak penguin, the gluonic penguin, and the color-suppressed electroweak penguin amplitudes. Of course, the two sets of NP operators, $\{\mathcal{A}'^{\text{comb}}e^{i\Phi'}, \mathcal{A}'^{C,u}e^{i\Phi'_u{}^C}, \mathcal{A}'^{C,d}e^{i\Phi'_d{}^C}\}$ and $\{P'_{EW,NP}, P'_{NP}, P'^C_{EW,NP}\}$, are equivalent. Depending on the situation, one set or the other may be used.

In the most general case, there are three independent NP operators that contribute to $B \rightarrow \pi K$ decays. Although their strong phases are negligible, they each have their own weak phase. There are therefore 12 parameters in the $B \rightarrow \pi K$ amplitudes: 6 magnitudes of diagrams, 2 relative strong phases, 3 NP weak phases, and the CKM phase γ . As before, the CKM phase β also contributes to the indirect CP asymmetry in $B_d^0 \rightarrow \pi^0 K^0$, so that there are a total of 13 unknown parameters. However, there are only 12 constraints: the 9 $B \rightarrow \pi K$ observables, the independent measurements of β and γ , and the theoretical input $|C'/T'| = 0.2$. With more unknowns than constraints, a fit cannot be done. However, this may be improved in the context of a specific model. There may be fewer unknown parameters, or they may not all be independent. We examine this possibility in the following subsections.

4.2 Z' models

For $B \rightarrow \pi K$, the relevant decay is $b \rightarrow s\bar{q}q$ ($q = u, d$). This can occur via the tree-level exchange of a Z' that has a flavor-changing coupling to $\bar{s}b$ and also couples to $\bar{q}q$. The exact form of the $\bar{s}bZ'$ coupling is unimportant, and for the light quarks, all four currents $g_{L(R)}^{qq} \bar{q}\gamma^\mu P_{L(R)}q$ are possible. Assuming the weak and mass eigenstates are the same (i.e., we neglect the off-diagonal terms in the CKM matrix), the only thing that is certain is that $g_L^{dd} = g_L^{uu}$ due to $\text{SU}(2)_L$ symmetry.

Still, how the Z' couples to $\bar{d}d$ and $\bar{u}u$ has direct consequences for the NP operators:

1. Suppose that the Z' couples only to left-handed $\bar{d}d$ and $\bar{u}u$, i.e., $g_R^{dd} = g_R^{uu} = 0$. Since $g_L^{dd} = g_L^{uu}$, the NP operators $\mathcal{A}'^{d,d}e^{i\Phi'_d}$ and $\mathcal{A}'^{u,u}e^{i\Phi'_u}$ are equal, as are $\mathcal{A}'^{C,d}e^{i\Phi'_d{}^C}$ and $\mathcal{A}'^{C,u}e^{i\Phi'_u{}^C}$. In this case, $P'_{EW,NP} = P'^C_{EW,NP} = 0$ in eq. (4.3); the only nonzero NP operator is P'_{NP} . This also holds if the Z' couples vectorially to $\bar{d}d$ and $\bar{u}u$, in which case $g_L^{dd} = g_R^{dd} = g_L^{uu} = g_R^{uu}$.
2. On the other hand, if g_R^{dd} and g_R^{uu} are nonzero, but $g_R^{uu} = -2g_R^{dd}$, then $P'_{NP} = 0$, but $P'_{EW,NP}$ and $P'^C_{EW,NP}$ are nonzero.
3. Alternatively, if only g_R^{dd} is nonzero, $\mathcal{A}'^{\text{comb}}e^{i\Phi'}$ and $\mathcal{A}'^{C,d}e^{i\Phi'_d{}^C}$ are nonzero (equivalently, $P'_{EW,NP}$ is nonzero and $P'_{NP}e^{i\Phi'_P} = -(2/3)P'^C_{EW,NP}e^{i\Phi'_{EW}{}^C}$).
4. Similarly, if only g_R^{uu} is nonzero, $\mathcal{A}'^{\text{comb}}e^{i\Phi'}$ and $\mathcal{A}'^{C,u}e^{i\Phi'_u{}^C}$ are nonzero (equivalently, $P'_{EW,NP}$ is nonzero and $P'_{NP}e^{i\Phi'_P} = (1/3)P'^C_{EW,NP}e^{i\Phi'_{EW}{}^C}$).
5. For all other choices of couplings, all three NP operators are nonzero.

The point is that, although we consider a specific NP model, it has many variations, so that its study includes a number of different NP scenarios. Below we consider each of the cases above (which are identified by their number in the list).

Note also that, for this particular kind of NP, we do naively expect the color-suppressed operators to be smaller than the color-allowed ones.

The four-fermion operator corresponding to $b \rightarrow s\bar{q}q$ is proportional to $g_L^{bs}g_{L(R)}^{qq}$. Note that $g_{L(R)}^{qq}$ must be real, since the light-quark current $\bar{q}\gamma^\mu P_{L(R)}q$ is self-conjugate. However, g_L^{bs} can be complex, i.e., it can contain a weak phase. This is the only source of CP violation in the model and, as it appears in all NP operators, the weak phases of all operators are equal, i.e., there is only a single NP weak phase. That is, $\Phi'_d = \Phi'_u = \Phi_d^{'C} = \Phi_u^{'C} \equiv \Phi'$ for $\{\mathcal{A}'^{\text{comb}}e^{i\Phi'}, \mathcal{A}'^{C,u}e^{i\Phi_u^{'C}}, \mathcal{A}'^{C,d}e^{i\Phi_d^{'C}}\}$ and $\Phi'_{EW} = \Phi'_P = \Phi_{EW}^{'C} \equiv \Phi'$ for $\{P'_{EW,NP}, P'_{NP}, P_{EW,NP}^{'C}\}$.

Now, Z' models with a flavor-changing coupling to $\bar{s}b$ also contribute to $B_s^0\text{--}\bar{B}_s^0$ mixing. The larger g_L^{bs} is, the more Z' models contribute to — and receive constraints from — this mixing. In particular, the phase of $B_s^0\text{--}\bar{B}_s^0$ mixing has been measured to be quite small: $\varphi_s^{c\bar{c}s} = -0.030 \pm 0.033$ [34]. Thus, if g_L^{bs} is big, its phase, which is the NP weak phase Φ' in $B \rightarrow \pi K$, must be small. Φ' can be large only if g_L^{bs} is small, which means $g_{L(R)}^{qq}$ is big. (This type of argument was first made in ref. [49].)

4.2.1 Case (5): all three NP operators nonzero

We begin with the general case, in which all three NP operators are nonzero. Since all three NP weak phases are equal in the Z' model, there are 11 unknown parameters. However, there are 12 constraints — the 9 $B \rightarrow \pi K$ observables, the independent measurements of β and γ , and the theoretical input $|C'/T'| = 0.2$ — so a fit can be done.

The results are shown in table 7 (left-hand table). With $\chi^2/\text{d.o.f.} = 0.41/1$ and a p-value of 52%, this is an excellent fit. However, there is a serious problem: the best fit has $P_{EW,NP}^{'C}/P'_{EW,NP} = 16$, whereas, in the Z' model, the color-suppressed NP operators are expected to be smaller than the color-allowed ones. We therefore conclude that this result cannot arise within a Z' NP model.

To focus on a possibility consistent with a Z' NP model, we impose an additional (theoretical) constraint: $|P_{EW,NP}^{'C}/P'_{EW,NP}| = 0.3$. The results of this fit are shown in table 7 (right-hand table). Here $\chi^2/\text{d.o.f.} = 1.85/2$, for a p-value of 40%, which is a good fit. Of course, this good p-value is a consequence of the fact that, with the constraint, the d.o.f. has increased from 1 to 2. If we consider instead d.o.f. = 1 (which essentially corresponds to the region in the space of the fit of table 7 (left-hand table) with $|P_{EW,NP}^{'C}/P'_{EW,NP}| = 0.3$), the p-value is 17%. Although far from an excellent fit, it is still better than that of the SM.

In the above fits, all NP operators are allowed. In the following, we examine whether a better fit can be found if the model contains only a subset of the operators. Clearly the minimum χ^2 will be larger than that found above, but since the d.o.f. will also be larger, a larger p-value may be found, indicative of a better fit.

4.2.2 Case (1): only P'_{NP} nonzero

The case where only P'_{NP} is nonzero arises when the Z' couplings to right-handed d and u quarks obey $g_R^{dd} = g_R^{uu}$ (these can both vanish or be nonzero). Since $g_L^{dd} = g_L^{uu}$ by weak isospin invariance, $\mathcal{A}'^{d,d}e^{i\Phi'_d} = \mathcal{A}'^{u,u}e^{i\Phi'_u}$ and $\mathcal{A}'^{C,d}e^{i\Phi_d^{'C}} = \mathcal{A}'^{C,u}e^{i\Phi_u^{'C}}$, so that

NP fit (5): $\chi^2/\text{d.o.f.} = 0.41/1$, p-value = 0.52		NP fit (5): $\chi^2/\text{d.o.f.} = 1.85/2$, p-value = 0.4	
Parameter	Best-fit value	Parameter	Best-fit value
γ	$(72.1 \pm 5.8)^\circ$	γ	$(70.8 \pm 5.3)^\circ$
β	$(21.85 \pm 0.68)^\circ$	β	$(21.80 \pm 0.68)^\circ$
Φ'	$(77.5 \pm 20.1)^\circ$	Φ'	$(34.7 \pm 12.8)^\circ$
$ T' $	19.8 ± 4.3	$ T' $	17.0 ± 8.6
$ P'_{tc} $	49.7 ± 0.9	$ P'_{tc} $	60.8 ± 10.5
P'_{NP}	7.4 ± 1.4	P'_{NP}	14.9 ± 13.7
$P'_{EW,NP}$	1.3 ± 8.6	$P'_{EW,NP}$	11.6 ± 8.5
$P'^C_{EW,NP}$	20.9 ± 4.2	$P'^C_{EW,NP}$	3.5 ± 2.7
$\delta_{P'_{tc}}$	$(257.4 \pm 7.1)^\circ$	$\delta_{P'_{tc}}$	$(177.9 \pm 2.1)^\circ$
$\delta_{C'}$	$(91.6 \pm 126.0)^\circ$	$\delta_{C'}$	$(64.1 \pm 58.3)^\circ$

Table 7. $\chi^2_{\min}/\text{d.o.f.}$ and best-fit values of unknown parameters for the Z' model where all three NP operators are present in $B \rightarrow \pi K$. Left-hand table has constraints: $B \rightarrow \pi K$ data, measurements of β and γ , $|C'/T'| = 0.2$. Right-hand table has constraints: $B \rightarrow \pi K$ data, measurements of β and γ , $|C'/T'| = 0.2$, and $|P'^C_{EW,NP}/P'_{EW,NP}| = 0.3$.

$P'_{EW,NP} = P'^C_{EW,NP} = 0$. In model-building terms, this corresponds to the case where the Z' couples only to left-handed $\bar{d}d$ and $\bar{u}u$, or where it couples vectorially to these quark pairs.

Assuming that only P'_{NP} is nonzero, we note that all four $B \rightarrow \pi K$ amplitudes of eq. (4.2) contain the following combination: $P'_{tc} - P'_{NP}e^{i\Phi'_P}$. Writing $P'_{tc} = |P'_{tc}|e^{i\delta_{P'_{tc}}}$, this contains the four quantities $|P'_{tc}|$, $\delta_{P'_{tc}}$, P'_{NP} and Φ'_P . However, here these are not all independent. One can see this by noting that the combinations that appear in B and $\bar{B}$ decays are:

$$\begin{aligned}
 |P'_{tc}|e^{i\delta_{P'_{tc}}} - P'_{NP}e^{i\Phi'_P} &\equiv z, \\
 |P'_{tc}|e^{i\delta_{P'_{tc}}} - P'_{NP}e^{-i\Phi'_P} &\equiv z'.
 \end{aligned}
 \tag{4.4}$$

z and z' are complex numbers; their four real and imaginary parts can be written in terms of the four theoretical parameters. However, it is clear from the above expressions that $\text{Re}(z) = \text{Re}(z')$.

In order to take this into account, we use $\text{Re}(z)$, $\text{Im}(z)$ and $\text{Im}(z')$ as unknown parameters in the fit. The results are shown in table 8.

With $\chi^2_{\min}/\text{d.o.f.} = 3.5/1$ and a p-value of 6%, this is a poor fit, despite the addition of NP. But we can understand what's going on here. We have used $\text{Re}(z)$, $\text{Im}(z)$ and $\text{Im}(z')$ as unknown parameters. Referring to eq. (4.4), we see that, if $P'_{NP} = 0$, $\text{Im}(z) = \text{Im}(z')$. But this is essentially what is found in table 8. That is, with this particular type of NP, we cannot do better than the SM. (This was also the conclusion of ref. [6].) Indeed, comparing

NP fit (1): $\chi^2/\text{d.o.f.} = 3.5/1$, p-value = 0.06	
Parameter	Best-fit value
γ	$(72.0 \pm 5.9)^\circ$
β	$(21.85 \pm 0.68)^\circ$
$ T' $	5.2 ± 1.5
$\text{Re}(z)$	-48.3 ± 2.0
$\text{Im}(z)$	15.4 ± 6.9
$\text{Im}(z')$	13.0 ± 9.0
$ C' $	3.9 ± 1.2
$ P'_{uc} $	0.2 ± 10.6
$\delta_{C'}$	$(29.8 \pm 21.3)^\circ$
$\delta_{P'_{uc}}$	$(333 \pm 360)^\circ$

Table 8. $\chi^2_{\min}/\text{d.o.f.}$ and best-fit values of unknown parameters [z and z' are defined in eq. (4.4)] for the Z' model where NP is present in $B \rightarrow \pi K$, but only P'_{NP} is nonzero. Constraints: $B \rightarrow \pi K$ data, measurements of β and γ .

with table 3, we see that the results are very similar. In particular, the $\chi^2_{\min}$ is identical (the p-values are different due to the different d.o.f.).

We therefore see that the only way for this NP to improve on the SM is if the Z' couplings to right-handed d and u quarks obey $g_R^{dd} \neq g_R^{uu}$.

4.2.3 Cases (3), (2), (4)

We begin with case (3), where only g_R^{dd} is nonzero. Now $P'_{EW,NP}$ is nonzero and $P'_{NP} = -(2/3)P'_{EW,NP}^C$. We also impose the constraint $|P'_{EW,NP}^C/P'_{EW,NP}| = 0.3$. The results of the fit for this case are shown in table 9. Here the p-value is 30%, which is not bad (and is far better than that of the SM).

One important question is: what values of the Z' mass and couplings are required to explain the $B \rightarrow \pi K$ puzzle? This can be deduced from table 9. The SM T' diagram involves the tree-level decay $\bar{b} \rightarrow \bar{u}W^{+*}(\rightarrow u\bar{s} = K^+)$. The NP $P'_{EW,NP}$ diagram looks very similar — we have the tree-level decay $\bar{b} \rightarrow \bar{s}Z'^*(\rightarrow d\bar{d} = \pi^0/\sqrt{2})$. Within factorization, the SM and NP diagrams involve $A_{\pi K} \equiv F_0^{B \rightarrow \pi}(0)f_K$ and $A_{K\pi} \equiv F_0^{B \rightarrow K}(0)f_\pi$, respectively, where $F_0^{B \rightarrow K,\pi}(0)$ are form factors and $f_{\pi,K}$ are decay constants. The hadronic factors are similar in size: $|A_{K\pi}/A_{\pi K}| = 0.9 \pm 0.1$ [39]. Taking central values, we have

$$\begin{aligned}
 \left| \frac{P'_{EW,NP}}{T'} \right| &\simeq \frac{A_{K\pi}|g_L^{bs}g_R^{dd}|/M_{Z'}^2}{A_{\pi K}(G_F/\sqrt{2})|V_{ub}^*V_{us}|} = \frac{14.8}{22.1} \\
 \Rightarrow \frac{|g_L^{bs}g_R^{dd}|}{M_{Z'}^2} &= 5.6 \times 10^{-3} \text{ TeV}^{-2}.
 \end{aligned} \tag{4.5}$$

NP fit (3): $\chi^2/\text{d.o.f.} = 3.67/3$, p-value = 0.30	
Parameter	Best-fit value
γ	$(68.1 \pm 3.7)^\circ$
β	$(21.80 \pm 0.68)^\circ$
Φ'	$(29.0 \pm 12.4)^\circ$
$ T' $	22.1 ± 10.7
$ P'_{tc} $	53.3 ± 2.0
$P'_{EW,NP}$	14.8 ± 9.3
$P'^C_{EW,NP}$	4.2 ± 2.9
$\delta_{P'_{tc}}$	$(176.5 \pm 2.4)^\circ$
$\delta_{C'}$	$(42.5 \pm 28.9)^\circ$

Table 9. $\chi^2_{\min}/\text{d.o.f.}$ and best-fit values of unknown parameters for the Z' model where only g_R^{dd} is nonzero. Constraints: $B \rightarrow \pi K$ data, measurements of β and γ , $|C'/T'| = 0.2$, $|P'^C_{EW,NP}/P'_{EW,NP}| = 0.3$.

This particular Z' has no couplings to leptons, i.e., it is leptophobic. The experimental limits on such Z' bosons are very weak. However, suppose they were stronger: say $M_{Z'} > 1 \text{ TeV}$ is required. The above constraint can still be satisfied with such values of $M_{Z'}$, while keeping perturbative couplings.

The only additional constraint comes from B_s^0 - $\bar{B}_s^0$ mixing. The formalism is described in ref. [49], to which we refer the reader for details. Briefly, in the presence of SM and NP contributions, B_s^0 - $\bar{B}_s^0$ mixing is due to the operator

$$NC_{VLL} (\bar{s}_L \gamma^\mu b_L) (\bar{s}_L \gamma_\mu b_L), \quad (4.6)$$

where

$$NC_{VLL} \equiv |NC_{VLL}^{\text{SM}}| e^{-2i\beta_s} + \frac{(g_L^{bs})^2}{2M_{Z'}^2}. \quad (4.7)$$

(Recall that $g_L^{bs} = |g_L^{bs}| e^{i\Phi'}$.) The B_s^0 - $\bar{B}_s^0$ mixing parameters are given by

$$\begin{aligned} \Delta M_s &= \frac{2}{3} m_{B_s} f_{B_s}^2 \hat{B}_{B_s} |NC_{VLL}|, \\ \varphi_s &= \arg(NC_{VLL}). \end{aligned} \quad (4.8)$$

Here, $f_{B_s} \sqrt{\hat{B}_{B_s}} = 270 \pm 16 \text{ MeV}$. The experimental measurements of the mixing parameters yield [34]

$$\begin{aligned} \Delta M_s^{\text{exp}} &= 17.757 \pm 0.021 \text{ ps}^{-1}, \\ \varphi_s^{c\bar{c}s} &= -0.030 \pm 0.033, \end{aligned} \quad (4.9)$$

while the SM predictions are

$$\begin{aligned}\Delta M_s^{\text{SM}} &= \frac{2}{3} m_{B_s} f_{B_s}^2 \hat{B}_{B_s} |NC_{VLL}^{\text{SM}}| = (17.9 \pm 2.4) \text{ ps}^{-1}, \\ \varphi_s^{\bar{c}\bar{c}s, \text{SM}} &= -2\beta_s = -0.03704 \pm 0.00064.\end{aligned}\quad (4.10)$$

With all of this, we can obtain an upper bound on the NP contribution. From eq. (4.7), we have

$$\frac{|g_L^{bs}|^2}{2M_{Z'}^2} \leq \sqrt{|NC_{VLL}|^2 + |NC_{VLL}^{\text{SM}}|^2 - 2|NC_{VLL}||NC_{VLL}^{\text{SM}}| \cos(\varphi_s^{\bar{c}\bar{c}s} - \varphi_s^{\bar{c}\bar{c}s, \text{SM}})}. \quad (4.11)$$

We allow the experimental quantities to vary by $\pm 2\sigma$, while $f_{B_s} \sqrt{\hat{B}_{B_s}}$ varies within its theoretical range. This leads to

$$\frac{|g_L^{bs}|^2}{M_{Z'}^2} \leq 3.56 \times 10^{-5} \text{ TeV}^{-2} \quad \implies \quad M_{Z'} \geq |g_L^{bs}| \times 168 \text{ TeV}. \quad (4.12)$$

(A similar result can be found in ref. [50].) Combining this with eq. (4.5), we have

$$\left| \frac{g_L^{bs}}{g_R^{dd}} \right| \leq 6.4 \times 10^{-3}. \quad (4.13)$$

With this condition, all the constraints can be satisfied. Therefore this Z' can indeed explain the $B \rightarrow \pi K$ puzzle.

The results for cases (2) (g_R^{dd} and g_R^{uu} are nonzero, but $g_R^{uu} = -2g_R^{dd}$) and (4) (only g_R^{uu} is nonzero) are similar. The fit for case (2) has a p-value of 28%, while that for case (4) has p-value = 26%.

We therefore conclude that a Z' model can explain the $B \rightarrow \pi K$ puzzle, but it is necessary that the Z' couple to right-handed d and/or u quarks, with $g_R^{dd} \neq g_R^{uu}$.

4.2.4 $Z' b \rightarrow s\mu^+\mu^-$ models and $b \rightarrow s\bar{q}q$

As noted in the introduction, there are currently several measurements that disagree with the predictions of the SM. These include (i) R_K (LHCb [7]) and (ii) R_{K^*} (LHCb [8]), (iii) the angular distribution of $B \rightarrow K^*\mu^+\mu^-$ (LHCb [9, 10], Belle [11], ATLAS [12] and CMS [13]), especially the observable P'_5 [51], and (iv) the branching fraction and angular distribution of $B_s^0 \rightarrow \phi\mu^+\mu^-$ (LHCb [14, 15]). The discrepancies in R_K and R_{K^*} are quite clean and are at the level of 2.2-2.6 σ . On the other hand, the discrepancies in $B \rightarrow K^*\mu^+\mu^-$ and $B_s^0 \rightarrow \phi\mu^+\mu^-$ have some amount of theoretical input. Depending on how one treats the hadronic uncertainties, the disagreement with the SM is in the 2.5–4 σ range.

Of these measurements, the most recent is that of R_{K^*} . Following its announcement, a number of papers appeared [16–26] computing the size of the discrepancy with the SM, and determining the general properties of the NP required to explain the results. Combining constraints from all measurements, the general consensus is that there is indeed a significant disagreement with the SM, somewhere in the range of 4–6 σ (this large range is due to the fact that different groups deal with the theoretical uncertainties in different ways). In order

to account for all measurements, the most probable explanation is that the NP primarily affects $b \rightarrow s\mu^+\mu^-$ transitions.

Arguably the simplest NP explanation is that the contribution to $b \rightarrow s\mu^+\mu^-$ arises due to the tree-level exchange of a Z' boson. Here the Z' has a flavor-changing coupling to $\bar{s}b$, and it also couples to $\bar{\mu}\mu$. Many models of this type have been proposed to explain the data. In some of these models, the Z' also has couplings to $\bar{u}u$ and/or $\bar{d}d$, so that it could potentially furnish an explanation of the $B \rightarrow \pi K$ puzzle. If so, in such models there would be a connection between the anomalies in $b \rightarrow s\mu^+\mu^-$ and $B \rightarrow \pi K$ decays, which is quite intriguing.

Many Z' models have been proposed to explain the $b \rightarrow s\mu^+\mu^-$ anomalies [52–89]. (Note that refs. [59–62] all discuss the 3-3-1 model. Here the Z' couples equally to e^+e^- and $\mu^+\mu^-$, so this model cannot explain $R_{K^{(*)}}$.) The question is: are there models in which the Z' couples to right-handed d and/or u quarks, with $g_R^{dd} \neq g_R^{uu}$? A survey of the models reveals the following:

- Models in which the Z' couplings to $u\bar{u}$ or $d\bar{d}$ either vanish or are very small include refs. [52, 76, 78, 83, 86–89].
- Models in which the Z' has vectorlike couplings to $u\bar{u}$ and $d\bar{d}$ include refs. [53, 65, 69, 74, 75, 77]. In this case the Z' model cannot explain the $B \rightarrow \pi K$ puzzle, see section 4.2.2.
- Models that focus only on $b \rightarrow s\mu^+\mu^-$ and say nothing about any other couplings include refs. [66, 82, 85].
- Models in which the Z' has explicit couplings to RH quarks, with different couplings to RH $u\bar{u}$ and $d\bar{d}$ include the 3-3-1 model [59–62] and refs. [67, 80].
- Many models have been proposed in which the Z' couples to LH quarks, but not RH quarks. A significant fraction of these can be easily modified to allow the Z' to couple to RH quarks. These include refs. [54–58, 63, 64, 68, 70–73, 79, 81, 84].

There are a total of 34 Z' models. Of these, 17 have, or can be modified to have, the Z' coupling to right-handed $u\bar{u}$ and/or $d\bar{d}$, with $g_R^{dd} \neq g_R^{uu}$. These models can therefore potentially also explain the $B \rightarrow \pi K$ puzzle.

It is necessary to check other constraints. In order to reproduce the $b \rightarrow s\mu^+\mu^-$ data, the Z' mass and couplings must satisfy

$$C_9^{\mu\mu}(\text{NP}) = -C_{10}^{\mu\mu}(\text{NP}) = - \left[\frac{\pi}{\sqrt{2}G_F\alpha V_{tb}V_{ts}^*} \right] \frac{g_L^{bs}g_L^{\mu\mu}}{M_{Z'}^2}. \quad (4.14)$$

For $C_9^{\mu\mu}(\text{NP}) = -C_{10}^{\mu\mu}(\text{NP}) = -0.8$ [49], we require

$$\frac{|g_L^{bs}g_L^{\mu\mu}|}{M_{Z'}^2} = 1.2 \times 10^{-3} \text{ TeV}^{-2}. \quad (4.15)$$

The Z' will also contribute to $\nu_\mu N \rightarrow \nu_\mu N \mu^+ \mu^-$ (neutrino trident production). Measurements of this process lead to the constraint [49]

$$\frac{|g_L^{\mu\mu}|^2}{M_{Z'}^2} < 1.6 \text{ TeV}^{-2}. \quad (4.16)$$

For example, in ref. [24], the $b \rightarrow s \mu^+ \mu^-$ data were analyzed in the context of Z' models. Good fits were found for $M_{Z'} = 1 \text{ TeV}$, with $g_L^{\mu\mu} \simeq 0.5$ and $g_L^{bs} \simeq -2.5 \times 10^{-3}$. From eq. (4.13), this implies that the Z' couplings to right-handed $u\bar{u}$ and/or $d\bar{d}$ must be on the large side, $O(1)$.

However, there is another, more important process to consider. A Z' that couples to both quarks and muons can be detected at the LHC via $pp \rightarrow Z' \rightarrow \mu^+ \mu^-$. If the Z' couples only to $\bar{s}b$, its production cross section may be small enough to escape detection (for example, see ref. [78]). On the other hand, if the Z' couples to $\bar{d}d$ and/or $\bar{u}u$, it will be produced plentifully in pp collisions. Recently, using the 2015 and 2016 data at $\sqrt{s} = 13 \text{ TeV}$, the ATLAS Collaboration searched for high-mass resonances decaying into dileptons, but found nothing [90]. They put an upper limit on the product of the production cross section and decay branching ratio, converting this to a lower limit on $M_{Z'} \gtrsim 4 \text{ TeV}$ for the Z' models analyzed. We expect that this limit applies to our Z' .

The conclusion is that some of the Z' models proposed to explain the $b \rightarrow s \mu^+ \mu^-$ data may also explain the $B \rightarrow \pi K$ puzzle. The Z' must be somewhat massive, with $M_{Z'} \gtrsim 4 \text{ TeV}$. On the other hand, its mass cannot be *much* larger than this lower limit, as the couplings would become nonperturbative in this case. So perhaps such a Z' will be observed at the LHC in the coming years.

4.3 Diquarks

Another NP particle that can contribute to $b \rightarrow s \bar{q} q$ ($q = u, d$) at tree level is a diquark [91]. Diquarks are scalar particles that couple to two quarks.⁵ Quarks are **3**s under $SU(3)_C$, so that the diquark must transform as a $\bar{\mathbf{3}}$ (triplet) or a **6** (sextet). Since diquark couplings are fermion-number-violating, the diquarks couple to two quarks of the same chirality. That is, they couple to $q_L^i q_L^j$, $u_R^i u_R^j$, $u_R^i d_R^j$, or $d_R^i d_R^j$, where $q_L = (u_L, d_L)$ is an $SU(2)_L$ doublet, and i, j are flavor indices. There are therefore a total of 8 different types of diquark. These are listed in table 10.

The triplets are antisymmetric in color, while sextets are symmetric. This implies that their couplings are also respectively antisymmetric and symmetric in flavor [91]. This has important consequences for flavor-changing neutral currents (FCNCs). Consider the diquarks that couple to ds (diquarks *I*, *II*, *VII* and *VIII*). In principle, they contribute at tree level to K^0 - $\bar{K}^0$ mixing via the t-channel exchange of a diquark. This involves the transitions $d \rightarrow \bar{d}$ and $\bar{s} \rightarrow s$, i.e., the flavor indices of the couplings have $i = j$. However, because the couplings of the triplet diquarks are antisymmetric in flavor, these couplings vanish for

⁵In principle, diquarks can also be vector particles. In this case, it is natural to consider them to be the gauge bosons of an extended gauge group (such as $SU(5)$). However, in general such diquarks also have leptoquark couplings, which leads to proton decay. Scalar diquarks do not have this problem.

Name	SU(3) _C	SU(2) _L	U(1) _Y	QQ Coupling
<i>I</i>	6	3	$\frac{2}{3}$	$q_L^i q_L^j$
<i>II</i>	$\bar{3}$	3	$\frac{2}{3}$	$q_L^i q_L^j$
<i>III</i>	6	1	$\frac{2}{3}$	$q_L^i q_L^j, u_R^i d_R^j$
<i>IV</i>	$\bar{3}$	1	$\frac{2}{3}$	$q_L^i q_L^j, u_R^i d_R^j$
<i>V</i>	6	1	$\frac{8}{3}$	$u_R^i u_R^j$
<i>VI</i>	$\bar{3}$	1	$\frac{8}{3}$	$u_R^i u_R^j$
<i>VII</i>	6	1	$-\frac{4}{3}$	$d_R^i d_R^j$
<i>VIII</i>	$\bar{3}$	1	$-\frac{4}{3}$	$d_R^i d_R^j$

Table 10. Scalar diquarks: quantum numbers and couplings.

these diquarks. That is, while sextet diquarks can generate $\Delta F = 2$ FCNCs, triplet diquarks cannot [91]. Thus, the measurements of neutral meson mixing constrain diquarks *I*, *V* and *VII* to be very massive, so that their effects in other low-energy processes are negligible. However, there are no similar constraints on diquarks *II*, *III*, *IV*, *VI* and *VIII*.

Diquarks contribute at tree level to $b \rightarrow s\bar{q}q$ ($q = u, d$) via $b \rightarrow \bar{q}D^*(\rightarrow sq)$. From this, we see that diquark *VI* does not contribute to $B \rightarrow \pi K$. Recall that in section 4.1 we noted that there are four NP matrix elements that contribute to $B \rightarrow \pi K$ decays: $\mathcal{A}'^q e^{i\Phi'_q}$ and $\mathcal{A}'^{C,q} e^{i\Phi'^C_q}$ ($q = u, d$). The color-allowed and color-suppressed matrix elements use the operators $\bar{s}_\alpha \Gamma_i b_\alpha \bar{q}_\beta \Gamma_j q_\beta$ and $\bar{s}_\alpha \Gamma_i b_\beta \bar{q}_\beta \Gamma_j q_\alpha$, respectively. Now consider diquark *VIII*. It contributes to $\mathcal{A}'^{C,d} e^{i\Phi'^C_d}$ and $\mathcal{A}'^{C,d} e^{i\Phi'_d}$. However, it is straightforward to see that these are not independent. The $b \rightarrow s\bar{q}q$ transition involves $b_\alpha \rightarrow \bar{q}_\beta D_{\alpha\beta}^*$. The virtual D^* then decays equally to $s_\beta d_\alpha$ and $s_\alpha d_\beta$. But the first decay creates the color-allowed operator, the second the color-suppressed operator. And since the triplet is antisymmetric, there is a relative minus sign. That is, we have $\mathcal{A}'^{C,d} e^{i\Phi'^C_d} = -\mathcal{A}'^{C,d} e^{i\Phi'_d}$, or $\Phi_d^C = \Phi'_d$ and $\mathcal{A}'^{C,d} = -\mathcal{A}'^{C,d}$. This same type of behavior holds for all diquarks, except that diquark *III* is a sextet, which is symmetric, so that $\mathcal{A}'^{C,u} e^{i\Phi'^C_u} = +\mathcal{A}'^{C,u} e^{i\Phi'_u}$.

The four diquarks that contribute to $B \rightarrow \pi K$ are *II*, *III*, *IV* and *VIII*. These have the following properties:

- *II*: decays to $q_L^i q_L^j$, which includes $u_L^i u_L^j$, $u_L^i d_L^j$ and $d_L^i d_L^j$. This implies that the amplitudes for $b \rightarrow s\bar{u}u$ and $b \rightarrow s\bar{d}d$ are equal, so that $\mathcal{A}'^{C,d} = \mathcal{A}'^{C,u}$ and $\mathcal{A}'^{C,d} = \mathcal{A}'^{C,u}$. We then have $P'_{EW,NP} = P'_{EW,NP} = 0$; the only nonzero NP operator is P'_{NP} . (We also have $\mathcal{A}'^{C,q} e^{i\Phi'^C_q} = -\mathcal{A}'^{C,q} e^{i\Phi'_q}$, but this is not important.)
- *III*: has $\mathcal{A}'^{C,d} e^{i\Phi'^C_d} = \mathcal{A}'^{C,d} e^{i\Phi'_d} = 0$ and $\mathcal{A}'^{C,u} e^{i\Phi'^C_u} = +\mathcal{A}'^{C,u} e^{i\Phi'_u}$.
- *IV*: has $\mathcal{A}'^{C,d} e^{i\Phi'^C_d} = \mathcal{A}'^{C,d} e^{i\Phi'_d} = 0$ and $\mathcal{A}'^{C,u} e^{i\Phi'^C_u} = -\mathcal{A}'^{C,u} e^{i\Phi'_u}$.
- *VIII*: has $\mathcal{A}'^{C,d} e^{i\Phi'^C_d} = -\mathcal{A}'^{C,d} e^{i\Phi'_d}$ and $\mathcal{A}'^{C,u} e^{i\Phi'^C_u} = \mathcal{A}'^{C,u} e^{i\Phi'_u} = 0$.

With this information, we can perform fits for the four diquark models.

Diquark <i>III</i> : $\chi^2/\text{d.o.f.} = 4.0/3$, p-value = 0.26	
Parameter	Best-fit value
γ	$(72.1 \pm 5.8)^\circ$
β	$(21.87 \pm 0.68)^\circ$
Φ'	$(273.2 \pm 10.5)^\circ$
$ T' $	7.8 ± 5.5
$ P'_{tc} $	50.6 ± 0.7
$P'_{EW,NP}$	4.6 ± 7.5
$\delta_{P'_{tc}}$	$(317.8 \pm 47.5)^\circ$
$\delta_{C'}$	$(128.0 \pm 329.0)^\circ$

Table 11. $\chi^2_{\min}/\text{d.o.f.}$ and best-fit values of unknown parameters for the model of diquark *III*. Constraints: $B \rightarrow \pi K$ data, measurements of β and γ , $|C'/T'| = 0.2$.

4.3.1 Diquark *II*

The scenario where P'_{NP} is the only nonzero NP operator was examined in section 4.2.2. There it was found that the fit was no better than that of the SM, and that the $B \rightarrow \pi K$ puzzle could not be explained.

4.3.2 Diquark *III*

The results of the fit with diquark *III* are shown in table 11. The best fit has $\chi^2/\text{d.o.f.} = 4.0/3$, for a p-value of 26%. Like the Z' model of section 4.2.3, the fit is not bad (and is far better than the SM). It could explain the $B \rightarrow \pi K$ puzzle.

In order to get a sense of what values are required for the diquark mass and couplings, we proceed as in section 4.2.3: we compare $P'_{EW,NP}$ and T' . Here, however, the form factors for the two diagrams are different, so this comparison will only give a rough idea. We have

$$\begin{aligned} \left| \frac{P'_{EW,NP}}{T'} \right| &\approx \frac{|g^{ub}g^{us}|/M_{DQ}^2}{(G_F/\sqrt{2})|V_{ub}^*V_{us}|} = \frac{4.6}{7.8} \\ \Rightarrow \frac{|g^{ub}g^{us}|}{M_{DQ}^2} &= 4.5 \times 10^{-3} \text{ TeV}^{-2}. \end{aligned} \quad (4.17)$$

This is similar to what was found for Z' models.

4.3.3 Diquark *IV*

The results of the fit with diquark *IV* are shown in table 12. The best fit has $\chi^2/\text{d.o.f.} = 5.4/3$, for a p-value of 15%. This is only so-so. Given that the best-fit values of the ratios $|T'/P'_{tc}|$ and $|P'_{EW,NP}/P'_{tc}|$ are both somewhat larger than expected, we conclude that this diquark does not provide a good explanation of the $B \rightarrow \pi K$ puzzle.

Diquark <i>IV</i> : $\chi^2/\text{d.o.f.} = 5.4/3$, p-value = 0.15	
Parameter	Best-fit value
γ	$(69.9 \pm 5.6)^\circ$
β	$(21.86 \pm 0.68)^\circ$
Φ'	$(0.6 \pm 6.8)^\circ$
$ T' $	31.5 ± 4.8
$ P'_{tc} $	50.3 ± 0.8
$P'_{EW,NP}$	22.6 ± 3.4
$\delta_{P'_{tc}}$	$(186.3 \pm 0.9)^\circ$
$\delta_{C'}$	$(57.5 \pm 17.0)^\circ$

Table 12. $\chi^2_{\min}/\text{d.o.f.}$ and best-fit values of unknown parameters for the model of diquark *IV*. Constraints: $B \rightarrow \pi K$ data, measurements of β and γ , $|C'/T'| = 0.2$.

Diquark <i>VIII</i> : $\chi^2/\text{d.o.f.} = 11.6/3$, p-value = 0.009	
Parameter	Best-fit value
γ	$(64.8 \pm 6.5)^\circ$
β	$(21.80 \pm 0.68)^\circ$
Φ'	$(-19.4 \pm 91.7)^\circ$
$ T' $	9.3 ± 3.2
$ P'_{tc} $	51.7 ± 2.1
$P'_{EW,NP}$	1.3 ± 2.1
$\delta_{P'_{tc}}$	$(-11.4 \pm 4.4)^\circ$
$\delta_{C'}$	$(250.2 \pm 24.8)^\circ$

Table 13. $\chi^2_{\min}/\text{d.o.f.}$ and best-fit values of unknown parameters for the model of diquark *VIII*. Constraints: $B \rightarrow \pi K$ data, measurements of β and γ , $|C'/T'| = 0.2$.

4.3.4 Diquark *VIII*

The results of the fit with diquark *VIII* are shown in table 13. The best fit has $\chi^2/\text{d.o.f.} = 11.6/3$, for a p-value of 0.9%. This is a very poor fit, worse than that of the SM. This diquark cannot explain the $B \rightarrow \pi K$ puzzle.

4.4 Summary

In the previous section, we saw that, if $|C'/T'| = 0.2$, the SM cannot explain the $B \rightarrow \pi K$ puzzle. The two types of NP that can contribute to $B \rightarrow \pi K$ at tree level are Z' bosons

and diquarks. In this section, we found that either NP model can explain the puzzle if $|C'/T'| = 0.2$. For Z' models, the Z' must couple to right-handed $u\bar{u}$ and/or $d\bar{d}$, with $g_R^{dd} \neq g_R^{uu}$. Half of the Z' models proposed to explain the $b \rightarrow s\mu^+\mu^-$ anomalies have the required Z' couplings to $u\bar{u}$ and/or $d\bar{d}$. As for diquarks, the only one that works is a color **6** that couples to ud . For both NP models, the fits have p-values in the range 25–40%.

5 Conclusions

There are four $B \rightarrow \pi K$ decays — $B^+ \rightarrow \pi^+ K^0$, $B^+ \rightarrow \pi^0 K^+$, $B_d^0 \rightarrow \pi^- K^+$ and $B_d^0 \rightarrow \pi^0 K^0$ — whose amplitudes obey a quadrilateral isospin relation. In the early 2000s, their branching ratios and CP asymmetries (direct and indirect) were measured, and it was noted that there was a tension between the measurements and the SM. This was referred to as the “ $B \rightarrow \pi K$ puzzle.” Over the years, a number of analyses were done, attempting to quantify the seriousness of the puzzle, and to identify the type of new physics that can ameliorate the problem.

In the present paper, we perform an update of the $B \rightarrow \pi K$ puzzle by doing fits to the data using a diagrammatic decomposition of the $B \rightarrow \pi K$ amplitudes. We find that the key unknown parameter is $|C'/T'|$, the ratio of color-suppressed and color-allowed tree amplitudes. Theoretically, this ratio is predicted to be $0.15 \lesssim |C'/T'| \lesssim 0.5$. If it is large, $|C'/T'| = 0.5$, we find that the SM can explain the data: the fit has a p-value of 43% (an excellent fit has p-value = 50%). On the other hand, if it is small, $|C'/T'| = 0.2$, the fit has a p-value of 4%, which is poor. Our conclusion is that, if $|C'/T'|$ is small, the SM cannot explain the $B \rightarrow \pi K$ puzzle — NP is needed.

The two types of NP that can contribute to $B \rightarrow \pi K$ at tree level are Z' bosons and diquarks. For the case of $|C'/T'| = 0.2$, we examine whether the $B \rightarrow \pi K$ puzzle can be explained with the inclusion of such NP. For both types of NP, the answer is yes.

In the case of Z' models, the decay $b \rightarrow s\bar{q}q$ ($q = u, d$) is produced via the tree-level exchange of a Z' that couples to $\bar{s}b$ and to $\bar{q}q$. We find that, if the Z' couples only to left-handed $\bar{q}q$, things do not work. The $B \rightarrow \pi K$ puzzle can be explained only if the Z' couples to right-handed $u\bar{u}$ and/or $d\bar{d}$, with $g_R^{dd} \neq g_R^{uu}$ — the p-values of the fits are in the range 25–40%.

This particular NP solution is intriguing because there are currently anomalies involving the process $b \rightarrow s\mu^+\mu^-$ that can also be explained by the addition of a Z' . We find that, of all the Z' models proposed for the $b \rightarrow s\mu^+\mu^-$ anomalies, half have the required Z' couplings to $u\bar{u}$ and/or $d\bar{d}$. Such models could potentially explain both the $b \rightarrow s\mu^+\mu^-$ anomalies and the $B \rightarrow \pi K$ puzzle.

Turning to diquarks, there are eight different types. Taking into account constraints from other processes, particularly $\Delta F = 2$ FCNCs, there is only one diquark that works. It is a color **6** that couples to ud . It contribute at tree level to $b \rightarrow s\bar{u}u$ via $b \rightarrow \bar{u}D^*(\rightarrow su)$. Its fit has a p-value of 26%.

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References

- [1] A.J. Buras, R. Fleischer, S. Recksiegel and F. Schwab, *The $B \rightarrow \pi K$ puzzle and its relation to rare B and K decays*, *Eur. Phys. J. C* **32** (2003) 45 [[hep-ph/0309012](#)] [[INSPIRE](#)].
- [2] A.J. Buras, R. Fleischer, S. Recksiegel and F. Schwab, *$B \rightarrow \pi\pi$, new physics in $B \rightarrow \pi K$ and implications for rare K and B decays*, *Phys. Rev. Lett.* **92** (2004) 101804 [[hep-ph/0312259](#)] [[INSPIRE](#)].
- [3] A.J. Buras, R. Fleischer, S. Recksiegel and F. Schwab, *Anatomy of prominent B and K decays and signatures of CP-violating new physics in the electroweak penguin sector*, *Nucl. Phys. B* **697** (2004) 133 [[hep-ph/0402112](#)] [[INSPIRE](#)].
- [4] S. Baek, P. Hamel, D. London, A. Datta and D.A. Suprun, *The $B \rightarrow \pi K$ puzzle and new physics*, *Phys. Rev. D* **71** (2005) 057502 [[hep-ph/0412086](#)] [[INSPIRE](#)].
- [5] S. Baek and D. London, *Is There Still a $B \rightarrow \pi K$ Puzzle?*, *Phys. Lett. B* **653** (2007) 249 [[hep-ph/0701181](#)] [[INSPIRE](#)].
- [6] S. Baek, C.-W. Chiang and D. London, *The $B \rightarrow \pi K$ Puzzle: 2009 Update*, *Phys. Lett. B* **675** (2009) 59 [[arXiv:0903.3086](#)] [[INSPIRE](#)].
- [7] LHCb collaboration, *Test of lepton universality using $B^+ \rightarrow K^+ \ell^+ \ell^-$ decays*, *Phys. Rev. Lett.* **113** (2014) 151601 [[arXiv:1406.6482](#)] [[INSPIRE](#)].
- [8] LHCb collaboration, *Test of lepton universality with $B^0 \rightarrow K^{*0} \ell^+ \ell^-$ decays*, *JHEP* **08** (2017) 055 [[arXiv:1705.05802](#)] [[INSPIRE](#)].
- [9] LHCb collaboration, *Measurement of Form-Factor-Independent Observables in the Decay $B^0 \rightarrow K^{*0} \mu^+ \mu^-$* , *Phys. Rev. Lett.* **111** (2013) 191801 [[arXiv:1308.1707](#)] [[INSPIRE](#)].
- [10] LHCb collaboration, *Angular analysis of the $B^0 \rightarrow K^{*0} \mu^+ \mu^-$ decay using 3 fb^{-1} of integrated luminosity*, *JHEP* **02** (2016) 104 [[arXiv:1512.04442](#)] [[INSPIRE](#)].
- [11] BELLE collaboration, A. Abdesselam et al., *Angular analysis of $B^0 \rightarrow K^*(892)^0 \ell^+ \ell^-$* , [arXiv:1604.04042](#) [[INSPIRE](#)].
- [12] ATLAS collaboration, *Angular analysis of $B_d^0 \rightarrow K^{*0} \mu^+ \mu^-$ decays in pp collisions at $\sqrt{s} = 8 \text{ TeV}$ with the ATLAS detector*, *ATLAS-CONF-2017-023* (2017).
- [13] CMS collaboration, *Measurement of the P_1 and P'_5 angular parameters of the decay $B^0 \rightarrow K^{*0} \mu^+ \mu^-$ in proton-proton collisions at $\sqrt{s} = 8 \text{ TeV}$* , *CMS-PAS-BPH-15-008* (2017).

- [14] LHCb collaboration, *Differential branching fraction and angular analysis of the decay $B_s^0 \rightarrow \phi \mu^+ \mu^-$* , *JHEP* **07** (2013) 084 [[arXiv:1305.2168](#)] [[INSPIRE](#)].
- [15] LHCb collaboration, *Angular analysis and differential branching fraction of the decay $B_s^0 \rightarrow \phi \mu^+ \mu^-$* , *JHEP* **09** (2015) 179 [[arXiv:1506.08777](#)] [[INSPIRE](#)].
- [16] B. Capdevila, A. Crivellin, S. Descotes-Genon, J. Matias and J. Virto, *Patterns of New Physics in $b \rightarrow s \ell^+ \ell^-$ transitions in the light of recent data*, [arXiv:1704.05340](#) [[INSPIRE](#)].
- [17] W. Altmannshofer, P. Stangl and D.M. Straub, *Interpreting Hints for Lepton Flavor Universality Violation*, *Phys. Rev. D* **96** (2017) 055008 [[arXiv:1704.05435](#)] [[INSPIRE](#)].
- [18] G. D’Amico et al., *Flavour anomalies after the R_{K^*} measurement*, *JHEP* **09** (2017) 010 [[arXiv:1704.05438](#)] [[INSPIRE](#)].
- [19] G. Hiller and I. Nisandzic, *R_K and R_{K^*} beyond the standard model*, *Phys. Rev. D* **96** (2017) 035003 [[arXiv:1704.05444](#)] [[INSPIRE](#)].
- [20] L.-S. Geng, B. Grinstein, S. Jäger, J. Martin Camalich, X.-L. Ren and R.-X. Shi, *Towards the discovery of new physics with lepton-universality ratios of $b \rightarrow s \ell \ell$ decays*, *Phys. Rev. D* **96** (2017) 093006 [[arXiv:1704.05446](#)] [[INSPIRE](#)].
- [21] M. Ciuchini et al., *On Flavourful Easter eggs for New Physics hunger and Lepton Flavour Universality violation*, *Eur. Phys. J. C* **77** (2017) 688 [[arXiv:1704.05447](#)] [[INSPIRE](#)].
- [22] A. Celis, J. Fuentes-Martin, A. Vicente and J. Virto, *Gauge-invariant implications of the LHCb measurements on lepton-flavor nonuniversality*, *Phys. Rev. D* **96** (2017) 035026 [[arXiv:1704.05672](#)] [[INSPIRE](#)].
- [23] D. Ghosh, *Explaining the R_K and R_{K^*} anomalies*, *Eur. Phys. J. C* **77** (2017) 694 [[arXiv:1704.06240](#)] [[INSPIRE](#)].
- [24] A.K. Alok, B. Bhattacharya, A. Datta, D. Kumar, J. Kumar and D. London, *New Physics in $b \rightarrow s \mu^+ \mu^-$ after the Measurement of R_{K^*}* , *Phys. Rev. D* **96** (2017) 095009 [[arXiv:1704.07397](#)] [[INSPIRE](#)].
- [25] W. Wang and S. Zhao, *Implications of the R_K and R_{K^*} anomalies*, *Chin. Phys. C* **42** (2018) 013105 [[arXiv:1704.08168](#)] [[INSPIRE](#)].
- [26] A. Datta, J. Kumar, J. Liao and D. Marfatia, *New light mediators for the R_K and R_{K^*} puzzles*, [arXiv:1705.08423](#) [[INSPIRE](#)].
- [27] M. Gronau, O.F. Hernández, D. London and J.L. Rosner, *Decays of B mesons to two light pseudoscalars*, *Phys. Rev. D* **50** (1994) 4529 [[hep-ph/9404283](#)] [[INSPIRE](#)].
- [28] M. Gronau, O.F. Hernández, D. London and J.L. Rosner, *Electroweak penguins and two-body B decays*, *Phys. Rev. D* **52** (1995) 6374 [[hep-ph/9504327](#)] [[INSPIRE](#)].
- [29] M. Neubert and J.L. Rosner, *New bound on γ from $B^\pm \rightarrow \pi K$ decays*, *Phys. Lett. B* **441** (1998) 403 [[hep-ph/9808493](#)] [[INSPIRE](#)].
- [30] M. Neubert and J.L. Rosner, *Determination of the weak phase γ from rate measurements in $B^\pm \rightarrow \pi K, \pi \pi$ decays*, *Phys. Rev. Lett.* **81** (1998) 5076 [[hep-ph/9809311](#)] [[INSPIRE](#)].
- [31] M. Gronau, D. Pirjol and T.-M. Yan, *Model independent electroweak penguins in B decays to two pseudoscalars*, *Phys. Rev. D* **60** (1999) 034021 [*Erratum ibid.* **D 69** (2004) 119901] [[hep-ph/9810482](#)] [[INSPIRE](#)].
- [32] G. Buchalla, A.J. Buras and M.E. Lautenbacher, *Weak decays beyond leading logarithms*, *Rev. Mod. Phys.* **68** (1996) 1125 [[hep-ph/9512380](#)] [[INSPIRE](#)].

- [33] CKMFITTER GROUP collaboration, J. Charles et al., *CP violation and the CKM matrix: Assessing the impact of the asymmetric B factories*, *Eur. Phys. J. C* **41** (2005) 1 [[hep-ph/0406184](#)] [[INSPIRE](#)].
- [34] HFLAV collaboration, Y. Amhis et al., *Averages of b-hadron, c-hadron and τ -lepton properties as of summer 2016*, *Eur. Phys. J. C* **77** (2017) 895 [[arXiv:1612.07233](#)] [[INSPIRE](#)].
- [35] F. James and M. Roos, *Minuit: A System for Function Minimization and Analysis of the Parameter Errors and Correlations*, *Comput. Phys. Commun.* **10** (1975) 343 [[INSPIRE](#)].
- [36] F. James and M. Winkler, *MINUIT User's Guide*, (2004) [[INSPIRE](#)].
- [37] F. James, *MINUIT Function Minimization and Error Analysis: Reference Manual Version 94.1*, CERN-D-506, CERN-D506 [[INSPIRE](#)].
- [38] M. Imbeault, A. S.-L. Lemerle, V. Pagé and D. London, *Obtaining the full unitarity triangle from $B \rightarrow \pi K$ decays*, *Phys. Rev. Lett.* **92** (2004) 081801 [[hep-ph/0309061](#)] [[INSPIRE](#)].
- [39] M. Beneke, G. Buchalla, M. Neubert and C.T. Sachrajda, *QCD factorization in $B \rightarrow \pi K, \pi\pi$ decays and extraction of Wolfenstein parameters*, *Nucl. Phys. B* **606** (2001) 245 [[hep-ph/0104110](#)] [[INSPIRE](#)].
- [40] G. Bell, *NNLO vertex corrections in charmless hadronic B decays: Imaginary part*, *Nucl. Phys. B* **795** (2008) 1 [[arXiv:0705.3127](#)] [[INSPIRE](#)].
- [41] G. Bell, *NNLO vertex corrections in charmless hadronic B decays: Real part*, *Nucl. Phys. B* **822** (2009) 172 [[arXiv:0902.1915](#)] [[INSPIRE](#)].
- [42] M. Beneke, T. Huber and X.-Q. Li, *NNLO vertex corrections to non-leptonic B decays: Tree amplitudes*, *Nucl. Phys. B* **832** (2010) 109 [[arXiv:0911.3655](#)] [[INSPIRE](#)].
- [43] G. Bell, M. Beneke, T. Huber and X.-Q. Li, *Two-loop current-current operator contribution to the non-leptonic QCD penguin amplitude*, *Phys. Lett. B* **750** (2015) 348 [[arXiv:1507.03700](#)] [[INSPIRE](#)].
- [44] G. Bell, M. Beneke, T. Huber and X.Q. Li, private communication.
- [45] H.-n. Li and S. Mishima, *Possible resolution of the $B \rightarrow \pi\pi, \pi K$ puzzles*, *Phys. Rev. D* **83** (2011) 034023 [[arXiv:0901.1272](#)] [[INSPIRE](#)].
- [46] C.S. Kim, S. Oh and Y.W. Yoon, *Analytic resolution of puzzle in $B \rightarrow K\pi$ decays*, *Phys. Lett. B* **665** (2008) 231 [[arXiv:0707.2967](#)] [[INSPIRE](#)].
- [47] A. Datta and D. London, *Measuring new physics parameters in B penguin decays*, *Phys. Lett. B* **595** (2004) 453 [[hep-ph/0404130](#)] [[INSPIRE](#)].
- [48] A. Datta, M. Imbeault, D. London, V. Pagé, N. Sinha and R. Sinha, *Methods for measuring new-physics parameters in B decays*, *Phys. Rev. D* **71** (2005) 096002 [[hep-ph/0406192](#)] [[INSPIRE](#)].
- [49] A.K. Alok, B. Bhattacharya, D. Kumar, J. Kumar, D. London and S.U. Sankar, *New physics in $b \rightarrow s\mu^+\mu^-$: Distinguishing models through CP-violating effects*, *Phys. Rev. D* **96** (2017) 015034 [[arXiv:1703.09247](#)] [[INSPIRE](#)].
- [50] W. Altmannshofer and D.M. Straub, *New physics in $b \rightarrow s$ transitions after LHC run 1*, *Eur. Phys. J. C* **75** (2015) 382 [[arXiv:1411.3161](#)] [[INSPIRE](#)].
- [51] S. Descotes-Genon, T. Hurth, J. Matias and J. Virto, *Optimizing the basis of $B \rightarrow K^*ll$ observables in the full kinematic range*, *JHEP* **05** (2013) 137 [[arXiv:1303.5794](#)] [[INSPIRE](#)].

- [52] L. Calibbi, A. Crivellin and T. Ota, *Effective Field Theory Approach to $b \rightarrow s\ell\ell^{(\prime)}$, $B \rightarrow K^{(*)}\nu\bar{\nu}$ and $B \rightarrow D^{(*)}\tau\nu$ with Third Generation Couplings*, *Phys. Rev. Lett.* **115** (2015) 181801 [[arXiv:1506.02661](#)] [[INSPIRE](#)].
- [53] A. Crivellin, G. D'Ambrosio and J. Heeck, *Addressing the LHC flavor anomalies with horizontal gauge symmetries*, *Phys. Rev. D* **91** (2015) 075006 [[arXiv:1503.03477](#)] [[INSPIRE](#)].
- [54] A. Greljo, G. Isidori and D. Marzocca, *On the breaking of Lepton Flavor Universality in B decays*, *JHEP* **07** (2015) 142 [[arXiv:1506.01705](#)] [[INSPIRE](#)].
- [55] D. Aristizabal Sierra, F. Staub and A. Vicente, *Shedding light on the $b \rightarrow s$ anomalies with a dark sector*, *Phys. Rev. D* **92** (2015) 015001 [[arXiv:1503.06077](#)] [[INSPIRE](#)].
- [56] C.-W. Chiang, X.-G. He and G. Valencia, *Z' model for $b \rightarrow s\ell\bar{\ell}$ flavor anomalies*, *Phys. Rev. D* **93** (2016) 074003 [[arXiv:1601.07328](#)] [[INSPIRE](#)].
- [57] S.M. Boucenna, A. Celis, J. Fuentes-Martin, A. Vicente and J. Virto, *Non-abelian gauge extensions for B -decay anomalies*, *Phys. Lett. B* **760** (2016) 214 [[arXiv:1604.03088](#)] [[INSPIRE](#)].
- [58] S.M. Boucenna, A. Celis, J. Fuentes-Martin, A. Vicente and J. Virto, *Phenomenology of an $SU(2) \times SU(2) \times U(1)$ model with lepton-flavour non-universality*, *JHEP* **12** (2016) 059 [[arXiv:1608.01349](#)] [[INSPIRE](#)].
- [59] R. Gauld, F. Goertz and U. Haisch, *On minimal Z' explanations of the $B \rightarrow K^*\mu^+\mu^-$ anomaly*, *Phys. Rev. D* **89** (2014) 015005 [[arXiv:1308.1959](#)] [[INSPIRE](#)].
- [60] R. Gauld, F. Goertz and U. Haisch, *An explicit Z' -boson explanation of the $B \rightarrow K^*\mu^+\mu^-$ anomaly*, *JHEP* **01** (2014) 069 [[arXiv:1310.1082](#)] [[INSPIRE](#)].
- [61] A.J. Buras and J. Girrbach, *Left-handed Z' and Z FCNC quark couplings facing new $b \rightarrow s\mu^+\mu^-$ data*, *JHEP* **12** (2013) 009 [[arXiv:1309.2466](#)] [[INSPIRE](#)].
- [62] A.J. Buras, F. De Fazio and J. Girrbach, *331 models facing new $b \rightarrow s\mu^+\mu^-$ data*, *JHEP* **02** (2014) 112 [[arXiv:1311.6729](#)] [[INSPIRE](#)].
- [63] W. Altmannshofer, S. Gori, M. Pospelov and I. Yavin, *Quark flavor transitions in $L_\mu - L_\tau$ models*, *Phys. Rev. D* **89** (2014) 095033 [[arXiv:1403.1269](#)] [[INSPIRE](#)].
- [64] A. Crivellin, G. D'Ambrosio and J. Heeck, *Explaining $h \rightarrow \mu^\pm\tau^\mp$, $B \rightarrow K^*\mu^+\mu^-$ and $B \rightarrow K\mu^+\mu^-/B \rightarrow Ke^+e^-$ in a two-Higgs-doublet model with gauged $L_\mu - L_\tau$* , *Phys. Rev. Lett.* **114** (2015) 151801 [[arXiv:1501.00993](#)] [[INSPIRE](#)].
- [65] A. Crivellin, G. D'Ambrosio and J. Heeck, *Addressing the LHC flavor anomalies with horizontal gauge symmetries*, *Phys. Rev. D* **91** (2015) 075006 [[arXiv:1503.03477](#)] [[INSPIRE](#)].
- [66] A. Crivellin, L. Hofer, J. Matias, U. Nierste, S. Pokorski and J. Rosiek, *Lepton-flavour violating B decays in generic Z' models*, *Phys. Rev. D* **92** (2015) 054013 [[arXiv:1504.07928](#)] [[INSPIRE](#)].
- [67] A. Celis, J. Fuentes-Martin, M. Jung and H. Serodio, *Family nonuniversal Z' models with protected flavor-changing interactions*, *Phys. Rev. D* **92** (2015) 015007 [[arXiv:1505.03079](#)] [[INSPIRE](#)].
- [68] G. Bélanger, C. Delaunay and S. Westhoff, *A Dark Matter Relic From Muon Anomalies*, *Phys. Rev. D* **92** (2015) 055021 [[arXiv:1507.06660](#)] [[INSPIRE](#)].
- [69] A. Falkowski, M. Nardecchia and R. Ziegler, *Lepton Flavor Non-Universality in B -meson Decays from a $U(2)$ Flavor Model*, *JHEP* **11** (2015) 173 [[arXiv:1509.01249](#)] [[INSPIRE](#)].

- [70] A. Carmona and F. Goertz, *Lepton Flavor and Nonuniversality from Minimal Composite Higgs Setups*, *Phys. Rev. Lett.* **116** (2016) 251801 [[arXiv:1510.07658](#)] [[INSPIRE](#)].
- [71] B. Allanach, F.S. Queiroz, A. Strumia and S. Sun, *Z' models for the $LHCb$ and $g - 2$ muon anomalies*, *Phys. Rev. D* **93** (2016) 055045 [[arXiv:1511.07447](#)] [[INSPIRE](#)].
- [72] A. Celis, W.-Z. Feng and D. Lüster, *Stringy explanation of $b \rightarrow s\ell^+\ell^-$ anomalies*, *JHEP* **02** (2016) 007 [[arXiv:1512.02218](#)] [[INSPIRE](#)].
- [73] K. Fuyuto, W.-S. Hou and M. Kohda, *Z' -induced FCNC decays of top, beauty and strange quarks*, *Phys. Rev. D* **93** (2016) 054021 [[arXiv:1512.09026](#)] [[INSPIRE](#)].
- [74] A. Celis, W.-Z. Feng and M. Vollmann, *Dirac dark matter and $b \rightarrow s\ell^+\ell^-$ with $U(1)$ gauge symmetry*, *Phys. Rev. D* **95** (2017) 035018 [[arXiv:1608.03894](#)] [[INSPIRE](#)].
- [75] A. Crivellin, J. Fuentes-Martin, A. Greljo and G. Isidori, *Lepton Flavor Non-Universality in B decays from Dynamical Yukawas*, *Phys. Lett. B* **766** (2017) 77 [[arXiv:1611.02703](#)] [[INSPIRE](#)].
- [76] I. Garcia Garcia, *$LHCb$ anomalies from a natural perspective*, *JHEP* **03** (2017) 040 [[arXiv:1611.03507](#)] [[INSPIRE](#)].
- [77] D. Bhatia, S. Chakraborty and A. Dighe, *Neutrino mixing and R_K anomaly in $U(1)_X$ models: a bottom-up approach*, *JHEP* **03** (2017) 117 [[arXiv:1701.05825](#)] [[INSPIRE](#)].
- [78] J.M. Cline, J.M. Cornell, D. London and R. Watanabe, *Hidden sector explanation of B -decay and cosmic ray anomalies*, *Phys. Rev. D* **95** (2017) 095015 [[arXiv:1702.00395](#)] [[INSPIRE](#)].
- [79] A. Datta, J. Liao and D. Marfatia, *A light Z' for the R_K puzzle and nonstandard neutrino interactions*, *Phys. Lett. B* **768** (2017) 265 [[arXiv:1702.01099](#)] [[INSPIRE](#)].
- [80] P. Ko, Y. Omura, Y. Shigekami and C. Yu, *$LHCb$ anomaly and B physics in flavored Z' models with flavored Higgs doublets*, *Phys. Rev. D* **95** (2017) 115040 [[arXiv:1702.08666](#)] [[INSPIRE](#)].
- [81] E. Megias, M. Quirós and L. Salas, *Lepton-flavor universality violation in R_K and $R_{D^{(*)}}$ from warped space*, *JHEP* **07** (2017) 102 [[arXiv:1703.06019](#)] [[INSPIRE](#)].
- [82] I. Ahmed and A. Rehman, *$LHCb$ anomaly in $B \rightarrow K^*\mu^+\mu^-$ optimised observables and potential of Z' Model*, [arXiv:1703.09627](#) [[INSPIRE](#)].
- [83] J.F. Kamenik, Y. Soreq and J. Zupan, *Lepton flavor universality violation without new sources of quark flavor violation*, [arXiv:1704.06005](#) [[INSPIRE](#)].
- [84] F. Sala and D.M. Straub, *A New Light Particle in B Decays?*, *Phys. Lett. B* **774** (2017) 205 [[arXiv:1704.06188](#)] [[INSPIRE](#)].
- [85] S. Di Chiara et al., *Minimal flavor-changing Z' models and muon $g - 2$ after the R_{K^*} measurement*, *Nucl. Phys. B* **923** (2017) 245 [[arXiv:1704.06200](#)] [[INSPIRE](#)].
- [86] R. Alonso, P. Cox, C. Han and T.T. Yanagida, *Anomaly-free local horizontal symmetry and anomaly-full rare B -decays*, *Phys. Rev. D* **96** (2017) 071701 [[arXiv:1704.08158](#)] [[INSPIRE](#)].
- [87] C. Bonilla, T. Modak, R. Srivastava and J.W.F. Valle, *$U(1)_{B_3-3L_\mu}$ gauge symmetry as the simplest description of $b \rightarrow s$ anomalies*, [arXiv:1705.00915](#) [[INSPIRE](#)].
- [88] R. Alonso, P. Cox, C. Han and T.T. Yanagida, *Flavoured $B - L$ local symmetry and anomalous rare B decays*, *Phys. Lett. B* **774** (2017) 643 [[arXiv:1705.03858](#)] [[INSPIRE](#)].

- [89] C.-W. Chiang, X.-G. He, J. Tandean and X.-B. Yuan, $R_{K^{(*)}}$ and related $b \rightarrow s\ell\bar{\ell}$ anomalies in minimal flavor violation framework with Z' boson, *Phys. Rev. D* **96** (2017) 115022 [[arXiv:1706.02696](#)] [[INSPIRE](#)].
- [90] ATLAS collaboration, Search for new high-mass phenomena in the dilepton final state using 36fb^{-1} of proton-proton collision data at $\sqrt{s} = 13\text{ TeV}$ with the ATLAS detector, *JHEP* **10** (2017) 182 [[arXiv:1707.02424](#)] [[INSPIRE](#)].
- [91] G.F. Giudice, B. Gripaios and R. Sundrum, Flavourful Production at Hadron Colliders, *JHEP* **08** (2011) 055 [[arXiv:1105.3161](#)] [[INSPIRE](#)].