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Asymptotic symmetries, holography and topological hair

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Abstract: Asymptotic symmetries of AdS_4 quantum gravity and gauge theory are derived by coupling the holographically dual CFT_3 to Chern-Simons gauge theory and 3D gravity in a "probe" (large-level) limit. Despite the fact that the three-dimensional AdS_4 boundary as a whole is consistent with only finite-dimensional asymptotic symmetries, given by AdS isometries, infinite-dimensional symmetries are shown to arise in circumstances where one is restricted to boundary subspaces with effectively two-dimensional geometry. A canonical example of such a restriction occurs within the 4D subregion described by a Wheeler-DeWitt wavefunctional of AdS_4 quantum gravity. An AdS_4 analog of Minkowski "super-rotation" asymptotic symmetry is probed by 3D Einstein gravity, yielding CFT_2 structure (in a large central charge limit), via AdS_3 foliation of AdS_4 and the $\mathrm{AdS}_3/\mathrm{CFT}_2$ correspondence. The maximal asymptotic symmetry is however probed by 3D *conformal* gravity. Both 3D gravities have Chern-Simons formulation, manifesting their topological character. Chern-Simons structure is also shown to be emergent in the Poincare patch of AdS_4, as soft/boundary limits of 4D gauge theory, rather than "put in by hand" as an external probe. This results in a finite effective Chern-Simons level. Several of the considerations of asymptotic symmetry structure are found to be simpler for AdS_4 than for Mink_4, such as non-zero 4D particle masses, 4D non-perturbative "hard" effects, and consistency with unitarity. The last of these in particular is greatly simplified because in some set-ups the time dimension is explicitly shared by each level of description: Lorentzian AdS_4, CFT_3 and CFT_2. Relatedly, the CFT_2 structure clarifies the sense in which the infinite asymptotic charges constitute a useful form of "hair" for black holes and other complex 4D states. An AdS_4 analog of Minkowski "memory" effects is derived, but with late-time memory of earlier events being replaced by (holographic) "shadow" effects. Lessons from AdS_4 provide hints for better understanding Minkowski asymptotic symmetries, the 3D structure of its soft limits, and Minkowski holography.

Keywords: AdS-CFT Correspondence, Chern-Simons Theories, Gauge Symmetry, Global Symmetries

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Contents

1 Introduction

In gravitational and gauge theories, Asymptotic Symmetries (AS) are diffeomorphisms and gauge transformations that preserve the asymptotic structure of spacetime while still acting non-trivially on asymptotic dynamical data. They include isometries of spacetime and the standard global charges arising from gauge theory, but they can be larger. Famously, 4D Minkowksi spacetime (Mink$_4$) has an infinite-dimensional spacetime AS algebra (see [1] for a recent review). This was originally identified as the BMS algebra of super-translations [2, 3], but has been extended more recently to include super-rotations as a subalgebra [4]. We refer to this extended algebra in 4D as XBMS$_4$. The ongoing challenge since discovery of these symmetries has been to understand their physical significance and utility.

Considerable progress has been made in this regard by the discovery that the associated large diffeomorphisms and gauge transformations arise as soft limits of physical gravitational and gauge fields emerging from scattering processes [5–15], as captured by the Weinberg Soft Theorems [16–20]. The infinite-dimensional AS then describe the soft field dressing of a hard process, and are sensitive to the passage of charge/energy-momentum as a function of angle, through "memory" effects [21–30]. This generalization of the usual overall charge/energy-momentum conservation laws has led to the suggestion that AS charges can act as a new subtle form of "hair" that can characterize black holes (or other complex states), giving a finer understanding of black hole entropy and information puzzles [31–34].

The fact that the super-rotation subalgebra of Mink$_4$ AS has a Virasoro $\times$ $\overline{\text{Virasoro}}$ form (Vir $\times$ $\overline{\text{Vir}}$), while gauge theory gives rise to Kac-Moody (KM) subalgebras, is highly reminiscent of Euclidean two-dimensional conformal field theories (ECFT$_2$) [4]. Indeed such a ECFT$_2$-like structure living on the celestial sphere was discovered [35, 36], AS charges arising as Laurent expansion coefficients of a 2D holomorphic stress tensor and other currents. A straightforward derivation [36] follows by foliating Mink$_4$ by 3D de Sitter spacetimes (dS$_3$) and hyperbolic spaces [37–41], more suggestively considered as the Euclidean continuation of 3D anti-de Sitter (EAdS$_3$). 4D fields can then be "Kaluza-Klein" (KK) reduced by separation of variables into 3D (EA)dS$_3$ fields, with a continuum of 3D masses, $m_3^{\mathrm{KK}} > 0$. In this language, 4D S-matrix elements map to boundary (EA)dS$_3$ correlators [37], the associated 4D LSZ-reduced Feynman diagrams mapping to 3D Witten diagrams (modulo superpositions). Most importantly, the 3D massless limit, $m_3 \to 0$, corresponds to 4D soft limits, in particular the soft limit of 4D gauge theory yielding 3D Chern-Simons (CS) gauge fields, and the subleading soft limit of 4D General Relativity (GR$_4$) fields yielding GR$_3$ (which also has a CS formulation [42]) on (EA)dS$_3$. The basic grammar of (EA)dS_3/$ECFT_2$ [43] then yields the $ECFT_2$-like structure. The 3D CS fields "live" on the boundary of 4D spacetime.

Despite these recent developments, several important questions and puzzles remain:

- A central question is how fully the axioms of CFT_2 are realized in the structure underlying AS. In particular, it has not been clear what the values of the associated central charge and KM levels are, whether zero, infinite or finite. This question is not answerable at the AS level of discussion which focuses on *external* CS/soft fields, since the central charge and levels are probed by internal CS/soft lines (at tree level). It was argued in ref. [36], that a central charge would be IR sensitive to the experimental delineation between "soft" and "hard", but this was not fully clarified.

- The $ECFT_2$ structure is not consistent with being the Euclidean continuation of a unitary CFT_2, much as in the dS/ECFT context. It is an open question as to how the unitarity of the $Mink_4$ quantum gravity (QG) S-matrix is encoded in the $ECFT_2$ correlators.

- The subleading soft limit of GR_4 leads to the super-rotation subalgebra of $Mink_4$ AS, and is elegantly encoded in GR_3, which has a $SO(3,1)$ CS formulation, but the leading soft limit and the associated super-translations do not have a CS formulation [36]. Naively, the $SO(3,1)$ Lorentz gauge group should be extended to the full Poincare group $ISO(3,1)$ as the CS gauge group in order to include (super-)translations, but $ISO(3,1)$ lacks the requisite quadratic invariant to construct a CS action. Relatedly, ref. [36] found that the $ECFT_2$ current, whose Laurent expansion yields super-translations, is non-primary. Therefore there is an open question as to what the 3D characterization of subleading and leading soft GR_4 fields is that leads to $XBMS_4$ in a unified way.

- Previous discussions of memory effects describe them in classical terms, while the hallmark of CS theories are quantum mechanical topological effects that generalize the Aharonov-Bohm effect [44–46]. These two views of memories need to be better reconciled.

- The connection of AS to 3D CS characterization of soft fields hints at a possible connection to a 3D holographic duality with $Mink_4$ QG, but this connection has not been spelled out.

- It is very attractive to contemplate AS charges as a new rich form of "hair" for black holes or other 4D states. But such a role is still unclear, and being debated [47, 48].

In this paper, we make some progress on all these fronts within a more transparent context, by generalizing the notion of AS to AdS_4 QG and gauge theory. Primarily this is because we know the 3D holographic dual of AdS_4 is CFT_3 [49–55], and there is a natural way to connect this to CS and GR_3, and from this to CFT_2 and infinite-dimensional AS. Yet by standard analysis the AS of asymptotically AdS_4 GR only consist of the finite-dimensional isometries [56], $SO(3,2)$, in sharp contrast to the infinite-dimensional AS of asymptotically $Mink_4$ GR. Let us sketch why this is the case.

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First consider Mink_4,

$$ds^2_{\mathrm{Mink}_4} = \frac{1}{\cos^2 u_+ \, \cos^2 u_-}\left(du_+\, du_- - \frac{1}{4}\sin^2(u_+ - u_-)\, d\Omega_2^2\right), \qquad u_\pm = \tan^{-1}(t \pm r), \tag{1.1}$$

where $d\Omega^2$ is the usual metric of the angular sphere. We see that at the boundary of Mink_4, $u_+ = \pm\pi/2$ and $u_- = \pm\pi/2$,

$$ds^2_{\partial \mathrm{Mink}_4} \underset{\mathrm{Weyl}}{\sim} d\Omega_2^2, \tag{1.2}$$

where $\underset{\mathrm{Weyl}}{\sim}$ refers to Weyl equivalence, modulo which the notion of conformal boundary is defined. While the boundary manifold is three-dimensional, because of the null direction the geometry degenerates to being effectively two-dimensional. A necessary condition for large diffeomorphisms to correspond to AS is that they preserve this boundary structure. In particular, these diffeomorphisms include those reducing to *conformal isometries* on the boundary geometry, namely the infinite-dimensional conformal symmetries of the 2D angular sphere, and correspond to the super-rotations. But in $\mathrm{AdS}_4^{\mathrm{global}}$,

$$ds^2_{\mathrm{AdS}_4} = \frac{1}{\cos^2\psi}\left(d\tau^2 - d\psi^2 - \sin^2\psi\, d\Omega_2^2\right), \tag{1.3}$$

the boundary at $\psi = \pi/2$ has a fully three-dimensional geometry

$$ds^2_{\partial \mathrm{AdS}_4^{\mathrm{global}}} \underset{\mathrm{Weyl}}{\sim} d\tau^2 - d\Omega_2^2. \tag{1.4}$$

The conformal isometries of this boundary $S^2 \times \mathbb{R}$, and hence AS of AdS_4, are just finite-dimensional $\mathrm{SO}(3,2)$. By contrast, in the case of AdS_3, $\partial\mathrm{AdS}_3$ is obviously two-dimensional, famously with infinite-dimensional conformal isometries and AS [57].

Nevertheless, there is a loop-hole to this no-go argument for infinite-dimensional AdS_4 AS if one is restricted to subspaces of $\partial\mathrm{AdS}_4$ with *two-dimensional* geometry, which we will see can happen for different physical reasons. Most straightforwardly, this is illustrated by the subregion of AdS_4 described by a Wheeler-DeWitt QG wavefunctional, holographically dual to a quantum state of CFT_3 at some fixed time, as depicted in figure 1. Its 3D boundary resembles the null boundary of Mink_4, with effectively 2-dimensional geometry, reflecting the two-dimensional holographic geometry of $\partial\mathrm{AdS}_4$ at $\tau = 0$. This has infinite-dimensional conformal isometries, leading to infinite-dimensional AS.

The basic strategy of this paper will be to study CS gauge theory and GR_3 coupled to CFT_3, where the CFT_3 is (in isolation) the holographic dual of AdS_4 QG, on a variety of 3D spacetimes $\mathcal{M}_3$:

$$S = S_{\mathrm{CS}} + S_{\mathrm{GR}_3} + S_{\mathrm{CFT}_3} + \text{UV-completion}. \tag{1.5}$$

The CFT_3 global internal symmetries are gauged by the CS sector, and the CFT_3 spacetime symmetries are gauged by GR_3. Such GR_3 and CS + matter theories are well-known to have infinite-dimensional AS [57–65]. In particular, when $\mathcal{M}_3 = \mathrm{AdS}_3$, $\mathrm{AdS}_3/\mathrm{CFT}_2$ implies this setup is dual to CFT_2, where there is a standard connection of the 2D chiral currents and stress-tensor with infinite-dimensional KM and $\mathrm{Vir}_+ \times \mathrm{Vir}_-$ symmetries (briefly reviewed

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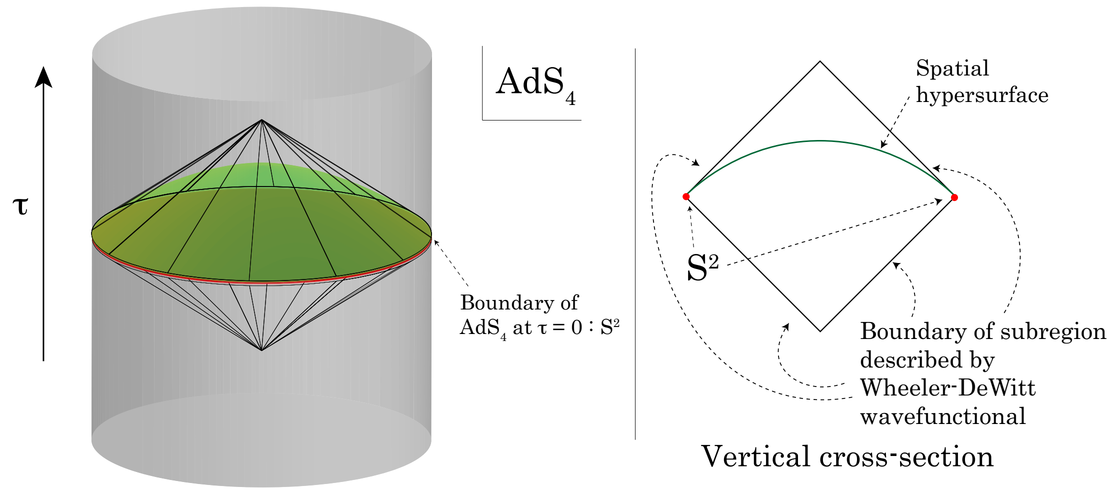

Figure 1. CFT_3 state living on S^2 at $\tau = 0$ on ∂AdS_4 (shown in red), dual to Wheeler-DeWitt wavefunctional describing the subregion of AdS_4 enclosed by the black cones. This subregion is spanned by all spacelike hypersurfaces ending on this boundary S^2. An example of such a hypersurface is shown in green. A vertical cross-section is shown on the right.

in section 2). The infinity of (AS) charges of CFT_2 (AdS_3) form a well-known type of 2D (3D) "hair", operating on and finely diagnosing quantum states, in a manner generalizing the action of ordinary conserved global charges. But now the CFT_3/AdS_4 duality of the 3D matter "lifts" the AS charges and their utility to 4D.

This construction yields three layers of description of the dynamics. The quarks and gluons of some large-N_{color} formulation of CFT_3 will be called for brevity, "quarks". The dual AdS_4 gravitons and matter are the "hadrons", composites of the 3D "quarks", the 4D fields being equivalent to KK towers of 3D "hadronic" states related by 3D conformal symmetry. The well-defined ∂AdS_3 correlators will involve external lines of these "hadrons", rather than "quarks" (as discussed in section 3). This is in complete analogy to the well-defined nature of hadronic S-matrix elements in Minkowski spacetimes, as compared with the provisional nature of the quark/gluon S-matrix. Even more fundamentally, the CFT_3 "quarks" and the CS + GR_3 fields themselves are composites of the CFT_2 degrees of freedom, which we call "preons". AS charges are simple moments of these local "preon" degrees of freedom. A nice feature here is that time persists at each layer of description, and hence unitarity is manifest at each stage. The 4D loop expansion (controlled by the expansion parameter $1/N_{\text{color}}$ in 3D) can be done to all orders without spoiling these results. Including 4D massive particles is straightforward, captured automatically by the CFT_3 description.

We will show that even in the large-level limit, in which the CS and GR_3 fields are decoupled, these AS remain as subtle charges of the matter sector, CFT_3 (see section 4). Because the CFT_3 on AdS_3 is dual to (half of) AdS_4, the 3D AS are inherited as AS of AdS_4 QG and gauge theory. From the 4D perspective (section 5), the "hadronic" ∂AdS_3 correlators which manifest the infinite-dimensional AS are also ∂AdS_4 correlators, but not of the standard form. In particular, the $\partial\mathrm{AdS}_3$ endpoints are restricted to a submanifold of $\partial\mathrm{AdS}_4$ with two-dimensional geometry, one natural realization of the loop-hole mentioned earlier in the no-go argument for infinite-dimensional AdS_4 AS (section 6).

The AS of AdS_4 are in fact closely analogous to those of Mink_4, in particular the $\mathrm{Vir}_+ \times \mathrm{Vir}_-$ can be viewed as analogous to Mink_4 super-rotations. The analog of Mink_4 super-translations is subtler. We will show (section 7) that these can be incorporated by replacing GR_3 by 3D *conformal* gravity (CGR_3) [66, 67], which also has a $SO(3,2)$ CS formulation [42, 68]. In the case of $\mathcal{M}_3 = \mathrm{AdS}_3$, this leads to an extension of the AS by a KM algebra [69]. But the full AS of AdS_4 is even larger, because CFT_3 on AdS_3 only projects half of AdS_4. The technically simplest approach to the full AS structure is taken by switching to $\mathcal{M}_3 = \mathrm{Mink}_3$, where the dual of the CFT_3 is given by the Poincare patch, $\mathrm{AdS}_4^{\mathrm{Poincare}}$ (section 8). While not the entirety of $\mathrm{AdS}_4^{\mathrm{global}}$, it shares all of its (infinitesimal) isometries, and hence exhibits the full AS algebra. This full AS structure allows us to run the connection to $\mathrm{AdS}_4/\mathrm{CFT}_3$ duality in reverse: if one begins by identifying the AS of AdS_4 in CGR_3 ($SO(3,2)$ CS) form, the only form of compatible matter that can couple to CGR_3, respecting its Weyl invariance, is CFT_3. In this sense, the holographic grammar follows from the AS structure.

The Poincare patch provides other simplifications. It gives the most straightforward 4D dual picture when GR_3 is *not yet decoupled* from CFT_3, namely a lower-dimensional Randall-Sundrum 2 (RS2) construction [70], with a 3D "Planck brane" in a 4D bulk [71]. The GR_3 then incarnates as the localized gravity of RS2. The familiarity of RS2 helps to make an important contrast. We have argued above, and in the body of this paper, that the infinite-dimensional AS are most readily recognized as coming from 3D GR_3/CS fields, and yet are interesting because we can "lift" them beyond three dimensions. But there appears to be an even easier way to arrange this, by just considering gravitational theories in higher-dimensional product spacetimes of the form $\mathrm{Mink}_3 \times X$ or $\mathrm{AdS}_3 \times X$, where X is some compact manifold. Under Kaluza-Klein reduction to Mink_3 or AdS_3, such theories would have a GR_3 3D-massless mode, which would again yield infinite-dimensional symmetries. The distinction with what we are doing here is that such product theories would not have a non-trivial decoupling limit for the GR_3 fields. That is, we cannot sensibly remove the GR_3 subsector in some limit while keeping the rest of the physics fixed. But RS2 with 4D bulk is dual to $\mathrm{GR}_3 + \mathrm{CFT}_3$, and there is a limit in which the 3D gravitational coupling vanishes, leaving a fixed limiting CFT_3, dual to $\mathrm{AdS}_4^{\mathrm{Poincare}}$ QG. In other words, we will argue that GR_3/CS has a tight connection with AS structure on the one hand and with 3D holography of the 4D QG on the other. But this only takes place in higher-dimensional theories where the GR_3 subsector has a decoupling limit. Higher-dimensional product spacetimes are not of this type.

The Poincare patch also provides the stage to simply derive the *emergence* of CS gauge fields as helicity-cut soft/boundary limits of AdS_4 gauge fields (CFT_3 composites), which couple to charged modes (section 9). In this way, the CS structure is not put in "by hand" and then removed by a large-level limit, but rather describes a subsector of the pure $\mathrm{CFT}_3/\mathrm{AdS}_4$, with a finite but subtle type of CS level. We will see that the effective CS gauge fields mediate analogs of the "memory" effects identified in Mink_4, which we call

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"shadow" effects since their relationship to the holographically emergent spatial direction is analogous to the relationship of memory effects to time.

For the CFT_3 to project all of AdS_4^{global}, we must choose $\mathcal{M}_3 = S^2 \times \mathbb{R}$ (section 10), but this closed universe does not have an asymptotic region or boundary to straightforwardly display AS. The AS arise by cutting at some point in time (say zero), so that the wavefunctional is given by functional integration up to that point, that is on $\mathcal{M}_3 = S^2 \times \mathbb{R}^-$ (where the last factor refers to only negative values of time). This yields precisely the holographic dual of the Wheeler-DeWitt wavefunctional in AdS_4, briefly discussed above.

Finally, it is obviously of interest to ask how to translate the insights of AdS_4 AS back to $Mink_4$ (section 11). A strategy is suggested by the argument of section 8 for deriving the holographic grammar of AdS_4 from its AS structure. In $Mink_4$ we are ignorant of the former but know the latter, so the analogous steps should yield new insight into $Mink_4$ holography. The first step is to give the 3D characterization of the full AS and soft fields of $Mink_4$ QG, in analogy to identifying CGR_3 for AdS_4. Currently this is not known for the $Mink_4$ super-translations, although super-rotations take a simple GR_3 form. We will provide some concrete guesses as to how to obtain the full 3D structure, which will then form the "mold" for a compatible holographic form of (hard) matter.

2 Lightning review of CS/GR_3 AS and CFT_2 currents

CS theories, including GR_3 in CS form, are famously gauge invariant and topological, insensitive to the geometry on 3D spacetime $\mathcal{M}_3$, except at the boundary $\partial\mathcal{M}_3$ where local degrees of freedom emerge, exhibiting infinite-dimensional AS. We briefly review how this happens for $\mathcal{M}_3 = AdS_3$, where the boundary structure and AS are just those of the dual CFT_2. Concretely, we write the metric in the form

$$ds^2_{AdS_3^{global}} = \frac{d\tau^2 - d\rho^2 - \cos^2\rho\, d\phi^2}{\sin^2\rho}, \qquad R_{AdS_3} \equiv 1. \tag{2.1}$$

A point y^μ in AdS_3 is represented by the coordinates (τ, ϕ, ρ), where $-\infty < \tau < \infty$, $0 \le \phi < 2\pi$ and $0 < \rho \le \pi/2$. The space of AdS_3 is conformally equivalent to $S^2/2 \times \mathbb{R}$, and the boundary ∂AdS_3 is at $\rho = 0$ in these coordinates.

2.1 Non-abelian CS gauge theory

We begin with internal CS gauge theory,

$$S_{CS} = \frac{\kappa}{4\pi} \int d^3y\, \epsilon^{\mu\nu\rho}\, \mathrm{Tr}\left(A_\mu \partial_\nu A_\rho + \frac{2}{3} A_\mu A_\nu A_\rho\right), \tag{2.2}$$

where $A_\mu \equiv A_\mu^a\, t^a$, t^a are the generators of the CS gauge group, $\mathrm{Tr}(t^a t^b) = \delta^{ab}$, and κ is the CS level.

This action is metric-independent and gauge-invariant in the AdS_3 "bulk", but since gauge-invariance depends on integration by parts it is violated on the boundary, ∂AdS_3. This implies that "gauge orbit" degrees of freedom "live" on this 2D boundary, $\rho = 0$, which is the root of the equivalence of the CS gauge sector to a 2D Wess-Zumino-Witten (WZW) current-algebra sector on the boundary [62–65].

It is convenient to use light-cone coordinates in the boundary directions,

$$z^{\pm} \equiv \tau \pm \phi. \tag{2.3}$$

The equations of motion read

$$\begin{aligned}\delta S_{\text{CS}} = \frac{\kappa}{4\pi}\,\text{Tr}\int d^3y\epsilon^{\mu\nu\rho}\left(\delta A_\mu F_{\nu\rho}\right) \\ +\frac{\kappa}{2\pi}\text{Tr}\int dz^+\,dz^-\,\Big(\delta A_-(\rho=0)\,A_+(\rho=0)-\delta A_+(\rho=0)\,A_-(\rho=0)\Big)=0\,.\end{aligned} \tag{2.4}$$

This implies boundary conditions, $A_{\pm}(\rho=0)=0$. Further, bulk gauge invariance can be used to go to the axial gauge: $A_\rho=0$. With this the boundary conditions are too stringent, giving $A_\mu=0$ throughout AdS_3 as the only solution to the first order equations.

We can modify the boundary conditions to constrain just one linear combination of boundary components of A, say A_-. To accomplish this we can add a boundary term to the action,

$$S_{\partial\text{AdS}_3} = -\frac{\kappa}{2\pi}\int dz^+\,dz^-\,\text{Tr}\,\Big(A_-(\rho=0)\,A_+(\rho=0)\Big)\,. \tag{2.5}$$

(While this explicitly violates gauge invariance, recall the bulk action is already not gauge-invariant on the boundary.) In the presence of this term, the total boundary contribution to the variation of the action is given by

$$\delta S_{\text{total}}\Big|_{\partial\text{AdS}_3} = -\frac{\kappa}{\pi}\int dz^+\,dz^-\,\text{Tr}\,\Big(\delta A_+\,A_-\Big), \tag{2.6}$$

implying the boundary condition $A_-(z^+,z^-,\rho=0)=0$. But now $A_+(z^+,z^-,\rho=0)$ is unconstrained, consistent with non-trivial solutions (in the presence of matter).

Even though we have fixed axial gauge $A_\rho=0$, we must retain the A_ρ equation of motion,

$$F_{+-}=0, \tag{2.7}$$

away from any matter sources, where F is the non-abelian field strength. Evaluating this on the boundary, and using the boundary condition $A_-=0$,

$$F_{+-}\underset{\rho\to 0}{\longrightarrow}\partial_- A_+=0. \tag{2.8}$$

The dual CFT$_2$ current,

$$j_+(z^+,z^-)=\lim_{\rho\to 0}A_+(z^+,z^-,\rho), \tag{2.9}$$

is therefore chirally conserved,

$$\partial_- j_+=0\,,\; j_+=j_+(z^+). \tag{2.10}$$

The Fourier components define AS charges,

$$j_+(z^+)=\sum_{n\in\mathbb{Z}} Q_n^{a+}(\tau)t^a\,e^{in\phi}\;\;, \tag{2.11}$$

which are angle-dependent "harmonics" of the conserved global charges, Q_0^{a+}. The τ dependence of $Q_n^{a+}(\tau)$ follows by the fact that j_+ is a function of $z^+ = \tau + \phi$ only, simply given by

$$Q_n^{a+}(\tau) \propto e^{in\tau} . \tag{2.12}$$

In the Q_n^+ basis, the simple structure of j correlators within $\partial\mathrm{AdS}_3$ Witten diagrams takes the form of a Kac-Moody algebra (at $\tau = 0$),

$$\left[Q_n^{a+},\, Q_m^{b+} \right] = \sum_c i\, f^{abc}\, Q_{n+m}^{c+} + \kappa\, n\, \delta^{ab}\, \delta_{n+m,0} \tag{2.13}$$

where f^{abc} are the structure constants, and the CS level κ sets the central extension. The non-abelian first term on the right-hand side reflects the non-abelian CS interaction, while the central extension second term on the right-hand side reflects the CS "propagation".

2.2 $\mathbf{GR_3}$

Consider next the case of 3D gravity on AdS_3, which can be formulated as a $\mathrm{SO}(2,2)$ CS theory in terms of dreibein and spin connection variables [42], with level

$$\kappa_{\mathrm{grav}} = \mathrm{M}_3^{\mathrm{Pl}}\, R_{\mathrm{AdS}_3} . \tag{2.14}$$

The dreibein VEVs lock the six $\mathrm{SO}(2,2)$ global generators $L^{\pm}_{n=-1,0,1}$ to the AdS_3 isometries. The action of these generators at the boundary of AdS_3 is given by

$$L_n^{\pm} \underset{\partial\mathrm{AdS}_3}{\longrightarrow} e^{inz^{\pm}}\, \partial_{\pm} \,, \qquad n = \pm 1, 0 \,. \tag{2.15}$$

Analogous to the case of internal CS gauge symmetries, the stress tensor components are chiral, and their Fourier modes give angle-dependent "harmonics" of the above $\mathrm{SO}(2,2)$ global symmetries,

$$t_{++}(z^+) = \sum_{n\in\mathbb{Z}} L_n^+(\tau) e^{in\phi} \,, \qquad t_{--}(z^-) = \sum_{n\in\mathbb{Z}} L_n^-(\tau) e^{-in\phi} \,, \tag{2.16}$$

where the τ dependence of $L_n^+(\tau)$ is fixed:

$$L_n^+(\tau) \propto e^{in\tau} . \tag{2.17}$$

These AS charges now form a $\mathrm{Vir}_+ \times \mathrm{Vir}_-$ algebra [57] generalizing the $\mathrm{SO}(2,2)$ isometries, as opposed to a KM algebra if there had been no VEVs (as reviewed in ref. [72]),

$$[L_m^{\pm}, L_n^{\pm}] = (m-n)\, L_{m+n}^{\pm} + \frac{c}{12}\, m(m^2-1)\, \delta_{m+n,0} \,, \qquad m, n \text{ integer}. \tag{2.18}$$

The central charge is given by

$$c = \kappa_{\mathrm{grav}} = \mathrm{M}_3^{\mathrm{Pl}} R_{\mathrm{AdS}_3} . \tag{2.19}$$

Again, the two terms on the right-hand side are the 2D reflection of the non-abelian interaction of GR_3 and the free "propagation".

The charges L_n have a non-zero commutator with internal KM charges Q_n,

$$[L_m^+, Q_n^+] = nQ_{m+n}^+ \tag{2.20}$$

while the commutator between + and − charges vanishes. Given that $m = 0$ measures the energy corresponding to τ translational symmetry: $E_\tau = L_0^+ + L_0^-$, it follows that

$$[L_0^+ + L_0^-, Q_n^+] = n\, Q_n^+, \tag{2.21}$$

matching our earlier observation that $Q_n^+ \propto e^{in\tau}$.

3 Holographic matter coupled to CS/GR$_3$ on AdS$_3$

We now couple CS and GR$_3$ to 3D matter in the form of CFT$_3$, all living on asymptotic AdS$_3$. The CFT$_3$ is chosen such that when living (in isolation) on ∂AdS$_4 = S^2 \times \mathbb{R}$ it is holographically dual to some AdS$_4$ QG and gauge theory.

3.1 CFT$_3$ in isolation on AdS$_3$

We begin by noting that

$$\mathrm{AdS}_3 \underset{\mathrm{Weyl}}{\sim} S^2/2 \times \mathbb{R}, \tag{3.1}$$

where $\underset{\mathrm{Weyl}}{\sim}$ denotes Weyl equivalence, and $S^2/2$ denotes the hemisphere. Since ∂AdS$_4$ $= S^2 \times \mathbb{R}$ is only defined up to Weyl equivalence, this suggests that CFT$_3$ on AdS$_3$ is holographically dual to *half* of AdS$_4$, as follows.

It is useful to use AdS$_4$ coordinates exhibiting an AdS$_3$ foliation [73],

$$\begin{aligned} ds^2_{\mathrm{AdS}_4} &= -dr^2 + \cosh^2 r\, ds^2_{\mathrm{AdS}_3}, && r \in \mathbb{R}, \qquad R_{\mathrm{AdS}_4} = 1 \\ ds^2_{\mathrm{AdS}_3} &= \frac{1}{\sin^2\rho}\left(d\tau^2 - d\rho^2 - \cos^2\rho\, d\phi^2\right), && \qquad\qquad R_{\mathrm{AdS}_3} = 1. \end{aligned} \tag{3.2}$$

The AdS$_3$ coordinates (τ, ϕ, ρ) have the ranges $-\infty < \tau < \infty, 0 < \rho \leq \pi/2$, $0 \leq \phi < 2\pi$, while the fourth dimension coordinate r takes all real values. Ref. [74] argued (translating their analysis down a dimension to the set-up of interest here) that CFT$_3$ states on AdS$_3$, reflecting off ∂AdS$_3$ are dual to AdS$_4$ particles in the region $r > 0$ reflecting off the $r = 0$ surface. The specific boundary condition at $r = 0$ is determined by the whether or not the CFT$_3$ ground state on AdS$_3$ preserves or spontaneously breaks the CFT$_3$ global symmetry. We will consider the case where the global symmetry is preserved, in which case we must choose Neumann boundary condition at $r = 0$. We denote the region $r > 0$, holographically projected by CFT$_3$, by "AdS$_4$/2". In the original AdS$_4$ global coordinates the AdS$_3$ foliation by constant r hypersurfaces is depicted in figure 2, where the restriction to AdS$_4$/2 corresponds to keeping only the northern half of the coordinate ball, $r = 0$ being the equatorial disc. The CFT$_3$ lives on the boundary of this region, the upper hemisphere.

∂AdS correlators are the classic diffeomorphism and gauge invariant observable in AdS QG, just as the S-matrix is in Mink QG. Here we are preparing to couple CFT$_3$ on AdS$_3$ to GR$_3$ and CS, so we are interested in ∂AdS$_3$ correlators. In this subsection however we

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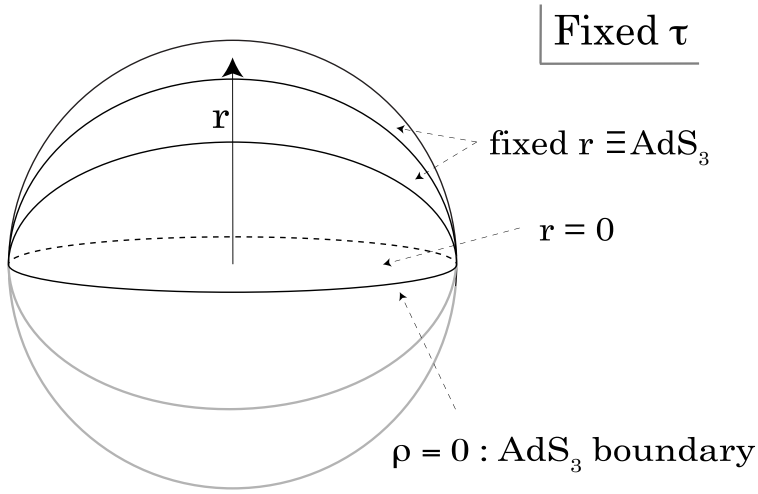

Figure 2. AdS_3 foliation of AdS_4 in global coordinates. A CFT_3 on AdS_3 projects only the upper half of AdS_4.

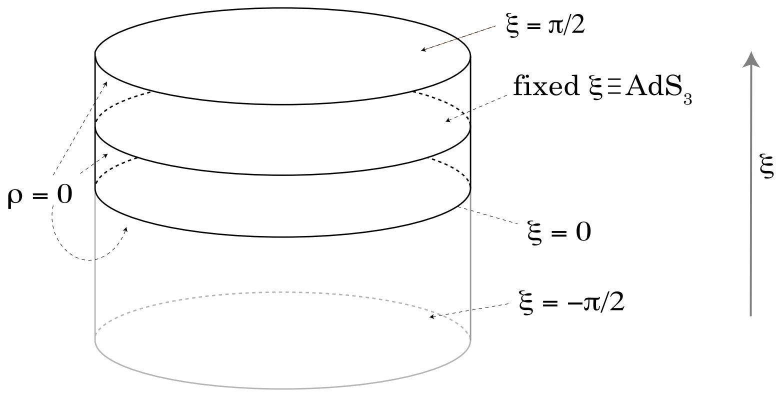

Figure 3. AdS_3 foliation of AdS_4 in "product-space" coordinates.

are not yet including the gauging by GR_3 and CS, focusing therefore on ∂AdS_3 correlators of just the CFT_3. In standard Minkowski QCD we have a provisional meaning for the S-matrix elements of quarks and gluons. But strictly speaking this is ill-defined because they are not asymptotic states. Instead we should more properly consider S-matrix elements of hadrons such as protons and pions. Similarly, with the CFT_3, instead of "quark" ∂AdS_3 correlators, we consider "hadron" ∂AdS_3 correlators. Of course these "hadrons" are given precisely by the AdS_4 dual. But now each AdS_4 field contains many "hadronic" AdS_3 mass eigenstates, which we can isolate by KK decomposition based on the AdS_3 foliation.

We illustrate this for the simple case of tree-level AdS_4 Yang-Mills theory, with 4D field $\mathcal{A}$. For this purpose it is convenient to adopt what we call "product-space" coordinates.

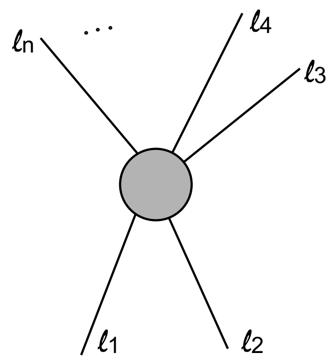

Figure 4. Witten diagrams for $\partial\mathrm{AdS}_3$ correlators of KK modes ($\equiv$ CFT_3 "hadrons"). External lines are AdS_3 bulk-boundary propagators for masses proportional to ℓ. Blob consists of AdS_3 KK interactions and bulk-bulk lines (Fourier transformed in ξ from AdS_4).

Using the change of variables from r to $\xi \equiv 2\tan^{-1}(\tanh(r/2))$, the metric changes to

$$\begin{aligned} ds^2_{\mathrm{AdS}_4} &= f^2(\xi)\,\left(-d\xi^2 + ds^2_{\mathrm{AdS}_3}\right) \\ \xi &= 2\tan^{-1}\left(\tanh\frac{r}{2}\right), \qquad f(\xi) = \cosh r\,, \end{aligned} \tag{3.3}$$

displaying Weyl-equivalence to the product geometry $\mathrm{AdS}_3\times$ Interval. The restricted region $\mathrm{AdS}_4/2$, $0 < r < \infty$ corresponds to $0 < \xi < \pi/2$ (in AdS units). Figure 3 shows this "product-space" representation of AdS_4.

Because of the Weyl invariance of classical 4D Yang-Mills, the factor $f^2(\xi)$ is irrelevant and the spacetime is effectively of product form. (Non-Weyl-invariant theories can also be KK-decomposed, but less straightforwardly.) In standard KK fashion, in axial gauge $\mathcal{A}_\xi = 0$, the 4D Maxwellian field decomposes as

$$\mathcal{A}_\mu(\tau,\phi,\rho,\xi) = \sum_{\ell\in\mathbb{Z}} A^\ell_\mu(\tau,\phi,\rho)\cos((2\ell+1)\xi)\,, \tag{3.4}$$

where the $A^\ell_\mu(\tau,\phi,\rho)$ are a tower of 3D Proca fields with AdS_3 masses $2\ell+1$ in units of AdS radius. It is AdS_3 Witten diagrams of these KK fields that correspond to "hadron" $\partial\mathrm{AdS}_3$ correlators. We depict such diagrams in figure 4.

3.2 CS and GR_3 coupled to CFT_3

We now switch on CS and GR_3, gauging any internal global symmetries of CFT_3 (dual to 4D gauge symmetries) and the global spacetime symmetries and stress tensor of CFT_3 (dual to 4D gravity), so that $\partial\mathrm{AdS}_3$ correlators include the dual 2D chiral currents $j_\pm$, and stress tensor $t_{\pm\pm}$. The 3D KK modes discussed above are dual to local 2D primary operators $\mathcal{O}^\ell_{2D}$. All the CFT_2 operators are composites of some 2D "preon" fields.

A typical Witten diagram is shown in figure 5, with CS and GR_3 lines decorating the earlier purely "hadronic" diagrams. Such 3D Witten diagrams yield general CFT_2 dual correlators, now including stress tensor and chiral currents, $\langle 0|\,T\{j_\pm\ldots t_{\pm\pm}\ldots\mathcal{O}^\ell\ldots\}\,|0\rangle$. As we reviewed in section 2, this CFT_2 has infinite-dimensional symmetries associated with its chiral currents and stress tensor.

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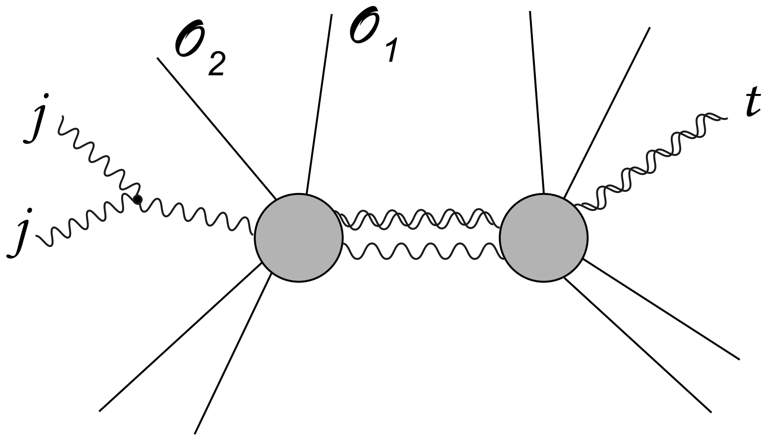


Figure 5. Witten diagram for ∂AdS_3 correlators, involving internal and external graviton/gluon lines.

4 The large-level "probe" limit

The decoupling of the CS and GR_3 sectors from CFT_3 is accomplished by simply taking the large CS-level limit, $\kappa \to \infty$. (For decoupling the GR_3 sector this is equivalent to the large-central-charge limit of the Virasoro symmetry of the CFT_2 dual.) We will show that in this limit there is a remnant of the AS algebra that survives for CFT_3 alone, providing a new form of "hair" for the dual $\text{AdS}_4/2$ states and black holes.

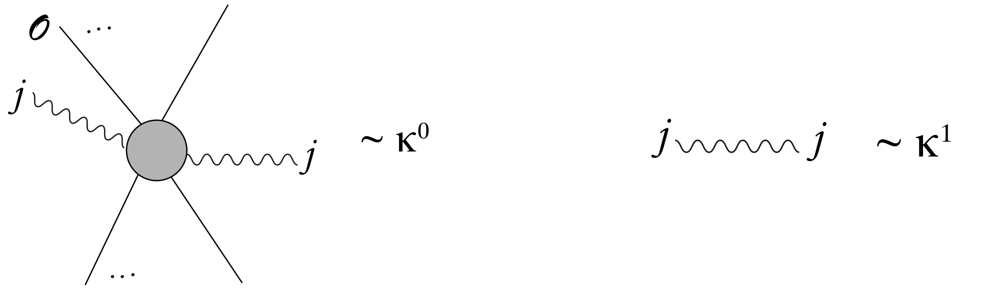
4.1 Abelian CS

The diagrammatics are very simple in the abelian CS case. The factor of $1/\kappa$ suppresses CS propagators, so the large level limit $\kappa \to \infty$ naively eliminates AdS_3 correlators involving CS lines altogether. However, choosing the normalization for the dual CFT_2 current according to

$$j_+ = \kappa \lim_{\rho \to 0} A_+(z^+, z^-, \rho)\,, \tag{4.1}$$

we effectively multiply CS endpoints in ∂AdS_3 Witten diagrams by κ, canceling the $1/\kappa$ of bulk-boundary propagators, so these survive the limit. Only bulk-bulk CS lines are suppressed. The surviving diagrams have the form shown in figure 6.

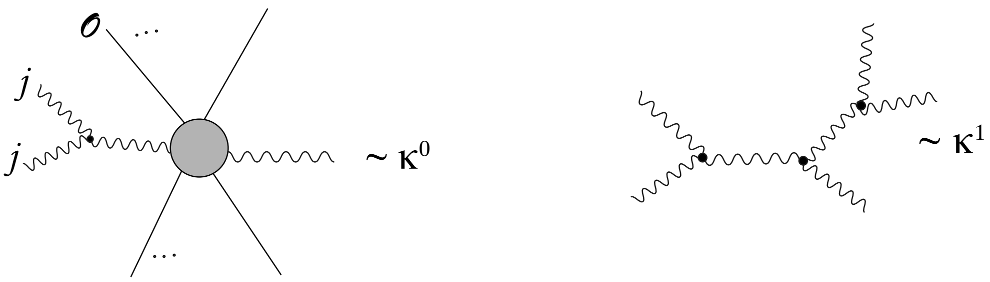
We see that correlators with the CFT_3 are $\mathcal{O}(\kappa^0)$ (figure 6a). However, the pure CS diagram shown in figure 6b corresponding to the correlator $\langle j_+ \, j_+ \rangle$ is special. While the propagator scales as $1/\kappa$, there are two factors of κ for the two end points, making this $\mathcal{O}(\kappa)$, dominating all other correlators as $\kappa \to \infty$. But, if we restrict our attention to correlations with CFT_3 "matter" (AdS_4 particles), then obviously this purely CS correlator drops out and we have a finite limit as $\kappa \to \infty$. This explains a puzzle regarding the CS level first seen in Mink_4 AS. For finite but large κ, κ appears in the central extension of the KM algebra as the KM face of the $\langle j_+ \, j_+ \rangle$ correlator. But if we are only tracking correlations that involve 4D particles (CFT_3), then we are blind to the purely CS correlator $\langle j_+ \, j_+ \rangle$



(a) Leading Witten diagrams including external CFT_3 lines.

(b) Leading Witten diagram without external CFT_3 lines.

Figure 6. Leading Witten diagrams in abelian CS theory for large κ.



(a) Leading diagrams with external CFT_3 lines.

(b) Leading diagrams without external CFT_3 lines.

Figure 7. Leading non-abelian CS diagrams for large κ.

and may mistakenly conclude that we are in the limit of vanishing KM level, when in fact we are in the limit of infinite KM level!

4.2 Non-abelian CS and GR_3

For the case of non-abelian CS and GR_3, $\kappa \to \infty$ correlators with CFT_3 hadrons have only tree like CS branches dressing KK $\partial \mathrm{AdS}_3$ Witten diagrams, such as in figure 7a. This is very similar to the CS/soft dressing of Mink_4 hard S-matrix elements [36]. While these diagrams are $\mathcal{O}(\kappa^0)$ for large κ, again there are $\mathcal{O}(\kappa)$ correlators given by the pure CS tree diagrams, such as in figure 7b. And again, focusing on correlations with the CFT_3 matter eliminates these, and gives a finite limit as $\kappa \to \infty$.

The fact that the CS/GR_3 branches attach *externally* to CFT_3 subdiagrams (blobs), rather than connecting different CFT_3 subdiagrams as in figure 5, means that the surviving diagrams are effectively purely CFT_3 correlators, with the branches just smearing the correlator point for CFT_3 currents/stress-tensor where they attach. It is these smeared correlators that manifest the CFT_2 and AS structure (in large-level limit). That is, in this limit the CS/GR_3 are just probes of the dynamical CFT_3, with no backreaction on it. We

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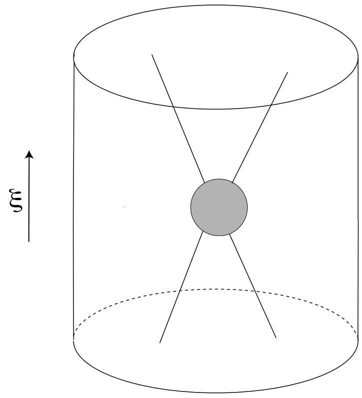
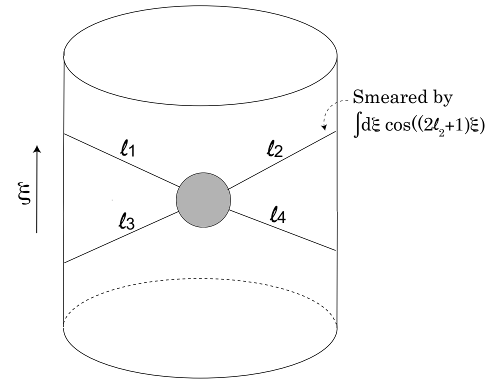


(a) Standard $\partial\mathrm{AdS}_4$ correlators $\neq \partial\mathrm{AdS}_3$ correlators. Here the external lines correspond to superpositions of 3D off-shell "hadrons".



(b) $\partial\mathrm{AdS}_3$ correlator as a non-standard $\partial\mathrm{AdS}_4$ correlator. External lines correspond to 3D on-shell "hadrons" (KK modes) of 3D mass $\propto \ell$.

Figure 8. Different types of Witten diagrams in AdS_4 in "product-space" coordinates.

discuss the structure and significance of the non-abelian branches as smearing functions in the next section, from the 4D viewpoint.

5 Non-standard $\partial\mathrm{AdS}_4/2$ correlators as CFT_2 correlators

A standard $\partial\mathrm{AdS}_4$ correlator is a gauge invariant correlator of local composite operators made of CFT_3 "quarks", but from the viewpoint of AdS_3 "hadron" mass eigenstates, they are off-shell correlators. Instead we are considering $\partial\mathrm{AdS}_3$ correlators of the "hadron" mass eigenstates. In 4D "product-space" coordinates (eq. (3.3), see figure 3) the distinction is shown in figure 8. These illustrate two alternative means of probing the bulk physics. In standard $\partial\mathrm{AdS}_4$ correlators we are putting sources and detectors on the ceiling and floor of AdS_4 (generic points on the $\partial\mathrm{AdS}_4$ in standard global coordinates) while having signals reflect off the walls with Dirichlet boundary conditions. In the KK-reduced $\partial\mathrm{AdS}_3$ correlators we have sources and detectors on the walls (only on $\partial\mathrm{AdS}_3 \equiv$ "equator" of $\partial\mathrm{AdS}_4$) with signals reflecting off the ceiling and floor with Dirichlet boundary conditions. (In the case of $\mathrm{AdS}_4/2$ we simply put the floor at $\xi = 0$, mid-level in the AdS_4 "product-space", with Neumann boundary conditions as discussed earlier.) Either way, no probability or energy is lost through the regions without sources because of the reflecting boundaries. We stress again that the reason we must insist on the non-standard form of $\partial\mathrm{AdS}_4$ correlators is because when $\mathrm{CS/GR}_3$ "emissions" are added, it is these that become $\mathrm{CS/GR}_3$ gauge/diffeomorphism invariant "on-shell" $\partial\mathrm{AdS}_3$ correlators. This is in contrast to the non-gauge/diffeomorphism invariance of standard $\partial\mathrm{AdS}_4$ correlators, which are "off-shell" from the AdS_3 viewpoint. The situation is entirely analogous to the gauge/diffeomorphism invariance of the Minkowski on-shell S-matrix in contrast to the non-invariance of off-shell Minkowski correlators in quantum field theory.

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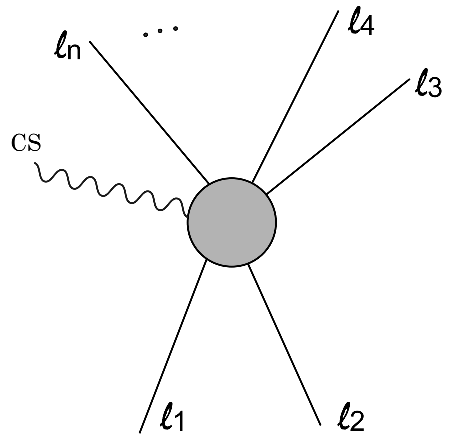

Figure 9. 3D Witten diagram with external abelian CS line.

5.1 Abelian gauge theory

For simplicity let us begin by considering U(1) CS coupled to a U(1) symmetry current of CFT_3, in turn dual to an AdS_4 U(1) gauge field. We focus on a 2D chiral current correlator of CFT_2 with other 2D operators in the large-κ limit. The 2D current of course contains the charges of a U(1) KM algebra by Laurent expansion. There are two equivalent ways of reading such CFT_2 correlators in the large-κ limit: (i) at face value, as a 3D "hadronic" correlator involving CS "emission" (see figure 9), or (ii) as a purely CFT_3 correlator involving a CFT_3 conserved current at a point y in the AdS_3 *bulk* (see figure 10), where this bulk point is "smeared" by a function of y given by the AdS_3 CS bulk-boundary propagator:

$$\langle 0|T\{j_+(z')\ldots\}|0\rangle_{\mathrm{CFT}_2} = \int d^3y\sqrt{g_{\mathrm{AdS}_3}}K^{\mathrm{CS}}_{+\mu}(z',y)\langle 0|T\{J^{\mu}_{\mathrm{CFT}_3}(y)\ldots\}|0\rangle_{\mathrm{CFT}_3}. \tag{5.1}$$

By standard $\mathrm{AdS}_4/\mathrm{CFT}_3$ diagrammatics, this lifts to 4D:

$$\langle 0|T\{j_+(z')\ldots\}|0\rangle_{\mathrm{CFT}_2} = \int d^3y\sqrt{g_{\mathrm{AdS}_3}}K^{\mathrm{CS}}_{+\mu}(z',y)\int d^4X\sqrt{-G_{\mathrm{AdS}_4}}\mathcal{K}^{\mu}_{N}(y,X)\mathcal{J}^N(X), \tag{5.2}$$

where $\mathcal{K}$ is an AdS_4 bulk-boundary propagator corresponding to the 4D photon line in figure 10, while $\mathcal{J}(X)$ is the bulk 4D current to which it couples, set up by the 4D matter.

We can write this compactly as

$$\langle 0|T\{j_+(z')\ldots\}|0\rangle_{\mathrm{CFT}_2} = \int d^4X\sqrt{-G_{\mathrm{AdS}_4}}\mathcal{A}_N(X)\mathcal{J}^N(X), \tag{5.3}$$

where

$$\mathcal{A}_N(X) \equiv \int d^3y\sqrt{g_{\mathrm{AdS}_3}}K^{\mathrm{CS}}_{+\mu}(z',y)\mathcal{K}^{\mu}_{N}(y,X). \tag{5.4}$$

By the defining properties of $\mathcal{K}$ in ξ-axial gauge, $\mathcal{A}_\mu(X)$ is that solution to the sourceless 4D Maxwell equations with boundary limit,

$$\mathcal{A}_\mu(y,\xi) \underset{\xi\to\pi/2}{\longrightarrow} K^{\mathrm{CS}}_{+\mu}(z',y). \tag{5.5}$$

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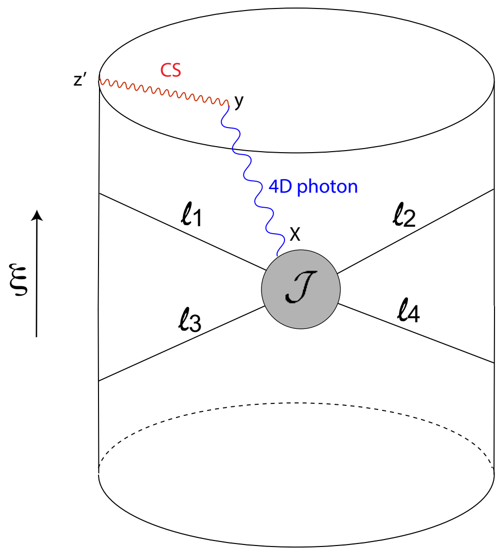

Figure 10. 4D Witten diagram in AdS_4, with the external lines ending on ∂AdS_3, including an external 3D CS line (red) "smearing" a 4D photon ∂AdS_4 correlator point.

That is, we deviate from the default Dirichlet boundary condition $\mathcal{A}_\mu = 0$ at $\xi = \pi/2$, corresponding to the unperturbed CFT_3, because K^{CS} acts as a perturbing source for the CFT_3 current.

It is straightforward to identify this $\mathcal{A}$. Since $K^{CS}_{+\mu}(z', y)$ is a solution to the free CS equation of motion as a function of y, it must be purely a (large) 3D gauge transformation, $K^{CS}_{+\mu}(z', y) = \partial_\mu \lambda(y)$, specified by its non-trivial boundary limit (at z'). This then clearly lifts to the simple 4D solution,

$$\mathcal{A}_N(y, \xi) = \partial_N \lambda(y), \qquad \mathcal{A}_\mu = \partial_\mu \lambda(y) = K^{CS}_{+\mu}(z', y), \qquad \mathcal{A}_\xi = 0. \tag{5.6}$$

The 3D large gauge transformation of CS is thereby lifted to a large 4D gauge transformation, such pure gauge configurations being at the root of traditional 4D AS analyses. Here, substituting eq. (5.6) into eq. (5.3) we see that

$$\langle 0 | T\{j_+(z') \ldots\} | 0 \rangle_{\mathrm{CFT}_2} = \int d^3y \sqrt{g_{\mathrm{AdS}_3}} K^{CS}_{+\mu}(z', y) \mathcal{J}^\mu_{\mathrm{eff}}(y), \tag{5.7}$$

where

$$\sqrt{g_{\mathrm{AdS}_3}} \mathcal{J}^\mu_{\mathrm{eff}}(y) \equiv \int d\xi \sqrt{-G_{\mathrm{AdS}_4}} \mathcal{J}^\mu(y, \xi). \tag{5.8}$$

In this way, we see that we can compute $\langle j_+(z') \rangle$ via a CS gauge field coupled either to the holographic CFT_3 current $\langle J_{\mathrm{CFT}_3}(y) \rangle$ or the effective "soft" current made from the 4D bulk, $\mathcal{J}_{\mathrm{eff}}(y)$.

5.2 Non-abelian gauge theory and gravity

Note that in non-abelian gauge theory and gravity, there will be non-abelian CS or GR_3 external branches in correlator diagrams, such as figure 11. Again, such correlators can

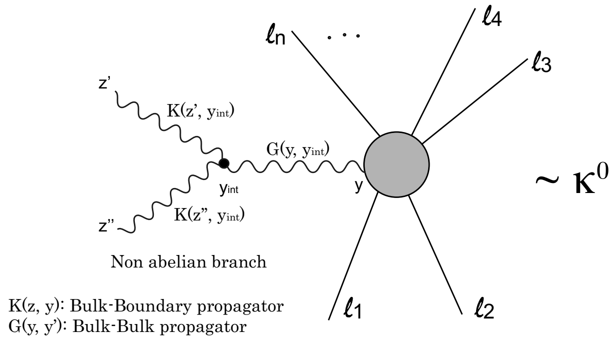

Figure 11. Surviving diagram as $\kappa \to \infty$ with non-abelian CS branch, smearing CFT_3 KK correlator.

be viewed as purely CFT_3 correlators, but with CFT_3 currents/stress-tensor in the AdS_3 bulk, at points y smeared by the non-abelian branch. These branches as functions of y are a non-abelian generalization of abelian CS bulk-boundary propagators, in that they just describe (large) gauge-transformations/diffeomorphisms, because they add up to solutions to the sourceless $\mathrm{CS/GR}_3$ equations of motion (with non-trivial boundary limits). The non-abelian interactions in the branches are just a diagrammatic representation of finding such large gauge-transformations/diffeomorphisms, which is a non-linear problem for non-abelian gauge/diffeomorphism symmetry. As for the abelian case, these are straightforwardly lifted into 4D large gauge-transformations/diffeomorphisms (as was done in Mink_4 [36]). Thus, once again we see that large gauge-transformations/diffeomorphisms are central to isolating the AS, by suitably smearing $\partial\mathrm{AdS}_4/\mathrm{CFT}_3$ correlators into the canonical form of CFT_2 correlators.

5.3 Compatibility with 4D quantum loops and masses

Note that while there are only tree-like CS and GR_3 branches dressing $\mathrm{CFT}_3/\mathrm{AdS}_4$ diagrams surviving in the large κ limit, the CFT_3 hadron (AdS_4) diagrams can be at full loop level, controlled by a separate parameter such as $1/N_{\mathrm{CFT}_3}$. In this sense, the AS we derive are an all-loop feature, in fact a non-perturbative feature, of AdS_4 QG. Furthermore, while it is technically easier to explicitly consider massless 4D fields, there is absolutely no obstruction to massive 4D fields, dual to high-dimension CFT_3 operators.

6 Evading the no-go for infinite-dimensional AS in $\mathrm{AdS}_4/2$

We have derived CFT_2 correlators for 2D currents/stress-tensor in the large level limit from purely CFT_3 ($\mathrm{AdS}_4/2$) correlators of "hadronic" (KK) modes and CFT_3 currents/stress-tensor, smeared by large gauge-transformations/diffeomorphisms. This gives rise to infinite dimensional AS of $\mathrm{Vir}_+ \times \mathrm{Vir}_-$ and KM type. The Virasoro symmetries are analogous to the super-rotations of Mink_4. Here, we show from the 4D viewpoint how we have evaded the no-go argument sketched in the introduction for such infinite-dimensional symmetries of AdS_4, which would equally apply to $\mathrm{AdS}_4/2$.

To understand this, note that in "product-space" coordinates (eq. (3.3)), there are two distinct $\partial\mathrm{AdS}_4$ regions, the ceiling/floor at $\xi = \pm\pi/2$ and the round wall at $\rho = 0$ (refer to figure 3). These two boundary regions have different conformal structure,

$$ds^2_{\partial\mathrm{AdS}_4} \underset{\mathrm{Weyl}}{\sim} \begin{cases} ds^2_{\mathrm{AdS}_3}, & \xi \to \pm\pi/2 \\ \\ ds^2_{\partial\mathrm{AdS}_3}, & \rho \to 0. \end{cases} \tag{6.1}$$

In standard global coordinates standard $\partial\mathrm{AdS}_4$ correlators only have sources on the ceiling/floor, and in this boundary region the geometry is fully three-dimensional, with only finite-dimensional conformal isometries as candidate AS. This is the no-go argument in "product-space" coordinates. However, we see that when we put sources only on the wall boundary region, as we have been led to do by the scaffolding of GR_3 and CS on AdS_3, the bulk geometry degenerates as we approach this boundary region to the 2D geometry of $\partial\mathrm{AdS}_3 \equiv S^1 \times \mathbb{R}$, which has infinite-dimensional conformal isometries, corresponding to $\mathrm{Vir}_+ \times \mathrm{Vir}_-$ AS.

Thus far $\mathrm{SO}(2,2)$ has played the analogous role of Lorentz transformations $\mathrm{SO}(3,1)$ in Mink_4, being extended to $\mathrm{Vir}_+ \times \mathrm{Vir}_-$ AS of AdS_4 analogously to the super-rotations $\mathrm{Vir} \times \overline{\mathrm{Vir}}$ of Mink_4. The analog of Mink_4 translation generators are the extra four generators of the AdS_4 isometries $\mathrm{SO}(3,2)$ which lie outside $\mathrm{SO}(2,2)$, just as Mink_4 translations are the Poincare generators outside $\mathrm{SO}(3,1)$. We would like to identify these extra generators and the full set of AS of AdS_4 that follows from them, in analogy to XBMS_4 incorporating translations to go beyond just the super-rotations in Mink_4. The problem is that the strategy we used to identify CFT_2 structure forced us to consider $\mathrm{AdS}_4/2$ (rather than AdS_4) and asymptotically AdS_3 GR_3, both of which respect only the $\mathrm{SO}(2,2)$ subgroup of the global $\mathrm{SO}(3,2)$. How can we recover some analog of "super-translations", and more generally the complete AS analog of XBMS_4?

7 Maximal spacetime AS from 3D conformal gravity

The infinite-dimensional extension of $\mathrm{SO}(2,2)$ isometry arose in our approach by gauging the CFT_3 by $\mathrm{SO}(2,2)$ CS $= \mathrm{GR}_3$ on AdS_3. This suggests that we may get the larger infinite-dimensional extension of $\mathrm{SO}(3,2)$ by gauging the CFT_3 by $\mathrm{SO}(3,2)$ CS instead. Remarkably, this is simply equivalent to 3D conformal gravity (CGR_3) [68].

7.1 A "super-translation"-like KM AS for $\mathrm{AdS}_4/2$

CGR_3 is compatible with asymptotically AdS_3 spacetime, even though AdS_3 does not have full $\mathrm{SO}(3,2)$ conformal isometry. CGR_3 is not only diffeomorphism invariant, but also Weyl invariant. The Weyl invariance shares much in common with an internal $\mathrm{U}(1)$ gauge invariance (not coincidentally given Weyl's original gauging of scale symmetry in the history of gauge theory and its similarity to QED's gauging of rephasing invariance). Therefore it is not surprising that the AS of CGR_3 (+ CFT_3) matter on AdS_3 are of the form of $\mathrm{Vir}_+ \times \mathrm{Vir}_-$ along with an abelian KM, the latter associated with Weyl symmetry [69]:

$$\begin{aligned}
\left[L^+_m, L^+_n\right] &= (m-n)L^+_{m+n} - \left(\kappa_{\rm grav} - \frac{1}{12}\right)\left(m^3 - m\right)\,\delta_{m+n,0} \\
\left[L^-_m, L^-_n\right] &= (m-n)L^-_{m+n} + \kappa_{\rm grav}\left(m^3 - m\right)\,\delta_{m+n,0} \\
\left[L^+_m, J^+_n\right] &= -nJ^+_{m+n} \\
\left[J^+_m, J^+_n\right] &= 2\kappa_{\rm grav}\, m\, \delta_{m+n,0}\,,
\end{aligned} \tag{7.1}$$

where $\kappa_{\rm grav}$ is the level of the CGR_3 theory in CS form, in the same manner as for GR_3. Note, it is critical that the "quark" sector is compatible with being gauged by CGR_3, precisely because it is 3D conformally invariant, so that it can be made Weyl-invariant once coupled to gravity. We can interpret the KM resulting from the Weyl invariance as an $\mathrm{AdS}_4/2$ analog of the Mink_4 super-translation KM.

7.2 Non-unitary nature of CGR_3

For large κ we see that the two Virasoro sub-algebras in eq. (7.1) have opposite sign central charges [69], $c_- \approx -c_+$, incompatible with unitarity [75]! This may be surprising because it only pertains to the $\mathrm{Vir}_+ \times \mathrm{Vir}_-$ subalgebra and might be thought to be the same as in GR_3. But crucially, GR_3 is *not* a truncation of CGR_3. They employ different quadratic invariants of the generators to define the trace in their CS formulations. Note that for $SO(2,2)$ there are two distinct quadratic invariants,

$$\epsilon^{IJKL} J_{IJ} J_{KL} \quad \text{and} \quad J_{IJ}J^{IJ}\,, \qquad I,J,K,L = 0,\dots,3. \tag{7.2}$$

The standard GR_3 formulation uses the first of these and it corresponds to $c_+ = c_-$, so they may both be positive. But the second alternative instead has $c_+ = -c_-$, at odds with that positivity. For the $SO(3,2)$ CS formulation of CGR_3, there is clearly only a single option,

$$J_{MN}J^{MN}\,, \qquad M,N = 0,\dots,4, \tag{7.3}$$

and the truncation to $SO(2,2)$ is then the non-positive choice for central charge. Nevertheless since we take $\kappa, c \to \pm\infty$ in our analysis of $\mathrm{CFT}_3/\mathrm{AdS}_4$ AS, this does not obstruct the unitarity of the target theory. It does however seem strangely at odds with our development so far, which has made physical sense for finite κ, c. Possibly, we must restrict to a single CS sector (4D helicity), say "+", with $c_+ > 0$ [75].

Although CGR_3 has led us to identify a KM "super-translation"-like extension of $\mathrm{AdS}_4/2$ AS, this extended algebra still does not contain all of global $SO(3,2)$, presumably because we are still explicitly breaking AdS_4 isometries by working with $\mathrm{AdS}_4/2$. We rectify this by first switching to the Poincare patch of AdS_4 in the next section, and then later to all of global AdS_4.

8 $\mathrm{AdS}_4^{\rm Poincare}$: AS from holography and holography from AS

We have accumulated a number of questions. Is there an AS algebra of AdS_4 that contains the isometry $SO(3,2)$ as a subgroup? While we are taking the large κ limit, what does the

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finite-κ set-up look like in the 4D dual prior to the limit? So far the CS and (C)GR$_3$ are added "by hand", even if then removed by $\kappa \to \infty$. Is there a sense in which such CS fields emerge as soft limits of the AdS$_4$ (hence CFT$_3$) fields themselves, as was the case in Mink$_4$? If so, do we get a finite emergent level, $\kappa < \infty$? These questions are most simply addressed within the Poincare patch of AdS$_4$, AdS$_4^{\text{Poincare}}$. The AdS$_4^{\text{Poincare}}$ metric is given by

$$ds^2 = \frac{\eta_{\mu\nu}\, dx^\mu\, dx^\nu - dw^2}{w^2}\,, \qquad 0 < w < \infty\,, \tag{8.1}$$

which manifests a Mink$_3$ foliation, where $\eta_{\mu\nu}$ is the Mink$_3$ metric. Although only a portion of AdS$_4^{\text{global}}$, it has the full AdS$_4$ isometry algebra of SO$(3,2)$, unlike AdS$_4/2$. We also know its holographic dual, namely CFT$_3$ on Mink$_3$, where SO$(3,2)$ are the conformal isometries.

Now we can couple this CFT$_3$ to GR$_3$. GR$_3$ on Mink$_3$ again has a CS formulation with gauge group ISO$(2,1)$, the 3D Poincare group. For finite κ_{grav} the 4D dual of GR$_3$ + CFT$_3$ (+ UV completion) is well known, namely it is the (UV completion of the) Randall-Sundrum 2 (RS2) model [70], but in one dimension lower than the originally formulated [71]. That is, the AdS$_4^{\text{Poincare}}$ boundary is cut off by a "Planck brane" whose 3D geometry is dynamical, dual to GR$_3$, and coupled to the 4D dynamical bulk geometry (dual to CFT$_3$).[1]

Rather than dwelling on finite κ_{grav}, we proceed with the strategy for the 4D theory to inherit the 3D AS of GR$_3$ in the large κ_{grav} limit. This AS of Mink$_3$ is XBMS$_3$ [58]. Here we review its derivation by a "contraction" of the Vir$_+ \times$ Vir$_-$ AS of AdS$_3$, essentially getting flat 3D by taking the $R_{\text{AdS}_3} \to \infty$ limit [59, 76–81].

8.1 XBMS$_3$ from Vir$_+ \times$ Vir$_-$

It is clear in what sense the "vacuum" geometry of AdS$_3$ approaches Mink$_3$ in the limit of large R_{AdS_3}, but we must study the GR$_3$ dynamics as well in this limit in order to understand the relationship of the two AS algebras. GR$_3$ on asymptotically AdS$_3$ can formulated in terms of SO$(2,2) \equiv$ SO$(2,1)_+ \times$ SO$(2,1)_-$ Chern-Simons gauge fields made from the dreibein e and spin connection ω as [42]

$$A^{\pm a}_\mu \equiv \omega^a_\mu \pm e^a_\mu / R_{\text{AdS}_3}. \tag{8.2}$$

If we plug this into the AdS$_3$ gravity action in CS form $\equiv S_{CS}(A^+) - S_{CS}(A^-)$, and keep the leading terms for large R_{AdS_3}, we find straightforwardly that it is the CS form of the gravity action in Mink$_3$ (with gauge group ISO$(2,1)$) written in terms of e and ω.

Staying in AdS$_3$, the asymptotic expansion of $A^\pm$ in terms of $L^\pm$ (reviewed in section 2) translates into an expansion for e given by $R_{\text{AdS}_3} \sum_n (L^+_n - L^-_{-n}) e^{in\phi}$, and for ω given by $\sum_n (L^+_n + L^-_{-n}) e^{in\phi}$. That is, the AS charges for e and ω respectively are

$$\begin{aligned} R_{\text{AdS}_3} l_n &= R_{\text{AdS}_3} (L^+_n - L^-_{-n}) \\ R_{\text{AdS}_3} T_n &= (L^+_n + L^-_{-n}), \end{aligned} \tag{8.3}$$

[1] The analogous dual in the case of $\mathcal{M}_3 = $ AdS$_3$ is less familiar, a 3D Planck brane in AdS$_4^{\text{global}}/2$. It is important to distinguish this from the Karch-Randall model [73], in this dimensionality a 3D Planck brane in all of AdS$_4^{\text{global}}$.

where the overall normalization of R_{AdS_3} on the left-hand side does not affect relative sizes of terms in the charge algebra, but does give a finite limit as $R_{\mathrm{AdS}_3} \to \infty$. Indeed, expressing the $\mathrm{Vir}_+ \times \mathrm{Vir}_-$ algebra in these variables and taking $R_{\mathrm{AdS}_3} \to \infty$ yields the centrally-extended XBMS_3:

$$
\begin{aligned}
&[l_m, l_n] = (m-n) l_{m+n} \\
&[l_m, T_n] = (m-n) T_{m+n} + \frac{2M_{\mathrm{Pl}}}{12}\, m^3\, \delta_{m+n,0}\,, \quad [T_m, T_n] = 0\,.
\end{aligned}
\tag{8.4}
$$

As in AdS_3, this XBMS_3 AS is symptomatic of the topological character of GR_3, the non-trivial topology arising from the "holes" drilled out by the matter world lines, where GR_3 reacts by introducing conical-type singularities.

We will think of XBMS_3 as an $\mathrm{AdS}_4^{\mathrm{Poincare}}$ analog of "super-rotations" in Mink_4 since they are the contraction of $\mathrm{Vir}_+ \times \mathrm{Vir}_-$ AS. The global subalgebra of XBMS_3 is the Poincare isometry $\mathrm{ISO}(2,1)$. But now $\mathrm{AdS}_4^{\mathrm{Poincare}}$ (Mink_3) has the larger (conformal) isometry algebra of $\mathrm{SO}(3,2)$, containing $\mathrm{ISO}(2,1)$ as a subalgebra. Therefore the extra generators of $\mathrm{SO}(3,2)$ can be (repeatedly) commuted with XBMS_3 to generate the full AS of $\mathrm{AdS}_4^{\mathrm{Poincare}}$, with global subgroup $\mathrm{SO}(3,2)$! This strategy was analogously followed in Mink_4 as one of the ways to (re-)derive super-translations by commuting ordinary translations with super-rotations [36].

8.2 $\mathbf{CGR_3}$ on $\mathbf{Mink_3}$

Above we outlined a strategy for finding the full AS of $\mathrm{AdS}_4^{\mathrm{Poincare}}$ by starting with its sub-algebra, XBMS_3, arising from gauging with GR_3. It would be more elegant and insightful if the entire AS emerged by the same procedure. This can now be done by replacing GR_3 by CGR_3 on Mink_3, coupled to CFT_3. Since Mink_3 has $\mathrm{SO}(3,2)$ conformal isometries, and CGR_3 is $\mathrm{SO}(3,2)$ CS, and our "quark" matter is also conformally invariant CFT_3, $\mathrm{SO}(3,2)$ is respected by each component, and therefore the infinite dimensional symmetries that arise from the CS structure must contain all of $\mathrm{SO}(3,2)$ as a global subalgebra. We will pursue the explicit form of this AS algebra elsewhere, just observing here that it is implicitly completely characterized by CGR_3 on Mink_3.

8.3 Holographic grammar from AS

While we have used the holographic grammar of $\mathrm{AdS}_4/\mathrm{CFT}_3$ in this paper to clarify the nature and utility of AS, we can run our arguments in a different order. Suppose that one did not know the holographic dual of AdS_4 QG, but was given the full AS structure of AdS_4 and learned to characterize it in terms of CGR_3 fields to capture the associated large gauge transformations. Then by the fact that matter compatible with coupling to 3D gravity must be a 3D local quantum field theory in order to have the requisite local stress tensor to source gravity, we can deduce that the holographic dual of AdS_4 must be such a 3D QFT. The fact that the 3D gravity is specifically conformal gravity implies that the dual 3D QFT must also be conformally invariant, that is CFT_3! It is just such a set of steps that awaits to be performed in the case of finding a holographic grammar behind Mink_4 QG.

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9 Emergent CS and "shadow" effects from boundary/soft limits

In Mink$_4$ gauge theory, it was shown that AS and memory effects arise from considering same-helicity gauge boson emissions in the soft limit [6, 7]. Ref. [36] showed that these features were captured by an emergent 3D CS description of the soft fields, "living" at ∂Mink$_4$, as well as on Rindler/Milne horizons. Here, we will demonstrate that analogous phenomena emerge within AdS$_4^{\text{Poincare}}$ U(1) gauge theory. (If AdS$_4^{\text{Poincare}}$ GR$_4$ is added, we can think of these phenomena as emerging from within the dual CFT$_3$ with U(1) global symmetry, even though the gravity will play no explicit role in our analysis.) While AdS$_4^{\text{global}}$ has a discrete spectrum, AdS$_4^{\text{Poincare}}$ has a continuous spectrum and a natural generalization of "soft" limit. We find emergent CS gauge fields localized on ∂AdS$_4^{\text{Poincare}}$ as well as on the Poincare horizon, connected by this soft limit. These CS fields connect to analogs of electromagnetic memory effects in Mink$_4$ [21–23, 36], which we will refer to as "shadow" effects, since they relate to the holographically emergent spatial direction rather than time. We will also see a sense in which a finite CS level emerges. While our approach here parallels similar steps in Mink$_4$ [36], the emergent CS structure in AdS$_4$ is closely related to "mirror" symmetry in the dual CFT$_3$ [82–84]. This aspect will be explored elsewhere [85].

9.1 Set-up

We consider an AdS$_4^{\text{Poincare}}$ U(1) Maxwell gauge field $\mathcal{A}_N$, coupled to a bulk 4D conserved source current $\mathcal{J}_N$, which is taken to implicitly describe interacting charged matter. The 4D gauge coupling is g. Because of the Weyl invariance of the 4D Maxwell action, AdS$_4^{\text{Poincare}}$ (eq. (8.1)) is effectively just Mink$_4$/2,

$$ds^2 \underset{\text{Weyl}}{\sim} \eta_{\mu\nu}\, dx^\mu\, dx^\nu - dw^2\,, \qquad w > 0,\ \mu,\nu = \{0,1,2\}. \tag{9.1}$$

The natural notion of "soft" in AdS$_4^{\text{Poincare}}$/CFT$_3$ is $m_3^2 \to 0$, where m_3^2 is 3D invariant mass-squared in the x^μ directions. This is obviously analogous to the observation of ref. [36] that Mink$_4$ soft limits correspond to $m_3 \to 0$ in the (EA)dS$_3$ foliation of Mink$_4$.

Maxwell radiation can be decomposed into positive and negative helicity components, $\mathcal{A}^\pm$. More generally, away from charged matter (away from the support of $\mathcal{J}$), we will decompose the electromagnetic field strength $\mathcal{F}_{MN}$ into self-dual and anti-self-dual components,

$$\mathcal{F}^\pm_{MN}(x, w = 0) \equiv \frac{1}{2}\left(\mathcal{F}_{MN} \pm i\widetilde{\mathcal{F}}_{MN}\right) \equiv \partial_M \mathcal{A}^\pm_N - \partial_N \mathcal{A}^\pm_M. \tag{9.2}$$

We will focus on the soft limit of $\mathcal{A}^+$. Let us first imagine that we are in full Mink$_4$ instead of Mink$_4$/2. In momentum space, (q_μ, q_w), $m_3^2 = q_\mu q^\mu$, so that for 4D on-shell radiation, $m_3^2 = q_w^2$. More precisely, the *leading* soft limit would be given by

$$\lim_{q_w \to 0} q_w\, \mathcal{A}^+(q_w) = \int_{-\infty}^{\infty} dw\, \partial_w \mathcal{A}^+(w), \tag{9.3}$$

where the 3D argument is implicit and can be either q_μ or x^μ. Within $\text{Mink}_4/2$, the analogous soft limit is truncated to[2]

$$\int_0^\infty dw\, \partial_w \mathcal{A}^+(w) = \mathcal{A}^+(w=\infty) - \mathcal{A}^+(w=0). \tag{9.4}$$

We will take this as our "soft limit".

In what follows, we will see that each of the 3D fields in this soft limit, $\mathcal{A}^+(x, w=\infty)$ and $\mathcal{A}^+(x, w=0)$ obeys an interesting CS-type equation.

9.2 CS on $\partial\text{AdS}_4^{\text{Poincare}}$ and a "holographic shadow" effect

Consider that the source current $\mathcal{J}$ emits radiation towards $\partial\text{AdS}_4^{\text{Poincare}}$. The positive helicity component at $\partial\text{AdS}_4^{\text{Poincare}}$ satisfies

$$\mathcal{F}^+_{\mu\nu}(x, w=0) = \frac{i}{2}\widetilde{\mathcal{F}}_{\mu\nu}(x, w=0) \equiv -\frac{i}{4}\epsilon_{\mu\nu\rho}\mathcal{F}^{w\rho}(x, w=0), \tag{9.5}$$

because the standard AdS_4 Dirichlet boundary condition, $\mathcal{A}_\mu(x, w=0) = 0$, implies $\mathcal{F}_{\mu\nu}(x, w=0) = 0$. In terms of the standard $\text{AdS}_4/\text{CFT}_3$ dictionary for the holographic symmetry current,

$$J^\rho_{\text{CFT}_3}(x) = \frac{1}{g}\mathcal{F}^{w\rho}(x, w=0), \tag{9.6}$$

we obtain

$$\mathcal{F}^+_{\mu\nu}(x, w=0) = -\frac{i}{4}g\, \epsilon_{\mu\nu\rho}J^\rho_{\text{CFT}_3}(x). \tag{9.7}$$

We can view this as the equation of motion for an emergent CS gauge field coupled to CFT_3 charged matter,

$$F^{\text{CS}}_{\mu\nu}(x) = g\, \epsilon_{\mu\nu\rho}J^\rho_{\text{CFT}_3}(x), \tag{9.8}$$

where the CS gauge field is identified with the helicity-cut boundary limit of the 4D gauge field in w-axial gauge,

$$A^{\text{CS}}_\mu(x) \equiv 4i\mathcal{A}^+_\mu(x, w=0). \tag{9.9}$$

(This does not vanish since only $\mathcal{A} = \mathcal{A}^+ + \mathcal{A}^-$ obeys the AdS Dirichlet boundary condition.) It was just such a CS field coupled to CFT_3 (but on AdS_3 instead of Mink_3) which was invoked in earlier sections to derive AS for AdS_4.

It is useful to cast the CS equation in integrated form, using Stokes' Theorem,

$$\oint_{\partial\Sigma} dx^\rho A^{\text{CS}}_\rho(x) = g\int_\Sigma d^2\Sigma^{\mu\nu}\epsilon_{\mu\nu\rho}J^\rho_{CFT_3}. \tag{9.10}$$

Here Σ is a finite two-dimensional surface in the $\partial\text{AdS}_4^{\text{Poincare}}$ x-spacetime, with boundary $\partial\Sigma$. For example, for purely spatial Σ, the right-hand side is the total CFT_3 "quark" charge lying inside Σ, a holographic "shadow" of the 4D bulk state.

[2] We can think of $\text{Mink}_4/2$ as the quotient space of Mink_4 under the identification $w \leftrightarrow -w$. If we imagine a "polarizer" projecting onto positive helicity in the physical region, $w > 0$, and its "mirror image" projecting onto negative helicity for $w < 0$, then the definition of leading soft limit in the Mink_4 covering space reduces to the truncated expression in $\text{Mink}_4/2$.

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9.3 Emergent CS level

As explained earlier, in CS theory, sensitivity to the CS level κ (in correlators with external matter lines) arises from diagrams with internal CS lines. In the present context, we have considered radiation emitted by a source $\mathcal{J}$. The CS gauge field is the boundary limit of the positive helicity component of this 4D radiation, $\mathcal{A}^+(w=0)$. To measure the associated CS level we imagine "detecting" this field with a probe charge localized near or at the boundary, $w=0$.

The subtlety is that physical charges couple to both positive and negative helicity components. We straightforwardly see that the boundary limit of the negative helicity component $\mathcal{A}^-(w=0)$ satisfies

$$\mathcal{F}^-_{\mu\nu}(x, w=0) = \frac{i}{4} g\, \epsilon_{\mu\nu\rho} J^\rho_{\mathrm{CFT}_3}(x). \tag{9.11}$$

That is, while the probe charge couples to the sum of the helicity components in the form, $g\mathcal{A} = g\mathcal{A}^+ + g\mathcal{A}^-$, the two helicities couple with *opposite* strength to the holographic current, in the form $\pm g J_{\mathrm{CFT}_3}$. Therefore the CS exchanges mediated by $\mathcal{A}^+$ and $\mathcal{A}^-$ have strengths $\pm g^2$, yielding a net cancelation. However, we can formally focus on just the $\mathcal{A}^+(w=0)$ CS exchange with strength $+g^2$, corresponding to CS level,

$$\kappa \sim \frac{1}{g^2}. \tag{9.12}$$

A similar result was anticipated in ref. [36] for Mink_4.

9.4 The soft limit, CS on the Poincare horizon, and a bulk "shadow" effect

The μ-component of the 4D Maxwell equations reads

$$\partial_w \mathcal{F}^{w\mu} + \partial_\nu \mathcal{F}^{\nu\mu} = g\mathcal{J}^\mu. \tag{9.13}$$

We again look at an integrated form of these equations, on a two-dimensional surface Σ in $\partial \mathrm{AdS}_4^{\mathrm{Poincare}}$ x-spacetime, and in our "soft limit" in w. That is, we integrate with respect to the three-volume, $\int_0^\infty dw \int d^2\Sigma^{\rho\sigma}\epsilon_{\rho\sigma\mu} \ldots$, to get

$$\begin{aligned}\int d^2\Sigma^{\rho\sigma}\epsilon_{\rho\sigma\mu}\Big[\mathcal{F}^{w\mu}(w=\infty) - \mathcal{F}^{w\mu}(w=0)\Big]& \\ + \int_0^\infty dw \oint_{\partial\Sigma} dx^\rho \epsilon_{\rho\mu\nu}\mathcal{F}^{\mu\nu} &= g\int_0^\infty dw \int d^2\Sigma^{\rho\sigma}\epsilon_{\rho\sigma\mu}\mathcal{J}^\mu, \end{aligned}\tag{9.14}$$

where we have used Stokes' Theorem to get the second line of the left-hand side. For the simple case of purely spatial Σ this is nothing but Gauss' Law, the right-hand side being just the total bulk charge lying inside the three-volume, while the left-hand side is the total electric flux through its boundary.

Taking the source current $\mathcal{J}$ to be localized at finite w at finite times, and Σ to only span finite times, we can drop the field strength at $w=\infty$ on the first line, by causality. The field strength at $w=0$ on the first line is just the holographic current again, so we have

$$\int_0^\infty dw \oint_{\partial\Sigma} dx^\rho \epsilon_{\rho\mu\nu}\mathcal{F}^{\mu\nu} = g\int d^2\Sigma^{\rho\sigma}\epsilon_{\rho\sigma\mu}\Big[J^\mu_{\mathrm{CFT}_3} + \int_0^\infty dw \mathcal{J}^\mu(w)\Big]. \tag{9.15}$$

This is closely analogous to the electromagnetic memory effect in Mink_4 for purely spatial Σ, with w now playing the role of time there. The total charge passing through Σ *regardless of when* in the memory effect is replaced here by the total charge in Σ *regardless of where in* w. We will refer to this as a "bulk shadow" effect.

As was done for the memory effect in ref. [36], we can write the bulk shadow effect in CS form. First note that the left-hand side can be re-expressed in terms of the dual field strength $\widetilde{\mathcal{F}}$ to give

$$2\oint_{\partial\Sigma} dx_\rho \int_0^\infty dw \widetilde{\mathcal{F}}^{w\rho} = g\int d^2\Sigma^{\rho\sigma}\epsilon_{\rho\sigma\mu}\Big[J^\mu_{\text{CFT}_3} + \mathcal{J}^\mu_{\text{eff}}\Big], \tag{9.16}$$

where we have defined a second 3D "shadow" current by taking the soft limit of the bulk 4D current,

$$\mathcal{J}^\mu_{\text{eff}}(x) \equiv \int_0^\infty dw \mathcal{J}^\mu(x,w). \tag{9.17}$$

We add zero to the bulk shadow effect in the form,

$$\begin{aligned} 0 &= 2i\oint_{\partial\Sigma} dx_\rho \int_0^\infty dw \mathcal{F}^{w\rho} \\ &= 2i\oint_{\partial\Sigma} dx_\rho \mathcal{A}^\rho(w=\infty) - 2i\oint_{\partial\Sigma} dx_\rho \mathcal{A}^\rho(w=0), \end{aligned} \tag{9.18}$$

where the term at $w=\infty$ is by Stokes' Theorem $= i\int d^2\Sigma_{\mu\nu}\mathcal{F}^{\mu\nu}(w=\infty)$, which vanishes by causality, and the term at $w=0$ vanishes by the standard AdS_4 Dirichlet boundary conditions on $\mathcal{A}$. Therefore we can write the bulk shadow effect in the form

$$-2i\oint_{\partial\Sigma} dx_\rho \int_0^\infty dw\big[\mathcal{F}^{w\rho} + i\widetilde{\mathcal{F}}^{w\rho}\big] = g\int d^2\Sigma^{\rho\sigma}\epsilon_{\rho\sigma\mu}\big[J^\mu_{\text{CFT}_3} + \mathcal{J}^\mu_{\text{eff}}\big]. \tag{9.19}$$

It is straightforward for $\partial\Sigma$ to avoid the support of $\mathcal{J}$ for all w, so that the self-dual component of the field strength on the left-hand side can be expressed in terms of the gauge potential $\mathcal{A}^+$. By Stokes' Theorem,

$$\oint_{\partial\Sigma} dx^\rho \mathcal{A}^+_\rho(w=\infty) - \oint_{\partial\Sigma} dx^\rho \mathcal{A}^+_\rho(w=0) = \frac{ig}{4}\int d^2\Sigma^{\rho\sigma}\epsilon_{\rho\sigma\mu}\Big[J^\mu_{\text{CFT}_3} + \mathcal{J}^\mu_{\text{eff}}\Big]. \tag{9.20}$$

We see that the term on the left at $w=0$ and the J_{CFT_3} term on the right are equal by the last subsection, so we isolate a new CS-type relation on the Poincare horizon,

$$\oint_{\partial\Sigma} dx^\rho \mathcal{A}^+_\rho(w=\infty) = \frac{ig}{4}\int d^2\Sigma^{\rho\sigma}\epsilon_{\rho\sigma\mu}\mathcal{J}^\mu_{\text{eff}}. \tag{9.21}$$

This is the (Σ-integrated) CS form of the bulk shadow effect, where the role of CS current is played by the shadow current, $\mathcal{J}_{\text{eff}}$.

In subsection 5.1, with CFT_3 on AdS_3, we saw that AS (CFT_2 chiral current j_+) could be derived by CS coupled to either J_{CFT_3} *or* $\mathcal{J}_{\text{eff}}$. But this equivalence required going to the boundary of AdS_3. For Σ in the "bulk" of Mink_3, the two CS relations at $w=0$ and $w=\infty$, with CS currents J_{CFT_3} and $\mathcal{J}_{\text{eff}}$ respectively, are distinct.

In the same sense as for the CS gauge field localized at the boundary, the CS gauge field on the Poincare horizon also has level $\kappa_{\text{eff}} \sim 1/g^2$.

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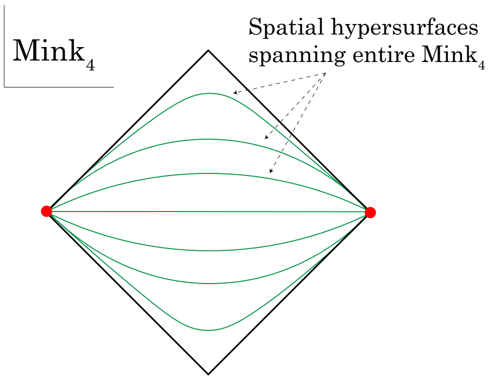

Figure 12. In asymptotic Mink_4 spacetime, different time-slices (given by green spatial hypersurfaces) are just related by diffeomorphisms within a single "timeless" Wheeler-DeWitt wavefunctional.

10 AS of Wheeler-DeWitt wavefunctionals on $\partial\text{AdS}_4^{\text{global}}$

The choices of $\mathcal{M} = \text{AdS}_3$ and Mink_3 have given an approach to AS on portions of $\text{AdS}_4^{\text{global}}$, but here we return to the full $\text{AdS}_4^{\text{global}}$. It is natural then to consider CS coupled to CFT_3 on the global boundary $S^2 \times \mathbb{R}$. However, space is then closed and there is no obvious asymptotic region to get AS or 2D chiral currents. Yet, it is well known from the CS viewpoint that there are effectively infinite-dimensional symmetries still at play, and these are revealed by cutting at a time slice to reveal a state [62]. Technically, this is clear if we consider the wavefunctional, say at time $\tau = 0$, to be determined by a 3D functional integral over all earlier times $\tau < 0$ and all of space, that is effectively $\mathcal{M}_3 \equiv S^2 \times \mathbb{R}^-$, where $\mathbb{R}^-$ is the negative-τ half-line. Once again, this spacetime has a boundary, the S^2 space at $\tau = 0$, on which AS appear in standard CS fashion. They act on the states of the theory.

Let us return to the no-go argument for infinite-dimensional AS of AdS_4 and the loop-hole pointed out in the introduction. CFT_3 states are dual to AdS_4 diffeomorphism-invariant Wheeler-DeWitt wavefunctionals. In particular they describe the state at $\tau = 0$ on the boundary, but on any interpolating spacelike hypersurface in the bulk. The collection of such hypersurfaces gives the 4D subregion of AdS_4 described by the quantum state, as depicted in figure 1. Its boundary geometry is effectively two-dimensional, compatible with infinite dimensional AS.

Such a restriction to a subregion does not occur for Mink_4. The quantum state at Minkowski time $= 0$ on the boundary describes the 4D region foliated by all interpolating spacelike hypersurfaces, as for AdS_4, but unlike AdS_4 this foliation covers *all* of Mink_4. See figure 12.

10.1 CS gauge theory on $\partial\text{AdS}_4^{\text{global}}$

Consider a U(1) CS field for simplicity. The CS field sees the U(1) charged CFT_3 state at $\tau = 0$ via an Aharonov-Bohm(AB) phase in Wilson loops. One can define associated charges,

$$Q_\Sigma \equiv \oint_{\partial\Sigma} d\ell \cdot A, \tag{10.1}$$

measuring the total "quark" charge inside subregion Σ of the spatial S^2 at $\tau = 0$, by the integrated form of equations of motion for CS coupled to CFT$_3$.

These contour-associated AS charges are related to the standard KM charges as follows. We use complex coordinates $z, \bar{z}$ on S^2 via sterographic projection. Out of the two boundary components of the CS gauge field, $A_z, A_{\bar{z}}$, one component is removed by a CS boundary condition, say $A_{\bar{z}} = 0$, while the other component is holomorphically conserved, $\partial_{\bar{z}} A_z = 0$ (refer to section 2). This holomorphic $A_z(z)$ is then completely determined by the poles at the location of charged CFT$_3$ "quarks", so that

$$Q_\Sigma \equiv \frac{1}{2\pi i} \oint_{\partial\Sigma} dz A_z(z) \tag{10.2}$$

as a complex contour integral. Laurent expanding about $z = 0$ say,

$$A_z(z) = \sum_n \frac{Q_n}{z^{n+1}}, \tag{10.3}$$

then determines the KM charges. Note that even as the CS is decoupled at $\kappa = \infty$, the Q_Σ, Q_n remain as non-gauged charges registering the location of charged "quarks", and therefore the holographic boundary "shadows" of 4D particles.

While this form of "hair" for 4D charges is amusing, the key question is whether it is *useful*, say in the sense that it makes time-evolution algebraic in terms of the charges, as opposed to having to solve complicated dynamics. We have already seen how such simple time-evolution of charges arises for $\mathcal{M} = \mathrm{AdS}_3$ in the context of AdS$_4/2$ (see eq. (2.12)). To see the analogous form of time-evolution of charges in the $S^2 \times \mathbb{R}$ setting we need the full power of CGR$_3$.

10.2 Time-evolution from AS algebra via CGR$_3$ on $\partial\mathrm{AdS}_4^{\mathrm{global}}$

Given the CS form of 3D gravity, one might think to just repeat the above steps performed for internal CS gauge symmetries. But now $S^2 \times \mathbb{R}$ geometry must be a solution to dynamical gravity. And yet, for standard GR$_3$ (with or without a cosmological constant) it is not a solution to 3D Einstein equations. The closest is GR$_3$ with positive cosmological constant, which has dS$_3$ solution. This is Weyl equivalent to $S^2\times$ timelike-interval. The Weyl equivalence is acceptable because $\partial\mathrm{AdS}_4$ is only defined within such Weyl rescaling. But to capture all of AdS$_4^{\mathrm{global}}$ we want CFT$_3$ on all of $S^2 \times \mathbb{R}$, not just a time interval.

Fortunately if we switch to CGR$_3$, then by its Weyl invariance, Weyl rescalings of GR$_3$ solutions are also solutions of CGR$_3$ [68]. In particular $S^2 \times$ time-interval must be a solution. By locality of CGR$_3$ equations of motions, this means $S^2 \times \mathbb{R}$ is also a solution. We can now couple CGR$_3$ to CFT$_3$ on $S^2 \times \mathbb{R}$. Given the CS form of CGR$_3$, we expect states at fixed time $\tau = 0$ to transform under AS charges arising on the S^2 boundary at $\tau = 0$ from CGR$_3$ = SO(3, 2) CS structure, and to persist in the $\kappa \to \infty$ limit. This gives AS charges acting on CFT$_3$ states. The full AS will contain the spacetime AS associated to

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CGR_3 as well as any related to internal (CS) symmetries. The former has $SO(3,2)$ global subalgebra. The SO(2) subgroup of SO(3,2) is just time translation in τ, that is, the global AdS_4/CFT_3 Hamiltonian H. In particular all AS charges will have commutation relations with H, determining their τ-dependence by the AS charge algebra.

11 Mink$_4$ and future directions

In this paper, we generalized the notion of asymptotic symmetries (AS) applied to AdS_4, so that infinite-dimensional symmetries arise, analogous to the AS of $Mink_4$. We found a tight connection between these AS and the 3D holographic dual, in this case CFT_3, coupled to 3D gravity and Chern-Simons topological sectors. In turn, the combined 3D theory is dual to a CFT_2 structure in the sense of the AdS_3/CFT_2 correspondence, whose chiral currents and stress tensor house the AS charges. Several issues remain in order to fill out this story. Also, having seen these interconnections in AdS_4 quantum gravity and gauge theory, it is worth seeing if a parallel understanding can be gained for other 4D spacetimes where holography is less well understood, including $Mink_4$.

11.1 (A)dS$_4$

It remains an important task to explore how 3D gravity emerges from AdS_4^{Poincare} gravity as a (helicity-cut) soft or boundary limit in analogy to our discussion of U(1) CS emerging from AdS_4^{Poincare} U(1) gauge theory. It will be interesting to see what type of 3D gravity emerges, GR_3 or CGR_3, or whether this depends on leading or subleading soft limits in some way. It will again be interesting to see if, and under what conditions, a finite effective level or central charge emerges. These gauge and gravitational exercises should be repeated for AdS_4^{global}. Here we do not have the notion of soft limit since the spectrum is discrete, but the helicity-cut boundary limit continues to make sense. The connection between the emergent CS gauge fields and AS with 3D "mirror" symmetry recast in dual 4D form [82–84] will be explored later [85].

We have explicitly given some infinite-dimensional subalgebras of the AS algebra acting on AdS_4 states, while we have argued that the full AS algebra is implicitly captured by CGR_3 on ∂AdS_4. It remains to explicitly describe this algebra of AS charges acting on states of CFT_3 living on the S^2 boundary space.

By comparison with $Mink_4$, where IR divergences at loop level affect and complicate the soft limit [86–90], it is possible that AdS_4 curvature IR-regulates and simplifies the considerations. This remains to be explored.

It appears feasible to do a similar analysis in dS_4 as done here for AdS_4, and thereby discover AS in that case. It would be interesting to compare this approach to that of ref. [91]. The approach suggested here would be compatible with the Poincare patch of dS.

11.2 AS as "hair"

We have argued that infinite-dimensional AdS_4 AS are a useful form of "hair" for 4D black holes and other complex states, very much in the manner that the infinite-dimensional symmetry charges of CFT_2 characterize 2D states. Given how explicit this is in AdS_4, it would be interesting to explore whether the AS *fully characterize* any AdS_4 quantum state. Even a partial but still rich characterization may be relevant to the information puzzles of quantum black holes. The fact that AdS_4 gives a new 4D example of how soft fields take a 3D CS topological form, suggests that this phenomenon is more general, and should be understood on less symmetric (black hole) 4D spacetimes.

11.3 $Mink_4$

In $Mink_4$, $Vir \times \overline{Vir}$ super-rotations from the subleading soft limit of GR_4 were shown to be captured by SO(3,1) CS = GR_3 on (EA)dS_3 [36]. But this CS description does not capture super-translations and the leading soft limit of GR_4, or even just Minkowski translations. It remains to find the 3D representation of all the soft limiting fields underlying the full $XBMS_4$ AS. Doing so would be the analog of finding CGR_3 = SO(3,2) CS for AdS_4.

The most obvious guess would be to try CS gauging of the 4D Poincare group, $ISO(3,1)$. But there is a simple no-go argument for this approach, in that there is no quadratic invariant to define the CS trace. The analogous GR_3 on $Mink_3$ is given by $ISO(2,1)$ CS, where the quadratic invariant is given by $\epsilon^{\mu\nu\rho} J_{\mu\nu} P_\rho$ [42], obviously lacking 4D generalization.

Nevertheless it is possible that a non-CS 3D characterization of $Mink_4$ soft fields exists, reducing to $SO(3,1)$ CS for the subleading soft limit and super-rotations. The fact that the 2D conserved current housing super-translation KM charges in $Mink_4$ was found to be a $ECFT_2$ descendent operator of a partially conserved operator [36] suggests a role for *partially massless* gauge fields [92, 93] in 3D, in turn coupled to *partially conserved* currents of a 3D holographic dual of $Mink_4$ QG.

One strategy to find this 3D characterization begins with the recently considered case of CGR_4 on $Mink_4$ [94]. Here we may guess that the soft fields are characterized by SO(4,2) CS on (EA)dS_3, which does have the requisite quadratic invariant $J_{\Phi\Omega} J^{\Phi\Omega}$, $\Phi, \Omega = 0, 1, 2, 3, 4, 5$. This suggests that the 3D $SO(4,2)$ CS theory might be truncated ("Higgsed") to the 3D characterization of just 4D Poincare symmetric soft fields in terms of massless and partially massless 3D fields. Another strategy is to see if there is a "contraction" procedure for the $SO(3,2)$ CS description of AdS_4 AS found here that yields the 3D description of $Mink_4$ AS, in rough analogy to the contraction of $SO(2,2)$ CS governing AS of AdS_3 to the $ISO(2,1)$ CS governing AS of $Mink_3$.

A full 3D characterization of the soft $Mink_4$ fields would strongly constrain the form of a 3D holographic dual of 4D Mink QG, since the latter would have to be able to be coupled to the soft fields. This is in analogy to the neat compatibility of CFT_3 with coupling to CGR_3 in the AdS_4 context. One can view such a connection in $Mink_4$ as a modern extension of Weinberg's classic derivation of consistency conditions on the S-matrix involving massless spin-1 and spin-2 particles. He showed [95–97] by studying soft limits that matter necessarily has to couple to soft spin-1 through conserved charges and to soft spin-2 through gravitational-form charges satisfying the Equivalence Principle. But the full soft field structure may in fact be strong enough to prescribe the full holographic grammar of the dynamics. Such a grammar would effectively have to force the precise vanishing of the 4D cosmological constant, perhaps in a novel way.

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