

Black string first order flow in $N = 2$, $d = 5$ abelian gauged supergravity

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ABSTRACT: We derive both BPS and non-BPS first-order flow equations for magnetically charged black strings in five-dimensional $N = 2$ abelian gauged supergravity, using the Hamilton-Jacobi formalism. This is first done for the coupling to vector multiplets only and U(1) Fayet-Iliopoulos (FI) gauging, and then generalized to the case where also hypermultiplets are present, and abelian symmetries of the quaternionic hyperscalar target space are gauged. We then use these results to derive the attractor equations for near-horizon geometries of extremal black strings, and solve them explicitly for the case where the constants appearing in the Chern-Simons term of the supergravity action satisfy an adjoint identity. This allows to compute in generality the central charge of the two-dimensional conformal field theory that describes the black strings in the infrared, in terms of the magnetic charges, the CY intersection numbers and the FI constants. Finally, we extend the r -map to gauged supergravity and use it to relate our flow equations to those in four dimensions.

KEYWORDS: Black Holes, Black Holes in String Theory, AdS-CFT Correspondence, Supergravity Models

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1 Introduction

Exact solutions to supergravity theories, like black holes, domain walls, plane-fronted waves etc. have been instrumental in various developments of string theory, for instance in holography or black hole microstate counting. Generically, one is interested both in supersymmetric backgrounds and in solutions that break supersymmetry, like nonextremal black holes. The latter play an important role for example in holographic descriptions of condensed matter systems at finite temperature, or of the quark-gluon plasma.

The supergravity equations of motion are coupled, nonlinear, second-order partial differential equations, and as such quite difficult to solve analytically, even in presence of a high degree of symmetry. A possible way out is to consider instead the Killing spinor equations, which are of first order in derivatives, and imply (at least when the Killing vector constructed as a bilinear from the Killing spinor is timelike) the second-order equations of motion. In this way, however, one obtains only supersymmetric solutions, and therefore interesting objects like nonextremal or extremal non-BPS black holes are excluded a priori.

A more general possibility that still involves solving first-order equations, is the Hamilton-Jacobi approach. This includes the (symmetry-reduced) Killing spinor equations as a special subcase, but is quite easily generalizable to extremal non-BPS- or even

nonextremal black holes. Using Hamilton-Jacobi theory is essentially¹ equivalent to writing the action as a sum of squares, which can be seen as follows: suppose that, owing to various symmetries (like e.g. staticity and spherical symmetry), the supergravity action can be dimensionally reduced to just one dimension,² such that

$$I = \int dr \left[\frac{1}{2} \mathcal{G}_{\Lambda\Sigma} \dot{q}^\Lambda \dot{q}^\Sigma - U(q) \right], \quad (1.1)$$

where r is a radial variable (the ‘flow’ direction), the $q^\Lambda(r)$ denote collectively the dynamical variables, $U(q)$ is the potential and $\mathcal{G}_{\Lambda\Sigma}(q)$ the metric on the target space parametrized by the q^Λ , with inverse $\mathcal{G}^{\Lambda\Sigma}$. Now suppose that U can be expressed in terms of a (fake) superpotential W as

$$U = E - \frac{1}{2} \mathcal{G}^{\Lambda\Sigma} \frac{\partial W}{\partial q^\Lambda} \frac{\partial W}{\partial q^\Sigma}, \quad (1.2)$$

where E is a constant. Then, the action (1.1) becomes

$$I = \int dr \left[\frac{1}{2} \mathcal{G}_{\Lambda\Sigma} \left(\dot{q}^\Lambda - \mathcal{G}^{\Lambda\Omega} \frac{\partial W}{\partial q^\Omega} \right) \left(\dot{q}^\Sigma - \mathcal{G}^{\Sigma\Delta} \frac{\partial W}{\partial q^\Delta} \right) + \frac{d}{dr} (W - Er) \right], \quad (1.3)$$

which is up to a total derivative equal to

$$I = \int dr \frac{1}{2} \mathcal{G}_{\Lambda\Sigma} \left(\dot{q}^\Lambda - \mathcal{G}^{\Lambda\Omega} \frac{\partial W}{\partial q^\Omega} \right) \left(\dot{q}^\Sigma - \mathcal{G}^{\Sigma\Delta} \frac{\partial W}{\partial q^\Delta} \right). \quad (1.4)$$

The latter is obviously stationary if the first-order flow equations

$$\dot{q}^\Lambda = \mathcal{G}^{\Lambda\Omega} \frac{\partial W}{\partial q^\Omega} \quad (1.5)$$

hold. But (1.2) is nothing else than the reduced Hamilton-Jacobi equation, with W Hamilton’s characteristic function, while (1.5) represents the expression for the conjugate momenta $p_\Lambda = \partial\mathcal{L}/\partial\dot{q}^\Lambda = \mathcal{G}_{\Lambda\Sigma}\dot{q}^\Sigma$ in Hamilton-Jacobi theory.³

First-order flow equations, derived either by writing the dimensionally reduced action as a sum of squares or from the Hamilton-Jacobi formalism, appear for many different settings in the literature, both in ungauged and gauged supergravity, and for BPS-, extremal non-BPS- and even nonextremal black holes, cf. e.g. [3–17] for an (incomplete) list of references. In particular, ref. [18] establishes general properties of supersymmetric flow equations for domain walls in five-dimensional $N = 2$ gauged supergravity coupled to vector- and hypermultiplets.

Here we shall consider magnetically charged black strings in five-dimensional $N = 2$ gauged supergravity, and obtain first-order flow equations for them. This is done first for

¹‘Essentially’ means that in many flow equations obtained in the literature by squaring an action, the r.h.s. of (1.5) is not a gradient, or, in other words, the flow is not driven by a (fake) superpotential (no gradient flow).

²When there is less symmetry, e.g. for rotating black holes, one obtains a field theory living in two or more dimensions, instead of a mechanical system [1, 2]. In this case, the Hamilton-Jacobi formalism has to be generalized to the so-called De Donder-Weyl-Hamilton-Jacobi theory [1].

³For further discussions of the relationship between the Hamilton-Jacobi formalism and the first-order equations derived from a (fake) superpotential cf. [3, 4].

the Fayet-Iliopoulos (FI)-gauged case, and then generalized to include also hypermultiplets, where abelian symmetries of the quaternionic hyperscalar target manifold are gauged. Extremal magnetic black strings interpolate between (so-called ‘magnetic’) AdS_5 at infinity and $\text{AdS}_3 \times \Sigma$ (with Σ a two-dimensional space of constant curvature) at the horizon. Holographically, this corresponds to an RG flow across dimensions from a 4d field theory to a two-dimensional CFT in the infrared [19–23]. By plugging the near-horizon data into the flow equations, one gets the attractor equations. We solve the latter in full generality under the additional assumption that the ‘adjoint identity’ (A.3) holds. This enables us to compute the central charge of the 2d CFT that describes the black strings in the IR, in terms of the string charges, the FI parameters (or, more generally, the moment maps, if also hypermultiplets are present), and the constants C_{IJK} that appear in the Chern-Simons term of the supergravity action.

The remainder of this paper is organized as follows: in the next section, we briefly review $N = 2, d = 5$ Fayet-Iliopoulos gauged supergravity. In section 3, we derive the BPS flow equations for static black strings, and generalize them in 4 to the non-supersymmetric case, by using a simple deformation of the BPS superpotential. After that, in section 5, the presence of (abelian) gauged hypermultiplets is taken into account as well. In 6, the flow equations are solved in the near-horizon limit, where the geometry contains an AdS_3 factor. This leads to the attractor equations for black strings, that we subsequently solve in full generality, and compute the central charge of the 2d CFT that describes the black strings in the IR. We conclude in section 7 with some proposals for extensions of our work. An appendix contains some useful relations in very special geometry and the construction of the r -map in the gauged case.

2 $N = 2, d = 5$ Fayet-Iliopoulos gauged supergravity

The bosonic Lagrangian of $N = 2, d = 5$ FI-gauged supergravity coupled to n_v vector multiplets is given by [24, 25]⁴

$$e^{-1} \mathcal{L} = \frac{1}{2} R - \frac{1}{2} \mathcal{G}_{ij} \partial_\mu \phi^i \partial^\mu \phi^j - \frac{1}{4} G_{IJ} F_{\mu\nu}^I F^{J\mu\nu} + \frac{e^{-1}}{48} C_{IJK} \epsilon^{\mu\nu\rho\sigma\lambda} F_{\mu\nu}^I F_{\rho\sigma}^J A_\lambda^K - g^2 V, \quad (2.1)$$

where the scalar potential reads

$$V = V_I V_J \left(\frac{9}{2} \mathcal{G}^{ij} \partial_i h^I \partial_j h^J - 6 h^I h^J \right). \quad (2.2)$$

Here, V_I are FI constants, ∂_i denotes a partial derivative with respect to the real scalar field ϕ^i , and $h^I = h^I(\phi^i)$ satisfy the condition

$$\mathcal{V} \equiv \frac{1}{6} C_{IJK} h^I h^J h^K = 1. \quad (2.3)$$

Moreover, G_{IJ} and \mathcal{G}_{ij} can be expressed in terms of the homogeneous cubic polynomial \mathcal{V} which defines a ‘very special geometry’ [26],

$$G_{IJ} = -\frac{1}{2} \frac{\partial}{\partial h^I} \frac{\partial}{\partial h^J} \log \mathcal{V}|_{\mathcal{V}=1}, \quad \mathcal{G}_{ij} = \partial_i h^I \partial_j h^J G_{IJ}|_{\mathcal{V}=1}. \quad (2.4)$$

⁴The indices I, J, \dots range from 1 to $n_v + 1$, while $i, j, \dots = 1, \dots, n_v$.

Further useful relations can be found in appendix A. We note that if the five-dimensional theory is obtained by gauging a supergravity theory coming from a Calabi-Yau compactification of M-theory, then \mathcal{V} is the intersection form, h^I and $h_I \equiv \frac{1}{6}C_{IJK}h^Jh^K$ correspond to the size of the two- and four-cycles and the constants C_{IJK} are the intersection numbers of the Calabi-Yau threefold [27].

3 BPS flow for a static black string

Very special real Kähler manifolds can be viewed as the pre-image of the supergravity r -map [28, 29]. This suggests to consider the five-dimensional spacetime as a Kaluza-Klein uplift of the usual static black holes in four dimensions. Moreover, a pure string solution in $d = 5$ supports only magnetic charges, thus the field configuration reads

$$\begin{aligned} ds^2 &= e^{2T(r)}dz^2 + e^{-T(r)}\left(-e^{2U(r)}dt^2 + e^{-2U(r)}dr^2 + e^{2\psi(r)-2U(r)}d\sigma_\kappa^2\right), \\ F^I &= p^I f_\kappa(\theta)d\theta \wedge d\phi, \quad \phi^i = \phi^i(r), \end{aligned} \tag{3.1}$$

where $d\sigma_\kappa^2 = d\theta^2 + f_\kappa^2(\theta)d\varphi^2$ is the metric on the two-dimensional surfaces $\Sigma = \{S^2, H^2\}$ of constant scalar curvature $R = 2\kappa$, with $\kappa \in \{1, -1\}$, and

$$f_\kappa(\theta) = \frac{1}{\sqrt{\kappa}} \sin(\sqrt{\kappa}\theta) = \begin{cases} \sin \theta & \kappa = 1, \\ \sinh \theta & \kappa = -1. \end{cases} \tag{3.2}$$

Plugging the ansatz (3.1) into the equations of motion following from (2.1) yields a set of ordinary differential equations that can be derived from the one-dimensional effective action

$$\begin{aligned} S_{\text{eff}} &= \int dr \left[e^{2\psi} \left(U'^2 + \frac{3}{4}T'^2 - \psi'^2 + \frac{1}{2}G_{ij}\phi'^i\phi'^j \right) - V_{\text{eff}} \right], \\ V_{\text{eff}} &= \kappa - e^{2\psi-2U-T}g^2V - \frac{1}{2}e^{2U+T-2\psi}G_{IJP}^I p^J, \end{aligned} \tag{3.3}$$

imposing in addition the zero energy condition $H_{\text{eff}} = 0$. In the Hamilton-Jacobi formulation the latter becomes the partial differential equation

$$e^{-2\psi} \left((\partial_U W)^2 - (\partial_\psi W)^2 + \frac{4}{3}(\partial_T W)^2 + 2G^{ij}\partial_i W \partial_j W \right) + V_{\text{eff}} = 0, \tag{3.4}$$

where W is Hamilton's characteristic function that we will sometimes refer to also as (fake) superpotential.

A solution of (3.4) permits to write the action as a sum of squares and to derive a set of first-order flow equations by setting the squares to zero.⁵ Guided by the four-dimensional case [9, 13, 17], the ansatz for the simplest non-trivial solution is

$$W = ae^{U+\frac{T}{2}}p^I h_I + be^{2\psi-U-\frac{T}{2}}V_I h^I, \tag{3.5}$$

⁵As explained in the introduction, these are of course equivalent to the usual first-order equations in the Hamilton-Jacobi formalism.

where a, b are constants. Using (A.1), one can show that (3.5) solves (3.4) if one imposes

$$a = -\frac{3}{4}, \quad b = \frac{3}{2}g, \quad V_I p^I = -\frac{\kappa}{3g}. \quad (3.6)$$

The last of (3.6) is a sort of Dirac quantization condition for the linear combination $V_I p^I$ of the magnetic charges in terms of the inverse gauge coupling constant g^{-1} . This solution for W leads to the first-order flow

$$\begin{aligned} U' &= -\frac{3}{4}e^{U+\frac{T}{2}-2\psi}p^I h_I - \frac{3}{2}g e^{-U-\frac{T}{2}}V_I h^I, \\ \psi' &= -3g e^{-U-\frac{T}{2}}V_I h^I, \quad T' = \frac{2}{3}U', \\ \phi'^i &= 3\mathcal{G}^{ij} \left(-\frac{1}{2}e^{U+\frac{T}{2}-2\psi}p^I \partial_j h_I + g e^{-U-\frac{T}{2}}V_I \partial_j h^I \right). \end{aligned} \quad (3.7)$$

One can check that (3.7) coincides with the system obtained in [30] from the Killing spinor equations. In particular, it is easy to verify that the supersymmetric magnetic black string solution of [31] satisfies (3.7). Moreover, introducing a new radial coordinate R and the warp factors f and ρ such that

$$U = \frac{3}{2}f, \quad \psi = 2f + \rho, \quad T = f, \quad \frac{dR}{dr} = e^{-3f}, \quad (3.8)$$

and specifying to the stu model, one shows that (3.7) is precisely the system of equations derived in appendix 7.1 of [19] from the Killing spinor equations.

4 Non-BPS flow

One of the main advantages of the Hamilton-Jacobi formalism is to allow for a simple generalization of the first-order flow driven by (3.5) to a non-BPS one. Similar to the case of $N = 2, d = 4$ abelian gauged supergravity [14, 17], one introduces a ‘field rotation matrix’ $S^I{}_J$ such that

$$G_{LK}S^L{}_I S^K{}_J = G_{IJ}. \quad (4.1)$$

A nontrivial S (different from $\pm \text{Id}$) allows to generate new solutions from known ones by ‘rotating charges’. This technique was first introduced in [6, 7], and generalizes the sign-flipping procedure of [32]. Using (4.1), one easily verifies that

$$\tilde{W} = -\frac{4}{3}e^{U+\frac{T}{2}}h_I S^I{}_J p^J + \frac{2}{3}g e^{2\psi-U-\frac{T}{2}}V_I h^I \quad (4.2)$$

satisfies again the Hamilton-Jacobi equation (3.4), provided the modified quantization condition

$$V_I S^I{}_J p^J = -\frac{\kappa}{3g} \quad (4.3)$$

holds. This leads to the first-order flow driven by \tilde{W} ,

$$\begin{aligned} U' &= -\frac{3}{4}e^{U+\frac{T}{2}-2\psi}h_I S^I{}_J p^J - \frac{3}{2}g e^{-U-\frac{T}{2}}V_I h^I, \\ \psi' &= -3g e^{-U-\frac{T}{2}}V_I h^I, \quad T' = \frac{2}{3}U', \\ \phi'^i &= 3\mathcal{G}^{ij} \left(-\frac{1}{2}e^{U+\frac{T}{2}-2\psi}\partial_j h_I S^I{}_J p^J + g e^{-U-\frac{T}{2}}V_I \partial_j h^I \right). \end{aligned} \quad (4.4)$$

An interesting example for which (4.1) admits nontrivial solutions, is the model $\mathcal{V} = h^1 h^2 h^3 = 1$ (cf. e.g. [30]), for which

$$G_{IJ} = \frac{\delta_{IJ}}{2(h^I)^2}. \tag{4.5}$$

In this case a particular solution of (4.1) is given by

$$S^I{}_J = \begin{pmatrix} \epsilon_1 & 0 & 0 \\ 0 & \epsilon_2 & 0 \\ 0 & 0 & \epsilon_3 \end{pmatrix}, \tag{4.6}$$

with $\epsilon_I = \pm 1$. These matrices form a discrete subgroup $D = (\mathbb{Z}_2)^3 \subset \text{GL}(3, \mathbb{R})$. Since there are two equivalent BPS branches, the independent solutions correspond to elements of the quotient group D/\mathbb{Z}_2 .

5 Inclusion of hypermultiplets

We now generalize our analysis to include also the coupling to n_{H} hypermultiplets. The charged hyperscalars q^u ($u = 1, \dots, 4n_{\text{H}}$) parametrize a quaternionic Kähler manifold with metric $h_{uv}(q)$, i.e., a $4n_{\text{H}}$ -dimensional Riemannian manifold admitting a locally defined triplet $\vec{K}_u{}^v$ of almost complex structures satisfying the quaternion relation

$$h^{st} K^x{}_{us} K^y{}_{tw} = -\delta^{xy} h_{uw} + \epsilon^{xyz} K^z{}_{uw}, \tag{5.1}$$

and whose Levi-Civita connection preserves \vec{K} up to a rotation,

$$\nabla_w \vec{K}_u{}^v + \vec{\omega}_w \times \vec{K}_u{}^v = 0, \tag{5.2}$$

where $\vec{\omega} \equiv \vec{\omega}_u(q) dq^u$ is the connection of the $\text{SU}(2)$ -bundle for which the quaternionic manifold is the base. The $\text{SU}(2)$ curvature is proportional to the complex structures,

$$\Omega^x \equiv d\omega^x + \frac{1}{2} \epsilon^{xyz} \omega^y \wedge \omega^z = -K^x. \tag{5.3}$$

Here we shall consider only gaugings of abelian isometries of the quaternionic Kähler metric h_{uv} . These are generated by commuting Killing vectors $k_I^u(q)$. For each Killing vector one can introduce a triplet of moment maps, P_I^x , such that

$$D_u P_I^x \equiv \partial_u P_I^x + \epsilon^{xyz} \omega^y{}_u P_I^z = -2\Omega^x{}_{uv} k_I^v. \tag{5.4}$$

One of the most important relations satisfied by the moment maps is the so-called equivariance relation. For abelian gaugings it has the form

$$\frac{1}{2} \epsilon^{xyz} P_I^y P_J^z - \Omega^x{}_{uv} k_I^u k_J^v = 0. \tag{5.5}$$

The bosonic Lagrangian is now given by⁶

$$e^{-1} \mathcal{L} = \frac{1}{2} R - \frac{1}{2} \mathcal{G}_{ij} \partial_\mu \phi^i \partial^\mu \phi^j - h_{uv} \hat{\partial}_\mu q^u \hat{\partial}^\mu q^v - \frac{1}{4} G_{IJ} F_{\mu\nu}^I F^{J\mu\nu} + \frac{e^{-1}}{48} C_{IJK} \epsilon^{\mu\nu\rho\sigma\lambda} F_{\mu\nu}^I F_{\rho\sigma}^J A_\lambda^K - g^2 V, \quad (5.6)$$

with the covariant derivative

$$\hat{\partial}_\mu q^u = \partial_\mu q^u + 3g A_\mu^I k_I^u, \quad (5.7)$$

and the scalar potential

$$V = P_I^x P_J^x \left(\frac{9}{2} \mathcal{G}^{ij} \partial_i h^I \partial_j h^J - 6h^I h^J \right) + 9h_{uv} k_I^u k_J^v h^I h^J. \quad (5.8)$$

Varying (5.6) w.r.t A_μ^I , one obtains the Maxwell equations

$$\partial_\mu (e \mathcal{G}_{IJ} F^{J\mu\nu}) + \frac{1}{4} \epsilon^{\mu\lambda\rho\sigma\nu} C_{IJK} \partial_\mu A_\lambda^J \partial_\rho A_\sigma^K = 6g e h_{uv} k_I^u \hat{\partial}^\nu q^v. \quad (5.9)$$

Imposing the ansatz (3.1), the t -, θ - and z -components of (5.9) are automatically satisfied, while the r - and φ -components become respectively

$$h_{uv} k_I^u q'^v = 0, \quad k_I^u p^I = 0. \quad (5.10)$$

The remaining equations of motion can be derived from the effective action

$$S_{\text{eff}} = \int dr \left[e^{2\psi} \left(U'^2 + \frac{3}{4} T'^2 - \psi'^2 + \frac{1}{2} \mathcal{G}_{ij} \phi'^i \phi'^j + h_{uv} q'^u q'^v \right) - V_{\text{eff}} \right], \quad (5.11)$$

$$V_{\text{eff}} = \kappa - e^{2\psi-2U-T} g^2 V - \frac{1}{2} e^{2U+T-2\psi} G_{IJ} p^I p^J,$$

supplemented by the Hamiltonian constraint $H_{\text{eff}} = 0$. The latter leads to the Hamilton-Jacobi equation

$$e^{-2\psi} \left((\partial_U W)^2 - (\partial_\psi W)^2 + \frac{4}{3} (\partial_T W)^2 + 2\mathcal{G}^{ij} \partial_i W \partial_j W + h^{uv} \partial_u W \partial_v W \right) + V_{\text{eff}} = 0. \quad (5.12)$$

Guided by the FI-gauged case and by previous work in four dimensions [17], we use the ansatz

$$W = c e^{U+\frac{T}{2}} \mathcal{Z} + d e^{2\psi-U-\frac{T}{2}} \mathcal{L}, \quad (5.13)$$

where

$$\mathcal{Z} = p^I h_I, \quad \mathcal{L} = Q^x \mathcal{W}^x, \quad Q^x = p^I P_I^x, \quad \mathcal{W}^x = h^J P_J^x. \quad (5.14)$$

Using some relations of very special geometry as well as (5.1), (5.3) and (5.5), one can show that (5.13) solves indeed (5.12) provided that

$$c = -\frac{3}{4}, \quad d = -\frac{9}{2} \kappa g^2, \quad Q^x Q^x = \frac{1}{9g^2}. \quad (5.15)$$

⁶(5.6) can be obtained from the Lagrangian in [18] by rescaling $a_{IJ} \rightarrow \frac{2}{3} G_{IJ}$, $C_{IJK} \rightarrow \frac{1}{6} C_{IJK}$, $k_I \rightarrow 2k_I$, $A^I \rightarrow \sqrt{\frac{3}{2}} A^I$, $g \rightarrow \sqrt{\frac{3}{2}} g$.

The solution (5.13) leads then to the first-order flow equations

$$\begin{aligned}
 U' &= -\frac{3}{4}e^{U+\frac{T}{2}-2\psi}\mathcal{Z} + \frac{9}{2}\kappa g^2 e^{-U-\frac{T}{2}}\mathcal{L}, & T' &= \frac{2}{3}U', \\
 \psi' &= 9\kappa g^2 e^{-U-\frac{T}{2}}\mathcal{L}, \\
 \phi'^i &= \mathcal{G}^{ij} \left(-\frac{3}{2}e^{U+\frac{T}{2}-2\psi}\partial_j\mathcal{Z} - 9\kappa g^2 e^{-U-\frac{T}{2}}\partial_j\mathcal{L} \right), \\
 q'^u &= -\frac{9}{2}\kappa g^2 e^{-U-\frac{T}{2}}h^{uv}\partial_v\mathcal{L}.
 \end{aligned} \tag{5.16}$$

One can recast (5.16) into a form very similar to that of the first-order flow in four dimensions, cf. eqs. (3.43) in [17]. Integrating $T' = \frac{2}{3}U'$ and plugging this into the remaining equations of (5.16), one gets

$$\begin{aligned}
 T' &= -\frac{1}{2}e^{2T-2\psi}\mathcal{Z} + 3\kappa g^2 e^{-2T}\mathcal{L}, \\
 \psi' &= 9\kappa g^2 e^{-2T}\mathcal{L}, \\
 \phi'^i &= \mathcal{G}^{ij} \left(-\frac{3}{2}e^{2T-2\psi}\partial_j\mathcal{Z} - 9\kappa g^2 e^{-2T}\partial_j\mathcal{L} \right), \\
 q'^u &= -\frac{9}{2}\kappa g^2 e^{-2T}h^{uv}\partial_v\mathcal{L}.
 \end{aligned} \tag{5.17}$$

Using the equation for ϕ'^i together with $(h^I)' = \phi'^i\partial_i h^I$ and (A.2), the equations for T and ϕ^i can be rewritten as

$$e^{2\psi} (e^{-2T}h^I)' + 9g^2\kappa e^{2\psi-4T}\mathcal{Q}^x P_J^x G^{IJ} - p^I = 0. \tag{5.18}$$

Note that the FI case can be recovered imposing $P_I^1 = P_I^2 = 0$ and $P_I^3 = V_I$. Then the charge quantization condition $\mathcal{Q}^x \mathcal{Q}^x = 1/(9g^2)$ boils down to $\mathcal{Q}^3 = p^I V_I = \pm\kappa/(3g)$ (use $\kappa^2 = 1$), while \mathcal{L} in (5.14) becomes $\mathcal{L} = \pm\frac{\kappa}{3g}h^J V_J$, which is the expression appearing in (3.5). The two signs correspond to the two equivalent BPS branches; in section 3 the lower sign was chosen.

6 Attractors and central charge of the dual CFT

In this section we want to investigate the near-horizon configurations of the black string. To keep things simple, we shall first concentrate on the hyperless FI-gauged case considered in section 3, and set $g = 1$. The geometry is of the type $\text{AdS}_3 \times \Sigma$ with $\Sigma = \{S^2, H^2\}$, and we assume that the scalars stabilize regularly at the horizon, i.e., $\phi'^i = 0$. Note that a similar problem was solved in four dimensions in [33] for the case of symmetric special Kähler manifolds with cubic prepotential.⁷

In the coordinates (t, R, z, θ, ϕ) , where R was introduced in (3.8), the metric (3.1) takes the form

$$ds^2 = e^{2f}(-dt^2 + dR^2 + dz^2) + e^{2\rho}d\sigma_\kappa^2, \tag{6.1}$$

⁷Supersymmetric Bianchi attractors in $N = 2$, $d = 5$ gauged supergravity coupled to vector- and hypermultiplets were analyzed recently in [34].

and the first-order flow equations (3.7) become

$$\begin{aligned} f' &= -e^f \left(h^I V_I + \frac{1}{2} e^{-2\rho} \mathcal{Z} \right), \\ \rho' &= -e^f (h^I V_I - e^{-2\rho} \mathcal{Z}), \\ \phi'^i &= 3\mathcal{G}^{ij} e^f \left(\partial_j h^I V_I - \frac{1}{2} e^{-2\rho} \partial_j \mathcal{Z} \right), \end{aligned} \tag{6.2}$$

where the primes now denote derivatives w.r.t. R . For a product space $\text{AdS}_3 \times \Sigma$ we have

$$e^{2f} = \frac{R_{\text{AdS}_3}^2}{R^2}, \quad e^{2\rho} = R_{\text{H}}^2. \tag{6.3}$$

Plugging this together with $\phi'^i = 0$ into (6.2), one obtains a system of algebraic equations whose solution fixes the near-horizon values of the scalars in terms of the charges and the FI parameters,

$$h^I V_I = \frac{2}{3R_{\text{AdS}_3}}, \quad \mathcal{Z} = R_{\text{H}}^2 h^I V_I, \quad \partial_i \mathcal{Z} = 2R_{\text{H}}^2 \partial_i h^I V_I. \tag{6.4}$$

For the ansatz (6.3), the FI-version of (5.18) (obtained by taking $Q^x P_J^x = Q^3 P_J^3 = -\kappa V_J / (3g)$) reduces to

$$e^{f+2\rho} (e^{-2f} h^I)' - 3e^{2\rho} G^{IJ} V_J - p^I = 0. \tag{6.5}$$

Using (6.3) and (6.4), this can be rewritten as

$$p^I + 3R_{\text{H}}^2 G^{IJ} V_J = 3\mathcal{Z} h^I. \tag{6.6}$$

We want to solve the attractor equations (6.4) (or equivalently (6.6)) in order to express R_{AdS_3} , R_{H} and h^I in terms of p^I and V_I . To this end, contract the third relation of (A.2) with V_I to get

$$\mathcal{G}^{ij} \partial_i h^I V_I \partial_j h_J = -\frac{2}{3} V_J + \frac{2}{3} h^I V_I h_J. \tag{6.7}$$

With (6.4), this becomes

$$R_{\text{H}}^2 V_J = -\frac{3}{4} \mathcal{G}^{ij} \partial_i \mathcal{Z} \partial_j h_J + \mathcal{Z} h_J. \tag{6.8}$$

Using $h_I = \frac{1}{6} C_{IJK} h^J h^K$ and (A.2), one obtains

$$R_{\text{H}}^2 V_J = \frac{1}{6} C_{JKLP} h^K h^L. \tag{6.9}$$

Let us introduce the charge-dependent matrix

$$C_{pIJ} \equiv C_{IJK} p^K. \tag{6.10}$$

Using the adjoint identity (A.3), one easily shows that C_{pIJ} is invertible, with inverse

$$C_p^{IJ} = 3 \frac{C^{IJK} C_{KMNP} p^M p^N - p^I p^J}{C_p}, \tag{6.11}$$

where $C_p = C_{IJK} p^I p^J p^K$. (6.9) implies then

$$h^I = 6R_{\text{H}}^2 C_p^{IJ} V_J. \quad (6.12)$$

Plugging (6.12) into (2.3), one can derive a general expression for R_{H} in terms of the intersection numbers, the charges and the FI parameters,

$$R_{\text{H}}^2 = (36C_{IJK} C_p^{IM} C_p^{JN} C_p^{KP} V_M V_N V_P)^{-\frac{1}{3}}. \quad (6.13)$$

Using this in (6.12) gives the values of the scalars at the horizon,

$$h^I = \frac{6C_p^{IJ} V_J}{(36C_{KLM} C_p^{KN} C_p^{LP} C_p^{MR} V_N V_P V_R)^{\frac{1}{3}}}. \quad (6.14)$$

Contracting (6.12) with V_I and using the first equation of (6.4) as well as (6.13), we obtain an expression for the AdS₃ curvature radius R_{AdS_3} ,

$$R_{\text{AdS}_3} = \frac{(36C_{IJK} C_p^{IM} C_p^{JN} C_p^{KP} V_M V_N V_P)^{\frac{1}{3}}}{9C_p^{RS} V_R V_S}. \quad (6.15)$$

Finally, one can plug (6.11) into (6.13), (6.14) and (6.15), and use (A.3) to write the solutions of (6.4) and (6.6) as

$$\begin{aligned} R_{\text{H}}^2 &= (\mathcal{C}^{IJK}(p) V_I V_J V_K)^{-\frac{1}{3}}, \\ h^I &= \frac{6\kappa p^I + 3\kappa C^{IJK} C_{KLM} p^L p^M V_J}{C_p (\mathcal{C}^{NPR}(p) V_N V_P V_R)^{\frac{1}{3}}}, \\ R_{\text{AdS}_3} &= \frac{C_p (\mathcal{C}^{IJK}(p) V_I V_J V_K)^{\frac{1}{3}}}{27 C_{LMN} C_{NRS} p^R p^S V_L V_M - \frac{1}{9}}, \end{aligned} \quad (6.16)$$

where

$$\mathcal{C}^{IJK}(p) = -\frac{108}{C_p} \left[2C^{IJK} - \frac{9}{C_p} p^{(I} C^{JK)M} C_{MNP} p^N p^P + \frac{9}{C_p} p^I p^J p^K \right]. \quad (6.17)$$

The central charge of the two-dimensional conformal field theory that describes the black strings in the infrared [19, 35, 36], is given by [37]

$$c = \frac{3R_{\text{AdS}_3}}{2G_3}, \quad (6.18)$$

where G_3 denotes the effective Newton constant in three dimensions, related to G_5 by

$$\frac{1}{G_3} = \frac{R_{\text{H}}^2 \text{vol}(\Sigma)}{G_5}. \quad (6.19)$$

In what follows, we assume Σ to be compactified to a Riemann surface of genus \mathfrak{g} , with $\mathfrak{g} = 0, 2, 3, \dots$. The unit Σ has Gaussian curvature $K = \kappa$, and thus the Gauss-Bonnet theorem gives

$$\text{vol}(\Sigma) = \frac{4\pi(1 - \mathfrak{g})}{\kappa}. \quad (6.20)$$

Using (6.19) and (6.20) in (6.18) yields for the central charge

$$c = \frac{6\pi(1 - \mathfrak{g})R_{\text{AdS}_3}R_{\text{H}}^2}{\kappa G_5}. \quad (6.21)$$

The curvature radii R_{AdS_3} and R_{H} can be expressed in terms of the constants C_{IJK} , the magnetic charges p^I and the FI parameters V_I by means of (6.16). This leads to

$$c = \frac{2\pi(1 - \mathfrak{g})C_p}{\kappa G_5(9C^{IJK}C_{KMN}p^M p^N V_I V_J - 1)}. \quad (6.22)$$

If the hyperscalars are running, one has to consider also the near-horizon limit of the last equation of (5.17). Assuming $q'^u = 0$ at the horizon and using (5.4), one easily derives the algebraic condition

$$k_I^u h^I = 0. \quad (6.23)$$

As far as the remaining equations of (5.17) are concerned, one can follow the same steps as in this section, with the only difference that V_I has to be replaced everywhere by $-3\kappa Q^x P_I^x$.

7 Final remarks

Let us conclude our paper with the following suggestions for possible extensions and questions for future work:

- Try to solve the flow equations in presence of hypermultiplets obtained in section 5 for some specific models, e.g. like those considered in [18], to explicitly construct black strings with running hyperscalars, similar in spirit to the black holes found in [38]. To the best of our knowledge, no such solutions are known up to now.
- Derive first-order equations for electrically charged black holes (rather than magnetically charged black strings) in five-dimensional matter-coupled gauged supergravity.
- Extend our work to the nonextremal case, similar to what was done in [5, 8, 10, 11, 15] in different contexts. Up to now, the only known nonextremal black string solutions in AdS₅ were constructed in [39] for minimal gauged supergravity.
- It would be interesting to see how the BPS flow equations derived in section 5 arise precisely in the general classification scheme of supersymmetric solutions obtained in [40].
- We have checked that our central charge (6.22) agrees with the results of [20, 35], where black string solutions corresponding to D3-branes at a Calabi-Yau singularity have been studied in detail. It may be of some interest to use the flow equations obtained in section 5 to study more complicated type IIB configurations, as was initiated in [23].

Work along these directions is in progress.

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A Useful relations in very special geometry

We list here some useful relations that can be proven using the techniques of very special geometry:

$$\partial_i h_I = -\frac{2}{3} G_{IJ} \partial_i h^J, \quad h_I = \frac{2}{3} G_{IJ} h^J, \quad G_{IJ} = \frac{9}{2} h_I h_J - \frac{1}{2} C_{IJK} h^K, \quad (\text{A.1})$$

$$\begin{aligned} \mathcal{G}^{ij} \partial_i h^I \partial_j h^J &= G^{IJ} - \frac{2}{3} h^I h^J, & \mathcal{G}^{ij} \partial_i h_I \partial_j h_J &= \frac{4}{9} G_{IJ} - \frac{2}{3} h_I h_J, \\ \mathcal{G}^{ij} \partial_i h^I \partial_j h_J &= -\frac{2}{3} \delta_J^I + \frac{2}{3} h^I h_J. \end{aligned} \quad (\text{A.2})$$

In the special case where the tensor T_{ijk} that determines the Riemann tensor of the vector multiplet scalar manifold \mathcal{M} (cf. [24] for details) is covariantly constant,⁸ one has also

$$C_{IJK} C_{J'(LM} C_{PQ)K'} \delta^{JJ'} \delta^{KK'} = \frac{4}{3} \delta_{I(L} C_{MPQ)}, \quad (\text{A.3})$$

which is the adjoint identity of the associated Jordan algebra [24]. Using (A.3) and defining $C^{IJK} \equiv \delta^{II'} \delta^{JJ'} \delta^{KK'} C_{I'J'K'}$, one can show that

$$G^{IJ} = -6C^{IJK} h_K + 2h^I h^J. \quad (\text{A.4})$$

B Down to $d = 4$ via r -map

A natural question arising in the discussion of (5.17) is the relation with the flow equations of [17, 41], coming from the abelian gauged supergravity theory in $d = 4$. An interesting way to answer this question is to extend the general r -map construction in ungauged supergravity [29] to the gauged case.

B.1 Construction of the r -map

The first step is a Kaluza-Klein reduction along the z -direction (i.e., along the string), by using the ansatz⁹

$$ds_5^2 = e^{\frac{\phi}{\sqrt{3}}} ds_4^2 + e^{-\frac{2}{\sqrt{3}}\phi} (dz + K_\mu dx^\mu)^2, \quad A^I = B^I dz + C_\mu^I dx^\mu + B^I K_\mu dx^\mu. \quad (\text{B.1})$$

⁸This implies that \mathcal{M} is a locally symmetric space.

⁹In this subsection μ, ν, \dots are curved indices for the four-dimensional theory, and the dilaton is related to the function T in (3.1) by $T = -\phi/\sqrt{3}$. Further details on the notation and the theory in $d = 4$ can be found in [17].

Defining $K_{\mu\nu} = \partial_\mu K_\nu - \partial_\nu K_\mu$ and $C_{\mu\nu}^I = \partial_\mu C_\nu^I - \partial_\nu C_\mu^I$, the five-dimensional Lagrangian (5.6) reduces to¹⁰

$$\begin{aligned}
 e_4^{-1} \mathcal{L}^{(4)} = & \frac{R^{(4)}}{2} - \frac{1}{8} e^{-\sqrt{3}\phi} K^{\mu\nu} K_{\mu\nu} - \frac{1}{4} G_{IJ} e^{-\frac{\phi}{\sqrt{3}}} (C^{I\mu\nu} + B^I K^{\mu\nu}) (C_{\mu\nu}^J + B^J K_{\mu\nu}) \\
 & - \frac{1}{2} e^{\frac{2\phi}{\sqrt{3}}} G_{IJ} \partial_\mu B^I \partial^\mu B^J - \frac{1}{2} G_{IJ} \partial_\mu h^I \partial^\mu h^J - \frac{1}{4} \partial_\mu \phi \partial^\mu \phi - h_{uv} \hat{\partial}_\mu q^u \hat{\partial}^\mu q^v \\
 & - \frac{e_4^{-1}}{16} \epsilon^{\mu\nu\rho\sigma} C_{IJK} \left(C_{\mu\nu}^I C_{\rho\sigma}^J B^K + \frac{1}{3} K_{\mu\nu} K_{\rho\sigma} B^I B^J B^K + C_{\mu\nu}^I K_{\rho\sigma} B^J B^K \right) \\
 & - e^{\sqrt{3}\phi} g^2 B^I k_I^u B^J k_J^v h_{uv} - g^2 e^{\frac{\phi}{\sqrt{3}}} V_5.
 \end{aligned} \tag{B.2}$$

Now we want to rewrite $\mathcal{L}^{(4)}$ in the language of $N = 2$, $d = 4$ supergravity, by using the identifications of the ungauged case [42]. The coordinates of the special Kähler manifold, Kähler potential, Kähler metric and electromagnetic field strengths are given in terms of five-dimensional data respectively by

$$\begin{aligned}
 z^I &= -B^I - i e^{-\frac{\phi}{\sqrt{3}}} h^I, & e^{\mathcal{K}} &= \frac{1}{8} e^{\sqrt{3}\phi}, \\
 g_{IJ} &= \frac{1}{2} e^{\frac{2\phi}{\sqrt{3}}} G_{IJ}, & F_{\mu\nu}^\Lambda &= \frac{1}{\sqrt{2}} (K_{\mu\nu}, C_{\mu\nu}^I),
 \end{aligned} \tag{B.3}$$

where capital greek indices Λ, Σ, \dots range from 0 to $n_v + 1$. If we introduce the matrices

$$R_{\Lambda\Sigma} = - \begin{pmatrix} \frac{1}{3} B & \frac{1}{2} B_J \\ \frac{1}{2} B_I & B_{IJ} \end{pmatrix}, \quad I_{\Lambda\Sigma} = -e^{-\sqrt{3}\phi} \begin{pmatrix} 1 + 4g & 4g_J \\ 4g_I & 4g_{IJ} \end{pmatrix}, \tag{B.4}$$

where we defined

$$\begin{aligned}
 B_{IJ} &= C_{IJK} B^K, & B_I &= C_{IJK} B^J B^K, & B &= C_{IJK} B^I B^J B^K, \\
 g &= g_{IJ} B^I B^J, & g_{IJ} B^J &= g_I = g_{\bar{I}} = g_{\bar{I}J} B^J,
 \end{aligned} \tag{B.5}$$

the Lagrangian (B.2) can be cast into the form

$$\begin{aligned}
 e_4^{-1} \mathcal{L}^{(4)} = & \frac{R}{2} - g_{I\bar{J}} \partial_\mu z^I \partial^\mu \bar{z}^{\bar{J}} - h_{uv} \hat{\partial}_\mu q^u \hat{\partial}^\mu q^v \\
 & + \frac{1}{4} I_{\Lambda\Sigma} F^{\Lambda\mu\nu} F_{\mu\nu}^\Sigma + \frac{1}{8} e_4^{-1} \epsilon^{\mu\nu\rho\sigma} R_{\Lambda\Sigma} F_{\mu\nu}^\Lambda F_{\rho\sigma}^\Sigma - \tilde{V},
 \end{aligned} \tag{B.6}$$

with the four-dimensional potential given by

$$\tilde{V} = g^2 e^{\frac{\phi}{\sqrt{3}}} V_5 + e^{\sqrt{3}\phi} g^2 h_{uv} k_I^u k_J^v B^I B^J. \tag{B.7}$$

The underlying prepotential of the special Kähler manifold turns out to be

$$F = \frac{1}{6} \frac{C_{IJK} X^I X^J X^K}{X^0}, \tag{B.8}$$

chosen the parametrization $X^I/X^0 = z^I = -B^I - i e^{-\phi/\sqrt{3}} h^I$ [42].

¹⁰We choose $\epsilon_5^{\mu\nu\rho\sigma z} = -\epsilon_4^{\mu\nu\rho\sigma}$.

The actual novelties with respect to the ungauged case are the potential and the covariant derivative acting on the hyperscalars. The former reads

$$\begin{aligned} \frac{\tilde{V}}{g^2} &= -9e^{\frac{\phi}{\sqrt{3}}} P_I^x P_J^x \left(h^I h^J - \frac{1}{2} G^{IJ} \right) + 9e^{\frac{\phi}{\sqrt{3}}} h_{uv} k_I^u k_J^v h^I h^J + 9e^{\sqrt{3}\phi} h_{uv} k_I^u k_J^v B^I B^J \\ &= 18P_I^x P_J^x \left(\frac{1}{4} e^{\frac{\phi}{\sqrt{3}}} G^{IJ} + \frac{1}{2} e^{\sqrt{3}\phi} B^I B^J - 4 \frac{e^{\sqrt{3}\phi}}{8} \left(e^{-\frac{\phi}{\sqrt{3}}} h^I \right) \left(e^{-\frac{\phi}{\sqrt{3}}} h^J \right) - \frac{1}{2} e^{\sqrt{3}\phi} B^I B^J \right) \\ &\quad + 72 \frac{e^{\sqrt{3}\phi}}{8} h_{uv} k_I^u k_J^v \left(e^{-\frac{2\phi}{\sqrt{3}}} h^I h^J + B^I B^J \right). \end{aligned} \tag{B.9}$$

Now the first two terms in the second line of (B.9) combine to give $-\frac{1}{2} I^{\Lambda\Sigma}$ (the inverse of $I_{\Lambda\Sigma}$ defined above), while the last two terms yield $-4X^I \bar{X}^J$. Fixing furthermore $g_4 = 3\sqrt{2}g$, one has thus

$$\begin{aligned} \tilde{V} &= g_4^2 \left[P_\Lambda^x P_\Sigma^x \left(-\frac{1}{2} I^{\Lambda\Sigma} - 4X^\Lambda \bar{X}^\Sigma \right) + 4h_{uv} k_\Lambda^u k_\Sigma^v X^\Lambda \bar{X}^\Sigma \right] \Big|_{P_0^x=0, k_0^u=0} \\ &= g_4^2 V_4 \Big|_{P_0^x=0, k_0^u=0}, \end{aligned} \tag{B.10}$$

which is precisely the truncated potential of the four-dimensional theory.

The final point to take care of is the covariant derivative of the hyperscalars,

$$\hat{\partial}_\mu q^u = \partial_\mu q^u + 3g C_\mu^I k_I^u = \partial_\mu q^u + g_4 A_{4\mu}^I k_I^u. \tag{B.11}$$

We have therefore shown that the r -map can be extended to the case of gauged supergravity, where the scalar fields have a potential.

B.2 Comparison with the flow in $N = 2$, $d = 4$ gauged supergravity

The result of the preceding subsection is completely general and interesting by itself, however our aim is to use this mapping to compare the flow (5.16) for a black string in $d = 5$ with the flow equations for black holes in four dimensions obtained in [17, 41]. The latter are driven by the Hamilton-Jacobi function

$$W_4 = e^U \text{Re}(e^{-i\alpha} \mathcal{Z}_4) - \kappa g_4 e^{2\psi-U} \text{Im}(e^{-i\alpha} \mathcal{L}_4), \tag{B.12}$$

where the phase α is defined by

$$e^{2i\alpha} = \frac{\mathcal{Z}_4 + i\kappa g_4 e^{2(\psi-U)} \mathcal{L}_4}{\bar{\mathcal{Z}}_4 - i\kappa g_4 e^{2(\psi-U)} \bar{\mathcal{L}}_4}. \tag{B.13}$$

Specifying to a purely magnetic charge configuration $\hat{p}^\Lambda = (0, p^I/\sqrt{2})$, purely electric couplings with $P_0^x = 0, k_0^u = 0$, and restricting to imaginary scalars, $z^I = -ie^{-\phi/\sqrt{3}} h^I$, the quantities defining (B.12) become

$$\begin{aligned} \mathcal{Z}_4 &= \frac{3}{2\sqrt{2}} e^{T/2} \hat{p}^I h_I = \frac{3}{4} e^{T/2} \mathcal{Z}, & \mathcal{Q}_4^x &= P_I^x \hat{p}^I = \frac{1}{\sqrt{2}} \mathcal{Q}^x, \\ \mathcal{W}_4^x &= -\frac{i}{2\sqrt{2}} e^{-T/2} P_I^x h^I = -\frac{i}{2\sqrt{2}} e^{-T/2} \mathcal{W}^x, & \mathcal{L}_4 &= \mathcal{Q}_4^x \mathcal{W}_4^x, \end{aligned} \tag{B.14}$$

where the quantities \mathcal{Z} , Q^x and W^x were defined in section 5. Note that axions are absent, since for magnetically charged black strings the z -components B^I of the five-dimensional gauge potentials vanish. For this choice, (B.13) becomes $e^{2i\alpha} = 1$. Moreover, taking in account that $g_4 = 3\sqrt{2}g$ and choosing $e^{i\alpha} = -1$, the function (B.12) boils down to (5.13). On the other hand, the Hamilton-Jacobi equation satisfied by (B.12), namely (3.40) of [17], becomes (5.12) once the dictionary is imposed. This proves the expected equivalence between the flows in five and four dimensions.

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