$\mathcal{N} = 6$ superconformal gravity in three dimensions from superspace

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ABSTRACT: A unique feature of $\mathcal{N} = 6$ conformal supergravity in three dimensions is that the super Cotton tensor W^{IJKL} can equivalently be viewed, via the Hodge duality, as the field strength of an Abelian vector multiplet, W^{IJ} . Using this observation and the conformal superspace techniques developed in arXiv:1305.3132 and arXiv:1306.1205, we construct the off-shell action for $\mathcal{N} = 6$ conformal supergravity. The complete component action is also worked out.

KEYWORDS: Extended Supersymmetry, Superspaces, Supergravity Models

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1 Introduction

Recently, we have constructed the superspace actions for three-dimensional (3D) conformal supergravity theories with $\mathcal{N} < 6$ [1], and also worked out the complete component actions. Our construction made use of the novel off-shell formulation for $\mathcal{N}$ -extended conformal supergravity presented in [2] and called 3D $\mathcal{N}$ -extended conformal superspace.¹ Upon degauging of certain local symmetries, conformal superspace reduces to the conventional formulation for $\mathcal{N}$ -extended conformal supergravity [5, 6] with the structure group $\text{SL}(2, \mathbb{R}) \times \text{SO}(\mathcal{N})$. However, the former formulation proves to be much more efficient for addressing certain problems such as the construction of conformal supergravity actions.

At the component level, the $\mathcal{N} = 1$ and $\mathcal{N} = 2$ supergravity actions given in [1] coincide with those constructed in the 1980s in [7] and [8], respectively, using the superconformal tensor calculus. But the off-shell $\mathcal{N} = 3$, $\mathcal{N} = 4$ and $\mathcal{N} = 5$ supergravity actions were derived in [1] for the first time. The method employed in [1] broke down for $\mathcal{N} > 5$ due to a technical reason to be discussed below. In this note we make use of a unique property of the $\mathcal{N} = 6$ case to construct the corresponding off-shell action from superspace. Upon reduction to components, our action coincides with that found a week ago by Nishimura and Tanii [9] who used completely different techniques.

This paper is organized as follows. In section 2 we briefly review $\mathcal{N} = 6$ conformal supergravity in superspace and then describe the unique feature of $\mathcal{N} = 6$ conformal supergravity. In section 3 we make use of this property to construct the off-shell action for $\mathcal{N} = 6$ conformal supergravity, both in superspace and in terms of the component fields. The main results and their implications are discussed in section 4. In appendix A

¹The 3D $\mathcal{N}$ -extended conformal superspace [2] is a generalization of the off-shell formulations for $\mathcal{N} = 1$ and $\mathcal{N} = 2$ conformal supergravity theories in four dimensions [3, 4].

we include the supersymmetry transformations of the component fields of the $\mathcal{N} = 6$ Weyl multiplet.

2 $\mathcal{N} = 6$ conformal supergravity

We begin with a brief review of $\mathcal{N} = 6$ conformal superspace following [2]. After that we describe the unique feature of $\mathcal{N} = 6$ conformal supergravity mentioned in the introduction.

We consider a curved three-dimensional $\mathcal{N} = 6$ superspace $\mathcal{M}^{3|12}$, parametrized by local bosonic (x^m) and fermionic coordinates (θ_I^μ) , $z^M = (x^m, \theta_I^\mu)$, where $m = 0, 1, 2$, $\mu = 1, 2$ and $I = 1, \dots, 6$. The $\mathcal{N} = 6$ conformal superspace [2] is obtained by gauging the $\mathcal{N} = 6$ superconformal algebra $\mathfrak{osp}(6|4, \mathbb{R})$ and then imposing appropriate constraints. The covariant derivatives have the form

$$\nabla_A = E_A^M \partial_M - \omega_A^b X_b = E_A^M \partial_M - \frac{1}{2} \Omega_A^{ab} M_{ab} - \frac{1}{2} \Phi_A^{PQ} N_{PQ} - B_A \mathbb{D} - \mathfrak{F}_A^B K_B. \quad (2.1)$$

Here $E_A = E_A^M(z) \partial_M$ is the inverse supervielbein, M_{ab} are the Lorentz generators, N_{IJ} are generators of the $\text{SO}(6)$ group, $\mathbb{D}$ is the dilatation generator and $K_A = (K_a, S_\alpha^I)$ are the special superconformal generators.² The complete set of the generators of $\mathfrak{osp}(6|4, \mathbb{R})$ consists of $X^{\tilde{a}} = (P_A, X_{\underline{a}})$, where $P_A = (P_a, Q_\alpha^I)$ are the super-Poincaré generators.

The Lorentz generators obey

$$[M_{ab}, M_{cd}] = 2\eta_{c[a} M_{b]d} - 2\eta_{d[a} M_{b]c}, \quad (2.2a)$$

$$[M_{ab}, \nabla_c] = 2\eta_{c[a} \nabla_{b]}, \quad [M_{\alpha\beta}, \nabla_\gamma^I] = \varepsilon_{\gamma(\alpha} \nabla_{\beta)}^I. \quad (2.2b)$$

The $\text{SO}(6)$ and dilatation generators obey

$$[N_{KL}, N^{IJ}] = 2\delta_{[K}^I N_{L]}^J - 2\delta_{[K}^J N_{L]}^I, \quad [N_{KL}, \nabla_\alpha^I] = 2\delta_{[K}^I \nabla_{\alpha L]}, \quad (2.3a)$$

$$[\mathbb{D}, \nabla_a] = \nabla_a, \quad [\mathbb{D}, \nabla_\alpha^I] = \frac{1}{2} \nabla_\alpha^I. \quad (2.3b)$$

The special conformal generators K_A transform under Lorentz and $\text{SO}(6)$ transformations as

$$[M_{ab}, K_c] = 2\eta_{c[a} K_{b]}, \quad [M_{\alpha\beta}, S_\gamma^I] = \varepsilon_{\gamma(\alpha} S_{\beta)}^I, \quad (2.4a)$$

$$[N_{KL}, S_\alpha^I] = 2\delta_{[K}^I S_{\alpha L]}, \quad (2.4b)$$

while under dilatations as

$$[\mathbb{D}, K_a] = -K_a, \quad [\mathbb{D}, S_\alpha^I] = -\frac{1}{2} S_\alpha^I. \quad (2.5)$$

Among themselves, the generators K_A obey the algebra

$$\{S_\alpha^I, S_\beta^J\} = 2i\delta^{IJ}(\gamma^c)_{\alpha\beta} K_c. \quad (2.6)$$

²As usual, we refer to K_a as the special conformal generator and S_α^I as the S -supersymmetry generator.

Finally, the algebra of K_A with ∇_A is given by

$$[K_a, \nabla_b] = 2\eta_{ab}\mathbb{D} + 2M_{ab}, \quad (2.7a)$$

$$[K_a, \nabla_\alpha^I] = -i(\gamma_a)_\alpha{}^\beta S_\beta^I, \quad (2.7b)$$

$$[S_\alpha^I, \nabla_a] = i(\gamma_a)_\alpha{}^\beta \nabla_\beta^I, \quad (2.7c)$$

$$\{S_\alpha^I, \nabla_\beta^J\} = 2\varepsilon_{\alpha\beta}\delta^{IJ}\mathbb{D} - 2\delta^{IJ}M_{\alpha\beta} - 2\varepsilon_{\alpha\beta}N^{IJ}. \quad (2.7d)$$

The remaining (anti-)commutators are not essential here and may be found in [2].

Under the supergravity gauge group, the covariant derivatives transform as

$$\delta_{\mathcal{G}}\nabla_A = [\mathcal{K}, \nabla_A], \quad (2.8)$$

where $\mathcal{K}$ denotes the first-order differential operator

$$\mathcal{K} = \xi^C \nabla_C + \frac{1}{2}\Lambda^{ab}M_{ab} + \frac{1}{2}\Lambda^{IJ}N_{IJ} + \Lambda_{\mathbb{D}}\mathbb{D} + \Lambda^A K_A. \quad (2.9)$$

Covariant (or tensor) superfields transform as

$$\delta_{\mathcal{G}}T = \mathcal{K}T. \quad (2.10)$$

The covariant derivatives obey the (anti-)commutation relations of the form

$$\begin{aligned} [\nabla_A, \nabla_B] &= -T_{AB}{}^C \nabla_C - R_{AB}{}^a X_a \\ &= -T_{AB}{}^C \nabla_C - \frac{1}{2}R(M)_{AB}{}^{cd}M_{cd} - \frac{1}{2}R(N)_{AB}{}^{PQ}N_{PQ} \\ &\quad - R(\mathbb{D})_{AB}\mathbb{D} - R(S)_{AB}{}^\gamma S_\gamma^K - R(K)_{AB}{}^c K_c, \end{aligned} \quad (2.11)$$

where T is the torsion, and $R(M)$, $R(N)$, $R(\mathbb{D})$, $R(K)$ are the curvatures.

The algebra of covariant derivatives corresponding to $\mathcal{N} = 6$ conformal supergravity is

$$\begin{aligned} \{\nabla_\alpha^I, \nabla_\beta^J\} &= 2i\delta^{IJ}\nabla_{\alpha\beta} + i\varepsilon_{\alpha\beta}W^{IJKL}N_{KL} - \frac{i}{3}\varepsilon_{\alpha\beta}(\nabla_K^\gamma W^{IJKL})S_{\gamma L} \\ &\quad + \frac{1}{24}\varepsilon_{\alpha\beta}(\gamma^c)_{\gamma\delta}(\nabla_{\gamma K}\nabla_{\delta L}W^{IJKL})K_c, \end{aligned} \quad (2.12a)$$

$$\begin{aligned} [\nabla_a, \nabla_\beta^J] &= \frac{1}{6}(\gamma_a)_{\beta\gamma}(\nabla_K^\gamma W^{JPQK})N_{PQ} \\ &\quad - \frac{1}{24}(\gamma_a)_{\beta\gamma}(\nabla_L^\gamma \nabla_P^\delta W^{JKLP})S_{\delta K} \\ &\quad - \frac{i}{240}(\gamma_a)_{\beta\gamma}(\gamma^c)_{\delta\rho}(\nabla_K^\gamma \nabla_L^\delta \nabla_P^\rho W^{JKLP})K_c, \end{aligned} \quad (2.12b)$$

$$\begin{aligned} [\nabla_a, \nabla_b] &= \frac{1}{48}\varepsilon_{abc}(\gamma^c)_{\alpha\beta}\left(i(\nabla_I^\alpha \nabla_J^\beta W^{PQIJ})N_{PQ} \right. \\ &\quad + \frac{i}{5}(\nabla_I^\alpha \nabla_J^\beta \nabla_K^\gamma W^{LIJK})S_{\gamma L} \\ &\quad \left. + \frac{1}{60}(\gamma^d)_{\gamma\delta}(\nabla_I^\alpha \nabla_J^\beta \nabla_K^\gamma \nabla_L^\delta W^{IJKL})K_d\right), \end{aligned} \quad (2.12c)$$

where $W^{IJKL} = W^{[IJKL]}$ is the super Cotton tensor, which is a completely antisymmetric primary superfield of dimension 1,

$$S_\alpha^P W^{IJKL} = 0, \quad \mathbb{D} W^{IJKL} = W^{IJKL}. \quad (2.13)$$

It satisfies the Bianchi identity

$$\nabla_\alpha^I W^{JKLP} = \nabla_\alpha^{[I} W^{JKLP]} - \frac{4}{3} \nabla_{\alpha Q} W^{Q[JKL} \delta^{P]I}. \quad (2.14)$$

The $\mathcal{N} = 6$ case is special because it has the important property that the Hodge dual of the Cotton tensor

$$W^{IJ} := \frac{1}{4!} \varepsilon^{IJKLPQ} W_{KLPQ} \quad (2.15)$$

satisfies the Bianchi identity for the field strength of an Abelian $\mathcal{N} = 6$ vector multiplet³

$$\nabla_\alpha^{[I} W^{JK]} = \nabla_\alpha^I W^{JK] - \frac{2}{5} \delta^{I[J} \nabla_{\alpha L} W^{K]L}. \quad (2.16)$$

Therefore, associated with the $\mathcal{N} = 6$ Weyl multiplet is a uniquely defined Abelian $\mathcal{N} = 6$ vector multiplet.

As a result we can use W^{IJ} to define a closed two-form $F = \frac{1}{2} E^B \wedge E^A F_{AB}$, $dF = 0$, with components

$$F_{\alpha\beta}^{IJ} = -2i\varepsilon_{\alpha\beta} W^{IJ}, \quad (2.17a)$$

$$F_{a\alpha}^I = \frac{1}{5} (\gamma_a)_\alpha{}^\beta \nabla_{\beta J} W^{IJ}, \quad (2.17b)$$

$$F_{ab} = -\frac{i}{120} \varepsilon_{abc} (\gamma^c)^{\alpha\beta} [\nabla_\alpha^K, \nabla_\beta^L] W_{KL}. \quad (2.17c)$$

It is the field strength of the vector multiplet. We associate with the field strength F a gauge one-form A ,

$$F = dA. \quad (2.18)$$

It is the existence of this gauge one-form which distinguishes the $\mathcal{N} = 6$ case from the $\mathcal{N} < 6$ cases.

3 Conformal supergravity action

In this section, we start by recalling the method to construct the $\mathcal{N} < 6$ conformal supergravity actions employed in [1]. After that, we present a generalization of the method that is suitable in the $\mathcal{N} = 6$ case.

³The $\mathcal{N}$ -extended vector multiplet in conformal superspace is described in [2].

3.1 Construction of $\mathcal{N} < 6$ conformal supergravity actions

The idea of the method employed in [1] is to look for two solutions, Σ_{CS} and Σ_R , to the superform equation

$$d\Sigma = \langle R^2 \rangle, \quad (3.1)$$

where⁴

$$\langle R^2 \rangle := R^{\tilde{b}} \wedge R^{\tilde{a}} \Gamma_{\tilde{a}\tilde{b}}, \quad d\langle R^2 \rangle = 0 \quad (3.2)$$

and $\Gamma_{\tilde{a}\tilde{b}}$ denotes a properly normalized Cartan-Killing metric of the $\mathcal{N}$ -extended superconformal algebra; the explicit form of $\langle R^2 \rangle$ is

$$\langle R^2 \rangle = -R(N)^{IJ} \wedge R(N)_{IJ}. \quad (3.3)$$

One solution is always the Chern-Simons form Σ_{CS} which exists for any $\mathcal{N}$. Its explicit form is found to be

$$\begin{aligned} \Sigma_{\text{CS}} = & -\hat{R}^a \wedge \Omega_a - \frac{1}{6} \Omega^c \wedge \Omega^b \wedge \Omega^a \varepsilon_{abc} - 4iE^a \wedge \mathfrak{F}^{\alpha I} \wedge \mathfrak{F}_I^{\beta} (\gamma_a)_{\alpha\beta} - \hat{R}^{IJ} \wedge \Phi_{IJ} \\ & + \frac{1}{3} \Phi^{IJ} \wedge \Phi_I^K \wedge \Phi_{KJ} + 2E^a \wedge \mathfrak{F}_a \wedge B - 2E_I^\alpha \wedge \mathfrak{F}_\alpha^I \wedge B + \text{exact form}, \end{aligned} \quad (3.4)$$

where

$$\hat{R}^{ab} := d\Omega^{ab} + \Omega^{ac} \wedge \Omega_c^b, \quad \hat{R}^{IJ} := d\Phi^{IJ} + \Phi^{IK} \wedge \Phi_K^J \quad (3.5)$$

correspond to the Riemann and $\text{SO}(\mathcal{N})$ curvature tensors. The other solution, the so-called curvature induced form Σ_R , is defined such that its components are constructed in terms of the super Cotton tensor and its covariant derivatives. It turns out that Σ_R exists for $\mathcal{N} < 6$ (in the cases $\mathcal{N} = 1$ and $\mathcal{N} = 2$, Σ_R vanishes), see [1] for more details.

The difference

$$\mathfrak{J} = \Sigma_{\text{CS}} - \Sigma_R \quad (3.6)$$

is a closed three-form which may be used to construct a locally supersymmetric action

$$S = \int_{\mathcal{M}^3} \mathfrak{J} = \int d^3x \, e \, {}^* \mathfrak{J}|_{\theta=0}, \quad {}^* \mathfrak{J} = \frac{1}{3!} \varepsilon^{mnp} \mathfrak{J}_{mnp}. \quad (3.7)$$

This action principle is not applicable for $\mathcal{N} \geq 6$ since Σ_R does not exist. However, in the $\mathcal{N} = 6$ case there is a way out and that is to make use of the closed two-form (2.17) constructed in terms of the vector multiplet field strength W^{IJ} , eq. (2.15).

⁴Following the notation of [1], we define $R^{\tilde{a}} := (R(P)^A, R^a)$ where $R(P)^A := T^A - \hat{T}^A$ and $\hat{T}^A$ is the flat superspace torsion.

3.2 Modified $\mathcal{N} = 6$ curvature induced form

In the $\mathcal{N} = 6$ case, we can modify the superform equation (3.3) to

$$d\Sigma = -R(N)^{IJ} \wedge R(N)_{IJ} - \mathcal{A}F \wedge F, \quad (3.8)$$

where $\mathcal{A}$ is some constant we will determine. Here F is the closed two-form (2.17). The Chern-Simons solution is now modified to

$$\begin{aligned} \Sigma_{\text{CS}} = & -\hat{R}^a \wedge \Omega_a - \frac{1}{6} \Omega^c \wedge \Omega^b \wedge \Omega^a \varepsilon_{abc} - 4iE^a \wedge \mathfrak{F}^{\alpha I} \wedge \mathfrak{F}_I^\beta (\gamma_a)_{\alpha\beta} - \hat{R}^{IJ} \wedge \Phi_{IJ} \\ & + \frac{1}{3} \Phi^{IJ} \wedge \Phi_I^K \wedge \Phi_{KJ} + 2E^a \wedge \mathfrak{F}_a \wedge B - 2E_I^\alpha \wedge \mathfrak{F}_\alpha^I \wedge B - \mathcal{A}F \wedge A \\ & + \text{exact form} . \end{aligned} \quad (3.9)$$

We may now attempt to find a covariant solution to eq. (3.8) which will also be called the curvature induced form and denoted Σ_R . We make the ansatz for the lowest components⁵

$$\Sigma_{\alpha\beta\gamma}^{IJK} = 0, \quad \Sigma_{a\beta\gamma}^{JK} = i(\gamma_a)_{\beta\gamma} (A\delta^{JK} W^{PQ} W_{PQ} + B W^{JP} W^K{}_P), \quad (3.10)$$

with A and B some constants to be determined, and turn to analyzing the superform equation (3.8) by increasing dimension.

At the lowest dimension we find that we must set $\mathcal{A} = -2$ and

$$\Sigma_{a\beta\gamma}^{JK} = 8i(\gamma_a)_{\beta\gamma} \left(W^{JP} W^K{}_P - \frac{1}{4} \delta^{JK} W^{PQ} W_{PQ} \right). \quad (3.11a)$$

The higher dimension components are found to be

$$\Sigma_{ab\gamma}^K = -2\varepsilon_{abc}(\gamma^c)_\gamma \delta \left(\nabla_\delta^{[K} W^{PQ]} W_{PQ} - \frac{2}{5} \nabla_\delta^P W_{QP} W^{QK} \right), \quad (3.11b)$$

$$\begin{aligned} \Sigma_{abc} = & i\varepsilon_{abc} \left(\frac{1}{5} \nabla^\gamma I \nabla_{\gamma K} W^{JK} W_{IJ} + \frac{1}{3} \nabla^\gamma [I W^{JK}] \nabla_{\gamma [I} W_{JK]} - \frac{2}{25} \nabla_I^\gamma W^{KI} \nabla_\gamma^J W_{KJ} \right. \\ & \left. - \frac{i}{3} \varepsilon^{IJKLPQ} W_{IJ} W_{KL} W_{PQ} \right). \end{aligned} \quad (3.11c)$$

To derive these results, we have made use of the following consequences of the Bianchi identity (2.16):

$$\nabla_\alpha^I \nabla_{\beta K} W^{JK} = \frac{1}{2} \varepsilon_{\alpha\beta} \nabla^\gamma [I \nabla_{\gamma K} W^{J]K} - 5i \nabla_{\alpha\beta} W^{IJ} + \frac{1}{6} \delta^{IJ} \nabla_{(\alpha K} \nabla_{\beta)L} W^{KL}, \quad (3.12a)$$

$$\nabla_P^\gamma \nabla_\gamma^P W^{IJ} = \frac{2}{5} \nabla^\gamma [I \nabla_{\gamma K} W^{J]K} - 4i W^{IJKL} W_{KL}, \quad (3.12b)$$

$$\begin{aligned} \nabla_\alpha^I \nabla_\beta^J W^{KL} = & \nabla_{(\alpha}^I \nabla_{\beta)}^J W^{KL} + \frac{3}{10} \varepsilon_{\alpha\beta} \delta^{IJ} \nabla^\gamma K \nabla_{\gamma P} W^{L]P} \\ & + 3i \delta^{IJ} \nabla_{\alpha\beta} W^{KL} + 6i \varepsilon_{\alpha\beta} W^{PI} [JK W^L]_P . \end{aligned} \quad (3.12c)$$

Now, the closed three-form $\mathfrak{J} = \Sigma_{\text{CS}} - \Sigma_R$ generates a locally supersymmetric action according to the rule (3.7).

⁵When referring to components of the curvature induced form we denote Σ_R by Σ to avoid awkward notation.

3.3 The component action

The complete component analysis of the $\mathcal{N}$ -extended Weyl multiplet was given in [1]. Here we specialize to the $\mathcal{N} = 6$ case where the auxiliary fields coming from the super Cotton tensor are defined as:

$$w_{IJKL} := W_{IJKL} \equiv \frac{1}{2} \varepsilon_{IJKLPQ} w^{PQ}, \quad (3.13a)$$

$$w_\alpha^{IJK} := -\frac{i}{6} \nabla_{\alpha L} W^{IJKL} \equiv \frac{1}{3!} \varepsilon^{IJKLPQ} \tilde{w}_{\alpha LPQ}, \quad (3.13b)$$

$$y^{IJKL} := \frac{i}{3} \nabla^{\gamma[I} \nabla_{\gamma P} W^{JKL]P} \equiv \frac{1}{2} \varepsilon^{IJKLPQ} y_{PQ}, \quad (3.13c)$$

$$X_\alpha^{I_1 \dots I_5} := i \nabla_\alpha^{[I_1} W^{I_2 \dots I_5]} \equiv \varepsilon^{I_1 \dots I_5 J} X_{\alpha J}. \quad (3.13d)$$

These definitions agree with [10, 11]. There is also an additional component field $X_{\alpha\beta}^{I_1 \dots I_6} := i \nabla_{(\alpha}^{[I_1} \nabla_{\beta}^{I_2} W^{I_3 \dots I_6]} = -\varepsilon^{I_1 \dots I_6} F_{\alpha\beta}$. However, this field turns out to be a composite object as it is the component U(1) field strength $\mathcal{F}_{ab}$ up to contributions involving the gravitino:

$$F_{ab} = \mathcal{F}_{ab} + i \psi_{[aJ}(\gamma_b)]_\beta \gamma^\beta X_\gamma^J - \frac{1}{2} i \psi_{[aI}^\alpha \psi_{b]\alpha J} w^{IJ}, \quad \mathcal{F}_{ab} := 2e_a^m e_b^n \partial_{[m} A_{n]}. \quad (3.14)$$

As the action is invariant with respect to the gauge transformations (2.8) up to a total derivative, it follows that the dependence on b_m must drop out. Equivalently, we can simply adopt the K -gauge $b_m = 0$. Using the action (3.7) and the Chern-Simons form (3.9), we find the Chern-Simons contribution to be

$$S_{CS} = \frac{1}{4} \int d^3x e \varepsilon^{abc} \left(\omega_a^{fg} \mathcal{R}_{bcfg} - \frac{2}{3} \omega_{af}^g \omega_{bg}^h \omega_{ch}^f - \frac{i}{2} \Psi_{bcI}^\alpha (\gamma_d)_\alpha^\beta (\gamma_a)_\beta^\gamma \varepsilon^{def} \Psi_{ef\gamma}^I \right. \\ \left. - 2\mathcal{R}_{ab}^{IJ} V_{cIJ} - \frac{4}{3} V_a^{IJ} V_{bI}^K V_{cKJ} + 4\mathcal{F}_{ab} A_c \right), \quad (3.15)$$

where the component curvatures $\mathcal{R}_{ab}^{cd}$ and $\mathcal{R}_{ab}^{IJ}$ are defined as

$$\mathcal{R}_{ab}^{cd} := 2e_a^m e_b^n \partial_{[m} \omega_{n]}^{ab} - 2\omega_{[a}^{cf} \omega_{b]}^d{}_f, \quad (3.16a)$$

$$\mathcal{R}_{ab}^{IJ} := 2e_a^m e_b^n \partial_{[m} V_{n]}^{IJ} - 2V_{[a}^{IK} V_{b]K}^J. \quad (3.16b)$$

Using the formula

$$\frac{1}{3!} \varepsilon^{mnp} \Sigma_{mnp} = \frac{1}{3!} \varepsilon^{mnp} E_p^C E_n^B E_m^A \Sigma_{ABC} \\ = \frac{1}{3!} \varepsilon^{abc} \left(\Sigma_{abc} + \frac{3}{2} \psi_a^\alpha \Sigma_{\alpha bc}^I + \frac{3}{4} \psi_b^\beta \psi_a^\alpha \Sigma_{\alpha\beta c}^{IJ} \right. \\ \left. + \frac{1}{8} \psi_c^\gamma \psi_b^\beta \psi_a^\alpha \Sigma_{\alpha\beta\gamma}^{IJK} \right) \quad (3.17)$$

for the component projection of a three-form along with the explicit expressions for the components of Σ_{ABC} , we find

$$\frac{1}{3!} \varepsilon^{mnp} \Sigma_{mnp} = y^{IJ} w_{IJ} + \frac{4i}{3} \tilde{w}^{\alpha IJK} \tilde{w}_{\alpha IJK} - 2i X_K^\gamma X_\gamma^K + \frac{2}{3} \varepsilon^{IJKLPQ} w_{IJ} w_{KL} w_{PQ} \\ + 2i \psi_a^\alpha (\gamma^a)_\alpha^\beta (\tilde{w}_\beta^{IJK} w_{JK} + X_{\beta J} w^{IJ}) \\ - i \varepsilon^{abc} (\gamma_a)_\alpha \beta \psi_b^\alpha \psi_c^\beta \left(w^{IK} w_K^J - \frac{1}{4} \delta^{IJ} w^{KL} w_{KL} \right), \quad (3.18)$$

where we have used the relations

$$\tilde{w}_\alpha^{IJK} = -\frac{i}{2} \nabla_\alpha^{[I} W^{JK]}, \quad (3.19a)$$

$$y^{IJ} = -\frac{i}{5} \nabla^{\gamma[I} \nabla_{\gamma^P} W^{J]P} - \frac{1}{2} \varepsilon^{IJKLPQ} W_{KL} W_{PQ}, \quad (3.19b)$$

$$X_\alpha^I = -\frac{i}{5} \nabla_{\alpha J} W^{IJ}. \quad (3.19c)$$

Combining this result with the Chern-Simons contribution gives the full action

$$\begin{aligned} S = \frac{1}{4} \int d^3x e \left\{ \varepsilon^{abc} \left(\omega_a^{fg} \mathcal{R}_{bcfg} - \frac{2}{3} \omega_{af}^g \omega_{bg}^h \omega_{ch}^f - \frac{i}{2} \Psi_{bcI}^\alpha (\gamma_d)_\alpha^\beta (\gamma_a)_\beta^\gamma \varepsilon^{def} \Psi_{ef\gamma}^I \right. \right. \\ \left. - 2\mathcal{R}_{ab}^{IJ} V_{cIJ} - \frac{4}{3} V_a^{IJ} V_{bI}^K V_{cKJ} + 4\mathcal{F}_{ab} A_c \right) \\ - 4y^{IJ} w_{IJ} - \frac{16i}{3} \tilde{w}^{\alpha IJK} \tilde{w}_{\alpha IJK} + 8i X_K^\gamma X_\gamma^K - \frac{8}{3} \varepsilon^{IJKLPQ} w_{IJ} w_{KL} w_{PQ} \\ - 8i \psi_{aI}^\alpha (\gamma^a)_\alpha^\beta (\tilde{w}_\beta^{IJK} w_{JK} + X_{\beta J} w^{IJ}) \\ \left. + 4i \varepsilon^{abc} (\gamma_a)_\alpha \beta \psi_{bI}^\alpha \psi_{cJ}^\beta \left(w^{IK} w_{JK} - \frac{1}{4} \delta^{IJ} w^{KL} w_{KL} \right) \right\}. \quad (3.20) \end{aligned}$$

Our choice of normalization for the auxiliary fields allows a simple truncation to lower values of $\mathcal{N}$. From the above action one can truncate the auxiliary fields to $\mathcal{N} = 5$ by taking (with $I, J, K = 1, 2, 3, 4, 5$)

$$\left. \begin{aligned} w_{IJ} &\longrightarrow 0, & \tilde{w}_\alpha^{IJK} &\longrightarrow 0, & X_\alpha^I &\longrightarrow 0, & y^{IJ} &\longrightarrow 0, \\ w^{I6} &\longrightarrow w^I, & \tilde{w}_\alpha^{IJ6} &\longrightarrow w_\alpha^{IJ}, & y^{I6} &\longrightarrow y^I \end{aligned} \right\}. \quad (3.21)$$

For the gauge fields one must switch off the U(1) gauge field $A_b \longrightarrow 0$, while truncation is obvious for the other gauge fields. The $\mathcal{N} < 5$ cases can be obtained via the truncation procedure given in [1].

4 Discussion

Our component action for $\mathcal{N} = 6$ conformal supergravity (3.20) agrees with that derived recently in [9], where alternative techniques were used involving the consistent truncation of the off-shell multiplet of $\mathcal{N} = 8$ conformal supergravity [12]. It also correctly reduces to the action for $\mathcal{N} = 5$ conformal supergravity [1] via the truncation procedure (3.21). Eliminating the auxiliary fields is equivalent to removing the last three lines in (3.20). The resulting on-shell action for $\mathcal{N} = 6$ conformal supergravity does not agree with that obtained in [13, 14] by gauging the $\mathcal{N} = 6$ superconformal algebra in x -space (the action given in [13, 14] does not contain the U(1) Chern-Simons term). Instead it coincides with the action given in [15].

In conclusion, we comment on the structure of a supercurrent multiplet associated with a superconformal matter theory coupled to $\mathcal{N}$ -extended conformal supergravity (see also [2]). In general, the supergravity-matter system is described by an action of the form

$$S = \frac{1}{\kappa} S_{\text{CSG}} + S_{\text{matter}}, \quad (4.1)$$

where S_{CSG} denotes the conformal supergravity action and S_{matter} the matter action. The conformal supergravity equation is

$$\frac{1}{\kappa}W + T = 0 . \quad (4.2)$$

Here W is the $\mathcal{N}$ -extended super Cotton tensor (with its indices suppressed) and T the matter supercurrent multiplet. The supercurrent has the same algebraic type as W and obeys the same differential constraints W is subject to. For any $\mathcal{N}$, the super Cotton tensor (and also the supercurrent) is a conformal primary superfield,

$$S_\alpha^I W = 0 , \quad \mathbb{D}W = \Delta_W W , \quad (4.3)$$

with Δ_W the dimension of W . We now recall the structure of W for various values of $\mathcal{N}$ following [2].

The $\mathcal{N} = 1$ super Cotton tensor [16] is a completely symmetric spinor $W_{\alpha\beta\gamma}$ of dimension 5/2. It obeys the conformally invariant constraint

$$\nabla^\alpha W_{\alpha\beta\gamma} = 0 . \quad (4.4)$$

In the $\mathcal{N} = 2$ case, the super Cotton tensor [17, 18] is a completely symmetric spinor $W_{\alpha\beta}$ of dimension 2. The corresponding conformally invariant constraint is

$$\nabla^{\alpha I} W_{\alpha\beta} = 0 . \quad (4.5)$$

In the $\mathcal{N} = 3$ case, the super Cotton tensor is a symmetric spinor W_α of dimension 3/2. It obeys the conformally invariant constraint

$$\nabla^{\alpha I} W_\alpha = 0 . \quad (4.6)$$

For $\mathcal{N} > 5$, the super Cotton tensor [5, 6] is a completely antisymmetric tensor W^{IJKL} of dimension 1. It obeys the conformally invariant constraint

$$\nabla_\alpha^I W^{JKLP} = \nabla_\alpha^{[I} W^{JKLP]} - \frac{4}{\mathcal{N}-3} \nabla_{\alpha Q} W^{Q[JKL} \delta^{P]I} . \quad (4.7)$$

In the $\mathcal{N} = 4$ case, the super Cotton tensor is equivalently described by a scalar primary dimension-1 superfield $W^{IJKL} := \varepsilon^{IJKL} W$. The corresponding conformally invariant constraint is

$$\nabla^{\alpha I} \nabla_\alpha^J W = \frac{1}{4} \delta^{IJ} \nabla_P^\alpha \nabla_\alpha^P W . \quad (4.8)$$

As we have demonstrated, the specific feature of the $\mathcal{N} = 6$ case is that the super Cotton tensor is equivalent to the $U(1)$ vector multiplet field strength (2.15). Therefore, the $\mathcal{N} = 6$ supercurrent T^{IJ} has the same multiplet structure. This agrees with the Nishimura-Tanii analysis [9] of the supercurrent of the ABJM model [19] coupled to conformal supergravity.

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A The supersymmetry transformations

In this appendix we present the complete Q - and S -supersymmetry transformations for the component fields of the Weyl multiplet for $\mathcal{N} = 6$. The component action (3.20) is manifestly supersymmetric by virtue of our superspace construction. We refer the reader to [1] for details on the component projection rules and the precise definition of the gauge component fields.

The Q -supersymmetry transformations of the connections are

$$\delta_Q(\xi)e_m^a = i\psi_m^{\alpha I}(\gamma^a)_\alpha{}^\beta \xi_{I\beta}, \quad (\text{A.1a})$$

$$\delta_Q(\xi)\psi_m^{\alpha I} = 2\mathcal{D}_m\xi^{\alpha I} = 2\partial_m\xi^{\alpha I} + \omega_m^{\alpha}{}_\beta \xi^{\beta I} + b_m\xi^{\alpha I} - 2V_m^{IJ}\xi^{\alpha J}, \quad (\text{A.1b})$$

$$\delta_Q(\xi)b_m = -\phi_m^{\alpha J}\xi_{\alpha J}, \quad (\text{A.1c})$$

$$\delta_Q(\xi)V_m^{IJ} = 2\phi_m^{\alpha[I}\xi_{\alpha}^{J]} - \frac{i}{2}\varepsilon^{IJKLPQ}\psi_m^\alpha \xi_{\alpha L} w_{PQ} - \frac{i}{3}\varepsilon^{IJKLPQ}\xi_K^\alpha (\gamma_m)_\alpha{}^\beta \tilde{w}_{\beta LPQ}, \quad (\text{A.1d})$$

$$\delta_Q(\xi)\omega_m^{ab} = -\varepsilon^{abc}\phi_m^{\alpha I}(\gamma_c)_\alpha{}^\beta \xi_{\beta I}, \quad (\text{A.1e})$$

$$\begin{aligned} \delta_Q(\xi)\phi_m^{\alpha I} = & -2i\xi^{\beta I}(\gamma_b)_\beta{}^\alpha \mathfrak{f}_m^b + \frac{1}{3}\varepsilon^{IJKLPQ}\psi_m^\beta \xi_{\beta K} \tilde{w}^\alpha{}_{LPQ} \\ & + i\xi^{\beta J}(\gamma_m)_\beta{}^\gamma (\gamma_c)_\gamma{}^\alpha R(N)^{cJI}, \end{aligned} \quad (\text{A.1f})$$

$$\delta_Q(\xi)\mathfrak{f}_m^a = \frac{i}{2}\psi_m^{\alpha I}\xi_{\alpha J}R(N)^{aIJ} + \xi_I^\alpha (\gamma_m)_\alpha{}^\beta R(S)^{aI}{}_\beta, \quad (\text{A.1g})$$

$$\delta_Q(\xi)A_m = -i\xi_K^\alpha (\gamma_m)_\alpha{}^\beta X_\beta^K - i\psi_m^\beta \xi_{\beta K} w^{JK}. \quad (\text{A.1h})$$

Here we have made use of the covariant derivative

$$\mathcal{D}_a = e_a^m \mathcal{D}_m = e_a^m \left(\partial_m - \frac{1}{2}\omega_m^{bc} M_{bc} - \frac{1}{2}V_m^{IJ} N_{IJ} - b_m \mathbb{D} \right) \quad (\text{A.2})$$

and the following results for the projection of the $\text{SO}(6)$ and S -supersymmetry curvatures

$$\begin{aligned} R(N)_{ab}{}^{IJ} = & \mathcal{R}_{ab}{}^{IJ} + 2\psi_{[a}^{\alpha[I}\phi_{b]\alpha}^{J]} + \frac{i}{3}\varepsilon^{IJKLPQ}\psi_{[aK}^\beta (\gamma_b)_{\beta}{}^\gamma \tilde{w}_{\gamma LPQ} \\ & - \frac{i}{4}\varepsilon^{IJKLPQ}\psi_{[aK}^\beta \psi_{b]\beta L} w_{PQ}, \end{aligned} \quad (\text{A.3a})$$

$$\begin{aligned} R(S)^{cI}{}_I = & e_{[a}^m e_{b]}^n \mathcal{D}_m \phi_n^\alpha + i\psi_{[a}^{\beta I} \mathfrak{f}_{b]}^c (\gamma_c)_{\beta}{}^\alpha - \frac{i}{2}\psi_{[a}^{\beta J} (\gamma_b)_{\beta}{}^\gamma (\gamma_c)_\gamma{}^\alpha R(N)^{cJI} \\ & + \frac{1}{12}\varepsilon^{IJKLPQ}\psi_{[a}^{\beta J} \psi_{b]\beta}^K \tilde{w}^\alpha{}_{LPQ}. \end{aligned} \quad (\text{A.3b})$$

The curvature tensors $R(N)^{cIJ}$ and $R(S)^{cI}{}_\alpha$ in eqs. (A.1f), (A.1g) and (A.3b) are the Hodge-duals of $R(N)_{ab}{}^{IJ}$ and $R(S)_{ab\alpha}^I$, respectively.⁶ The S -supersymmetry transformations of

⁶Given a two-form F_{ab} , its Hodge-dual is $F^c = \frac{1}{2}\varepsilon^{cab}F_{ab}$.

the connections are

$$\delta_S(\eta)e_m^a = 0, \quad (\text{A.4a})$$

$$\delta_S(\eta)\psi_m^{\alpha I} = -2i\eta^{\beta I}(\gamma_m)_\beta^\alpha, \quad (\text{A.4b})$$

$$\delta_S(\eta)b_m = \psi_m^{\alpha I}\eta_{\alpha I}, \quad (\text{A.4c})$$

$$\delta_S(\eta)V_m^{IJ} = 2\psi_m^{\alpha[I}\eta_{\alpha}^{J]}, \quad (\text{A.4d})$$

$$\delta_S(\eta)\omega_m^{ab} = -\varepsilon^{abc}\psi_m^{\alpha I}(\gamma_c)_\alpha^\beta\eta_{\beta I}, \quad (\text{A.4e})$$

$$\delta_S(\eta)\phi_m^{\alpha I} = 2\mathcal{D}_m\eta^{\alpha I} = 2\partial_m\eta^{\alpha I} + \omega_m^\alpha{}_\beta\eta^{\beta I} - b_m\eta^{\alpha I} - 2V_m^I{}_J\eta^{\alpha J}, \quad (\text{A.4f})$$

$$\delta_S(\eta)\mathfrak{f}_m^a = i\phi_m^{\alpha I}(\gamma^a)_\alpha^\beta\eta_{\beta I}, \quad (\text{A.4g})$$

$$\delta_S(\eta)A_m = 0. \quad (\text{A.4h})$$

It should be pointed out that all the auxiliary fields defined in (3.13), as well as the Cottino $w_{\alpha\beta\gamma}^I$ and Cotton $w_{\alpha\beta\gamma\delta}$ tensors [1], are ordinary primary fields annihilated by the special conformal generator K_a .

Along with the transformation laws of the gauge fields of the Weyl multiplet it is necessary to have the transformation rules of the auxiliary fields given by eqs. (3.13) and (3.19). Their Q - and S -supersymmetry transformations are

$$\delta_Q(\xi)w^{IJ} = 2i\xi_K^\alpha\tilde{w}_\alpha^{KIJ} - 2i\xi^{\alpha[I}X_\alpha^{J]}, \quad (\text{A.5a})$$

$$\delta_S(\eta)w^{IJ} = 0, \quad (\text{A.5b})$$

$$\delta_Q\tilde{w}_\alpha^{IJK} = \frac{1}{4}(\gamma^a)_\alpha{}^\beta\xi_P^\beta\varepsilon^{PIJKST}R(N)_{aST} + \frac{3}{2}(\gamma^a)_{\alpha\beta}\xi^\beta[I\hat{\nabla}_aw^{JK]} - \frac{3}{4}\xi_\alpha^{[I}y^{JK]}, \quad (\text{A.5c})$$

$$\delta_S\tilde{w}_\alpha^{IJK} = 3i\eta_\alpha^{[I}w^{JK]}, \quad (\text{A.5d})$$

$$\delta_QX_\alpha^I = \xi^{\gamma I}F_{\gamma\alpha} - \frac{1}{2}\xi_{\alpha K}y^{KI} - \xi_K^\gamma\hat{\nabla}_\gamma w^{KI} + \frac{1}{4}\varepsilon^{IJKLPQ}\xi_{\alpha J}w_{KL}w_{PQ}, \quad (\text{A.5e})$$

$$\delta_SX_\alpha^I = 2i\eta_{\alpha K}w^{IK}, \quad (\text{A.5f})$$

$$\begin{aligned} \delta_Qy^{IJ} &= -4i\xi^{\gamma[I}\hat{\nabla}_{\gamma\beta}X^{\beta J]} + 4i\xi_K^\gamma\hat{\nabla}_\gamma^\beta\tilde{w}_\beta^{KIJ} + \frac{4i}{3}\xi_K^\gamma\tilde{w}_\gamma^{MNP}\varepsilon_{MNP}{}^{K[I}Qw^{J]Q} \\ &\quad - \frac{4i}{3}\tilde{w}_{\gamma MNP}w_{QS}\varepsilon^{MNPQS[I}\xi^{\gamma J]}, \end{aligned} \quad (\text{A.5g})$$

$$\delta_Sy^{IJ} = -4\eta^{\delta[I}X_\delta^{J]} - 4\eta_L^\delta\tilde{w}_\delta^{LIJ}, \quad (\text{A.5h})$$

where we have defined

$$\hat{\nabla}_aw^{IJ} := \mathcal{D}_aw^{IJ} - i\psi_a^\gamma\tilde{w}_\gamma^{KIJ} + i\psi_a^{\gamma[I}X_\gamma^{J]}, \quad (\text{A.6a})$$

$$\begin{aligned} \hat{\nabla}_aX^{\beta J} &:= \mathcal{D}_aX^{\beta J} + \frac{1}{2}\psi_a^\gamma F^{\beta\gamma} + \frac{1}{4}\psi_a^\beta y^{KJ} - \frac{1}{2}(\gamma^b)^{\beta\gamma}\psi_{a\gamma L}\hat{\nabla}_bw^{LJ} \\ &\quad - \frac{1}{8}\varepsilon^{JKLPQS}\psi_a^\beta w_{LP}w_{QS} - i\phi_{aL}^\beta w^{JL}, \end{aligned} \quad (\text{A.6b})$$

$$\begin{aligned} \hat{\nabla}_a\tilde{w}_\beta^{IJK} &:= \mathcal{D}_a\tilde{w}_\beta^{IJK} + \frac{1}{8}(\gamma^b)_{\beta\gamma}\varepsilon^{IJKLPQ}\psi_a^\gamma R(N)_{bPQ} + \frac{3}{8}\psi_a^{[I}y^{JK]} \\ &\quad - \frac{3}{4}(\gamma^b)_{\beta\delta}\psi_a^{\delta[I}\hat{\nabla}_bw^{JK]} - \frac{3i}{2}\phi_{a\beta}^{[I}w^{JK]}. \end{aligned} \quad (\text{A.6c})$$

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References

- [1] D. Butter, S.M. Kuzenko, J. Novak and G. Tartaglino-Mazzucchelli, *Conformal supergravity in three dimensions: off-shell actions*, *JHEP* **10** (2013) 073 [[arXiv:1306.1205](https://arxiv.org/abs/1306.1205)] [[INSPIRE](#)].
- [2] D. Butter, S.M. Kuzenko, J. Novak and G. Tartaglino-Mazzucchelli, *Conformal supergravity in three dimensions: new off-shell formulation*, *JHEP* **09** (2013) 072 [[arXiv:1305.3132](https://arxiv.org/abs/1305.3132)] [[INSPIRE](#)].
- [3] D. Butter, $\mathcal{N} = 1$ conformal superspace in four dimensions, *Annals Phys.* **325** (2010) 1026 [[arXiv:0906.4399](https://arxiv.org/abs/0906.4399)] [[INSPIRE](#)].
- [4] D. Butter, $\mathcal{N} = 2$ conformal superspace in four dimensions, *JHEP* **10** (2011) 030 [[arXiv:1103.5914](https://arxiv.org/abs/1103.5914)] [[INSPIRE](#)].
- [5] P.S. Howe, J.M. Izquierdo, G. Papadopoulos and P.K. Townsend, *New supergravities with central charges and Killing spinors in $(2+1)$ -dimensions*, *Nucl. Phys. B* **467** (1996) 183 [[hep-th/9505032](https://arxiv.org/abs/hep-th/9505032)] [[INSPIRE](#)].
- [6] S.M. Kuzenko, U. Lindström and G. Tartaglino-Mazzucchelli, *Off-shell supergravity-matter couplings in three dimensions*, *JHEP* **03** (2011) 120 [[arXiv:1101.4013](https://arxiv.org/abs/1101.4013)] [[INSPIRE](#)].
- [7] P. van Nieuwenhuizen, $D = 3$ conformal supergravity and Chern-Simons terms, *Phys. Rev. D* **32** (1985) 872 [[INSPIRE](#)].
- [8] M. Roček and P. van Nieuwenhuizen, $\mathcal{N} \geq 2$ supersymmetric Chern-Simons terms as $D = 3$ extended conformal supergravity, *Class. Quant. Grav.* **3** (1986) 43 [[INSPIRE](#)].
- [9] M. Nishimura and Y. Tanii, $\mathcal{N} = 6$ conformal supergravity in three dimensions, *JHEP* **10** (2013) 123 [[arXiv:1308.3960](https://arxiv.org/abs/1308.3960)] [[INSPIRE](#)].
- [10] J. Greitz and P.S. Howe, *Maximal supergravity in three dimensions: supergeometry and differential forms*, *JHEP* **07** (2011) 071 [[arXiv:1103.2730](https://arxiv.org/abs/1103.2730)] [[INSPIRE](#)].
- [11] U. Gran, J. Greitz, P.S. Howe and B.E.W. Nilsson, *Topologically gauged superconformal Chern-Simons matter theories*, *JHEP* **12** (2012) 046 [[arXiv:1204.2521](https://arxiv.org/abs/1204.2521)] [[INSPIRE](#)].
- [12] M. Nishimura and Y. Tanii, *Coupling of the BLG theory to a conformal supergravity background*, *JHEP* **01** (2013) 120 [[arXiv:1206.5388](https://arxiv.org/abs/1206.5388)] [[INSPIRE](#)].
- [13] U. Lindström and M. Roček, *Superconformal gravity in three-dimensions as a gauge theory*, *Phys. Rev. Lett.* **62** (1989) 2905 [[INSPIRE](#)].
- [14] H. Nishino and S.J. Gates Jr., *Chern-Simons theories with supersymmetries in three-dimensions*, *Int. J. Mod. Phys. A* **8** (1993) 3371 [[INSPIRE](#)].
- [15] X. Chu and B.E.W. Nilsson, *Three-dimensional topologically gauged $\mathcal{N} = 6$ ABJM type theories*, *JHEP* **06** (2010) 057 [[arXiv:0906.1655](https://arxiv.org/abs/0906.1655)] [[INSPIRE](#)].
- [16] S.M. Kuzenko and G. Tartaglino-Mazzucchelli, *Conformal supergravities as Chern-Simons theories revisited*, *JHEP* **03** (2013) 113 [[arXiv:1212.6852](https://arxiv.org/abs/1212.6852)] [[INSPIRE](#)].

- [17] B. Zupnik and D. Pak, *Superfield formulation of the simplest three-dimensional gauge theories and conformal supergravities*, *Theor. Math. Phys.* **77** (1988) 1070 [*Teor. Mat. Fiz.* **77** (1988) 97] [[INSPIRE](#)].
- [18] S.M. Kuzenko, *Prepotentials for $\mathcal{N} = 2$ conformal supergravity in three dimensions*, *JHEP* **12** (2012) 021 [[arXiv:1209.3894](#)] [[INSPIRE](#)].
- [19] O. Aharony, O. Bergman, D.L. Jafferis and J. Maldacena, *$\mathcal{N} = 6$ superconformal Chern-Simons-matter theories, M2-branes and their gravity duals*, *JHEP* **10** (2008) 091 [[arXiv:0806.1218](#)] [[INSPIRE](#)].