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Linking natural supersymmetry to flavour physics

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ABSTRACT: With the aim of linking natural supersymmetry to flavour physics, a model is proposed based on a family symmetry $G \times U(1)$, where G is a discrete nonabelian subgroup of $SU(2)$, with both F-term and (abelian) D-term supersymmetry breaking. A good fit to the fermion masses and mixing is obtained with the same $U(1)$ charges for the left- and right-handed quarks of the first two families and the right-handed bottom quark, and with zero charge for the left-handed top-bottom doublet and the right-handed top. The model shows an interesting indirect correlation between the correct prediction for the V_{ub}/V_{cb} ratio and large right-handed rotations in the (s, b) sector, required to diagonalise the Yukawa matrix. For the squarks, one obtains almost degenerate first two generations. The main source of the FCNC and CP violation effects is the splitting between the first two families and the right-handed sbottom determined by the relative size of F-term and D-term supersymmetry breaking. The presence of the large right-handed rotation implies that the bounds on the masses of the first two families of squarks and the right-handed sbottom are in a few to a few tens TeV range. The picture that emerges is light stops and left-handed sbottom and much heavier other squarks.

KEYWORDS: Supersymmetry Breaking, Quark Masses and SM Parameters, Supersymmetric Standard Model

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1 Introduction

According to the hypothesis of “minimal” supersymmetry [1], now termed as natural supersymmetry, the only light states in the supersymmetric spectrum are those that have the largest effect on the quadratic terms in the Higgs potential, and the rest is much heavier. Thus, the only light superpartners are the stops, the left-handed sbottom and the higgsino. Also the gluino is expected to be moderately light, as it enters the Higgs potential at the 2-loop level. The heavy set includes, in particular, the right handed sbottom. The new LHC lower bounds on the superpartner masses [2, 3], stronger for the first two generations of squarks than for the third one, revive the interest in such a spectrum.¹ Recently, several new ideas have been proposed to explain such a spectrum [7–12] and detailed LHC analysis were performed [13–15].

It is an obvious and interesting theoretical question whether such hierarchical squark masses can be linked to the fermion mass hierarchies in one framework of some theory of

¹For related ideas on flavor-dependent spectra see ref. [4–6].

flavour, combined with a mechanism of supersymmetry breaking. In this paper, we show that the spectrum of minimal supersymmetry is predicted by the flavour theory based on family symmetry $G \times U(1)$, where G is a discrete nonabelian subgroup of $SU(2)$, with both F -term and D -term [16, 17] supersymmetry breaking. Essentially all our phenomenological considerations are the same as in the $SU(2)_{global}$ case. We also point out that in addition to the well known bound on the splitting between the first two and the third family masses [15, 18–20], there is in general also an upper bound on the splitting between the left and right handed sbottoms, dependent on the value of $\tan \beta$. It is a combined effect of the 2-loop and 1-loop contributions to the RG equations for the Higgs mass parameter m_H^2 and the scalar masses, mainly the CP-odd Higgs scalar, respectively, that depend on the bottom Yukawa coupling and are proportional to the right-handed sbottom mass. Thus, if the RG evolution from the high scale to the electroweak scale is involved, the “natural” spectrum is generically incompatible with very large values of $\tan \beta$ and with any flavour model that implies such values, the one based on the horizontal $U(2)$ symmetry in particular. The flavour model proposed in this paper works for a large range of moderate values of $\tan \beta$.

The issues we address in this paper have a long research history. Already in the early days of supersymmetric phenomenology, a large splitting in mass between the third and the first two families of sfermions has been proposed as a way to reconcile the naturalness of the Higgs potential with the suppression of new potential sources of the FCNC and CP violation effects generated by the superpartner sector [1, 21, 22]. A very simple and attractive possibility of linking the supersymmetric spectrum to the fermion one is offered by flavour theories with a single horizontal $U(1)$ gauge symmetry [23–25]. With both F -term and D -term [16, 17] supersymmetry breaking, one indeed predicts an inverted hierarchy of sfermion masses as a consequence of the hierarchical pattern of fermion masses [26–31]. The scales in the soft masses depend on the relative magnitude of the F - and D -term breaking.

This attractive mechanism has been, unfortunately, facing certain phenomenological problems. In models based on a single $U(1)$ horizontal symmetry, the compatibility with the fermion mass spectrum implies not only the 3rd generation lighter than the first two but also a large splitting between the first two. Thus, after the rotation from the electroweak basis to the SCKM basis, flavour off-diagonal entries are generated and one needs $\mathcal{O}(1000)$ TeV first two generation squarks to suppress FCNC effects to acceptable levels.² Such large values are not compatible with the proper electroweak breaking at the level of 2-loop quantum corrections. The constraints on the splitting between the left- and right-handed sbottom has neither been given a sufficient attention so far.

In the present paper we readdress the issue of obtaining natural supersymmetry from a flavour theory and D -term supersymmetry breaking, inspired by the phenomenological success of the model for fermion masses and mixing based on the global horizontal $U(2)$ symmetry [34, 35]. The $U(2)$ model is very predictive and describes well the bulk of the data, with several very interesting relations between the masses and mixings. However, the price for good predictivity is that there is also some tension with the data. It predicts

²In models with several $U(1)$ factors the FCNC constraints can be eased by alignment, see e.g. refs. [25, 28, 32, 33].

$m_s/m_b = m_c/m_t$ (up to order one coefficients). Moreover, as pointed out in ref. [36], the prediction for $|V_{ub}/V_{cb}|$ is off by a factor of 2. This tension can be traced back to the left-right symmetric structure of Yukawa matrices and the presence of texture zeros in the (1,3),(3,1) entries in the quark Yukawa matrices. As has been already proposed in ref. [36, 37], at least one of these conditions has to be relaxed in order to achieve consistency with experiment. Since the up and down Yukawa matrices have the same pattern, the model naturally works only for large $\tan\beta$ values.

In our present case the left-right symmetric structure in the down sector is broken by the U(1) charge assignment.³ A good fit to the fermion masses and mixings is obtained for several sets of the horizontal charges and in particular for the same U(1) charges for the left- and right- handed quarks of the first two families and the right-handed bottom quark, and with zero charge for the left-handed top-bottom doublet and the the right handed top. This is the case that we investigate in detail. The pattern of Yukawa matrices in the $SU(2)_{global} \times U(1)_{local}$ model shows an interesting indirect correlation between the presence of large corrections to the V_{ub}/V_{cb} ratio and large right-handed rotations in the (s, b) sector required to diagonalise the Yukawas. In the limit when the Yukawa matrices become left-right symmetric, as in the U(2) model, the right and left rotations become similar and small, both of the order of magnitude of the corresponding elements of the CKM matrix, and the correction to the V_{ub}/V_{cb} ratio becomes negligibly small.

In the squark sector, the $SU(2)_{global}$ symmetry ensures almost degenerate squark masses of the first two generations. The main contribution to the FCNC and CP violation in the kaon system comes from the splitting between the first two families and the right-handed sbottom determined by the relative size of F-term and D-term supersymmetry breaking. An interesting conclusion is that, in the presence of the discussed above large right handed rotations in the (s, b) sector, the experimental bounds on the right-handed sbottom and the first two families of squarks are of the order of a few to a few tens TeV, depending on various parameters of the model and on the gluino mass. We discuss in detail that dependence, including the dependence on the structure of the Kähler potential. The picture that emerges is then light stops and left-handed sbottom and much heavier other squarks.

Finally, the $SU(2)_{global} \times U(1)_{local}$ model gives us an opportunity to investigate the mechanism of the D -term supersymmetry breaking in a scheme going beyond the simplest U(1) case. The presence of the additional SU(2) global symmetry, to be spontaneously broken by a second flavon, imposes non-trivial constraints on the D -term supersymmetry breaking mechanism, particularly if a hierarchy between the F -term and D -term breaking scales is to be obtained. Actually, we discuss all those issues with the continuous SU(2) replaced by a nonabelian discrete subgroup G , to avoid the problem of Goldstone bosons. A discrete symmetry is also more natural from a string theory perspective. Another possibility would be to consider a weakly gauged SU(2), with a gauge coupling small enough in order to suppress the non-universal D-term contribution to sfermion masses, as proposed in ref. [38].

The outline of this paper is as follows. In section 2 we formulate the flavour model. In section 3 we discuss the predictions of the model for quark masses and mixings. In section 4

³With the chosen symmetry group, contrary to the U(2) model, the U(1) charges are free to choose, of course the same for the whole SU(2) multiplets.

	10_a	10_3	$\bar{5}_a$	$\bar{5}_3$	H_u	H_d	ϕ^a	χ
SU(2)	2	1	2	1	1	1	$\bar{2}$	1
U(1)	X_{10}	0	$X_{\bar{5}}$	X_3	0	0	X_ϕ	-1

Table 1. Flavor group representations of the model.

we discuss the soft susy breaking masses arising from both D and F terms. Section 5 is devoted to the bounds on the squark sector obtained from the FCNC and CP violation effects and in the leptonic sector from $\mu \rightarrow e\gamma$. In section 6 we discuss the effects of the renormalization group running starting from a high (GUT) scale and resulting constraints at low-energy. In section 7 we discuss the main requirements on models that break spontaneously both horizontal symmetries and have D and F term supersymmetry breaking, with a hierarchy of scales. In section 8 we give our conclusions. In two appendices we present more details of the model and of the calculation of the bounds on the squark masses.

2 The model

In the present paper we propose a flavour model based on $G \times \text{U}(1)_{\text{local}}$ horizontal symmetry, where G is a discrete nonabelian subgroup of $\text{SU}(2)_{\text{global}}$. The discrete version (or a weakly gauged $\text{SU}(2)$) allows first of all to avoid potential Goldstone bosons after spontaneous symmetry breaking. From a UV quantum gravity perspective, whereas we can imagine continuous abelian symmetries in string theory, broken only by nonperturbative effects, there are no similar continuous nonabelian symmetries. On the other hand, all string theory constructions are naturally endowed with discrete symmetries. For some recent dedicated discussions on discrete symmetries in string theory, see e.g. [39–41] for heterotic examples and [42–46] for D-brane examples.

We have to specify the representations of the various fields under the flavour group $G \times \text{U}(1)$, where G is a discrete subgroup of $\text{SU}(2)$. However, it turns out that the flavour phenomenology can be largely decoupled from the choice of G , so we will postpone any issues related to the global part of the flavour group to section 7. The simplest choice for the flavour charges is to consider $\text{SU}(5)$ invariant charges X_{10} and $X_{\bar{5}}$, with Higgs uncharged. We need two flavons, an $\text{SU}(2)$ doublet ϕ with charge X_ϕ and an $\text{SU}(2)$ singlet χ with charge -1 . The total field content is summarized in table 1. The zero $\text{U}(1)$ charge of the 3rd generation ten-plet takes account of the large top quark Yukawa coupling, whereas the X_3 is left free, to accommodate different values of $\tan \beta$.

The relevant part of the superpotential is given by

$$\begin{aligned}
 W = & h_{33}^u H_u Q_3 U_3 + h_{23}^u Q_a U_3 H_u \frac{\phi^a}{\Lambda} \left(\frac{\chi}{\Lambda}\right)^{X_{10}+X_\phi} + h_{32}^u Q_3 U_a H_u \frac{\phi^a}{\Lambda} \left(\frac{\chi}{\Lambda}\right)^{X_{10}+X_\phi} \\
 & + h_{12}^u H_u Q_a U_b \epsilon^{ab} \left(\frac{\chi}{\Lambda}\right)^{2X_{10}} + h_{22}^u Q_a U_b H_u \frac{\phi^a}{\Lambda} \frac{\phi^b}{\Lambda} \left(\frac{\chi}{\Lambda}\right)^{2X_{10}+2X_\phi} \\
 & + h_{33}^d H_d Q_3 D_3 \left(\frac{\chi}{\Lambda}\right)^{X_3} + h_{23}^d Q_a D_3 H_d \frac{\phi^a}{\Lambda} \left(\frac{\chi}{\Lambda}\right)^{X_{10}+X_3+X_\phi} + h_{32}^d Q_3 D_a H_d \frac{\phi^a}{\Lambda} \left(\frac{\chi}{\Lambda}\right)^{X_{10}+X_3+X_\phi} \\
 & + h_{12}^d H_d Q_a D_b \epsilon^{ab} \left(\frac{\chi}{\Lambda}\right)^{X_{10}+X_3} + h_{22}^d Q_a D_b H_d \frac{\phi^a}{\Lambda} \frac{\phi^b}{\Lambda} \left(\frac{\chi}{\Lambda}\right)^{X_{10}+X_3+2X_\phi} .
 \end{aligned} \tag{2.1}$$

We have imposed here that all exponents are non-negative

$$X_{10} \geq 0, X_3 \geq 0, X_{10} + X_\phi \geq 0, X_{\bar{5}} + X_\phi \geq 0, X_{\bar{10}} + X_{\bar{5}} \geq 0. \quad (2.2)$$

The h 's are complex $O(1)$ coefficients, Λ is a high flavour scale and a, b are the $SU(2)$ indices. The structure of the Kähler potential is discussed in section 4. In the leading order in small parameters, its details do not affect the predictions in the fermion sector. Using the flavon vevs

$$\langle \phi^a \rangle = \epsilon_\phi \Lambda \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \quad \langle \chi \rangle = \epsilon_\chi \Lambda, \quad (2.3)$$

one can calculate masses and mixings in terms of the original parameters.

The Yukawa matrices are given by

$$Y_u = \begin{pmatrix} 0 & h_{12}^u \epsilon'_u & 0 \\ -h_{12}^u \epsilon'_u & h_{22}^u \epsilon_u^2 & h_{23}^u \epsilon_u \\ 0 & h_{32}^u \epsilon_u & h_{33}^u \end{pmatrix}, \quad (2.4)$$

$$Y_d = \begin{pmatrix} 0 & h_{12}^d \epsilon'_u \epsilon_d / \epsilon_u & 0 \\ -h_{12}^d \epsilon'_u \epsilon_d / \epsilon_u & h_{22}^d \epsilon_u \epsilon_d & h_{23}^d \epsilon_3 \epsilon_u \\ 0 & h_{32}^d \epsilon_d & h_{33}^d \epsilon_3 \end{pmatrix}, \quad (2.5)$$

with

$$\epsilon_u \equiv \epsilon_\phi \epsilon_\chi^{X_{10}+X_\phi}, \quad \epsilon_d \equiv \epsilon_\phi \epsilon_\chi^{X_{\bar{5}}+X_\phi}, \quad \epsilon'_u \equiv \epsilon_\chi^{2X_{10}}, \quad \epsilon_3 \equiv \epsilon_\chi^{X_3}. \quad (2.6)$$

It was noticed some time ago that in models based on Abelian gauge symmetries of the Froggatt-Nielsen type [23] with one flavon, there are simple relations between the mass matrices and the mixed gauged anomalies $U(1) \times G_a^2$ of the flavour $U(1)$ and the SM gauge group factors G_a [47–53]. These relations clearly point towards an anomaly of the Abelian flavor gauge group. Moreover, even without the $SU(5)$ assumption on the Abelian charges, as done here for simplicity, they also predict the value of the weak angle $\sin^2 \theta_w = 3/8$ at the high scale [54]. Those predictions rely on Yukawa couplings generated by one flavon field and are usually violated if there are more flavons. Our present model has three flavons from the $U(1)$ viewpoint, or in $U(1) \times SU(2)$ terms, one $SU(2)$ singlet and one doublet. Interestingly enough, however, the determinants of the mass matrices depend only on the singlet flavon field

$$\begin{aligned} \det Y_U &= (h_{12}^u)^2 h_{33}^u \epsilon_\chi^{4X_{10}}, \\ \det Y_D &= (h_{12}^d)^2 h_{33}^d \epsilon_\chi^{2(X_{10}+X_{\bar{5}})+X_3}, \\ \det Y_L &= (h_{12}^e)^2 h_{33}^e \epsilon_\chi^{2(X_{10}+X_{\bar{5}})+X_3}. \end{aligned} \quad (2.7)$$

By combining these mass determinants and using the values of the the anomaly coefficients (in the notation of ref. [30, 31]),

$$C_3 = C_2 = \frac{3}{5} C_1 = X_3 + 6X_{10} + 2X_{\bar{5}}, \quad (2.8)$$

we find the same mass-anomaly connections as in the pure U(1) case with one flavon, in particular the relation

$$\det Y_U Y_D = (h_{12}^u h_{12}^d)^2 h_{33}^u h_{33}^d \epsilon_\chi^{C_3}, \quad (2.9)$$

which clearly displays the need for an anomalous U(1). One crucial ingredient in deriving these relations in our case is the vanishing of the 13 element in the Yukawa matrices. Therefore, models in which the 13 element is non-vanishing, with multiple SU(2) flavons with no alignment, will violate the relations above. The anomaly of this abelian flavor symmetry has then to be cancelled by the Green-Schwarz mechanism [55]. This has several well-known consequences, among which:

- there is typically an induced Fayet-Iliopoulos term close to the string scale, which has the consequence that generically the symmetry is broken at high scale.
- such anomalous symmetries can naturally implement supersymmetry breaking at hierarchically small scales, combined with nonperturbative effects, which are natural in this context.

3 Quark masses and mixings

In this section we discuss the predictions of our model for quark masses and mixings. The model is defined at a high scale and any comparison with experimental data is subject to renormalisation effects. Such effects give important corrections to small CKM matrix elements due to the large top quark Yukawa coupling [56], but not to their ratios and to the ratios of the first two generations Yukawa couplings to the third one. Therefore, the scale dependence of the predictions discussed in this section is negligible. It can only affect the fit of the small order parameters, which is anyway made with random O(1) coefficients.

The diagonalisation of the Yukawa matrices given in the previous section by left and right rotations on the quark fields is performed in appendix A. The rotation matrices, the Yukawa eigenvalues and the CKM matrix are explicitly given there. Here we just mention that, using the freedom of phase rotations on the quark fields, one can as usual bring the mass matrices to the diagonal form with real eigenvalues and the CKM mixing matrix can be written in the standard form, with one physical phase. For the future calculation of the soft sfermion masses, it is also important that the left and right rotations depend altogether on four phases, which cannot be removed by phase redefinitions.

3.1 Predictions

Before giving the results of our fit of the parameters in the Yukawa matrices to fermion masses and mixings, we discuss the predictions of the model that do not depend on the details of that fit. Using the results in appendix A one finds, in particular, the following relations:

$$|V_{us}| \approx \sqrt{m_d/m_s} \sqrt{c_d}, \quad (3.1)$$

$$|V_{ub}/V_{cb}| \approx |\sqrt{m_u/m_c} + e^{i\beta} \Delta t_d \sqrt{c_d}|, \quad |V_{td}/V_{ts}| \approx |\sqrt{m_d/m_s} + e^{i\beta'} \Delta t_d \sqrt{c_d}|, \quad (3.2)$$

where

$$t_d \equiv \tan \theta_d \equiv \frac{|h_{32}^d| \epsilon_d}{|h_{33}^d| \epsilon_3}, \quad \Delta \equiv \frac{\sqrt{m_s m_d}}{|V_{cb}| m_b} \approx 0.09, \quad (3.3)$$

and β, β' are phases that are given in eq. (A.38). In deriving these results we have used that $\epsilon'_u \ll \epsilon_u^2$ (as confirmed by the fit), but made no assumption yet on the relative size of ϵ_d and ϵ_3 . Notice that the relations in eq. (3.2) do not involve any unknown $\mathcal{O}(1)$ factors but only receive corrections of the order $\sim \epsilon_u^2, \epsilon'_u/\epsilon_u$. At this point, it is interesting to notice that to obtain the same Yukawa structure as in the U(2) models [34, 35] one needs⁴

$$\epsilon_d = \epsilon_u, \quad \epsilon_3 = 1. \quad (3.4)$$

In this case one has $\epsilon_d \ll \epsilon_3$ and it follows that $t_d \approx 0$, $c_d \approx 1$. As a consequence one obtains the stronger predictions [34, 35]

$$|V_{us}| \approx \sqrt{m_d/m_s}, \quad |V_{ub}/V_{cb}| \approx \sqrt{m_u/m_c}, \quad |V_{td}/V_{ts}| \approx \sqrt{m_d/m_s}, \quad (3.5)$$

which only involve measured quantities. However our analysis shows that these predictions do not follow alone from the zero textures in the Yukawa matrices but also require $\epsilon_d \ll \epsilon_3$, or, equivalently, $Y_{i2}^d \ll Y_{i3}^d$ (see also ref. [36]).

Numerically one has, taking mass ratios at M_Z and CKM elements from a global fit [57],

$$\sqrt{m_d/m_s} = 0.22 \pm 0.02, \quad \sqrt{m_u/m_c} = 0.046 \pm 0.008, \quad (3.6)$$

$$|V_{us}| = 0.2253 \pm 0.0007, \quad |V_{ub}/V_{cb}| = 0.085 \pm 0.004, \quad |V_{td}/V_{ts}| = 0.22 \pm 0.01, \quad (3.7)$$

so that the relations in eq. (3.5) work well except the second one. Turning to the more general relations in eq. (3.2), assuming positive interference ($\beta = 0$) and comparing with the experimental values, we see that we need approximately $t_d \approx 0.5$ in order to account for the discrepancy in the second relation in eq. (3.5). This implies $\sqrt{c_d} \approx 0.95$, leading to $\sim 5\%$ corrections for the first and $\sim 14\%$ corrections for the third relation. We observe an interesting correlation between the magnitude of the correction to the second relation in eq. (3.5) which goes as $\sim t_d$ and the size of the right-handed rotation matrix element $|V_{32}^d| = s_d$ (see appendix A), which has very important implications for the discussion of the FCNC effects in section 5.

In addition to the accurate relations in eq. (3.5), in U(2) models one also gets predictions which are valid only up to combinations of $\mathcal{O}(1)$ numbers

$$V_{cb} \sim \sqrt{m_c/m_t}, \quad m_d/m_s \sim m_u/m_c, \quad (3.8)$$

besides the SU(5) relations for the masses

$$m_b \sim m_\tau, \quad m_s \sim m_\mu, \quad m_d \sim m_e. \quad (3.9)$$

⁴Note, however, that in our case the flavon representations and vev's are different, such that scalar masses and FCNC effects in traditional U(2) models are different from the ones one would get from our models in the particular case (3.4). In particular, in our case the flavon vev's can be bigger. In what follows, when we refer to predictions of U(2) models and compare to the models in the present paper, we refer to the original class of models in [34, 35].

Most of these predictions valid up to $\mathcal{O}(1)$ factors work pretty well, since

$$V_{cb} \sim 0.04, \quad \sqrt{m_c/m_t} \sim 0.06, \quad (3.10)$$

$$m_b/m_\tau \sim 2, \quad m_s/m_\mu \sim 0.5 \quad m_d/m_e \sim 5. \quad (3.11)$$

Only the second relation in eq. (3.8) requires a large numerical factor $\mathcal{O}(20)$ (RG effects improve the agreement in the first relation in eq. (3.8) [56]). One possibility is to have $|h_{22}^u h_{33}^u - h_{23}^u h_{32}^u| \sim 4.5$ (see (A.5) and (A.6)), which is easy to achieve with moderate deviations from $h_{ij}^u \sim 1$. This would indicate that y_u/y_c is accidentally small in our parametrization while y_d/y_s is at its natural value. In fact this is exactly what we need in order to explain the relative importance of the corrections to the exact relations in eq. (3.5).

Notice that the U(2) relations in eq. (3.4) imply that $\tan\beta$ is fixed to be large and another order-of-magnitude prediction is made

$$m_u/m_t \sim m_d/m_b, \quad (3.12)$$

which does not work as well as the other relations, being off by a factor $\mathcal{O}(120)$. The two extra parameters that we have in the $SU(2) \times U(1)$ model therefore allow to accommodate also small values of $\tan\beta$ and the different ratios m_u/m_t and m_d/m_b .

3.2 Numerical fit

The small order parameters can be fixed by a fit to fermion masses and mixings for random $\mathcal{O}(1)$ coefficients. In a common approach to Yukawa hierarchies arising from spontaneously broken horizontal symmetries one would typically scan over $\mathcal{O}(1)$ “bare” Yukawa couplings h_{ij} and perform a fit for the order parameter(s) and charges. However, because of the highly constrained nature of the Yukawa couplings (which besides phases only comprise five real parameters in each sector) we have proceeded differently. In a first step, we perform a χ^2 fit of the Yukawas entries Y_{ij}^d and Y_{ij}^u in eqs. (2.4) and (2.5) to the masses and mixings.⁵ This results in typical values Y_{ij}^q needed in any model satisfying the texture $Y_{13}^q = Y_{31}^q = Y_{11}^q = 0$ and $Y_{12}^q = -Y_{21}^q$. In a second step we would like to estimate the ϵ_i of our specific model. We assume that the h_{ij}^q are log normal distributed with mean 1 and variance $\sigma = 0.55$ (this particular value of σ corresponds to the assumption that the h_{ij} lie between 1/3 and 3 at 95% C.L.). We thus fit the ϵ_i by minimizing

$$\chi^2 = \sum_{h_{ij}^q} \frac{[\log h_{ij}^q(Y_{ij}^q, \epsilon_i)]^2}{2\sigma^2}, \quad (3.13)$$

where the “experimental” Y_{ij}^q are the values resulting from the first step. We obtain the result

$$\log_{10} \epsilon_u = -1.5 \pm 0.15, \quad \log_{10} \epsilon'_u = -3.8 \pm 0.35,$$

⁵We do this using real Y_{ij} . We do not expect the values of the charges and order parameters to depend on this simplifying assumption. We thus have 10 real parameters for 9 masses and mixings, and hence expect one flat direction, which turns out to be roughly aligned with the Y_{32}^u direction.

Model	ϵ_ϕ	ϵ_χ	$\tan\beta$	X_ϕ	X_{10}	X_5	X_3
A	0.02	0.02	5	-1	1	1	1
B	0.1	0.2	5	-2	3	3	2
B'	0.1	0.2	20	-2	3	2	1
C	0.2	0.1	50	-1	2	1	0

Table 2. Possible choices of parameters compatible with the fit to fermion masses and mixings.

$$\log_{10} \epsilon_d/t_\beta = -2.0 \pm 0.28, \quad \log_{10} \epsilon_3/t_\beta = -2.0 \pm 0.32. \quad (3.14)$$

Interestingly the correlations between the ϵ_i are very small. The goodness of fit is $\chi^2_{\min}/(\text{d.o.f.}) = 0.8$. At the best fit point, all h_{ij} are indeed close to unity, with the largest deviation occurring in the parameter $h_{22}^u \sim 2.8$.⁶ These values in turn determine the original parameters according to eq. (2.6)

$$\begin{aligned} X_{10} &= \frac{1.9 \pm 0.17}{|\log_{10} \epsilon_\chi|} & X_5 &= \frac{2.4 \pm 0.36 - \log_{10} t_\beta}{|\log_{10} \epsilon_\chi|} \\ X_\phi &= -\frac{0.4 \pm 0.23 + |\log_{10} \epsilon_\phi|}{|\log_{10} \epsilon_\chi|} & X_3 &= \frac{2.0 \pm 0.32 - \log_{10} t_\beta}{|\log_{10} \epsilon_\chi|}. \end{aligned} \quad (3.15)$$

From the imposed relations $X_{10} + X_\phi \geq 0$ and $X_5 + X_\phi \geq 0$, one also obtains from eqs. (2.6) and (3.14) the lower bound (at 95% C.L.)

$$\epsilon_\phi > \max(0.016, 0.0028 t_\beta). \quad (3.16)$$

Imposing that the charges are integers then gives a series of possibilities. Here are a few examples, indicating the acceptable range of values for the small parameters. A particularly simple possibility is for instance

$$\epsilon_\chi \sim \epsilon_\phi \sim 0.02 \quad X_{10} = X_5 = X_3 = -X_\phi = 1 \quad t_\beta = 5. \quad (3.17)$$

But the relations in eq. (3.15) also allow for several other choices, a non exhaustive list is given in table 2.

Clearly, the range of values for $\epsilon_\phi = 0.02 - 0.2$ and similarly for ϵ_χ is acceptable. Notice that the choice $X_5 = X_3$ is allowed for any value of ϵ_χ and in fact remains the only possibility for $\epsilon_\chi \lesssim 0.05$. This will turn out to be an important source of FCNC suppression as will be explained in detail in section 5. In particular, it means that in models where D -term breaking dominates, the RH sbottom mass cannot be split very much from the RH down squark masses of the first two generations. For this reason we will mostly focus on the model defined by eq. (3.17), which has the additional advantage of a very small SU(2) breaking order parameter, $\epsilon_\phi \sim 0.02$.

⁶We could use the same values for the Y_{ij}^q obtained in the first step and apply it to the U(2) model defined by eq. (3.4). Taking as parameters ϵ_u, ϵ'_u and t_β we find a much worse fit with $\chi^2_{\min}/(\text{d.o.f.}) = 3$. Indeed the h_{ij} deviate much more from unity, with typical values $h_{32}^d \sim 10, h_{33}^d \sim 0.2$ etc. Put differently, with h_{ij} closer to one it is impossible to reproduce the quantities Y^u and Y^d needed to fit the data.

4 Soft SUSY breaking terms

The purpose of our model is to link natural supersymmetry to flavour physics. The structure of the soft supersymmetry breaking terms is defined at a high scale (of the order of the GUT scale) by the horizontal symmetry group and the family charges. It is then subject to the RG evolution to the electroweak scale and constrained by the FCNC and CP violation effects at that scale. Those constraints depend on the fits to the fermion sector and on the general structure of soft terms, which is not affected by the RG running. In this section we discuss that structure, in the next one we investigate the experimental constraints on the various low energy scales present, once the UV structure is imposed and in section 6 we include the RG running effects to map the low energy picture to the UV completion, within the considered framework.

In our scenario soft masses receive contributions both from F - and D -terms. The contributions from the $U(1)_H$ D -term are characterized by the scale

$$\tilde{m}_D = \sqrt{g_H \langle D_H \rangle}, \quad (4.1)$$

while the F -term contributions that arise from higher-dimensional spurion operators in the potential suppressed by some SUSY messenger scale M have a typical size

$$\tilde{m}_F = F/M. \quad (4.2)$$

Gaugino masses, A -terms, the μ -term and all masses of scalars uncharged under $U(1)_H$ are generated from F -term contributions,⁷ while charged scalars receive an additional contribution from the D -term vev. Let us first discuss the flavour structure of the D terms. Using that $X_{10} + X_\phi \geq 0$, the Kähler potential for Q can be given as follows:

$$\begin{aligned} K \supset & |Q_a|^2 + |Q_3|^2 + z_{11}^q \left| Q^{\dagger a} \epsilon_{ab} \frac{\phi^b}{\Lambda} \right|^2 + z_{22}^q \left| Q_b \frac{\phi^b}{\Lambda} \right|^2 + \left(z_{33}^q \frac{\phi^\dagger \phi}{\Lambda^2} + z_{33}'^q \frac{\chi^\dagger \chi}{\Lambda^2} \right) |Q_3|^2 \\ & + z_{11}'^q \frac{\chi^\dagger \chi}{\Lambda^2} |Q_a|^2 + \left(z_{12}^q Q^{\dagger a} Q_b \epsilon_{ac} \frac{\phi^c \phi^b}{\Lambda^2} \left(\frac{\chi^\dagger}{\Lambda} \right)^{2|X_\phi|} + z_{13}^q Q^{\dagger a} Q_3 \epsilon_{ab} \frac{\phi^b}{\Lambda} \left(\frac{\chi^\dagger}{\Lambda} \right)^{X_{10} - X_\phi} \right. \\ & \left. + Q^{\dagger a} Q_3 \left(z_{23}^q \frac{\phi^\dagger \phi}{\Lambda^2} + z_{23}'^q \frac{\chi^\dagger \chi}{\Lambda^2} \right) \frac{\phi_a^\dagger}{\Lambda} \left(\frac{\chi^\dagger}{\Lambda} \right)^{X_{10} + X_\phi} + h.c. \right) + \dots, \end{aligned} \quad (4.3)$$

where $\dots$ denote operators which break the $SU(2)$ symmetry down to a discrete subgroup $\tilde{D}_n$.⁸ The last operator has an additional suppression factor, as otherwise it will be removed

⁷As will be shown explicitly, uncharged scalars can also get D -term contributions to their mass from higher-dimensional operators in the Kähler potential. These contributions are however suppressed by flavon vev's and for small enough vev's they are smaller than the F -term contributions.

⁸There are several such operators. One example, for the case of the discrete subgroups $\tilde{D}_n$ (see e.g. ref. [58] for the group theory), is $|Q_1|^2 |\phi^1|^2 + |Q_2|^2 |\phi^2|^2$. We have checked that their effects is to redefine coefficients of some of the operators in what follows, without changing our conclusions. Consequently, we will ignore writing explicitly such operators in what follows.

by a holomorphic field redefinition of Q_3 . One finds the Kähler metric

$$K_{ij}^q = \begin{pmatrix} 1 + z_{11}^q \epsilon_\phi^2 + z_{11}^{q'} \epsilon_\chi^2 & z_{12}^q \epsilon_\phi^2 \epsilon_\chi^{2|X_\phi|} & z_{13}^q \epsilon_\phi \epsilon_\chi^{X_{10}-X_\phi} \\ z_{12}^{q*} \epsilon_\phi^2 \epsilon_\chi^{2|X_\phi|} & 1 + z_{22}^q \epsilon_\phi^2 + z_{11}^{q'} \epsilon_\chi^2 & \left(z_{23}^q \epsilon_\phi^2 + z_{23}^{q'} \epsilon_\chi^2 \right) \epsilon_\phi \epsilon_\chi^{X_{10}+X_\phi} \\ z_{13}^{q*} \epsilon_\phi \epsilon_\chi^{X_{10}-X_\phi} & \left(z_{23}^{q*} \epsilon_\phi^2 + z_{23}^{q'*} \epsilon_\chi^2 \right) \epsilon_\phi \epsilon_\chi^{X_{10}+X_\phi} & 1 + \left(z_{33}^q \epsilon_\phi^2 + z_{33}^{q'} \epsilon_\chi^2 \right) \end{pmatrix}. \quad (4.4)$$

The soft mass terms are found by expanding the Kähler potential in eq. (4.3) to linear order in $\langle D_H \rangle$ and going to canonical normalization using the Kähler metric eq. (4.4). One obtains

$$\begin{aligned} (\tilde{m}_{q,D}^2)_{11} &= (X_{10} + z_{11}^q X_\phi \epsilon_\phi^2) \tilde{m}_D^2 \\ (\tilde{m}_{q,D}^2)_{22} &= (X_{10} + z_{22}^q X_\phi \epsilon_\phi^2) \tilde{m}_D^2 \\ (\tilde{m}_{q,D}^2)_{33} &= (z_{33}^q X_\phi \epsilon_\phi^2 - z_{33}^{q'} \epsilon_\chi^2) \tilde{m}_D^2 \\ (\tilde{m}_{q,D}^2)_{12} &= 2 z_{12}^q X_\phi \epsilon_\phi^2 \epsilon_\chi^{2|X_\phi|} \tilde{m}_D^2 \\ (\tilde{m}_{q,D}^2)_{13} &= z_{13}^q (X_\phi - \tfrac{1}{2} X_{10}) \epsilon_\phi \epsilon_\chi^{X_{10}-X_\phi} \tilde{m}_D^2 \\ (\tilde{m}_{q,D}^2)_{23} &= [z_{23}^q (X_\phi - \tfrac{1}{2} X_{10}) \epsilon_\phi^2 - z_{23}^{q'} (1 + \tfrac{1}{2} X_{10}) \epsilon_\chi^2] \epsilon_\phi \epsilon_\chi^{X_{10}+X_\phi} \tilde{m}_D^2 \end{aligned} \quad (4.5)$$

The U sector works in the same way. For the D sector we assume $|X_{\bar{5}} - X_3| \leq |X_\phi|$, leading to

$$\begin{aligned} K \supset & |D_a|^2 + |D_3|^2 + z_{11}^d \left| D^{\dagger a} \epsilon_{ab} \frac{\phi^b}{\Lambda} \right|^2 + z_{22}^d \left| D_b \frac{\phi^b}{\Lambda} \right|^2 + \left(z_{33}^d \frac{\phi^\dagger \phi}{\Lambda^2} + z_{33}^{d'} \frac{\chi^\dagger \chi}{\Lambda^2} \right) |D_3|^2 \\ & + z_{11}^{d'} \frac{\chi^\dagger \chi}{\Lambda^2} |D_a|^2 + \left(z_{12}^d D^{\dagger a} D_b \epsilon_{ac} \frac{\phi^c \phi^b}{\Lambda^2} \left(\frac{\chi^\dagger}{\Lambda} \right)^{2|X_\phi|} + z_{23}^d D^{\dagger a} D_3 \frac{\phi_a^\dagger}{\Lambda} \left(\frac{\chi}{\Lambda} \right)^{-X_{53}-X_\phi} \right. \\ & \left. + z_{13}^d D^{\dagger a} D_3 \epsilon_{ab} \frac{\phi^b}{\Lambda} \left(\frac{\chi^\dagger}{\Lambda} \right)^{X_{53}-X_\phi} + h.c. \right) + \dots \end{aligned} \quad (4.6)$$

where we have defined $X_{53} = X_{\bar{5}} - X_3$. Note that unlike the Q and U sectors, the z_{23} term scales with a power of χ rather than $\chi^\dagger$. One finds the Kähler metric

$$K_{ij}^d = \begin{pmatrix} 1 + z_{11}^d \epsilon_\phi^2 + z_{11}^{d'} \epsilon_\chi^2 & z_{12}^d \epsilon_\phi^2 \epsilon_\chi^{2|X_\phi|} & z_{13}^d \epsilon_\phi \epsilon_\chi^{X_{53}-X_\phi} \\ z_{12}^{d*} \epsilon_\phi^2 \epsilon_\chi^{2|X_\phi|} & 1 + z_{22}^d \epsilon_\phi^2 + z_{11}^{d'} \epsilon_\chi^2 & z_{23}^d \epsilon_\phi \epsilon_\chi^{-X_{53}-X_\phi} \\ z_{13}^{d*} \epsilon_\phi \epsilon_\chi^{X_{53}-X_\phi} & z_{23}^{d*} \epsilon_\phi \epsilon_\chi^{-X_{53}-X_\phi} & 1 + \left(z_{33}^d \epsilon_\phi^2 + z_{33}^{d'} \epsilon_\chi^2 \right) \end{pmatrix} \quad (4.7)$$

After going to canonical normalization one obtains the soft mass terms:

$$\begin{aligned} (\tilde{m}_{d,D}^2)_{11} &= (X_{\bar{5}} + z_{11}^d X_\phi \epsilon_\phi^2) \tilde{m}_D^2, \\ (\tilde{m}_{d,D}^2)_{22} &= (X_{\bar{5}} + z_{22}^d X_\phi \epsilon_\phi^2) \tilde{m}_D^2, \\ (\tilde{m}_{d,D}^2)_{33} &= (X_3 + (z_{33}^d X_\phi \epsilon_\phi^2 - z_{33}^{d'} \epsilon_\chi^2)) \tilde{m}_D^2, \\ (\tilde{m}_{d,D}^2)_{12} &= 2 z_{12}^d X_\phi \epsilon_\phi^2 \epsilon_\chi^{2|X_\phi|} \tilde{m}_D^2, \end{aligned}$$

$$\begin{aligned}
 (\tilde{m}_{d,D}^2)_{13} &= z_{13}^d (X_\phi - \tfrac{1}{2}X_{53}) \epsilon_\phi \epsilon_\chi^{X_{53}-X_\phi} \tilde{m}_D^2, \\
 (\tilde{m}_{d,D}^2)_{23} &= z_{23}^d (X_\phi + \tfrac{1}{2}X_{53}) \epsilon_\phi \epsilon_\chi^{-X_{53}-X_\phi} \tilde{m}_D^2.
 \end{aligned} \tag{4.8}$$

In principle the whole Kähler potential in eq. (4.3) and eq. (4.6) can be multiplied by $X^\dagger X$, giving rise to F term contributions to the soft masses with identical scalings with $\epsilon_{\phi,\chi}$, but with an additional suppression $\tilde{m}_F^2/\tilde{m}_D^2$. However, there are a few cases where the F terms can be relevant. First notice that the leading D term contribution vanishes for $(\tilde{m}_{I,D}^2)_{33}$, $I = q, u$, resulting in an additional suppression $\sim \epsilon_\phi^2, \epsilon_\chi^2$, as is explicit in the above expressions. This suppression, for small flavon vev's, is bigger than the one from the hierarchy $\tilde{m}_F^2 \ll \tilde{m}_D^2$. In particular this means that the stop, the right handed stau and the left handed sbottom masses are mainly due to F terms. Second, in the particular case $X_3 = X_5$, all the diagonal elements $(\tilde{m}_{d,D}^2)_{ii}$ are degenerate and one has to take into account the splitting induced by the F terms. We then only need to consider the F term contributions

$$\begin{aligned}
 (\tilde{m}_{I,F}^2)_{23} &= d_{23}^I \epsilon_\phi \epsilon_\chi^{X_{10}+X_\phi} \tilde{m}_F^2, & I = q, u \\
 (\tilde{m}_{I,F}^2)_{33} &= d_{33}^I \tilde{m}_F^2, & I = q, u, d
 \end{aligned} \tag{4.9}$$

with other $\mathcal{O}(1)$ coefficients d^I . All other F term contributions can be neglected.

Let us pause a moment and discuss the various effects of the flavour breaking terms in the Kähler potential. First of all, we have checked that the off-diagonal terms present in eqs. (4.5) and (4.8) give only non-leading contributions to the exact rotation matrices diagonalising squark masses. Thus, a very good approximation, neglecting the LR contribution, the squark mass matrices are diagonal in the original basis. The natural basis choice is then to perform rotations only on the fermion fields to diagonalise Yukawa matrices, so that the flavour changing effects will appear in the quark-squark-gluino vertices, controlled by the latter rotation angles. A splitting of the squark masses of the first two generations, introduced by the non-diagonal Kähler terms, renders the FCNCs sensitive to the large 12 rotation angles of the quark sector, which for exactly degenerate first two generations drops out. For ϵ_ϕ in the range 0.02 to 0.2, as obtained from the fit in the previous section, these effects are often subleading to the effects generated by the splitting between the first two and the third generation. A detailed analysis of various effects is presented in the next section.

To summarize, we now collect the relevant sfermion mass matrices in an effective parametrization, keeping only the entries that are most relevant for the SUSY spectrum and dominantly contribute to flavor changing effects. For $\tilde{u}_L, \tilde{d}_L, \tilde{u}_R, \tilde{e}_R$ sfermions the diagonal terms also provide the main source of flavor violation

$$\tilde{m}_{10}^2 = \tilde{m}_D^2 \begin{pmatrix} X_{10} & 0 & 0 \\ 0 & X_{10} + c_{22}^I X_\phi \epsilon_\phi^2 & 0 \\ 0 & 0 & c_{33}^I X_\phi \epsilon_\phi^2 + \tilde{c}_{33}^I \epsilon_\chi^2 \end{pmatrix} + \tilde{m}_F^2 \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & d_{33}^I \end{pmatrix}_{I=q,u,e}, \tag{4.10}$$

while for $\tilde{e}_L, \tilde{\nu}_L, \tilde{d}_R$ sfermions also the off-diagonal terms in eqs. (4.8) can be relevant if ϵ_ϕ is particularly large. Still, provided that

$$\epsilon_\phi^2 < \epsilon_u \quad (X_3 \neq X_{\bar{5}}), \quad \epsilon_\phi^2 < \epsilon_u \tilde{m}_F^2 / \tilde{m}_D^2 \quad (X_3 = X_{\bar{5}}), \tag{4.11}$$

the diagonal elements also dominate flavor violating effects

$$\tilde{m}_{\bar{5}}^2 = \tilde{m}_D^2 \begin{pmatrix} X_{\bar{5}} & 0 & 0 \\ 0 & X_{\bar{5}} + c_{22}^I X_{\phi} \epsilon_{\phi}^2 & 0 \\ 0 & 0 & X_3 + c_{33}^I X_{\phi} \epsilon_{\phi}^2 + \tilde{c}_{33}^I \epsilon_{\chi}^2 \end{pmatrix} + \tilde{m}_F^2 \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & d_{33}^I \end{pmatrix}_{I=d,l} \quad (4.12)$$

The structure of the A -terms follows the structure of the Yukawas in eqs. (2.4), (2.5)

$$A_u = \tilde{m}_F \begin{pmatrix} 0 & a_{12}^u \epsilon'_u & 0 \\ -a_{12}^u \epsilon'_u & a_{22}^u \epsilon_u^2 & a_{23}^u \epsilon_u \\ 0 & a_{32}^u \epsilon_u & a_{33}^u \end{pmatrix}, \quad (4.13)$$

$$A_d = \tilde{m}_F \begin{pmatrix} 0 & a_{12}^d \epsilon'_u \epsilon_d / \epsilon_u & 0 \\ -a_{12}^d \epsilon'_u \epsilon_d / \epsilon_u & a_{22}^d \epsilon_u \epsilon_d & a_{23}^d \epsilon_3 \epsilon_u \\ 0 & a_{32}^d \epsilon_d & a_{33}^d \epsilon_3 \end{pmatrix}, \quad (4.14)$$

with some complex $\mathcal{O}(1)$ coefficients $a_{ij}^{u,d}$.

5 Flavor constraints

We have shown that the hierarchy of fermion masses generates certain (inverted) hierarchy for sfermions, whose actual magnitude would depend on the relative magnitude of the scales $\tilde{m}_D$ and $\tilde{m}_F$ and on the RG renormalisation effects. In more detail, the picture that emerges is the following one: both stop masses, the left-handed sbottom and the right-handed stau (we have neglected the LR mixing) are controlled by the scale $\tilde{m}_F$, the first two generation squark masses are controlled by $\tilde{m}_D$ (assuming $\tilde{m}_D > \tilde{m}_F$) and are necessarily heavier, whereas the mass of the right-handed sbottom and left-handed stau depend on the scale $\tilde{m}_D$ through the charge X_3 . Their dependence on $\tilde{m}_F$ also cannot be a priori neglected. Given this general hierarchical structure of the squark masses, in this section we investigate if the constraints from the FCNC and CP violation effects allow for the physical stops and the left-handed sbottom to be below or around 1 TeV and, if taken so low, what are then the bounds on the other squark masses. Then, in the next section we take into account the RG evolution to map the low energy bounds into the initial conditions for $\tilde{m}_D$ and $\tilde{m}_F$ at the high scale, where the model is defined. In the following we restrict to the most relevant observables ϵ_K and $\Gamma(\mu \rightarrow e\gamma)$ and leave a detailed analysis of the phenomenology to a future publication.

5.1 Constraints from ϵ_K

The strongest constraints on the sfermion masses in this model come from ϵ_K mediated by squark-gluino exchange. In our phenomenological analysis, we take the gluino mass in the range (1.5–3) TeV. Since the bounds on the squark masses scale inversely proportional to the gluino mass, the quoted bounds can vary by a factor of two. For simplicity⁹ we use as an estimate the bounds on the relevant Wilson coefficients from ref. [60], at the

⁹For a thorough discussion of Kaon mixing in natural SUSY see e.g. ref. [59].

scale of the soft masses. For the bound on the left-handed sbottom only ΔC_1 is relevant (see appendix B). Since it is proportional to the product of the left-handed rotations, which are small in the model (of the order of the corresponding CKM matrix elements), it is not surprising that the bound on the left-handed sbottom is weak; for the gluino mass of 1.5 TeV it is generically below 1 TeV (it depends on the assumed values of the phases). This means that a necessary condition for natural supersymmetry is consistent in this model with the flavour data and we take in the following steps and the left handed sbottom to be in the TeV range. Large rotation angles entering into the Wilson coefficients are the right-handed rotations in the (2,3) sector, so we expect non-trivial bounds from $\tilde{C}_1$ and C_4 . Although in the latter case, one angle in the product is left-handed and small, the much stronger experimental bound on C_4 than on $\tilde{C}_1$, makes C_4 (corresponding to the LLRR amplitude in the mass insertion language) the most relevant coefficient for our discussion. The imaginary part of C_4 is bounded by [60]

$$-\frac{3.0 \times 10^{-12}}{\text{TeV}^2} \leq \text{Im } C_4 \leq \frac{4.7 \times 10^{-12}}{\text{TeV}^2} . \quad (5.1)$$

We will now turn to the analysis of the FCNC bounds in our model guided by the general structure of the soft squark masses, as given by eqs. (4.5), (4.8) and (4.9). We calculate the supersymmetric contribution to the Wilson coefficient C_4 in the basis in which both quark and squark masses are diagonal. Since in model A both 1-2 and 1-3 splittings are small, we expand the masses in eq. (B.7) around the common values and use unitarity of the rotations. One obtains

$$\begin{aligned} \Delta C_4 = & \frac{\alpha_s^2}{m_g^2} \left(\hat{\delta}_{12}^{d,RR} \Delta_{31}^R + \tilde{\delta}_{12}^{d,RR} \Delta_{21}^R \right) \left\{ -\frac{1}{3} \left[\hat{\delta}_{12}^{d,LL} x_1^R \partial_R \left(\tilde{f}_4(x_3^L, x_1^R) - \tilde{f}_4(x_1^L, x_1^R) \right) \right. \right. \\ & \left. \left. + \tilde{\delta}_{12}^{d,LL} \Delta_{21}^L x_1^L x_1^R \partial_L \partial_R \tilde{f}_4(x_1^L, x_1^R) \right] + \frac{7}{3} \left[\tilde{f}_4 \rightarrow f_4 \right] \right\} , \end{aligned} \quad (5.2)$$

where $x_i^{L,R} = m_{d_{i,L,R}}^2 / m_g^2$ and $\Delta_{i1}^A = x_i^A / x_1^A - 1$ for $A = L, R$. The loop functions $f_4, \tilde{f}_4$ are defined in [61, 62] and given explicitly in eq. (B.28). The details of the calculation can be found in appendix B. The flavor suppression is encoded in the following quantities, defined as

$$\tilde{\delta}_{12}^{d,RR} \equiv (V_R^d)_{21} (V_R^d)_{22}^* , \quad \hat{\delta}_{12}^{d,RR} \equiv (V_R^d)_{31} (V_R^d)_{32}^* = \tilde{\delta}_{12}^{d,RR} t_d^2 , \quad (5.3)$$

$$\tilde{\delta}_{12}^{d,LL} \equiv (V_L^d)_{21}^* (V_L^d)_{22} , \quad \hat{\delta}_{12}^{d,LL} \equiv (V_L^d)_{31}^* (V_L^d)_{32} = \tilde{\delta}_{12}^{d,LL} |V_{23}^d| \left(|V_{23}^d| - \frac{m_s}{m_b} t_d e^{i\alpha_d} \right) . \quad (5.4)$$

The product relevant for eq. (5.2) is given by

$$\tilde{\delta}_{12}^{d,LL} \tilde{\delta}_{12}^{d,RR} = -\frac{m_d}{m_s} c_d^2 e^{-2i\tilde{\alpha}_{12}} . \quad (5.5)$$

The case of exactly degenerate first two generations corresponds to the limit $\Delta_{21}^{L,R} = 0$. Due to the unitarity of the rotation matrices that diagonalise the Yukawa matrices, those contributions are always proportional to a product of two rotation angles $(V_{L,R}^d)_{3i}$ with

Figure 1. Bounds on the masses of the gluino and the approximately degenerate right handed down squark sector for various choices of the parameters. The region below each line is excluded. The three lines correspond to different choices of the dominant 3-1 splitting, namely $\tilde{m}_{d_R}^2 - \tilde{m}_{b_R}^2 = (1.5, 2.5, 4.0 \text{ TeV})^2$. The remaining parameters are chosen as $|V_{23}^d| = 0.04$, $\sin(\alpha_{12}) = 0.5$ and $s_d^2 = 0.2$. The decoupling of the gluino occurs outside the displayed range of the gluino mass.

$i = 1, 2$ and nicely demonstrate the supersymmetric GIM mechanism [63]: they vanish in the limit of the (relevant for a given contribution) degenerate first two and the third generation squark masses. Another interesting limit is the decoupling limit for the first two generations [64], where they depend only on the third generation squark masses. It is interesting to observe how the experimental bounds on the Wilson coefficients result in the bounds for squark masses as a function of the splitting between generations.

As already mentioned in section 4, the effect of the 1–2 splitting in (5.2) is often negligible. As a rough estimate, if the splitting of the first two generation squarks is smaller than

$$\Delta_{21}^L \lesssim 3 \log \frac{\tilde{m}_D^2}{\tilde{m}_F^2} |V_{23}^d|^2, \quad \Delta_{21}^R \lesssim \Delta_{31}^R t_d^2 \quad (5.6)$$

the corresponding terms become subleading. This is a common situation, in particular in scenario A where the smallness of the SU(2) breaking results in $\Delta_{21}^{L,R} \sim \mathcal{O}(10^{-4})$. However, one should keep in mind that this is not always the case, and for some parameter choices in the other scenarios in table 2 they can become the dominant source of flavour violation.

In figure 1 we plot the bounds arising from imposing eq. (5.1) on (5.2) in the $m_{\tilde{g}}$ and $\tilde{m}_{b_R} \simeq \tilde{m}_{d_R}$ plane, for various choices of the remaining parameters. In the section 6 we will see that a natural stop/gluino spectrum in the TeV range will imply a high scale value of $\tilde{m}_F$ and hence a splitting $\tilde{m}_{d_R}^2 - \tilde{m}_{b_R}^2$ slightly larger than 1 TeV. As is clear from the various lines, the heavy squarks (i.e. the right handed down squarks and the first two generation left handed down squarks) should have masses around 10 – 20 TeV. The bounds do not depend on the much smaller 1-2 splitting, as in model A the hierarchies in eq. (5.6) are very strong.

If we for simplicity set

$$\tilde{m}_{b_L} = m_{\tilde{g}}, \quad \tilde{m}_{d_L} = \tilde{m}_{d_R} \quad (5.7)$$

we can obtain a simple estimate for ΔC_4 as a function of the remaining free masses and splittings, as well as the other parameters. Expanding eq. (5.2) for large $\tilde{m}_{d_R}$ one gets¹⁰

$$\begin{aligned} \text{Im } \Delta C_4 &\approx \frac{2}{3} \alpha_s^2 \frac{m_d}{m_s} |V_{23}^d|^2 s_d^2 \sin 2\tilde{\alpha}_{12} (\tilde{m}_{d_R}^2 - \tilde{m}_{b_R}^2) \frac{\log\left(\frac{\tilde{m}_{d_R}}{m_{\tilde{g}}}\right) + \frac{1}{4}}{(\tilde{m}_{d_R})^4} \\ &\approx 1.6 \times 10^{-8} \left(\frac{|V_{23}^d|}{0.04}\right)^2 \left(\frac{s_d^2}{0.2}\right) \left(\frac{\sin \alpha_{12}}{0.5}\right) (\tilde{m}_{d_R}^2 - \tilde{m}_{b_R}^2) \frac{\log\left(\frac{\tilde{m}_{d_R}}{m_{\tilde{g}}}\right) + \frac{1}{4}}{(\tilde{m}_{d_R})^4} \end{aligned} \quad (5.8)$$

where we took $\alpha_s(\mu)$ at $\mu = 1.5 \text{ TeV}$ and used eq. (B.20).

Up to now we have been working in model A. For the other models consistent with the quark masses and mixings, summarized in table 2, the bounds are much stronger. The reason is that the 3–1 splitting is not set by $\tilde{m}_F^2$ but instead by $\tilde{m}_D^2$ (as $X_3 \neq X_5$). Moreover, the 2–1 degeneracy is also less exact due to the much larger $\text{SU}(2)$ breaking, $\epsilon_\phi \sim 0.1 - 0.2$. Barring some fine tuning of parameters, we then need to go to scales at least of the order of $\tilde{m}_{b_R} \sim \mathcal{O}(100 \text{ TeV})$. Given that these models also require generally large $\tan \beta$, such hierarchies can easily run into problems of RG induced tachyonic masses for squarks and sleptons. We will quantify this statement in the next section.

5.2 Constraints from $\mu \rightarrow e\gamma$

Another important effect in these class of models is the contribution to $\text{BR}(\mu \rightarrow e\gamma)$ through the exchange of charginos and sneutrinos. Although all three sneutrinos are at the heavy scale $\tilde{m}_D$, this contribution is enhanced by large LH mixing angles in the charged lepton sector (that are of the same order as the RH mixing angles in the down sector). Therefore it dominates over the contribution from neutralinos and right-handed staus in the loop, which is strongly suppressed by CKM-like mixing angles, and typically also over the contribution from neutralinos and left-handed staus, which has the same flavor and mass suppression, but smaller numerical coefficients.

For the calculation of the decay rate we use the general results of ref. [65]. We then expand the chargino mixing matrices in leading order in m_W/M_i and m_W/μ , neglect LR mixing and use approximate degeneracy of the first two sneutrino generations to carry out the flavor summation. The result for the decay rate is

$$\Gamma(\mu \rightarrow e\gamma) \approx \frac{\alpha}{4} m_\mu^5 |A_2^{(c)R}|^2, \quad (5.9)$$

with

$$A_2^{(c)R} \approx -\frac{\alpha}{8\pi s_W^2} (Z_\nu)_{31} (Z_\nu)^*_{32} \left[\frac{f(\mu, M_2, \tan \beta, \tilde{m}_{\nu_3}^2)}{\tilde{m}_{\nu_3}^2} - \frac{f(\mu, M_2, \tan \beta, \tilde{m}_{\nu_1}^2)}{\tilde{m}_{\nu_1}^2} \right], \quad (5.10)$$

and the loop function can be found at the end of appendix B. Here the matrix Z_ν diagonalizes the sneutrino mass matrix in the super-CKM basis and is therefore given by the LH charged lepton rotation matrix in the limit where sneutrino masses are diagonal in the

¹⁰Here we have neglected the term proportional to the phase $e^{i\alpha_d}$, since for typical values $V_{23}^d \sim V_{cb} \approx 0.04$ and $t_d \approx 0.5$, the second term in the last relation in eq. (5.4) is subleading.

Figure 2. Allowed region from $BR(\mu \rightarrow e\gamma)$ constraint in the $m_{\bar{\nu}_3}/\mu$ plane for $\mu = M_2$. The region below each line is excluded. The three lines correspond to different choices of the 3–1 splitting, $\Delta m^2 = (1.5, 2.5, 4.0 \text{ TeV})^2$, and $\tan \beta = 5$ (left panel), $\tan \beta = 50$ (right panel).

original basis (i.e. the diagonal D-term contribution dominates), $Z_\nu \approx V_L^e$. Then using the approximate SU(5) relation $V_L^e \approx V_R^d$ one finds for the relevant flavor transitions

$$|(Z_\nu)_{31}(Z_\nu)_{32}| \approx |V_{32}^d|^2 |V_{12}^d| / c_d \approx \sqrt{\frac{m_e}{m_\mu}} \frac{s_d^2}{\sqrt{c_d}} \approx 0.15, \quad (5.11)$$

where we used $c_d \approx 0.9$. Similarly the loop function can be estimated very roughly by neglecting the weak dependence on M_2 for $M_2 \approx \mu \ll \tilde{m}_{\nu_3}$. Introducing the parametrization $\tilde{m}_{\nu_1}^2 = \tilde{m}_{\nu_3}^2 + \Delta m^2$, the 3–1 splitting is approximately the same as in the right-handed sdown sector, $\Delta m^2 \approx \tilde{m}_{d_R}^2 - \tilde{m}_{b_R}^2$, so that the loop function can be expanded for small splittings $\Delta m^2 \ll \tilde{m}_{\nu_3}^2$ corresponding to $X_3 = X_5$. Within these rough approximations the branching ratio is given by

$$BR(\mu \rightarrow e\gamma) \approx 1.1 \times 10^{-11} (11 + 3 \tan \beta)^2 \left(\frac{\Delta m^2}{\tilde{m}_{\nu_3}^2} \right)^2 \left(\frac{\text{TeV}}{\tilde{m}_{\nu_3}} \right)^4, \quad (5.12)$$

to be compared with the bound provided by the MEG Collaboration [66]

$$BR(\mu \rightarrow e\gamma) < 5.7 \times 10^{-13}. \quad (5.13)$$

For the exact loop function, the bounds in the $M_2 - \tilde{m}_{\nu_3}$ plane for $\mu = M_2$ and different values of Δm^2 are shown in figure 2.

6 Mapping to the high scale¹¹

A brief summary of our results from the previous sections is as follows. We have defined a flavour theory at a high (of the order of the GUT) scale by the underlying flavour symmetry.

¹¹This section is written in collaboration with M.Badziak.

The horizontal charges have been determined by a fit to the quark masses and mixing. This procedure does not introduce any relevant scale dependence. We have chosen Model A in table 2 with $X_{10} = X_5 = X_3 = 1$ and $\epsilon_\phi \approx \epsilon_\chi \approx 0.02$ as our reference model. This model gives the structure of the squark masses at the high scale in terms of the two unknown mass scales $\tilde{m}_D$ and $\tilde{m}_F$. The stops, the left-handed sbottom and the right-handed stau masses, as well as the gaugino masses depend only on $\tilde{m}_F$. The $\tilde{m}_D$ contributions to the remaining sfermion masses are universal (in Model A) and the first two generations can be split from the right-handed sbottom/left-handed stau by a contribution of order $\tilde{m}_F$. Apart from quantitative aspects of the RG evolution from the high to the electroweak scale, the basic structure at the low scale remains: the stops, the left-handed sbottom and the gluino physical masses continue to be of the same order of magnitude; the first two generations and right-handed sbottom/left-handed stau are heavier, with a splitting of the latter from the first two (degenerate) generations of the order of the lower mass scale.

Within that structure, we have investigated the constraints from the CP violation in the Kaon system on the above two mass scales set by the physical squark masses, for gluino mass in the range 1 – 3 TeV. We have found that the bounds on the stop and the left-handed sbottom mass is below 1 TeV. Taking those masses to be indeed in the 1 TeV range, as suggested by naturalness, we have then found that the lower bound on the physical mass of the heavy set is in the range of 10 – 20 TeV. Thus, our model accommodates — in fact hints to — the desired spectrum of minimal supersymmetry by linking it to a flavour theory, with a sizable splitting by a factor 10 – 20 between the physical masses of the light and heavy sets of superpartners.

The questions we are now facing are: i) what are the initial conditions in terms of the high scale parameters $\tilde{m}_D$ and $\tilde{m}_F$ for such a hierarchical spectrum and ii) can they be consistent with the proper electroweak breaking and the absence of any tachyonic states? It is well known that in models defined at a high scale, the initial splitting of the first two families of squarks (let their mass be $\tilde{m}_{1,2}$) from the third one (with mass $\tilde{m}_3$) cannot be arbitrarily large (see refs. [15, 18–20]). This is because of the 2-loop sensitivity to the heavy masses in the RG equations for the Higgs mass parameter $m_{H_u}^2$ and for the stop masses. The numerical solutions to the 2-loop RG equations in terms of the initial values of the SO(10) symmetric soft mass parameters at the GUT scale read (obtained in ref. [20] with the SOFTSUSY code [67]):

$$\begin{aligned}
m_{H_u}^2 &\approx -1.3M_{1/2}^2 - 0.1A_0^2 + 0.35M_{1/2}A_0 + 0.01\tilde{m}_3^2 + 0.006\tilde{m}_{1,2}^2 \\
\tilde{m}_{q_3}^2 &\approx 3.1M_{1/2}^2 - 0.04A_0^2 + 0.1M_{1/2}A_0 + 0.65\tilde{m}_3^2 - 0.03\tilde{m}_{1,2}^2 \\
\tilde{m}_{t_R}^2 &\approx 2.3M_{1/2}^2 - 0.07A_0^2 + 0.2M_{1/2}A_0 + 0.35\tilde{m}_3^2 - 0.02\tilde{m}_{1,2}^2 \\
\tilde{m}_{\tau_R}^2 &\approx 0.13M_{1/2}^2 - 0.055A_0^2 + 0.035M_{1/2}A_0 + 0.98\tilde{m}_3^2 - 0.002\tilde{m}_{1,2}^2.
\end{aligned} \tag{6.1}$$

The coefficients in the above equations have been computed at the scale $Q = 1.5$ TeV, $\tan\beta = 10$, $M_{1/2} = 700$ GeV, $A_0 = -3$ TeV, $\tilde{m}_3 = 3$ TeV and $\tilde{m}_{1,2} = 10$ TeV. Their precise values depend slightly on that choice but the qualitative features of the solutions remain unchanged. To a very good approximation, the coefficients in eq. (6.1) are as obtained in the limit of vanishing bottom Yukawa coupling (the effects due to the Yukawa couplings of the

first two generations are totally negligible anyway). Its effect is small since it is multiplied by small masses of the light set (and introduces only a very mild $\tan\beta$ dependence).

The positive contribution of the $\tilde{m}_{1,2}$ to the running of $m_{H_u}^2$ may destroy electroweak breaking whereas the negative contribution of $\tilde{m}_{1,2}$ to the running of the stops and left-handed sbottom masses can drive them tachyonic. For a fixed value of $\tilde{m}_3$, the synergy of both effects leads to an upper bound on the mass $\tilde{m}_{1,2}$ and for a fixed large $\tilde{m}_{1,2}$, a lower and upper bound on $\tilde{m}_3$, constraining the allowed parameter space. Fixing the physical stop masses to be around 1 TeV, one obtains an upper bound on the hierarchy of the physical masses (since the first two generation masses run very weakly). This bound is rising with the gluino mass, as the gluino contribution eases both effects of $\tilde{m}_{1,2}$.

In our present case the initial spectrum is only SU(5) symmetric and also the right-handed sbottom and the left-handed stau are heavy. This splitting introduces important corrections to the solutions in eq. (6.1) that lead to a $\tan\beta$ dependence in both effects above, strengthening them for larger values of $\tan\beta$. This information is encoded already in the 1-loop RG equations, which include the terms $Y_b^2 \tilde{m}_{b_R}^2$ and $Y_\tau^2 \tilde{m}_{\tau_L}^2$:

$$\begin{aligned}\frac{d}{dt}\tilde{m}_{q_3}^2 &= \frac{16}{3}g_3^2 M_3^2 + \dots - Y_b^2 \tilde{m}_{b_R}^2 \\ \frac{d}{dt}\tilde{m}_{\tau_R}^2 &= \frac{12}{5}g_1^2 M_1^2 + \dots - 2Y_\tau^2 \tilde{m}_{\tau_L}^2\end{aligned}\quad (6.2)$$

and

$$\frac{d}{dt}m_{H_u}^2 = 3g_2^2 M_2^2 + \frac{3}{5}g_1^2 M_1^2 - 3Y_t^2(\tilde{m}_{q_3}^2 + \tilde{m}_{t_R}^2 + m_{H_u}^2 + A_t^2) \quad (6.3)$$

where $t = \frac{1}{8\pi^2} \log \frac{M_{GUT}}{Q}$ and M_i are gaugino masses.

Adopting the notation of this paper, we shall denote the light masses at high scale by $\tilde{m}_F$ and the heavy ones by $\tilde{m}_D$. The convolution of the equation for $m_{H_u}^2$ with the equation for $\tilde{m}_{q_3}^2$ gives the net positive contribution¹² to the former, approximately $+0.005(\tan\beta/30)^2 \tilde{m}_D^2$, whereas the contributions to $\tilde{m}_{t_L}^2$, $\tilde{m}_{t_R}^2$ and $\tilde{m}_{\tau_R}^2$ are approximately $-0.03(\tan\beta/30)^2 \tilde{m}_D^2$, $+0.003(\tan\beta/30)^2 \tilde{m}_D^2$ and $-0.055(\tan\beta/30)^2 \tilde{m}_D^2$, respectively. These are additional terms that have to be added to the solutions in eq. (6.1), with the obvious identification $\tilde{m}_{1,2}^2 = \tilde{m}_D^2$ and $\tilde{m}_3^2 = \tilde{m}_F^2$.¹³

Finally, a very important constraint on the high energy parameter space comes from the experimental bounds on the CP-odd Higgs scalar with mass m_A . Using the weak scale condition for the correct electroweak symmetry breaking (at intermediate and large $\tan\beta$ values)

$$m_{H_u}^2 \approx -\mu^2, \quad (6.4)$$

one has the following relation at the electroweak scale for m_A :

$$m_A^2 = m_{H_u}^2 + m_{H_d}^2 + 2\mu^2 \approx m_{H_d}^2 - m_{H_u}^2. \quad (6.5)$$

¹²This and the following numbers are an approximate parametrization of the numerical results obtained with the SOFTSUSY code.

¹³To be precise, the shift of right-handed sbottom and left-handed stau from the light to the heavy set leads also to a very mild change in the coefficients of the terms already present in eq. (6.1).

Thus, one can express the CP-odd scalar mass in terms of the high energy parameters by solving the RG equation for $(m_{H_d}^2 - m_{H_u}^2)$

$$\frac{d}{dt}(m_{H_d}^2 - m_{H_u}^2) = 3Y_t^2 \tilde{m}_{q3}^2 + \dots - 3Y_b^2 \tilde{m}_{bR}^2 - Y_\tau^2 \tilde{m}_{\tau L}^2. \quad (6.6)$$

The net $\tilde{m}_D^2$ dependent contribution to m_A^2 is approximately $-0.12(\tan\beta/30)^2 \tilde{m}_D^2$, so it is pushed by the terms proportional to $\tilde{m}_D^2$ into the “tachyonic” region more effectively than the sfermions. The experimental bounds on m_A as a function of $\tan\beta$ therefore put an important lower bound on $\tilde{m}_F$ (that controls the positive contributions to m_A), for a large fixed value of $\tilde{m}_D$.

The synergy of the required proper electroweak breaking and the experimental limits on m_A leads now to much stronger bounds in the space $(\tilde{m}_D, \tilde{m}_F)$ as a function of $\tan\beta$. In consequence, one gets strong bounds on the hierarchy of the physical masses of the two sets of superpartners. Turning it around, the natural physical spectrum of our model, with stops and the left-handed sbottom around 1 TeV and with the other squarks a factor 10 – 20 heavier, cannot be realized for arbitrarily large values of $\tan\beta$.

Those qualitative considerations are nicely illustrated by the two plots in figure 3. In both plots the value of $\tilde{m}_D$ is fixed to 15 TeV, and A-terms are chosen as $A_0 = -3$ TeV (left plot) and $A_0 = -2$ TeV (right plot). The initial gluino mass is fixed to 0.6 (1.0) TeV in the left (right) plot respectively, corresponding to approximately 1.5 and 2.5 TeV physical gluino masses. Since the first two generation and the R-handed sbottom masses run very weakly, their physical masses remain around 15 TeV. The blue, green and red colours describe the lighter stop, the lighter stau and the Higgs mass values, respectively. In both plots we see the $\tan\beta$ dependent upper bound on $\tilde{m}_F$ coming from the requirement of proper electroweak breaking. The lower bound on $\tilde{m}_F$ does not depend on $\tan\beta$ for certain range of its values for which it is due to the 2-loop stop “tachyonic” constraint that is independent of the left- and right- handed sbottom splitting. For large enough $\tan\beta$ the 1-loop splitting effects take over, and the lower bound on $\tilde{m}_F$ increases with $\tan\beta$ as a result from the experimental limit on m_A . The intersection of the upper and lower bound on $\tilde{m}_F$ then determines the allowed upper value of $\tan\beta$. There is also an exclusion region for very low $\tan\beta$ which depends on the details of the RG equations.

Finally, it is interesting to see that, for the parameter values of the plots, the lighter right-handed stau mass remain close to its initial value $\tilde{m}_F$ because for $\tilde{m}_{\tau_R}^2$ in eq. (6.1) the coefficient of $\tilde{m}_F^2$ is close to 1 and the negative 2-loop effect from $\tilde{m}_{1,2}^2$ is an order of magnitude smaller than for stops. Although the $\tan\beta$ dependent effects summarized below (6.3) are similar, as the net result the right-handed stau mass runs very little. Therefore the constraints from LFV are typically satisfied, as the stau is significantly heavier than the stop. For larger values of the gluino mass, however, the stau mass gets closer to the stop mass since the latter gets a larger contribution from gluino renormalisation.

Summarizing, the minimal supersymmetry spectrum of our model fits nicely a two mass scale initial conditions with a moderate hierarchy $\frac{\tilde{m}_D}{\tilde{m}_F} \approx (3 - 5)$. However, the range of $\tan\beta$ for which such a spectrum can be obtained is limited to small and intermediate values as a result from proper EWSB. This is an interesting constraint on minimal natural

Figure 3. Parameter region in the $\tilde{m}_F/\tan\beta$ plane for fixed $\tilde{m}_D = 15$ TeV and $M_{1/2} = 0.6$ TeV, $A_0 = -3.0$ TeV (left panel) and $M_{1/2} = 1.0$ TeV, $A_0 = -2.0$ TeV (right panel). The contour lines correspond to the masses of $\tilde{t}_1$ (blue), $\tilde{\tau}_1$ (green) and h^0 (red).

supersymmetry, with only stops and the left-handed sbottom light and the rest of squarks heavy, almost totally model independent. In the framework of our $SU(2) \times U(1)$ flavour model, the constraints discussed in this section strongly point to an almost unique choice of universal $U(1)$ charges $X_{10} = X_5 = X_3$, since larger mass splittings would significantly rise the lower bound on the heavy set masses (see section 5), in conflict with the results of the present section.

7 Model building requirements

All our discussion of fermion and superpartner masses in the previous sections are compatible with a flavor symmetry based on $U(1) \times SU(2)$, with broken supersymmetry, and where $U(1)$ is gauged in order to provide large D -terms $D > (F/M)^2$ by a factor of $3 - 5$, whereas $SU(2)$ is a global symmetry. The model would link then flavour symmetry with natural supersymmetry in a simple and economical way. It is clearly an interesting challenge to put these ideas on a more firm theoretical footing.

From a theoretical viewpoint a continuous $SU(2)$ is problematic, since after spontaneous symmetry breaking it leads to massless goldstone bosons. In a string theory setup, there are no obvious ways to obtain continuous non-abelian symmetries, whereas discrete nonabelian symmetries are typically present and related to the geometry of the internal space [39–41] and/or of the fluxes needed for generating chirality in realistic models [42–46]. Another potential possibility would be to consider a gauged $SU(2)$ with a gauge coupling small enough in order to prevent non-universal D -term contributions to soft masses. Both options are worth further exploration towards realistic UV completions. Here we limit ourselves to some remarks on discrete symmetries.

The simplest discrete symmetries that do the job seem to be the groups $\tilde{D}_n$, also called D'_n in the literature. However, most renormalizable operators preserve $SU(2)$ and typically at renormalizable tree-level there are still massless goldstones. Most lower higher-dimensional operators that break $SU(2) \rightarrow \tilde{D}_n$ preserve actually the continuous subgroup $U(1) \in SU(2)$ and as such, there is still one massless goldstone. Finding higher-dimensional operators that break also this Abelian subgroup and give mass to the goldstone is therefore necessary.

Moreover, the model should induce the structure of vevs that we need: large scalar vevs for the flavons and alignment in the flavor space in order to provide successful fermion, scalar mass matrices, A -terms and gaugino masses.

To summarize, the minimal setup that seems viable is based on supersymmetric models with flavor symmetries $U(1)_X \times \tilde{D}_n$, satisfying the following requirements:

- Supersymmetry is broken with D and F-terms, such that $D > (F/M)^2$ (by a factor of 3 – 5).
- The dynamics of the model generate appropriate vev's and vacuum alignment.
- There are higher-dimensional operators that break explicitly $SU(2) \rightarrow \tilde{D}_n$, in particular breaking the subgroup $U(1) \in SU(2)$.

A complete theoretical framework should be a string theory with an anomalous flavor-dependent $U(1)$ gauged symmetry and non-abelian discrete subgroup of $SU(2)$. The vacuum structure is typically tied also to moduli stabilization, due to the anomalous nature of $U(1)$ and the field-dependence of the Fayet-Iliopoulos terms in string theory. Whereas a complete string theory model along these lines is beyond the goals of our paper, it is worth mentioning here that the main ingredients of our construction are present in most current realistic string theory constructions.

8 Conclusions

The first round of the LHC experiments suggests a change of perspective on supersymmetric models. If low energy supersymmetry is realized in Nature and if it is to be “natural”, the simplest (universal) pattern of soft supersymmetry breaking terms looks less plausible. In this paper we have explored the possibility of linking natural supersymmetry to flavour physics. We have revived the idea of obtaining natural supersymmetry from a flavour theory based on a horizontal symmetry. The proposed model is based on spontaneously broken $G \times U(1)_{local}$ family symmetry, where G is a discrete non-abelian subgroup of $SU(2)$, with both F-term and D-term supersymmetry breaking.¹⁴ The soft masses depend then on the two mass scales and their flavour dependence is predicted as a consequence of the hierarchical pattern of fermion masses. A fit to fermion masses and mixings gives a small number of sets of the horizontal charges consistent with the data. The bounds on the sfermion sector from the CP-violation in the kaon system and the other phenomenological

¹⁴Another possibility is that the $SU(2)$ group is weakly gauged, with sufficiently suppressed D-terms.

constraints discussed in section 6 point then almost uniquely to the set with the same U(1) charges for all fermion fields except the left-handed top-bottom quark doublet and the right-handed top (with the interchange left $\leftrightarrow$ right for charged leptons, if the charges respect SU(5) symmetry), which remain zero.

The spectrum of sfermions appears as two sets of particles. The light one, with zero horizontal charges, obtain their mass from the F-term supersymmetry breaking. The dominant contribution to the mass of the heavy ones comes from the D-term breaking. Insisting, for naturalness, on the light masses to be in the 1 TeV range, for the heavy masses one obtains a narrow range of a few to a few tens of TeV, with the lower bound determined by the CP-violation constraint and the upper bound given by the constraints on the left- versus right-handed sbottom mass splitting discussed in section 6. We emphasize the latter point as a model independent constraint on the spectra of minimal supersymmetry, in which only stops and the left-handed sbottom are light. The required high scale hierarchy of $\frac{D}{(F/M)^2}$ is a factor of (3-5).

Besides linking natural supersymmetry to flavour physics, the model combines the advantages and minimizes the disadvantages of the abelian U(1) and non-abelian U(2) symmetries as theories of flavour in supersymmetric models. For instance, U(2) models are in some tension with the fermions mass and mixing data. In particular, the relation $|V_{ub}/V_{cb}| \simeq \sqrt{m_u/m_c}$ predicted in this framework is off by a factor of two compared to the data. We found an interesting indirect correlation between correcting that result and the magnitude of the right-handed rotations in the (3,2) sector required to diagonalise the down quark Yukawa matrix. This, in turn has a strong impact on the bounds obtained from the FCNC and CP violation data on the masses of first two families of squarks and the right-handed sbottom mass. Another point worth mentioning is that U(2) models predict generically large $\tan \beta$, which together with constraints from the RG running from a high (GUT) scale forces the right-handed sbottom to stay light, leading therefore to a non-minimal natural supersymmetry spectrum.

The phenomenological success of the model makes it worth exploring in detail the UV mechanisms of the D-term supersymmetry breaking in the presence of symmetry groups beyond simple U(1), which have to be spontaneously broken with a proper alignment of the flavon vevs. In the present case these are either gauged SU(2) with suppressed non-abelian D-terms or a discrete subgroup of global SU(2). The latter may fit very well into the theoretical framework of string theory. We discussed general requirements for such a mechanism.

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A Fermion sector

The Yukawa matrices as obtained from the superpotential in eq. (2.1) are given by

$$Y_u = \begin{pmatrix} 0 & h_{12}^u \epsilon'_u & 0 \\ -h_{12}^u \epsilon'_u & h_{22}^u \epsilon_u^2 & h_{23}^u \epsilon_u \\ 0 & h_{32}^u \epsilon_u & h_{33}^u \end{pmatrix}, \quad (\text{A.1})$$

$$Y_d = \begin{pmatrix} 0 & h_{12}^d \epsilon'_u \epsilon_d / \epsilon_u & 0 \\ -h_{12}^d \epsilon'_u \epsilon_d / \epsilon_u & h_{22}^d \epsilon_u \epsilon_d & h_{23}^d \epsilon_3 \epsilon_u \\ 0 & h_{32}^d \epsilon_d & h_{33}^d \epsilon_3 \end{pmatrix}, \quad (\text{A.2})$$

These matrices are diagonalized by unitary matrices $V_{L,R}^I$ according to

$$(V_L^I)^T Y_I V_R^I = Y_I^{\text{diag}}, \quad I = u, d, \quad (\text{A.3})$$

such that the mass eigenstates are related to the original quarks (denoted by ')

$$u'_L = V_L^u u_L, \quad d'_L = V_L^d d_L, \quad u'_R = V_R^{u*} u_R, \quad d'_R = V_R^{d*} d_R. \quad (\text{A.4})$$

The eigenvalues are approximately¹⁵ given by

$$Y_t \approx |Y_{33}^u|, \quad Y_c \approx \frac{|Y_{22}^u Y_{33}^u - Y_{23}^u Y_{32}^u|}{Y_t}, \quad Y_u \approx \frac{|(Y_{12}^u)^2 Y_{33}^u|}{Y_t Y_c}, \quad (\text{A.5})$$

$$Y_b \approx \sqrt{|Y_{33}^d|^2 + |Y_{32}^d|^2}, \quad Y_s \approx \frac{|Y_{22}^d Y_{33}^d - Y_{23}^d Y_{32}^d|}{Y_b}, \quad Y_d \approx \frac{|(Y_{12}^d)^2 Y_{33}^d|}{Y_b Y_s}, \quad (\text{A.6})$$

The unitary rotations have the form

$$V_L^{u,d} = P_L' \hat{V}_L^{u,d} P_L, \quad V_R^I = P_R^I \hat{V}_R^I P_R^I, \quad (\text{A.7})$$

where $P_{L,R}^I$ and $P_{L,R}^I$ are diagonal phase matrices to be fixed below, and $\hat{V}_{L,R}^I$ are approximately given by

$$\hat{V}_L^u = \begin{pmatrix} 1 & V_{12}^u & 0 \\ -V_{12}^{u*} & 1 & V_{23}^u \\ V_{12}^{u*} V_{23}^{u*} & -V_{23}^{u*} & 1 \end{pmatrix}, \quad \hat{V}_R^u = \begin{pmatrix} 1 & -V_{12}^u & 0 \\ V_{12}^{u*} & 1 & V_{32}^u \\ -V_{12}^{u*} V_{32}^{u*} & -V_{32}^{u*} & 1 \end{pmatrix}, \quad (\text{A.8})$$

¹⁵All expressions receive multiplicative corrections of the form $(1 + \mathcal{O}(\epsilon_u^2, \epsilon'_u / \epsilon_u))$.

$$\hat{V}_L^d = \begin{pmatrix} 1 & V_{12}^d & V_{13}^d \\ -V_{12}^{d*} & 1 & V_{23}^d \\ V_{23}^{d*}V_{12}^{d*} - V_{13}^{d*} & -V_{23}^{d*} & 1 \end{pmatrix}, \quad \hat{V}_R^d = \begin{pmatrix} 1 & -V_{12}^d/c_d & 0 \\ V_{12}^{d*} & c_d & V_{32}^d \\ -V_{12}^{d*}V_{32}^{d*}/c_d & -V_{32}^{d*} & c_d \end{pmatrix}, \quad (\text{A.9})$$

with

$$V_{12}^u \approx \left(\frac{Y_{12}^u Y_{33}^u}{Y_{22}^u Y_{33}^u - Y_{23}^u Y_{32}^u} \right)^* \equiv |V_{12}^u| e^{i\alpha_{12}^u} = e^{i\alpha_{12}^u} \sqrt{\frac{m_u}{m_c}}, \quad (\text{A.10})$$

$$V_{23}^u \approx \frac{Y_{23}^{u*} Y_{33}^u}{Y_t^2} \equiv e^{i\alpha_{23}^u} |V_{23}^u|, \quad (\text{A.11})$$

$$V_{32}^u \approx \frac{Y_{32}^{u*} Y_{33}^u}{Y_t^2} \equiv e^{i\alpha_{32}^u} |V_{32}^u|, \quad (\text{A.12})$$

$$V_{12}^d \approx \left(\frac{Y_{12}^d Y_{33}^d}{Y_{22}^d Y_{33}^d - Y_{23}^d Y_{32}^d} \right)^* \equiv e^{i\alpha_{12}^d} |V_{12}^d| = e^{i\alpha_{12}^d} \sqrt{\frac{m_d}{m_s}} \sqrt{c_d}, \quad (\text{A.13})$$

$$V_{13}^d \approx \frac{Y_{12}^{d*} Y_{32}^d}{Y_b^2} \equiv e^{i\alpha_{13}^d} |V_{13}^d| = e^{i\alpha_{13}^d} \sqrt{\frac{m_d m_s}{m_b^2}} \frac{s_d}{\sqrt{c_d}}, \quad (\text{A.14})$$

$$V_{23}^d \approx \frac{Y_{23}^{d*} Y_{33}^d + Y_{22}^{d*} Y_{32}^d}{Y_b^2} \equiv e^{i\alpha_{23}^d} |V_{23}^d|, \quad (\text{A.15})$$

$$V_{32}^d \approx \frac{Y_{32}^{d*} |Y_{33}^d|}{Y_{33}^{d*} Y_b} \equiv e^{i\alpha_{32}^d} |V_{32}^d| = e^{i\alpha_{32}^d} s_d \quad (\text{A.16})$$

Here we neglected contribution to the 1-3 entries of the rotations that are of order

$$(\hat{V}_{R,L}^u)_{13} = \max \mathcal{O} \left(\epsilon_u \epsilon'_u, \epsilon_\phi \epsilon_\chi^{X_{10}-X_\phi} \right), \quad (\hat{V}_R^d)_{13} = \max \mathcal{O} \left(\epsilon'_u, \epsilon_\phi \epsilon_\chi^{X_{53}-X_\phi} \right), \quad (\text{A.17})$$

and we defined $c_d \equiv \cos \theta_d$, $s_d \equiv \sin \theta_d$, where the angle θ_d is defined by

$$\tan \theta_d \equiv \frac{|Y_{32}^d|}{|Y_{33}^d|}, \quad (\text{A.18})$$

This angle parametrizes the all-order corrections in ϵ_d/ϵ_3 , which typically is not a very small expansion parameter. The twelve angles present in a general set of quark rotations are thus reduced to only four, given by the free parameters $|V_{23}^{u,d}|$ and $|V_{32}^{u,d}|$. This is due to the three textures zeroes and the relation $Y_{12} = -Y_{21}$ in the Yukawas, which indeed remove eight real degrees of freedom. Below we will see that the phase structure is also greatly simplified, as there are only four phases that can be physically relevant. Matching this to the degrees of freedom in the CKM matrix, we see that we have one angle and three phases as free parameters.

The left-handed phases matrices can be chosen to bring the CKM matrix to the standard PDG form with one physical phase, while the right-handed ones render the diagonal Yukawas real and positive. This gives

$$P_L = \begin{pmatrix} e^{i(\tilde{\alpha}_{12} + \alpha_{12}^d)} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & e^{-i(\tilde{\alpha}_{23} + \alpha_{23}^d)} \end{pmatrix}, \quad (\text{A.19})$$

$$P_R^u = e^{-i \arg m_c} \begin{pmatrix} e^{-i(\tilde{\alpha}_{12} + \alpha_{12} - \alpha_{12}^u)} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & e^{i(\tilde{\alpha}_{23} + \alpha_{23}^d + \arg m_c - \arg m_t)} \end{pmatrix}, \quad (\text{A.20})$$

$$P_R^d = e^{-i \arg m_s} \begin{pmatrix} e^{-i(\tilde{\alpha}_{12} - \alpha_{12}^d)} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & e^{i(\tilde{\alpha}_{23} + \alpha_{23}^d + \arg m_s - \arg m_b)} \end{pmatrix}, \quad (\text{A.21})$$

with

$$\alpha_{12} \equiv \alpha_{12}^d - \alpha_{12}^u, \quad \alpha_{23} \equiv \alpha_{23}^d - \alpha_{23}^u, \quad (\text{A.22})$$

$$\tilde{\alpha}_{12} = \arg \left(1 - \frac{|V_{12}^u|}{|V_{12}^d|} e^{-i\alpha_{12}} \right), \quad \tilde{\alpha}_{23} = \arg \left(1 - \frac{|V_{23}^u|}{|V_{23}^d|} e^{-i\alpha_{23}} \right), \quad (\text{A.23})$$

and we used the fact that under the rotations $\hat{V}_{L,R}^{u,d}$ the eigenvalues have phases given by

$$\arg m_t = \arg h_{33}^u, \quad \arg m_c = \arg h_{12}^u + \alpha_{12}^u, \quad \arg m_u = -2\alpha_{12}^u + \arg m_c \quad (\text{A.24})$$

$$\arg m_b = \arg h_{33}^d, \quad \arg m_s = \arg h_{12}^d + \alpha_{12}^d, \quad \arg m_d = -2\alpha_{12}^d + \arg m_s. \quad (\text{A.25})$$

For later use, we will also make the definitions

$$\alpha_u \equiv \arg m_t - \arg m_c - \alpha_{32}^u - \alpha_{23}^u \quad (\text{A.26})$$

$$\begin{aligned} \alpha_d &\equiv \arg m_b - \arg m_s - \alpha_{32}^d - \alpha_{23}^d \\ &= \alpha_{13}^d - \alpha_{23}^d - \alpha_{12}^d \end{aligned} \quad (\text{A.27})$$

Finally we can multiply our rotations $\hat{V}_{L,R}^{u,d}$ by phases P_R^I, P_L^I from the left without any physical effect (as we can absorb these phases by a redefinition of the original fields $q', u',$ and d'). Using this freedom, we can bring the quark rotations to their final form:

$$V_L^d = \begin{pmatrix} 1 & |V_{12}^d| & |V_{13}^d| e^{i\alpha_d} \\ -|V_{12}^d| & 1 & |V_{23}^d| \\ |V_{12}^d V_{23}^d| - |V_{13}^d| e^{-i\alpha_d} & -|V_{23}^d| & 1 \end{pmatrix} \begin{pmatrix} e^{i\tilde{\alpha}_{12}} \\ 1 \\ e^{-i\tilde{\alpha}_{23}} \end{pmatrix}, \quad (\text{A.28})$$

$$V_L^u = \begin{pmatrix} e^{-i\alpha_{12}} \\ 1 \\ e^{i\alpha_{23}} \end{pmatrix} \begin{pmatrix} 1 & |V_{12}^u| & 0 \\ -|V_{12}^u| & 1 & |V_{23}^u| \\ |V_{12}^u V_{23}^u| - |V_{23}^u| & 1 & 1 \end{pmatrix} \begin{pmatrix} e^{i(\tilde{\alpha}_{12} + \alpha_{12})} \\ 1 \\ e^{-i(\tilde{\alpha}_{23} + \alpha_{23})} \end{pmatrix}, \quad (\text{A.29})$$

$$V_R^d = \begin{pmatrix} 1 & -|V_{12}^d|/c_d & 0 \\ |V_{12}^d| & c_d & |V_{32}^d| \\ -|V_{12}^d V_{32}^d|/c_d & -|V_{32}^d| & c_d \end{pmatrix} \begin{pmatrix} e^{-i\tilde{\alpha}_{12}} \\ 1 \\ e^{i(\tilde{\alpha}_{23} - \alpha_d)} \end{pmatrix}, \quad (\text{A.30})$$

$$V_R^u = \begin{pmatrix} 1 & -|V_{12}^u| & 0 \\ |V_{12}^u| & 1 & |V_{32}^u| \\ -|V_{12}^u V_{32}^u| & -|V_{32}^u| & 1 \end{pmatrix} \begin{pmatrix} e^{-i(\tilde{\alpha}_{12} + \alpha_{12})} \\ 1 \\ e^{i(\tilde{\alpha}_{23} + \alpha_{23} - \alpha_u)} \end{pmatrix}. \quad (\text{A.31})$$

Any physical observable can thus only depend on the four phases α_{12} (also through $\tilde{\alpha}_{12}$), α_{23} (also through $\tilde{\alpha}_{23}$), α_u and α_d . The CKM matrix $V_{CKM} = (V_L^u)^\dagger V_L^d$ is then given by

$$V_{CKM} = \begin{pmatrix} 1 & |V_{12}| & |V_{13}|e^{-i\delta} \\ -|V_{12}| & 1 & |V_{23}| \\ |V_{12}V_{23}| - |V_{13}|e^{i\delta} & -|V_{23}| & 1 \end{pmatrix} \quad (\text{A.32})$$

with

$$V_{12} = V_{12}^d - V_{12}^u, \quad V_{23} = V_{23}^d - V_{23}^u, \quad (\text{A.33})$$

$$V_{13} = V_{13}^d - V_{12}^u V_{23}, \quad |V_{td}| = |V_{12}V_{23} - V_{13}|, \quad (\text{A.34})$$

and the CKM phase is given by

$$\delta = \alpha_{12} + \tilde{\alpha}_{12} - \arg \left(\frac{|V_{13}^d|}{|V_{12}^u V_{23}|} e^{i(\alpha_d + \alpha_{12} - \tilde{\alpha}_{23})} - 1 \right). \quad (\text{A.35})$$

The above form of the CKM matrix gives rise to the relations

$$|V_{us}| \approx \sqrt{m_d/m_s} \sqrt{c_d}, \quad (\text{A.36})$$

$$|V_{ub}/V_{cb}| \approx |\sqrt{m_u/m_c} + e^{i\beta} \Delta t_d \sqrt{c_d}|, \quad |V_{td}/V_{ts}| \approx |\sqrt{m_d/m_s} + e^{i\beta'} \Delta t_d| \sqrt{c_d}, \quad (\text{A.37})$$

where $t_d \equiv \tan \theta_d$ and

$$\Delta = \frac{\sqrt{m_s m_d}}{|V_{cb}| m_b} \approx 0.09, \quad \beta = \alpha_d + \alpha_{12} - \tilde{\alpha}_{23} + \pi, \quad \beta' = \beta - \alpha_{12}. \quad (\text{A.38})$$

The eigenvalues and rotations for the charged lepton sector can be obtained by replacing everywhere in the above the index d by e , with the exception of the phases, which in the leptonic sector depend on the model of neutrino masses.

B SUSY contributions to flavor observables

The new SUSY contributions to FCNC processes due to squark-gluino loops have been calculated in the literature in terms of the flavor-changing unitary matrices $Z_{U,D}$ appearing at the gluino vertex in the notation of ref. [68]

$$\mathcal{L} = \tilde{u}_L^* \bar{\lambda} (Z_U^L)^\dagger u_L + \tilde{u}_R^* \bar{\lambda} (Z_U^R)^\dagger u_R + \tilde{d}_L^* \bar{\lambda} (Z_D^L)^T d_L + \tilde{d}_R^* \bar{\lambda} (Z_D^R)^T d_R + h.c. \quad (\text{B.1})$$

where we have used that the LR flavour violation is negligible because of the suppression of the A terms. The flavour violation in the Kaon sector can then be encoded in the Wilson coefficients of the operators

$$Q_1 = (\bar{d}_L \gamma_\mu s_L)^2, \quad Q_2 = (\bar{d}_R s_L)^2, \quad Q_3 = \bar{d}_R^\beta s_L^\alpha \bar{d}_R^\alpha s_L^\beta, \quad (\text{B.2})$$

$$\tilde{Q}_1 = (\bar{d}_R \gamma_\mu s_R)^2, \quad \tilde{Q}_2 = (\bar{d}_L s_R)^2, \quad \tilde{Q}_3 = \bar{d}_L^\beta s_R^\alpha \bar{d}_L^\alpha s_R^\beta, \quad (\text{B.3})$$

$$Q_4 = (\bar{d}_R s_L) (\bar{d}_L s_R), \quad Q_5 = \bar{d}_R^\beta s_L^\alpha \bar{d}_L^\alpha s_R^\beta, \quad (\text{B.4})$$

For example the SUSY contribution to the Wilson coefficient of the effective operator Q_1 , $\tilde{Q}_1$, Q_4 and Q_5 is given by [61, 62]

$$\Delta C_1 = \frac{\alpha_s^2}{m_{\tilde{g}}^2} (Z_D^L)_{1i}^* (Z_D^L)_{2i} (Z_D^L)_{1j}^* (Z_D^L)_{2j} \left(\frac{11}{36} \tilde{f}_4(x_i^L, x_j^L) + \frac{1}{9} f_4(x_i^L, x_j^L) \right), \quad (\text{B.5})$$

$$\Delta \tilde{C}_1 = \frac{\alpha_s^2}{m_{\tilde{g}}^2} (Z_D^R)_{1i}^* (Z_D^R)_{2i} (Z_D^R)_{1j}^* (Z_D^R)_{2j} \left(\frac{11}{36} \tilde{f}_4(x_i^R, x_j^R) + \frac{1}{9} f_4(x_i^R, x_j^R) \right), \quad (\text{B.6})$$

$$\Delta C_4 = \frac{\alpha_s^2}{m_{\tilde{g}}^2} (Z_D^L)_{1i}^* (Z_D^L)_{2i} (Z_D^R)_{1j}^* (Z_D^R)_{2j} \left(-\frac{1}{3} \tilde{f}_4(x_i^L, x_j^R) + \frac{7}{3} f_4(x_i^L, x_j^R) \right), \quad (\text{B.7})$$

$$\Delta C_5 = \frac{\alpha_s^2}{m_{\tilde{g}}^2} (Z_D^L)_{1i}^* (Z_D^L)_{2i} (Z_D^R)_{1j}^* (Z_D^R)_{2j} \left(\frac{5}{9} \tilde{f}_4(x_i^L, x_j^R) + \frac{1}{9} f_4(x_i^L, x_j^R) \right) \quad (\text{B.8})$$

where $f_4, \tilde{f}_4$ are some loop functions given below and $x_i^L = m_{d_L}^2/m_{\tilde{g}}^2$, where $m_{\tilde{g}}$ is the gluino mass and m_{d_L} is the mass of the d-squark. In the limit where the first two sfermion generations are degenerate, we can carry out the summation over i, j and use unitarity of Z_D^L to obtain a compact form for the Wilson coefficient in which the flavor suppression and the loop functions are factorized, e.g.

$$\Delta C_1 = \frac{\alpha_s^2}{m_{\tilde{g}}^2} [(Z_D^L)_{13}^* (Z_D^L)_{23}]^2 \left[\frac{11}{36} \left(\tilde{f}_4(x_1^L, x_1^L) - 2\tilde{f}_4(x_1^L, x_3^L) + \tilde{f}_4(x_3^L, x_3^L) \right) + \frac{1}{9} \left(\tilde{f}_4 \rightarrow f_4 \right) \right]. \quad (\text{B.9})$$

The suppression of the $i \rightarrow j$ flavor transition is therefore entirely determined by the factor $(Z_D^L)_{i3}^* (Z_D^L)_{j3}$. In principle the unitary matrices Z_I are a product of quark and squark rotations. In the case where all flavour violation comes from the quark rotations we can simply read them off eq. (A.4),

$$Z_U^L = (V_L^u)^\dagger, \quad Z_U^R = (V_R^u)^T, \quad (\text{B.10})$$

$$Z_D^L = (V_L^d)^T, \quad Z_D^R = (V_R^d)^\dagger. \quad (\text{B.11})$$

which in turn only depend on the unitary superfield rotations V_L^d . We therefore define the quantities¹⁶

$$\hat{\delta}_{ij}^{d,LL} \equiv (Z_D^L)_{i3}^* (Z_D^L)_{j3} = (V_L^d)_{3i}^* (V_L^d)_{3j}, \quad \hat{\delta}_{ij}^{d,RR} \equiv (Z_D^R)_{i3}^* (Z_D^R)_{j3} = (V_R^d)_{3i} (V_R^d)_{3j}^*, \quad (\text{B.12})$$

$$\hat{\delta}_{ij}^{u,LL} \equiv (Z_U^L)_{i3}^* (Z_U^L)_{j3} = (V_L^u)_{3i}^* (V_L^u)_{3j}, \quad \hat{\delta}_{ij}^{u,RR} \equiv (Z_U^R)_{i3}^* (Z_U^R)_{j3} = (V_R^u)_{3i} (V_R^u)_{3j}^*. \quad (\text{B.13})$$

Using the explicit expressions for $V_{L,R}^{u,d}$ in appendix A, we obtain

$$\hat{\delta}_{23}^{d,LL} = -|V_{23}^d| e^{-i\tilde{\alpha}_{23}}, \quad (\text{B.14})$$

$$\hat{\delta}_{13}^{d,LL} = \left(|V_{23}^d| \sqrt{m_d/m_s} \sqrt{c_d} - \sqrt{\frac{m_d m_s}{m_b^2}} \frac{s_d}{\sqrt{c_d}} e^{i\alpha_d} \right) e^{-i(\tilde{\alpha}_{23} + \tilde{\alpha}_{12})}, \quad (\text{B.15})$$

$$\hat{\delta}_{12}^{d,LL} = \left(-|V_{23}^d|^2 \sqrt{m_d/m_s} \sqrt{c_d} + \sqrt{\frac{m_d m_s}{m_b^2}} \frac{s_d}{\sqrt{c_d}} |V_{23}^d| e^{i\alpha_d} \right) e^{-i\tilde{\alpha}_{12}}, \quad (\text{B.16})$$

¹⁶The analogue expressions of ΔC_1 in the up sector are obtained with $Z_D \rightarrow Z_U^*$.

$$\hat{\delta}_{23}^{d,RR} = -s_d c_d e^{-i(\tilde{\alpha}_{23}-\alpha_d)}, \quad (\text{B.17})$$

$$\hat{\delta}_{13}^{d,RR} = -\sqrt{m_d/m_s}\sqrt{c_d} s_d e^{-i(\tilde{\alpha}_{12}+\tilde{\alpha}_{23}-\alpha_d)}, \quad (\text{B.18})$$

$$\hat{\delta}_{12}^{d,RR} = \sqrt{m_d/m_s} \frac{s_d^2}{\sqrt{c_d}} e^{-i\tilde{\alpha}_{12}}. \quad (\text{B.19})$$

Notice that the phase $\tilde{\alpha}_{12}$ that appears in the LL and RR 1-2 sector is small:

$$\tilde{\alpha}_{12} = \sqrt{\frac{m_u m_s}{m_d m_c}} \sin \alpha_{12} + \mathcal{O}\left(\frac{m_u m_s}{m_d m_c}\right) \lesssim 0.2. \quad (\text{B.20})$$

In terms of these quantities the relevant Wilson coefficients are given by

$$\Delta C_1 = \frac{\alpha_s^2}{m_g^2} \left(\hat{\delta}_{12}^{LL} \right)^2 \left[\frac{11}{36} \left(\tilde{f}_4(x_1^L, x_1^L) - 2\tilde{f}_4(x_1^L, x_3^L) + \tilde{f}_4(x_3^L, x_3^L) \right) + \frac{1}{9} \left(\tilde{f}_4 \rightarrow f_4 \right) \right], \quad (\text{B.21})$$

$$\Delta \tilde{C}_1 = \frac{\alpha_s^2}{m_g^2} \left(\hat{\delta}_{12}^{RR} \right)^2 \left[\frac{11}{36} \left(\tilde{f}_4(x_1^R, x_1^R) - 2\tilde{f}_4(x_1^R, x_3^R) + \tilde{f}_4(x_3^R, x_3^R) \right) + \frac{1}{9} \left(\tilde{f}_4 \rightarrow f_4 \right) \right], \quad (\text{B.22})$$

$$\Delta C_4 = \frac{\alpha_s^2}{m_g^2} \hat{\delta}_{12}^{LL} \hat{\delta}_{12}^{RR} \left[-\frac{1}{3} \left(\tilde{f}_4(x_1^L, x_1^R) - \tilde{f}_4(x_1^L, x_3^R) - \tilde{f}_4(x_3^L, x_1^R) + \tilde{f}_4(x_3^L, x_3^R) \right) + \frac{7}{3} \left(\tilde{f}_4 \rightarrow f_4 \right) \right], \quad (\text{B.23})$$

$$\Delta C_5 = \frac{\alpha_s^2}{m_g^2} \hat{\delta}_{12}^{LL} \hat{\delta}_{12}^{RR} \left[\frac{5}{9} \left(\tilde{f}_4(x_1^L, x_1^R) - \tilde{f}_4(x_1^L, x_3^R) - \tilde{f}_4(x_3^L, x_1^R) + \tilde{f}_4(x_3^L, x_3^R) \right) + \frac{1}{9} \left(\tilde{f}_4 \rightarrow f_4 \right) \right], \quad (\text{B.24})$$

with the loop functions

$$f_4(x, x) = \frac{2 - 2x + (1+x) \log x}{(x-1)^3}, \quad (\text{B.25})$$

$$f_4(x, y) = \frac{x(y-1)^2 \log x - y(x-1)^2 \log y - (x-1)(y-1)(y-x)}{(x-1)^2(y-1)^2(y-x)}, \quad (\text{B.26})$$

$$\tilde{f}_4(x, x) = \frac{1 - x^2 + 2x \log x}{(x-1)^3}, \quad (\text{B.27})$$

$$\tilde{f}_4(x, y) = \frac{x^2(y-1)^2 \log x - y^2(x-1)^2 \log y - (x-1)(y-1)(y-x)}{(x-1)^2(y-1)^2(y-x)}. \quad (\text{B.28})$$

Finally we give the loop function used for the calculation of BR ($\mu \rightarrow e\gamma$)

$$f(\mu, M_2, \tan \beta, \tilde{m}_{\nu_k}^2) = f_R^c(x_{2k}) + \frac{\mu(\mu + M_2 \tan \beta)}{M_2^2 - \mu^2} f_{LR}^c(x_{\mu k}) - \frac{M_2(M_2 + \mu \tan \beta)}{M_2^2 - \mu^2} f_{LR}^c(x_{2k}), \quad (\text{B.29})$$

where

$$f_L^c(x) = \frac{2 + 3x - 6x^2 + x^3 + 6x \log x}{6(1-x)^4}, \quad f_{LR}^c(x) = \frac{-3 + 4x - x^2 - 2 \log x}{(1-x)^3} \quad (\text{B.30})$$

and $x_{2k} = M_2^2/\tilde{m}_{\nu_k}^2$, $x_{\mu k} = \mu^2/\tilde{m}_{\nu_k}^2$.

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