An explicit Z' -boson explanation of the $B \rightarrow K^* \mu^+ \mu^-$ anomaly

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ABSTRACT: A global fit to the recent $B \rightarrow K^* \mu^+ \mu^-$ data shows indications for a large new-physics contribution to the Wilson coefficient of the semi-leptonic vector operator. In this article we consider a simple Z' -boson model of 3-3-1 type that can accommodate such an effect without violating any other constraint from quark-flavour physics. Implications for yet unobserved decay modes such as $B \rightarrow X_s \nu \bar{\nu}$ and longstanding puzzles like $B \rightarrow \pi K$ are also discussed. The Z' -boson masses required to address the observed anomaly lie in the range of 7 TeV. Such heavy Z' bosons evade the existing bounds from precision data and direct searches, and will remain difficult to discover even at a high-luminosity LHC. The potential of an ILC as well as the next generation of low-energy parity-violation experiments in constraining the Z' -boson parameter space is also examined.

KEYWORDS: Rare Decays, Beyond Standard Model, B-Physics

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Contents

1	Introduction	1
2	Flavour-changing quark interactions	3
3	Couplings in $\beta = -\sqrt{3}$ model	4
4	Dipole and semi-leptonic operators	5
5	Mass difference in B_s-$\bar{B}_s$ mixing	7
6	Rare decays: $B \rightarrow X_s \ell^+ \ell^-$ and $B \rightarrow K \mu^+ \mu^-$	7
7	Rare decays: $B_s \rightarrow \mu^+ \mu^-$ and $B \rightarrow K^{(*)}, X_s \nu \bar{\nu}$	9
8	Non-leptonic decays	10
9	Precision measurements	11
10	Direct searches	13
11	Discussion	16

1 Introduction

The success of the LHCb experiment has so far been a nightmare for all flavour physicists that were hoping to see signs of new physics popping up in B_s - $\bar{B}_s$ mixing and the rare $B_s \rightarrow \mu^+ \mu^-$ decay. This situation might have changed with the latest measurements [1, 2] of the angular correlations of the decay products in $B \rightarrow K^* \mu^+ \mu^-$ that display several deviations from the standard model (SM) predictions. The largest discrepancy of 3.7σ arises in the variable P'_5 [3] (the analogue of S_5 in [4]), which has specifically been designed to combine theoretical and experimental benefits, while retaining a high sensitivity to new-physics effects. Further LHCb analyses combined with a critical assessment of theoretical uncertainties (see in particular [5] as well as very recently [6]) will be necessary to clarify whether the observed deviations call for the presence of new physics or are simply flukes.

A first attempt to shed light on the $B \rightarrow K^* \mu^+ \mu^-$ anomaly has been made in [7] (with subsequent new-physics analyses presented in [8–10]). By performing a global analysis of $B \rightarrow K^* \mu^+ \mu^-$, $B \rightarrow X_s \ell^+ \ell^-$ and $B \rightarrow X_s \gamma$ data, model-independent constraints on the effective couplings of higher-dimensional operators have been derived. While the fit does

not provide a unique best solution, a particularly simple scenario of new physics emerges as a possible explanation, that features the following modifications of Wilson coefficients [7]¹

$$\Delta C_7^\gamma \sim 0, \quad \Delta C_9^\ell \sim -1.5, \quad \Delta C_{10}^\ell \sim 0. \quad (1.1)$$

Given that within the SM one has $(C_9^\ell)_{\text{SM}} \simeq 4.1$ (see e.g. [11]) the solution (1.1) should originate from new flavour dynamics that induces a large destructive contribution to the semi-leptonic vector operator, while leaving the electromagnetic dipole and the semi-leptonic axial-vector operator essentially SM-like.

A glimpse at the extensive literature on quark-flavour physics readily shows that the pattern seen in (1.1) is highly non-standard, and that the usual suspects — such as the minimal supersymmetric SM [12, 13], warped extra dimension scenarios [14, 15] or models with partial compositeness [16, 17] to just name a few — cannot accommodate the observed deviations (this point has also been stressed in [8, 9]). An apparent though ad hoc way to obtain (1.1) is to postulate the existence of a Z' boson with mass in the TeV range and specific couplings to fermions [7]: the new neutral gauge boson should couple only to the left-handed $\bar{s}b$ current and proportionally to the product $V_{ts}^* V_{tb}$ of Cabibbo-Kobayashi-Maskawa (CKM) matrix elements so that excessive CP-violating contributions to $B_s - \bar{B}_s$ mixing are avoided; the Z' boson should furthermore couple to left-handed and right-handed muons with close to equal strength, since the $B \rightarrow K^* \mu^+ \mu^-$ data seem to prefer a vector rather than an axial-vector coupling to the $\bar{\mu}\mu$ current.

In this article we discuss a Z' -boson model based on the gauge group $\text{SU}(3)_c \times \text{SU}(3)_L \times \text{U}(1)_X$, which has the above properties. Many variants of these so-called 3-3-1 scenarios have been considered in the literature, but we will focus on the original proposal [18, 19] in which the electric charge is given by $Q = T^3 - \sqrt{3}T^8 + X$. Here $T^a = \lambda^a/2$ denotes the $\text{SU}(3)_L$ generators with λ^a the Gell-Mann matrices and X is the $\text{U}(1)_X$ quantum number. This model (for obvious reasons referred to as “ $\beta = -\sqrt{3}$ ” hereafter) is the only 3-3-1 scenario that has the desired feature that the Z' boson couples much more strongly to the vector rather than axial-vector component of the charged-lepton current. As it turns out, the choice $\beta = -\sqrt{3}$ also makes the 3-3-1 model leptophilic/hadrophobic leading to a rich and interesting phenomenology.

We find that correlations between the quark-flavour-changing $b \rightarrow s \ell^+ \ell^-$, $\nu \bar{\nu}$ transitions and modifications in muon decay, quark β -decay as well as parity-violating $e^- \rightarrow e^-$ observables are unavoidable in the considered Z' -boson scenario. Advances in the low-energy measurements of the properties of charged leptons can hence provide valuable insights into the structure of the underlying theory, if the deviations in $B \rightarrow K^* \mu^+ \mu^-$ as seen by LHCb are indeed due to interactions of a new neutral gauge boson. We also stress the importance of improved lattice-QCD determinations of the hadronic parameters entering the SM prediction for the mass difference in $B_s - \bar{B}_s$ mixing, which provides the strongest constraint on the simplest explanations of the $B \rightarrow K^* \mu^+ \mu^-$ anomaly in the 3-3-1 model with $\beta = -\sqrt{3}$. From the experimental side, updated BaBar and Belle measurements of the decay distributions in $B \rightarrow X_s \ell^+ \ell^-$ would be essential to corroborate or challenge

¹See (4.2) for the definition of the corresponding operators.

the case for new physics in $|\Delta B| = 1$ transitions. We finally highlight the complementarity between low- p_T and high- p_T measurements by considering the present and future constraints on the Z' -boson mass $M_{Z'}$ arising from direct Z' -boson searches at the LHC, its high-luminosity upgrade (HL-LHC) and an ILC.

Our work is organised as follows. In section 2 we review the generic structure of flavour-changing Z' -boson interactions in 3-3-1 scenarios and discuss viable flavour alignments necessary to curb the amount of CP violation in the quark-flavour sector. The explicit expression for the Z' -boson couplings relevant for our analysis are given in section 3. Our phenomenological analysis that ranges from the study of B -physics observables over precision measurements to direct searches starts with section 4 and ends with section 10. We conclude in section 11 by summarising our main findings and discussing further ways to cast light on the origin of the deviations seen in the recent $B \rightarrow K^* \mu^+ \mu^-$ data.

2 Flavour-changing quark interactions

The left-handed couplings between the Z' boson and the SM quarks have in 3-3-1 scenarios the generic form

$$\mathcal{L} \supset \sum_{q=d,u} (\mathcal{G}_{qL})_{ij} \bar{q}_L^i Z' q_L^j. \quad (2.1)$$

In the weak interaction basis indicated by a superscript “I”, the couplings are diagonal 3×3 matrices in flavour space

$$\mathcal{G}_{qL}^I = \text{diag} \left(g_q^l, g_q^l, g_q^h \right). \quad (2.2)$$

Since $g_q^l \neq g_q^h$ the strength of the Z' -boson couplings to the third and the first two generations are different. Note that we do not consider the right-handed Z' -boson couplings to the SM quarks in this section, since it is always possible to make these interactions flavour diagonal by an appropriate choice of quantum numbers.

By appropriate unitary rotations $U_{u,d}$ of the chiral quark fields, one can always choose a basis where the mass matrices of the SM quarks are diagonal. In such a basis the Z' -boson couplings are given by

$$\mathcal{G}_{qL} = U_q^\dagger \mathcal{G}_{qL}^I U_q, \quad (2.3)$$

where the matrices $U_{d,u}$ have to fulfil the constraint

$$V = U_u^\dagger U_d, \quad (2.4)$$

with V denoting the CKM matrix. Notice that in the mass eigenstate basis the matrices in (2.3) will generically contain off-diagonal elements which signals the presence flavour-changing Z' -boson tree-level interactions.

The flavour-changing quark interactions can however be confined to the sector of down-type quarks by choosing $U_u = 1$ (or equivalently $U_d = V$), i.e. by alignment in the up-type quark sector. In such a case, one obtains

$$\mathcal{G}_{dL} = g_q^l \mathbf{1}_3 + (g_q^h - g_q^l) \begin{pmatrix} |V_{td}|^2 & V_{td}^* V_{ts} & V_{td}^* V_{tb} \\ V_{ts}^* V_{td} & |V_{ts}|^2 & V_{ts}^* V_{tb} \\ V_{tb}^* V_{td} & V_{tb}^* V_{ts} & |V_{tb}|^2 \end{pmatrix}, \quad \mathcal{G}_{uL} = \text{diag} \left(g_q^l, g_q^l, g_q^h \right). \quad (2.5)$$

This implies that for up-type alignment the $s_L \rightarrow d_L Z'$, $b_L \rightarrow d_L Z'$ and $b_L \rightarrow s_L Z'$ amplitudes are proportional to the combinations $V_{td}^* V_{ts}$, $V_{td}^* V_{tb}$ and $V_{ts}^* V_{tb}$ of CKM elements. The flavour-changing Z' -boson contributions hence follow the pattern of minimal-flavour violation (MFV). In particular, there will be no new sources of CP violation beyond the CKM phase.

Given the freedom in the choice of down-type misalignment U_d it is also possible to set the contributions to the $s_L \rightarrow d_L Z'$ and $b_L \rightarrow d_L Z'$ to zero. This is achieved by the texture (see e.g. [20])

$$U_d \simeq \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & V_{ts}^* \\ 0 & V_{ts} & V_{tb} \end{pmatrix}. \quad (2.6)$$

It follows that

$$(\mathcal{G}_{d_L})_{23} \simeq (g_q^h - g_q^l) V_{ts}^* V_{tb}, \quad (\mathcal{G}_{d_L})_{12} \simeq (\mathcal{G}_{d_L})_{13} \simeq 0, \quad (2.7)$$

to leading power in the Cabibbo angle $\lambda \simeq 0.23$, while the $c_L \rightarrow u_L Z'$ transition is governed by the coupling

$$\begin{aligned} (\mathcal{G}_{u_L})_{12} &\simeq (g_q^h - g_q^l) \left[V_{cb}^* V_{ub} |V_{tb}|^2 + V_{cb}^* V_{us} V_{ts}^* V_{tb} + V_{cs}^* V_{us} |V_{ts}|^2 + V_{cs}^* V_{ub} V_{tb}^* V_{ts} \right] \\ &\simeq (g_q^h - g_q^l) \frac{A^2 \lambda^7}{2} (\bar{\rho} - i\bar{\eta} - 1). \end{aligned} \quad (2.8)$$

This result has to be compared with the expression $(\mathcal{G}_{u_L})_{12} \simeq (g_q^h - g_q^l) A^2 \lambda^5 (\bar{\rho} - i\bar{\eta})$ obtained in the case of down-type alignment (i.e. $U_d = 1$). We see that enforcing flavour-changing effects in the B_s -meson sector only leads to new CP violation in the D -meson sector with respect to MFV. The resulting effects are however strongly Cabibbo suppressed and we will not consider them any further.

In the following we will assume that a mechanism is at work that leads to a flavour structure like (2.5) or (2.7) suitable to explain the pattern (1.1). In fact, the flavour-changing interactions will necessarily have a non-trivial structure when there are gauge quantum numbers that distinguish generations. This is the case in Z' -boson models of the 3-3-1 type in which the third-generation fermions are treated differently from the second and first generation (see (2.2)). The textures of flavour-changing neutral currents will then ultimately be controlled by the symmetry breaking patterns of the horizontal global flavour symmetries. We leave the underlying mechanism and the ultraviolet (UV) origin that gives rise to the flavour alignment unspecified.

3 Couplings in $\beta = -\sqrt{3}$ model

Assuming up-type alignment (2.5), the Z' -boson couplings in the 3-3-1 model with $\beta = -\sqrt{3}$ relevant for our analysis are ($i \neq j$)

$$(\mathcal{G}_{d_L})_{ij} = \frac{g_{CW}}{\sqrt{3} \sqrt{1 - 4s_W^2}} V_{ti}^* V_{tj}, \quad (\mathcal{G}_{u_L})_{ij} = 0, \quad (3.1)$$

and (see e.g. [21])

$$\begin{aligned}
 (\mathcal{G}_{d_L})_{ii} &= \frac{g(-1 + 2c_W^2 |V_{ti}|^2)}{2\sqrt{3}c_W \sqrt{1 - 4s_W^2}}, & (\mathcal{G}_{d_R})_{ii} &= \frac{gs_W^2}{\sqrt{3}c_W \sqrt{1 - 4s_W^2}}, \\
 (\mathcal{G}_{u_L})_{ii} &= \frac{g(-1 + 2c_W^2 \delta_{ti})}{2\sqrt{3}c_W \sqrt{1 - 4s_W^2}}, & (\mathcal{G}_{u_R})_{ii} &= \frac{-2gs_W^2}{\sqrt{3}c_W \sqrt{1 - 4s_W^2}}, \\
 (\mathcal{G}_{\ell_L})_{ii} &= \frac{g(1 + 2s_W^2)}{2\sqrt{3}c_W \sqrt{1 - 4s_W^2}}, & (\mathcal{G}_{\ell_R})_{ii} &= \frac{\sqrt{3}gs_W^2}{c_W \sqrt{1 - 4s_W^2}}, \\
 (\mathcal{G}_{\nu_L})_{ii} &= (\mathcal{G}_{\ell_L})_{ii}, & (\mathcal{G}_{\nu_R})_{ii} &= 0,
 \end{aligned} \tag{3.2}$$

where g is the usual $SU(2)_L$ coupling, while s_W and c_W denote the sine and cosine of the weak mixing angle. For partial misalignment in the down-type quark with a texture (2.7), the coupling $(\mathcal{G}_{d_L})_{23}$ is also given by (3.1), while the remaining off-diagonal entries $(\mathcal{G}_{d_L})_{ij}$ vanish to excellent approximation. The diagonal quark couplings in (3.2) are to first order independent of such a change.

Notice that for the choice $\beta = -\sqrt{3}$, the vector and axial-vector couplings of the Z' -boson to charge leptons take the form

$$\begin{aligned}
 (\mathcal{G}_{\ell_V})_{ii} &= (\mathcal{G}_{\ell_R})_{ii} + (\mathcal{G}_{\ell_L})_{ii} = \frac{g(1 + 8s_W^2)}{2\sqrt{3}c_W \sqrt{1 - 4s_W^2}} \simeq 3.30g, \\
 (\mathcal{G}_{\ell_A})_{ii} &= (\mathcal{G}_{\ell_R})_{ii} - (\mathcal{G}_{\ell_L})_{ii} = -\frac{g\sqrt{1 - 4s_W^2}}{2\sqrt{3}c_W} \simeq -0.09g.
 \end{aligned} \tag{3.3}$$

Electrons, muons and taus hence couple essentially vectorially to the new gauge boson. For other common choices of β (i.e. $\sqrt{3}$, $\pm 1/\sqrt{3}$) this is not the case, which renders these models uninteresting for our purposes.

4 Dipole and semi-leptonic operators

In order to calculate the various $b \rightarrow s\gamma, \mu^+\mu^-$ observables in the 3-3-1 model, one has to determine the Wilson coefficients of the operators that enter the effective Hamiltonian

$$\mathcal{H}_{\text{eff}} = -\frac{4G_F}{\sqrt{2}} V_{ts}^* V_{tb} \left(C_7^\gamma Q_7^\gamma + C_9^\ell Q_9^\ell + C_{10}^\ell Q_{10}^\ell \right) + \text{h.c.}, \tag{4.1}$$

where $G_F \simeq 1.167 \cdot 10^{-5} \text{ GeV}^{-2}$ is the Fermi constant and $(\ell = e, \mu, \tau)$

$$\begin{aligned}
 Q_7^\gamma &= \frac{e}{(4\pi)^2} m_b (\bar{s}_L \sigma_{\alpha\beta} b_R) F^{\alpha\beta}, \\
 Q_9^\ell &= \frac{e^2}{(4\pi)^2} (\bar{s}_L \gamma_\alpha b_L) (\bar{\ell} \gamma^\alpha \ell), \\
 Q_{10}^\ell &= \frac{e^2}{(4\pi)^2} (\bar{s}_L \gamma_\alpha b_L) (\bar{\ell} \gamma^\alpha \gamma_5 \ell).
 \end{aligned} \tag{4.2}$$

Figure 1. Tree-level contributions to $B \rightarrow K^* \mu^+ \mu^-$ (left diagram) and $B_s - \bar{B}_s$ mixing (right diagram) from Z' -boson exchange.

A straightforward matching calculation gives

$$\begin{aligned}\Delta C_7^\gamma &\simeq \frac{8s_W^2}{27(1-4s_W^2)} \frac{M_W^2}{M_{Z'}^2}, \\ \Delta C_9^\ell &= -\frac{2\pi}{3\alpha} \frac{1+8s_W^2}{1-4s_W^2} \frac{M_W^2}{M_{Z'}^2}, \\ \Delta C_{10}^\ell &= \frac{2\pi}{3\alpha} \frac{M_W^2}{M_{Z'}^2},\end{aligned}\tag{4.3}$$

in agreement with the results presented in [21]. To obtain the result for ΔC_7^γ we have only kept the terms that are not CKM suppressed. The tree-level diagram giving rise to $\Delta C_{9,10}^\ell$ is shown on the left-hand side in figure 1. Interestingly, our 3-3-1 model predicts $\Delta C_9^\ell < 0$ and $\Delta C_{10}^\ell / \Delta C_9^\ell = -(1-4s_W^2)/(1+8s_W^2) \ll 1$.

Employing $\alpha = \alpha(M_Z) \simeq 1/128$, $s_W^2 = s_W^2(M_Z) \simeq 0.23$ and $M_W \simeq 80.4 \text{ GeV}$, it hence follows that the 68% confidence level (CL) range

$$\Delta C_9^\ell \in [-1.9, -1.3],\tag{4.4}$$

found in [7] from a fit to the present $b \rightarrow s\gamma, \mu^+ \mu^-$ data, can be achieved for Z' -boson masses

$$M_{Z'} \in [5.7, 6.9] \text{ TeV}.\tag{4.5}$$

Such large Z' -boson masses lead to new-physics effects of

$$\Delta C_7^\gamma = \mathcal{O}(10^{-4}),\tag{4.6}$$

i.e. negligible corrections with respect to $(C_7^\gamma)_{\text{SM}} \simeq -0.19$ [22]. Likewise, the Z' -boson contributions to the semi-leptonic axial-vector operator are very small, amounting to

$$\Delta C_{10}^\ell \in [0.04, 0.05].\tag{4.7}$$

The smallness of the coefficient ΔC_7^γ (ΔC_{10}^ℓ) is of course a result of the one-loop suppression of dipole interactions (the vector-like nature of the Z' -boson couplings to charged leptons).

5 Mass difference in B_s - $\bar{B}_s$ mixing

Tree-level exchange of Z' bosons also leads to a shift in the mass difference of neutral mesons. From the right graph in figure 1 we find, in the case of B_s - $\bar{B}_s$ mixing

$$\Delta_{B_s} = \frac{\Delta M_{B_s}}{(\Delta M_{B_s})_{\text{SM}}} - 1 = \eta_{B_s} \frac{16\pi}{3\alpha} \frac{c_W^2 s_W^2}{1 - 4s_W^2} \frac{M_W^2}{M_{Z'}^2}, \quad (5.1)$$

with

$$\eta_{B_s} \simeq 0.41 \left[1 - 0.03 \ln \left(\frac{M_{Z'}}{5 \text{ TeV}} \right) \right]. \quad (5.2)$$

These results agree with [21] (also [20]). Similar expressions apply in the case of up-type alignment for B_d - $\bar{B}_d$ and K - $\bar{K}$ mixing. Notice that Δ_{B_s} is strictly positive, meaning that the Z' -boson corrections necessarily enhance the mass difference ΔM_{B_s} with respect to the SM prediction. This general feature has also been stressed recently in [10].

Using the available experimental information on $|\Delta F| = 2$ processes, the UT_{fit} collaboration obtains the following 95% CL bound [23]

$$\Delta_{B_s} \in [-0.16, 0.26]. \quad (5.3)$$

These limits imply that Z' -boson masses

$$M_{Z'} > 6.9 \text{ TeV}, \quad (5.4)$$

are compatible with the current bounds on B_s - $\bar{B}_s$ mixing. The remaining $|\Delta F| = 2$ constraints turn out to be less restrictive and only apply if the flavour structure (2.5) is realised.

By comparing (5.4) to (4.5) we see that the bound from B_s -meson mixing already cuts into the parameter region favoured by $B \rightarrow K^* \mu^+ \mu^-$. In the 3-3-1 model with $\beta = -\sqrt{3}$, the constraint from $\bar{B}_s$ - B_s mixing hence effectively limits the possible new-physics corrections in the Wilson coefficient C_9^ℓ . Future improvements in lattice-QCD determinations of the B_s -meson decay constant f_{B_s} , the hadronic parameter $\hat{B}_{B_s}$ as well as V_{cb} , which represent the dominant sources of uncertainty in $(\Delta M_{B_s})_{\text{SM}}$ [24], will therefore be crucial to cross-check Z' -boson explanations of the $B \rightarrow K^* \mu^+ \mu^-$ anomaly. In fact, in our scenario one finds

$$\Delta C_9^\ell = -\frac{1 + 8s_W^2}{8c_W^2 s_W^2} \frac{\Delta_{B_s}}{\eta_{B_s}} \simeq -4.9 \Delta_{B_s}, \quad (5.5)$$

where in the final step we have neglected the weak logarithmic dependence of η_{B_s} on $M_{Z'}$. An upper bound on Δ_{B_s} hence translates into a lower bound on ΔC_9^ℓ , which makes the relation (5.5) useful to monitor the effect of future improvements in the SM prediction of ΔM_{B_s} on destructive new physics in $B \rightarrow K^* \mu^+ \mu^-$.

6 Rare decays: $B \rightarrow X_s \ell^+ \ell^-$ and $B \rightarrow K \mu^+ \mu^-$

The large destructive contribution to the semi-leptonic vector operator (1.1) will leave an imprint in both the inclusive $B \rightarrow X_s \ell^+ \ell^-$ decay ($\ell = e, \mu$) as well as the exclusive

$B \rightarrow K\mu^+\mu^-$ channels. For the relative shift in the branching ratio of $B \rightarrow X_s\ell^+\ell^-$ integrated over low invariant di-lepton masses, i.e. $q^2 \in [1, 6] \text{ GeV}^2$, we find by employing the results of [11], the following approximate formula

$$\Delta_{X_s} = \frac{\text{Br}_{\text{low}}(B \rightarrow X_s\ell^+\ell^-)}{\text{Br}_{\text{low}}(B \rightarrow X_s\ell^+\ell^-)_{\text{SM}}} - 1 \simeq 0.19 \Delta C_9^\ell + 0.04 (\Delta C_9^\ell)^2. \quad (6.1)$$

Here we have only included the dominant corrections due to ΔC_9^ℓ . Given that $\Delta C_7^\gamma \simeq 0$ and $\Delta C_{10}^\ell/\Delta C_9^\ell \simeq -0.03$, the expression (6.1) is a very good approximation to the full result. From (4.4) it then follows that the low- q^2 branching ratio of $B \rightarrow X_s\ell^+\ell^-$ should receive a modification of

$$\Delta_{X_s} \in -[22, 18] \%, \quad (6.2)$$

relative to the SM expectation $\text{Br}_{\text{low}}(B \rightarrow X_s\ell^+\ell^-)_{\text{SM}} = (1.68 \pm 0.17) \cdot 10^{-6}$ [15].

In this context it is important to emphasise that the global analyses [7, 8] both apply the experimental value $\text{Br}_{\text{low}}(B \rightarrow X_s\ell^+\ell^-) = (1.6 \pm 0.5) \cdot 10^{-6}$ obtained in [25] from a naive average of the BaBar [26] and Belle [27] measurements. The latter results are by now almost ten years old, but an update of $B \rightarrow X_s\ell^+\ell^-$ decay distributions that has been published in a peer-reviewed journal does not exist. There is however a recent preliminary result from Belle [28] which is included in the latest HFAG compendium [29] of heavy-flavour averages. It reads $\text{Br}_{\text{low}}(B \rightarrow X_s\ell^+\ell^-) = (0.99 \pm 0.22) \cdot 10^{-6}$ and when averaged with the BaBar measurement [26] leads to a value of $\text{Br}_{\text{low}}(B \rightarrow X_s\ell^+\ell^-) = (1.04 \pm 0.21) \cdot 10^{-6}$, which is slightly more than 2σ below the SM prediction. Since a shift $\Delta C_9^\ell \sim -1.5$ results in $\text{Br}_{\text{low}}(B \rightarrow X_s\ell^+\ell^-) \sim 1.3 \cdot 10^{-6}$, the newest Belle results on $B \rightarrow X_s\ell^+\ell^-$ could also be interpreted as a hint for the presence of a destructive new-physics contribution to the Wilson coefficient of the semi-leptonic vector operator.

We now turn our attention to the exclusive $b \rightarrow s\mu^+\mu^-$ channels, focusing on the $B^+ \rightarrow K^+\mu^+\mu^-$ mode which has recently been measured precisely by LHCb [30]. In the high- q^2 region $[14.18, 22] \text{ GeV}^2$, we obtain in agreement with [8]

$$\Delta_{K^+} = \frac{\text{Br}_{\text{high}}(B^+ \rightarrow K^+\mu^+\mu^-)}{\text{Br}_{\text{high}}(B^+ \rightarrow K^+\mu^+\mu^-)_{\text{SM}}} - 1 \simeq 0.24 \Delta C_9^\ell + 0.03 (\Delta C_9^\ell)^2, \quad (6.3)$$

where ΔC_7^γ and ΔC_{10}^ℓ contributions have again been neglected. Using (4.4) it then follows that

$$\Delta_{K^+} \in -[35, 26] \%. \quad (6.4)$$

The corresponding SM prediction is $\text{Br}_{\text{high}}(B^+ \rightarrow K^+\mu^+\mu^-)_{\text{SM}} = (1.10 \pm 0.25) \cdot 10^{-7}$ [8] (see also [31]) and lies on top of the value $\text{Br}_{\text{high}}(B^+ \rightarrow K^+\mu^+\mu^-) = (1.04 \pm 0.12) \cdot 10^{-7}$ measured by LHCb. A correction $\Delta C_9^\ell \sim -1.5$ leads to $\text{Br}_{\text{high}}(B^+ \rightarrow K^+\mu^+\mu^-) \sim 0.8 \cdot 10^{-7}$, corresponding to a tension between theory and experiment of close to 1σ . Given the recent observation of a charmonium resonance in the high- q^2 di-muon spectrum of $B^+ \rightarrow K^+\mu^+\mu^-$ [32], the theoretical uncertainty plaguing $\text{Br}_{\text{high}}(B^+ \rightarrow K^+\mu^+\mu^-)_{\text{SM}}$ deserves further study.

The above discussion makes clear that the outcome of global analyses of $b \rightarrow s$ data and their physics interpretations depend significantly on various factors, most importantly on which observables are included in the fit (or excluded from the fit) and the size and

treatment of theoretical uncertainties. Keeping this in mind it is not difficult to understand why [7] and [8] do not obtain identical results. The main differences in the two analyses are: in the former work the constraint from $B^+ \rightarrow K^+ \mu^+ \mu^-$ is not included, while the latter analysis considers this exclusive channel; the global fit in [7] is based on $B \rightarrow K^* \mu^+ \mu^-$ data from LHCb alone, whereas [8] incorporates the available results from ATLAS, BaBar, Belle, CDF and CMS; for what concerns the low- q^2 region the work [7] considers the bins $[0.1, 2] \text{ GeV}^2$, $[2, 4.3] \text{ GeV}^2$ and $[4.3, 8.68] \text{ GeV}^2$, while the authors of [8] perform averages to obtain values for the observables integrated over the $[1, 6] \text{ GeV}^2$ bin. These differences in combination with the fact that the most significant deviation of 3.7σ appears in P'_5 in the interval $[4.3, 8.68] \text{ GeV}^2$ [1, 2] explains why, in contrast to [7], the article [8] finds that a new-physics contribution $\Delta C_9^\ell \sim -1.5$ alone does not lead to a good description of the data.

We argued above that also the treatment of $B \rightarrow X_s \ell^+ \ell^-$ in the global fit is likely to have an important impact on the physics implications of the $B \rightarrow K^* \mu^+ \mu^-$ anomaly (and may in fact also point to a sizeable negative contribution ΔC_9^ℓ). Since the theoretical predictions for $B \rightarrow X_s \ell^+ \ell^-$ are rather sound, BaBar and Belle measurements of the inclusive rare semi-leptonic B decay based on the full data sets are urgently needed to clarify the situation. We also add that the preference for a small negative new-physics contribution to the Wilson coefficient C_7^γ , as found in both analyses [7, 8], is a simple consequence [33] of the fact that the SM value of the branching ratio of $B \rightarrow X_s \gamma$ [34, 35] is around 1σ below the experimental world average [29].

7 Rare decays: $B_s \rightarrow \mu^+ \mu^-$ and $B \rightarrow K^{(*)}, X_s \nu \bar{\nu}$

Besides $B \rightarrow K^{(*)} \mu^+ \mu^-$ and $B \rightarrow X_s \ell^+ \ell^-$ the LHC measurements of $B_s \rightarrow \mu^+ \mu^-$ [36, 37] also put tight constraints on the Wilson coefficients of the semi-leptonic operators. In fact, only the axial-vector operator enters the prediction for the $B_s \rightarrow \mu^+ \mu^-$ rate

$$\Delta_{\mu^+ \mu^-} = \frac{\text{Br}(B_s \rightarrow \mu^+ \mu^-)}{\text{Br}(B_s \rightarrow \mu^+ \mu^-)_{\text{SM}}} - 1 = \frac{|(C_{10}^\ell)_{\text{SM}} + \Delta C_{10}^\ell|^2}{|(C_{10}^\ell)_{\text{SM}}|^2} - 1. \quad (7.1)$$

Recalling that $(C_{10}^\ell)_{\text{SM}} \simeq -4.3$ (see e.g. [11]), the range (4.7) translates into

$$\Delta_{\mu^+ \mu^-} \in -[2.5, 1.7] \%, \quad (7.2)$$

corresponding to suppressions of the branching fraction by $\mathcal{O}(2\%)$ relative to the SM. Such small effects are (and will remain) unobservable given the theoretical uncertainties inherent in $B_s \rightarrow \mu^+ \mu^-$. Similarly, the decay modes $B_d \rightarrow \mu^+ \mu^-$ and $K_L \rightarrow \pi^0 \mu^+ \mu^-$ will receive only very small or no corrections in our 3-3-1 model depending on whether the flavour structure (2.5) or (2.7) is realised.

In the future the rare decays $B \rightarrow K^{(*)}, X_s \nu \bar{\nu}$ may also allow for transparent studies of electroweak penguin effects. Normalised to the SM rates, one finds for the branching ratios of these modes

$$\Delta_{\nu \bar{\nu}} = \frac{\text{Br}(B \rightarrow K^{(*)}, X_s \nu \bar{\nu})}{\text{Br}(B \rightarrow K^{(*)}, X_s \nu \bar{\nu})_{\text{SM}}} - 1 = \frac{|X_{\text{SM}} + \Delta X|^2}{|X_{\text{SM}}|^2} - 1, \quad (7.3)$$

with $X_{\text{SM}} \simeq 1.47$ [38]. The coefficient ΔX takes the form (cf. also [21])

$$\Delta X = \frac{2\pi}{3\alpha} \frac{s_W^2 (1 + 2s_W^2)}{1 - 4s_W^2} \frac{M_W^2}{M_{Z'}^2}. \quad (7.4)$$

For Z' -boson masses in the range (4.5), one obtains

$$\Delta_{\nu\bar{\nu}} \in [22, 33] \%, \quad (7.5)$$

implying enhancements of the $b \rightarrow s\nu\bar{\nu}$ branching ratios of $\mathcal{O}(25\%)$. Note that this is a rather model-independent conclusion following from $\text{SU}(2)_L$ invariance, lepton-flavour universality and the absence/smallness of right-handed currents [8, 9, 39]. If the up-type sector is fully aligned, effects of the same size arise in $B \rightarrow X_d \nu\bar{\nu}$, while the corrections in $K \rightarrow \pi \nu\bar{\nu}$ are smaller and follow the MFV pattern, i.e. $\Delta\text{Br}(K^+ \rightarrow \pi^+ \nu\bar{\nu}) / \Delta\text{Br}(K_L \rightarrow \pi^0 \nu\bar{\nu}) \simeq 2.2$. Misaligning the down-type sector will break the correlations between $b \rightarrow s\nu\bar{\nu}$ and the $b \rightarrow d\nu\bar{\nu}$, $s \rightarrow d\nu\bar{\nu}$ transitions.

8 Non-leptonic decays

As another application, we consider the puzzle of the difference $\Delta A_{\text{CP}} = (12.6 \pm 2.2)\%$ [40] in the direct CP asymmetries $A_{\text{CP}}(B^- \rightarrow \pi^0 K^-)$ and $A_{\text{CP}}(B^0 \rightarrow \pi^+ K^-)$. In the SM, theoretical expectations for ΔA_{CP} are typically no more than a few percent (e.g. [15] quotes $(\Delta A_{\text{CP}})_{\text{SM}} = (0.7 \pm 2.9)\%$). Within the 3-3-1 model under consideration, the non-leptonic B -meson decays receive corrections from the effective Hamiltonian

$$\mathcal{H}_{\text{eff}} = -\frac{4G_F}{\sqrt{2}} V_{ts}^* V_{tb} \sum_{i=3,5,7,9} C_i Q_i + \text{h.c.}, \quad (8.1)$$

which contains the usual QCD and electroweak penguin operators²

$$\begin{aligned} Q_3 &= (\bar{s}_L \gamma_\alpha b_L) \sum_{q=u,d,s,c,b} (\bar{q}_L \gamma^\alpha q_L), & Q_5 &= (\bar{s}_L \gamma_\alpha b_L) \sum_{q=u,d,s,c,b} (\bar{q}_R \gamma^\alpha q_R), \\ Q_7 &= \frac{3}{2} (\bar{s}_L \gamma_\alpha b_L) \sum_{q=u,d,s,c,b} e_q (\bar{q}_R \gamma^\alpha q_R), & Q_9 &= \frac{3}{2} (\bar{s}_L \gamma_\alpha b_L) \sum_{q=u,d,s,c,b} e_q (\bar{q}_L \gamma^\alpha q_L), \end{aligned} \quad (8.2)$$

with e_q denoting the electric charge of the quark q . Tree-level Z' -boson exchange leads to the following matching corrections

$$\Delta C_3 = \frac{1}{3(1 - 4s_W^2)} \frac{M_W^2}{M_{Z'}^2}, \quad \Delta C_7 = \frac{4s_W^2}{3(1 - 4s_W^2)} \frac{M_W^2}{M_{Z'}^2}, \quad (8.3)$$

$\Delta C_5 = 0$ and $\Delta C_9 = 0$. Employing the results of [15], we find

$$\Delta A_{\text{CP}} - (\Delta A_{\text{CP}})_{\text{SM}} \simeq 3.3 \left[1 + 0.07 \ln \left(\frac{M_{Z'}}{5 \text{ TeV}} \right) \right] \Delta C_7 \simeq 13 \frac{M_W^2}{M_{Z'}^2}, \quad (8.4)$$

²As the couplings $(\mathcal{G}_{d_L})_{ii}$ in (3.2) are flavour non-universal also the operator $Q_1^b = (\bar{s}_L \gamma_\alpha b_L)(\bar{b}_L \gamma^\alpha b_L)$ [11] will get a non-zero initial condition. We neglect this effect since its impact on the processes under consideration is very small.

where the tiny effects due to ΔC_3 have been neglected. The logarithmic dependence on the Z' -boson mass arises from renormalisation group running between the scales $M_{Z'}$ and M_W . For $M_{Z'} = \mathcal{O}(5 \text{ TeV})$ one arrives at $\Delta A_{\text{CP}} - (\Delta A_{\text{CP}})_{\text{SM}} = \mathcal{O}(0.3\%)$, i.e. the Z' -boson corrections in ΔA_{CP} are clearly insufficient to account for the $\mathcal{O}(10\%)$ discrepancy in $B \rightarrow \pi K$.

The parameter ϵ'/ϵ that measures direct CP violation in $K \rightarrow \pi\pi$ is also a very sensitive probe of modifications in the electroweak penguin sector. Assuming complete up-type alignment in our 3-3-1 model, we have to excellent approximation (see e.g. [15])

$$\Delta_{\epsilon'/\epsilon} = \frac{\epsilon'/\epsilon}{(\epsilon'/\epsilon)_{\text{SM}}} - 1 \simeq -2479 \left[1 + 0.1 \ln \left(\frac{M_{Z'}}{5 \text{ TeV}} \right) \right] \Delta C_7 \simeq -9504 \frac{M_W^2}{M_{Z'}^2}. \quad (8.5)$$

Here we have employed the vacuum-insertion approximation $B_{6,8} = 1$ for the hadronic parameters and in the final step neglected the weak logarithmic dependence. We see that even for $M_{Z'} = \mathcal{O}(5 \text{ TeV})$ the numerical shifts $\Delta_{\epsilon'/\epsilon}$ reach (or exceed) the level of -200% . Given the expected progress in the lattice calculations of ϵ'/ϵ such large effects should become clearly visible in the near future. Since the effects in $b \rightarrow sq\bar{q}$ and $s \rightarrow dq\bar{q}$ can be decoupled by down-type misalignment, an order of magnitude suppression of ϵ'/ϵ is however clearly not a model-independent prediction in the new-physics scenario under study.

9 Precision measurements

Until now we have only considered the impact of Z' -boson exchange in quark-flavour physics. Given that the flavour-diagonal Z' -boson couplings to both quarks and leptons are large and non-universal (cf. (3.2)), current precision measurements already impose lower limits on $M_{Z'}$. In the following we will neglect effects due to Z - Z' mixing that are known to be very tightly bounded (see e.g. [41]).

We first discuss the modifications $\Delta g_{L\ell}$, $\Delta g_{R\ell}$ of the couplings $g_{L\ell}$, $g_{R\ell}$ of the Z boson to charged leptons. In our 3-3-1 model this is a one-loop effect that in the limit of $M_{Z'}^2 \gg M_W^2$ gives rise to a shift [20]

$$\frac{\Delta g_{L\ell}}{g_{L\ell}} \simeq 0.01 \frac{M_W^2}{M_{Z'}^2} \left[1 - 0.6 \ln \left(\frac{M_W^2}{M_{Z'}^2} \right) \right], \quad (9.1)$$

in the left-handed coupling of the charged leptons to the Z boson. The experimental constraint $|\Delta g_{L\ell}|/|g_{L\ell}| \lesssim 1\%$ [42] is hence safely fulfilled for $M_{Z'} > 0.5 \text{ TeV}$. The constraints that arise from the right-handed charged-lepton couplings as well as the total width Γ_Z of the Z boson and the hadronic pole cross-section σ_{had} are similar in strength.

Bounds from atomic parity violation and polarised electron-electron Møller scattering asymmetries are also known to provide powerful constraints on heavy Z' bosons if they violate parity (which is the case in our 3-3-1 model). For example, the effect due to a Z' boson on the weak charge of a nucleus of Z protons and N neutrons can be written as [43]

$$\Delta Q_W(Z, N) = \frac{\sqrt{2}}{2G_F} \frac{(\mathcal{G}_{\ell_A})_{11} \mathcal{G}_{q_V}(Z, N)}{M_{Z'}^2} 3(Z + N), \quad (9.2)$$

Figure 2. Examples of one-loop box corrections to muon (left) and down-quark (right) decays involving W and Z' bosons. In the case of V_{us} (V_{ub}), the d quark in the right graph has to be replaced by a s (b) quark.

where

$$\mathcal{G}_{qV}(Z, N) = \frac{(2Z + N)(\mathcal{G}_{uV})_{11} + (Z + 2N)(\mathcal{G}_{dV})_{11}}{3(Z + N)}. \quad (9.3)$$

Numerically, one finds in the case of Cesium ($^{133}_{55}\text{Cs}$)

$$\Delta Q_W^{\text{Cs}} = \Delta Q_W(55, 78) \simeq 206 \frac{M_W^2}{M_{Z'}^2}, \quad (9.4)$$

which combined with the 90% CL limit $|\Delta Q_W^{\text{Cs}}| < 0.6$ [44] implies $M_{Z'} > 1.5 \text{ TeV}$. Notice that this is a non-trivial bound, although the new-physics effect ΔQ_W^{Cs} is suppressed in our model by the small axial-vector coupling of the electron, $(\mathcal{G}_{\ell_A})_{11} \simeq -0.06$.

Similarly to (9.2), shifts from the expected electron-electron Møller scattering asymmetry can effectively be encoded in the weak charge of the electron

$$\Delta Q_W^e = \frac{\sqrt{2}}{2G_F} \frac{(\mathcal{G}_{\ell_A})_{11}(\mathcal{G}_{\ell_V})_{11}}{M_{Z'}^2} \simeq -1.2 \frac{M_W^2}{M_{Z'}^2}, \quad (9.5)$$

where the final result corresponds to our 3-3-1 model with $\beta = -\sqrt{3}$. The 90% CL limit $|\Delta Q_W^e| < 0.016$ [45] is satisfied for $M_{Z'} > 0.7 \text{ TeV}$, meaning that at present the weak charge of the electron does not pose a very strong bound on the considered scenario.

Constraints on additional neutral gauge bosons also derive from violations of first row CKM unitarity as parametrised by

$$\Delta_{\text{CKM}} = 1 - \sum_{q=d,s,b} |V_{uq}|^2. \quad (9.6)$$

The coefficient Δ_{CKM} is determined from the difference of the one-loop Z' -boson corrections to quark β -decay amplitudes from which the CKM elements are extracted as well as muon decay which normalises those amplitudes. The corresponding Feynman diagrams are depicted in figure 2. Generalising the classic results of [46], we find the following expression³

$$\Delta_{\text{CKM}} \simeq -\frac{3}{4\pi^2} \frac{M_W^2}{M_{Z'}^2} \ln \left(\frac{M_W^2}{M_{Z'}^2} \right) (\mathcal{G}_{\ell_L})_{11} \left[(\mathcal{G}_{\ell_L})_{11} - \frac{(\mathcal{G}_{d_L})_{11} + (\mathcal{G}_{u_L})_{11}}{2} \right]. \quad (9.7)$$

³Only the logarithmically enhanced terms of the one-loop box corrections to muon and quark decay have been included here. Furthermore, subleading effects involving CKM elements in (3.2) have been neglected.

Figure 3. Upper limits as a function of Z' -boson mass on the production ratio R of cross section times branching fraction into muon pairs. The green and yellow band corresponds to the 68% and 95% CLs for the expected limits obtained in [50]. The prediction in our 3-3-1 model is indicated by the dotted red line.

Since the $SU(2)_L$ relation $(\mathcal{G}_{d_L})_{11} = (\mathcal{G}_{u_L})_{11}$ is broken by small mixing angles only, our expression for Δ_{CKM} agrees with the result presented recently in [10]. Inserting the couplings (3.2) into the above formula gives

$$\Delta_{\text{CKM}} \simeq -0.16 \frac{M_W^2}{M_{Z'}^2} \ln \left(\frac{M_W^2}{M_{Z'}^2} \right). \quad (9.8)$$

Employing the experimental bound $|\Delta_{\text{CKM}}| < 1\%$ [47] it then follows that

$$M_{Z'} > 2.7 \text{ TeV}, \quad (9.9)$$

at 90% CL. Notice that while the individual constraints from (9.2), (9.5) and (9.7) can be evaded by suitable choices of the lepton and quark couplings, dodging all bounds at once is not possible. This shows the complementarity of present and upcoming $\Delta Q_W^{\text{Cs},p,e}$ and Δ_{CKM} precision measurements (see section 11) in extracting information on Z' -boson models.

We add finally that the presence of a new neutral gauge boson with close to vector-like couplings to muons leads to a positive shift in the muon anomalous magnetic moment (see e.g. [48]). In the 3-3-1 scenario with $\beta = -\sqrt{3}$, one finds $\Delta a_\mu \simeq 2.7 \cdot 10^{-9} (0.2 \text{ TeV}/M_{Z'})^2$. Clearly, for $M_{Z'}$ values in the TeV range no improvement of the infamous tension between the experimental result and the SM prediction for a_μ can be achieved.

10 Direct searches

In order to determine the best direct search strategy for a Z' boson, one has to consider the partial Z' -boson decay widths. From (3.2) we find that the ratio of charged leptonic

Figure 4. Expected event yield in the invariant mass distribution of muon pairs from DY production at the LHC with $\sqrt{s} = 14$ TeV, assuming 0.3 ab^{-1} and 3 ab^{-1} of integrated luminosity. The SM expectation and deviations arising from the specific 3-3-1 model are plotted for varying Z' -boson mass $M_{Z'}$ as indicated in the legend. See text for further explanations.

to hadronic decays is given in the considered 3-3-1 model by

$$\frac{\Gamma(Z' \rightarrow \ell^+ \ell^-)}{\Gamma(Z' \rightarrow \text{hadrons})} \simeq 0.52, \quad (10.1)$$

which is a factor of around 3 larger than what would be obtained if the Z' boson would couple with universal strength to all fermion chiralities. This implies that the most promising channel for an LHC discovery is provided by Drell-Yan (DY) Z' -boson production followed by same-flavour di-lepton decays. Given the significantly larger QCD backgrounds, searches for di-jet resonances and the measurements of the di-jet angular distributions have less potential and thus will not be examined in what follows.

The latest resonance searches in the di-lepton invariant mass spectrum by ATLAS [49] and CMS [50] both include approximately 20 fb^{-1} of $\sqrt{s} = 8 \text{ TeV}$ data. As illustrated in figure 3, the most recent CMS search in the di-muon channel allows to derive a 95% CL bound of

$$M_{Z'} > 3.9 \text{ TeV}. \quad (10.2)$$

In order to derive this number we have implemented the 3-3-1 model into MadGraph5 [51] using CTEQ6l1 parton distribution functions [52] for the event generation while imposing the relevant CMS cuts. We then extrapolated the expected CMS limit on $R = \sigma(pp \rightarrow Z' + X \rightarrow \mu^+ \mu^- + X) / \sigma(pp \rightarrow Z + X \rightarrow \mu^+ \mu^- + X)$ linearly beyond 3.5 TeV to set the bound. Slightly weaker exclusions are obtained from the di-electron channel of CMS as well as the ATLAS searches.

The above limit makes clear that direct Z' -boson searches can at present not probe the mass range of $\mathcal{O}(7 \text{ TeV})$ favoured by the $B \rightarrow K^* \mu^+ \mu^-$ anomaly. To estimate the future

Figure 5. Predictions for the $e^+e^- \rightarrow \mu^+\mu^-$ cross section σ and the forward-backward asymmetry A_{FB} in the SM (blue) and assuming a Z' boson of mass 7 TeV (green) and 9 TeV (red). The light (dark) coloured regions represent 68% (95%) CL contours.

LHC prospects we study the expected kinematic and statistical reach in $pp \rightarrow Z' \rightarrow \mu^+\mu^-$ at $\sqrt{s} = 14$ TeV, assuming integrated luminosities of 0.3 ab^{-1} and 3 ab^{-1} . The muons are required to be within $|\eta| < 2.1$ and that at least one of the muons has $p_T > 45$ GeV. Our results are shown in figure 4. The dotted red line corresponds to the expected event yield at 0.3 ab^{-1} for a Z' -boson with mass of 5 TeV, while the blue hatched band indicates the prediction for $M_{Z'} = 7$ TeV, including statistical errors, assuming the same amount of integrated luminosity. We see that a Z' boson of 5 TeV should be clearly visible with 0.3 ab^{-1} of $\sqrt{s} = 14$ TeV data. However, in the case of $M_{Z'} = 7$ TeV, we expect only a small excess of events in the tail beyond $m(\mu^+\mu^-) > 3$ TeV where the statistics are low. To conclusively probe Z' -boson masses up to 7 TeV in the considered 3-3-1 model the full HL-LHC data set of 3 ab^{-1} will be necessary — illustrated by the green hatched band in the plot. Of course, our results should be taken with a grain of salt, since our calculations are simple minded as they are performed at the leading order and do not include the effects of parton showering and hadronisation nor detector response. Yet, we are optimistic that in the 3-3-1 model with $\beta = -\sqrt{3}$, a discovery of a Z' -boson with mass of $\mathcal{O}(7 \text{ TeV})$, though challenging, should be ultimately possible at the HL-LHC.

Since a Z' boson interferes with the photon and the Z boson, its couplings to fermions can also be probed at a high-luminosity e^+e^- collider. Below we study the mass reach in our 3-3-1 scenario of an ILC with $\sqrt{s} = 500$ GeV CM energy and beam polarisations $(P_{e^+}, P_{e^-}) = (0.3, 0.8)$. For simplicity we consider only the di-muon final state, which we simulate using MadGraph5. Following [53] only muons with a polar angle $\theta \in [10^\circ, 170^\circ]$ are accepted, and we assume a total beam polarisation uncertainty of 0.25% and an intrinsic error of 0.20% (0.06%) associated to the measurement of symmetric (asymmetric) leptonic observables. Our results are displayed in figure 5. We see that in the presence of a Z' boson with $M'_Z = 7$ TeV ($M'_Z = 9$ TeV) the predicted di-muon cross sections σ are smaller than the SM expectation by 9.9% (6.1%). In view of the high-precision measurements possible at an ILC, a reduction of the $e^+e^- \rightarrow \mu^+\mu^-$ cross section by a few percent should be clearly

visible. In the context of the 3-3-1 scenario with $\beta = -\sqrt{3}$ this implies that an 500 GeV e^+e^- machine can probe Z' -boson masses up to $\mathcal{O}(10 \text{ TeV})$. Further information on the Z' -boson couplings to leptons can be obtained by measuring the forward-backward asymmetry A_{FB} . It is evident from the figure that in the model under consideration the predicted deviations in A_{FB} are by a factor of around 3 smaller than those in σ . Numerically, we find that a 7 TeV (9 TeV) Z' boson leads to a shift of +4.0% (+2.4%) in A_{FB} relative to the SM. In contrast to many other commonly studied Z' -boson scenarios (see e.g. [53]) the forward-backward asymmetry is however enhanced in our 3-3-1 model relative to the SM prediction.

11 Discussion

Motivated by the recent LHCb results [1, 2] that show a discrepancy of 3.7σ in one of the angular observables in $B \rightarrow K^*\mu^+\mu^-$, we have studied in this article the Z' -boson contributions to various low- p_T and high- p_T observables that arise in the 3-3-1 model with $\beta = -\sqrt{3}$. We found that in this specific model, assuming a suitable flavour alignment in the down-type quark sector, a Z' boson with mass of $\mathcal{O}(7 \text{ TeV})$ leads to a negative shift in the Wilson coefficient of the semi-leptonic vector operator, which could cause the observed anomaly and also explain other smaller inconsistencies [7].

At present the most stringent constraint on the considered new-physics scenario arises from the mass difference in $B_s - \bar{B}_s$ mixing, which effectively leads to a lower bound of $\Delta C_9^\ell \sim -1.3$ on the modification in the relevant Wilson coefficient. Negative shifts of this size would induce effects of $\mathcal{O}(-20\%)$ in $B \rightarrow X_s \ell^+ \ell^-$ and $\mathcal{O}(-25\%)$ in $B^+ \rightarrow K^+ \mu^+ \mu^-$ without affecting $B_s \rightarrow \mu^+ \mu^-$. Updated analyses of the theoretically clean $B \rightarrow X_s \ell^+ \ell^-$ mode seem hence more important than ever, and we encourage both the BaBar and the Belle collaborations to analyse their full data sets. In the case of the exclusive channel a better theoretical understanding and a careful assessment of form-factor uncertainties and the impact of contributions from charmonium resonances is needed to fully exploit the power of the $B^+ \rightarrow K^+ \mu^+ \mu^-$ constraint. This makes clear that a combined experimental and theoretical effort is indispensable to draw more definitive conclusions about the deviations in $B \rightarrow K^* \mu^+ \mu^-$ as reported by LHCb.

We also showed that the pattern (1.1) of new-physics effects is correlated to enhancements of $\mathcal{O}(20\%)$ in the rates of $B \rightarrow K^{(*)}, X_s \nu \bar{\nu}$ (see also [8, 9]). More than 50 ab^{-1} of SuperKEKB data would however be needed to detect corrections of this size [54]. The considered Z' -boson effects have the correct sign to improve the notorious deviations in the $B \rightarrow \pi K$ sector, but they are numerically far too small to provide an full explanation.

Since the large corrections ΔC_9^ℓ require that the Z' boson couples strongly to both the $\bar{s}b$ and $\bar{\mu}\mu$ currents, possible physics interpretations of the $B \rightarrow K^* \mu^+ \mu^-$ anomaly can also be cross-checked with precision measurements of charged-lepton properties. Here the most stringent constraint arises at present from first row CKM unitarity, which gives a bound of $M_{Z'} > 2.7 \text{ TeV}$. The upgrade of the JLAB polarized electron beam and the further development of the MAMI facility in Mainz will advance the precision frontier for low-energy parity violation. The Qweak experiment will probe the weak charge of

the proton Q_W^p in electron-proton scattering, anticipating a relative experimental error of around 4% [55], while the P2 experiment aims for a relative precision of 2% [56]. Using (9.2) and (9.3) these accuracies translate into a reach on $M_{Z'}$ of 2.1 TeV and 2.9 TeV, respectively. The goal of the proposed MOLLER experiment is to measure the weak charge of the electron Q_W^e to about 2% [57]. From (9.5) we see that this precision would allow to exclude $M_{Z'}$ values up to 2.9 TeV. The next generation of low-energy parity-violation experiments will hence start to become sensitive to the range of Z' -boson masses that can already now be probed by first row CKM unitarity.

Given the leptophilic character of the Z' boson in the 3-3-1 model with $\beta = -\sqrt{3}$, the most recent resonance searches in the di-lepton invariant mass spectrum at LHC already exclude values $M_{Z'} > 3.9$ TeV at 95% CL. This bound is expected to significantly improve at the 14 TeV LHC: while it should be possible with 0.3 ab^{-1} of integrated luminosity to discover Z' bosons with masses up to $\mathcal{O}(5 \text{ TeV})$, a HL-LHC delivering 3 ab^{-1} of data would be needed to see evidence for Z' bosons as heavy as (4.5). An ILC with a CM energy of 500 GeV would be superior in this context and should be able to probe values of $M_{Z'}$ up to $\mathcal{O}(10 \text{ TeV})$ by a precision measurement of the $e^+e^- \rightarrow \mu^+\mu^-$ cross section.

We have seen that achieving a large negative correction ΔC_9^ℓ requires the presence of a Z' boson with a mass in the ballpark of $\mathcal{O}(7 \text{ TeV})$. Such large values of $M_{Z'}$ are potentially problematic in the 3-3-1 scenario with $\beta = -\sqrt{3}$, since the Z' -boson couplings to fermions will develop a Landau pole at high enough energies. In the original model [18, 19] with minimal particle content, the relevant couplings become non-perturbative for energy scales of $\mathcal{O}(5 \text{ TeV})$, but the turn-on of non-perturbative dynamics can be delayed by suitably enlarging the particle content of the 3-3-1 model [58, 59]. To which extend such an UV deformation modifies the physics at low energies is an very interesting question, that is however beyond the scope of the present paper.

Altogether we believe that the 3-3-1 scenario with $\beta = -\sqrt{3}$ studied in our work is a prototype of an explicit new-physics scenario that can successfully address the $B \rightarrow K^*\mu^+\mu^-$ anomaly. The real question is now whether the observed deviations are a manifestation of beyond the SM physics or a simple result of statistical fluctuations combined with too optimistic theory errors. The final verdict is still out!

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