Higher spin currents in the holographic $\mathcal{N} = 1$ coset minimal model

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ABSTRACT: In the $\mathcal{N} = 1$ supersymmetric coset minimal model based on $(B_N^{(1)} \oplus D_N^{(1)}, D_N^{(1)})$ at level $(k, 1)$ studied recently, the standard $\mathcal{N} = 1$ super stress tensor of spins $(\frac{3}{2}, 2)$ is reviewed. By considering the stress tensor in the coset $(B_N^{(1)}, D_N^{(1)})$ at level k , the higher spin- $2'$ Casimir current was obtained previously. By acting the above spin- $\frac{3}{2}$ current on the higher spin- $2'$ Casimir current, its superpartner, the higher spin- $\frac{5}{2}$ current, can be generated and combined as the first higher spin supercurrent with spins $(2', \frac{5}{2})$. By calculating the operator product expansions (OPE) between the higher spin supercurrent and itself, the next higher spin supercurrent can be generated with spins $(\frac{7}{2}, 4)$. Moreover, the other higher spin supercurrent with spins $(4', \frac{9}{2})$ can be generated by calculating the OPE between the first higher spin supercurrent with spins $(2', \frac{5}{2})$ and the second higher spin supercurrent with spins $(\frac{7}{2}, 4)$. Finally, the higher spin supercurrent, $(\frac{11}{2}, 6)$, can be extracted from the right hand side of OPE between the higher spin supercurrents, $(2', \frac{5}{2})$ and $(4', \frac{9}{2})$.

KEYWORDS: AdS-CFT Correspondence, Conformal and W Symmetry

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Contents

1	Introduction	1
2	The GKO coset construction: review	4
3	The construction of higher spin supercurrents	5
3.1	The OPEs between the higher spin currents of spins- $(2', \frac{5}{2})$ and itself	5
3.2	The OPEs between the higher spin currents of spins- $(2', \frac{5}{2})$ and the higher spin currents of spins- $(\frac{7}{2}, 4)$	14
3.3	The OPEs between the higher spin currents of spins- $(2', \frac{5}{2})$ and the higher spin currents of spins- $(4', \frac{9}{2})$	21
4	Conclusions and outlook	37
A	The coefficient functions in (3.30) related to the spin-4 current	38
B	The coefficient functions in (3.38) associated with the spin-$\frac{9}{2}$ current	39
C	The coefficient functions in (3.46) corresponding to the spin-4' current	41
D	The coefficient functions in (3.66) related to the spin-$\frac{11}{2}$ current	43
E	The coefficient functions in (3.75) associated with the spin-6 current	49
F	The $\mathcal{N} = 1$ superspace description	70
G	The OPEs between the stress energy tensor $T(z)$ and the quasi-primary (or primary) fields	73
H	The OPEs between $G(z)$ and the quasi-primary (or primary) fields	75

1 Introduction

One example of the minimal model holography [1–4] is studied in [5]. The gravity theory is the $\mathcal{N} = 1$ truncation of the $\mathcal{N} = 2$ higher spin supergravity on AdS_3 space [6]. The $\mathcal{N} = 1$ truncation of the matter fields provides the $\mathcal{N} = 1$ hypermultiplet with two complex massive scalars and two massive fermions. The 2-dimensional dual conformal field theory is described by the following coset minimal model

$$\frac{G}{H} = \frac{\widehat{SO}(2N+1)_k \oplus \widehat{SO}(2N)_1}{\widehat{SO}(2N)_{k+1}}. \quad (1.1)$$

The diagonal denominator current of spin-1 is the sum of two numerator currents where the fermions of spin- $\frac{1}{2}$ belong to the second numerator factor. The higher spin currents including the $\mathcal{N} = 1$ super stress tensor with spins- $(\frac{3}{2}, 2)$ are given by

$$\left(\frac{3}{2}, 2\right) : \left(2', \frac{5}{2}\right), \left(\frac{7}{2}, 4\right), \left(4', \frac{9}{2}\right), \left(\frac{11}{2}, 6\right), \dots, \left(n - \frac{1}{2}, n\right), \left(n', n' + \frac{1}{2}\right), \dots \quad (1.2)$$

The 't Hooft coupling constant is given by $\lambda = \frac{2N}{(2N+k-1)}$ in the coset model (1.1) and corresponds to the mass parameter in the AdS_3 bulk theory. Recently, in [7], the $\mathcal{N} = 1$ $\mathcal{W}_\infty[\mu]$ algebra with $\mu = (1 - 2N)$ by taking the field contents in (1.2) is found.

In this paper, we would like to construct the higher spin currents for the coset model (1.1). For example, in order to understand this duality, the three-point functions can be compared to each other. Once the higher spin currents with bosonic spins are known completely, then in principle, the three point functions can be obtained.¹ The direct construction using the Jacobi identities in [7] doesn't tell us what the central charge is. Only after the isomorphism between the Drinfeld-Sokolov reduction and the coset construction is used, the central charge in the direct construction [7] can be identified with the one in the above coset model (1.1) where the central charge c is equal to $c = \frac{3Nk}{(2N+k-1)}$. Furthermore, if one considers more general coset where the second level 1 is replaced by an arbitrary integer l , then the extra current will appear in general. The above isomorphism cannot be used in this case also. This is one of the reasons why we are interested in the construction of higher spin currents for the coset model explicitly.

The main procedure in this coset construction is based on the fact that once the lower higher spin currents are found, then the next undetermined higher spin currents can be generated, in principle, by calculating the operator product expansions (OPE) between the known higher spin currents. The lowest component spin- $2'$ in the $\mathcal{N} = 1$ multiplet $(2', \frac{5}{2})$ of (1.2) with known realization of $\mathcal{N} = 1$ stress tensor is the fundamental higher spin current because this current generates all the higher spin currents. It turns out that once the spin- $2'$ current in the multiplet $(2', \frac{5}{2})$ is found, then the spin- $\frac{3}{2}$ current in the $\mathcal{N} = 1$ super stress tensor determines the spin- $\frac{5}{2}$ in the above multiplet. Then one considers the OPE between the above spin- $2'$ current and the spin- $\frac{5}{2}$ current. The first-order pole of this OPE determines the spin- $\frac{7}{2}$ current in the $\mathcal{N} = 1$ multiplet $(\frac{7}{2}, 4)$ of (1.2). Now one can continue to

¹We will determine the complete expressions for the bosonic higher spin currents of spins $s = 2', 4, 4'$ and 6. Then one can proceed the previous analysis done in different coset model [8]. The zero mode for each bosonic current acts on the vector representation. The zero mode of the spin-1 current with level k in the numerator of the coset (1.1) acting on the state $|0; v\rangle$ vanishes while the zero mode of the diagonal spin-1 current in the denominator of the coset acting on the state $|v; 0\rangle$ vanishes. For the former, the zero mode for higher spin current consists of multiple product of quadratic in the fermions of the numerator current (i.e. $K^a K^b(z)$ in section 2) and for the latter, the zero mode can be written in terms of the numerator current with level k (i.e. $J^{AB}(z)$ in section 2) (or the combination of fermions in the numerator current (i.e. $K^a K^b(z)$) and the numerator current with level k where one index is fixed by $(2N+1)$ (i.e. $J^{a2N+1}(z)$ in section 2)). Some identities in the trace of the generators of $SO(2N)$ or $SO(2N+1)$ can be used in the eigenvalue equations. One should take into account the fact that the first factor of the numerator group of the coset has different from other factor groups in the coset and there exist spin- $\frac{1}{2}$ current in the second factor of the numerator group, compared to the previous results in [8]. It would be interesting to obtain the three point functions from the findings of this paper explicitly.

calculate the OPE between the spin-2' current and the above spin- $\frac{7}{2}$ current and it turns out that the spin- $\frac{9}{2}$ current that is the second component in $(4', \frac{9}{2})$ appears in the first-order pole of this OPE. Furthermore, the OPE between the spin-2' and the spin- $\frac{9}{2}$ current determines the spin- $\frac{11}{2}$ current that is the first component in $(\frac{11}{2}, 6)$. In this way, all the half-integer spin currents can be obtained. What about the bosonic higher spin currents? They can be determined by calculating the OPE between the spin- $\frac{3}{2}$ current of $\mathcal{N} = 1$ stress tensor and any known higher spin current of half-integer spin due to the $\mathcal{N} = 1$ supersymmetry.

Then how one can extract the correct primary or quasi-primary fields in the given singular terms in the OPE? It is known that the OPE of two quasi-primary fields, of spins h_i and h_j respectively, takes the form [9–13]

$$\begin{aligned} \Phi_i(z) \Phi_j(w) = & \frac{1}{(z-w)^{h_i+h_j}} \gamma_{ij} \\ & + \sum_k C_{ijk} \sum_{n=0}^{\infty} \frac{1}{(z-w)^{h_i+h_j-h_k-n}} \left[\frac{1}{n!} \prod_{x=0}^{n-1} \frac{(h_i-h_j+h_k+x)}{(2h_k+x)} \right] \partial^n \Phi_k(w). \end{aligned} \quad (1.3)$$

γ_{ij} corresponds to a metric on the space of quasi-primary fields. The structure constant C_{ijk} appears in the three-point function between the quasi-primary fields, $\Phi_i(z), \Phi_j(z)$ and $\Phi_k(z)$. The index k specifies all the quasi-primary fields occurring in the right hand side of (1.3). The descendant fields for the quasi-primary field $\Phi_k(w)$ of spin h_k are multiple derivatives of $\Phi_k(w)$. The relative coefficient functions $\left[\frac{1}{n!} \prod_{x=0}^{n-1} \frac{(h_i-h_j+h_k+x)}{(2h_k+x)} \right]$ in the descendant fields depend on the spins and number of derivatives. Since the higher spin currents can be written in terms of WZW currents, the above structure constant C_{ijk} can be determined. For the fixed C_{ijk} , the relative coefficient functions in the descendant fields using (1.3) can be obtained. In general, the singular terms of the OPE are written in terms of WZW currents in complicated way. It is quite nontrivial to rearrange those expressions in terms of determined (and known) higher spin currents. This rearrangement can be done using the primary or quasi-primary condition under the spin-2 current of $\mathcal{N} = 1$ super stress tensor and superprimary condition under the spin- $\frac{3}{2}$ current of $\mathcal{N} = 1$ stress tensor. The details can be seen in the next section.

In section 2, the fundamental OPEs between the WZW currents living in the coset model (1.1) are given and the spin-2 stress tensor and its superpartner with Sugawara construction are reviewed.

In section 3, from the observation of [5], the lowest higher spin $\mathcal{N} = 1$ multiplet $(2, \frac{5}{2})$ in (1.2) is obtained. The additional three higher spin supercurrents in the list (1.2) are constructed very explicitly.

In section 4, we summarize what we have found in this paper and discuss the future directions.

In appendices, some results from the detailed calculations in section 3 are provided.

The mathematica package by Thielemans [14] is used.

2 The GKO coset construction: review

For the diagonal coset model [5]

$$\frac{G}{H} = \frac{\widehat{SO}(2N+1)_k \oplus \widehat{SO}(2N)_1}{\widehat{SO}(2N)_{k+1}}, \quad (2.1)$$

the antisymmetric spin-1 fields, $J^{AB}(z)$ with level k , and the spin- $\frac{1}{2}$ fields, $K^a(z)$ with level 1, generate the affine Lie algebra $G = \widehat{SO}(2N+1)_k \oplus \widehat{SO}(2N)_1$. The index a runs from 1 to $2N$ and the index A runs from 1 to $(2N+1)$. The number of independent fields of J^{AB} and $K^a(z)$ are given by $N(2N+1)$ and $2N$ respectively. Note that this coset model (2.1) is a little different from the ones studied in [3, 8, 15, 16]. The fundamental OPE of the fermion fields $K^a(z)$ is given by

$$K^a(z)K^b(w) = \frac{1}{(z-w)} \delta^{ab} + \dots \quad (2.2)$$

The standard OPE of the spin-1 currents is expressed as

$$\begin{aligned} J^{AB}(z)J^{CD}(w) = & -\frac{1}{(z-w)^2} k(-\delta^{BC}\delta^{AD} + \delta^{AC}\delta^{BD}) \\ & + \frac{1}{(z-w)} [\delta^{BC}J^{AD}(w) + \delta^{AD}J^{BC}(w) - \delta^{AC}J^{BD}(w) - \delta^{BD}J^{AC}(w)] + \dots \end{aligned} \quad (2.3)$$

The diagonal spin-1 field $J'^{ab}(z)$ with level $(k+1)$ generates the affine Lie algebra $H = \widehat{SO}(2N)_{k+1}$. It is expressed as

$$J'^{ab}(z) = J^{ab}(z) + (K^a K^b)(z). \quad (2.4)$$

Note that we are using the double index notation for the spin-1 current rather than a single index used in [5]. Among $(2N+1) \times (2N+1)$ matrix, the first $(2N) \times (2N)$ matrix corresponds to the spin-1 current $J^{ab}(z)$ and the remaining matrix elements correspond to other spin-1 current $J^{a2N+1}(z)$. The spin-2 stress energy tensor with (2.4), via the Sugawara construction in the coset model (2.1), is written as

$$\begin{aligned} T(z) = & -\frac{1}{4(k+2N-1)} (J^{AB}J^{AB})(z) - \frac{1}{4(1+2N-2)} ((K^a K^b)(K^a K^b))(z) \\ & + \frac{1}{4(1+k+2N-2)} (J'^{ab}J'^{ab})(z). \end{aligned} \quad (2.5)$$

Since $J^{AB}(z)$ can be decomposed into $J^{a2N+1}(z)$, and $J^{ab}(z)$ which belongs to $SO(2N)$ subgroup, $(J^{AB}J^{AB})(z)$ can be written as

$$(J^{AB}J^{AB})(z) = (J^{ab}J^{ab})(z) + 2(J^{a2N+1}J^{a2N+1})(z). \quad (2.6)$$

Therefore, by plugging (2.6) into the stress tensor (2.5), the spin-2 stress tensor can be expressed concisely as

$$T(z) = \frac{1}{(2k+4N-2)} \left[K^a K^b J^{ab} - k K^a \partial K^a - J^{a2N+1} J^{a2N+1} \right] (z). \quad (2.7)$$

Note that there is no $J^{ab}J^{ab}(z)$ in (2.7). One can easily check that there is no singular term in the OPE between $T(z)$ and $J'^{ab}(w)$. The OPE between the stress energy tensor $T(z)$ and itself, from (2.2), (2.3) and (2.7), is given by

$$T(z) T(w) = \frac{1}{(z-w)^4} \frac{c}{2} + \frac{1}{(z-w)^2} 2T(w) + \frac{1}{(z-w)} \partial T(w) + \dots \quad (2.8)$$

The central charge in the highest singular term of (2.8) is given by

$$\begin{aligned} c &= \frac{1}{2}(2N+1)(2N) \frac{k}{(k+2N+1-2)} + \frac{1}{2}(2N)(2N-1) \frac{1}{(1+2N-2)} \\ &\quad - \frac{1}{2}(2N)(2N-1) \frac{(1+k)}{(1+k+2N-2)} = \frac{3Nk}{(k+2N-1)} \leq 3N. \end{aligned} \quad (2.9)$$

The superpartner of the spin-2 current $T(z)$ is the spin- $\frac{3}{2}$ current $G(z)$ [17]. They can be combined into a single $\mathcal{N} = 1$ multiplet as $(\frac{3}{2}, 2)$ in (1.2). The OPE between $G(z)$ and itself reads as

$$G(z) G(w) = \frac{1}{(z-w)^3} \frac{2}{3} c + \frac{1}{(z-w)} 2T(w) + \dots \quad (2.10)$$

One can construct $G(z)$ from the WZW currents $K^a(z)$ of spin- $\frac{1}{2}$ and $J^{AB}(z)$ of spin-1. Since $K^a(z)$ has one index, $(K^a J^{a2N+1})(z)$ is the only candidate for $G(z)$. The normalization for $G(z)$ can be fixed from (2.10) and the explicit form of $G(z)$ is given by

$$G(z) = \frac{i}{\sqrt{k+2N-1}} (K^a J^{a2N+1})(z). \quad (2.11)$$

One checks that the OPE between $G(z)$ and the diagonal spin-1 current $J'^{ab}(w)$ does not contain any singular terms. As we expect, it satisfies the following OPE:

$$T(z) G(w) = \frac{1}{(z-w)^2} \frac{3}{2} G(w) + \frac{1}{(z-w)} \partial G(w) + \dots \quad (2.12)$$

The standard $\mathcal{N} = 1$ superconformal algebra consists of (2.8), (2.10) and (2.12). In the next section, the fundamental OPEs (2.2) and (2.3) are used heavily and the coset stress tensor (2.7) and its superpartner (2.11) with central charge (2.9) will be used all the times.

3 The construction of higher spin supercurrents

In this section, the higher spin currents will be constructed for general N explicitly from the fermion fields $K^a(z)$ of spin- $\frac{1}{2}$ and the antisymmetric spin-1 currents $J^{AB}(z)$.

3.1 The OPEs between the higher spin currents of spins- $(2', \frac{5}{2})$ and itself

• **The OPE $O_2(z) O_2(w)$.** Now, let us consider the stress energy tensor of the coset $\frac{\widehat{SO}(2N+1)_k}{\widehat{SO}(2N)_k}$ [5], which is

$$\tilde{T}(z) = -\frac{1}{4(k+2N-1)} (J^{AB} J^{AB})(z) + \frac{1}{4(k+2N-2)} (J^{ab} J^{ab})(z). \quad (3.1)$$

Note that the denominator current is simply given by $J^{ab}(z)$. This stress tensor $\tilde{T}(z)$ obeys the following OPEs from (3.1) and (2.7):

$$\tilde{T}(z) \tilde{T}(w) = \frac{1}{(z-w)^4} \frac{\tilde{c}}{2} + \frac{1}{(z-w)^2} 2\tilde{T}(w) + \frac{1}{(z-w)} \partial\tilde{T}(w) + \dots, \quad (3.2)$$

$$T(z) \tilde{T}(w) = \frac{1}{(z-w)^4} \frac{\tilde{c}}{2} + \frac{1}{(z-w)^2} 2\tilde{T}(w) + \frac{1}{(z-w)} \partial\tilde{T}(w) + \dots, \quad (3.3)$$

where $\tilde{c}$ is the central charge of coset $\frac{\widehat{SO}(2N+1)_k}{\widehat{SO}(2N)_k}$ and can be expressed as

$$\tilde{c} = \frac{kN(-3+2k+2N)}{(-2+k+2N)(-1+k+2N)}. \quad (3.4)$$

The stress tensor $\tilde{T}(z)$ is a quasi-primary field under the stress tensor (2.7). From (2.8) and (3.3), one can easily figure out that the following combination of $T(z)$ and $\tilde{T}(z)$ with (2.9) and (3.4) gives a spin-2' primary field $O_{2'}(z)$ under the stress tensor $T(z)$:

$$O_{2'}(z) = c\tilde{T}(z) - \tilde{c}T(z). \quad (3.5)$$

This is because the OPE between $T(z)$ and $O_{2'}(w)$ does not contain the fourth-order pole. In terms of $J^{AB}(z)$ and $K^a(z)$, the spin-2 current (3.5) is expressed as²

$$O_2(z) = \frac{kN}{4(-2+k+2N)(-1+k+2N)^2} \left[3J^{ab}J^{ab} + 2k(-3+2k+2N)K^a\partial K^a + 2(3-2k-2N)K^aK^bJ^{ab} - 2(-3+k+4N)J^{a2N+1}J^{a2N+1} \right] (z). \quad (3.6)$$

Compared to (2.7), the first term of (3.6) appears newly.

The OPE between $G(z)$ and $O_2(w)$ generates a primary spin- $\frac{5}{2}$ current $O_{\frac{5}{2}}(z)$ under the stress tensor $T(z)$ as follows:

$$G(z) O_2(w) = \frac{1}{(z-w)} O_{\frac{5}{2}}(w) + \dots. \quad (3.7)$$

The explicit form of spin- $\frac{5}{2}$ current $O_{\frac{5}{2}}(z)$ is given by

$$O_{\frac{5}{2}}(z) = \frac{ikN}{2(-2+k+2N)(-1+k+2N)\sqrt{k+2N-1}} \left[2(-3+2k+2N)\partial K^a J^{a2N+1} - (-3+2k+2N)K^a\partial J^{a2N+1} + 3K^a(J^{ab}J^{b2N+1} + J^{b2N+1}J^{ab}) \right] (z). \quad (3.8)$$

The OPE between $G(z)$ and $O_{\frac{5}{2}}(w)$ with (2.11) and (3.8) is described as

$$G(z) O_{\frac{5}{2}}(w) = \frac{1}{(z-w)^2} 4O_2(w) + \frac{1}{(z-w)} \partial O_2(w) + \dots. \quad (3.9)$$

From (3.7) and (3.9), one can clearly see that the currents $O_{\frac{5}{2}}(z)$ and $O_2(z)$ are superpartners each other.

²We denote $O_{2'}(z)$ as $O_2(z)$. This is O.K. because the other spin-2 current is denoted by $T(z)$.

Now let us consider the OPE between $O_2(z)$ and itself. From (2.8), (3.2), and (3.3), one can show that the OPE between $O_2(z)$ and itself is given by

$$O_2(z) O_2(w) = \frac{c_{22}}{(z-w)^4} + \frac{1}{(z-w)^2} [c_{22}^o O_2 + c_{22}^t T](w) + \frac{1}{(z-w)} \left[\frac{1}{2} c_{22}^o \partial O_2 + \frac{1}{2} c_{22}^t \partial T \right](w) \cdots \quad (3.10)$$

The relative coefficient $\frac{1}{2}(= \frac{2-2+2}{2 \times 2})$ in the descendant fields can be understood from the general formula in (1.3). There are no new primary fields in the right hand side of (3.10). The structure constants in (3.10) are

$$c_{22} = -\frac{c^3(2c(-1+N) + 3(3-4N)N)(c + 2cN + 3N(-3+2N))}{18(c + 6(-1+N)N)^2},$$

$$c_{22}^o = \frac{2c(c - 4cN + 6N^2)}{3(c + 6(-1+N)N)},$$

$$c_{22}^t = -\frac{2c^2(2c(-1+N) + 3(3-4N)N)(c + 2cN + 3N(-3+2N))}{9(c + 6(-1+N)N)^2}.$$

The fusion rule can be summarized by $[O_2][O_2] = [I] + [O_2]$.

• **The OPE $O_2(z) O_{\frac{5}{2}}(w)$.** Now let us move to the OPE between $O_2(z)$ and $O_{\frac{5}{2}}(w)$.³ We follow the method used in [18] to find the complete structure of the OPE. The OPE between $O_2(z)$ and $O_{\frac{5}{2}}(w)$ for $N = 2$ shows that

$$O_2(z) O_{\frac{5}{2}}(w) = \frac{1}{(z-w)^3} c_{2\frac{5}{2}}^g G(w) + \frac{1}{(z-w)^2} \left[\frac{1}{3} c_{2\frac{5}{2}}^g \partial G + c_{2\frac{5}{2}}^o O_{\frac{5}{2}} \right](w) + \frac{1}{(z-w)} \left[\frac{1}{12} c_{2\frac{5}{2}}^g \partial^2 G + \frac{2}{5} c_{2\frac{5}{2}}^o \partial O_{\frac{5}{2}} + c_{2\frac{5}{2}}^{go} \left(G O_2 - \frac{2}{5} \partial O_{\frac{5}{2}} \right) + c_{2\frac{5}{2}}^{tg} \left(T G - \frac{3}{8} \partial^2 G \right) + O_{\frac{7}{2}} \right](w) + \cdots, \quad (3.11)$$

where the structure constants for fixed $N = 2$ are given by

$$c_{2\frac{5}{2}}^g(N=2) = -\frac{12k^2(5+k)(1+2k)}{(2+k)^2(3+k)^2}, \quad c_{2\frac{5}{2}}^o(N=2) = -\frac{2(-4+k)k}{(2+k)(3+k)},$$

$$c_{2\frac{5}{2}}^{go}(N=2) = \frac{6(-4+k)k}{(2+k)(1+2k)}, \quad c_{2\frac{5}{2}}^{tg}(N=2) = -\frac{24k^2(5+k)(1+2k)}{(2+k)^2(3+k)(7+5k)}. \quad (3.12)$$

³From now on, when we compute the OPEs, we compute them for $N = 2$ case in order to determine the algebraic structures of the OPEs firstly. Then, we will compute them for general N . The reason is as follows. The $SO(2N+1)$ or $SO(2N)$ invariant tensor of rank 2, Kronecker delta, appears in the fundamental OPEs of (2.2) and (2.3). In any OPE between any two higher spin currents, the group index structure in the right hand side of this OPE appears in the fields using the Kronecker delta's. That is, the coefficients in front of the fields do not have tensorial structures as one varies the N . Therefore, it is possible to extract the N -dependence of these coefficients from those with several fixed N . This feature is different from the $SU(N)$ coset model where there are tensorial structures between f -symbol and d -symbol as well as Kronecker delta.

Moreover, the summation indices (appearing in $O_2(z), O_{\frac{5}{2}}(z)$ and $G(w)$) $a, b = 1, \dots, 4$ because $N = 2$. For the primary field $G(w)$ with the structure constant $c_{2\frac{5}{2}}^g(N = 2)$ with (3.12) in the right hand side, the relative coefficients, $\frac{1}{3}$ and $\frac{1}{12}$, for its descendant fields appearing in the second- and first-order pole can be read off from (1.3). Refer to [18] for details on how to compute the relative coefficients of descendant fields. For the second-order pole in (3.11), the first term descending from $G(w)$ is fixed. To check if there are other fields besides $\partial G(w)$ in the second-order pole, we compute $[\{O_2 O_{\frac{5}{2}}\}_{-2} - (\frac{1}{3})c_{2\frac{5}{2}}^g(N = 2)\partial G](w)$.⁴ It turns out that this doesn't vanish implying that there should be extra fields besides $\partial G(w)$. The spin of fields in the second-order pole should be $\frac{5}{2}$ and the only candidate for this extra field is the primary field $O_{\frac{5}{2}}(w)$ in (1.2) with $c_{2\frac{5}{2}}^o(N = 2)$. Therefore, the extra field is proportional to $O_{\frac{5}{2}}(w)$.

For the first-order pole, the first- and second-term descending from $G(w)$ and $O_{\frac{5}{2}}(w)$ respectively are completely fixed. The relative coefficient $\frac{2}{5}$ in front of $\partial O_{\frac{5}{2}}(w)$ in the first-order pole is also determined from (1.3). As we did for the second-order pole, to find if there are other fields besides $\partial^2 G(w)$ and $\partial O_{\frac{5}{2}}(w)$, we compute $[\{O_2 O_{\frac{5}{2}}\}_{-1} - \text{first line}](w)$ where the first line contains two terms in (3.11). It doesn't vanish and there should be extra fields. To find the extra fields in the first-order pole, let us consider the OPE between $T(z)$ and $[\{O_2 O_{\frac{5}{2}}\}_{-1} - \text{first line}](w)$ and the OPE between $G(z)$ and $[\{O_2 O_{\frac{5}{2}}\}_{-1} - \text{first line}](w)$. The OPE $T(z)$ with $[\{O_2 O_{\frac{5}{2}}\}_{-1} - \text{first line}](w)$ is given by $\frac{1}{(z-w)^4} \left[\frac{(-27k^2(5+k)(1+2k))}{(2+k)^2(3+k)^2} \right] G(w) + \mathcal{O}((z-w)^{-2})$. This means that a quasi-primary field, $TG(w)$ plus derivative terms, should be considered to cancel the fourth-order term of this OPE for the primary condition. Moreover, the OPE between $G(z)$ and $[\{O_2 O_{\frac{5}{2}}\}_{-1} - \text{first line}](w)$ leads to $\frac{1}{(z-w)^3} \left[-\frac{36k^2(5+k)(1+2k)}{(2+k)^2(3+k)^2} T + \frac{72k(-4+k)}{5(2+k)(3+k)} O_2 \right](w) + \mathcal{O}((z-w)^{-2})$. This means that we should consider another quasi-primary field, $GO_2(w)$ plus derivative terms, to remove the third-order terms of this OPE. With the help of superprimary condition (primary under the stress tensor and the equation (3.14)), the consistent coefficients, $c_{2\frac{5}{2}}^{go}(N = 2)$ and $c_{2\frac{5}{2}}^{tg}(N = 2)$, in the second line of the first-order pole in (3.11) are determined.

Then we subtract the first and second line in the first-order pole from $\{O_2 O_{\frac{5}{2}}\}_{-1}(w)$. We are left with a new primary spin- $\frac{7}{2}$ current $O_{\frac{7}{2}}(z)$ in the first-order pole. The explicit form of the $O_{\frac{7}{2}}(z)$ can be expressed as

$$O_{\frac{7}{2}}(w) = \{O_2 O_{\frac{5}{2}}\}_{-1}(w) - \left[\frac{1}{12} c_{2\frac{5}{2}}^g \partial^2 G + \frac{2}{5} c_{2\frac{5}{2}}^o \partial O_{\frac{5}{2}} + c_{2\frac{5}{2}}^{go} \left(GO_2 - \frac{2}{5} \partial O_{\frac{5}{2}} \right) + c_{2\frac{5}{2}}^{tg} \left(TG - \frac{3}{8} \partial^2 G \right) \right](w). \quad (3.13)$$

As expected, $O_{\frac{7}{2}}(z)$ obeys the following OPE:

$$G(z) O_{\frac{7}{2}}(w) = \frac{1}{(z-w)} O_4(w) + \dots, \quad (3.14)$$

⁴Here one uses the notation [11] for the n -th order pole in the OPE $\Phi_i(z) \Phi_j(w)$ as follows: $\{\Phi_i \Phi_j\}_{-n}(w)$ which is written in terms of WZW currents.

where $O_4(z)$ is a spin-4 primary field in (1.2) which will appear in (3.20) when the OPE between $O_{\frac{5}{2}}(z)$ and itself is computed later.

To determine the structure constants in (3.11) for general N , we compute the OPE between $O_2(z)$ and $O_{\frac{5}{2}}(z)$ by hand explicitly with the help of (3.11). Explicit calculations for the third- and second-order pole show that the coefficients $c_{2\frac{5}{2}}^g$ and $c_{2\frac{5}{2}}^o$ for general N are

$$\begin{aligned} c_{2\frac{5}{2}}^g &= \frac{c^2(2c(-1+N) + 3(3-4N)N)(c + 2cN + 3N(-3+2N))}{3(c + 6(-1+N)N)^2}, \\ c_{2\frac{5}{2}}^o &= \frac{c(c - 4cN + 6N^2)}{3(c + 6(-1+N)N)}. \end{aligned} \quad (3.15)$$

For the first-order pole, we find

$$\begin{aligned} \{O_2 O_{\frac{5}{2}}\}_{-1}(w) &= c_1 K^a J^{ab} J^{cb} J^{cN'}(w) + c_2 K^a J^{ab} \partial J^{bN'}(w) + c_3 K^a \partial J^{ab} J^{bN'}(w) \\ &\quad + c_4 K^a \partial^2 J^{aN'}(w) + c_5 \partial K^a J^{ab} \partial J^{bN'}(w) + c_6 \partial K^a \partial J^{aN'}(w) \\ &\quad + c_7 \partial^2 K^a J^{aN'}(w), \quad N' \equiv 2N + 1, \end{aligned} \quad (3.16)$$

where the coefficients are given by

$$\begin{aligned} c_1 &= -9Dc^2(c - 3N)^2, \quad c_2 = -6Dc^2(c - 3N)(2c(-1+N) + 3(3-4N)N), \\ c_3 &= -3Dc^2(c - 3N)(-6N^2 + c(-1+4N)), \quad c_4 = Dc^2(2c(-1+N) + 3(3-4N)N)^2, \\ c_5 &= -6Dc^2(c - 3N)(c + 2cN + 3N(-3+2N)), \\ c_6 &= 2Dc^2(2c(-1+N) + 3(3-4N)N)(c + 2cN + 3N(-3+2N)), \\ c_7 &= Dc^2(c + 2cN + 3N(-3+2N))^2, \quad D \equiv \frac{-\sqrt{3N-c}}{9(c+6(-1+N)N)^2\sqrt{3(1-2N)N}}. \end{aligned} \quad (3.17)$$

To fix the remaining undetermined coefficients $c_{2\frac{5}{2}}^{go}$ and $c_{2\frac{5}{2}}^{tg}$, we apply $G(z)$ to the both sides of (3.13). From the superprimary condition (3.14), we find that

$$\begin{aligned} \{G \{O_2 O_{\frac{5}{2}}\}_{-1}\}_{-3}(w) &= \left\{ \frac{1}{12} c_{2\frac{5}{2}}^g G \partial^2 G + \frac{2}{5} c_{2\frac{5}{2}}^o G \partial O_{\frac{5}{2}} + c_{2\frac{5}{2}}^{go} G \left(GO_2 - \frac{2}{5} \partial O_{\frac{5}{2}} \right) \right. \\ &\quad \left. + c_{2\frac{5}{2}}^{tg} G \left(TG - \frac{3}{8} \partial^2 G \right) \right\}_{-3}(w). \end{aligned} \quad (3.18)$$

The first two coefficients in the right hand side of (3.18) are known from (3.15). From the equation (3.18), we find that

$$\begin{aligned} c_{2\frac{5}{2}}^{go} &= \frac{18c(-6N^2 + c(-1+4N))}{(6+5c)(c+6(-1+N)N)}, \\ c_{2\frac{5}{2}}^{tg} &= \frac{6c^2(2c(-1+N) + 3(3-4N)N)(c + 2cN + 3N(-3+2N))}{(21+4c)(c+6(-1+N)N)^2}. \end{aligned} \quad (3.19)$$

Therefore, the spin- $\frac{7}{2}$ current is given by (3.13), (3.15), (3.16), (3.17) and (3.19). The OPE $O_2(z) O_{\frac{5}{2}}(w)$ for general N is given by (3.11) with (3.15) and (3.19). The fusion rule can be summarized by $[O_2][O_{\frac{5}{2}}] = [I] + [O_{\frac{5}{2}}] + [O_{\frac{7}{2}}]$.

• **The OPE $O_{\frac{5}{2}}(z) O_{\frac{5}{2}}(w)$.** The computation of OPEs is getting more and more complicated and difficult as we move to the next higher spin currents. From now on, we compute the OPEs by the mathematica package [14] exclusively instead of computing by hand. Our strategy is as follows. First we compute the OPEs between the higher spin currents for $N = 2$ to find the complete structures of the OPEs. Then we continue to compute the OPEs for different $N = 3, 4, 5, \dots$, until we can find the N -dependence of structure constants fully. Our method will be clear as we compute the simplest OPE between $O_{\frac{5}{2}}(z)$ and itself. From the computation of the OPE between $O_{\frac{5}{2}}(z)$ and itself for $N = 2$, we find that

$$\begin{aligned} O_{\frac{5}{2}}(z) O_{\frac{5}{2}}(w) = & \frac{c_{\frac{5}{2}\frac{5}{2}}}{(z-w)^5} + \frac{1}{(z-w)^3} \left(c_{\frac{5}{2}\frac{5}{2}}^o O_2 + c_{\frac{5}{2}\frac{5}{2}}^t T \right) + \frac{1}{(z-w)^2} \left[\frac{1}{2} c_{\frac{5}{2}\frac{5}{2}}^o \partial O_2 + \frac{1}{2} c_{\frac{5}{2}\frac{5}{2}}^t \partial T \right] (w) \\ & + \frac{1}{(z-w)} \left[\frac{3}{20} c_{\frac{5}{2}\frac{5}{2}}^o \partial^2 O_2 + \frac{3}{20} c_{\frac{5}{2}\frac{5}{2}}^t \partial^2 T + c_{\frac{5}{2}\frac{5}{2}}^{tt} \left(TT - \frac{3}{10} \partial^2 T \right) \right. \\ & + c_{\frac{5}{2}\frac{5}{2}}^{to} \left(TO_2 - \frac{3}{10} \partial^2 O_2 \right) + c_{\frac{5}{2}\frac{5}{2}}^{go} \left(GO_{\frac{5}{2}} - \frac{1}{5} \partial^2 O_2 \right) + c_{\frac{5}{2}\frac{5}{2}}^{gg} \left(G\partial G - \frac{7}{10} \partial^2 T \right) \\ & \left. + O_4 \right] (w) + \dots, \end{aligned} \quad (3.20)$$

with the structure constants for fixed $N = 2$ are

$$\begin{aligned} c_{\frac{5}{2}\frac{5}{2}}(N=2) &= -\frac{48k^3(5+k)(1+2k)}{(2+k)^2(3+k)^3}, & c_{\frac{5}{2}\frac{5}{2}}^o(N=2) &= \frac{8(-4+k)k}{(2+k)(3+k)}, \\ c_{\frac{5}{2}\frac{5}{2}}^t(N=2) &= -\frac{40k^2(5+k)(1+2k)}{(2+k)^2(3+k)^2}, & c_{\frac{5}{2}\frac{5}{2}}^{tt}(N=2) &= -\frac{48k^2(5+k)(1+2k)}{(2+k)^2(3+k)(7+5k)}, \\ c_{\frac{5}{2}\frac{5}{2}}^{to}(N=2) &= \frac{12(-4+k)k}{(2+k)(1+2k)}, & c_{\frac{5}{2}\frac{5}{2}}^{go}(N=2) &= -\frac{6(-4+k)k}{(2+k)(1+2k)}, \\ c_{\frac{5}{2}\frac{5}{2}}^{gg}(N=2) &= \frac{12k^2(5+k)(1+2k)}{(2+k)^2(3+k)(7+5k)}. \end{aligned} \quad (3.21)$$

Of course, all the summation indices (appearing in the currents) run as $a, b = 1, \dots, 4$. As before, the relative coefficients of descendent fields of $O_2(z)$ and $T(z)$ are fixed by (1.3). To check if there are extra fields in the second-order pole, we compute $[\{O_{\frac{5}{2}} O_{\frac{5}{2}}\}_{-2} - (\frac{1}{2} c_{\frac{5}{2}\frac{5}{2}}^o \partial O_2 + \frac{1}{2} c_{\frac{5}{2}\frac{5}{2}}^t \partial T)](w)$. It turns out that it is identically zero. There are no extra fields besides $\partial O_2(w)$ and $\partial T(w)$. The presence of the extra fields in the first-order pole can be checked by calculating the following quantity $[\{O_{\frac{5}{2}} O_{\frac{5}{2}}\}_{-1} - (\frac{3}{20} c_{\frac{5}{2}\frac{5}{2}}^o \partial^2 O_2 + \frac{3}{20} c_{\frac{5}{2}\frac{5}{2}}^t \partial^2 T)](w)$. Because this doesn't vanish, we should compute the following two OPEs:

$$\begin{aligned} T(z) & \left[\{O_{\frac{5}{2}} O_{\frac{5}{2}}\}_{-1} - \left(\frac{3}{20} c_{\frac{5}{2}\frac{5}{2}}^o \partial^2 O_2 + \frac{3}{20} c_{\frac{5}{2}\frac{5}{2}}^t \partial^2 T \right) \right] (w), \\ G(z) & \left[\{O_{\frac{5}{2}} O_{\frac{5}{2}}\}_{-1} - \left(\frac{3}{20} c_{\frac{5}{2}\frac{5}{2}}^o \partial^2 O_2 + \frac{3}{20} c_{\frac{5}{2}\frac{5}{2}}^t \partial^2 T \right) \right] (w). \end{aligned} \quad (3.22)$$

These OPEs (3.22) have higher-order poles with order $n > 2$. Looking at these higher-order poles enables us to find the extra four quasi-primary fields in the first-order pole in (3.20). Then we compute $[\{O_{\frac{5}{2}} O_{\frac{5}{2}}\}_{-1} - (\text{first and second line in the first-order pole})](w)$ and it turns out that this doesn't vanish and there exists a new primary spin-4 field $O_4(z)$ which

is the superpartner of $O_{\frac{7}{2}}(z)$ given in (3.13). As expected, the $O_4(z)$ obeys the following OPEs:

$$\begin{aligned} T(z) O_4(w) &= \frac{1}{(z-w)^2} 4 O_4(w) + \frac{1}{(z-w)} \partial O_4(w) + \dots, \\ G(z) O_4(w) &= \frac{1}{(z-w)^2} 7 O_{\frac{7}{2}}(w) + \frac{1}{(z-w)} \partial O_{\frac{7}{2}}(w) + \dots. \end{aligned} \quad (3.23)$$

Therefore, the two currents $O_{\frac{7}{2}}(z)$ and $O_4(z)$ provide the correct $\mathcal{N} = 1$ supermultiplet. One can read off the explicit form of $O_4(z)$ from (3.20) as follows:

$$\begin{aligned} O_4(w) &= \{O_{\frac{5}{2}} O_{\frac{5}{2}}\}_{-1}(w) - \left[\frac{3}{20} c_{\frac{5}{2}\frac{5}{2}}^o \partial^2 O_2 + \frac{3}{20} c_{\frac{5}{2}\frac{5}{2}}^t \partial^2 T + c_{\frac{5}{2}\frac{5}{2}}^{tt} \left(TT - \frac{3}{10} \partial^2 T \right) \right. \\ &\quad \left. + c_{\frac{5}{2}\frac{5}{2}}^{to} \left(TO_2 - \frac{3}{10} \partial^2 O_2 \right) + c_{\frac{5}{2}\frac{5}{2}}^{go} \left(GO_{\frac{5}{2}} - \frac{1}{5} \partial^2 O_2 \right) + c_{\frac{5}{2}\frac{5}{2}}^{gg} \left(G\partial G - \frac{7}{10} \partial^2 T \right) \right] (w). \end{aligned} \quad (3.24)$$

Next we compute $O_{\frac{5}{2}}(z) O_{\frac{5}{2}}(w)$ for $N = 3$ (where the summation indices for the various currents run as $a, b = 1, \dots, 6$) with the help of the package. To find the structure constants $c_{\frac{5}{2}\frac{5}{2}}^o$, $c_{\frac{5}{2}\frac{5}{2}}^t$, and $c_{\frac{5}{2}\frac{5}{2}}^{tt}$, we calculate $[\{O_{\frac{5}{2}} O_{\frac{5}{2}}\}_{-5} - c_{\frac{5}{2}\frac{5}{2}}^o](w)$ and $[\{O_{\frac{5}{2}} O_{\frac{5}{2}}\}_{-3} - c_{\frac{5}{2}\frac{5}{2}}^o O_2 - c_{\frac{5}{2}\frac{5}{2}}^t T](w)$ that should vanish. Then we find the correct values for $c_{\frac{5}{2}\frac{5}{2}}^o$, $c_{\frac{5}{2}\frac{5}{2}}^t$ and $c_{\frac{5}{2}\frac{5}{2}}^{tt}$ when $N = 3$. For the remaining four structure constants, we use the equations (3.24) and (3.23) which should satisfy for any N . Then the final structure constants for $N = 3$ are given by

$$\begin{aligned} c_{\frac{5}{2}\frac{5}{2}}^o &= -\frac{162k^3(9+k)(3+2k)}{(4+k)^2(5+k)^3}, & c_{\frac{5}{2}\frac{5}{2}}^t &= \frac{12(-6+k)k}{(4+k)(5+k)}, & c_{\frac{5}{2}\frac{5}{2}}^{tt} &= -\frac{90k^2(9+k)(3+2k)}{(4+k)^2(5+k)^2}, \\ c_{\frac{5}{2}\frac{5}{2}}^{to} &= -\frac{324k^2(9+k)(3+2k)}{(4+k)^2(5+k)(35+19k)}, & c_{\frac{5}{2}\frac{5}{2}}^{go} &= \frac{108(-6+k)k}{(4+k)(10+17k)}, \\ c_{\frac{5}{2}\frac{5}{2}}^{gg} &= -\frac{54(-6+k)k}{(4+k)(10+17k)}, & c_{\frac{5}{2}\frac{5}{2}}^{gg} &= \frac{81k^2(9+k)(3+2k)}{(4+k)^2(5+k)(35+19k)}. \end{aligned} \quad (3.25)$$

We repeat the above procedures for $N = 4$ and $N = 5$ (the indices $a, b = 1, \dots, 8$ and the indices $a, b = 1, \dots, 10$ respectively) and put all the results together.⁵ Looking into the above results (3.26) and (3.27), one can easily figure out that, for general N ,

$$\begin{aligned} c_{\frac{5}{2}\frac{5}{2}}^o &= \frac{4k(k-2N)N}{(-2+k+2N)(-1+k+2N)}, \\ c_{\frac{5}{2}\frac{5}{2}}^t &= -\frac{10k^2N^2(-3+2k+2N)(-3+k+4N)}{(-2+k+2N)^2(-1+k+2N)^2}. \end{aligned} \quad (3.28)$$

⁵For $c_{\frac{5}{2}\frac{5}{2}}^o$, we obtain the following structure constants for different values of N

$$\begin{aligned} c_{\frac{5}{2}\frac{5}{2}}^o(N=2) &= \frac{8(-4+k)k}{(2+k)(3+k)}, & c_{\frac{5}{2}\frac{5}{2}}^o(N=3) &= \frac{12(-6+k)k}{(4+k)(5+k)}, \\ c_{\frac{5}{2}\frac{5}{2}}^o(N=4) &= \frac{16(-8+k)k}{(6+k)(7+k)}, & c_{\frac{5}{2}\frac{5}{2}}^o(N=5) &= \frac{20(-10+k)k}{(8+k)(9+k)}. \end{aligned} \quad (3.26)$$

For $c_{\frac{5}{2}\frac{5}{2}}^t$, the following structure constants for each value of N can be obtained

$$\begin{aligned} c_{\frac{5}{2}\frac{5}{2}}^t(N=2) &= \frac{(-40k^2(5+k)(1+2k))}{(2+k)^2(3+k)^2}, & c_{\frac{5}{2}\frac{5}{2}}^t(N=3) &= \frac{(-90k^2(9+k)(3+2k))}{(4+k)^2(5+k)^2}, \\ c_{\frac{5}{2}\frac{5}{2}}^t(N=4) &= \frac{(-160k^2(13+k)(5+2k))}{(6+k)^2(7+k)^2}, & c_{\frac{5}{2}\frac{5}{2}}^t(N=5) &= \frac{(-250k^2(17+k)(7+2k))}{(8+k)^2(9+k)^2}. \end{aligned} \quad (3.27)$$

In this way, one can find all the structure constants for general N in (3.20). By converting k into the central charge c using (2.9), the structure constants of (3.20) for general N (which contains (3.21) and (3.25)) in terms of c and N (rather than k and N) are

$$\begin{aligned}
 c_{\frac{5}{2}\frac{5}{2}} &= \frac{2c^3(2c(-1+N) + 3(3-4N)N)(c+2cN+3N(-3+2N))}{9(c+6(-1+N)N)^2}, \\
 c_{\frac{5}{2}\frac{5}{2}}^o &= \frac{4c(-6N^2 + c(-1+4N))}{3(c+6(-1+N)N)}, \\
 c_{\frac{5}{2}\frac{5}{2}}^t &= \frac{10c^2(2c(-1+N) + 3(3-4N)N)(c+2cN+3N(-3+2N))}{9(c+6(-1+N)N)^2}, \\
 c_{\frac{5}{2}\frac{5}{2}}^{tt} &= \frac{12c^2(2c(-1+N) + 3(3-4N)N)(c+2cN+3N(-3+2N))}{(21+4c)(c+6(-1+N)N)^2}, \\
 c_{\frac{5}{2}\frac{5}{2}}^{to} &= \frac{36c(-6N^2 + c(-1+4N))}{(6+5c)(c+6(-1+N)N)}, \quad c_{\frac{5}{2}\frac{5}{2}}^{go} = -\frac{18c(-6N^2 + c(-1+4N))}{(6+5c)(c+6(-1+N)N)}, \\
 c_{\frac{5}{2}\frac{5}{2}}^{gg} &= -\frac{3c^2(2c(-1+N) + 3(3-4N)N)(c+2cN+3N(-3+2N))}{(21+4c)(c+6(-1+N)N)^2}, \tag{3.29}
 \end{aligned}$$

where the second and third coefficients of (3.29) are the same as the ones in (3.28).

Now let us compute the explicit form of $\{O_{\frac{5}{2}} O_{\frac{5}{2}}\}_{-1}(w)$ for general N . To find all the operators and their coefficients in $\{O_{\frac{5}{2}} O_{\frac{5}{2}}\}_{-1}(w)$, we use the same method as we did for the structure constants. First we find all the operators in $\{O_{\frac{5}{2}} O_{\frac{5}{2}}\}_{-1}(w)$ for $N = 2$. In this case, there are three hundred twenty terms. Then we investigate them and write down the possible various spin-4 fields with coefficients as follows:

$$\begin{aligned}
 \{O_{\frac{5}{2}} O_{\frac{5}{2}}\}_{-1}(w) &= c_1 J^{ab} J^{ad} J^{b2N+1} J^{dN'}(w) + c_2 J^{ab} J^{aN'} \partial J^{bN'}(w) + c_3 \partial J^{aN'} \partial J^{aN'}(w) \\
 &\quad + c_4 \partial J^{ab} \partial J^{ab}(w) + c_5 \partial^2 J^{ab} J^{ab}(w) + c_6 \partial^2 J^{aN'} J^{bN'}(w) \\
 &\quad + c_7 K^a K^b J^{ac} \partial J^{bc}(w) + c_8 K^a K^b \partial^2 J^{ab}(w) + c_9 K^a \partial K^b J^{ac} J^{bc}(w) \\
 &\quad + c_{10} K^a \partial K^b \partial J^{ab}(w) + c_{11} K^a \partial^2 K^b J^{ab}(w) + c_{12} \partial^3 K^a K^a(w) \\
 &\quad + c_{13} K^a K^b J^{ab} J^{cN'} J^{cN'}(w) + c_{14} K^a K^b J^{ac} J^{bd} J^{cd}(w) \\
 &\quad + c_{15} \partial K^a K^a J^{bc} J^{bc}(w) + c_{16} K^a K^b J^{aN'} \partial J^{bN'}(w) + c_{17} K^a \partial K^b J^{aN'} J^{bN'}(w) \\
 &\quad + c_{18} \partial K^a K^a J^{bN'} J^{bN'}(w) + c_{19} \partial K^a \partial K^b J^{ab}(w) + c_{20} \partial^2 K^a \partial K^a(w), \tag{3.30}
 \end{aligned}$$

where $N' \equiv 2N + 1$ and the dummy indices run as $a, b, c, d = 1, \dots, 2N$. Of course, for $N = 2$, the expression (3.30) consists of 320 independent terms (by expanding them out completely) as mentioned before. With the tensorial structures in (3.30), there exist only twenty coefficient functions for general N . Then we solve $[\{O_{\frac{5}{2}} O_{\frac{5}{2}}\}_{-1} - (\text{the right hand side of (3.30)})](w) = 0$ for the coefficients. The left hand side of (3.30) is given by too many WZW currents and we want to write it using the tensorial structure to simplify. We repeat this procedure for $N = 3, 4, 5$.⁶ From the above results (3.31)

⁶For the coefficient c_1 appearing in the first term of (3.30), each coefficient function can be obtained

$$\begin{aligned}
 c_1(N=2) &= \frac{-36k^2}{(2+k)^2(3+k)^3}, & c_1(N=3) &= \frac{-81k^2}{(4+k)^2(5+k)^3}, \\
 c_1(N=4) &= \frac{-144k^2}{(6+k)^2(7+k)^3}, & c_1(N=5) &= \frac{-225k^2}{(8+k)^2(9+k)^3}. \tag{3.31}
 \end{aligned}$$

and (3.32), one can easily figure out the general forms of c_1 and c_2 . For general N , the coefficients c_1 and c_2 are given by

$$c_1(N, k) = -\frac{9k^2 N^2}{(-2+k+2N)^2(-1+k+2N)^3},$$

$$c_2(N, k) = -\frac{18k^2 N^2(-2+k+3N)}{(-2+k+2N)^2(-1+k+2N)^3}.$$

In this way one can find all coefficients in (3.30) for general N . We also put the equations of the coefficients for $N = 2$ in (A.1) of appendix A. The general forms of coefficients are presented in (A.2) of appendix A. All results regarding the OPE $O_{\frac{5}{2}}(z)O_{\frac{5}{2}}(w)$ have been checked up to $N = 8$ by mathematica package [14]. The OPE $\hat{O}_{\frac{5}{2}}(z)\hat{O}_{\frac{5}{2}}(w)$ for general N can be obtained from (3.20) with (3.29). The fusion rule can be summarized by $[O_{\frac{5}{2}}][O_{\frac{5}{2}}] = [I] + [O_2] + [O_4]$.

Then the $\mathcal{N} = 1$ fusion rule is summarized by $[\hat{O}_2][\hat{O}_2] = [\hat{I}] + [\hat{O}_2] + [\hat{O}_{\frac{7}{2}}]$. The explicit OPE for this is given by (F.4). By rescaling the currents as $O_2(z) \rightarrow N_2 O_2(z)$ and $O_{\frac{5}{2}}(z) \rightarrow N_{\frac{5}{2}} O_{\frac{5}{2}}(z)$, where

$$N_2^2 = -\frac{9(c+6(-1+N)N)^2}{c^2(2c(-1+N)+3(3-4N)N)(c+2cN+3N(-3+2N))},$$

$$N_{\frac{5}{2}}^2 = -\frac{1}{5}N_2^2,$$

one has the standard normalizations: $O_2(z)O_2(w) \rightarrow \frac{1}{(z-w)^4}\frac{c}{2} + \dots$ and $O_{\frac{5}{2}}(z)O_{\frac{5}{2}}(w) \rightarrow \frac{1}{(z-w)^5}\frac{2c}{5} + \dots$. Therefore all the previous OPEs can be rewritten in terms of these rescaled currents. For example, in the OPE of (3.11), the spin- $\frac{7}{2}$ is simply taken from the first-order pole subtracted by the descendant fields and quasi-primary fields in the right hand side. Strictly speaking, one should consider the structure constant coming from the spin-2 current, the spin- $\frac{5}{2}$ current and the spin- $\frac{7}{2}$ current. We will see that the normalization of spin- $\frac{7}{2}$ current is needed also. Then there should exist a nontrivial structure constant in front of spin- $\frac{7}{2}$ current.⁷

Similarly, for the coefficient c_2 appearing in the second term of (3.30), the coefficient functions for different N -values are given by

$$c_2(N=2) = \frac{-72k^2(4+k)}{(2+k)^2(3+k)^3}, \quad c_2(N=3) = \frac{-162k^2(7+k)}{(4+k)^2(5+k)^3},$$

$$c_2(N=4) = \frac{-288k^2(10+k)}{(6+k)^2(7+k)^3}, \quad c_2(N=5) = \frac{-450k^2(13+k)}{(8+k)^2(9+k)^3}. \quad (3.32)$$

⁷The reason why we do not continue to do in this direction is that once we obtain the higher spin current for given spin, then one should determine the highest singular term of this OPE between this current and itself in order to fix the normalization. For lower higher spin currents, this can be done by hand (or by package). However, as the spin increases, it is hard to obtain the highest singular term for this OPE even by the package (it takes too much time). Note that in this paper, we have considered the normalization of higher spin current up to the spin $\frac{9}{2}$ (See the end of the section 3).

3.2 The OPEs between the higher spin currents of spins- $(2', \frac{5}{2})$ and the higher spin currents of spins- $(\frac{7}{2}, 4)$

• **The OPE $O_2(z) O_{\frac{7}{2}}(w)$.** Now we compute the OPE between $O_2(z)$ and $O_{\frac{7}{2}}(w)$ in order to calculate the OPEs between the two $\mathcal{N} = 1$ multiplets, $(2', \frac{5}{2})$ and $(\frac{7}{2}, 4)$, in (1.2). To find the structure of the OPE, we compute it for $N = 2$ as mentioned before. The final result is presented first which explains how the result can be obtained explicitly. This OPE is described as

$$\begin{aligned} O_2(z) O_{\frac{7}{2}}(w) = & \frac{1}{(z-w)^3} c_{2\frac{7}{2}}^o O_{\frac{5}{2}}(w) + \frac{1}{(z-w)^2} \left[\frac{1}{5} c_{2\frac{7}{2}}^o \partial O_{\frac{5}{2}} + c_{2\frac{7}{2}}^{o'} O_{\frac{7}{2}} + c_{2\frac{7}{2}}^{go} \left(GO_2 - \frac{2}{5} \partial O_{\frac{5}{2}} \right) \right] (w) \\ & + \frac{1}{(z-w)} \left[\frac{1}{30} c_{2\frac{7}{2}}^o \partial^2 O_{\frac{5}{2}} + \frac{2}{7} c_{2\frac{7}{2}}^{o'} \partial O_{\frac{7}{2}} + \frac{2}{7} c_{2\frac{7}{2}}^{go} \partial \left(GO_2 - \frac{2}{5} \partial O_{\frac{5}{2}} \right) \right. \\ & + c_{2\frac{7}{2}}^{go'} \left(\partial GO_2 - \frac{3}{4} G \partial O_2 + \frac{1}{8} \partial^2 O_{\frac{5}{2}} \right) + c_{2\frac{7}{2}}^{to} \left(TO_{\frac{5}{2}} - \frac{1}{4} \partial^2 O_{\frac{5}{2}} \right) \\ & \left. + O_{\frac{9}{2}} \right] (w) \cdots \end{aligned} \quad (3.33)$$

All the coefficients for $N = 2$ in the right hand side of (3.33) are determined. The relative coefficients of descendent fields of $O_{\frac{5}{2}}(w)$, $O_{\frac{7}{2}}(w)$, and $\left(GO_2 - \frac{2}{5} \partial O_{\frac{5}{2}} \right) (w)$ are determined by (1.3). For the second-order pole, we compute $T(z) [\{O_2 O_{\frac{7}{2}}\}_{-2} - \frac{1}{5} c_{2\frac{7}{2}}^o \partial O_{\frac{5}{2}}](w)$ and $G(z) [\{O_2 O_{\frac{7}{2}}\}_{-2} - \frac{1}{5} c_{2\frac{7}{2}}^o \partial O_{\frac{5}{2}}](w)$ to find extra fields in the second-order pole. These OPEs have higher-order poles with order $n > 2$. Adding $\left(GO_2 - \frac{2}{5} \partial O_{\frac{5}{2}} \right) (w)$ with the coefficient $c_{2\frac{7}{2}}^{go}$ to the second-order pole makes the higher-order poles of the above OPEs disappear. Then we compute $[\{O_2 O_{\frac{7}{2}}\}_{-2} - \frac{1}{5} c_{2\frac{7}{2}}^o \partial O_{\frac{5}{2}} - c_{2\frac{7}{2}}^{go} \left(GO_2 - \frac{2}{5} \partial O_{\frac{5}{2}} \right)](w)$ which doesn't vanish implying that there exists the primary spin- $\frac{7}{2}$ field $O_{\frac{7}{2}}(z)$ (that has been found in (3.13)) in the second-order pole).

For the first-order pole, we compute $[\{O_2 O_{\frac{7}{2}}\}_{-1} - \text{first line}](w)$ leading to the nonvanishing quantity. So let us compute $T(z) [\{O_2 O_{\frac{7}{2}}\}_{-1} - \text{first line}](w)$ and $G(z) [\{O_2 O_{\frac{7}{2}}\}_{-1} - \text{first line}](w)$ as before. It turns out that the OPEs have higher-order poles with order $n > 2$. To remove the higher-order poles, we should add the quasi-primary fields in the second line of the first-order pole in (3.33). Then we compute $[\{O_2 O_{\frac{7}{2}}\}_{-1} - \text{first line} - \text{second line}](w)$ which doesn't vanish. This implies that there exists a new primary spin- $\frac{9}{2}$ field $O_{\frac{9}{2}}(z)$. As expected, the spin- $\frac{9}{2}$ current $O_{\frac{9}{2}}(z)$ obeys the following OPEs:

$$\begin{aligned} T(z) O_{\frac{9}{2}}(w) &= \frac{1}{(z-w)^2} \frac{9}{2} O_{\frac{9}{2}}(w) + \frac{1}{(z-w)} \partial O_{\frac{9}{2}}(w) + \cdots, \\ G(z) O_{\frac{9}{2}}(w) &= \frac{1}{(z-w)^2} 8 O_{4'}(w) + \frac{1}{(z-w)} \partial O_{4'}(w) + \cdots, \end{aligned} \quad (3.34)$$

where $O_{4'}(z)$ is a new spin-4 primary field that will appear in (3.42) in the next OPE. The explicit form of $O_{\frac{9}{2}}(z)$ can be derived as follows:

$$\begin{aligned} O_{\frac{9}{2}}(w) = & \{O_2 O_{\frac{7}{2}}\}_{-1}(w) - \left[\frac{1}{30} c_{2\frac{5}{2}}^o \partial^2 O_{\frac{5}{2}} + \frac{2}{7} c_{2\frac{5}{2}}^{o'} \partial O_{\frac{7}{2}} + \frac{2}{7} c_{2\frac{5}{2}}^{go} \partial \left(GO_2 - \frac{2}{5} \partial O_{\frac{5}{2}} \right) \right. \\ & \left. + c_{2\frac{5}{2}}^{go'} \left(\partial GO_2 - \frac{3}{4} G \partial O_2 + \frac{1}{8} \partial^2 O_{\frac{5}{2}} \right) + c_{2\frac{5}{2}}^{to} \left(TO_{\frac{5}{2}} - \frac{1}{4} \partial^2 O_{\frac{5}{2}} \right) \right] (w). \end{aligned} \quad (3.35)$$

To find the structure constants in (3.33) for general N , we compute the OPE for $N = 3, 4, 5$ as we did before. To find the structure constants $c_{2\frac{7}{2}}^o$, $c_{2\frac{7}{2}}^{o'}$, and $c_{2\frac{7}{2}}^{go}$, we solve the following equations:

$$\begin{aligned} & \left[\{O_2 O_{\frac{7}{2}}\}_{-3} - c_{2\frac{5}{2}}^o O_{\frac{5}{2}} \right] (w) = 0, \\ & \left[\{O_2 O_{\frac{7}{2}}\}_{-2} - \frac{1}{5} c_{2\frac{5}{2}}^o \partial O_{\frac{5}{2}} - c_{2\frac{5}{2}}^{o'} O_{\frac{7}{2}} - c_{2\frac{5}{2}}^{go} \left(G O_2 - \frac{2}{5} \partial O_{\frac{5}{2}} \right) \right] (w) = 0. \end{aligned} \quad (3.36)$$

Then from these relations (3.36), the three structure constants can be determined for $N = 3, 4, 5$ and therefore the first three coefficients in (3.35) are determined completely. To find the remaining undetermined coefficients, $c_{2\frac{7}{2}}^{go'}$ and $c_{2\frac{7}{2}}^{to}$, we compute $T(z) O_{\frac{9}{2}}(w)$ and $G(z) O_{\frac{9}{2}}(w)$ and use the fact that there should be no higher-order poles with order $n > 2$ as in (3.34). We repeat this procedure for $N = 3, 4, 5$ cases.

Then we put all the results together, and find the general forms of structure constants in (3.33). The structure constants for general N in (3.33) are given by

$$\begin{aligned} c_{2\frac{7}{2}}^o &= -\frac{6c^2(-3+2c)(1+N)(c^2-3c(-1+N)+18(1-3N)N)(6N+c(-3+2N))}{(21+4c)(6+5c)(c+6(-1+N)N)^2}, \\ c_{2\frac{7}{2}}^{o'} &= \frac{2c(21+4c)(c-4cN+6N^2)}{3(6+5c)(c+6(-1+N)N)}, \\ c_{2\frac{7}{2}}^{go} &= -\frac{108c^2(-3+2c)(1+N)(c^2-3c(-1+N)+18(1-3N)N)(6N+c(-3+2N))}{(21+4c)(6+5c)^2(c+6(-1+N)N)^2}, \\ c_{2\frac{7}{2}}^{go'} &= -\frac{48c^2(-45+2c)(3+c-9N)(1+N)(c+6N)(6N+c(-3+2N))}{7(29+2c)(21+4c)(6+5c)(c+6(-1+N)N)^2}, \\ c_{2\frac{7}{2}}^{to} &= -\frac{12c^2(-3+10c)(3+c-9N)(1+N)(c+6N)(6N+c(-3+2N))}{(29+2c)(21+4c)(6+5c)(c+6(-1+N)N)^2}. \end{aligned} \quad (3.37)$$

To find the explicit structure of $\{O_2 O_{\frac{7}{2}}\}_{-1}(w)$ for general N , we examine the operators in $\{O_2 O_{\frac{7}{2}}\}_{-1}(w)$ for $N = 2$ and write down the possible various spin- $\frac{9}{2}$ fields with undetermined coefficients. Then we solve $[\{O_2 O_{\frac{7}{2}}\}_{-1} - (\text{the possible various spin-}\frac{9}{2} \text{ fields})](w) = 0$ for these coefficients. We repeat this procedure for $N = 3, 4, 5$ cases. The first-order pole $\{O_2 O_{\frac{7}{2}}\}_{-1}(w)$, which consists of thirty-two terms, is presented as follows:

$$\begin{aligned} \{O_2 O_{\frac{7}{2}}\}_{-1}(w) &= c_1 K^a J^{ab} J^{bc} J^{dc} J^{dN'}(w) + c_2 K^a J^{ab} J^{cd} J^{cd} J^{bN'}(w) + c_3 K^a J^{ab} J^{cd} J^{bd} J^{cN'}(w) \\ &+ c_4 K^a J^{bc} J^{bc} \partial J^{aN'}(w) + c_5 K^a J^{ab} J^{bc} \partial J^{cN'}(w) + c_6 K^a J^{ab} J^{bN'} J^{cN'} J^{cN'}(w) \\ &+ c_7 K^a J^{ab} \partial^2 J^{bN'}(w) + c_8 K^a J^{aN'} \partial J^{bN'} J^{bN'}(w) + c_9 K^a K^b K^c J^{ab} J^{cd} J^{dN'}(w) \\ &+ c_{10} K^a J^{bc} \partial J^{ac} J^{bN'}(w) + c_{11} K^a J^{bN'} J^{bN'} \partial J^{aN'}(w) + c_{12} K^a K^b K^c J^{ab} \partial J^{cN'}(w) \\ &+ c_{13} K^a K^b \partial K^c J^{ab} J^{cN'}(w) + c_{14} K^a K^b K^c \partial J^{ab} J^{cN'}(w) + c_{15} K^a K^b \partial K^c J^{ac} J^{bN'}(w) \\ &+ c_{16} K^a \partial J^{ab} J^{bc} J^{cN'}(w) + c_{17} K^a \partial J^{bc} J^{bc} J^{aN'}(w) + c_{18} K^a \partial J^{ab} \partial J^{bN'}(w) \\ &+ c_{19} K^a \partial K^b K^b J^{ca} J^{cN'}(w) + c_{20} K^a \partial K^b K^b \partial J^{aN'}(w) + c_{22} K^a \partial^2 K^b K^b J^{aN'}(w) \\ &+ c_{23} K^a \partial^3 J^{aN'}(w) + c_{24} \partial K^a J^{bc} J^{bc} J^{aN'}(w) + c_{25} \partial K^a J^{ab} J^{bc} J^{cN'}(w) \\ &+ c_{26} \partial K^a J^{ab} \partial J^{bN'}(w) + c_{27} \partial K^a J^{aN'} J^{bN'} J^{bN'}(w) + c_{28} \partial K^a \partial J^{ab} J^{bN'}(w) \\ &+ c_{29} \partial K^a K^a \partial K^b J^{bN'}(w) + c_{30} \partial K^a \partial^2 J^{aN'}(w) + c_{31} \partial^2 K^a J^{ab} J^{bN'}(w) \\ &+ c_{32} \partial^2 K^a \partial J^{aN'}(w) + c_{33} \partial^3 K^a J^{aN'}(w), \end{aligned} \quad (3.38)$$

where $N' \equiv 2N + 1$ and the summation indices run $a, b, c, d = 1, \dots, 2N$. The coefficients in (3.38) are presented in (B.1) of appendix B. All the results regarding the OPE between $O_2(z)$ and $O_{\frac{7}{2}}(z)$ are checked up to $N = 7$ case by mathematica package [14].

Therefore, the spin- $\frac{9}{2}$ current $O_{\frac{9}{2}}(z)$ is summarized by (3.35), (3.37), and (3.38). The OPE $O_2(z)O_{\frac{7}{2}}(w)$ for general N is given by (3.33) with (3.37). The fusion rule is summarized by $[O_2][O_{\frac{7}{2}}] = [I] + [O_{\frac{5}{2}}] + [O_{\frac{7}{2}}] + [O_{\frac{9}{2}}]$.

• **The OPE $O_2(z)O_4(w)$.** Now we move to the OPE between the $O_2(z)$ and $O_4(z)$. From the computation of the OPE for $N = 2$, we find that this OPE takes the form

$$\begin{aligned} O_2(z)O_4(w) = & \frac{1}{(z-w)^4}c_{24}^o O_2 \\ & + \frac{1}{(z-w)^2} \left[c_{24}^{go} \left(GO_{\frac{5}{2}} - \frac{1}{5}\partial^2 O_2 \right) + c_{24}^{to} \left(TO_2 - \frac{3}{10}\partial^2 O_2 \right) + c_{24}' O_4 + c_{24}'' O_{4'} \right] (w) \\ & + \frac{1}{(z-w)} \left[\frac{1}{4}c_{24}^{go}\partial \left(GO_{\frac{5}{2}} - \frac{1}{5}\partial^2 O_2 \right) + \frac{1}{4}c_{24}^{to}\partial \left(TO_2 - \frac{3}{10}\partial^2 O_2 \right) + \frac{1}{4}c_{24}'\partial O_4 \right. \\ & + \frac{1}{4}c_{24}''\partial O_{4'} + c_{24}^{to'} \left(T\partial O_2 - \partial TO_2 - \frac{1}{6}\partial^3 O_2 \right) + c_{24}^{go'} \left(G\partial O_{\frac{5}{2}} - \frac{5}{3}\partial GO_{\frac{5}{2}} - \frac{1}{9}\partial^3 O_2 \right) \\ & \left. + c_{24}^{go''} \left(GO_{\frac{7}{2}} - \frac{1}{4}\partial O_4 \right) \right] (w) + \dots \end{aligned} \quad (3.39)$$

At the moment, the right hand side holds for $N = 2$ case only. We would like to obtain this OPE for general N . In particular, the structure constants for general N . The relative coefficients of descendants fields are found by (1.3) once again. According to (1.3), there is no third-order pole because the coefficient for the descendant field of $O_2(w)$ is zero in the OPE. To find the fields in the second-order pole, first we compute $T(z)\{O_2 O_4\}_{-2}(w)$ and $G(z)\{O_2 O_4\}_{-2}(w)$ as before. The two OPEs have higher-order poles with order $n > 2$. We look at the higher-order poles and find that the following two quasi-primary fields can remove all the higher-order poles in the above OPEs: $\left(GO_{\frac{5}{2}} - \frac{1}{5}\partial^2 O_2\right)(w)$ and $\left(TO_2 - \frac{3}{10}\partial^2 O_2\right)(w)$ with appropriate coefficients c_{24}^{go} and c_{24}^{to} respectively. Then we subtract $c_{24}^{go}\left(GO_{\frac{5}{2}} - \frac{1}{5}\partial^2 O_2\right)(w)$ and $c_{24}^{to}\left(TO_2 - \frac{3}{10}\partial^2 O_2\right)(w)$ from the second-order pole term $\{O_2 O_4\}_{-2}(w)$. This doesn't vanish implying that the second-order pole has at least one primary spin-4 field. So we solve the following equation for c_{24}' :

$$[\{O_2 O_4\}_{-2} - c_{24}^{go}\left(GO_{\frac{5}{2}} - \frac{1}{5}\partial^2 O_2\right) - c_{24}^{to}\left(TO_2 - \frac{3}{10}\partial^2 O_2\right) - c_{24}' O_4](w) = 0. \quad (3.40)$$

It turns out that there is no solution for (3.40). This implies that there should be a new primary spin-4 field and that this new spin-4 field would be the superpartner of $O_{\frac{9}{2}}(z)$.

To check this, we consider the following equation for c_{24}' and c' :

$$\begin{aligned} & [\{O_2 O_4\}_{-2} - c_{24}^{go}\left(GO_{\frac{5}{2}} - \frac{1}{5}\partial^2 O_2\right) - c_{24}^{to}\left(TO_2 - \frac{3}{10}\partial^2 O_2\right) - c_{24}' O_4](w) + \\ & + \frac{c' c_{24}''}{8} \{G O_{\frac{9}{2}}\}_{-2}(w) = 0. \end{aligned} \quad (3.41)$$

See also the equation (3.34). It turns out that there exists a solution for (3.41) when $c' = -1$. So we can be sure about that the new spin-4 field is nothing but $O_{4'}(z)$.

Therefore, the explicit form of $O_{4'}(z)$, from (3.41), can be written as

$$O_{4'}(z) = \frac{1}{c_{24}^{o''}} \left[\{O_2 O_4\}_{-2}(w) - c_{24}^{go} \left(GO_{\frac{5}{2}} - \frac{1}{5} \partial^2 O_2 \right) - c_{24}^{to} \left(TO_2 - \frac{3}{10} \partial^2 O_2 \right) - c_{24}^{o'} O_4 \right](z). \quad (3.42)$$

As expected, the spin-4 current, $O_{4'}(z)$, obeys the following OPEs:

$$\begin{aligned} T(z) O_{4'}(w) &= \frac{1}{(z-w)^2} 4 O_{4'}(w) + \frac{1}{(z-w)} \partial O_{4'}(w) + \dots, \\ G(z) O_{4'}(w) &= \frac{1}{(z-w)} O_{\frac{9}{2}}(w) + \dots, \end{aligned} \quad (3.43)$$

where the spin- $\frac{9}{2}$ current is given by (3.35). Therefore, the $O_{4'}(z)$ and $O_{\frac{9}{2}}(z)$ provide the correct $\mathcal{N} = 1$ multiplet.

To check if there are extra fields besides the descendant fields in the first-order pole, we compute $[\{O_2 O_4\}_{-1} - (\text{desc. fields in the 1st-order pole})](w)$. Because this doesn't vanish, we should compute the following two OPEs:

$$\begin{aligned} T(z) [\{O_2 O_4\}_{-1} - (\text{desc. fields in the 1st-order pole})](w), \\ G(z) [\{O_2 O_4\}_{-1} - (\text{desc. fields in the 1st-order pole})](w). \end{aligned} \quad (3.44)$$

Here the descendant fields in the 1st-order pole contain the first four terms in (3.39). The above OPEs (3.44) have higher-order poles with order $n > 2$ and these higher-order poles are removed by the following two quasi-primary fields: $(T\partial O_2 - \partial TO_2 - \frac{1}{6}\partial^3 O_2)(w)$ and $(G\partial O_{\frac{5}{2}} - \frac{5}{3}\partial GO_{\frac{5}{2}} - \frac{1}{9}\partial^3 O_2)(w)$ with the coefficients $c_{24}^{to'}$ and $c_{24}^{go'}$ respectively. Furthermore, there exists $GO_{\frac{7}{2}}$ term with derivative. Then we compute the following expression:

$$[\{O_2 O_4\}_{-1} - (\text{desc. fields} + \text{quasi. fields})](w),$$

which vanishes and there are no extra fields in the first-order pole.

To find the structure constants in (3.39) for general N , we compute the OPEs for $N = 3, 4, 5$ cases. We use $[\{O_2 O_4\}_{-4} - c_{24}^o O_2](w) = 0$ to find the structure constant c_{24}^o . To find the next coefficients $c_{24}^{go}, c_{24}^{to}, c_{24}^{o'}$, and $c_{24}^{o''}$, the equations (3.42) and (3.43) can be used. To find the remaining structure constants $c_{24}^{to'}, c_{24}^{go'}$ and $c_{24}^{go''}$, we solve the following equation for them:

$$[\{O_2 O_4\}_{-1} - (\text{the 1st-order pole})](w) = 0.$$

It turns out that the structure constants in (3.39) for general N are given by

$$\begin{aligned} c_{24}^o &= \frac{24c^2(-3+2c)(3+c-9N)(1+N)(c+6N)(6N+c(-3+2N))}{(21+4c)(6+5c)(c+6(-1+N)N)^2}, \\ c_{24}^{go} &= \frac{18c^2(-3+2c)(156+35c)(3+c-9N)(1+N)(c+6N)(6N+c(-3+2N))}{(29+2c)(21+4c)(6+5c)^2(c+6(-1+N)N)^2}, \\ c_{24}^{to} &= \frac{24c^2(-3+2c)(-123+40c)(3+c-9N)(1+N)(c+6N)(6N+c(-3+2N))}{(29+2c)(21+4c)(6+5c)^2(c+6(-1+N)N)^2}, \\ c_{24}^{o'} &= -\frac{10c(21+4c)(-6N^2+c(-1+4N))}{21(6+5c)(c+6(-1+N)N)}, \quad c_{24}^{o''} = -8, \end{aligned}$$

$$\begin{aligned}
 c_{24}^{to'} &= -\frac{6c^2(-33+40c)(3+c-9N)(1+N)(c+6N)(6N+c(-3+2N))}{(21+4c)(6+5c)^2(c+6(-1+N)N)^2}, \\
 c_{24}^{go'} &= -\frac{9c^2(-24+7c)(3+c-9N)(1+N)(c+6N)(6N+c(-3+2N))}{2(21+4c)(6+5c)^2(c+6(-1+N)N)^2}, \\
 c_{24}^{go''} &= \frac{18c(-6N^2+c(-1+4N))}{(6+5c)(c+6(-1+N)N)}. \tag{3.45}
 \end{aligned}$$

To find the explicit form of $\{O_2 O_4\}_{-2}(w)$ for general N , we examine the operators in $\{O_2 O_4\}_{-2}(w)$ for $N = 2$ and write down the possible various spin-4 fields with unknown coefficients. Then we solve $[\{O_2 O_4\}_{-2} - (\text{the various spin-4 fields})](w) = 0$ for unknown coefficients when $N = 2, 3, 4, 5$ cases. The second-order pole $\{O_2 O_4\}_{-2}(w)$, which consists of twenty-nine terms, is presented as follows:

$$\begin{aligned}
 \{O_2 O_4\}_{-2}(w) &= c_1 J^{ab} J^{ab} J^{cd} J^{cd}(w) + c_2 J^{ab} J^{ab} J^{cN'} J^{cN'}(w) + c_3 J^{ab} J^{ac} J^{bd} J^{cd}(w) \\
 &\quad + c_4 J^{ab} J^{ac} J^{bN'} J^{bN'}(w) + c_5 J^{ab} J^{aN'} \partial J^{bN'}(w) + c_6 J^{aN'} J^{aN'} J^{bN'} J^{bN'}(w) \\
 &\quad + c_7 \partial J^{ab} \partial J^{ab}(w) + c_8 \partial^2 J^{ab} J^{ab}(w) + c_9 \partial^2 J^{aN'} J^{aN'}(w) \\
 &\quad + c_{10} K^a K^b J^{ab} J^{cd} J^{cd}(w) + c_{11} K^a K^b J^{ab} J^{cN'} J^{cN'}(w) + c_{12} K^a K^b J^{ac} J^{bd} J^{cd}(w) \\
 &\quad + c_{13} K^a K^b J^{ac} J^{bN'} J^{cN'}(w) + c_{14} K^a K^b J^{aN'} \partial J^{bN'}(w) + c_{15} K^a K^b K^c K^d J^{ab} J^{cd}(w) \\
 &\quad + c_{16} K^a K^b \partial K^c K^c J^{ab}(w) + c_{17} K^a \partial K^b J^{ac} J^{bc}(w) + c_{18} K^a \partial K^b J^{aN'} J^{bN'}(w) \\
 &\quad + c_{19} K^a \partial K^b \partial J^{ab}(w) + c_{20} K^a \partial^2 K^b J^{ab}(w) + c_{21} \partial J^{aN'} \partial J^{aN'}(w) \\
 &\quad + c_{22} \partial K^a K^a J^{bc} J^{bc}(w) + c_{23} \partial K^a K^a J^{bN'} J^{bN'}(w) + c_{24} K^a K^b \partial^2 J^{ab}(w) \\
 &\quad + c_{25} K^a K^b J^{ac} \partial J^{bc}(w) + c_{26} \partial K^a K^a \partial K^b K^b(w) + c_{27} \partial K^a \partial K^b J^{ab}(w) \\
 &\quad + c_{28} \partial^2 K^a \partial K^a(w) + c_{29} \partial^3 K^a K^a(w), \tag{3.46}
 \end{aligned}$$

where $N' \equiv 2N + 1$. The coefficients in (3.46) for general N are presented in (C.1) of appendix C. All of the results regarding the OPE $O_2(z) O_4(w)$ are checked for $N = 6$ again.

Therefore, the spin-4 current $O_{4'}(z)$ is summarized by (3.42) and (3.45). The OPE $O_2(z) O_4(w)$ for general N is obtained from (3.39) with (3.45). The fusion rule is summarized by $[O_2][O_4] = [I] + [O_2] + [O_4] + [O_{4'}]$.

• **The OPE $O_{\frac{5}{2}}(z) O_{\frac{7}{2}}(w)$.** Now we move to the OPE between $O_{\frac{5}{2}}(z)$ and $O_{\frac{7}{2}}(z)$. The computation of the OPE for $N = 2$ shows the following OPE

$$\begin{aligned}
 O_{\frac{5}{2}}(z) O_{\frac{7}{2}}(w) &= \frac{1}{(z-w)^4} c_{\frac{5}{2}\frac{7}{2}}^o O_2(w) + \frac{1}{(z-w)^3} \frac{1}{4} c_{\frac{5}{2}\frac{7}{2}}^{o'} \partial O_2(w) \\
 &\quad + \frac{1}{(z-w)^2} \left[\frac{1}{20} c_{\frac{5}{2}\frac{7}{2}}^o \partial^2 O_2 + c_{\frac{5}{2}\frac{7}{2}}^{o'} O_4 + c_{\frac{5}{2}\frac{7}{2}}^{o''} O_{4'} \right. \\
 &\quad \left. + c_{\frac{5}{2}\frac{7}{2}}^{to'} \left(T O_2 - \frac{3}{10} \partial^2 O_2 \right) + c_{\frac{5}{2}\frac{7}{2}}^{go'} \left(G O_{\frac{5}{2}} - \frac{1}{5} \partial^2 O_2 \right) \right] (w) \\
 &\quad + \frac{1}{(z-w)} \left[\frac{1}{120} c_{\frac{5}{2}\frac{7}{2}}^o \partial^3 O_2 + \frac{3}{8} c_{\frac{5}{2}\frac{7}{2}}^{o'} \partial O_4 + \frac{3}{8} c_{\frac{5}{2}\frac{7}{2}}^{o''} \partial O_{4'} \right. \\
 &\quad \left. + \frac{3}{8} c_{\frac{5}{2}\frac{7}{2}}^{to'} \partial \left(T O_2 - \frac{3}{10} \partial^2 O_2 \right) + \frac{3}{8} c_{\frac{5}{2}\frac{7}{2}}^{go'} \partial \left(G O_{\frac{5}{2}} - \frac{1}{5} \partial^2 O_2 \right) \right. \\
 &\quad \left. + c_{\frac{5}{2}\frac{7}{2}}^{to'} \left(T \partial O_2 - \partial T O_2 - \frac{1}{6} \partial^3 O_2 \right) + c_{\frac{5}{2}\frac{7}{2}}^{go'} \left(G \partial O_{\frac{5}{2}} - \frac{5}{3} \partial G O_{\frac{5}{2}} - \frac{1}{9} \partial^3 O_2 \right) \right. \\
 &\quad \left. + c_{\frac{5}{2}\frac{7}{2}}^{go''} \left(G O_{\frac{7}{2}} - \frac{1}{4} \partial O_{4'} \right) \right] (w) + \dots \tag{3.47}
 \end{aligned}$$

The structure of this OPE is very similar to (3.39) except that it has the third-order pole and there are the descendant fields of $O_2(z)$ in the right hand side of the OPE. The relative coefficients of descendant fields are fixed by (1.3). No new primary fields appear in (3.47). The quasi-primary and primary fields are obtained by the same method we used in the previous OPEs. To find the structure constants for general N , we continue to compute the OPE for $N = 3, 4, 5$ cases. Then we solve the following equations for the structure constants when $N = 3, 4, 5$:

$$\begin{aligned} \left[\{O_{\frac{5}{2}} O_{\frac{7}{2}}\}_{-4} - c_{\frac{5}{2} \frac{7}{2}}^o O_2 \right] (w) &= 0, \\ \left[\{O_{\frac{5}{2}} O_{\frac{7}{2}}\}_{-2} - \text{the 2nd-order pole} \right] (w) &= 0, \\ \left[\{O_{\frac{5}{2}} O_{\frac{7}{2}}\}_{-1} - \text{the 1st-order pole} \right] (w) &= 0. \end{aligned}$$

Here the 2nd-order pole and 1st-order pole are given in the right hand side of (3.47) respectively. After that, we put all the results together and find the general forms of structure constants. The general expression of structure constants in (3.47) which holds for general N are given by

$$\begin{aligned} c_{\frac{5}{2} \frac{7}{2}}^o &= -\frac{24c^2(-3+2c)(3+c-9N)(1+N)(c+6N)(6N+c(-3+2N))}{(21+4c)(6+5c)(c+6(-1+N)N)^2}, \\ c_{\frac{5}{2} \frac{7}{2}}^{o'} &= -\frac{4c(21+4c)(-6N^2+c(-1+4N))}{21(6+5c)(c+6(-1+N)N)}, \quad c_{\frac{5}{2} \frac{7}{2}}^{o''} = 8, \\ c_{\frac{5}{2} \frac{7}{2}}^{to} &= -\frac{48c^2(-3+2c)(69+29c)(3+c-9N)(1+N)(c+6N)(6N+c(-3+2N))}{(29+2c)(21+4c)(6+5c)^2(c+6(-1+N)N)^2}, \\ c_{\frac{5}{2} \frac{7}{2}}^{go} &= -\frac{18c^2(-3+2c)(-18+23c)(3+c-9N)(1+N)(c+6N)(6N+c(-3+2N))}{(29+2c)(21+4c)(6+5c)^2(c+6(-1+N)N)^2}, \\ c_{\frac{5}{2} \frac{7}{2}}^{to'} &= \frac{6c^2(-51+25c)(3+c-9N)(1+N)(c+6N)(6N+c(-3+2N))}{(21+4c)(6+5c)^2(c+6(-1+N)N)^2}, \\ c_{\frac{5}{2} \frac{7}{2}}^{go'} &= \frac{9c^2(-30+29c)(3+c-9N)(1+N)(c+6N)(6N+c(-3+2N))}{4(21+4c)(6+5c)^2(c+6(-1+N)N)^2}, \\ c_{\frac{5}{2} \frac{7}{2}}^{go''} &= -\frac{18c(-6N^2+c(-1+4N))}{(6+5c)(c+6(-1+N)N)}. \end{aligned} \quad (3.48)$$

All of the results regarding the OPE $O_{\frac{5}{2}}(z)O_{\frac{7}{2}}(w)$ are checked for $N = 6$ again. The OPE $O_{\frac{5}{2}}(z)O_{\frac{7}{2}}(w)$ for general N is obtained from (3.47) with (3.48). The fusion rule is summarized by $[O_{\frac{5}{2}}][O_{\frac{7}{2}}] = [I] + [O_2] + [O_4] + [O_{4'}]$.

• **The OPE $O_{\frac{5}{2}}(z)O_4(w)$.** Now let us move to the OPE between $O_{\frac{5}{2}}(z)$ and $O_4(z)$. The computation of the OPE for $N = 2$ turns out to be

$$\begin{aligned} O_{\frac{5}{2}}(z)O_4(w) &= \frac{1}{(z-w)^4} c_{\frac{5}{2} 4}^o O_{\frac{5}{2}}(w) \\ &\quad + \frac{1}{(z-w)^3} \left[\frac{1}{5} c_{\frac{5}{2} 4}^o \partial O_{\frac{5}{2}} + c_{\frac{5}{2} 4}^{o'} O_{\frac{7}{2}} + c_{\frac{5}{2} 4}^{go} \left(GO_2 - \frac{2}{5} \partial O_{\frac{5}{2}} \right) \right] (w) \\ &\quad + \frac{1}{(z-w)^2} \left[\frac{1}{30} c_{\frac{5}{2} 4}^o \partial^2 O_{\frac{5}{2}} + \frac{2}{7} c_{\frac{5}{2} 4}^{o'} \partial O_{\frac{7}{2}} + \frac{2}{7} c_{\frac{5}{2} 4}^{go} \partial \left(GO_2 - \frac{2}{5} \partial O_{\frac{5}{2}} \right) \right] \end{aligned} \quad (3.49)$$

$$\begin{aligned}
 & + c_{\frac{5}{2}4}^{go'} \left(\partial G O_2 - \frac{3}{4} G \partial O_2 + \frac{1}{8} \partial^2 O_{\frac{5}{2}} \right) + c_{\frac{5}{2}4}^{to} \left(T O_{\frac{5}{2}} - \frac{1}{4} \partial^2 O_{\frac{5}{2}} \right) + c_{\frac{5}{2}4}^{o''} O_{\frac{9}{2}} \Big] (w) \\
 & + \frac{1}{(z-w)} \left[\frac{1}{210} c_{\frac{5}{2}4}^o \partial^3 O_{\frac{5}{2}} + \frac{3}{56} c_{\frac{5}{2}4}^{o'} \partial^2 O_{\frac{7}{2}} + \frac{3}{56} c_{\frac{5}{2}4}^{go} \partial^2 \left(G O_2 - \frac{2}{5} \partial O_{\frac{5}{2}} \right) \right. \\
 & + \frac{1}{3} c_{\frac{5}{2}4}^{go'} \partial \left(\partial G O_2 - \frac{3}{4} G \partial O_2 + \frac{1}{8} \partial^2 O_{\frac{5}{2}} \right) + \frac{1}{3} c_{\frac{5}{2}4}^{to} \partial \left(T O_{\frac{5}{2}} - \frac{1}{4} \partial^2 O_{\frac{5}{2}} \right) + \frac{1}{3} c_{\frac{5}{2}4}^{o''} \partial O_{\frac{9}{2}} \\
 & + c_{\frac{5}{2}4}^{go''} \left(\partial^2 G O_2 + \frac{3}{5} G \partial^2 O_2 - 2 \partial G \partial O_2 - \frac{2}{35} \partial^3 O_{\frac{5}{2}} \right) + c_{\frac{5}{2}4}^{to'} \left(T O_{\frac{7}{2}} - \frac{3}{16} \partial^2 O_{\frac{7}{2}} \right) \\
 & \left. + c_{\frac{5}{2}4}^{go'''} \left(G O_4 - \frac{1}{8} \partial^2 O_{\frac{7}{2}} \right) + c_{\frac{5}{2}4}^{to''} \left(T \partial O_{\frac{5}{2}} - \frac{5}{4} \partial T O_{\frac{5}{2}} - \frac{1}{7} \partial^3 O_{\frac{5}{2}} \right) \right] (w) + \dots
 \end{aligned}$$

As done before, the relative coefficients of descendant fields are fixed by (1.3), and the quasi-primary, and primary fields are found by the same method we used before. No new primary field is found in this case. The field contents up to the second-order pole are the same as the one in (3.33). To find the general forms of structure constants, we continue to compute the OPE between $O_{\frac{5}{2}}(z)$ and $O_4(w)$ using the package for $N = 3, 4, 5$ cases. After computing the OPEs for $N = 3, 4, 5$, we solve the following equations for structure constants when $N = 3, 4, 5$:

$$\begin{aligned}
 & \left[\{O_{\frac{5}{2}} O_4\}_{-4} - c_{\frac{5}{2}4}^o O_{\frac{5}{2}} \right] (w) = 0, \\
 & \left[\{O_{\frac{5}{2}} O_4\}_{-3} - \frac{1}{5} c_{\frac{5}{2}4}^o \partial O_{\frac{5}{2}} - c_{\frac{5}{2}4}^{o'} O_{\frac{7}{2}} - c_{\frac{5}{2}4}^{go} \left(G O_2 - \frac{2}{5} \partial O_{\frac{5}{2}} \right) \right] (w) = 0, \\
 & \left[\{O_{\frac{5}{2}} O_4\}_{-2} - \text{the 2nd-order pole} \right] (w) = 0, \\
 & \left[\{O_{\frac{5}{2}} O_4\}_{-1} - \text{the 1st-order pole} \right] (w) = 0.
 \end{aligned}$$

Here the 2nd-order pole and 1st-order pole are given in the right hand side of (3.49) respectively. Then we put all the results together and find the structure constants for general N . The general expressions of structure constants in (3.49) are given by

$$\begin{aligned}
 c_{\frac{5}{2}4}^o &= \frac{42c^2(-3+2c)(3+c-9N)(1+N)(c+6N)(6N+c(-3+2N))}{(21+4c)(6+5c)(c+6(-1+N)N)^2}, \\
 c_{\frac{5}{2}4}^{o'} &= \frac{4c(21+4c)(-6N^2+c(-1+4N))}{3(6+5c)(c+6(-1+N)N)}, \\
 c_{\frac{5}{2}4}^{go} &= \frac{216c^2(-3+2c)(3+c-9N)(1+N)(c+6N)(6N+c(-3+2N))}{(21+4c)(6+5c)^2(c+6(-1+N)N)^2}, \\
 c_{\frac{5}{2}4}^{go'} &= \frac{72c^2(-67+26c)(3+c-9N)(1+N)(c+6N)(6N+c(-3+2N))}{7(29+2c)(21+4c)(6+5c)(c+6(-1+N)N)^2}, \\
 c_{\frac{5}{2}4}^{to} &= \frac{72c^2(-19+14c)(3+c-9N)(1+N)(c+6N)(6N+c(-3+2N))}{(29+2c)(21+4c)(6+5c)(c+6(-1+N)N)^2}, \\
 c_{\frac{5}{2}4}^{o''} &= -9, \quad c_{\frac{5}{2}4}^{go''} = \frac{45c^2(-3+2c)(3+c-9N)(1+N)(c+6N)(6N+c(-3+2N))}{(21+4c)(6+5c)^2(c+6(-1+N)N)^2}, \\
 c_{\frac{5}{2}4}^{to'} &= \frac{36c(-6N^2+c(-1+4N))}{(6+5c)(c+6(-1+N)N)}, \quad c_{\frac{5}{2}4}^{go'''} = -\frac{18c(-6N^2+c(-1+4N))}{(6+5c)(c+6(-1+N)N)}, \quad (3.50) \\
 c_{\frac{5}{2}4}^{to''} &= -\frac{144c^2(-3+2c)(1+N)(c^2-3c(-1+N)+18(1-3N)N)(6N+c(-3+2N))}{(21+4c)(6+5c)^2(c+6(-1+N)N)^2}.
 \end{aligned}$$

All of the results regarding the OPE $O_{\frac{5}{2}}(z)O_4(w)$ are checked for $N = 6$ again. The OPE $O_{\frac{5}{2}}(z)O_4(w)$ for general N is obtained from (3.49) with (3.50). The fusion rule is summarized by $[O_{\frac{5}{2}}][O_4] = [I] + [O_{\frac{5}{2}}] + [O_{\frac{7}{2}}] + [O_{\frac{9}{2}}]$.

The $\mathcal{N} = 1$ fusion rule is summarized by $[\hat{O}_2][\hat{O}_{\frac{7}{2}}] = [\hat{O}_2] + [\hat{O}_{\frac{7}{2}}] + [\hat{O}_4]$. The explicit OPE is given by (F.5). Among four OPEs between this $\mathcal{N} = 1$ supermultiplet, half of them are quite related to the other because they have common field contents. By rescaling the currents as $O_{\frac{7}{2}}(z) \rightarrow N_{\frac{7}{2}}O_{\frac{7}{2}}(z)$ and $O_4(z) \rightarrow N_4O_4(z)$, where

$$\begin{aligned} N_{\frac{7}{2}}^2 &= 3M(21 + 4c)(6 + 5c)(c + 6(-1 + N)N)^4, & N_4^2 &= \frac{1}{8}N_{\frac{7}{2}}^2, \\ \frac{1}{M} &\equiv (14c^4(-3 + 2c)(3 + c - 9N)(1 + N)(c + 6N) \\ &\quad \times (-3c + 2(3 + c)N)(2c(-1 + N) + 3(3 - 4N)N)(c + 2cN + 3N(-3 + 2N))), \end{aligned}$$

the standard normalizations arise: $O_{\frac{7}{2}}(z)O_{\frac{7}{2}}(w) \rightarrow \frac{1}{(z-w)^7} \frac{2c}{7} + \dots$ and $O_4(z)O_4(w) \rightarrow \frac{1}{(z-w)^8} \frac{c}{4} + \dots$. We have checked this up to $N = 5$. Therefore all the previous OPEs can be rewritten in terms of these rescaled currents.

3.3 The OPEs between the higher spin currents of spins- $(2', \frac{5}{2})$ and the higher spin currents of spins- $(4', \frac{9}{2})$

• **The OPE $O_2(z)O_{4'}(w)$.** Now let us consider the OPE between $O_2(z)$ and $O_{4'}(z)$. The final result for $N = 2$ is presented first, which explains how this result can be obtained explicitly

$$\begin{aligned} O_2(z)O_{4'}(w) &= \frac{1}{(z-w)^4}c_{24'}^o O_2 + \frac{1}{(z-w)^2} \left[c_{24'}^{o'} O_4 + c_{24'}^{o''} O_{4'} + A_4 \right. \\ &\quad \left. + c_{24'}^{to} \left(TO_2 - \frac{3}{10} \partial^2 O_2 \right) + c_{24'}^{go} \left(GO_{\frac{5}{2}} - \frac{1}{5} \partial^2 O_2 \right) \right] (w) \\ &\quad + \frac{1}{(z-w)} \left[\frac{1}{4} c_{24'}^{o'} \partial O_4 + \frac{1}{4} c_{24'}^{o''} \partial O_{4'} + \frac{1}{4} \partial A_4 + \frac{1}{4} c_{24'}^{to} \partial \left(TO_2 - \frac{3}{10} \partial^2 O_2 \right) \right. \\ &\quad \left. + \frac{1}{4} c_{24'}^{go} \partial \left(GO_{\frac{5}{2}} - \frac{1}{5} \partial^2 O_2 \right) + c_{24'}^{to'} \left(T \partial O_2 - \partial T O_2 - \frac{1}{6} \partial^3 O_2 \right) + \right. \\ &\quad \left. + c_{24'}^{go'} \left(G \partial O_{\frac{5}{2}} - \frac{5}{3} \partial G O_{\frac{5}{2}} - \frac{1}{9} \partial^3 O_2 \right) + c_{24'}^{go''} \left(G O_{\frac{7}{2}} - \frac{1}{4} \partial O_4 \right) \right] (w) + \dots \end{aligned} \quad (3.51)$$

The structure of this OPE appears similar to (3.39) except that the OPE (3.51) has a composite spin-4 primary field $A_4(z)$. The relative coefficients of descendant fields appearing in the first-order pole are fixed by the formula (1.3). For the second-order pole, as performed before, we compute $T(z)\{O_2 O_{4'}\}_{-2}(w)$ and $G(z)\{O_2 O_{4'}\}_{-2}(w)$ to find the quasi-primary fields in the second-order pole. By subtracting two candidates from the second-order singular terms, we want to check if the following equation holds:

$$\left[\{O_2 O_{4'}\}_{-2} - c_{24'}^{o'} O_4 - c_{24'}^{o''} O_{4'} - c_{24'}^{to} \left(TO_2 - \frac{3}{10} \partial^2 O_2 \right) - c_{24'}^{go} \left(GO_{\frac{5}{2}} - \frac{1}{5} \partial^2 O_2 \right) \right] (w) = 0. \quad (3.52)$$

It turns out that there is no solution for (3.52). This implies that there should be another primary spin-4 field $A_4(z)$. Since we already have found two spin-4 primary fields, we expect that the new spin-4 field would be a composite field in terms of known currents (we have determined so far). Otherwise, the spin contents of (1.2) will not be correct. To check this, let us first express $A_4(z)$ as

$$A_4(z) = \{O_2 O_{4'}\}_{-1}(z) - \left[c_{24'}^{o'} O_4 + c_{24'}^{o''} O_{4'} + c_{24'}^{to} \left(T O_2 - \frac{3}{10} \partial^2 O_2 \right) + c_{24'}^{go} \left(G O_{\frac{5}{2}} - \frac{1}{5} \partial^2 O_2 \right) \right](z). \quad (3.53)$$

Then we try to express $A_4(z)$ in terms of other possible spin-4 fields. It turns out that $A_4(z)$ can be written as

$$\begin{aligned} A_4(z) = & c_4^{gg} \left(G \partial G - \frac{7}{10} \partial^2 T \right)(z) + c_4^{go} \left(G O_{\frac{5}{2}} - \frac{1}{5} \partial^2 O_2 \right)(z) \\ & + c_4^{tt} \left(T T - \frac{3}{10} \partial^2 T \right)(z) + c_4^{to} \left(T O_2 - \frac{3}{10} \partial^2 O_2 \right)(z) \\ & + c_4^{oo} \left(O_2 O_2 - \frac{3}{10} (-2\tilde{c} + c) \partial^2 O_2 + \frac{3}{10} (\tilde{c} - c) \partial^2 T \right)(z) + c_4^2 O_4(z), \end{aligned} \quad (3.54)$$

where $\tilde{c}$ is the central charge of coset $\frac{\widehat{SO}(2N+1)_k}{\widehat{SO}(2N)_k}$. This field (3.54) containing $O_2 O_2(z)$ corresponds to the $A^{(4,0)}$ in [7]. As expected it obeys the following OPEs:

$$\begin{aligned} T(z) A_4(w) &= \frac{1}{(z-w)^2} 4 A_4(w) + \frac{1}{(z-w)} \partial A_4(w) + \dots, \\ G(z) A_4(w) &= \frac{1}{(z-w)} A_{\frac{9}{2}}(w) + \dots, \end{aligned} \quad (3.55)$$

where $A_{\frac{9}{2}}(z)$ is a composite primary spin- $\frac{9}{2}$ field and the superpartner of $A_4(z)$. It will appear in (3.58) or (3.59) in the next OPE soon. For the first-order pole, the derivative terms are completely determined and one can find the extra quasi-primary fields by the same procedure which we have performed.

We continue to compute the OPE between $O_2(z)$ and $O_{4'}(w)$ by the package [14] for $N = 3, 4, 5$ cases to find the general forms of structure constants. To find the structure constant $c_{24'}^o$, we should solve $[\{O_2 O_{4'}\}_{-4} - c_{24'}^o O_2](w) = 0$. To find the other structure constants $c_{24'}^{o'}$, $c_{24'}^{o''}$, $c_{24'}^{to}$, and $c_{24'}^{go}$, we use (3.53), and (3.55). Moreover, we solve $[\{O_2 O_{4'}\}_{-1} - (\text{the 1st-order pole})](w) = 0$ to find the remaining structure constants $c_{24'}^{to'}$, $c_{24'}^{go'}$, and $c_{24'}^{go''}$. To find the coefficients in (3.54), we use (3.54) and (3.55). We do this procedure for $N = 3, 4, 5$ cases also. Then the general expressions of the coefficients in (3.54) are given by

$$\begin{aligned} c_4^{gg} &= -336F(-33+c)c^4(29+2c)(6+5c)(3+c-9N)(1+N)(c+6N) \\ &\quad \times (-3c+2(3+c)N)(2c(-1+N)+3(3-4N)N)(c+2cN+3N(-3+2N)), \\ c_4^{go} &= 1008c^3F(-33+c)(21+4c)(-7+10c)(3+c-9N)(1+N)(c+6N) \\ &\quad \times (-3c+2(3+c)N)(c+6(-1+N)N)(-6N^2+c(-1+4N)), \end{aligned}$$

$$\begin{aligned}
 c_4^{tt} &= 112c^4 F(29+2c)(6+5c)(3+88c)(3+c-9N)(1+N)(c+6N) \\
 &\quad \times (-3c+2(3+c)N)(2c(-1+N)+3(3-4N)N)(c+2cN+3N(-3+2N)), \\
 c_4^{to} &= 672c^3 F(21+4c)(-7+10c)(129+22c)(3+c-9N)(1+N)(c+6N) \\
 &\quad \times (-3c+2(3+c)N)(c+6(-1+N)N)(-6N^2+c(-1+4N)), \\
 c_4^o &= 72c^2 F(29+2c)(21+4c)(6+5c)(-7+10c)(3+c-9N)(1+N) \\
 &\quad \times (c+6N)(-3c+2(3+c)N)(c+6(-1+N)N)^2, \\
 c_4^{oo} &= 504c^2 F(29+2c)(21+4c)(6+5c)(-7+10c)(3+c-9N)(1+N) \\
 &\quad \times (c+6N)(-3c+2(3+c)N)(c+6(-1+N)N)^2, \\
 F &\equiv \frac{1}{21(29+2c)^2(21+4c)(6+5c)^3(c+6(-1+N)N)^4}.
 \end{aligned}$$

The structure constants in (3.51) for general N are given by

$$\begin{aligned}
 c_{24'}^o &= 210c^3 B(-3+2c)(29+2c)(6+5c)(1+N)(6N+c(-3+2N))(108N^3(-1+3N) \\
 &\quad +c^3(-1+4N)-3c^2(1-5N+6N^2)-18cN(1-6N+11N^2)), \\
 c_{24'}^{o'} &= 3c^2 B(29+2c)(21+4c)(6+5c)(c+6(-1+N)N)(10c^3(-6-N+2N^2) \\
 &\quad -126N^2(25-50N+12N^2)-c^2(42-623N+496N^2+60N^3) \\
 &\quad -3cN(-350+1155N-1246N^2+720N^3)), \\
 c_{24'}^{o''} &= 28cB(29+2c)^2(6+5c)(84+25c)(c+6(-1+N)N)^2(c-4cN+6N^2), \\
 c_{24'}^{to} &= 840c^3 BB'(-3+2c)(69+10c)(3+c-9N)(1+N)(c+6N)(6N+c(-3+2N)), \\
 c_{24'}^{go} &= -23940c^3 BB'(-3+2c)(3+c-9N)(1+N)(c+6N)(6N+c(-3+2N)), \\
 c_{24'}^{to'} &= -210c^3 B(29+2c)(-3+10c)(1+N)(6N+c(-3+2N))(108N^3(-1+3N) \\
 &\quad +c^3(-1+4N)-3c^2(1-5N+6N^2)-18cN(1-6N+11N^2)), \\
 c_{24'}^{go'} &= 945c^3 B(29+2c)(1+N)(6N+c(-3+2N))(108N^3(-1+3N)+c^3(-1+4N) \\
 &\quad -3c^2(1-5N+6N^2)-18cN(1-6N+11N^2)), \\
 c_{24'}^{go''} &= 63c^2 B(29+2c)(6+5c)(c+6(-1+N)N)(10c^3(-6-N+2N^2) \\
 &\quad -126N^2(25-50N+12N^2)-c^2(42-623N+496N^2+60N^3) \\
 &\quad -3cN(-350+1155N-1246N^2+720N^3)), \\
 B &\equiv \frac{1}{147(29+2c)^2(6+5c)^2(c+6(-1+N)N)^3}, \quad B' \equiv (-6N^2+c(-1+4N)). \quad (3.56)
 \end{aligned}$$

All of the results regarding the OPE $O_2(z)O_{4'}(w)$ are checked for $N=6$ again. The OPE for general N is obtained from (3.51) with (3.56). The fusion rule is summarized by $[O_2][O_{4'}] = [I] + [O_2] + [O_4] + [O_{4'}]$.

• **The OPE $O_2(z)O_{\frac{9}{2}}(w)$.** Now let us compute the OPE between $O_2(z)$ and $O_{\frac{9}{2}}(z)$. The computation of the OPE for $N=2$ leads to the OPE

$$\begin{aligned}
 O_2(z)O_{\frac{9}{2}}(w) &= \frac{1}{(z-w)^4} c_{2\frac{9}{2}}^o O_{\frac{5}{2}}(w) \\
 &\quad + \frac{1}{(z-w)^3} \left[c_{2\frac{9}{2}}^{o'} O_{\frac{7}{2}} + c_{2\frac{9}{2}}^{go} \left(GO_2 - \frac{2}{5} \partial O_{\frac{5}{2}} \right) \right] (w)
 \end{aligned} \quad (3.57)$$

$$\begin{aligned}
 & + \frac{1}{(z-w)^2} \left[\frac{1}{7} c_{2\frac{9}{2}}^{o'} \partial O_{\frac{7}{2}} + \frac{1}{7} c_{2\frac{9}{2}}^{go} \partial \left(GO_2 - \frac{2}{5} \partial O_{\frac{5}{2}} \right) + c_{2\frac{9}{2}}^{to} \left(TO_{\frac{5}{2}} - \frac{1}{4} \partial^2 O_{\frac{5}{2}} \right) \right. \\
 & \left. + c_{2\frac{9}{2}}^{go'} \left(\partial GO_2 - \frac{3}{4} G \partial O_2 + \frac{1}{8} \partial^2 O_{\frac{5}{2}} \right) + c_{2\frac{9}{2}}^{o''} O_{\frac{9}{2}} + c_{2\frac{9}{2}}^a A_{\frac{9}{2}} \right] (w) \\
 & + \frac{1}{(z-w)} \left[\frac{1}{56} c_{2\frac{9}{2}}^{o'} \partial^2 O_{\frac{7}{2}} + \frac{1}{56} c_{2\frac{9}{2}}^{go} \partial^2 \left(GO_2 - \frac{2}{5} \partial O_{\frac{5}{2}} \right) + \frac{2}{9} c_{2\frac{9}{2}}^{to} \partial \left(TO_{\frac{5}{2}} - \frac{1}{4} \partial^2 O_{\frac{5}{2}} \right) \right. \\
 & \left. + \frac{2}{9} c_{2\frac{9}{2}}^{go'} \partial \left(\partial GO_2 - \frac{3}{4} G \partial O_2 + \frac{1}{8} \partial^2 O_{\frac{5}{2}} \right) + \frac{2}{9} c_{2\frac{9}{2}}^{o''} \partial O_{\frac{9}{2}} + \frac{2}{9} c_{2\frac{9}{2}}^a \partial A_{\frac{9}{2}} \right. \\
 & \left. + c_{2\frac{9}{2}}^{go''} \left(\partial^2 GO_2 + \frac{3}{5} G \partial^2 O_2 - 2 \partial G \partial O_2 - \frac{2}{35} \partial^3 O_{\frac{5}{2}} \right) + c_{2\frac{9}{2}}^{to'} \left(TO_{\frac{7}{2}} - \frac{3}{16} \partial^2 O_{\frac{7}{2}} \right) \right. \\
 & \left. + c_{2\frac{9}{2}}^{o'''} \left(T \partial O_{\frac{5}{2}} - \frac{5}{4} \partial T O_{\frac{5}{2}} - \frac{1}{7} \partial^3 O_{\frac{5}{2}} \right) + c_{2\frac{9}{2}}^{go'''} \left(GO_{4'} - \frac{2}{9} \partial O_{\frac{9}{2}} \right) \right. \\
 & \left. + c_{2\frac{9}{2}}^{ga} \left(GA_4 - \frac{2}{9} \partial A_{\frac{9}{2}} \right) + c_{2\frac{9}{2}}^{go''''} \left(GO_4 - \frac{1}{8} \partial^2 O_{\frac{7}{2}} \right) + c_{2\frac{9}{2}}^q Q_{\frac{11}{2}} + O_{\frac{11}{2}} \right] (w) + \dots
 \end{aligned}$$

The relative coefficients of descendant fields are fixed by (1.3). As predicted by (1.3), there are no descendant fields of $O_{\frac{5}{2}}(z)$ in the OPE. For the second-order pole, there is a composite primary spin- $\frac{9}{2}$ field $A_{\frac{9}{2}}(z)$ which is the superpartner $A_4(z)$ in (3.51). It is found by the same method we used to find $A_4(z)$. The explicit form of $A_{\frac{9}{2}}(z)$ is

$$\begin{aligned}
 A_{\frac{9}{2}}(z) = & \frac{1}{c_{2\frac{9}{2}}^a} \{O_2 O_{\frac{9}{2}}\}_{-2}(z) - \frac{1}{c_{2\frac{9}{2}}^a} \left[\frac{1}{7} c_{2\frac{9}{2}}^{o'} \partial O_{\frac{7}{2}} + \frac{1}{7} c_{2\frac{9}{2}}^{go} \partial \left(GO_2 G - \frac{2}{5} \partial O_{\frac{5}{2}} \right) \right. \\
 & \left. + c_{2\frac{9}{2}}^{to} \left(TO_{\frac{5}{2}} - \frac{1}{4} \partial^2 O_{\frac{5}{2}} \right) + c_{2\frac{9}{2}}^{go'} \left(\partial GO_2 - \frac{3}{4} G \partial O_2 + \frac{1}{8} \partial^2 O_{\frac{5}{2}} \right) + c_{2\frac{9}{2}}^{o''} O_{\frac{9}{2}} \right] (z).
 \end{aligned} \tag{3.58}$$

In terms of other spin- $\frac{9}{2}$ fields, it is expressed as

$$\begin{aligned}
 A_{\frac{9}{2}}(z) = & c_{\frac{9}{2}}^{go} \left(\partial GO_2 - \frac{3}{4} G \partial O_2 + \frac{1}{8} \partial^2 O_{\frac{5}{2}} \right) (z) + c_{2\frac{9}{2}}^{to} \left(TO_{\frac{5}{2}} - \frac{1}{4} \partial^2 O_{\frac{5}{2}} \right) (z) \\
 & + c_{2\frac{9}{2}}^{tg} \left(T \partial G - \frac{3}{4} \partial T G - \frac{1}{5} \partial^3 G \right) (z) \\
 & + c_{\frac{9}{2}}^{oo} \left(O_2 O_{\frac{5}{2}} - \frac{3}{4} c_1' \partial T G - \frac{3}{4} c_2' G \partial O_2 - \frac{1}{120} (2c_4' - 9c_1') \partial^3 G \right. \\
 & \left. - \frac{1}{40} (4c_3' - 9c_2') \partial^2 O_{\frac{5}{2}} - \frac{3}{7} \partial O_{\frac{7}{2}} \right) (z),
 \end{aligned} \tag{3.59}$$

where the structure constants are

$$\begin{aligned}
 c_1' &= \frac{6c^2(2c(-1+N) + 3(3-4N)N)(c+2cN+3N(-3+2N))}{(21+4c)(c+6(-1+N)N)^2}, \\
 c_2' &= \frac{18c(-6N^2+c(-1+4N))}{(6+5c)(c+6(-1+N)N)}, \quad c_3' = \frac{c(c-4cN+6N^2)}{3(c+6(-1+N)N)}, \\
 c_4' &= \frac{c^2(2c(-1+N) + 3(3-4N)N)(c+2cN+3N(-3+2N))}{3(c+6(-1+N)N)^2}.
 \end{aligned}$$

The spin- $\frac{9}{2}$ field $A_{\frac{9}{2}}(z)$ obeys the following OPEs:

$$T(z) A_{\frac{9}{2}}(w) = \frac{1}{(z-w)^2} \frac{9}{2} A_{\frac{9}{2}}(w) + \frac{1}{(z-w)} \partial A_{\frac{9}{2}}(w) + \dots,$$

$$G(z) A_{\frac{9}{2}}(w) = \frac{1}{(z-w)^2} 8 A_4(w) + \frac{1}{(z-w)} \partial A_4(w) + \dots \quad (3.60)$$

Then, the fields $A_4(z)$ and $A_{\frac{9}{2}}(z)$ consist of $\mathcal{N} = 1$ multiplet. For the first-order pole, we compute the following OPEs:

$$\begin{aligned} T(z)[\{O_2 O_{\frac{9}{2}}\}_{-1} - (\text{desc. fields in the 1st pole})](w), \\ G(z)[\{O_2 O_{\frac{9}{2}}\}_{-1} - (\text{desc. fields in the 1st pole})](w). \end{aligned}$$

Here the descendant fields in the first-order pole contains the first six terms in (3.57). Then we examine higher-order poles with order $n > 2$ and add extra spin- $\frac{11}{2}$ quasi-primary fields to the first-order pole, and compute the OPEs with $T(z)$ and $G(z)$ again to check whether the higher-order poles are removed or not. If there are still higher-order poles, we can add another quasi-primary field and compute the OPEs with $T(z)$ and $G(z)$ again. We continue this procedure until the higher-order poles with order $n > 2$ are completely removed. In this case, removing the higher-order poles was very complicated. When we added the final quasi-primary field, $Q_{\frac{11}{2}}(z)$, we could successfully remove all higher-order poles. The $Q_{\frac{11}{2}}(z)$ is given by

$$\begin{aligned} Q_{\frac{11}{2}}(z) = & O_2 O_{\frac{7}{2}}(z) + 315c^2 H(-117 + 4(-36 + c)c) \partial^2 G O_2(z) \\ & + 1260c^2 H(-36 + c(75 + 2c)) \partial G \partial O_2(z) \\ & + 252c^2 H(6 + 5c)(-3 + 10c) T \partial O_{\frac{5}{2}}(z) \\ & + 2c^2 H(2304 + 5c(-654 + c(-247 + 2c))) \partial^3 O_{\frac{5}{2}}(z) \\ & - \frac{c(21 + 4c)(c - 4cN + 6N^2)}{28(6 + 5c)(c + 6(-1 + N)N)} \partial^2 O_{\frac{7}{2}}(z) - \frac{1}{3} \partial O_{\frac{9}{2}}(z), \end{aligned} \quad (3.61)$$

where $H \equiv \frac{(3+c-9N)(1+N)(c+6N)(-3c+2(3+c)N)}{35(29+2c)(21+4c)(6+5c)^2(c+6(-1+N)N)^2}$. This quasi-primary field (3.61) containing $O_2 O_{\frac{7}{2}}(z)$ is related to the primary field $A^{(\frac{11}{2}, 2)}$ of [7]. The coefficients are determined by the fact that the third-pole of the OPE between $T(z)$ and $Q_{\frac{11}{2}}(w)$ must vanish. As one can see, $Q_{\frac{11}{2}}(w)$ is very different from the other quasi-primary fields in the sense that the nonderivative term doesn't contain $T(z)$ or $G(z)$.⁸ The OPEs $T(z) Q_{\frac{11}{2}}(w)$ and $G(z) Q_{\frac{11}{2}}(w)$ are put in the appendix *G* and appendix *H* respectively. After finding $Q_{\frac{11}{2}}(z)$, we subtract all the derivative terms and quasi-primary fields with their coefficients in (3.57) from $\{O_2 O_{\frac{9}{2}}\}_{-1}(w)$. It doesn't vanish meaning there is a new primary spin- $\frac{11}{2}$ field. It turns out that the new primary spin- $\frac{11}{2}$ field cannot be expressed in terms of other spin- $\frac{11}{2}$ fields meaning it is not a composite field that can be written in terms of known currents. The explicit form of new primary spin- $\frac{11}{2}$, $O_{\frac{11}{2}}(z)$, is given by

$$\begin{aligned} O_{\frac{11}{2}}(w) = & \{O_2 O_{\frac{9}{2}}\}_{-1}(w) - \left[\frac{1}{56} c_{2\frac{9}{2}}^{o'} \partial^2 O_{\frac{7}{2}} + \frac{1}{56} c_{2\frac{9}{2}}^{go} \partial^2 \left(G O_2 - \frac{2}{5} \partial O_{\frac{5}{2}} \right) + \frac{2}{9} c_{2\frac{9}{2}}^{to} \partial \left(T O_{\frac{5}{2}} - \frac{1}{4} \partial^2 O_{\frac{5}{2}} \right) \right. \\ & \left. + \frac{2}{9} c_{2\frac{9}{2}}^{g'o'} \partial \left(\partial G O_2 - \frac{3}{4} G \partial O_2 + \frac{1}{8} \partial^2 O_{\frac{5}{2}} \right) + \frac{2}{9} c_{2\frac{9}{2}}^{o''} \partial O_{\frac{9}{2}} + \frac{2}{9} c_{2\frac{9}{2}}^a \partial A_{\frac{9}{2}} \right] \end{aligned}$$

⁸We believe that this should come from the general formula in (B.4) of [18]. See also the original paper [10].

$$\begin{aligned}
 & +c_{2\frac{9}{2}}^{go''} \left(\partial^2 GO_2 + \frac{3}{5} G \partial^2 O_2 - 2 \partial G \partial O_2 - \frac{2}{35} \partial^3 O_{\frac{5}{2}} \right) + c_{2\frac{9}{2}}^{to'} \left(TO_{\frac{7}{2}} - \frac{3}{16} \partial^2 O_{\frac{7}{2}} \right) \\
 & + c_{2\frac{9}{2}}^{to''} \left(TO_{\frac{5}{2}} - \frac{5}{4} \partial TO_{\frac{5}{2}} - \frac{1}{7} \partial^3 O_{\frac{5}{2}} \right) + c_{2\frac{9}{2}}^{go'''} \left(GO_{4'} - \frac{2}{9} \partial O_{\frac{9}{2}} \right) \\
 & + c_{2\frac{9}{2}}^{ga} \left(GA_4 - \frac{2}{9} \partial A_{\frac{9}{2}} \right) + c_{2\frac{9}{2}}^{go''''} \left(GO_4 - \frac{1}{8} \partial^2 O_{\frac{7}{2}} \right) + c_{2\frac{9}{2}}^g Q_{\frac{11}{2}} \Big] (w). \tag{3.62}
 \end{aligned}$$

The spin- $\frac{11}{2}$ current $O_{\frac{11}{2}}(z)$ obeys the following OPEs:

$$\begin{aligned}
 T(z) O_{\frac{11}{2}}(w) &= \frac{1}{(z-w)^2} \frac{11}{2} O_{\frac{11}{2}}(w) + \frac{1}{(z-w)} \partial O_{\frac{11}{2}}(w) + \dots, \\
 G(z) O_{\frac{11}{2}}(w) &= \frac{1}{(z-w)} O_6(w) + \dots, \tag{3.63}
 \end{aligned}$$

where $O_6(z)$ is a primary spin-6 field and the superpartner of $O_{\frac{11}{2}}(z)$. The $O_6(w)$ will appear in (3.72) in the OPE between $O_{\frac{5}{2}}(z)$ and $O_{\frac{9}{2}}(w)$ soon. To find the N -dependence of the structure constants, we compute the OPE between $O_2(z)$ and $O_{\frac{9}{2}}(w)$ for $N = 3, 4, 5, 6$ cases. In the previous OPEs, it was enough to compute OPEs up to $N = 5$ to find the N -dependence of the structure constants. But in this case, we had to compute the OPE between $O_2(z)$ and $O_{\frac{9}{2}}(w)$ up to $N = 6$ to find the N -dependence of the structure constants completely. To find the structure constants $c_{2\frac{9}{2}}^o$, $c_{2\frac{9}{2}}^{o'}$, and $c_{2\frac{9}{2}}^{go}$, we should solve the following equations:

$$\begin{aligned}
 & \left[\{O_2 O_{\frac{9}{2}}\}_{-4} - c_{2\frac{9}{2}}^o O_{\frac{5}{2}} \right] (w) = 0, \\
 & \left[\{O_2 O_{\frac{9}{2}}\}_{-3} - c_{2\frac{9}{2}}^{o'} O_{\frac{7}{2}} - c_{2\frac{9}{2}}^{go} \left(GO_2 - \frac{2}{5} \partial O_{\frac{5}{2}} \right) \right] (w) = 0.
 \end{aligned}$$

To find the structure constants $c_{2\frac{9}{2}}^{to}$, $c_{2\frac{9}{2}}^{go'}$, $c_{2\frac{9}{2}}^{o''}$ and $c_{2\frac{9}{2}}^a$, we use the equations (3.58) and (3.60). To find the coefficients in (3.59), the equations (3.58) and (3.59) can be used. To find the remaining structure constants in (3.57), we use the equations (3.62) and (3.63). Then the structure constants in (3.57) for general N are as follows:

$$\begin{aligned}
 c_{2\frac{9}{2}}^o &= 2520c^3 BB'(-3+2c)(29+2c)(53+2c)(20+3c)(6+5c)(1+N)(6N+c(-3+2N)), \\
 c_{2\frac{9}{2}}^{o'} &= -252c^2 B(29+2c)(53+2c)(20+3c)(21+4c)(6+5c)(4+73c)(c+6(-1+N)N) \\
 &\quad \times (10c^3(-6-N+2N^2) - 126N^2(25-50N+12N^2) \\
 &\quad - c^2(42-623N+496N^2+60N^3) - 3cN(-350+1155N-1246N^2+720N^3)), \\
 c_{2\frac{9}{2}}^{go} &= -75600c^3 BB'(-3+2c)(29+2c)(53+2c)(20+3c)(1+N)(6N+c(-3+2N)), \\
 c_{2\frac{9}{2}}^{go'} &= -5760c^3 BB'(53+2c)^2(20+3c)(6+5c)(1+N)(6N+c(-3+2N)), \\
 c_{2\frac{9}{2}}^{to} &= 20160c^3 BB'(5+2c)(53+2c)(20+3c)(6+5c)(1+N)(6N+c(-3+2N)), \\
 c_{2\frac{9}{2}}^{o''} &= 252cB(29+2c)^2(53+2c)(20+3c)(6+5c)(84+25c) \\
 &\quad \times (4+73c)(c+6(-1+N)N)^2(c-4cN+6N^2), \quad c_{2\frac{9}{2}}^a = \frac{3}{4}, \\
 c_{2\frac{9}{2}}^{go''} &= 2450c^3 B(53+2c)(20+3c)(23601-27059c+17768c^2+100c^3)(1+N) \\
 &\quad \times (c^2-3c(-1+N)+18(1-3N)N)(6N+c(-3+2N))(-6N^2+c(-1+4N)),
 \end{aligned}$$

$$\begin{aligned}
 c_{2\frac{9}{2}}^{to'} &= 588c^2B(29+2c)(20+3c)(6+5c)(c+6(-1+N)N) \\
 &\quad \times (-2268N^2(-100+200N+5447N^2) + 20c^5(4402-3023N+6046N^2) \\
 &\quad + c^4(274408-1893242N+2911384N^2-362760N^3) \\
 &\quad + 216cN(-350+2135N+35259N^2+76222N^3) \\
 &\quad + 6c^3(76222-900143N+2198091N^2-1893242N^3+528240N^4) \\
 &\quad + 9c^2(-38129+141036N+908438N^2-3600572N^3+1097632N^4)), \\
 c_{2\frac{9}{2}}^{to''} &= -784c^3B(53+2c)(20+3c)(2703-180995c+104480c^2+16900c^3)(1+N) \\
 &\quad \times (c^2-3c(-1+N)+18(1-3N)N)(6N+c(-3+2N))(-6N^2+c(-1+4N)), \\
 c_{2\frac{9}{2}}^{go'''} &= 3528cB(29+2c)(53+2c)(6+5c)(32718+357445c+113528c^2+4300c^3) \\
 &\quad \times (c+6(-1+N)N)^2(-6N^2+c(-1+4N)), \\
 c_{2\frac{9}{2}}^{ga} &= -37044B(29+2c)^2(53+2c)(6+5c)^2(4+73c)(c+6(-1+N)N)^3, \\
 c_{2\frac{9}{2}}^{go''''} &= -126c^2B(29+2c)(20+3c)(6+5c)(c+6(-1+N)N) \\
 &\quad \times (100c^5(-1334-59N+118N^2)-5292N^2(800-1600N+2799N^2) \\
 &\quad -10c^4(148336-41789N-78272N^2+3540N^3) \\
 &\quad -252cN(-5600+343485N-703756N^2+95568N^3) \\
 &\quad -6c^3(111496-2344454N+1931133N^2-417890N^3+800400N^4) \\
 &\quad -3c^2(137151-9852584N+31095988N^2-28133448N^3+17800320N^4)), \\
 c_{2\frac{9}{2}}^q &= 4410cB(29+2c)(53+2c)(20+3c)(6+5c)^2(-147+182c+40c^2) \\
 &\quad \times (c+6(-1+N)N)^2(-6N^2+c(-1+4N)), \\
 \frac{1}{B} &\equiv 1764(29+2c)(53+2c)(6+5c)(4+73c) \\
 &\quad \times (c+6(-1+N)N)^2(20+3c)(29+2c)(6+5c)(c+6(-1+N)N), \\
 B' &\equiv (4+73c)(108N^3(-1+3N)+c^3(-1+4N) \\
 &\quad -3c^2(1-5N+6N^2)-18cN(1-6N+11N^2)).
 \end{aligned} \tag{3.64}$$

The coefficients in (3.59) are given by

$$\begin{aligned}
 c_{\frac{9}{2}}^{go} &= -96c^3F(21+4c)(-7+10c)(327+16c)(3+c-9N)(1+N)(c+6N) \\
 &\quad \times (-3c+2(3+c)N)(c+6(-1+N)N)(-6N^2+c(-1+4N)), \\
 c_{2\frac{9}{2}}^{to} &= 480c^3F(21+4c)(6+5c)(-7+10c)(3+c-9N)(1+N)(c+6N) \\
 &\quad \times (-3c+2(3+c)N)(c+6(-1+N)N)(-6N^2+c(-1+4N)), \\
 c_{2\frac{9}{2}}^{tg} &= -32c^4F(29+2c)(6+5c)(-195+94c)(3+c-9N)(1+N)(c+6N) \\
 &\quad \times (-3c+2(3+c)N)(2c(-1+N)+3(3-4N)N)(c+2cN+3N(-3+2N)), \\
 c_{\frac{9}{2}}^{oo} &= 144c^2F(29+2c)(21+4c)(6+5c)(-7+10c)(3+c-9N)(1+N) \\
 &\quad \times (c+6N)(-3c+2(3+c)N)(c+6(-1+N)N)^2, \\
 F &\equiv \frac{1}{3(29+2c)^2(21+4c)(6+5c)^3(c+6(-1+N)N)^4}.
 \end{aligned} \tag{3.65}$$

The equations (3.64) and (3.65) are checked for $N = 7$ again.

To find the explicit form of $\{O_2 O_{\frac{9}{2}}\}_{-1}(w)$ for general N , we examine the operators in $\{O_2 O_{\frac{9}{2}}\}_{-1}(w)$ for $N = 2$ and write down the possible various spin- $\frac{11}{2}$ fields with unknown coefficients. Then we subtract them from the first-order pole $\{O_2 O_{\frac{9}{2}}\}_{-1}(w)$ and solve it for the coefficients for $N = 2, 3, 4, 5$ cases. The explicit expression for the first-order pole $\{O_2 O_{\frac{9}{2}}\}_{-1}(w)$ is presented as follows:

$$\begin{aligned}
 \{O_2 O_{\frac{9}{2}}\}_{-1}(w) = & c_1 K^a J^{bc} J^{bc} J^{ad} J^{de} J^{eN'}(w) + c_2 K^a J^{bc} J^{bc} J^{ad} \partial J^{dN'}(w) \\
 & + c_3 K^a J^{ab} J^{bc} J^{cd} J^{de} J^{eN'}(w) + c_4 K^a J^{bc} J^{bc} \partial J^{ad} J^{dN'}(w) + c_5 K^a J^{bc} J^{bc} \partial^2 J^{aN'}(w) \\
 & + c_6 K^a J^{ab} J^{bc} J^{cd} \partial J^{dN'}(w) + c_7 K^a J^{ab} J^{bc} J^{dN'} J^{dN'} J^{cN'}(w) + c_8 K^a J^{ab} J^{bc} \partial^2 J^{cN'}(w) \\
 & + c_9 K^a J^{ab} J^{bc} \partial J^{cd} J^{dN'}(w) + c_{10} K^a J^{ab} J^{cN'} J^{cN'} \partial J^{bN'}(w) + c_{11} K^a J^{ab} J^{bN'} \partial J^{cN'} J^{cN'}(w) \\
 & + c_{12} K^a J^{ab} \partial J^{cd} J^{cd} J^{bN'}(w) + c_{13} K^a J^{bc} \partial J^{ad} J^{cd} J^{bN'}(w) + c_{15} K^a J^{ab} \partial J^{bc} \partial J^{cN'}(w) \\
 & + c_{16} K^a J^{ab} \partial^2 J^{bc} J^{cN'}(w) + c_{17} K^a J^{ab} \partial J^{bc} J^{dc} J^{dN'}(w) + c_{18} K^a J^{ab} \partial^3 J^{bN'}(w) \\
 & + c_{19} K^a J^{aN'} \partial J^{bN'} \partial J^{bN'}(w) + c_{20} K^a J^{aN'} \partial^2 J^{bN'} J^{bN'}(w) + c_{21} K^a K^b K^c J^{ab} J^{de} J^{ce} J^{dN'}(w) \\
 & + c_{23} K^a K^b K^c J^{ab} J^{cd} \partial J^{dN'}(w) + c_{25} K^a K^b K^c J^{ab} \partial^2 J^{cN'}(w) + c_{27} K^a K^b K^c \partial^2 J^{ab} J^{cN'}(w) \\
 & + c_{31} K^a K^b \partial K^d J^{ab} J^{cd} J^{cN'}(w) + c_{32} K^a K^b \partial^2 K^c J^{ac} J^{bN'}(w) + c_{33} K^a K^b \partial K^c \partial J^{ab} J^{cN'}(w) \\
 & + c_{34} K^a K^b \partial^2 K^c J^{ab} J^{cN'}(w) + c_{35} K^a K^b \partial K^c \partial J^{ac} J^{bN'}(w) + c_{36} K^a K^b \partial K^c J^{bc} \partial J^{aN'}(w) \\
 & + c_{37} K^a K^b \partial K^c J^{ac} J^{bd} J^{dN'}(w) + c_{38} K^a K^b \partial K^c J^{ab} \partial J^{cN'}(w) + c_{39} K^a K^b K^c \partial J^{ab} J^{cd} J^{dN'}(w) \\
 & + c_{42} K^a \partial J^{ab} J^{bc} \partial J^{cN'}(w) + c_{43} K^a \partial J^{ab} J^{cN'} J^{cN'} J^{bN'}(w) + c_{44} K^a \partial J^{ab} \partial J^{cb} J^{cN'}(w) \\
 & + c_{45} K^a \partial J^{ab} \partial^2 J^{bN'}(w) + c_{46} K^a \partial J^{aN'} \partial J^{bN'} J^{bN'}(w) + c_{47} K^a \partial J^{bc} \partial J^{bc} J^{aN'}(w) \\
 & + c_{48} K^a \partial J^{bc} J^{bc} \partial J^{aN'}(w) + c_{49} K^a \partial K^b K^b J^{ac} J^{cd} J^{dN'}(w) + c_{50} K^a \partial K^b K^b J^{ac} \partial J^{cN'}(w) \\
 & + c_{51} K^a \partial K^b K^b \partial J^{ac} J^{cN'}(w) + c_{52} K^a \partial K^b K^b \partial^2 J^{aN'}(w) + c_{53} K^a \partial K^b \partial K^c J^{ac} J^{bN'}(w) \\
 & + c_{54} K^a \partial K^b \partial K^c J^{bc} J^{aN'}(w) + c_{55} K^a \partial^2 J^{ab} J^{bc} J^{cN'}(w) + c_{56} K^a \partial^2 J^{ab} \partial J^{bN'}(w) \\
 & + c_{57} K^a \partial^2 J^{bc} J^{bc} J^{aN'}(w) + c_{58} K^a \partial^2 J^{aN'} J^{bN'} J^{bN'}(w) + c_{59} K^a \partial^2 K^b K^b \partial J^{aN'}(w) \\
 & + c_{60} K^a \partial^2 K^b K^b J^{ac} J^{cN'}(w) + c_{61} K^a \partial^2 K^b \partial K^b J^{aN'}(w) + c_{62} K^a \partial^3 J^{ab} J^{bN'}(w) \\
 & + c_{63} K^a \partial^3 K^b K^b J^{aN'}(w) + c_{64} K^a \partial^4 J^{aN'}(w) + c_{65} \partial^2 K^a J^{ba} J^{bc} J^{cN'}(w) \\
 & + c_{66} \partial K^a J^{ab} J^{bc} J^{cd} J^{dN'}(w) + c_{67} \partial K^a J^{ab} J^{bc} \partial J^{cN'}(w) + c_{68} \partial K^a J^{ab} J^{cd} J^{cd} J^{bN'}(w) \\
 & + c_{69} \partial K^a J^{ab} J^{bN'} J^{cN'} J^{cN'}(w) + c_{70} \partial K^a J^{ab} \partial J^{bc} J^{cN'}(w) + c_{71} \partial K^a J^{ab} \partial^2 J^{bN'}(w) \\
 & + c_{72} \partial K^a J^{bc} J^{bc} \partial J^{aN'}(w) + c_{73} \partial K^a J^{aN'} \partial J^{bN'} J^{bN'}(w) + c_{74} \partial K^a K^a \partial K^b J^{bc} J^{cN'}(w) \\
 & + c_{75} \partial K^a \partial J^{ab} J^{bc} J^{cN'}(w) + c_{76} \partial K^a K^a \partial K^b \partial J^{bN'}(w) + c_{77} \partial K^a K^a \partial^2 K^b J^{bN'}(w) \\
 & + c_{78} \partial K^a \partial J^{ab} \partial J^{bN'}(w) + c_{79} \partial K^a J^{aN'} \partial J^{bN'} J^{bN'}(w) + c_{80} \partial K^a \partial J^{bc} J^{bc} J^{aN'}(w) \\
 & + c_{81} \partial K^a \partial^2 J^{ab} J^{bN'}(w) + c_{82} \partial K^a \partial^2 K^b K^b J^{aN'}(w) + c_{83} \partial K^a \partial^3 J^{aN'}(w) \\
 & + c_{84} \partial^2 K^a J^{bc} J^{bc} J^{aN'}(w) + c_{85} \partial^2 K^a J^{ab} \partial J^{bN'}(w) + c_{86} \partial^2 K^a J^{aN'} J^{bN'} J^{bN'}(w) \\
 & + c_{87} \partial^2 K^a \partial J^{ab} J^{bN'}(w) + c_{88} \partial^2 K^a \partial^2 J^{aN'}(w) + c_{89} \partial^3 K^a J^{ab} J^{bN'}(w) + c_{90} \partial^3 K^a \partial J^{aN'}(w) \\
 & + c_{91} K^a K^b K^c J^{ab} \partial J^{dc} J^{dN'}(w) + c_{92} \partial^4 K^a J^{aN'}(w) + c_{93} K^a K^b K^c \partial J^{ab} \partial J^{cN'}(w), \quad (3.66)
 \end{aligned}$$

where $N' \equiv 2N + 1$. The coefficients in (3.66) are presented in (D.1) of appendix D. This result is checked for $N = 6$ again. The spin- $\frac{11}{2}$ current $O_{\frac{11}{2}}(z)$ is given by (3.62), (3.64) and (3.66). The OPE for general N is obtained from (3.57) with (3.64). The fusion rule is summarized by $[O_2][O_{\frac{9}{2}}] = [I] + [O_{\frac{5}{2}}] + [O_{\frac{7}{2}}] + [O_{\frac{9}{2}}] + [O_{\frac{11}{2}}]$.

• **The OPE $O_{\frac{5}{2}}(z) O_{4'}(w)$.** Now let us compute the OPE between $O_{\frac{5}{2}}(z)$ and $O_{4'}(z)$. The computation of the OPE for $N = 2$ provides

$$O_{\frac{5}{2}}(z) O_{4'}(w) = \frac{1}{(z-w)^3} \left[c_{\frac{5}{2}4'}^o O_{\frac{7}{2}} + c_{\frac{5}{2}4'}^{go} \left(GO_2 - \frac{2}{5} \partial O_{\frac{5}{2}} \right) \right] (w)$$

$$\begin{aligned}
 & + \frac{1}{(z-w)^2} \left[\frac{2}{7} c_{\frac{5}{2}4'}^o \partial O_{\frac{7}{2}} + \frac{2}{7} c_{\frac{5}{2}4'}^{go} \partial \left(GO_2 - \frac{2}{5} \partial O_{\frac{5}{2}} \right) \right. \\
 & + c_{\frac{5}{2}4'}^{go'} \left(\partial GO_2 - \frac{3}{4} G \partial O_2 + \frac{1}{8} \partial^2 O_{\frac{5}{2}} \right) + c_{\frac{5}{2}4'}^{to} \left(TO_{\frac{5}{2}} - \frac{1}{4} \partial^2 O_{\frac{5}{2}} \right) \\
 & \left. + c_{\frac{5}{2}4'}^{o'} O_{\frac{9}{2}} + c_{\frac{5}{2}4'}^a A_{\frac{9}{2}} \right] (w) \\
 & + \frac{1}{(z-w)} \left[\frac{3}{56} c_{\frac{5}{2}4'}^o \partial^2 O_{\frac{7}{2}} + \frac{3}{56} c_{\frac{5}{2}4'}^{go} \partial^2 \left(GO_2 - \frac{2}{5} \partial O_{\frac{5}{2}} \right) \right. \\
 & + \frac{1}{3} c_{\frac{5}{2}4'}^{go'} \partial \left(\partial GO_2 - \frac{3}{4} G \partial O_2 + \frac{1}{8} \partial^2 O_{\frac{5}{2}} \right) + \frac{1}{3} c_{\frac{5}{2}4'}^{to} \partial \left(TO_{\frac{5}{2}} - \frac{1}{4} \partial^2 O_{\frac{5}{2}} \right) \\
 & + \frac{1}{3} c_{\frac{5}{2}4'}^{o'} \partial O_{\frac{9}{2}} + \frac{1}{3} c_{\frac{5}{2}4'}^a \partial A_{\frac{9}{2}} + c_{\frac{5}{2}4'}^{go''} \left(\partial^2 GO_2 + \frac{3}{5} G \partial^2 O_2 - 2 \partial G \partial O_2 - \frac{2}{35} \partial^3 O_{\frac{5}{2}} \right) \\
 & + c_{\frac{5}{2}4'}^{to'} \left(TO_{\frac{7}{2}} - \frac{3}{16} \partial^2 O_{\frac{7}{2}} \right) + c_{\frac{5}{2}4'}^{to''} \left(T \partial O_{\frac{5}{2}} - \frac{5}{4} \partial TO_{\frac{5}{2}} - \frac{1}{7} \partial^3 O_{\frac{5}{2}} \right) \\
 & + c_{\frac{5}{2}4'}^{go'''} \left(GO_{4'} - \frac{2}{9} \partial O_{\frac{9}{2}} \right) + c_{\frac{5}{2}4'}^{ga} \left(GA_4 - \frac{2}{9} \partial A_{\frac{9}{2}} \right) + c_{\frac{5}{2}4'}^{go''''} \left(GO_4 - \frac{1}{8} \partial^2 O_{\frac{7}{2}} \right) \\
 & \left. + c_{\frac{5}{2}4'}^q Q_{\frac{11}{2}} + c_{\frac{5}{2}4'}^{o''} O_{\frac{11}{2}} \right] (w) + \dots
 \end{aligned} \tag{3.67}$$

The field contents of (3.67) are almost the same as the ones in (3.57). The relative coefficients of descendant fields are determined from (1.3). The quasi-primary and primary fields are found by the same method we have used so far. The quasi-primary field $Q_{\frac{11}{2}}(w)$ is given in (3.61). From the above OPE, $O_{\frac{11}{2}}(z)$ can be expressed as

$$\begin{aligned}
 O_{\frac{11}{2}}(w) = & \frac{1}{c_{\frac{5}{2}4'}^{o''}} \{O_{\frac{5}{2}} O_{4'}\}_{-1}(w) - \frac{1}{c_{\frac{5}{2}4'}^{o''}} \left[\frac{3}{56} c_{\frac{5}{2}4'}^o \partial^2 O_{\frac{7}{2}} + \frac{3}{56} c_{\frac{5}{2}4'}^{go} \partial^2 \left(GO_2 - \frac{2}{5} \partial O_{\frac{5}{2}} \right) \right. \\
 & + \frac{1}{3} c_{\frac{5}{2}4'}^{go'} \partial \left(\partial GO_2 - \frac{3}{4} G \partial O_2 + \frac{1}{8} \partial^2 O_{\frac{5}{2}} \right) + \frac{1}{3} c_{\frac{5}{2}4'}^{to} \partial \left(TO_{\frac{5}{2}} - \frac{1}{4} \partial^2 O_{\frac{5}{2}} \right) \\
 & + \frac{1}{3} c_{\frac{5}{2}4'}^{o'} \partial O_{\frac{9}{2}} + \frac{1}{3} c_{\frac{5}{2}4'}^a \partial A_{\frac{9}{2}} + c_{\frac{5}{2}4'}^{go''} \left(\partial^2 GO_2 + \frac{3}{5} G \partial^2 O_2 - 2 \partial G \partial O_2 - \frac{2}{35} \partial^3 O_{\frac{5}{2}} \right) \\
 & + c_{\frac{5}{2}4'}^{to'} \left(TO_{\frac{7}{2}} - \frac{3}{16} \partial^2 O_{\frac{7}{2}} \right) + c_{\frac{5}{2}4'}^{to''} \left(T \partial O_{\frac{5}{2}} - \frac{5}{4} \partial TO_{\frac{5}{2}} - \frac{1}{7} \partial^3 O_{\frac{5}{2}} \right) \\
 & + c_{\frac{5}{2}4'}^{go'''} \left(GO_{4'} - \frac{2}{9} \partial O_{\frac{9}{2}} \right) + c_{\frac{5}{2}4'}^{ga} \left(GA_4 - \frac{2}{9} \partial A_{\frac{9}{2}} \right) + c_{\frac{5}{2}4'}^{go''''} \left(GO_4 - \frac{1}{8} \partial^2 O_{\frac{7}{2}} \right) \\
 & \left. + c_{\frac{5}{2}4'}^q Q_{\frac{11}{2}} \right] (w).
 \end{aligned} \tag{3.68}$$

Note that this is equal to (3.62) found before. As done before, we compute the OPE $O_{\frac{5}{2}}(z) O(w)_{4'}$ for $N = 3, 4, 5$ cases to find the general forms of structure constants. To find the structure constants $c_{\frac{5}{2}4'}^o$, $c_{\frac{5}{2}4'}^{go}$, $c_{\frac{5}{2}4'}^{go'}$, $c_{\frac{5}{2}4'}^{to}$, $c_{\frac{5}{2}4'}^{o'}$, and $c_{\frac{5}{2}4'}^{ga}$, we use the following equations:

$$\begin{aligned}
 & \left[\{O_{\frac{5}{2}} O_{4'}\}_{-3} - c_{\frac{5}{2}4'}^o O_{\frac{7}{2}} - c_{\frac{5}{2}4'}^{go} \left(GO_2 - \frac{2}{5} \partial O_{\frac{5}{2}} \right) \right] (w) = 0, \\
 & \left[\{O_{\frac{5}{2}} O_{4'}\}_{-2} - (\text{the 2nd-order pole}) \right] (w) = 0.
 \end{aligned}$$

Here the 2nd-order pole contains six terms in (3.67). The remaining structure constants are determined completely from (3.62) and (3.68). The structure constants in (3.67) for general N are given by

$$\begin{aligned}
 c_{\frac{5}{2}4'}^o &= 168c^2B(29+2c)(53+2c)(20+3c)(21+4c)(6+5c)(4+73c) \\
 &\quad \times (c+6(-1+N)N)(10c^3(-6-N+2N^2)-126N^2(25-50N+12N^2) \\
 &\quad -c^2(42-623N+496N^2+60N^3)-3cN(-350+1155N-1246N^2+720N^3)), \\
 c_{\frac{5}{2}4'}^{go} &= 50400c^3BB'(-3+2c)(29+2c)(53+2c)(20+3c)(4+73c)(1+N)(6N+c(-3+2N)), \\
 c_{\frac{5}{2}4'}^{go'} &= 7680c^3BB'(25+2c)(53+2c)(20+3c)(6+5c)(4+73c)(1+N)(6N+c(-3+2N)), \\
 c_{\frac{5}{2}4'}^{to} &= -107520c^3BB'(53+2c)(20+3c)(6+5c)(4+73c)(1+N)(6N+c(-3+2N)), \\
 c_{\frac{5}{2}4'}^{o'} &= 56cB(29+2c)^2(53+2c)(20+3c)(6+5c)(84+25c)(4+73c) \\
 &\quad \times (c+6(-1+N)N)^2(c-4cN+6N^2), \quad c_{\frac{5}{2}4'}^a = \frac{1}{4}, \\
 c_{\frac{5}{2}4'}^{go''} &= 700c^3BB'(53+2c)(20+3c)(-56229+42661c-28752c^2+740c^3) \\
 &\quad \times (1+N)(6N+c(-3+2N)), \\
 c_{\frac{5}{2}4'}^{to'} &= -56c^2B(29+2c)(20+3c)(6+5c)(c+6(-1+N)N) \\
 &\quad \times (-11340N^2(-1200+2400N+7117N^2)+20c^5(38698-19847N+39694N^2) \\
 &\quad +c^4(6218392-14192078N+20287456N^2-2381640N^3) \\
 &\quad +108cN(-42000+2187745N-3709300N^2+2120432N^3) \\
 &\quad +6c^3(1060216-14000654N+22660563N^2-14192078N^3+4643760N^4) \\
 &\quad +9c^2(-249095-7418600N+34356780N^2-56002616N^3+24873568N^4)), \\
 c_{\frac{5}{2}4'}^{to''} &= 224c^3B(53+2c)(20+3c)(17907-217555c+116720c^2+29700c^3) \\
 &\quad \times (1+N)(6N+c(-3+2N))(108N^3(-1+3N)+c^3(-1+4N) \\
 &\quad -3c^2(1-5N+6N^2)-18cN(1-6N+11N^2)), \\
 c_{\frac{5}{2}4'}^{go'''} &= -2352cB(29+2c)(53+2c)(6+5c)(32718+357445c+113528c^2+4300c^3) \\
 &\quad \times (c+6(-1+N)N)^2(-6N^2+c(-1+4N)), \\
 c_{\frac{5}{2}4'}^{ga} &= 24696B(29+2c)^2(53+2c)(6+5c)^2(4+73c)(c+6(-1+N)N)^3, \\
 c_{\frac{5}{2}4'}^{go''''} &= -84c^2B(29+2c)(20+3c)(6+5c)(c+6(-1+N)N) \\
 &\quad \times (20c^5(4042-143N+286N^2)+2268N^2(100-200N+5683N^2) \\
 &\quad -2c^4(-25424+52381N+375988N^2+8580N^3) \\
 &\quad -216cN(350-41090N+120561N^2+64058N^3) \\
 &\quad +9c^2(39781-482244N+1032758N^2+888388N^3+203392N^4) \\
 &\quad +6c^3(-64058+222097N-493509N^2-104762N^3+485040N^4)), \\
 c_{\frac{5}{2}4'}^q &= -2940cB(29+2c)(53+2c)(20+3c)(6+5c)^2(-147+182c+40c^2) \\
 &\quad \times (c+6(-1+N)N)^2(-6N^2+c(-1+4N)), \quad c_{\frac{5}{2}4'}^{o''} = -1, \\
 \frac{1}{B} &\equiv 1176(29+2c)^2(6+5c)^2(20+3c)(4+73c)(53+2c)(c+6(-1+N)N)^3, \\
 B' &\equiv (108N^3(-1+3N)+c^3(-1+4N)-3c^2(1-5N+6N^2)-18cN(1-6N+11N^2)).
 \end{aligned} \tag{3.69}$$

All results regarding the OPE $O_{\frac{5}{2}}(z)O_{4'}(w)$ are checked for $N = 6$ again. The spin- $\frac{11}{2}$ current $O_{\frac{11}{2}}(z)$ is given by (3.68) and (3.69). Again the OPE $O_{\frac{5}{2}}(z)O_{4'}(w)$ is obtained from (3.67) with (3.69). The fusion rule is summarized by $[O_{\frac{5}{2}}][O_{4'}] = [I] + [O_{\frac{7}{2}}] + [O_{\frac{9}{2}}] + [O_{\frac{11}{2}}]$.

• **The OPE $O_{\frac{5}{2}}(z)O_{\frac{9}{2}}(w)$.** Now we compute the OPE between $O_{\frac{5}{2}}(z)$ and $O_{\frac{9}{2}}(w)$, which is the final and most complicated OPE in this work. From the computation of the OPE for $N = 2$, we find that

$$\begin{aligned}
 O_{\frac{5}{2}}(z)O_{\frac{9}{2}}(w) = & \frac{1}{(z-w)^5}c_{\frac{5}{2}\frac{9}{2}}^o O_2(w) + \frac{1}{(z-w)^3} \left[c_{\frac{5}{2}\frac{9}{2}}^{go} \left(GO_{\frac{5}{2}} - \frac{1}{5}\partial^2 O_2 \right) \right. \\
 & + c_{\frac{5}{2}\frac{9}{2}}^{to} \left(TO_2 - \frac{3}{10}\partial^2 O_2 \right) + c_{\frac{5}{2}\frac{9}{2}}^{o'} O_4 + c_{\frac{5}{2}\frac{9}{2}}^{o''} O_{4'} + c_{\frac{5}{2}\frac{9}{2}}^a A_4 \left. \right] (w) \\
 & + \frac{1}{(z-w)^2} \left[\frac{1}{4}c_{\frac{5}{2}\frac{9}{2}}^{go} \partial \left(GO_{\frac{5}{2}} - \frac{1}{5}\partial^2 O_2 \right) + \frac{1}{4}c_{\frac{5}{2}\frac{9}{2}}^{to} \partial \left(TO_2 - \frac{3}{10}\partial^2 O_2 \right) + \frac{1}{4}c_{\frac{5}{2}\frac{9}{2}}^{o'} \partial O_4 \right. \\
 & + \frac{1}{4}c_{\frac{5}{2}\frac{9}{2}}^{o''} \partial O_{4'} + \frac{1}{4}c_{\frac{5}{2}\frac{9}{2}}^a \partial A_4 + c_{\frac{5}{2}\frac{9}{2}}^{to'} \left(T\partial O_2 - \partial O_2 T - \frac{1}{6}\partial^3 O_2 \right) \\
 & + c_{\frac{5}{2}\frac{9}{2}}^{go'} \left(G\partial O_{\frac{5}{2}} - \frac{5}{3}\partial GO_{\frac{5}{2}} - \frac{1}{9}\partial^3 O_2 \right) + c_{\frac{5}{2}\frac{9}{2}}^{go''} \left(GO_{\frac{7}{2}} - \frac{1}{4}\partial O_4 \right) \left. \right] (w) \\
 & + \frac{1}{(z-w)} \left[\frac{1}{24}c_{\frac{5}{2}\frac{9}{2}}^{go} \partial^2 \left(GO_{\frac{5}{2}} - \frac{1}{5}\partial^2 O_2 \right) + \frac{1}{24}c_{\frac{5}{2}\frac{9}{2}}^{to} \partial^2 \left(TO_2 - \frac{3}{10}\partial^2 O_2 \right) \right. \\
 & + \frac{1}{24}c_{\frac{5}{2}\frac{9}{2}}^{o'} \partial^2 O_4 + \frac{1}{24}c_{\frac{5}{2}\frac{9}{2}}^{o''} \partial^2 O_{4'} + \frac{1}{24}c_{\frac{5}{2}\frac{9}{2}}^a \partial^2 A_4 + \frac{3}{10}c_{\frac{5}{2}\frac{9}{2}}^{to'} \partial \left(T\partial O_2 - \partial O_2 T - \frac{1}{6}\partial^3 O_2 \right) \\
 & + \frac{3}{10}c_{\frac{5}{2}\frac{9}{2}}^{go'} \partial \left(G\partial O_{\frac{5}{2}} - \frac{5}{3}\partial GO_{\frac{5}{2}} - \frac{1}{9}\partial^3 O_2 \right) + \frac{3}{10}c_{\frac{5}{2}\frac{9}{2}}^{go''} \partial \left(GO_{\frac{7}{2}} - \frac{1}{4}\partial O_4 \right) \\
 & + c_{\frac{5}{2}\frac{9}{2}}^{go'''} \left(GO_{\frac{9}{2}} - \frac{1}{9}\partial^2 O_{4'} \right) + c_{\frac{5}{2}\frac{9}{2}}^{ga} \left(GA_{\frac{9}{2}} - \frac{1}{9}\partial^2 A_4 \right) \\
 & + c_{\frac{5}{2}\frac{9}{2}}^{to''} \left(T\partial^2 O_2 - \frac{5}{2}\partial T\partial O_2 + \partial^2 TO_2 - \frac{3}{28}\partial^4 O_2 \right) \\
 & + c_{\frac{5}{2}\frac{9}{2}}^{go''''} \left(G\partial^2 O_{\frac{5}{2}} - 4\partial G\partial O_{\frac{5}{2}} + \frac{5}{2}\partial^2 GO_{\frac{5}{2}} - \frac{1}{14}\partial^4 O_2 \right) \\
 & + c_{\frac{5}{2}\frac{9}{2}}^{go'''''} \left(G\partial O_{\frac{7}{2}} - \frac{7}{3}\partial GO_{\frac{7}{2}} - \frac{1}{9}\partial^2 O_4 \right) + c_{\frac{5}{2}\frac{9}{2}}^{to'''} \left(TO_4 - \frac{1}{6}\partial^2 O_4 \right) \\
 & + c_{\frac{5}{2}\frac{9}{2}}^{to''''} \left(TO_{4'} - \frac{1}{6}\partial^2 O_{4'} \right) + c_{\frac{5}{2}\frac{9}{2}}^{ta} \left(TA_4 - \frac{1}{6}\partial^2 A_4 \right) + c_{\frac{5}{2}\frac{9}{2}}^q Q_6 + O_6 \left. \right] (w) + \dots
 \end{aligned} \tag{3.70}$$

The relative coefficients of descendant fields are fixed by (1.3). The fifth-, third- and second-order pole of (3.70) have the same structure to the fourth-, second- and first-order pole of (3.51) respectively. The spin-6 quasi primary fields in (3.70) are found by the same method we used in the previous OPEs. The explicit form of quasi-primary field $Q_6(z)$ is given by

$$\begin{aligned}
 Q_6(z) = & O_2 O_4(z) + O_{\frac{5}{2}} O_{\frac{7}{2}}(z) \\
 & + 70c^2 H(1629 + c(-2982 + c(-911 + 10c)))\partial^2 GO_{\frac{5}{2}}(z) \\
 & - 14c^2 H(7839 + c(-25155 - 7714c + 80c^2))\partial G\partial O_{\frac{5}{2}}(z) \\
 & + 7c^2 H(6507 + 4c(-4245 + 2c(-484 + 5c)))G\partial^2 O_{\frac{5}{2}}(z) \\
 & + 42c^2 H(21 + 4c)(-13 + 62c)T\partial^2 O_2(z) \\
 & + 84c^2 H(21 + 4c)(-79 + 26c)\partial^2 TO_2(z)
 \end{aligned}$$

$$\begin{aligned}
 & + \frac{18c(-6N^2 + c(-1 + 4N))}{7(6 + 5c)(c + 6(-1 + N)N)} G\partial O_{\frac{7}{2}}(z) \\
 & + \frac{c(3 + 2c)(-6N^2 + c(-1 + 4N))}{14(6 + 5c)(c + 6(-1 + N)N)} \partial^2 O_4(z) - \frac{1}{3} \partial^2 O_{4'}(z), \quad (3.71)
 \end{aligned}$$

where $H \equiv \frac{(3+c-9N)(1+N)(c+6N)(-3c+2(3+c)N)}{35(29+2c)(21+4c)(6+5c)^2(c+6(-1+N)N)^2}$. This quasi-primary field (3.71) containing $O_2 O_4(z)$ and $O_{\frac{5}{2}} O_{\frac{7}{2}}(z)$ is related to the primary field $A^{(6,2)}$ in [7]. The coefficients in (3.71) are determined by the fact that the third-order pole of the OPE between $T(z)$ and $Q_6(w)$ should vanish. The OPEs $T(z) Q_6(w)$ and $G(z) Q_6(w)$ are put in the appendix G and appendix H respectively. The $O_6(w)$ is a spin-6 primary field and the superpartner of spin- $\frac{11}{2}$ primary field $O_{\frac{11}{2}}(z)$. The $O_6(w)$ was found by the same procedure used in finding $O_{\frac{11}{2}}(z)$. The explicit form of the $O_6(w)$ is expressed as

$$\begin{aligned}
 O_6(w) = & \{O_{\frac{5}{2}} O_{\frac{9}{2}}\}_{-1}(w) - \left[\frac{1}{24} c_{\frac{5}{2}\frac{9}{2}}^{go} \partial^2 \left(G O_{\frac{5}{2}} - \frac{1}{5} \partial^2 O_2 \right) + \frac{1}{24} c_{\frac{5}{2}\frac{9}{2}}^{to} \partial^2 \left(T O_2 - \frac{3}{10} \partial^2 O_2 \right) \right. \\
 & + \frac{1}{24} c_{\frac{5}{2}\frac{9}{2}}^{o'} \partial^2 O_4 + \frac{1}{24} c_{\frac{5}{2}\frac{9}{2}}^{o''} \partial^2 O_{4'} + \frac{1}{24} c_{\frac{5}{2}\frac{9}{2}}^a \partial^2 A_4 + \frac{3}{10} c_{\frac{5}{2}\frac{9}{2}}^{to'} \partial \left(T \partial O_2 - \partial O_2 T - \frac{1}{6} \partial^3 O_2 \right) \\
 & + \frac{3}{10} c_{\frac{5}{2}\frac{9}{2}}^{go'} \partial \left(G \partial O_{\frac{5}{2}} - \frac{5}{3} \partial G O_{\frac{5}{2}} - \frac{1}{9} \partial^3 O_2 \right) + \frac{3}{10} c_{\frac{5}{2}\frac{9}{2}}^{go''} \partial \left(G O_{\frac{7}{2}} - \frac{1}{4} \partial O_4 \right) \\
 & + c_{\frac{5}{2}\frac{9}{2}}^{go'''} \left(G O_{\frac{9}{2}} - \frac{1}{9} \partial^2 O_{4'} \right) + c_{\frac{5}{2}\frac{9}{2}}^{ga} \left(G A_{\frac{9}{2}} - \frac{1}{9} \partial^2 A_4 \right) \\
 & + c_{\frac{5}{2}\frac{9}{2}}^{to''} \left(T \partial^2 O_2 - \frac{5}{2} \partial T \partial O_2 + \partial^2 T O_2 - \frac{3}{28} \partial^4 O_2 \right) \\
 & + c_{\frac{5}{2}\frac{9}{2}}^{go'''} \left(G \partial^2 O_{\frac{5}{2}} - 4 \partial G \partial O_{\frac{5}{2}} + \frac{5}{2} \partial^2 G O_{\frac{5}{2}} - \frac{1}{14} \partial^4 O_2 \right) \\
 & + c_{\frac{5}{2}\frac{9}{2}}^{go''''} \left(G \partial O_{\frac{7}{2}} - \frac{7}{3} \partial G O_{\frac{7}{2}} - \frac{1}{9} \partial^2 O_4 \right) + c_{\frac{5}{2}\frac{9}{2}}^{to'''} \left(T O_4 - \frac{1}{6} \partial^2 O_4 \right) \\
 & \left. + c_{\frac{5}{2}\frac{9}{2}}^{to''''} \left(T O_{4'} - \frac{1}{6} \partial^2 O_{4'} \right) + c_{\frac{5}{2}\frac{9}{2}}^{ta} \left(T A_4 - \frac{1}{6} \partial^2 A_4 \right) + c_{\frac{5}{2}\frac{9}{2}}^a Q_6 \right] (w). \quad (3.72)
 \end{aligned}$$

As expected, the spin-6 current $O_6(z)$ obeys the following OPEs:

$$\begin{aligned}
 T(z) O_6(w) &= \frac{1}{(z-w)^2} 6 O_6(w) + \frac{1}{(z-w)} \partial O_6(w) + \dots, \\
 G(z) O_6(w) &= \frac{1}{(z-w)} 11 O_{\frac{11}{2}}(w) + \frac{1}{(z-w)} \partial O_{\frac{11}{2}}(w) \dots
 \end{aligned}$$

Therefore, the currents $O_{\frac{11}{2}}(z)$ and $O_6(z)$ consist of the correct $\mathcal{N} = 1$ supermultiplet. Finding the general forms of structure constants is much more complicated than the previous OPEs. First we determine the structure constants $c_{\frac{5}{2}\frac{9}{2}}^{o'}$, $c_{\frac{5}{2}\frac{9}{2}}^{go}$, $c_{\frac{5}{2}\frac{9}{2}}^{to}$, $c_{\frac{5}{2}\frac{9}{2}}^{o'}$, $c_{\frac{5}{2}\frac{9}{2}}^{o''}$, $c_{\frac{5}{2}\frac{9}{2}}^a$, $c_{\frac{5}{2}\frac{9}{2}}^{to'}$, $c_{\frac{5}{2}\frac{9}{2}}^{go'}$, and $c_{\frac{5}{2}\frac{9}{2}}^{go''}$ that appear in the fifth-, third-, and second-order pole. To find them, we solve following equations for $N = 3, 4, 5$ cases:

$$\begin{aligned}
 & \left[\{O_{\frac{5}{2}} O_{\frac{9}{2}}\}_{-5} - c_{\frac{5}{2}\frac{9}{2}}^o O_2 \right] (w) = 0, \\
 & \left[\{O_{\frac{5}{2}} O_{\frac{9}{2}}\}_{-3} - (\text{the 2nd-order pole}) \right] (w) = 0, \\
 & \left[\{O_{\frac{5}{2}} O_{\frac{9}{2}}\}_{-2} - (\text{the 1st-order pole}) \right] (w) = 0.
 \end{aligned}$$

Here the 2nd-order pole and 1st-order pole are given in the right hand side of (3.70) respectively. Then we put all results together and find the general forms of them. The general forms of them are checked for $N = 6$ again.

To find the remaining structure constants appearing only in the first-order pole, we use the fact that the four OPEs (3.51), (3.57), (3.67) and (3.70) should be expressed as a single $\mathcal{N} = 1$ super OPE. For $N = 2$ and $N = 3$ cases, we checked that they are expressed as one single super OPE. The expression of the super OPE is given in (F.6) of the appendix F. Moreover, the following relations are found for $N = 2, 3$ cases:

$$\begin{aligned}
 & \frac{1}{24}c_{\frac{5}{2}\frac{9}{2}}^{go} - \frac{1}{2}c_{\frac{5}{2}\frac{9}{2}}^{go'} + \frac{5}{2}c_{\frac{5}{2}\frac{9}{2}}^{go''''} + \frac{1}{6}c_{\frac{5}{2}4'}^{to} - \frac{5}{8}c_{\frac{5}{2}4'}^{to''} - \frac{3}{56}c_{\frac{5}{2}4'}^{go} + \frac{1}{3}c_{\frac{5}{2}4'}^{go'} + c_{\frac{5}{2}4'}^{go''} \\
 & + 315c^2H(-117 + 4(-36 + c)c)c_{\frac{5}{2}4'}^q \\
 & + 70c^2H(1629 + c(-2982 + c(-911 + 10c)))c_{\frac{5}{2}\frac{9}{2}}^q = 0, \\
 & \frac{1}{12}c_{\frac{5}{2}\frac{9}{2}}^{to} - \frac{5}{2}c_{\frac{5}{2}\frac{9}{2}}^{to''} + \frac{1}{3}c_{\frac{5}{2}4'}^{to} - \frac{5}{4}c_{\frac{5}{2}4'}^{to''} + \frac{3}{14}c_{\frac{5}{2}4'}^{go} + \frac{1}{6}c_{\frac{5}{2}4'}^{go'} - 4c_{\frac{5}{2}4'}^{go''} \\
 & + 2520c^2H(-36 + c(75 + 2c))(3 + c - 9N)c_{\frac{5}{2}4'}^q = 0, \\
 & \frac{3}{10}c_{\frac{5}{2}\frac{9}{2}}^{go''} + c_{\frac{5}{2}\frac{9}{2}}^{go''''} - c_{\frac{5}{2}4'}^{go''''} + \frac{18c(-6N^2 + c(-1 + 4N))}{7(6 + 5c)(c + 6(-1 + N)N)}c_{\frac{5}{2}\frac{9}{2}}^q = 0, \\
 & c_{\frac{5}{2}\frac{9}{2}}^{go''''} - c_{\frac{5}{2}4'}^{go''''} = 0, \quad c_{\frac{5}{2}\frac{9}{2}}^{ga} - c_{\frac{5}{2}4'}^{ga} = 0, \quad c_{\frac{5}{2}\frac{9}{2}}^{to''''} + 2c_{\frac{5}{2}4'}^{go''''} + c_{\frac{5}{2}4'}^{to} = 0, \\
 & c_{\frac{5}{2}\frac{9}{2}}^{to''''} + 2c_{\frac{5}{2}4'}^{go''''} = 0, \quad c_{\frac{5}{2}\frac{9}{2}}^{ta} + 2c_{\frac{5}{2}4'}^{ga} = 0, \quad c_{\frac{5}{2}\frac{9}{2}}^q + c_{\frac{5}{2}4'}^q = 0, \tag{3.73}
 \end{aligned}$$

where $H \equiv \frac{(3+c-9N)(1+N)(c+6N)(-3c+2(3+c)N)}{35(29+2c)(21+4c)(6+5c)^2(c+6(-1+N)N)^2}$. Since the four OPEs (3.51), (3.57), (3.67) and (3.70) should be expressed as a single $\mathcal{N} = 1$ super OPE for any N , we safely can assume that the equations in (3.73) hold for beyond $N = 3$ case. By solving the equations in (3.73), one can find the structure constants $c_{\frac{5}{2}\frac{9}{2}}^{go''''}$, $c_{\frac{5}{2}\frac{9}{2}}^{ga}$, $c_{\frac{5}{2}\frac{9}{2}}^{to''}$, $c_{\frac{5}{2}\frac{9}{2}}^{go''''}$, $c_{\frac{5}{2}\frac{9}{2}}^{go''''}$, $c_{\frac{5}{2}\frac{9}{2}}^{to''''}$, $c_{\frac{5}{2}\frac{9}{2}}^{to''''}$, $c_{\frac{5}{2}\frac{9}{2}}^{ta}$, and $c_{\frac{5}{2}\frac{9}{2}}^q$ for general N completely. The followings are the general forms of structure constants of (3.70):

$$\begin{aligned}
 c_{\frac{5}{2}\frac{9}{2}}^o &= -8400c^3BB'(-3 + 2c)(29 + 2c)(53 + 2c)(20 + 3c)(6 + 5c)(4 + 73c), \\
 c_{\frac{5}{2}\frac{9}{2}}^{go} &= 12600c^3BB'(-3 + 2c)(53 + 2c)(20 + 3c)(183 + 10c)(4 + 73c), \\
 c_{\frac{5}{2}\frac{9}{2}}^{to} &= -8400c^3BB'(-3 + 2c)(53 + 2c)(20 + 3c)(573 + 50c)(4 + 73c), \\
 c_{\frac{5}{2}\frac{9}{2}}^{o'} &= -270c^2B(29 + 2c)(53 + 2c)(20 + 3c)(21 + 4c)(6 + 5c)(4 + 73c)(c + 6(-1 + N)N) \\
 & \quad \times (10c^3(-2 + N)(3 + 2N) - 126N^2(25 + 2N(-25 + 6N))) \\
 & \quad + 3cN(350 + N(-1155 + 2(623 - 360N)N)) - c^2(42 + N(-623 + 496N + 60N^2))), \\
 c_{\frac{5}{2}\frac{9}{2}}^{o''} &= -560cB(29 + 2c)^2(53 + 2c)(20 + 3c)(6 + 5c)(84 + 25c)(4 + 73c) \\
 & \quad \times (c + 6(-1 + N)N)^2(c - 4cN + 6N^2), \quad c_{\frac{5}{2}\frac{9}{2}}^a = -2, \\
 c_{\frac{5}{2}\frac{9}{2}}^{to'} &= 10500c^3BB'(29 + 2c)(53 + 2c)(20 + 3c)(9 + 10c)(4 + 73c), \\
 c_{\frac{5}{2}\frac{9}{2}}^{go'} &= -15750c^3BB'(3 + 2c)(29 + 2c)(53 + 2c)(20 + 3c)(4 + 73c), \\
 c_{\frac{5}{2}\frac{9}{2}}^{go''} &= -630c^2B(29 + 2c)(53 + 2c)(20 + 3c)(6 + 5c)(4 + 73c)
 \end{aligned}$$

$$\begin{aligned}
 & \times (c + 6(-1 + N)N)(10c^3(-2 + N)(3 + 2N) - 126N^2(25 + 2N(-25 + 6N)) \\
 & + 3cN(350 + N(-1155 + 2(623 - 360N)N)) - c^2(42 + N(-623 + 496N + 60N^2))), \\
 c_{\frac{5}{2}\frac{9}{2}}^{go'''} &= -2940cB(29 + 2c)(53 + 2c)(6 + 5c)(32718 + c(357445 + 4c(28382 + 1075c))) \\
 & \times (c + 6(-1 + N)N)^2(-6N^2 + c(-1 + 4N)), \\
 c_{\frac{5}{2}\frac{9}{2}}^{ga} &= 30870B(29 + 2c)^2(53 + 2c)(6 + 5c)^2(4 + 73c)(c + 6(-1 + N)N)^3, \\
 c_{\frac{5}{2}\frac{9}{2}}^{to''} &= -980c^3B(-3 + 2c)(53 + 2c)(20 + 3c)(16851 + c(33139 + 3730c)) \\
 & \times (3 + c - 9N)(1 + N)(c + 6N)(-3c + 2(3 + c)N)(-6N^2 + c(-1 + 4N)), \\
 c_{\frac{5}{2}\frac{9}{2}}^{go''''} &= -1470c^3BB'(53 + 2c)(20 + 3c)(-2817 + c(74535 + 2c(-29859 + 10c(-983 + 10c))))), \\
 c_{\frac{5}{2}\frac{9}{2}}^{to''''} &= -21c^2B(29 + 2c)(20 + 3c)(21 + 4c)(6 + 5c)(c + 6(-1 + N)N) \\
 & \times (10c^4(12976 + 7229N(-1 + 2N)) - 756N^2(-450 + N(900 + 3709N)) \\
 & + c^3(198962 + N(-2104453 + 4(717164 - 108435N)N)) \\
 & + 18cN(-6300 + N(310765 - 492478N + 397924N^2)) \\
 & + 3c^2(-25963 + N(-492478 + N(2293561 + 2N(-2104453 + 778560N))))), \\
 c_{\frac{5}{2}\frac{9}{2}}^{to'''} &= 140c^2B(29 + 2c)(20 + 3c)(21 + 4c)(6 + 5c)^2 \\
 & \times (c + 6(-1 + N)N)(-252N^2(-225 + 450N + 662N^2) + 2c^3(12706 + 5069N(-1 + 2N)) \\
 & + 3cN(-6300 + N(328405 - 594538N + 304944N^2)) \\
 & - c^2(4634 + N(297269 + 4N(-80002 + 15207N))))), \\
 c_{\frac{5}{2}\frac{9}{2}}^{to''''} &= 5880cB(29 + 2c)(53 + 2c)(6 + 5c) \\
 & \times (32718 + c(357445 + 4c(28382 + 1075c)))(c + 6(-1 + N)N)^2(-6N^2 + c(-1 + 4N)), \\
 c_{\frac{5}{2}\frac{9}{2}}^{ta} &= -61740B(29 + 2c)^2(53 + 2c)(6 + 5c)^2(4 + 73c)(c + 6(-1 + N)N)^3, \\
 c_{\frac{5}{2}\frac{9}{2}}^a &= 3675cB(29 + 2c)(53 + 2c)(20 + 3c)(21 + 4c) \\
 & \times (6 + 5c)^2(-7 + 10c)(c + 6(-1 + N)N)^2(-6N^2 + c(-1 + 4N)), \\
 \frac{1}{B} &\equiv 735(29 + 2c)^2(53 + 2c)(20 + 3c)(6 + 5c)^2(8 + 146c)(c + 6(-1 + N)N)^3, \\
 B' &\equiv (1 + N)(3 + c - 9N)(c + 6N)(-3c + 2(3 + c)N)(-6N^2 + c(-1 + 4N)). \tag{3.74}
 \end{aligned}$$

To find the explicit form of $\{O_{\frac{5}{2}} O_{\frac{9}{2}}\}_{-1}(w)$ for general N , we examine the operators in $\{O_{\frac{5}{2}} O_{\frac{9}{2}}\}_{-1}(w)$ for $N = 2$ and write down the possible spin-6 fields with undetermined coefficients. Then we subtract them from the first-order pole $\{O_{\frac{5}{2}} O_{\frac{9}{2}}\}_{-1}(w)$ and solve it for the coefficients when $N = 2, 3, 4, 5, 6$ cases. The explicit expression of $\{O_{\frac{5}{2}} O_{\frac{9}{2}}\}_{-1}(w)$, which consist of one hundred eighty seven terms, is presented as follows:

$$\begin{aligned}
 \{O_{\frac{5}{2}} O_{\frac{9}{2}}\}_{-1}(w) &= c_1 J^{ab} J^{ab} J^{cd} \partial J^{cN'} J^{dN'}(w) + c_2 J^{ab} J^{ac} \partial J^{bd} \partial J^{cd}(w) + c_4 J^{ab} J^{ab} \partial J^{cd} \partial J^{cd}(w) \\
 &+ c_5 J^{ab} J^{ab} \partial^2 J^{cd} J^{cd}(w) + c_6 J^{ab} J^{ab} \partial^2 J^{cN'} J^{cN'}(w) + c_7 J^{ab} J^{ac} J^{bd} J^{ce} J^{dN'} J^{eN'}(w) \\
 &+ c_9 K^a K^b K^c K^d J^{cd} \partial J^{aN'} J^{bN'}(w) + c_{11} J^{ab} J^{ac} J^{dc} J^{bN'} \partial J^{dN'}(w) + c_{12} J^{ab} J^{cb} J^{aN'} J^{cN'} J^{dN'} J^{dN'}(w) \\
 &+ c_{13} J^{ab} J^{cb} \partial J^{ad} J^{cN'} J^{dN'}(w) + c_{15} J^{ab} J^{ab} \partial J^{cN'} \partial J^{cN'}(w) + c_{16} J^{ab} J^{cb} \partial^2 J^{aN'} J^{cN'}(w) \\
 &+ c_{17} J^{ab} J^{ac} J^{bd} \partial^2 J^{cd}(w) + c_{18} J^{ab} J^{aN'} \partial J^{bN'} J^{cN'} J^{cN'}(w) + c_{19} J^{ab} J^{aN'} \partial^3 J^{bN'}(w) \\
 &+ c_{20} J^{ab} \partial J^{ac} J^{bN'} \partial J^{cN'}(w) + c_{21} J^{ab} \partial J^{ac} \partial J^{bd} J^{cd}(w) + c_{22} J^{ab} \partial J^{ac} \partial J^{bN'} J^{cN'}(w) \\
 &+ c_{23} J^{ab} \partial J^{ac} \partial^2 J^{bc}(w) + c_{24} J^{ab} \partial J^{aN'} \partial^2 J^{bN'}(w) + c_{25} J^{ab} \partial^2 J^{ac} J^{bN'} J^{cN'}(w) \\
 &+ c_{26} J^{aN'} J^{aN'} \partial J^{bN'} \partial J^{bN'}(w) + c_{27} J^{aN'} J^{aN'} \partial^2 J^{bN'} J^{bN'}(w) + c_{28} J^{ab} J^{ac} J^{bd} J^{ed} J^{cN'} J^{eN'}(w) \\
 &+ c_{29} K^a K^b J^{ac} J^{bd} J^{cd} J^{ef} J^{ef}(w) + c_{30} K^a K^b J^{cd} J^{bc} J^{ad} J^{ef} J^{ef}(w) + c_{31} K^a K^b J^{ad} J^{cd} J^{cb} J^{ef} J^{ef}(w)
 \end{aligned}$$

$$\begin{aligned}
 & +c_{32}K^aK^bJ^{ae}J^{cd}J^{ce}J^{bf}J^{df}(w) + c_{34}K^aK^bJ^{ab}J^{cd}J^{cd}J^{eN'}J^{eN'}(w) \\
 & +c_{35}K^aK^bJ^{ac}J^{bd}J^{cd}J^{eN'}J^{eN'}(w) + c_{36}K^aK^bJ^{ad}J^{ce}J^{bd}J^{cN'}J^{eN'}(w) \\
 & +c_{38}K^aK^bJ^{ab}J^{cd}J^{de}J^{cN'}J^{eN'}(w) + c_{41}K^aK^bJ^{cd}J^{cd}J^{aN'}\partial J^{bN'}(w) + c_{43}K^aK^bJ^{ac}J^{cd}\partial J^{be}J^{de}(w) \\
 & +c_{44}K^aK^bJ^{cb}J^{ad}J^{de}J^{cN'}J^{eN'}(w) + c_{45}K^aK^bJ^{ab}J^{cd}J^{cN'}\partial J^{dN'}(w) \\
 & +c_{46}K^aK^bJ^{ac}J^{bd}\partial J^{cN'}J^{dN'}(w) + c_{47}K^aK^bJ^{ac}J^{cd}\partial J^{bN'}J^{dN'}(w) \\
 & +c_{48}K^aK^bJ^{ab}J^{cN'}J^{cN'}J^{dN'}J^{dN'}(w) + c_{49}K^aK^bJ^{ab}\partial J^{cN'}\partial J^{cN'}(w) + c_{52}K^aK^bJ^{ab}\partial J^{cd}\partial J^{cd}(w) \\
 & +c_{53}K^aK^bJ^{ab}\partial^2J^{cN'}J^{cN'}(w) + c_{54}K^aK^bK^cK^dJ^{ad}J^{bc}J^{eN'}J^{eN'}(w) \\
 & +c_{56}K^aK^bJ^{ac}J^{de}J^{bf}J^{ef}(w) + c_{57}K^aK^bJ^{ac}\partial J^{bN'}\partial J^{cN'}(w) + c_{58}K^aK^bJ^{ab}\partial^2J^{cd}J^{cd}(w) \\
 & +c_{60}K^aK^bJ^{ac}\partial^2J^{bd}J^{cd}(w) + c_{61}K^aK^bJ^{cd}\partial^2J^{ab}J^{cd}(w) + c_{62}K^aK^bJ^{ac}J^{bN'}\partial^2J^{cN'}(w) \\
 & +c_{63}K^aK^bJ^{ac}J^{bd}J^{cN'}\partial J^{dN'}(w) + c_{64}K^aK^bJ^{ac}J^{bd}\partial^2J^{cd}(w) + c_{65}K^aK^bJ^{ac}\partial J^{bc}J^{dN'}J^{dN'}(w) \\
 & +c_{66}K^aK^bJ^{ac}J^{cd}J^{bN'}\partial J^{dN'}(w) + c_{67}K^aK^bJ^{ac}\partial J^{cd}J^{bN'}J^{dN'}(w) + c_{68}K^aK^bJ^{ac}\partial^2J^{bN'}J^{cN'}(w) \\
 & +c_{69}K^aK^bJ^{ad}\partial J^{bc}J^{cN'}J^{dN'}(w) + c_{70}K^aK^bJ^{ad}\partial J^{bc}\partial J^{cd}(w) + c_{71}K^aK^bJ^{aN'}J^{cN'}J^{cN'}\partial J^{bN'}(w) \\
 & +c_{72}K^aK^bJ^{cN'}J^{aN'}J^{cN'}\partial J^{bN'}(w) + c_{73}K^aK^bJ^{cN'}J^{cN'}J^{aN'}\partial J^{bN'}(w) \\
 & +c_{74}K^aK^bK^c\partial K^dJ^{ab}J^{cN'}J^{dN'}(w) + c_{77}K^aK^bK^c\partial^2K^dJ^{ab}J^{cd}(w) + c_{78}K^aK^b\partial^3K^cK^cJ^{ab}(w) \\
 & +c_{79}K^aK^b\partial^3J^{aN'}J^{bN'}(w) + c_{80}K^aK^b\partial^2K^c\partial K^cJ^{ab}(w) + c_{81}K^aK^b\partial^2K^cK^c\partial J^{ab}(w) \\
 & +c_{82}K^aK^b\partial^2J^{bc}J^{aN'}J^{cN'}(w) + c_{83}K^aK^b\partial^2J^{ac}\partial J^{bc}(w) + c_{84}K^aK^b\partial^2J^{ab}J^{cN'}J^{cN'}(w) \\
 & +c_{85}K^aK^b\partial K^cK^c\partial^2J^{ab}(w) + c_{86}K^aK^b\partial K^cK^c\partial J^{aN'}J^{bN'}(w) + c_{87}K^aK^b\partial K^cK^c\partial J^{ad}J^{bd}(w) \\
 & +c_{89}K^aK^b\partial K^cK^d\partial J^{ad}J^{bc}(w) + c_{90}K^aK^b\partial K^cK^cJ^{ad}J^{be}J^{ed}(w) \\
 & +c_{91}K^aK^b\partial K^cK^eJ^{ad}J^{bc}J^{ed}(w) + c_{92}K^aK^b\partial K^cK^cJ^{ab}J^{dN'}J^{dN'}(w) \\
 & +c_{94}K^aK^b\partial K^cK^cJ^{ab}J^{de}J^{de}(w) + c_{95}K^aK^b\partial K^cK^eJ^{ab}J^{dc}J^{de}(w) \\
 & +c_{97}K^aK^b\partial K^c\partial K^dJ^{ab}J^{cd}(w) + c_{99}K^aK^b\partial J^{bc}J^{dc}J^{aN'}J^{dN'}(w) + c_{100}K^aK^b\partial J^{ac}\partial J^{bN'}J^{cN'}(w) \\
 & +c_{101}K^aK^b\partial J^{ac}\partial J^{bd}J^{dc}(w) + c_{103}K^a\partial K^cJ^{ab}J^{bd}J^{ce}J^{de}(w) + c_{104}K^a\partial K^cJ^{ab}J^{bd}J^{de}J^{ce}(w) \\
 & +c_{105}K^a\partial K^b\partial^2K^cK^cJ^{ab}(w) + c_{106}K^a\partial K^b\partial^2J^{aN'}J^{bN'}(w) + c_{107}K^a\partial K^b\partial^2J^{ac}J^{bc}(w) \\
 & +c_{108}K^a\partial K^b\partial K^cK^c\partial J^{ab}(w) + c_{109}K^a\partial K^b\partial K^cK^cJ^{aN'}J^{bN'}(w) + c_{110}K^a\partial K^b\partial K^cK^cJ^{ad}J^{bd}(w) \\
 & +c_{111}K^a\partial K^b\partial K^cK^dJ^{ac}J^{bd}(w) + c_{112}K^a\partial K^b\partial J^{aN'}\partial J^{bN'}(w) + c_{114}K^a\partial K^b\partial J^{ac}\partial J^{bc}(w) \\
 & +c_{115}K^a\partial K^b\partial J^{ac}J^{bN'}J^{cN'}(w) + c_{116}K^a\partial K^b\partial J^{ac}J^{bd}J^{dc}(w) + c_{118}K^a\partial K^b\partial J^{ab}J^{cN'}J^{cN'}(w) \\
 & +c_{119}K^a\partial K^b\partial J^{cb}J^{aN'}J^{cN'}(w) + c_{120}K^a\partial K^bJ^{ac}J^{db}J^{dN'}J^{cN'}(w) \\
 & +c_{122}K^a\partial K^bJ^{dc}J^{db}J^{aN'}J^{cN'}(w) + c_{123}K^a\partial K^bJ^{ac}J^{dc}J^{dN'}J^{bN'}(w) \\
 & +c_{125}K^a\partial K^bJ^{dc}J^{dc}J^{aN'}J^{bN'}(w) + c_{126}K^a\partial K^b\partial J^{ab}J^{cd}J^{cd}(w) + c_{127}K^a\partial K^bK^b\partial^2K^cJ^{ac}(w) \\
 & +c_{128}K^a\partial K^bJ^{aN'}\partial^2J^{bN'}(w) + c_{129}K^a\partial K^bJ^{aN'}J^{bN'}J^{cN'}J^{cN'}(w) + c_{130}K^a\partial K^bJ^{bc}\partial J^{aN'}J^{cN'}(w) \\
 & +c_{131}K^a\partial K^bJ^{bc}J^{aN'}\partial J^{cN'}(w) + c_{132}K^a\partial K^bJ^{ac}\partial J^{bN'}J^{cN'}(w) + c_{133}K^a\partial K^bJ^{ac}J^{bN'}\partial J^{cN'}(w) \\
 & +c_{134}K^a\partial K^bJ^{ac}J^{bc}J^{dN'}J^{dN'}(w) + c_{135}K^a\partial K^bJ^{ab}\partial J^{cN'}J^{cN'}(w) + c_{136}K^a\partial^4K^bJ^{ab}(w) \\
 & +c_{137}K^a\partial^3K^b\partial J^{ab}(w) + c_{138}K^a\partial^3K^bJ^{aN'}J^{bN'}(w) + c_{141}K^a\partial^3K^bJ^{ac}J^{cb}(w) \\
 & +c_{142}K^a\partial^2K^b\partial^2J^{ab}(w) + c_{143}K^a\partial^2K^b\partial J^{aN'}J^{bN'}(w) + c_{144}K^a\partial^2K^b\partial J^{ac}J^{cb}(w) \\
 & +c_{145}K^a\partial^2K^bJ^{aN'}\partial J^{bN'}(w) + c_{146}K^a\partial^2K^bJ^{cb}J^{aN'}J^{cN'}(w) + c_{147}K^a\partial^2K^bJ^{ab}J^{cN'}J^{cN'}(w) \\
 & +c_{148}K^a\partial^2K^bJ^{ac}J^{bN'}J^{cN'}(w) + c_{149}K^a\partial^2K^bJ^{ab}J^{cd}J^{cd}(w) + c_{150}K^a\partial^2K^bJ^{ad}J^{cb}J^{cd}(w) \\
 & +c_{151}\partial^5K^aK^a(w) + c_{152}\partial^4K^a\partial K^a + c_{153}\partial^4J^{aN'}J^{aN'}(w) + c_{154}\partial^3K^aK^a\partial K^bK^b(w) \\
 & +c_{155}\partial^3K^aK^aJ^{bN'}J^{bN'}(w) + c_{156}\partial^3K^a\partial^2K^a(w) + c_{157}\partial^3K^a\partial K^bJ^{ab}(w) + c_{158}\partial^3K^aK^aJ^{bc}J^{bc}(w) \\
 & +c_{159}\partial^3J^{aN'}\partial J^{aN'}(w) + c_{160}\partial^2K^a\partial K^aJ^{bN'}J^{bN'}(w) + c_{161}\partial^2K^a\partial K^bJ^{aN'}J^{bN'}(w) \\
 & +c_{163}\partial^2K^a\partial K^aJ^{bc}J^{bc}(w) + c_{164}\partial^2K^a\partial K^bJ^{ac}J^{bc}(w) + c_{165}\partial^2K^a\partial K^bJ^{bc}J^{ac}(w) \\
 & +c_{166}\partial^2K^a\partial K^a\partial K^bK^b(w) + c_{167}\partial^2K^aK^a\partial^2K^bK^b(w) + c_{168}\partial^2K^aK^a\partial J^{bN'}J^{bN'}(w) \\
 & +c_{169}\partial^2K^a\partial^2K^bJ^{ab}(w) + c_{170}\partial K^aK^aJ^{bc}\partial J^{bN'}J^{cN'}(w) + c_{171}\partial K^aK^aJ^{bc}J^{dc}J^{bN'}J^{dN'}(w)
 \end{aligned}$$

$$\begin{aligned}
 & +c_{173}\partial K^a K^a J^{bc} J^{bc} J^{dN'} J^{dN'}(w) + c_{174}\partial K^a K^a J^{bc} J^{bc} J^{de} J^{de}(w) \\
 & +c_{175}\partial K^a \partial K^b \partial^2 J^{ab}(w) + c_{176}\partial K^a \partial K^b \partial J^{aN'} J^{bN'}(w) + c_{178}\partial K^a \partial K^b J^{ab} J^{cN'} J^{cN'}(w) \\
 & +c_{179}\partial K^a \partial K^b J^{ac} J^{bN'} J^{cN'}(w) + c_{180}\partial K^a \partial K^c J^{ac} J^{bd} J^{bd}(w) + c_{181}\partial K^a \partial K^b J^{cb} J^{cN'} J^{aN'}(w) \\
 & +c_{182}\partial J^{ab} J^{ab} \partial J^{cN'} J^{cN'}(w) + c_{183}\partial J^{ab} J^{aN'} \partial^2 J^{bN'}(w) + c_{184}\partial J^{ab} \partial J^{ab} J^{cN'} J^{cN'}(w) \\
 & +c_{185}\partial^3 J^{ab} \partial J^{ab}(w) + c_{186}J^{ab} J^{ca} \partial J^{cN'} \partial J^{bN'}(w) + c_{187}\partial J^{aN'} J^{aN'} \partial J^{bN'} J^{bN'}(w) \\
 & +c_{188}\partial^2 J^{ab} J^{aN'} \partial J^{bN'}(w) + c_{189}\partial^2 J^{ab} \partial^2 J^{ab}(w) + c_{190}\partial^2 J^{aN'} \partial^2 J^{aN'}(w) \\
 & +c_{191}K^a K^b \partial J^{ab} \partial J^{cN'} J^{cN'}(w) + c_{192}K^a \partial K^b J^{ab} \partial J^{cd} J^{cd}(w) + c_{194}K^a \partial^2 K^b J^{ac} \partial J^{bc}(w) \\
 & +c_{196}\partial K^a K^b J^{bc} J^{ac} J^{de} J^{de}(w) + c_{197}\partial K^a K^a J^{bc} J^{bd} J^{ce} J^{de}(w) + c_{198}\partial K^a K^b J^{ac} J^{bd} J^{ce} J^{de}(w) \\
 & +c_{199}\partial K^a K^a J^{bN'} J^{bN'} J^{cN'} J^{cN'}(w) + c_{200}\partial K^a K^a \partial J^{bN'} \partial J^{bN'}(w) \\
 & +c_{201}\partial K^a K^a \partial K^b K^b J^{cd} J^{cd}(w) + c_{202}\partial K^a K^a \partial K^b K^b J^{cN'} J^{cN'}(w) + c_{203}\partial K^a K^a \partial K^b \partial K^c J^{bc}(w) \\
 & +c_{204}\partial K^a K^a \partial^2 J^{bc} J^{bc}(w) + c_{205}\partial K^a K^a \partial^2 J^{bN'} J^{bN'}(w) + c_{206}\partial K^a \partial K^b J^{ac} J^{bd} J^{cd}(w) \\
 & +c_{207}\partial^2 K^a K^a \partial J^{bc} J^{bc}(w) + c_{209}J^{ab} J^{ac} J^{de} J^{de} J^{bN'} J^{cN'}(w) + c_{211}J^{ab} J^{cb} J^{de} J^{de} J^{cN'} J^{aN'}(w) \\
 & +c_{212}J^{ab} J^{ab} J^{cd} J^{ed} J^{ef} J^{ef} J^{cN'}(w) + c_{213}J^{ac} J^{bd} J^{aN'} J^{cN'} J^{bN'} J^{dN'}(w) \\
 & +c_{217}K^a K^b K^c K^d J^{ae} J^{ef} J^{ef} J^{bd}(w) + c_{222}K^a K^b K^c K^d \partial J^{ac} \partial J^{bd}(w) \\
 & +c_{224}K^a \partial K^b J^{ac} \partial^2 J^{cb}(w) + c_{225}K^a \partial K^b \partial^3 J^{ab}(w) + c_{227}\partial K^a K^a \partial J^{bc} \partial J^{bc}(w) \\
 & +c_{228}K^a K^b K^c K^d J^{ad} \partial^2 J^{bc}(w) + c_{229}K^a K^b K^c K^d J^{ec} \partial J^{ed} J^{ab}(w), \tag{3.75}
 \end{aligned}$$

where $N' \equiv 2N + 1$. The coefficients in (3.75) are presented in (E.1) of appendix E. The spin-6 current $O_6(z)$ is given by (3.72), (3.74) and (3.75). So far, the spin-4 and the spin-5 currents in different coset model were constructed in [19, 20]. One can analyze the three-point functions of the spin-6 current with scalars. The OPE for general N is obtained from (3.70) with (3.74). The fusion rule is summarized by $[O_{\frac{5}{2}}][O_{\frac{5}{2}}] = [I] + [O_2] + [O_4] + [O_{4'}] + [O_6]$.

The $\mathcal{N} = 1$ fusion rule is summarized by $[\hat{O}_2][\hat{O}_4] = [\hat{O}_2] + [\hat{O}_{\frac{7}{2}}] + [\hat{O}_4] + [\hat{O}_{\frac{11}{2}}]$. The explicit OPE is given by (F.6). As described before, among four OPEs between this $\mathcal{N} = 1$ supermultiplet, half of them are quite related to the others because they have common field contents. By rescaling the currents as $O_{4'}(z) \rightarrow N_{4'} O_{4'}(z)$ and $O_{\frac{9}{2}}(z) \rightarrow N_{\frac{9}{2}} O_{\frac{9}{2}}(z)$, where

$$\begin{aligned}
 N_{4'}^2 &= -189M(29 + 2c)(6 + 5c)^2 N(c - 6N + 6N^2)^6, & N_{\frac{9}{2}}^2 &= -\frac{1}{9}N_{4'}^2, \\
 \frac{1}{M} &\equiv c^6(-3 + 2c)(3 + c - 9N)(1 + N)(c + 6N) \\
 &\quad \times (-3c + 2(3 + c)N)(2c(-1 + N) + 3(3 - 4N)N)(c + 2cN + 3N(-3 + 2N)) \\
 &\quad \times (-2268N^3(25 + 2N(-25 + 6N)) + 5c^3(-2 + N)(-420 + N(437 + N(-12 + 7N)))) \\
 &\quad + 9cN^2(6300 + N(-12320 + N(9961 + 5N(-962 + 7N)))) \\
 &\quad - 6c^2N(4326 + N(-7259 + N(3973 + 5N(-62 + 7N))))),
 \end{aligned}$$

the standard normalizations arise: $O_{4'}(z)O_{4'}(w) \rightarrow \frac{1}{(z-w)^8} \frac{c}{4} + \dots$ and $O_{\frac{9}{2}}(z)O_{\frac{9}{2}}(w) \rightarrow \frac{1}{(z-w)^9} \frac{2c}{9} + \dots$. We have checked this up to $N = 5$. Then all the previous OPEs can be rewritten in terms of these rescaled currents.

4 Conclusions and outlook

We have found the first four higher spin supercurrents with spins $(2', \frac{5}{2}), (\frac{7}{2}, 4), (4', \frac{9}{2})$ and $(\frac{11}{2}, 6)$ in (1.2) including the super stress tensor with spins $(\frac{3}{2}, 2)$ in terms of the WZW currents in the coset model (1.1). Some of the OPEs between these supercurrents are determined. In the right hand side of these OPEs, one sees various kinds of quasi-primary (and primary) fields with given spins that can be written in terms of the above higher spin currents.⁹

- So far, the level of the second numerator current of the coset model (1.1) is fixed by 1. It would be interesting to study the higher spin currents for general levels (k, l) . Or what happens when the level k is replaced by $2N$ or $(2N + 1)$ in the coset model?
- According to recent work in [21], the large $\mathcal{N} = 4$ minimal model holography is an interesting subject. For example, there exists a particular $\mathcal{N} = 4$ coset theory related to the orthogonal group as follows: $W \times \text{SU}(2) \times \text{U}(1) = \frac{\text{SO}(N+4)}{\text{SO}(N) \times \text{SU}(2)} \times \text{U}(1)$ where W is a Wolf space. Simple computation for the central charge in this model leads to $c = \frac{6(k+1)(N+1)}{(k+N+2)}$ which is exactly the same as the central charge studied in [21]. The immediate step is how to construct the large $\mathcal{N} = 4$ superconformal algebra in this particular coset theory. For the Wolf space itself, the subgroup of $\text{SO}(N + 4)$ is realized by $\text{SO}(N) \times \text{SO}(4) =$

⁹One might ask what is the spin dependence of the maximal degree of the polynomials appearing in the structure constants which can be expressed as a ratio of two polynomials in N . For given higher spin current written in terms of WZW currents, the N dependence arises in many places. That is, the overall factor and the relative coefficient functions between various independent terms. As one calculates the particular OPE between the higher spin currents with spins s_1 and s_2 , these N dependences occur in each singular term of the OPE. Furthermore, each multiple product of WZW currents of spin s_1 and those of spin s_2 can produce the N dependence also by contracting the group indices during the OPE calculation.

For example, in the OPE of $O_2(z)O_2(w)$ given in (3.10), the large N behavior of c_{22} can be analyzed as follows. The maximal degrees of each polynomial in the numerator and denominator are given by 4 and 4 respectively from c_{22} . The overall factor of $O_2(z)$ contributes to $\frac{1}{N^2}$ and therefore by considering the other $O_2(w)$, the total contributions in the OPE are given by $\frac{1}{N^4}$. How does one obtain the extra N^4 behavior? The $O_2(z)$ contains four independent terms. One realizes that the N^4 behavior arises in the OPE between the third term and itself (i.e. $K^a K^b J^{ab}(z) K^c K^d J^{cd}(w)$) and in the OPE between the last term and itself (i.e. $J^{a2N+1} J^{a2N+1}(z) J^{b2N+1} J^{b2N+1}(w)$). The fourth-order pole terms of these OPEs behave as N^2 and the relative coefficient functions of third and fourth terms of $O_2(z)$ behave as N . Therefore, the total contribution is $N^2 \times N \times N = N^4$ as above. Note that the contribution from the OPE between the first term and itself in $O_2(z)$ is given by N^3 (the relative coefficient function in this case is a constant).

For the structure constants we have found in this paper, the maximal degree of polynomial in the numerator is the same as the one in the denominator. Let us denote the maximal degree of polynomial of numerator (or denominator) by $\text{deg}(N)$ and then one realizes that $\text{deg}(N) \leq s_1 + s_2$. The c dependent coefficients appearing in the numerator and denominator can be determined by $\text{deg}(N) + 1$ linear equations which can be obtained from the expressions for lower $N = 2, 3, \dots, \text{deg}(N) + 2$. Most of the structure constants have their factorized forms and therefore, we do not need all the above $\text{deg}(N) + 1$ linear equations to determine the c -dependent coefficients. Of course, as the spins s_1 and s_2 increase, the $\text{deg}(N)$ becomes large and it will take too much time (by package) to obtain the complete OPEs for low several N values. The spin s_3 of higher spin current appearing in the right hand side of above OPE is less than $s_1 + s_2$: $s_3 < s_1 + s_2$. There is no definite relation between the s_3 and $\text{deg}(N)$. In some examples, $\text{deg}(N)$ is greater than s_3 but in other examples, $\text{deg}(N)$ is less than or equal to s_3 . Therefore, the final c -dependent coefficient functions (i.e. structure constants of the OPEs) for general N can be obtained.

$SO(N) \times SU(2) \times SU(2)$. One can study this coset theory for fixed N in order to see the structure of an extended version of large $\mathcal{N} = 4$ superconformal algebra.

- Furthermore, one of the Kazama-Suzuki models has the following coset model $\frac{SO(N+2)}{SO(N) \times SO(2)}$ where the central charge is given by $c = \frac{3Nk}{(N+k)}$. It would be interesting to find the higher spin currents, along the line of [22, 23]. First of all, the $\mathcal{N} = 2$ superconformal algebra should be realized in this coset model from the $\mathcal{N} = 2$ WZW currents with constraints. After this is done, then the extension of the $\mathcal{N} = 2$ superconformal algebra can be obtained by constructing the higher spin currents with spins greater than 2.

- In [24], the generalization of the coset (1.1) is suggested and it has the following coset $\frac{SO(2N+M)_k \oplus SO(2NM)_1}{SO(2N)_{k+M} \oplus SO(M)_{k+2N}}$. When $M = 1$, this reduces to (1.1). When $M = 2$, this coset looks like the Kazama-Suzuki model above except the level 1 factor in the numerator. One can calculate the central charge and it is given by $c = \frac{3kMN}{(-2+k+M+2N)}$. For the stringy limit where M, N , and k are taken to be very large at the same time, the central charge c goes to N^2 due to the extra degree of freedom by M . It would be interesting to construct the higher spin currents in this generalized coset model, along the line of [18, 25].

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A The coefficient functions in (3.30) related to the spin-4 current

The equations for coefficients in (3.30) for $N = 2$ are given by

$$\begin{aligned}
 & \frac{36k^2}{(2+k)^2(3+k)^3} + c_1 = 0, & \frac{72k^2}{(2+k)^2(3+k)^3} - 2c_{14} = 0, & \frac{72k^2}{(2+k)^2(3+k)^3} + 2c_{13} = 0, \\
 & \frac{36(-2+k)k^2}{(2+k)^2(3+k)^3} + c_7 + c_{14} = 0, & \frac{36k^3}{(2+k)^2(3+k)^3} + c_{18} = 0, \\
 & -\frac{6k^2(7+k(13+4k))}{(2+k)^2(3+k)^3} + c_7 - 2c_8 - c_{14} + \frac{c_{16}}{2} = 0, & \frac{6k^2(1+2k)(7+5k)}{(2+k)^2(3+k)^3} + c_{11} = 0, \\
 & \frac{36k^2(4+3k)}{(2+k)^2(3+k)^3} + c_9 = 0, & \frac{36k^2}{(2+k)^2(3+k)^2} + c_{17} = 0, & \frac{24k^2(1+2k)^2}{(2+k)^2(3+k)^3} - 2c_{19} = 0, \\
 & \frac{72k^2}{(2+k)^2(3+k)^3} + 2c_{15} = 0, & \frac{6k^3(1+2k)}{(2+k)(3+k)^3} + c_{12} = 0, & \frac{6k^3(1+2k)^2}{(2+k)^2(3+k)^3} - c_{20} = 0, \\
 & \frac{6k^2(5+k)(11+4k)}{(2+k)^2(3+k)^3} - c_6 = 0, & \frac{36k^2}{(2+k)^2(3+k)^2} + c_{16} = 0, \\
 & 54(2c_9 - c_{10} + c_{17}) + k(81(2c_9 - c_{10} + c_{17}) + k(84 + 90c_9 - 45c_{10} + 45c_{17} \\
 & + k(-12 + 2(11+k)c_9 - (11+k)c_{10} + (11+k)c_{17}))) = 0, \\
 & \frac{12k^2(5+k)^2}{(2+k)^2(3+k)^3} + c - 3 = 0, & \frac{72k^2(4+k)}{(2+k)^2(3+k)^3} + c_2 = 0,
 \end{aligned}$$

$$\begin{aligned}
 & -108(2c_1 - c_2 + 4c_5) + k(k(-2(171 + k(67 + k(13 + k)))c_1 + 9(-24 + 19c_2 - 76c_5) \\
 & + k(67 + k(13 + k))(c_2 - 4c_5)) - 216(2c_1 - c_2 + 4c_5)) = 0, \\
 & \frac{72k^2}{(2+k)^2(3+k)^3} + 2(c_1 + c_4) = 0.
 \end{aligned} \tag{A.1}$$

One performs similar analysis for $N = 3, 4, 5$. Then the coefficients in (3.30) for general N (in terms of N and c) are given by

$$\begin{aligned}
 c_1 &= 6c^2 D(c - 3N)^3, & c_2 &= 12c^2 D(c - 3N)^2(c(-1 + N) + 3(2 - 3N)N), \\
 c_3 &= 2c^2 D(c - 3N)(2c(-1 + N) + 3(3 - 4N)N)^2, & c_4 &= 0, \\
 c_5 &= 3c^2 D(c - 6N)(c - 3N)^2(-1 + 2N), \\
 c_6 &= -c^2 D(c - 3N)(2c(-1 + N) + 3(3 - 4N)N)(3(9 - 10N)N + c(-5 + 2N)), \\
 c_7 &= -6c^2 D(-3 + 2c)(c - 3N)^2 N, \\
 c_8 &= -c^2 D(c - 3N)(-5c^2 + (45 - 2c)cN + 4(-9 + c)(3 + c)N^2 + 6(27 - 2c)N^3 - 72N^4), \\
 c_9 &= -6c^2 D(c - 3N)^2(c + 2cN + 12(-1 + N)N), \\
 c_{10} &= -2c^2 D(c - 3N)(6c(-1 + N)N(-9 + 2N) \\
 & \quad + 4c^2(-1 + N)(1 + 2N) - 9N^2(15 + 2N(-15 + 8N))), \\
 c_{11} &= c^2 D(c - 3N)(c + 2cN + 3N(-3 + 2N))(2c(2 + N) + 3N(-9 + 8N)), \\
 c_{12} &= -c^3 D(-1 + 2N)(c + 6(-1 + N)N)(c + 2cN + 3N(-3 + 2N)), \\
 c_{13} &= 6c^2 D(c - 3N)^3, & c_{14} &= -6c^2 D(c - 3N)^3, & c_{15} &= 6c^2 D(c - 3N)^3, \\
 c_{16} &= -18c^2 D(c - 3N)^2 N(-1 + 2N), & c_{17} &= -18c^2 D(c - 3N)^2 N(-1 + 2N), \\
 c_{18} &= -6c^3 D(c - 3N)^2(-1 + 2N), & c_{19} &= -2c^2 D(c - 3N)(c + 2cN + 3N(-3 + 2N))^2, \\
 c_{20} &= c^3 D(-1 + 2N)(c + 2cN + 3N(-3 + 2N))^2, \\
 D &\equiv \frac{1}{18N(-1 + 2N)(c + 6(-1 + N)N)^2}.
 \end{aligned} \tag{A.2}$$

B The coefficient functions in (3.38) associated with the spin- $\frac{9}{2}$ current

After the analysis for $N = 3, 4, 5$ in (3.38), the coefficients in (3.38) for general N are given by

$$\begin{aligned}
 c_1 &= D(c - 3N)^2(18(3 - 2c)c^2 + 5c(144 + (159 - 8c)c)N - 3(1350 + c(9 + 8c)(81 + 10c))N^2 \\
 & \quad + 2(2160 + c(4401 + 16c(27 + 2c)))N^3 - 24(171 + c(123 + 4c))N^4), \\
 c_2 &= -9D(21 + 4c)(c - 3N)^3(-1 + N)(-6N^2 + c(-1 + 4N)), \\
 c_3 &= D(c - 3N)^2(-54N^2(-33 - 46N + 8N^2) + 4c^3(9 + 2N(20 + N(-15 + 22N))) \\
 & \quad - 9cN(-4 + N(267 + 4N(9 + 47N))) - 3c^2(18 + N(7 + 4N(39 + N(-147 + 52N))))), \\
 c_4 &= 3D(21 + 4c)(c - 3N)^2(-1 + N)(c - 4cN + 6N^2)(-2c(-1 + N) + 3N(-3 + 4N)), \\
 c_5 &= 2D(c - 3N)^2(-1 + N)(108c^2(3 + c) - 2c(360 + c(681 + 34c))N \\
 & \quad - 3(-3 + 2c)(-54 + c(111 + 40c))N^2 + 2(3 + 4c)(414 + c(339 + 22c))N^3 \\
 & \quad - 24(207 + 8c(33 + 7c))N^4), \\
 c_6 &= 18D(3 + c - 9N)(c - 3N)^2(-1 + N)(-3c + 2(3 + c)N)(2c(-1 + N) + 3(3 - 4N)N),
 \end{aligned}$$

$$\begin{aligned}
 c_7 &= -D(3+c-9N)(c-3N)(-1+N)(-3c+2(3+c)N)(2c(-1+N) \\
 &\quad +3(3-4N)N)(3N(11+2N)+c(-33+4N(5+11N))), \\
 c_8 &= -12D(3+c-9N)(c-3N)(-1+N)(-3c+2(3+c)N)(2c(-1+N)+3(3-4N)N)^2, \\
 c_9 &= -18D(-3+2c)(c-3N)^2(-1+N)(1+N)(c+6N)(c+2cN+3N(-3+2N)), \\
 c_{10} &= -2D(c-3N)(2c^4(18+(-1+N)N(55+4N(-3+11N))) \\
 &\quad +162N^3(-9+N(81+4N(-37+14N))) +27cN^2(117+N(-719+2N(585-286N \\
 &\quad +76N^2))) -3c^3(-36+N(319+N(-763+2N(395+2N(-89+22N)))))) \\
 &\quad +9c^2N(-110+N(701+N(-1307+4N(283+N(-163+28N)))))), \\
 c_{11} &= -6D(3+c-9N)(c-3N)(-1+N)(-3c+2(3+c)N)(2c(-1+N)+3(3-4N)N)^2, \\
 c_{12} &= 6D(-3+2c)(c-3N)(-1+N)(1+N)(c+6N)(2c(-1+N) \\
 &\quad +3(3-4N)N)(c+2cN+3N(-3+2N)), \\
 c_{13} &= 6D(-3+2c)(c-3N)(-1+N)(1+N)(c+6N)(c+2cN+3N(-3+2N))^2, \\
 c_{14} &= 6D(-3+2c)(c-3N)(-1+N)(1+N)(c+6N)(c+2cN+3N(-3+2N))^2, \\
 c_{15} &= -12D(-3+2c)(c-3N)(-1+N)(1+N)(c+6N)(c+2cN+3N(-3+2N))^2, \\
 c_{16} &= 2D_3(c-3N)(1+N)(2c^4N(-5+2N)(-19+22N)+324N^3(-12+N(-7+24N)) \\
 &\quad +27cN^2(1+N(511-682N+96N^2))-3c^3(54+N(-239+4N(117+2N(-45+7N)))) \\
 &\quad -18c^2N(-38+N(279+8N(-30+N(-13+9N))))), \\
 c_{17} &= 6D(c-3N)^2(-1+N)(2c^3(7+11N(-1+2N))+54N^2(9+2N(-9+11N)) \\
 &\quad +3c^2(11+N(-97+4(40-11N)N))+9cN(-18+N(89+2N(-97+28N))))), \\
 c_{18} &= -2D(c-3N)(-1+N)(324N^3(12+N(-65+74N+4N^2))+4c^4(-18+N(43+N \\
 &\quad -76N^2+44N^3))+54cN^2(-95+N(506+N(-577+2N(-47+80N)))) \\
 &\quad +9c^2N(190+N(-1087+4N(304+N(139+4N(-61+10N)))))) \\
 &\quad -24c^3(9+N(-70+N(106+N(29+3N(-49+22N))))), \\
 c_{19} &= -18D(-3+2c)(c-3N)(-1+N)(1+N)(c+6N) \\
 &\quad \times (-3c+2(3+c)N)(c+2cN+3N(-3+2N)), \\
 c_{20} &= -6D(-3+2c)(-1+N)(1+N)(c+6N)(-3c+2(3+c)N) \\
 &\quad \times (2c(-1+N)+3(3-4N)N)(c+2cN+3N(-3+2N)), \\
 c_{22} &= -6D(-3+2c)(-1+N)(1+N)(c+6N)(-3c+2(3+c)N)(c+2cN+3N(-3+2N))^2, \\
 c_{23} &= 2cD(3+c-9N)(-1+N)(1+N)(-1+2N)(-3c+2(3+c)N) \\
 &\quad \times (2c(-1+N)+3(3-4N)N)^2, \\
 c_{24} &= -3D(21+4c)(c-3N)^2(-1+N)(c-4cN+6N^2)(c+2cN+3N(-3+2N)), \\
 c_{25} &= 2D(c-3N)^2(-1+N)(-54N^2(-9+4N(29+5N)) \\
 &\quad +4c^3(-9+2N(-22+N(-3+22N)))+9cN(10+N(273+4N(-18+67N))) \\
 &\quad +c^2(54+3N(-13+4N(-30+N(3+68N))))), \\
 c_{26} &= -4D(c-3N)(-1+N)(1+N)(-3c+2(3+c)N)(2c^3(-5+11N(-1+2N)) \\
 &\quad +54N^2(-3+4N(-3+7N))+9cN(24+N(-103+2(107-92N)N)) \\
 &\quad -3c^2(4+N(-35+8N+44N^2))), \\
 c_{27} &= -6D(3+c-9N)(c-3N)(-1+N)(-3c+2(3+c)N)(2c(-1+N) \\
 &\quad +3(3-4N)N)(c+2cN+3N(-3+2N)),
 \end{aligned}$$

$$\begin{aligned}
 c_{28} &= -2D(c-3N)(-1+N)(324N^3(12+N(23-46N+48N^2)) \\
 &\quad -54cN^2(7+N(198+N(-163+36N(5+N)))) \\
 &\quad +4c^4(-3+N(-9+N(-47+4N(-3+11N)))) \\
 &\quad -27c^2N(14+N(-71+4N(-8+N(-5+8N(-3+2N)))) \\
 &\quad +3c^3(6+N(75+2N(-3+4N(-6+11N(-1+2N))))), \\
 c_{29} &= -6D(-3+2c)(-1+N)(1+N)(c+6N)(-3c+2(3+c)N)(c+2cN+3N(-3+2N))^2, \\
 c_{30} &= D(3+c-9N)(-1+N)(-1+2N)(-3c+2(3+c)N)(2c(-1+N) \\
 &\quad +3(3-4N)N)(-45N^2(1+2N)+6cN(-7+5N(1+2N))+c^2(3+4N(4+3N))), \\
 c_{31} &= -D(-3+2c)(c-3N)(-1+N)(1+N)(c+6N)(1+22N) \\
 &\quad \times (-3c+2(3+c)N)(c+2cN+3N(-3+2N)), \\
 c_{32} &= D(-3+2c)(-1+N)(1+N)(-1+2N)(c+6N)(-3c+2(3+c)N) \\
 &\quad \times (c+2cN+3N(-3+2N))(3(7-16N)N+c(-2+6N)), \\
 c_{33} &= D(-3+2c)(-1+N)(1+N)(-1+2N)(c+6N) \\
 &\quad \times (-3c+2(3+c)N)(c+2cN+3N(-3+2N))^2, \\
 D &\equiv \frac{ic^3\sqrt{3c-9N}}{18(-1+N)(21+4c)(6+5c)N(-1+2N)(c+6(-1+N)N)^3\sqrt{N-2N^2}}. \tag{B.1}
 \end{aligned}$$

C The coefficient functions in (3.46) corresponding to the spin-4' current

After analyzing for fixed $N = 3, 4, 5$, one obtains that the coefficients in (3.46) for general N are given by

$$\begin{aligned}
 c_1 &= -27c^3D(21+4c)(c-3N)^4(-6N^2+c(-1+4N)), \\
 c_2 &= 18c^3D(c-3N)^3(8c^3(5-9N+6N^2)+54N^2(11-64N+86N^2) \\
 &\quad +3c^2(43+N(-259+2(179-72N)N))+9cN(-64+N(359+2N(-259+80N)))), \\
 c_3 &= 18c^3D(21+4c)(6+5c)(c-3N)^4N(-1+2N), \\
 c_4 &= -12c^3D(21+4c)(c-3N)^3N(-1+2N) \\
 &\quad \times (20c^2(-1+N)+18(6-17N)N-3c(17-74N+40N^2)), \\
 c_5 &= -12c^3D(3+c-9N)(c-3N)^2(1+N)(432(9-14N)N^3+32c^3(-1+N)^2(-9+10N) \\
 &\quad -36cN^2(157+N(-377+222N))+3c^2N(803-4N(596+5N(-113+32N)))), \\
 c_6 &= -72c^3D(3+c-9N)(c-3N)^2(-3c+2(3+c)N)(2c(-1+N)+3(3-4N)N)^2, \\
 c_7 &= -6c^3D(c-3N)^2N(-1+2N)(4c^4(-7+N(-19+8N))+162N^2(15+N(-51+88N)) \\
 &\quad +6c^3(17+N(-46+(87-16N)N))+27cN(-30+N(233+N(-659+136N))) \\
 &\quad -9c^2(-44+N(307+N(-615+14N(-1+8N))))), \\
 c_8 &= -12c^3D(3+c-9N)(c-3N)^2(1+N)(108(9-14N)N^3-9cN^2(157+133N(-3+2N)) \\
 &\quad +3c^2N(220+N(-695+6(119-36N)N))+4c^3(-18+N(52+N(-49+18N)))), \\
 c_9 &= 24c^3D(3+c-9N)(c-3N)(-1+2N)(-3c+2(3+c)N) \\
 &\quad \times (2c(-1+N)+3(3-4N)N)^2(3N+c(3+4N)), \\
 c_{10} &= -18c^3D(c-3N)^3(8c^3(2+3N+6N^2)+54N^2(11+2N(10+N)) \\
 &\quad +3c^2(1+N(-43+70N+48N^2))+9cN(20+N(-145-86N+64N^2))), \\
 c_{11} &= -12c^3D(c-3N)^2(-6N^2+c(-1+4N))(8c^3(1+N)(-3+2N) \\
 &\quad -27N^2(-41+82N+60N^2)-3c^2(5+2N)(3+N(-13+8N)))
 \end{aligned}$$

$$\begin{aligned}
 & -9cN(41 + N(1 + 2N)(-83 + 48N))) \\
 c_{12} &= 12c^3 D(-3 + 2c)(21 + 4c)(c - 3N)^3 N(-1 + 2N)(5c(1 + 2N) + 6N(6 + 5N)), \\
 c_{13} &= 108c^3 D(c - 3N)^2 N(-1 + 2N)(-108N^2(2 + N)(-2 + 5N) + c^3(1 + 27(1 - 2N)N) \\
 & \quad + 18cN(-8 + N(19 + 2N(7 + N))) + 3c^2(-5 + N(14 + N(-59 + 54N))))), \\
 c_{14} &= -36c^3 D(3 + c - 9N)(c - 3N)N(1 + N)(-1 + 2N)(c + 2cN - 12N^2) \\
 & \quad \times (-9c^2 + 2c(24 + 11c)N - 12(12 + c)N^2), \\
 c_{15} &= -72c^3 D(-3 + 2c)(c - 3N)^2(1 + N)(c + 6N)(c + 2cN + 3N(-3 + 2N))^2, \\
 c_{16} &= 144c^3 D(-3 + 2c)(c - 3N)(1 + N)(c + 6N) \\
 & \quad \times (-3c + 2(3 + c)N)(c + 2cN + 3N(-3 + 2N))^2, \\
 c_{17} &= 12c^3 D(c - 3N)^2(c + 6N)(-3c + 2(3 + c)N)(8(2 + 5N)(c + 2cN)^2 \\
 & \quad - 9N(-36 + N(-98 + N(21 + 62N))) + 3c(-8 + N(-115 + N(-335 + 2N(-19 + 80N))))), \\
 c_{18} &= -72c^3 D(c - 3N)N(-1 + 2N)(c + 6N)(-3c + 2(3 + c)N) \\
 & \quad \times (c(-1 + N) + 3N^2)(c(-1 + 11N) - 3(4 + N(-8 + 9N))), \\
 c_{19} &= 12c^3 D(-3 + 2c)(c - 3N)(c + 6N)(-3c + 2(3 + c)N)(8(c + 2cN)^2(-2 + N(-2 + 3N)) \\
 & \quad + 9N^2(-108 + N(2 + N(155 + 2N - 64N^2))) \\
 & \quad + 3cN(88 + N(205 + N(-111 + 2N(-83 + 16N))))), \\
 c_{20} &= -48c^3 D(-3 + 2c)(c - 3N)(1 + N)(-1 + 2N)(c + 6N) \\
 & \quad \times (-3c + 2(3 + c)N)(c + 2cN + 3N(-3 + 2N))^2 \\
 & \quad \times (2c(-1 + N) + 3(3 - 4N)N)(c + 2cN + 3N(-3 + 2N)), \\
 c_{21} &= -144c^4 D(3 + c - 9N)(c - 3N)(1 + N)(-1 + 2N) \\
 & \quad \times (-3c + 2(3 + c)N)(2c(-1 + N) + 3(3 - 4N)N)^2, \\
 c_{22} &= -6c^3 D(c - 3N)^2(-1 + 2N)(c + 6N)(-3c + 2(3 + c)N) \\
 & \quad \times (8c^2(2 + N) + 63N(6 + 5N) - 3c(-1 + N(97 + 88N))), \\
 c_{23} &= 12c^3 D(c - 3N)(-1 + 2N)(c + 6N)(-3c + 2(3 + c)N) \\
 & \quad \times (8c^3(1 + N)(-1 + 4N) - 81N^2(2 + N)(-2 + 5N) + 18cN(-6 + N(-2 + N(43 + 24N))) \\
 & \quad - 3c^2(5 + N(-48 + N(83 + 112N))))), \\
 c_{24} &= 12c^3 D(c - 3N)(1 + N)(4c^5(-2 + N)(1 + 2N)(-3 + 4N(-2 + 3N)) \\
 & \quad + 1944N^4(18 + N(-31 + 2N(5 + 4N))) \\
 & \quad + 324cN^3(-43 + N(-95 + N(285 + N(-201 + 38N)))) \\
 & \quad - 18c^3 N(-20 + N(-10 + N(387 + 2N(-165 + 8N(-9 + 5N)))))) \\
 & \quad + 3c^4(-12 + N(-130 + N(-53 + 4N(128 + N(-95 + 16N)))))) \\
 & \quad - 27c^2 N^2(-13 + N(-602 + N(117 + 4N(254 + N(-153 + 32N)))))), \\
 c_{25} &= 12c^3 D(-3 + 2c)(c - 3N)^2(-54N^3(174 + N(1 + 6N)(-23 + 16N)) \\
 & \quad + 6c^2 N(-1 + 2N)(3 + 2N)(-16 + N(-29 + 20N)) + 9cN^2(107 + 8N(78 + (24 - 35N)N)) \\
 & \quad + 8c^3(1 + 2N)(-3 + N(-9 + N(-11 + 10N))))), \\
 c_{26} &= -72c^4 D D'(-3 + 2c)(-1 + 2N), \quad c_{27} = 144D D' c^3(-3 + 2c)(c - 3N)N, \\
 c_{28} &= -36c^4 D D'(-3 + 2c)(1 - 2N)^2, \quad c_{29} = 4c^4 D D'(-3 + 2c)(1 - 2N)^2, \\
 D &\equiv \frac{1}{54N^2(-1 + 2N)(21 + 4c)(6 + 5c)(-1 + 2N)(c + 6(-1 + N)N)^3}, \\
 D' &\equiv (1 + N)(c + 6N)(6N + c(-3 + 2N))(c + 2cN + 3N(-3 + 2N))^2.
 \end{aligned} \tag{C.1}$$

D The coefficient functions in (3.66) related to the spin- $\frac{11}{2}$ current

Similar analysis leads to the fact that the coefficients in (3.66) for general N are given by

$$\begin{aligned}
 c_1 &= -567D(29+2c)(c-3N)^4N(-6N^2+c(-1+4N)), \\
 c_2 &= 54D(c-3N)^3N(8c^2(49+c)-c(1071+2c(791+60c))N \\
 &\quad +2(-1134+c(4557+4c(187+9c)))N^2-6(1071+2c(791+60c))N^3+288(49+c)N^4), \\
 c_3 &= -378D(29+2c)(6+5c)(c-3N)^4N^2(-1+2N), \\
 c_4 &= 27D(c-3N)^3N(2c^3(-13+36N(-1+2N))+252N^2(-18+N(36+25N)) \\
 &\quad -24cN(-63+N(154+N(119+39N))))+c^2(175-4N(119+2N(-185+54N)))), \\
 c_5 &= -9D(c-3N)^2N(2c(-1+N)+3(3-4N)N)(4c^3(-3+N(-25+8N)) \\
 &\quad +378N^2(-12+N(-5+36N))-9cN(35+48N(-14+N(14+N))) \\
 &\quad +6c^2(63-4N(42+N(-22+25N)))), \\
 c_6 &= 36(c-3N)^3N(504c^2(3+c)+8c(-462+c(-568+5c))N \\
 &\quad -3(2394+c(-1323+2182c+248c^2))N^2+4(5418+5c(47+2c)(39+8c))N^3 \\
 &\quad -12(2163+25c(103+10c))N^4), \\
 c_7 &= -378D(3+c-9N)(c-3N)^3N(-3c+2(3+c)N)(2c(4+N)+3N(-13+16N)), \\
 c_8 &= -18D(c-3N)^2N(162N^3(-98+N(577+2N(-843+748N))) \\
 &\quad +2c^4(-492+N(478+N(597+4N(-187+45N)))) \\
 &\quad +27cN^2(-1131+2N(1605+4N(897+N(-2353+1070N)))) \\
 &\quad -3c^3(984+N(-5918+N(2725+2N(4535+6N(-743+190N)))) \\
 &\quad +9c^2N(2286+N(-9376+N(-1147+4N(5563+N(-4079+840N)))))), \\
 c_9 &= 18D(29+2c)(c-3N)^3N^2(-1+2N)(252(3-8N)N \\
 &\quad +5c^2(-29+32N)-6c(56+5N(-49+29N))), \\
 c_{10} &= 108D(3+c-9N)(c-3N)^2N(-3c+2(3+c)N)(2c(-1+N) \\
 &\quad +3(3-4N)N)(2c(8+3N)+N(-49+96N)), \\
 c_{11} &= 72D(3+c-9N)(c-3N)^2N(-3c+2(3+c)N)(2c(-1+N) \\
 &\quad +3(3-4N)N)(c(34+4N)+3N(-77+68N)), \\
 c_{12} &= 108D(c-3N)^3N(23c^2(7+4c)-c(1386+c(1603+36c))N \\
 &\quad + (4158+c(5649+2210c+72c^2))N^2-6(1386+c(1603+36c))N^3+828(7+4c)N^4), \\
 c_{13} &= 18D(c-3N)^3N(-504N^2(15+N(-59+13N)) \\
 &\quad +6cN(2044+N(-7056+(1699-1102N)N))+2c^3(-504+N(145+54N+320N^2)) \\
 &\quad +c^2(-3024+N(14789+2N(-4593+2906N+780N^2)))), \\
 c_{15} &= -36D(c-3N)^2N(-984c^3(3+c)+c^2(22737+c(20329+1206c))N \\
 &\quad +3c(-20490+c(-39767+c(-6205+178c)))N^2 \\
 &\quad + (71442+c(272709+c(108279-14c(887+84c))))N^3 \\
 &\quad +12(-3+c)(7410+c(9085+c(1537+30c)))N^4 \\
 &\quad -12(-14661+c(4038+c(6197+410c)))N^5+72(1203+5c(127+34c))N^6), \\
 c_{16} &= -9D(c-3N)^2N(108N^3(-1974+N(5643-6806N+4456N^2)) \\
 &\quad +6c^4(384+N(-23+4N(51-86N+30N^2))) \\
 &\quad +18cN^2(9801+N(-36408+N(53177+2N(-22567+6292N)))) \\
 &\quad +3c^2N(-18370+N(92811+2N(-61705+4N(11891-4714N+780N^2)))) \\
 &\quad +c^3(5532+N(-44257+2N(20115-2N(8585+12N(-577+125N))))), \\
 c_{17} &= -180D(29+2c)(21+4c)(c-3N)^3N^2(-1+2N)(-6N^2+c(-1+4N)),
 \end{aligned}$$

$$\begin{aligned}
 c_{18} &= 12D(3+c-9N)(c-3N)N(1+N)(-3c+2(3+c)N)(2c(-1+N) \\
 &\quad +3(3-4N)N)(86c^2-15c(31+10c)N+18(21+c(55+2c))N^2-216(3+2c)N^3), \\
 c_{19} &= -36D(3+c-9N)N(-c+3N)(-3c+2(3+c)N)(2c(-2+N) \\
 &\quad +3(21-8N)N)(2c(-1+N)+3(3-4N)N)^2, \\
 c_{20} &= -36D(3+c-9N)N(-c+3N)(-3c+2(3+c)N)(2c(-2+N) \\
 &\quad +3(21-8N)N)(2c(-1+N)+3(3-4N)N)^2, \\
 c_{21} &= -378D(c-3N)^3N(1+N)(c+6N)(3(5-6c)c+(-117+4c(30+c))N+18(5-6c)N^2), \\
 c_{23} &= -108D(c-3N)^2N(1+N)(c+6N)(4c^3(-2+N)(-11+6N) \\
 &\quad -9N^2(399-862N+416N^2)+3cN(431+2N(127+12N(-79+44N))) \\
 &\quad -4c^2(26+N(237+N(-431+138N))))), \\
 c_{25} &= 6D(-3+2c)(c-3N)N(1+N)(c+6N)(2c(-1+N)+3(3-4N)N)(9N^2(525 \\
 &\quad +46N(-25+12N))+2c^2(69+N(-61+22N))-3cN(575+4N(-230+61N))), \\
 c_{27} &= 18DN(1+N)(-c+3N)(c+6N)(c+2cN+3N(-3+2N))^2(9(21-34N)N \\
 &\quad +c^2(6+4N)+c(-51+36N(1+N))), \\
 c_{31} &= 108D(c-3N)^2N(1+N)(c+6N)(c+2cN+3N(-3+2N)) \\
 &\quad \times(3N(-49+2N)+2c^2(-19+6N)+c(1+76(4-3N)N)), \\
 c_{32} &= -36DN(1+N)(-c+3N)(c+6N)(c+2cN+3N(-3+2N))^2 \\
 &\quad \times(9(21-34N)N+c^2(6+4N)+c(-51+36N(1+N))), \\
 c_{33} &= -12DN(1+N)(-c+3N)(c+6N)(c+2cN+3N(-3+2N))^2 \\
 &\quad \times(4c^2(-15+4N)+9N(-35+18N)+c(27+6(73-60N)N)), \\
 c_{34} &= -6DN(1+N)(-c+3N)(c+6N)(c+2cN+3N(-3+2N))^2 \\
 &\quad \times(2c^2(-51+22N)-9N(7+66N)+c(-99+12(82-51N)N)), \\
 c_{35} &= -72DN(1+N)(-c+3N)(c+6N)(c+2cN+3N(-3+2N))^2 \\
 &\quad \times(9(21-34N)N+c^2(6+4N)+c(-51+36N(1+N))), \\
 c_{36} &= 24D(c-3N)N(1+N)(c+6N)(c+2cN+3N(-3+2N)) \\
 &\quad \times(4c^3(39+8(-4+N)N)-27N^2(357-854N+480N^2) \\
 &\quad -6c^2(60+N(227+2N(-229+64N)))+9cN(427+4N(-64+N(-227+156N))))), \\
 c_{37} &= 72D(c-3N)^2N(1+N)(c+6N)(c+2cN+3N(-3+2N)) \\
 &\quad \times(8c^2(-9+N)+9N(-77+86N)+c(129+6(61-72N)N)), \\
 c_{38} &= 12D(c-3N)N(1+N)(c+6N)(c+2cN+3N(-3+2N))(4c^3(60+N(-67+22N)) \\
 &\quad -27N^2(273+2N(-245+72N))+9cN(245+2N(341+4N(-257+120N))) \\
 &\quad -12c^2(9+N(257+N(-425+134N))))), \\
 c_{39} &= -36D(c-3N)^2N(1+N)(c+6N)(c+2cN+3N(-3+2N)) \\
 &\quad \times(8c^2(-9+N)+9N(-77+86N)+c(129+6(61-72N)N)), \\
 c_{42} &= -36D(c-3N)^2N(504c^3(3+c)-c^2(15771+13c(847+10c))N \\
 &\quad +3c(16422+c(27620+c(3593+2c)))N^2 \\
 &\quad -(48762+c(227997+c(101709+170c(49+4c))))N^3 \\
 &\quad +6(36036+c(44577+2c(6320+c(557+30c))))N^4 \\
 &\quad -12(19467+c(13098+125c(17+2c)))N^5+288(210+(217-20c)c)N^6), \\
 c_{43} &= 54D(3+c-9N)(c-3N)^2N(-3c+2(3+c)N)(c^2(-8+6N(9+4N)) \\
 &\quad -6N^2(-126+N(125+48N))+cN(-125+4N(-67+81N))), \\
 c_{44} &= -18D(c-3N)^2N(1872c^3(3+c)-c^2(47424+c(37951+1558c))N
 \end{aligned}$$

$$\begin{aligned}
& +2c(62820 + c(109767 + c(17813 + 14c)))N^2 \\
& -2(37422 + c(266247 + 2c(35124 + c(3017 + 158c))))N^3 \\
& +8(42633 + c(36792 + c(-3081 + c(-167 + 70c))))N^4 \\
& +144(-1251 + c(1326 + 5c(35 + 2c)))N^5 - 144(942 + c(533 + 50c))N^6, \\
c_{45} = & 6D(c - 3N)N(-324cN^3(1608 + N(-10316 + N(17687 - 4518N - 9268N^2 + 4248N^3))) \\
& -972N^4(-294 + N(2040 + N(-3707 + 2N(725 + 396N)))) \\
& +c^5(912 + 2N(-1174 + N(415 + 4N(388 + N(-409 + 108N)))) \\
& -27c^2N^2(-8639 + 2N(27044 + N(-40858 + N(-3123 + 4N(9903 + N(-4495 + 72N)))))) \\
& +3c^4(912 + N(-8842 + N(15705 + 2N(333 - 4N(3006 + N(-2633 + 702N)))))) \\
& +9c^3N(-4586 + N(31236 + N(-45873 + 2N(-6287 + 4N(8373 + N(-5471 + 1098N)))))), \\
c_{46} = & 24D(3 + c - 9N)N(-c + 3N)(-3c + 2(3 + c)N)(2c(-1 + N) \\
& +3(3 - 4N)N)^2(c(26 + 8N) + 3N(-35 + 52N)), \\
c_{47} = & 54D(c - 3N)^2N(c - 4cN + 6N^2)(c^2(65 + 14c) - c(1161 + c(283 + 6c))N \\
& + (3483 + c(2391 + 4c(104 + 3c)))N^2 - 6(1161 + c(283 + 6c))N^3 + 36(65 + 14c)N^4), \\
c_{48} = & -108D(c - 3N)^2N(-4c^3(16 + 13c) + c^2(633 + 2c(545 + 22c))N \\
& -c(4608 + c(5193 + 2c(965 + 24c)))N^2 + (12474 + c(12231 + 4c(3645 + c(205 + 4c))))N^3 \\
& -6(4608 + c(5193 + 2c(965 + 24c)))N^4 + 36(633 + 2c(545 + 22c))N^5 - 864(16 + 13c)N^6), \\
c_{49} = & -378D(c - 3N)^2N(1 + N)(c + 6N)(-3c + 2(3 + c)N)(3(5 - 6c)c \\
& + (-117 + 4c(30 + c))N + 18(5 - 6c)N^2), \\
c_{50} = & -108DN(1 + N)(-c + 3N)(c + 6N)(-3c + 2(3 + c)N)(4c^3(-2 + N)(-11 + 6N) \\
& -9N^2(399 - 862N + 416N^2) + 3cN(431 + 2N(127 + 12N(-79 + 44N))) \\
& -4c^2(26 + N(237 + N(-431 + 138N)))), \\
c_{51} = & -54DN(1 + N)(-c + 3N)(c + 6N)(-3c + 2(3 + c)N)(2c^3(25 + 6N(-13 + 4N)) \\
& -18N^2(126 + N(-379 + 206N)) + 3cN(379 + 8N(-94 + 75(-1 + N)N)) \\
& -c^2(103 + 4N(75 + 26N(-17 + 9N)))), \\
c_{52} = & -6D(-3 + 2c)N(1 + N)(c + 6N)(-3c + 2(3 + c)N)(2c(-1 + N) \\
& +3(3 - 4N)N)(9N^2(525 + 46N(-25 + 12N)) + 2c^2(69 + N(-61 + 22N)) \\
& -3cN(575 + 4N(-230 + 61N))), \\
c_{53} = & 24DN(1 + N)(-c + 3N)(c + 6N)(c + 2cN + 3N(-3 + 2N))^2(4c^2(-15 + 4N) \\
& +9N(-35 + 18N) + c(27 + 6(73 - 60N)N)), \\
c_{54} = & 36DN(1 + N)(-c + 3N)(c + 6N)(c + 2cN + 3N(-3 + 2N))^2(9(21 - 34N)N \\
& +c^2(6 + 4N) + c(-51 + 36N(1 + N))), \\
c_{55} = & -9D(c - 3N)^2N(108N^3(1050 + N(47 - 2978N + 2288N^2)) \\
& +c^4(864 + 2N(-1145 + 4N(-39 - 46N + 90N^2))) \\
& +18cN^2(-79 + N(-12724 + N(17249 + 2N(-5367 + 404N)))) \\
& +3c^2N(-6786 + N(32719 + 2N(-14725 + 4N(4801 - 2654N + 300N^2)))) \\
& +3c^3(1324 + N(-5703 + 10N(861 + 2N(-443 + 4N(37 + 7N))))), \\
c_{56} = & 36D(c - 3N)N(-324N^4(-378 + N(1666 + N(-2077 + 458N + 316N^2))) \\
& +2c^5(24 + N(-53 + N(13 + 2N(43 - 86N + 36N^2)))) \\
& +54cN^3(-1477 + N(7198 + N(-10925 + 8N(821 + N(-240 + 59N)))) \\
& -9c^2N^2(-2552 + N(16442 + N(-30725 + 2N(7517 + 2N(3527 - 3530N + 552N^2)))) \\
& +3c^3N(-1061 + N(9239 + 2N(-8552 + N(135 + 2N(5345 + 6N(-555 + 58N))))))
\end{aligned}$$

$$\begin{aligned}
 & +c^4(144+N(-2087+N(3699+2N(691-6N(577+2N(-233+72N)))))), \\
 c_{57} = & 9D(c-3N)^2N(540N^3(42+N(-1+6N)(-67+26N)) \\
 & +c^4(-924+2N(-41+2N(81+88N))) \\
 & +c^3(-2634+N(14555+8N(-33+N(-616+117N)))) \\
 & -3c^2N(-4790+N(14051+8N(1667+3N(-482+251N)))) \\
 & +18cN^2(-1303+N(-580+N(12787+2N(-2707+504N))))), \\
 c_{58} = & -3D(3+c-9N)(c-3N)(-3c+2(3+c)N)(2c(-1+N) \\
 & +3(3-4N)N)^2(c(40+22N)+3N(-7+80N)), \\
 c_{59} = & -12D(-3+2c)N(1+N)(c+6N)(-3c+2(3+c)N) \\
 & \times(c+2cN+3N(-3+2N))(2c^2(-2+N)(-23+8N) \\
 & -3cN(-7+2N)(-51+64N)+27N^2(105+8N(-28+13N))), \\
 c_{60} = & -36DN(1+N)(-c+3N)(c+6N)(-3c+2(3+c)N)(c+2cN+3N(-3+2N)) \\
 & \times(c^2(-86+8N)+9N(-63+44N)+c(87+6(89-72N)N)), \\
 c_{61} = & 6D(-3+2c)N(1+N)(c+6N)(-3c+2(3+c)N) \\
 & \times(c+2cN+3N(-3+2N))^2(c(23+6N)+3N(-35+18N)), \\
 c_{62} = & 6D(c-3N)N(324N^4(-210+N(-41+2N(1650+N(-2579+1032N)))) \\
 & +4c^5(-252+N(335+N(-113+N(253-250N+72N^2)))) \\
 & +54cN^3(1807+N(-2474+N(-17042+N(27203+2N(-5353+732N)))) \\
 & -9c^2N^2(9886+N(-41452+N(23217+4N(2116+N(-865+36N(1+14N)))))) \\
 & -2c^4(1512+N(-11839+2N(7862+N(-5173+N(5281+12N(-256+27N)))))) \\
 & +3c^3N(9620+N(-53049+N(60735+2N(-16767+4N(3551+3N(-793+186N)))))), \\
 c_{63} = & 6D(-3+2c)N(1+N)(c+6N)(-3c+2(3+c)N) \\
 & \times(c+2cN+3N(-3+2N))^2(c(23+6N)+3N(-35+18N)), \\
 c_{64} = & -D(3+c-9N)N(-1+2N)(-3c+2(3+c)N)(2c(-1+N)+3(3-4N)N)^2 \\
 & \times(-64c^2+(435-34c)cN+3(-189+4(-2+c)c)N^2+72(3-2c)N^3), \\
 c_{65} = & 9D(c-3N)^2N(324N^3(-98+N(1719+4N(-282+25N))) \\
 & +2c^4(-183+N(-1439+2N(-423+10N(43+18N)))) \\
 & +54cN^2(284+N(-6409+4N(866+N(-1039+456N)))) \\
 & +3c^3(71+N(1797+8N(105+2N(-154+15N(1+13N)))) \\
 & +9c^2N(-491+N(4339+2N(2835+2N(-143+10N(-65+72N))))), \\
 c_{66} = & 36D(c-3N)^3N(126N^2(57+4N(-101+37N)) \\
 & +3cN(-392+N(8421-8948N+5492N^2))+c^3(-378+8N(-58+N(33+40N))) \\
 & +c^2(-1+2N)(-315+2N(655+N(509+510N)))), \\
 c_{67} = & -108D(c-3N)^2N(-378N^3(41+2N(32+3N(-45+14N))) \\
 & +10c^4(32+3N(7+N(-13+4(-1+N)N))) \\
 & -9cN^2(-2170+N(1841+8N(973+N(-1173+569N)))) \\
 & -c^3(-224+N(2981+N(1373+90N(-41+2N(5+2N)))) \\
 & -3c^2N(1673+N(-4445+N(-3427+12N(588+5N(-87+44N))))), \\
 c_{68} = & 54D(c-3N)^3N(c^3(-34+24N(2+3N))-126N^2(18+N(-123+62N)) \\
 & +c^2(-217+2N(553+8N(-1+18N)))+3cN(861-2N(2135+2N(-553+102N)))), \\
 c_{69} = & 108D(3+c-9N)(c-3N)^2N(-3c+2(3+c)N)(2c^2(3+2N)(4+3N) \\
 & +3N^2(399-734N+288N^2)+cN(-367+4N(40+51N))),
 \end{aligned}$$

$$\begin{aligned}
 c_{70} &= -36D(c-3N)^2N(378N^3(-129+2N(-4+N(39+22N))) \\
 &\quad +c^4(378+2N(227+N(-237+20N(-1+9N)))) \\
 &\quad -9cN^2(-1414+N(-7147+2N(2511+4N(-215+334N)))) \\
 &\quad +c^3(-315+N(-1367+N(-2961+2N(4075+6N(-83+70N)))) \\
 &\quad -3c^2N(-427+N(5990+N(-7617+4N(4670+N(-3403+960N))))), \\
 c_{71} &= 6D(c-3N)N(-3c+2(3+c)N)(-81N^3(2667+N(-8301+4N(1556+N(59+24N)))) \\
 &\quad +2c^4(520+N(149+2N(-467+2N(-89+54N)))) \\
 &\quad +3c^3(488+N(-6019+4N(695+N(2161-8N(41+54N)))) \\
 &\quad +27cN^2(5610+N(-22421+4N(6233+N(239+16N(-266+153N)))) \\
 &\quad -9c^2N(3317+N(-17773+2N(8661+4N(1447+N(-1901+306N))))), \\
 c_{72} &= -18D(c-3N)^2N(c-4cN+6N^2)(8c^3(6+N-2N^2) \\
 &\quad +189N^2(99+2N(-99+40N))+6c^2(70+N(-91+82N+8N^2)) \\
 &\quad +9cN(-693+4N(252+N(-91+48N)))), \\
 c_{73} &= -24D(3+c-9N)(c-3N)N(-3c+2(3+c)N)(2c(-1+N) \\
 &\quad +3(3-4N)N)(9N^2(357-574N+200N^2) \\
 &\quad +2c^2(25+8N(3+N))+3cN(-287+2N(65+48N))), \\
 c_{74} &= -36DN(1+N)(-c+3N)(c+6N)(-3c+2(3+c)N)((87-86c)c \\
 &\quad +9(-77+4c(16+c))N+18(43-38c)N^2)(c+2cN+3N(-3+2N)), \\
 c_{75} &= -36D(c-3N)^2N(-54N^3(-1323+2N(554+N(-1857+886N))) \\
 &\quad +6c^4(-62+N(-173+N(-115+76N+60N^2))) \\
 &\quad -9cN^2(2998+N(355+4N(6321+2N(-2461+689N)))) \\
 &\quad +c^3(264+N(4037+N(-2811+10N(-173+6N(7+46N)))) \\
 &\quad +3c^2N(-313+N(1421+N(22269+4N(-3424+N(671+60N))))), \\
 c_{76} &= -12D(-3+2c)N(1+N)(c+6N)(-3c+2(3+c)N) \\
 &\quad \times (c+2cN+3N(-3+2N))(9N^2(357-854N+480N^2) \\
 &\quad +2c^2(46+N(-53+22N))-3cN(385+4N(-176+67N))), \\
 c_{77} &= -6D(-3+2c)N(1+N)(c+6N)(-3c+2(3+c)N) \\
 &\quad \times (c+2cN+3N(-3+2N))^2(9(21-34N)N+c(-23+22N)), \\
 c_{78} &= 36D(c-3N)N(c-4cN+6N^2)(-2c^4(48+N(-1+2N)(-47+9N(-1+2N))) \\
 &\quad -162N^3(-683+2N(673+5N(-55+6N))) \\
 &\quad +27cN^2(-2752+N(4015+4N(221-692N+310N^2))) \\
 &\quad +3c^3(-280+N(305+N(225+2N(-321+2N(53+18N)))) \\
 &\quad +9c^2N(1681+N(-1777+N(-1083+4N(462+N(-305+156N))))), \\
 c_{79} &= -12D(3+c-9N)(c-3N)N(-3c+2(3+c)N)(2c(-1+N) \\
 &\quad +3(3-4N)N)(c^2(64+90N+44N^2) \\
 &\quad +9N^2(273-602N+256N^2)+3cN(-301+20N(10+9N))), \\
 c_{80} &= -108D(c-3N)^2N(8c^4(5+N(1+N)(1+2N)) \\
 &\quad +54N^3(-231+2N(437+N(-573+194N))) \\
 &\quad +3c^2N(-573+4N(920+N(-1493+4N(131+6N)))) \\
 &\quad +9cN^2(874+5N(-851+4N(368+3N(-71+16N)))) \\
 &\quad +c^3(97+N(-1065+4N(262+N(-11+36N))))), \\
 c_{81} &= 36D(c-3N)N(-324N^4(378+N(-1547+2N(1381+N(-1229+386N))))
 \end{aligned}$$

$$\begin{aligned}
 & -108cN^3(-679 + N(3004 + N(-6917 + N(7903 - 3582N + 520N^2)))) \\
 & + 2c^5(-3 + N(-39 + 2N(13 + N(-39 + 4N(-1 + 9N)))) \\
 & - 3c^3N(23 + N(2095 + N(-3797 + 4N(-1920 + N(3463 + 24N(-61 + 8N)))))) \\
 & + c^4(51 + N(403 + 2N(519 + 2N(-994 + 3N(147 + 4N(8 + 9N)))))) \\
 & + 9c^2N^2(-1153 + N(6847 + N(-18715 + 4N(3218 + N(1945 + 16N(-148 + 33N)))))), \\
 c_{82} = & -12D(-3 + 2c)N(1 + N)(c + 6N)(-3c + 2(3 + c)N) \\
 & \times (c + 2cN + 3N(-3 + 2N))^2(3(49 - 60N)N + c(-23 + 8N)), \\
 c_{83} = & -2D(3 + c - 9N)N(-1 + 2N)(-3c + 2(3 + c)N)(2c(-1 + N) \\
 & + 3(3 - 4N)N)(81N^3(119 + 2N(-89 + 40N)) \\
 & - 3c^2N(-935 + 4N(-23 + 18N(8 + N))) + 2c^3(-82 + N(-153 + 8N(-7 + 3N))) \\
 & - 18cN^2(627 + N(-563 + 16N(-14 + 15N))))), \\
 c_{84} = & -9D(c - 3N)^2N(c + 2cN + 3N(-3 + 2N))(c^3(-54 + 4N(17 + 8N)) \\
 & - 378N^2(12 + N(-53 + 22N)) - 9cN(-371 + 4N(441 - 280N + 54N^2)) \\
 & + 3c^2(-77 + 8N(70 + N(-41 + 17N)))), \\
 c_{85} = & 6D(c - 3N)N(1 + N)(-3c + 2(3 + c)N)(162N^3(735 + 2N(-549 - 214N + 228N^2)) \\
 & + 4c^4(-44 + N(-413 + 4N(31 + 27N))) \\
 & + 12c^3(-44 + N(567 + N(607 + 4N(-314 + 27N)))) \\
 & - 27cN^2(2421 + 4N(-1958 + N(3859 + 28N(-167 + 54N)))) \\
 & - 9c^2N(-1115 + 4N(1608 + N(-2007 + 4N(19 + 288N))))), \\
 c_{86} = & -6D(3 + c - 9N)(c - 3N)N(-3c + 2(3 + c)N)(c + 2cN + 3N(-3 + 2N)) \\
 & \times (c^2(88 + 78N + 44N^2) + 9N^2(525 - 950N + 352N^2) + 3cN(-475 + 4N(80 + 39N))), \\
 c_{87} = & 3D(c - 3N)N(1944N^4(-294 + N(-864 + N(3601 + 4N(-950 + 291N)))) \\
 & + 2c^5(-381 + N(371 + 2N(-1063 + 2N(-617 + 386N + 216N^2)))) \\
 & + 324cN^3(690 + N(6893 + N(-16301 + 4N(3438 + N(-743 + 12N)))) \\
 & + 3c^4(549 + N(2255 + 4N(-510 + N(1359 + 2N(93 - 652N + 756N^2)))) \\
 & - 54c^2N^2(-718 + N(13556 + N(-9361 + 2N(-5691 + 2N(7209 - 4042N + 864N^2)))) \\
 & + 9c^3N(-2213 + N(5427 + 4N(2025 + N(-2275 + 2N(999 + 4N(-68 + 9N))))))), \\
 c_{88} = & -9DN(-1 + 2N)(-3c + 2(3 + c)N)(2c^5(16 + N(-77 + 2N(-49 + 4N + 8N^2))) \\
 & + c^4(-180 + N(133 + 2N(1537 - 2N(55 + 6N)(-5 + 8N)))) \\
 & - 81N^4(-525 + N(-225 + 4N(1117 + N(-1499 + 504N)))) \\
 & + 27cN^3(900 + N(-10375 + 2N(8891 + 4N(-331 + 8N(-166 + 63N)))) \\
 & + 3c^3N(1099 - 2N(2005 + 2N(1244 + N(-1223 + 4N(-196 + 87N)))) \\
 & + 9c^2N^2(-2092 + N(12167 + 2N(-3459 + 4N(-1346 + N(883 + 186N))))), \\
 c_{89} = & 6D(-3 + 2c)(c - 3N)N(1 + N)(-1 + 6N)(c + 6N)(-3c + 2(3 + c)N) \\
 & \times (c + 2cN + 3N(-3 + 2N))(c(23 + 6N) + 3N(-35 + 18N)), \\
 c_{90} = & -2D(-3 + 2c)N(1 + N)(-1 + 2N)(c + 6N)(-3c + 2(3 + c)N) \\
 & \times (4c(-1 + N) + 3(9 - 14N)N)(c + 2cN + 3N(-3 + 2N)) \\
 & \times (c(23 + 6N) + 3N(-35 + 18N)), \\
 c_{91} = & -54D(c - 3N)^2N(1 + N)(c + 6N)(-18N^2(-49 + 2N)(-3 + 2N) \\
 & + 2c^3(101 + 4(-19 + N)N) + 3cN(377 + 4N(314 + 3N(-181 + 76N))) \\
 & - c^2(107 + 2N(963 + 2N(-587 + 186N))))), \\
 c_{92} = & -D(-3 + 2c)N(1 + N)(-1 + 2N)(c + 6N)(-3c + 2(3 + c)N)
 \end{aligned}$$

$$\begin{aligned}
 & \times (c + 2cN + 3N(-3 + 2N))^2 (c(23 + 6N) + 3N(-35 + 18N)), \\
 c_{93} = & 12D(c - 3N)N(1 + N)(c + 6N)(c + 2cN + 3N(-3 + 2N))(4c^3(39 + 8(-4 + N)N) \\
 & - 27N^2(357 - 854N + 480N^2) - 6c^2(60 + N(227 + 2N(-229 + 64N))) \\
 & + 9cN(427 + 4N(-64 + N(-227 + 156N))))), \\
 D \equiv & -\frac{c^4\sqrt{9N-3c}}{1134N(29+2c)(6+5c)N(-1+2N)(c+6(-1+N)N)^4\sqrt{N-2N^2}}. \tag{D.1}
 \end{aligned}$$

E The coefficient functions in (3.75) associated with the spin-6 current

As in previous examples, the coefficients in (3.75) for general N are

$$\begin{aligned}
 c_1 = & -2520c^4D(29+2c)(c-3N)^4(-1+N)N(-1+2N)(c^2(3-10N+2N^2) \\
 & + 18N^2(-1-4N+9N^2) - 6cN(2-15N+16N^2)), \\
 c_2 = & -360c^4D(c-3N)^3(-1+N)N(-1+2N)(2c^4(132-267N+28N^2+52N^3) \\
 & + 108N^3(-448+2671N-4642N^2+2592N^3) \\
 & + c^3(792-7189N+12742N^2-5644N^3+168N^4) \\
 & + 18cN^2(2398-14907N+27696N^2-20132N^3+5184N^4) \\
 & - 3c^2N(3487-23223N+41556N^2-27500N^3+7248N^4)), \\
 c_4 = & \frac{1}{77}c^4D(c-3N)^2(-561330N^5(136519124-190461085N+99067643N^2 \\
 & - 23286276N^3+2158900N^4-567cN^4(-250673172795+400721752191N \\
 & - 253411549210N^2+80285572310N^3-12942693020N^4+864045664N^5) \\
 & - 27c^2N^3(3876057446115-6879443743236N+4921753433246N^2 \\
 & - 1810191324180N^3+357244952000N^4-34749846384N^5+1137076064N^6) \\
 & - 9c^3N^2(-4253484112800+8207608700457N-6410229831502N^2 \\
 & + 2596155612624N^3-577800178760N^4+68021343488N^5-3542911968N^6 \\
 & + 39700096N^7)+4c^5(126017881920-276009625179N+243262702680N^2 \\
 & - 111677980521N^3+28803431305N^4-4158821396N^5+307777160N^6-8476504N^7 \\
 & + 960N^8)-6c^4N(1160425676115-2398645828212N+2003282395938N^2 \\
 & - 870443206969N^3+210688708860N^4-27974557408N^5+1818472272N^6 \\
 & - 38133616N^7+1920N^8)), \\
 c_5 = & -90Dc^4(c-3N)^4(-1+N)N(-1+2N)(2c^3(-39-94N+20N^2) \\
 & + 252N^2(-25-37N+75N^2)+c^2(525-1442N+520N^2-1128N^3) \\
 & - 6cN(259-2499N+1442N^2+468N^3)), \\
 c_6 = & 90Dc^4(c-3N)^3(-1+N)N(-1+2N)(c^3(2142-9850N+10092N^2-4832N^3) \\
 & + 12c^4(25-48N+8N^3)+378N^3(132+343N-1610N^2+1224N^3) \\
 & - 3c^2N(5635-31432N+39184N^2-20184N^3+6912N^4) \\
 & + 9cN^2(2401-31892N+62864N^2-39400N^3+7200N^4)), \\
 c_7 = & -\frac{8}{231}c^4D(c-3N)^3(1122660N^4(3731003-5401448N+2978485N^2-768196N^3 \\
 & + 82236N^4)+378cN^3(-16624591875+27777490776N-18670540180N^2+6440887055N^3 \\
 & - 1171822700N^4+93062044N^5)+9c^2N^2(387960125940-717486059313N \\
 & + 536660981687N^2-207226798130N^3+43229166720N^4-4499331832N^5+161167408N^6) \\
 & + 3c^3N(-284527613520+567704947041N-453994190967N^2+185882259358N^3
 \end{aligned}$$

$$\begin{aligned}
 & -41032516240N^4 + 4619758344N^5 - 207952688N^6 + 10112N^7) + 2c^4(38801306880 \\
 & -82132149096N + 68767904655N^2 - 29188848039N^3 + 6642884380N^4 - 772039724N^5 \\
 & + 36579920N^6 - 87616N^7 + 3840N^8)), \\
 c_9 = & -7560c^4D(c-3N)^3(1-2N)^2(-1+N)N^2(1+N)(c+6N)(2c^2(-9+2N) \\
 & + 9N(-13+10N) - 3c(-5-40N+36N^2)), \\
 c_{11} = & 5040c^4D(c-3N)^4(-1+N)N(-1+2N)(-36N^2(19-67N+88N^2) \\
 & + 3cN(-127+126N+1245N^2-1066N^3) + c^3(48-58N^2+20N^3) \\
 & + c^2(144-490N-437N^2+728N^3-300N^4)), \\
 c_{12} = & 2520c^4D(3+c-9N)(c-3N)^4(-1+N)N(-1+2N)(6N+c(-3+2N)) \\
 & \times (2c(4+N) + 3N(-13+16N)), \\
 c_{13} = & \frac{8}{231}c^4D(c-3N)^3(-1122660N^4(3732858-5413293N+3004199N^2 \\
 & -791064N^3+89380N^4) - 756cN^3(-8314464240+13904327007N-9373319260N^2 \\
 & + 3261733685N^3 - 606720860N^4 + 50693908N^5) - 9c^2N^2(387994080780 \\
 & -717804244827N + 537525927493N^2 - 208223832850N^3 + 43752437220N^4 \\
 & -4610679848N^5 + 165318752N^6) + 3c^3N(284520798720 - 567743058189N \\
 & + 454189530093N^2 - 186163376912N^3 + 41204518640N^4 - 4667744496N^5 \\
 & + 214167472N^6 + 10112N^7) + 2c^4(-38796952320 + 82114755264N - 68739047235N^2 \\
 & + 29163318141N^3 - 6631094030N^4 + 769829836N^5 - 36185080N^6 - 87616N^7 + 3840N^8)), \\
 c_{15} = & -180c^4D(c-3N)^3(-1+N)N(-1+2N)(8c^4(81-27N-56N^2+22N^3) \\
 & + 1134N^3(42-45N-88N^2+144N^3) + 36c^2N(-462+1824N-157N^2-666N^3+64N^4) \\
 & -12c^3(-231+1017N-3N^2-514N^3+168N^4) \\
 & -27cN^2(-413+3654N-2664N^2+928N^3+192N^4)), \\
 c_{16} = & 180c^4D(c-3N)^3(-1+N)N(-1+2N)(378N^3(-231+1022N-1616N^2+872N^3) \\
 & + 4c^4(-252+271N+387N^2-376N^3+60N^4) + 9cN^2(-448-6321N+51892N^2 \\
 & -86084N^3+38464N^4) - 2c^3(1512-8747N+4358N^2+15634N^3-15408N^4 \\
 & + 3480N^5) + 3c^2N(6412-23075N-14300N^2+81828N^3-60832N^4+16320N^5)), \\
 c_{17} = & -180c^4D(c-3N)^4(-1+N)N(-1+2N)(-252N^2(19-96N+146N^2) \\
 & + 2c^3(168-5N-208N^2+100N^3) - 6cN(343+371N-6022N^2+4624N^3) \\
 & + c^2(1008-3533N-3540N^2+6620N^3-2400N^4)), \\
 c_{18} = & 5040c^4D(3+c-9N)(c-3N)^3(-1+N)N(-1+2N)(c(-1+N) \\
 & + 3(2-3N)N)(6N+c(-3+2N))(2c(4+N) + 3N(-13+16N)), \\
 c_{19} = & 40c^4D(3+c-9N)(c-3N)^2(-1+N)N(-1+2N)(810N^4(252-479N \\
 & + 46N^2+168N^3) + 27cN^3(-12902+30095N-9982N^2-17504N^3+10464N^4) \\
 & + c^4(2424-4574N+206N^2+3704N^3-2000N^4+240N^5) \\
 & -3c^3N(12225-28081N+9210N^2+19744N^3-15888N^4+2880N^5) \\
 & + 9c^2N^2(20282-49937N+22270N^2+28480N^3-27664N^4+7680N^5)), \\
 c_{20} = & 720c^4D(c-3N)^3(-1+N)N(-1+2N)(378N^3(65-269N+170N^2+156N^3) \\
 & + 2c^4(-168+208N+93N^2-208N^3+60N^4) + 9cN^2(-3626+17003N-14129N^2 \\
 & -11028N^3+8628N^4) - 3c^3(336-2654N+2833N^2+1652N^3-2804N^4+880N^5) \\
 & + 3c^2N(3759-20438N+19432N^2+13778N^3-18016N^4+3960N^5)), \\
 c_{21} = & -\frac{8}{231}c^4D(c-3N)^3(561330N^4(3731003-5401245N
 \end{aligned}$$

$$\begin{aligned}
 & +2977470N^2 - 766572N^3 + 81424N^4) + 189cN^3(-16624491390 + 27776385441N \\
 & - 18666608395N^2 + 6435095060N^3 - 1168504220N^4 + 92608624N^5) \\
 & + 9c^2N^2(193979359575 - 358736302359N + 268306431616N^2 - 103572896680N^3 \\
 & + 21582178680N^4 - 2239312496N^5 + 80167904N^6) + 6c^3N(-71131903380 \\
 & + 141926038389N - 113496736413N^2 + 46464724582N^3 - 10250000170N^4 \\
 & + 1150414296N^5 - 51364472N^6 + 2528N^7) + 8c^4(4850163360 - 10266518637N \\
 & + 8595981585N^2 - 3648547533N^3 + 830178635N^4 - 96271078N^5 + 4468540N^6 \\
 & - 10952N^7 + 480N^8)), \\
 c_{22} = & 2160c^4D(c-3N)^3(1-2N)^2(-1+N)N^2(10c^4(2-5N+2N^2) \\
 & + 252N^2(63-184N+101N^2) + c^3(818-1103N+692N^2-300N^3) + 6cN(-1694 \\
 & + 6853N-6533N^2+2458N^3) + c^2(1722-11167N+12617N^2-4770N^3-120N^4)), \\
 c_{23} = & \frac{4}{231}c^4D(c-3N)^2(1683990N^5(102382230-141015895N \\
 & + 71750931N^2-16195600N^3+1385308N^4) + 567cN^4(-557957088765+870822849609N \\
 & - 531891742640N^2+159776917720N^3-23551346320N^4+1325324816N^5) \\
 & + 27c^2N^3(8547376777470-14717811122874N+10128970212019N^2-3531905290290N^3 \\
 & + 641895014440N^4-53978593416N^5+1355007376N^6) + 9c^3N^2(-9303024429315 \\
 & + 17351338484853N-13020185959823N^2+5008576828836N^3-1034298518680N^4 \\
 & + 107748672472N^5-4675753872N^6+96793184N^7) + 4c^5(-271832863680 \\
 & + 573501628566N-485728888725N^2+212717494344N^3-51583421780N^4 \\
 & + 6834595654N^5-454097800N^6+13200536N^7+960N^8) - 6c^4N(-2519584749900 \\
 & + 5023025276328N-4030065284037N^2+1666868473781N^3-377003205180N^4 \\
 & + 45201848972N^5-2568568848N^6+70805984N^7+1920N^8)), \\
 c_{24} = & -360c^4D(3+c-9N)(c-3N)^2(-1+N)N(-1+2N)(54N^4(980-2123N \\
 & + 614N^2+672N^3) + 27cN^3(-3867+10156N-4550N^2-6172N^3+4064N^4) \\
 & + 2c^4(444-995N+197N^2+810N^3-536N^4+80N^5) - c^3N(12207-31319N+13282N^2 \\
 & + 21932N^3-19792N^4+4080N^5) + 3c^2N^2(19235-52456N+27978N^2+30668N^3 \\
 & - 31696N^4+8640N^5)), \\
 c_{25} = & 720c^4D(3+c-9N)(c-3N)^3(-1+N)N^2(-1+2N)(2c^3(51-116N+49N^2+6N^3) \\
 & - 18N^2(-49+700N-1432N^2+864N^3) + 3c^2(77-806N+1473N^2-768N^3+124N^4) \\
 & - 3cN(301-3581N+7529N^2-5782N^3+1728N^4)), \\
 c_{26} = & 120c^4D(3+c-9N)(c-3N)^2(-1+N)N(-1+2N)(2c(-1+N) \\
 & + 3(3-4N)N)^2(6N+c(-3+2N))(c(40+22N)+3N(-7+80N)), \\
 c_{27} = & 60c^4D(3+c-9N)(c-3N)^2(-1+N)N(-1+2N)(2c(-1+N) \\
 & + 3(3-4N)N)(6N+c(-3+2N))(2c^2(172-69N+2N^2) \\
 & - 3cN(1391-1816N+276N^2)+9N^2(1323-2782N+1376N^2)), \\
 c_{28} = & -\frac{1}{693}c^4D(c-3N)^3(-1+N)N(1+N)(56133N^4(-2980308+3590060N \\
 & - 946135N^2-182630N^3+8653N^4) - 27cN^3(-9365853492+14104771440N \\
 & - 7431900457N^2+1340713637N^3+12733180N^4+432772N^5) - 9c^2N^2(15393018348 \\
 & - 26130193080N+17307715327N^2-5674703221N^3+1000138398N^4-113096996N^5 \\
 & + 5081264N^6) + 6c^3N(5376847824-9259611516N+5668320348N^2-1284880711N^3 \\
 & - 73815653N^4+91749600N^5-16045372N^6+910720N^7) + 8c^4(-329760720
 \end{aligned}$$

$$\begin{aligned}
 & +474667560N - 6601644N^2 - 376267854N^3 + 309667435N^4 - 118352105N^5 \\
 & +24304394N^6 - 2586556N^7 + 111930N^8)), \\
 c_{29} = & -\frac{1}{16}c^4D(c-3N)^2(-9720N^5(-293424 + 113244N + 8504N^2 \\
 & -82673N^3 + 42827N^4) + 162cN^4(-33943080 + 21794844N - 3311584N^2 - 989489N^3 \\
 & -4812139N^4 + 2068962N^5) - 27c^2N^3(-149031120 + 109666764N - 5854968N^2 \\
 & -7511485N^3 - 4319005N^4 - 2045984N^5 + 58588N^6) - 9c^3N^2(144114960 - 28575924N \\
 & -216116504N^2 + 209095967N^3 - 88408193N^4 + 17110896N^5 - 737604N^6 + 106080N^7) \\
 & +12c^4N(13847400 + 27499116N - 99100876N^2 + 92049550N^3 - 39372751N^4 \\
 & +8542056N^5 - 1135527N^6 + 43212N^7 + 468N^8) + 4c^5(-1321920 - 13946256N \\
 & +31802712N^2 - 10045048N^3 - 23534512N^4 + 25902661N^5 - 11624171N^6 + 2679564N^7 \\
 & -312132N^8 + 14496N^9)), \\
 c_{30} = & \frac{1}{16}c^4D(c-3N)^2(-9720N^5(-293424 + 157092N - 108424N^2 \\
 & -24209N^3 + 42827N^4) + 162cN^4(-34381560 + 27056604N - 14601184N^2 + 4171471N^3 \\
 & -5054059N^4 + 2068962N^5) - 27c^2N^3(-151369680 + 129141324N - 43110648N^2 \\
 & +8777795N^3 - 6254365N^4 - 2045984N^5 + 58588N^6) - 9c^3N^2(146161200 - 43091124N \\
 & -189908504N^2 + 197161247N^3 - 85988993N^4 + 17110896N^5 - 737604N^6 + 106080N^7) \\
 & +12c^4N(13993560 + 26516316N - 97205836N^2 + 90900430N^3 - 39130831N^4 + 8542056N^5 \\
 & -1135527N^6 + 43212N^7 + 468N^8) + 4c^5(-1321920 - 13916016N + 31621272N^2 \\
 & -9803128N^3 - 23534512N^4 + 25902661N^5 - 11624171N^6 + 2679564N^7 \\
 & -312132N^8 + 14496N^9)), \\
 c_{31} = & \frac{1}{8}c^4D(c-3N)^2(-9720N^5(-293424 + 149784N - 93808N^2 \\
 & -24209N^3 + 42827N^4) + 162cN^4(-34308480 + 26325804N - 13693984N^2 + 4695631N^3 \\
 & -5054059N^4 + 2068962N^5) - 27c^2N^3(-151077360 + 127357164N - 42808248N^2 \\
 & +13092035N^3 - 6496285N^4 - 2045984N^5 + 58588N^6) - 9c^3N^2(146015040 - 42738324N \\
 & -187348184N^2 + 191314847N^3 - 85021313N^4 + 17110896N^5 - 737604N^6 + 106080N^7) \\
 & +12c^4N(13993560 + 26450796N - 96782476N^2 + 90164590N^3 - 38828431N^4 \\
 & +8542056N^5 - 1135527N^6 + 43212N^7 + 468N^8) + 4c^5(-1321920 - 13916016N \\
 & +31606152N^2 - 9712408N^3 - 23655472N^4 + 25902661N^5 - 11624171N^6 + 2679564N^7 \\
 & -312132N^8 + 14496N^9)), \\
 c_{32} = & 3780c^4D(29+2c)(6+5c)(c-3N)^4N^3(-1+2N)^3, \\
 c_{34} = & 3780c^4D(29+2c)(c-3N)^5(-1+N)N(-1+2N)(-6N^2+c(-1+4N)), \\
 c_{35} = & -2520c^4D(c-3N)^4(-1+N)N(-1+2N)(24c^3(-1+N^2)+c^2(-72+254N+56N^2) \\
 & -18N^2(10-107N+144N^2)-3cN(-107+510N-508N^2+288N^3)), \\
 c_{36} = & -2c^4D(c-3N)^3(-1+N)(486N^4(170320-9676N-1976N^2-25851N^3 \\
 & +249N^4)+27cN^3(-4721220+2364672N-1195744N^2-412299N^3+108591N^4 \\
 & +1214N^5)-54c^2N^2(-838490-1069044N+2231184N^2-1423452N^3+384633N^4 \\
 & -50073N^5+2534N^6)-12c^3N(-331200+7129221N-13046236N^2+9734314N^3 \\
 & -3873994N^4+815765N^5-90053N^6+4022N^7)+8c^4(-304560+2236392N \\
 & -3888153N^2+2509841N^3-501941N^4-117054N^5+71651N^6-11723N^7+654N^8)), \\
 c_{38} = & 2520c^4D(c-3N)^4(-1+N)N(-1+2N)(6c^3(-3-4N+4N^2) \\
 & +18N^2(-10-67N+30N^2)+c^2(15-130N+272N^2-144N^3))
 \end{aligned}$$

$$\begin{aligned}
 & -3cN(67 - 534N + 260N^2 + 216N^3)), \\
 c_{41} = & -11340c^4D(29 + 2c)(c - 3N)^4(1 - 2N)^2(-1 + N)N^2(-6N^2 + c(-1 + 4N)), \\
 c_{43} = & -2520c^4D(c - 3N)^4(-1 + N)N(-1 + 2N)(18N^2(-11 - 268N + 4N^2) \\
 & + 4c^3(-18 - 19N + 9N^2 + 10N^3) + 3cN(22 + 901N - 286N^2 + 248N^3) \\
 & + 10c^2(6 - 17N - 4N^2 + 16N^3 + 24N^4)), \\
 c_{44} = & 45360c^4D(29 + 2c)(c - 3N)^4(1 - 2N)^2(-1 + N)N^2(-6N^2 + c(-1 + 4N)), \\
 c_{45} = & -5040c^4D(c - 3N)^3(-1 + N)N(1 + N)(-1 + 2N)(c^4(66 - 80N + 24N^2) \\
 & - 54N^3(98 - 237N + 90N^2) + c^3(129 - 1040N + 1184N^2 - 360N^3) + 6c^2N(-145 + 616N \\
 & - 442N^2 + 108N^3) + 27cN^2(95 - 144N - 248N^2 + 216N^3)), \\
 c_{46} = & -2160c^4D(c - 3N)^2(1 - 2N)^2(-1 + N)N^2(c^4(222 + 34N - 212N^2 + 96N^3) \\
 & + 108N^3(-161 + 220N + 78N^2 + 306N^3) + c^3(321 - 2729N - 884N^2 + 3168N^3 - 792N^4) \\
 & - 3c^2N(1322 - 4373N - 1876N^2 + 2856N^3 + 72N^4) - 18cN^2(-920 + 2117N - 89N^2 \\
 & + 474N^3 + 216N^4)), \\
 c_{47} = & -1080c^4D(c - 3N)^3(1 - 2N)^2(-1 + N)N^2(2c^3(12 + 65N + 38N^2) \\
 & - 126N^2(-101 + 28N + 216N^2) + 3cN(-196 - 2121N + 1834N^2 + 288N^3) \\
 & + c^2(-756 + 917N + 1352N^2 + 780N^3)), \\
 c_{48} = & 2520c^4D(3 + c - 9N)(c - 3N)^4(-1 + N)N(-1 + 2N)(6N + c(-3 + 2N)) \\
 & \times (2c(4 + N) + 3N(-13 + 16N)), \\
 c_{49} = & -120c^4D(c - 3N)^2(-1 + N)N(-1 + 2N)(4c^5(-390 + 641N + 271N^2) \\
 & - 596N^3 + 164N^4) + 486N^4(-1834 + 5011N - 1574N^2 - 4384N^3 + 2208N^4) \\
 & + 27c^2N^2(-4189 + 12254N - 9038N^2 - 6416N^3 + 568N^4 + 1024N^5) - 6c^4(366 - 5057N \\
 & + 7909N^2 + 3524N^3 - 7220N^4 + 2048N^5) + 18c^3N(1111 - 8894N + 14104N^2 \\
 & + 4534N^3 - 11752N^4 + 3512N^5) - 81cN^3(-6103 + 11194N + 6992N^2 - 14192N^3 \\
 & - 10688N^4 + 8832N^5)), \\
 c_{52} = & -\frac{1}{(21 + 4c)N(-1 + 2N)}60c^4D(c - 3N)^3(3402N^5(44351447 \\
 & - 61956177N + 32308358N^2 - 7638900N^3 + 718952N^4) + 81cN^4(-3301821036 \\
 & + 5013869837N - 2980498275N^2 + 882161030N^3 - 133961484N^4 + 8856008N^5) \\
 & + 54c^2N^3(3490542895 - 5645277003N + 3613609675N^2 - 1159782701N^3 + 190838642N^4 \\
 & - 13647676N^5 + 96008N^6) - 9c^3N^2(7337907348 - 12435294511N + 8243780357N^2 \\
 & - 2624168622N^3 + 358445964N^4 + 6294712N^5 - 7469504N^6 + 745536N^7) \\
 & + 8c^5(-99688320 + 178269984N - 115164108N^2 + 25437843N^3 + 5387531N^4 \\
 & - 4228834N^5 + 954208N^6 - 100208N^7 + 4704N^8) - 6c^4N(-1917613440 \\
 & + 3358682364N - 2232994015N^2 + 641613101N^3 - 28782294N^4 - 29210940N^5 \\
 & + 7594600N^6 - 787424N^7 + 37248N^8)), \\
 c_{53} = & 60c^4D(c - 3N)^2(-1 + N)N(-1 + 2N)(4c^5(-543 + 734N + 592N^2) \\
 & - 824N^3 + 176N^4) + 486N^4(-2016 + 2921N + 4408N^2 - 6604N^3 + 624N^4) \\
 & + 54c^2N^2(-599 - 8250N + 23696N^2 - 17524N^3 - 512N^4 + 2000N^5) \\
 & - 6c^4(465 - 6287N + 8108N^2 + 8072N^3 - 11040N^4 + 2960N^5) \\
 & + 9c^3N(1667 - 12696N + 13892N^2 + 31312N^3 - 41696N^4 + 12160N^5) \\
 & - 81cN^3(-4013 - 16312N + 64500N^2 - 34560N^3 - 30368N^4 + 18624N^5)), \\
 c_{54} = & -2520c^4D(c - 3N)^4(-1 + N)N(1 + N)(-1 + 2N)(c + 6N) \\
 & \times (2c^2(-9 + 2N) + 9N(-13 + 10N) - 3c(-5 - 40N + 36N^2)),
 \end{aligned}$$

$$\begin{aligned}
c_{56} &= -\frac{1}{2}c^4 D(29+2c)(c-3N)^4(-1+N)N^2(1+N) \\
&\quad \times (-1+2N)(189N^2(1356-620N+157N^2-20N^3+N^4) \\
&\quad +3cN(-156384+288772N-152842N^2+40377N^3-5198N^4+260N^5) \\
&\quad +10c^2(16200-43956N+40432N^2-18553N^3+4473N^4-542N^5+26N^6)), \\
c_{57} &= -2160c^4 D(c-3N)^2(1-2N)^2(-1+N)N^2(8c^4(-3+8N-2N^2 \\
&\quad +2N^3)-162N^3(315-400N-390N^2+528N^3)-9c^2N(-580+2571N-254N^2 \\
&\quad -1212N^3+24N^4)-6c^3(150-388N-135N^2-32N^3+44N^4) \\
&\quad +27cN^2(218+1679N-2160N^2-732N^3+1152N^4)), \\
c_{58} &= -180c^4 D(c-3N)^3(-1+N)N(-1+2N)(-54N^3(399-1618N \\
&\quad -1384N^2+24N^3)+2c^4(93-349N-150N^2+232N^3)+c^3(1317-7311N \\
&\quad +4286N^2+6392N^3-3648N^4)-3c^2N(2619-12662N-3658N^2+6604N^3+1488N^4) \\
&\quad +9cN^2(1884-8065N-9914N^2+296N^3+5472N^4)), \\
c_{60} &= -180c^4 D(c-3N)^3(-1+N)N(-1+2N)(-756N^3(24-281N \\
&\quad +162N^2+32N^3)+6c^4(-42-175N+54N^2+12N^3+40N^4)-36cN^2(-504+5495N \\
&\quad -5115N^2+2256N^3+212N^4)+c^3(210-881N+10572N^2-14824N^3+6096N^4 \\
&\quad +2160N^5)+3c^2N(-1652+15459N-20034N^2+18700N^3-7800N^4+3360N^5)), \\
c_{61} &= -15c^4 D(c-3N)^2(-1+N)N(-1+2N)(1944N^4(-1071+390N \\
&\quad +530N^2-1858N^3+900N^4)+4c^5(-1707-551N+434N^2+1154N^3-704N^4 \\
&\quad +120N^5)-27c^2N^2(2550+66013N+7580N^2-84772N^3+47808N^4+4256N^5) \\
&\quad -162cN^3(-5295-27170N+27256N^2+2840N^3-23968N^4+8544N^5) \\
&\quad +6c^4(933+7217N-6247N^2+8966N^3-14160N^4+5608N^5+720N^6) \\
&\quad +9c^3N(-2784+11609N+45634N^2-63688N^3+35192N^4-18832N^5+6720N^6)), \\
c_{62} &= -540c^4 D(c-3N)^2(1-2N)^2(-1+N)N^2(6c^4(46-47N-66N^2+32N^3) \\
&\quad +54N^3(1295-1318N-948N^2+1056N^3)+c^3(2070-6461N-2546N^2+4872N^3 \\
&\quad -2304N^4)+6c^2N(-1672+6235N+4007N^2-5898N^3+288N^4) \\
&\quad -9cN^2(884+6361N+134N^2-7728N^3+2304N^4)), \\
c_{63} &= -6480c^4 D(c-3N)^2(1-2N)^2(-1+N)N^2(4c^4(1-3N-6N^2+8N^3) \\
&\quad +54N^3(63+9N-256N^2+204N^3)-3c^2N(254-1145N+70N^2+1148N^3+24N^4) \\
&\quad -2c^3(-29+99N+75N^2-220N^3+132N^4) \\
&\quad -9cN^2(-135+1325N-1296N^2-356N^3+144N^4)), \\
c_{64} &= 180c^4 D(c-3N)^2(-1+N)N(-1+2N)(4c^5(-189-182N+117N^2 \\
&\quad -30N^3+40N^4)-162N^4(1001+2116N-2920N^2+240N^3+2448N^4)+c^4(630 \\
&\quad +3904N+5988N^2-12024N^3+8944N^4-3552N^5)+18c^2N^2(-69-16735N+17100N^2 \\
&\quad -22224N^3+14192N^4+144N^5)+81cN^3(1306+6393N-5554N^2+3880N^3-1424N^4 \\
&\quad +576N^5)+3c^3N(-2018+14019N-19838N^2+37476N^3-29624N^4+5088N^5)), \\
c_{65} &= -2520c^4 D(c-3N)^3(-1+N)N(-1+2N)(54N^3(166-245N+372N^2) \\
&\quad +c^3(60+288N+172N^2-56N^3)+8c^4(-9-13N+2N^2+6N^3)+9cN^2(-557+746N \\
&\quad -2216N^2+720N^3)-6c^2N(-75+92N-554N^2-28N^3+288N^4)), \\
c_{66} &= 6480c^4 D(c-3N)^3(1-2N)^2(-1+N)N^2(126N^2(9-18N+2N^2) \\
&\quad +10c^3(1-N+2N^2)+c^2(7-287N+484N^2-60N^3) \\
&\quad +3cN(-126+511N-574N^2+120N^3)), \\
c_{67} &= 6480c^4 D(c-3N)^3(1-2N)^2(-1+N)N^2(126N^2(9-18N+2N^2)+10c^3(1-N+2N^2)
\end{aligned}$$

$$\begin{aligned}
 & +c^2(7-287N+484N^2-60N^3)+3cN(-126+511N-574N^2+120N^3)), \\
 c_{68} = & 540c^4D(c-3N)^2(1-2N)^2(-1+N)N^2(-378N^3(245-338N-256N^2+240N^3) \\
 & +c^4(540-342N-500N^2+352N^3)+c^3(378-5763N+4770N^2+6456N^3-3264N^4) \\
 & -6c^2N(-168+1663N-229N^2-3434N^3+912N^4) \\
 & +9cN^2(1708+7777N-13834N^2-12496N^3+12096N^4)), \\
 c_{69} = & -15120c^4D(c-3N)^3(1-2N)^2(-1+N)N^2(c+6N)(c^2(-24-34N+20N^2) \\
 & -3N(73-59N+42N^2)+c(66+11N+236N^2-180N^3)), \\
 c_{70} = & -360c^4D(c-3N)^3(-1+N)N(-1+2N)(-378N^3(9+566N-644N^2+104N^3) \\
 & +2c^4(378+147N-540N^2+76N^3+160N^4)-9cN^2(-560-16625N+20040N^2 \\
 & -14036N^3+9232N^4)+c^3(-630-225N-3670N^2+5584N^3-1336N^4+1200N^5) \\
 & -3c^2N(-168+9971N-9996N^2+8340N^3-7072N^4+3360N^5)), \\
 c_{71} = & -\frac{24c^4D(c-3N)}{77(21+4c)N(-1+2N)}(23575860N^7(1126813235-1527071265N \\
 & +759680090N^2-166434912N^3+13757504N^4)-561330cN^6(102620164907 \\
 & -123595173143N+48476464330N^2-5061841190N^3-885867856N^4+166616376N^5) \\
 & -93555c^2N^5(-549219821813+479770622798N+64405558815N^2-201395927508N^3 \\
 & +84931075160N^4-15015570864N^5+1013139248N^6)-81c^3N^4(295533212163005 \\
 & -60387392248366N-430899208834271N^2+429913090406336N^3-181198035913120N^4 \\
 & +39497729492016N^5-4377838656464N^6+194983987904N^7) \\
 & -27c^4N^3(-222749015462150-279226708420137N+1019026358801235N^2 \\
 & -935734392571366N^3+424069024043788N^4-107491950631488N^5+15337847831760N^6 \\
 & -1124081229664N^7+30718352192N^8)-9c^5N^2(79367760102600+434729432049855N \\
 & -1107648815321053N^2+1024087154548724N^3-498970966102000N^4 \\
 & +141671684535808N^5-23817722027152N^6+2262818278016N^7-105784835200N^8 \\
 & +1738017792N^9)-6c^6N(-2224499448960-129005374954188N+295599570320055N^2 \\
 & -281055430434627N^3+146084030570018N^4-45363379310320N^5 \\
 & +8592780382176N^6-966953944848N^7+59550909472N^8-1697215232N^9 \\
 & +20373504N^{10})+8c^7(390475572480-7111268919936N+15701367135828N^2 \\
 & -15435645414755N^3+8491816854263N^4-2837477110790N^5+589509643852N^6 \\
 & -74913386952N^7+5475060192N^8-199216064N^9+2542080N^{10}+33792N^{11})), \\
 c_{72} = & \frac{24c^4D(c-3N)}{77(21+4c)N(-1+2N)}(70727580N^7(770560200-1045761679N+521386199N^2 \\
 & -114631208N^3+9532252N^4)-561330cN^6(212337984624-260083236667N \\
 & +106175428227N^2-13376117508N^3-1205357188N^4+294464496N^5) \\
 & -93555c^2N^5(-1154223354007+1090794720138N-3454441429N^2 \\
 & -330987561762N^3+148023472440N^4-26524787464N^5+1793727760N^6) \\
 & -81c^3N^4(639054650848785-260893941509182N-682629183140027N^2 \\
 & +735544886230612N^3-312833104771100N^4+67654896396832N^5-7369356044288N^6 \\
 & +319572284288N^7)-27c^4N^3(-510138272224550-335609082910749N \\
 & +1703514781620605N^2-1600957803836012N^3+721306278372156N^4 \\
 & -179798187943296N^5+25068646615920N^6-1789749821568N^7+48006105664N^8) \\
 & -9c^5N^2(210659897248200+666390601225695N-1873780811938661N^2 \\
 & +1740755708001688N^3-836731301667660N^4+232477255533376N^5
 \end{aligned}$$

$$\begin{aligned}
 & -38059018444784N^6 + 3509192992512N^7 - 158519085120N^8 + 2439085824N^9) \\
 & -6c^6N(-15902848566720 - 208656503319036N + 500842481949735N^2 \\
 & -473910220538759N^3 + 241830961073706N^4 - 73243434739540N^5 + 13471444419952N^6 \\
 & -1464747924576N^7 + 86215439264N^8 - 2254189824N^9 + 20640768N^{10}) \\
 & +8c^7(246864360960 - 11739195624672N + 26544109049436N^2 - 25812369240635N^3 \\
 & +13894173768421N^4 - 4516732135790N^5 + 908737641454N^6 - 111068377304N^7 \\
 & +7680580264N^8 - 249939168N^9 + 1831680N^{10} + 67584N^{11})), \\
 c_{73} = & \frac{24c^4D(c-3N)(-1+N)}{77(21+4c)N(-1+2N)}(-11787930N^7(222611143 - 310059419N \\
 & +160857158N^2 - 37656004N^3 + 3468600N^4) + 280665cN^6(20095612135 - 24578607139N \\
 & +9830633640N^2 - 1022607960N^3 - 217061616N^4 + 43691216N^5) \\
 & +93555c^2N^5(-53841298366 + 49362458293N + 1661028992N^2 - 16270753720N^3 \\
 & +7132300096N^4 - 1291668592N^5 + 91656576N^6) + 81c^3N^4(29545065638225 \\
 & -11254867766623N - 30544289028438N^2 + 31426817196548N^3 - 12793486806880N^4 \\
 & +2637352195888N^5 - 271080981472N^6 + 10767842752N^7) + 27c^4N^3(-23631732425640 \\
 & -14264786376501N + 70436561046360N^2 - 62300918312088N^3 + 26041125407744N^4 \\
 & -5905987618224N^5 + 725769138880N^6 - 42995357312N^7 + 822612736N^8) \\
 & +18c^5N^2(5029522790400 + 13561743637500N - 36500948626907N^2 \\
 & +31312837087266N^3 - 13588335894280N^4 + 3309594596112N^5 - 453650045488N^6 \\
 & +32148525344N^7 - 887392000N^8 + 186368N^9) - 24c^6N(222551435520 \\
 & +2047213653696N - 4629172979700N^2 + 3973818277659N^3 - 1787643788006N^4 \\
 & +459732168360N^5 - 67629598272N^6 + 5209647536N^7 - 145014304N^8 - 2913536N^9 \\
 & +133632N^{10}) + 64c^7(-136080 + 55639689156N - 116897577588N^2 + 101490793455N^3 \\
 & -47170882708N^4 + 12688396525N^5 - 1989595672N^6 + 178776112N^7 - 11918872N^8 \\
 & +1477984N^9 - 177600N^{10} + 8448N^{11})), \\
 c_{74} = & 7560c^4D(c-3N)^3(1-2N)^2(-1+N)N^2(1+N)(c+6N)(2c^2(-9+2N) \\
 & +9N(-13+10N) - 3c(-5-40N+36N^2)), \\
 c_{77} = & 60c^4D(-3+2c)(c-3N)^2(-1+N)N(1+N)(-1+2N)(c+6N)(c+2cN \\
 & +3N(-3+2N))(2c^2(138+67N+2N^2) + 3cN(-1255+1000N+268N^2) \\
 & +9N^2(1323-2510N+1104N^2)), \\
 c_{78} = & -20c^4D(-3+2c)(c-3N)(-1+N)N(1+N)(-1+2N)(c+6N)(6N \\
 & +c(-3+2N))(c+2cN+3N(-3+2N))(c^2(253+110N+48N^2) \\
 & +3cN(-1123+912N+108N^2) + 18N^2(567-1039N+450N^2)), \\
 c_{79} = & 360c^4D(3+c-9N)(c-3N)(1-2N)^2(-1+N)N^2(1+N)(108N^4(231-664N \\
 & +408N^2) + c^3N(2651-4314N+2220N^2-648N^3) + c^4(-288+518N-352N^2+72N^3) \\
 & -36cN^3(10-119N-186N^2+144N^3) - 3c^2N^2(2455-3644N+948N^2+720N^3)), \\
 c_{80} = & -60c^4D(-3+2c)(c-3N)(-1+N)N(1+N)(-1+2N)(c+6N)(6N+c(-3+2N)) \\
 & \times (c+2cN+3N(-3+2N))^2(9(21-34N)N+c(-23+22N)), \\
 c_{81} = & 60c^4D(-3+2c)(c-3N)(-1+N)N(1+N)(-1+2N)(c+6N)(c+2cN \\
 & +3N(-3+2N))(-27N^3(903-2014N+960N^2) + 2c^3(69-43N+70N^2+24N^3) \\
 & +18cN^2(766-1221N+158N^2+72N^3) + 3c^2N(-815+802N-72N^2+192N^3)), \\
 c_{82} = & 1080c^4D(c-3N)^2(1-2N)^2(-1+N)N^2(4c^4(3+68N-46N^2+24N^3)
 \end{aligned}$$

$$\begin{aligned}
 & +54N^3(-238 + 41N - 852N^2 + 696N^3) - 4c^3(198 - 781N + 836N^2 - 534N^3 + 108N^4) \\
 & + 9cN^2(517 + 1268N + 4504N^2 - 3240N^3 + 288N^4) - 3c^2N(-925 + 5518N - 1600N^2 \\
 & + 696N^3 + 432N^4)), \\
 c_{83} = & \frac{60c^4D(c-3N)}{(21+4c)N(-1+2N)}(30618N^7(1879156195 - 2552308699N + 74066316N^2 \\
 & - 280658444N^3 + 23411216N^4) + 729cN^6(-194696510634 + 285867969601N \\
 & - 161778693257N^2 + 44463727254N^3 - 6000700796N^4 + 324597784N^5) \\
 & - 243c^2N^5(-611248780951 + 941044890863N - 551579169616N^2 + 148343739040N^3 \\
 & - 15163385212N^4 - 615248320N^5 + 172707152N^6) - 81c^3N^4(1054441730907 \\
 & - 1652051776145N + 910697130696N^2 - 150626552798N^3 - 46144176868N^4 \\
 & + 24060216336N^5 - 3891142928N^6 + 228965536N^7) - 54c^4N^3(-539128068255 \\
 & + 831459452981N - 366140697300N^2 - 70575832003N^3 + 120519241062N^4 \\
 & - 46021509048N^5 + 8422933568N^6 - 744785648N^7 + 23445024N^8) \\
 & - 9c^5N^2(652627588086 - 949225519313N + 196210000785N^2 + 428583964264N^3 \\
 & - 378317491640N^4 + 143339605976N^5 - 29240429248N^6 + 3209784288N^7 - 164554432N^8 \\
 & + 2314240N^9) + 8c^7(-3759851520 + 3788666784N + 5088306162N^2 - 11129158163N^3 \\
 & + 8411293753N^4 - 3418763020N^5 + 816269610N^6 - 114568952N^7 + 8826872N^8 \\
 & - 302464N^9 + 768N^{10}) - 6c^6N(-108015768000 + 138879230190N + 36859111663N^2 \\
 & - 166824261639N^3 + 130102162300N^4 - 50883535168N^5 + 11321335464N^6 \\
 & - 1432639280N^7 + 94135072N^8 - 2439168N^9 + 21504N^{10})), \\
 c_{84} = & -60c^4D(c-3N)^2(-1+N)N(-1+2N)(4c^5(564 - N - 713N^2 + 16N^3 + 164N^4)] \\
 & + 486N^4(1015 - 3408N + 7394N^2 - 6360N^3 + 1920N^4) - 12c^4(-81 + 2043N - 1121N^2 \\
 & - 1138N^3 + 488N^4 + 136N^5) + 27c^2N^2(8359 - 31420N + 60808N^2 - 47112N^3 + 9424N^4 \\
 & + 2656N^5) - 9c^3N(3683 - 20105N + 29100N^2 - 13536N^3 - 3392N^4 + 3344N^5) \\
 & + 81cN^3(-6964 + 23723N - 51024N^2 + 47380N^3 - 20240N^4 + 3840N^5)), \\
 c_{85} = & 60c^4D(c-3N)(-1+N)N(1+N)(-1+2N)(c+6N)(c^5(4050 - 8436N \\
 & + 6112N^2 - 2096N^3 + 352N^4) + 486N^4(-1197 + 2241N - 344N^2 - 1204N^3 + 528N^4) \\
 & - 3c^4(849 + 22764N - 60892N^2 + 55312N^3 - 19104N^4 + 2304N^5) + 81cN^3(9087 - 8986N \\
 & - 22192N^2 + 39448N^3 - 20464N^4 + 3264N^5) + 9c^3N(5175 + 39944N - 139868N^2 \\
 & + 153512N^3 - 63808N^4 + 7008N^5) - 27c^2N^2(10843 + 18486N - 117332N^2 + 155432N^3 \\
 & - 80288N^4 + 13952N^5)), \\
 c_{86} = & 7560c^4D(c-3N)^2(1-2N)^2(-1+N)N^2(1+N)(c+6N)(12N + c(-5+2N)) \\
 & \times (2c^2(-9+2N) + 9N(-13+10N) - 3c(-5-40N+36N^2)), \\
 c_{87} = & 120c^4D(c-3N)^2(-1+N)N(1+N)(-1+2N)(c+6N) \\
 & \times (4c^4(-51+952N-368N^2+88N^3) - 81N^3(-1785+3036N-1948N^2+528N^3) \\
 & - 6c^3(33+767N+6704N^2-6132N^3+800N^4) - 54cN^2(1305+967N-1036N^2 \\
 & - 1636N^3+816N^4) + 9c^2N(907+6580N+5964N^2-14048N^3+3136N^4)), \\
 c_{89} = & -\frac{120c^4D(c-3N)^2(-1+N)(1+N)}{(21+4c)N(-1+2N)}(10206N^6(17893080 - 24439185N + 12259768N^2 \\
 & - 2700208N^3 + 221584N^4) + 243cN^5(-2162240736 + 4287473215N - 3334762892N^2 \\
 & + 1281132480N^3 - 246201968N^4 + 19254544N^5) + 81c^2N^4(7281651520 - 17539261449N \\
 & + 16471340390N^2 - 7855180400N^3 + 2042274960N^4 - 280227280N^5 + 16294368N^6) \\
 & + 54c^3N^3(-6262982621 + 17106009721N - 18109557310N^2 + 9857352680N^3
 \end{aligned}$$

$$\begin{aligned}
 & -3013592448N^4 + 517744720N^5 - 45340768N^6 + 1421824N^7) - 18c^4N^2(-5876547021 \\
 & + 17572662207N - 20305606131N^2 + 12137100162N^3 - 4129969160N^4 + 812334064N^5 \\
 & - 87262896N^6 + 4098976N^7 + 2560N^8) + 16c^6(71850240 - 243701568N + 318950691N^2 \\
 & - 216782257N^3 + 84751421N^4 - 19616924N^5 + 2627536N^6 - 185400N^7 + 5136N^8) \\
 & - 12c^5N(1436659200 - 4606587915N + 5700740241N^2 - 3659607901N^3 + 1346504472N^4 \\
 & - 290728400N^5 + 35603168N^6 - 2213424N^7 + 54592N^8)), \\
 c_{90} = & 2520c^4D(c-3N)^3(-1+N)N(1+N)(-1+2N)(c+6N) \\
 & \times (12N+c(-5+2N))(2c^2(-9+2N)+9N(-13+10N)-3c(-5-40N+36N^2)), \\
 c_{91} = & 120c^4D(c-3N)^2(-1+N)N(1+N)(-1+2N)(c+6N)(4c^4(-51+7N+220N^2+4N^3) \\
 & - 81N^3(-693+2196N-1948N^2+528N^3)+6c^3(-33+703N-1790N^2+420N^3 \\
 & + 712N^4)+18c^2N(191-1246N+4116N^2-5176N^3+1568N^4) \\
 & - 27cN^2(825-2014N+2212N^2-3272N^3+1632N^4)), \\
 c_{92} = & 5040c^4D(c-3N)^3(-1+N)N(1+N)(-1+2N)(c+6N)(6N+c(-3+2N)) \\
 & \times (2c^2(-9+2N)+9N(-13+10N)-3c(-5-40N+36N^2)), \\
 c_{94} = & -2520c^4D(c-3N)^4(-1+N)N(1+N)(-1+2N)(c+6N) \\
 & \times (2c^2(-9+2N)+9N(-13+10N)-3c(-5-40N+36N^2)), \\
 c_{95} = & -2520c^4D(c-3N)^3(-1+N)N(1+N)(-1+2N)(c+6N)(c+2cN \\
 & + 12(-1+N)N)(2c^2(-9+2N)+9N(-13+10N)-3c(-5-40N+36N^2)), \\
 c_{97} = & 120c^4D(c-3N)^2(-1+N)N(1+N)(-1+2N)(c+6N)(c+2cN \\
 & + 3N(-3+2N))^2(2c^2(-51+22N)-9N(7+66N)+c(-99+984N-612N^2)), \\
 c_{99} = & 6480c^4D(c-3N)^3(1-2N)^2(-1+N)N^2(126N^2(9-47N+2N^2) \\
 & + c^3(24-66N+20N^2)+c^2(210-1141N+736N^2-60N^3) \\
 & + 3cN(-329+1729N-658N^2+120N^3)), \\
 c_{100} = & 120c^4D(c-3N)(-2916N^6(38106977-53043766N+27498349N^2-6436712N^3 \\
 & + 595164N^4)+486cN^5(371362536-345913714N+31665557N^2+58140419N^3 \\
 & - 21702822N^4+2377144N^5)+81c^2N^4(-1380185935+21861735N \\
 & + 1823118762N^2-1384713438N^3+430669664N^4-62207024N^5+3473424N^6) \\
 & + 27c^3N^3(1140198925+2601110708N-5765670627N^2+3917400066N^3 \\
 & - 1282992992N^4+218777456N^5-18042288N^6+479584N^7)+2c^6(34145280 \\
 & - 318753852N+508171813N^2-359262379N^3+137626932N^4-30304928N^5 \\
 & + 3784712N^6-243456N^7+5568N^8)+18c^4N^2(-140155451-2099436457N \\
 & + 3687470901N^2-2496491546N^3+867354436N^4-165291200N^5+16651520N^6 \\
 & - 745344N^7+9792N^8)-3c^5N(138290040-2693063119N+4402083017N^2 \\
 & - 3036819792N^3+1111878548N^4-229862352N^5+26270960N^6 \\
 & - 1474208N^7+26112N^8)), \\
 c_{101} = & -360c^4D(c-3N)^3(-1+N)N(-1+2N)(-378N^3(-61-242N \\
 & + 128N^2+48N^3)+2c^4(-126-385N-158N^2+136N^3+80N^4)-6c^2N(434-3655N \\
 & + 175N^2-1730N^3+864N^4)-9cN^2(-84+13153N-5590N^2-1880N^3+2880N^4) \\
 & + c^3(210+1571N+1742N^2-4040N^3+592N^4+1440N^5)), \\
 c_{103} = & -\frac{4}{77}c^4D(c-3N)^3(-1+2N)(1309770N^4(41+86N-836N^2+656N^3) \\
 & - 31185cN^3(-4326+9933N-24296N^2+7500N^3+6368N^4) \\
 & - 18c^2N^2(4877190+618411N-1430930N^2+2041840N^3-3057520N^4+3873824N^5))
 \end{aligned}$$

$$\begin{aligned}
 & -6c^3N(-2700495 - 9200196N + 18224908N^2 - 16032350N^3 + 6579680N^4 + 586616N^5 \\
 & + 19392N^6) + 4c^4(-241920 - 1090971N - 1166052N^2 + 5022886N^3 - 3836580N^4 \\
 & + 993616N^5 - 132768N^6 + 6784N^7)), \\
 c_{104} = & -180c^4D(c-3N)^3N(-1+2N)(-378N^3(-231+322N-448N^2+304N^3) \\
 & + 4c^4(-63-175N-129N^2+88N^3+60N^4) - 9cN^2(4326-7301N+28120N^2 \\
 & - 22324N^3+2000N^4) + 6c^3(35+751N-795N^2-480N^3-60N^4+640N^5) \\
 & + 6c^2N(-210-408N+12307N^2-11152N^3+3052N^4+1680N^5)), \\
 c_{105} = & 240c^4D(-3+2c)(c-3N)(-1+N)N(1+N)(-1+2N)(c+6N) \\
 & \times (6N+c(-3+2N))(c+2cN+3N(-3+2N))^2(c(23+6N)+3N(-35+18N)), \\
 c_{106} = & 180c^4D(c-3N)(1-2N)^2(-1+N)N^2(4c^5(21-86N-214N^2-94N^3 \\
 & + 148N^4) + 486N^4(-917+771N+1886N^2-2400N^3+1056N^4) - 6c^4(-663+2499N \\
 & - 1887N^2-1460N^3+716N^4+160N^5) + 54c^2N^2(911-11736N+16411N^2-2588N^3 \\
 & - 6148N^4+3520N^5) - 9c^3N(4615-23999N+25082N^2+5048N^3-18008N^4 \\
 & + 6160N^5) + 81cN^3(3781-515N-7262N^2+9120N^3-13696N^4+6528N^5)), \\
 c_{107} = & \frac{4}{77}c^4D(c-3N)^2(1122660N^5(34175025-47493304N+24542619N^2 \\
 & - 5702504N^3+517524N^4) + 187110cN^4(-367403364+562199399N-336217961N^2 \\
 & + 99285338N^3-14644828N^4+870056N^5) - 27c^2N^3(-1775992371990 \\
 & + 2785716629931N-1564546209467N^2+314684799230N^3+27095663760N^4 \\
 & - 19614412856N^5+2176569872N^6) - 27c^3N^2(603184238670-907088343479N \\
 & + 352626759469N^2+107907557168N^3-125049798260N^4+39061512544N^5 \\
 & - 5496174944N^6+312372992N^7) + 4c^5(-43030189440+42053655483N \\
 & + 48678116280N^2-92590179965N^3+57274065706N^4-18051115988N^5 \\
 & + 3117707380N^6-280385360N^7+9708784N^8) - 6c^4N(-448247199015 \\
 & + 591513419034N+19044018070N^2-402862378847N^3+272830776122N^4 \\
 & - 83887701924N^5+13443154720N^6-1083809248N^7+38267168N^8)), \\
 c_{108} = & -120c^4D(c-3N)(-1+N)N(1+N)(-1+2N)(c+6N) \\
 & \times (-972N^4(-1155+3285N-3008N^2+900N^3) + 4c^5(-405-144N+968N^2 \\
 & - 640N^3+176N^4) + 81cN^3(-14613+30842N-13220N^2-6408N^3+4128N^4) \\
 & - 18c^3N(2823+12888N-37806N^2+29892N^3-8472N^4+560N^5) \\
 & - 6c^4(-363-6378N+8178N^2+2316N^3-5464N^4+1264N^5) \\
 & + 27c^2N^2(14729-1222N-64852N^2+85272N^3-44160N^4+8832N^5)), \\
 c_{109} = & -7560c^4D(c-3N)^2(1-2N)^2(-1+N)N^2(1+N)(c+6N) \\
 & \times (6N+c(-3+2N))(2c^2(-9+2N)+9N(-13+10N)-3c(-5-40N+36N^2)), \\
 c_{110} = & -2520c^4D(c-3N)^2(-1+N)N(1+N)(-1+2N)(c+6N) \\
 & \times (c+2cN+12(-1+N)N)(6N+c(-3+2N))(2c^2(-9+2N) \\
 & + 9N(-13+10N)-3c(-5-40N+36N^2)), \\
 c_{111} = & 720c^4D(c-3N)^2(-1+N)N(1+N)(-1+2N)(c+6N) \\
 & \times (c+2cN+3N(-3+2N))^2(9(21-34N)N+c^2(6+4N)+c(-51+36N+36N^2)), \\
 c_{112} = & -720c^4D(c-3N)(1-2N)^2(-1+N)N^2(6N+c(-3+2N)) \\
 & \times (2c^4(8-71N-41N^2+74N^3)-9c^2N(-571+2290N+1044N^2-1654N^3+172N^4) \\
 & - 162N^3(266-215N-374N^2-22N^3+480N^4) \\
 & - 3c^3(260-441N-1205N^2-346N^3+560N^4)
 \end{aligned}$$

$$\begin{aligned}
 & +27cN^2(89 + 2063N - 1559N^2 - 2490N^3 + 2600N^4)), \\
 c_{114} = & \frac{8}{77}c^4D(c-3N)^2(4490640N^5(14721197 - 20535689N + 10681372N^2 \\
 & -2511568N^3 + 233088N^4) + 1496880cN^4(-77099786 + 114631341N - 66007098N^2 \\
 & +18566610N^3 - 2575093N^4 + 141426N^5) - 54c^2N^3(-1466209709190 \\
 & +2214447910908N - 1214642811541N^2 + 255067120555N^3 + 8530456440N^4 \\
 & -11454947908N^5 + 1340361376N^6) - 27c^3N^2(988407540210 - 1447959585159N \\
 & +621188559129N^2 + 58838623408N^3 - 125696666020N^4 + 39465602384N^5 \\
 & -5181645344N^6 + 249269952N^7) + 4c^5(-71650535040 + 80805966003N \\
 & +31308958320N^2 - 88319669105N^3 + 54396568006N^4 - 16201803908N^5 \\
 & +2560988140N^6 - 204469520N^7 + 6936784N^8) - 6c^4N(-736080466275 \\
 & +988543139814N - 165713029790N^2 - 371661474497N^3 + 267228917552N^4 \\
 & -78882116724N^5 + 11562371200N^6 - 778408768N^7 + 12210368N^8)), \\
 c_{115} = & -1080c^4D(c-3N)^2(1-2N)^2(-1+N)N^2(c^4(324-202N-44N^2 \\
 & +32N^3) - 108N^3(189+214N-1142N^2+660N^3) + c^3(6-3521N+6340N^2-3792N^3 \\
 & +696N^4) - 18cN^2(-1186+1641N+873N^2-2926N^3+1872N^4) \\
 & -3c^2N(1658-7695N+13740N^2-10504N^3+3384N^4)), \\
 c_{116} = & \frac{8}{77}c^4D(c-3N)^3(2619540N^4(80522-116869N \\
 & +65865N^2-18662N^3+2584N^4) + 124740cN^3(-2928876+5595281N-4326337N^2 \\
 & +1727974N^3-367862N^4+35340N^5) - 9c^2N^2(-23557535550+47188850859N \\
 & -32268867313N^2+7401593860N^3+1104317280N^4-814885264N^5+122416048N^6) \\
 & -6c^3N(8470667835-15987679002N+6588462910N^2+3316995473N^3-3472049160N^4 \\
 & +1088840152N^5-158304880N^6+13047792N^7) + 4c^4(1077874560-1714907817N \\
 & -307364880N^2+1663675885N^3-1055778944N^4+293841412N^5-37923560N^6 \\
 & +1249840N^7+6784N^8)), \\
 c_{118} = & -120c^4D(c-3N)^2(-1+N)N(-1+2N)(6N+c(-3+2N))(8c^4(-146-97N \\
 & +131N^2+82N^3) + 6c^3(60+1080N-465N^2-1010N^3+232N^4) - 54cN^2(794+464N \\
 & +4069N^2-6226N^3+688N^4) - 9c^2N(-678+529N-6098N^2+2600N^3+3344N^4) \\
 & +81N^3(553+1857N+1082N^2-7344N^3+3840N^4)), \\
 c_{119} = & 6480c^4D(3+c-9N)(c-3N)^2(1-2N)^2(-1+N)N^2 \\
 & \times(2c(-2+N)+3(21-8N)N)(2c(-1+N)+3(3-4N)N)(6N+c(-3+2N)), \\
 c_{120} = & 15120c^4D(c-3N)^3(1-2N)^2(-1+N)N^2(6N+c(-3+2N)) \\
 & \times(10c^2(1+N)+c(7-59N+42N^2)-3N(73-233N+216N^2)), \\
 c_{122} = & 6480c^4D(c-3N)^3(1-2N)^2(-1+N)N^2(126N^2(9-18N+2N^2) \\
 & +10c^3(1-N+2N^2)+c^2(7-287N+484N^2-60N^3) \\
 & +3cN(-126+511N-574N^2+120N^3)), \\
 c_{123} = & 1080c^4D(c-3N)^3(1-2N)^2(-1+N)N^2(2c^3(54-103N \\
 & +38N^2)+126N^2(101-376N+132N^2)+c^2(462-4459N+4376N^2 \\
 & -1236N^3)+3cN(-2632+12495N-8918N^2+1296N^3)), \\
 c_{125} = & -11340c^4D(29+2c)(c-3N)^4(1-2N)^2(-1+N)N^2(-6N^2+c(-1+4N)), \\
 c_{126} = & \frac{4}{77}c^4D(c-3N)^3(-1309770N^4(622278-901903N+498045N^2 \\
 & -128024N^3+13284N^4)+31185cN^3(34417063-49030109N+26353862N^2
 \end{aligned}$$

$$\begin{aligned}
 & -6501160N^3 + 625608N^4 + 9936N^5) + 18c^2N^2(-29451109755 + 41244298176N \\
 & -21578463917N^2 + 5089232180N^3 - 426586380N^4 - 34917536N^5 + 7442912N^6) \\
 & -12c^3N(-9698153220 + 13347002913N - 6785560445N^2 + 1521690038N^3 \\
 & -117136830N^4 - 7236368N^5 + 1574840N^6 + 19392N^7) + 16c^4(-598691520 \\
 & +808526754N - 395608740N^2 + 78197555N^3 - 164197N^4 - 2894344N^5 \\
 & +627020N^6 - 68080N^7 + 3392N^8)), \\
 c_{127} = & -60c^4D(-3+2c)(c-3N)(-1+N)N(1+N)(-1+2N)(c+6N) \\
 & \times(6N+c(-3+2N))(c+2cN+3N(-3+2N))(2c^2(138+67N+2N^2) \\
 & +3cN(-1255+1000N+268N^2)+9N^2(1323-2510N+1104N^2)), \\
 c_{128} = & -540c^4D(c-3N)^2(1-2N)^2(-1+N)N^2(12c^4(11+32N \\
 & -28N^2-22N^3+12N^4)+54N^3(-1617+3571N+610N^2-5448N^3+2784N^4) \\
 & -2c^3(423-587N+1177N^2-1104N^3-24N^4+504N^5)+9cN^2(3347-2973N \\
 & -10386N^2+14552N^3-5232N^4+1152N^5)-3c^2N(-181+2517N-4608N^2 \\
 & +3464N^3-5016N^4+3168N^5)), \\
 c_{129} = & -7560c^4D(3+c-9N)(c-3N)^3(1-2N)^2(-1+N)N^2 \\
 & \times(6N+c(-3+2N))(2c(4+N)+3N(-13+16N)), \\
 c_{130} = & 1080c^4D(c-3N)^2(1-2N)^2(-1+N)N^2(2c^4(228+57N \\
 & -170N^2+16N^3)+54N^3(-483+412N-2924N^2+2880N^3)+c^3(-288-2039N \\
 & +2938N^2+120N^3-1440N^4)-6c^2N(388-561N+5439N^2-4778N^3+1632N^4) \\
 & +9cN^2(2162-2211N+15894N^2-17200N^3+5760N^4)), \\
 c_{131} = & 6480c^4D(29+2c)(c-3N)^2(1-2N)^2(-1+N)N^2(-6N^2+c(-1+4N)) \\
 & \times(-18N^2(8-9N+4N^2)+c^2(-9-2N+4N^2)-3cN(-23+8N+4N^2)), \\
 c_{132} = & 2160c^4D(c-3N)^2(1-2N)^2(-1+N)N^2(-6N^2+c(-1+4N)) \\
 & \times(c^3(-42-22N+44N^2)-189N^2(-45+32N+48N^2) \\
 & +3c^2(119+57N+76N^2-44N^3)-9cN(518-461N-282N^2+336N^3)), \\
 c_{133} = & -1080c^4(c-3N)^2(1-2N)^2(-1+N)N^2(c^4(-432-202N \\
 & +292N^2+32N^3)+54N^3(483-736N-1300N^2+528N^3)+c^3(-1296+7439N \\
 & +2510N^2-6264N^3+1152N^4)-9cN^2(2486-4803N-8922N^2+4208N^3+1152N^4) \\
 & -6c^2N(-1768+6537N+1953N^2-5710N^3+2928N^4)), \\
 c_{134} = & -2520c^4D(c-3N)^3(-1+N)N(-1+2N)(6N+c(-3+2N)) \\
 & \times(8c^3(4+7N+3N^2)+2c^2(2-93N+4N^2+72N^3) \\
 & -18N^2(-98+303N-616N^2+288N^3)-3cN(155-354N+580N^2+288N^3)), \\
 c_{135} = & -360c^4D(c-3N)^2(-1+N)N(-1+2N)(4c^5(18+9N-17N^2-4N^3+4N^4) \\
 & -162N^4(-2275+9100N-10352N^2+2504N^3+480N^4)-2c^4(30+1105N-1062N^2 \\
 & -800N^3+440N^4+48N^5)+18c^2N^2(2588-13713N+22182N^2-15616N^3+4248N^4 \\
 & +432N^5)-3c^3N(626-9181N+15616N^2-5224N^3-3200N^4+816N^5) \\
 & +27cN^3(-9100+39135N-54852N^2+36724N^3-17680N^4+3456N^5)), \\
 c_{136} = & 5c^4D(-3+2c)(c-3N)(1-2N)^2(-1+N)N(1+N)(c+6N)(6N+c(-3+2N)) \\
 & \times(c+2cN+3N(-3+2N))(c(23+6N)+3N(-35+18N)) \\
 & \times(2c(5+N)+3N(-27+32N)), \\
 c_{137} = & -20c^4D(-3+2c)(c-3N)(-1+N)N(-1+2N)(c+6N)(6N+c(-3+2N)) \\
 & \times(2c^3(-138-586N-545N^2+400N^3+548N^4+96N^5)+3c^2N(1988+4567N
 \end{aligned}$$

$$\begin{aligned}
 & -2812N^2 - 6628N^3 + 1520N^4 + 1440N^5) + 9cN^2(-4529 - 1968N + 11286N^2 + 3000N^3 \\
 & - 9928N^4 + 1824N^5) - 27N^3(-3255 + 3899N + 3208N^2 - 2060N^3 - 4336N^4 + 2304N^5)), \\
 c_{138} = & -180c^4D(c-3N)(1-2N)^2(-1+N)N^2(c+6N)(6N+c(-3+2N)) \\
 & \times (2c^3(-9+61N+136N^2+36N^3) - c^2(-107+283N+600N^2+2844N^3) \\
 & - 6cN(319-1317N+1307N^2-2400N^3+684N^4) + 9N^2(735-3593N+5728N^2 \\
 & - 5268N^3+1728N^4)), \\
 c_{141} = & 20c^4D(c-3N)^2(-1+N)N(-1+2N)(c+6N)(6N+c(-3+2N)) \\
 & (10c^3(23+148N+286N^2+176N^3+24N^4) - 54N^2(357+1621N-3031N^2 \\
 & + 364N^3+396N^4) + 3c^2(-115-2058N-7322N^2-2928N^3+6168N^4+2400N^5) \\
 & + 9cN(619+5022N+4148N^2-7460N^3-8192N^4+5040N^5)), \\
 c_{142} = & 30c^4D(c-3N)(-1+N)N^2(-1+2N)(4c^6(-93+1361N-836N^2 \\
 & - 1556N^3+752N^4+288N^5) - 2916N^4(-623+4630N-10446N^2+11948N^3-8416N^4 \\
 & + 2304N^5) + 6c^5(-252-143N-4422N^2+7200N^3-2360N^4-2928N^5+3072N^6) \\
 & + 486cN^3(-4077+33908N-80059N^2+88604N^3-47148N^4+544N^5+4608N^6) \\
 & - 108c^3N(1722-14042N+26152N^2-7925N^3-25694N^4+28676N^5-9544N^6 \\
 & + 1344N^7) + 9c^4(1317-8478N+10606N^2+13164N^3-32936N^4+28368N^5 \\
 & - 15104N^6+4224N^7) + 81c^2N^2(11781-101906N+233586N^2-204800N^3 \\
 & - 11624N^4+153792N^5-92672N^6+18432N^7)), \\
 c_{143} = & -180c^4D(c-3N)(1-2N)^2(-1+N)N^2(c^5(-690+892N-448N^2 \\
 & - 808N^3+592N^4) + 972N^4(-595+1746N+2266N^2-6072N^3+1920N^4) \\
 & - 18c^3N(-1732+9161N-641N^2-3470N^3-580N^4+1784N^5) - 3c^4(483-5070N \\
 & + 3144N^2-2776N^3-1808N^4+2048N^5) + 162cN^3(3797-13138N-8941N^2 \\
 & + 33366N^3-16520N^4+3840N^5) + 27c^2N^2(-8063+32586N+9692N^2-46144N^3 \\
 & + 12256N^4+3872N^5)), \\
 c_{144} = & -180c^4D(c-3N)^2(-1+N)N(-1+2N)(648N^4(-434-872N+1307N^2 \\
 & - 10N^3+72N^4) + 2c^5(-69-692N-1332N^2-152N^3+672N^4+160N^5) \\
 & - 18c^2N^2(-1816+19833N+8928N^2-14758N^3-648N^4+6168N^5) \\
 & - 54cN^3(-69-26298N+21973N^2+6836N^3-14204N^4+6336N^5) + c^4(207+3858N \\
 & + 14104N^2-3812N^3-18448N^4+5840N^5+6720N^6) + 3c^3N(-1791+942N \\
 & + 11314N^2+28840N^3-4360N^4-22432N^5+11520N^6)), \\
 c_{145} = & -540c^4D(c-3N)(1-2N)^2(-1+N)N^2(-324N^4(-945+3146N \\
 & - 2722N^2+1104N^3) + 2c^5(171+88N-272N^2-12N^3+72N^4) \\
 & - 54cN^3(5197-17938N+14987N^2-7250N^3+360N^4) - c^4(-267+5432N+2492N^2 \\
 & - 8712N^3+2400N^4+288N^5) - 6c^3N(1351-7120N-2569N^2+8830N^3-5556N^4 \\
 & + 1944N^5) + 9c^2N^2(8701-30414N+8136N^2+9904N^3-11184N^4+2592N^5)), \\
 c_{146} = & -540c^4D(c-3N)^2(1-2N)^2(-1+N)N^2(6c^4(-10-89N+18N^2+32N^3) \\
 & + 54N^3(-1295+6886N-5316N^2+1728N^3) + c^3(1062-7027N+6866N^2 \\
 & - 1032N^3+576N^4) - 6c^2N(2330-14873N+12791N^2-5178N^3+1152N^4) \\
 & + 9cN^2(6452-38183N+35846N^2-18672N^3+3456N^4)), \\
 c_{147} = & 60c^4D(c-3N)^2(-1+N)N(-1+2N)(8c^5(129-440N-379N^2+278N^3 \\
 & + 88N^4) - 486N^4(1470-12241N+27122N^2-19272N^3+3840N^4) - 18c^3N(-2745 \\
 & + 16740N-12160N^2-8556N^3+5840N^4+688N^5) + 12c^4(-225+755N+2144N^2
 \end{aligned}$$

$$\begin{aligned}
 & -284N^3 - 1212N^4 + 776N^5) - 81cN^3(-10057 + 77854N - 156816N^2 + 105696N^3 \\
 & - 35584N^4 + 7680N^5) - 27c^2N^2(11629 - 81012N + 120764N^2 - 23776N^3 \\
 & - 24896N^4 + 13376N^5)), \\
 c_{148} = & 540c^4D(c-3N)^2(1-2N)^2(-1+N)N^2(c^4(-288-578N-28N^2+352N^3) \\
 & - 378N^3(-245+1226N-1868N^2+576N^3) + 3c^3(-196+1889N-310N^2 \\
 & + 88N^3 + 32N^4) - 6c^2N(-2331+12779N-15347N^2+2114N^3+2592N^4) \\
 & - 9cN^2(7924-41951N+68294N^2-33616N^3+8064N^4)), \\
 c_{149} = & -90c^4D(c-3N)^3(-1+N)N(-1+2N)(16c^4(-33-56N-2N^2+6N^3) \\
 & + 378N^3(24+1171N-1466N^2+432N^3) - 2c^3(-588+1097N-1830N^2+2896N^3 \\
 & + 432N^4) - 9cN^2(-6013+58688N-55304N^2+11800N^3+576N^4) \\
 & - 3c^2N(5383-34120N+15448N^2-5304N^3+6048N^4)), \\
 c_{150} = & -180c^4D(c-3N)^3(-1+N)N^2(-1+2N)(4c^4(-357-101N+256N^2+60N^3) \\
 & + 378N^2(-309+1432N-1136N^2+168N^3) + c^3(-30+6164N-7444N^2+3840N^3 \\
 & + 5520N^4) + 9cN(3514-26383N+36616N^2-45660N^3+19056N^4) \\
 & + 3c^2(-714+10713N-14096N^2+18716N^3-11984N^4+10080N^5)), \\
 c_{151} = & -3c^5D(-3+2c)(-1+N)N(1+N)(-1+2N)^3(c+6N)(6N+c(-3+2N)) \\
 & \times (c+2cN+3N(-3+2N))(c+2N(-4+5N))(c(23+6N)+3N(-35+18N)), \\
 c_{152} = & 5c^5D(-3+2c)(-1+N)N(1+N)(-1+2N)^3(c+6N)(6N+c(-3+2N)) + (c+2cN \\
 & + 3N(-3+2N))(c(23+6N)+3N(-35+18N))(c(7+2N)+3N(-19+22N)), \\
 c_{153} = & -30c^4D(3+c-9N)(c-3N)(1-2N)^2(-1+N)N(1+N)(2c(-1+N) \\
 & + 3(3-4N)N)(6N+c(-3+2N))(72N^3(-21+23N)+c^2N(-1049+1304N-252N^2) \\
 & + c^3(80-34N+4N^2)+3cN^2(1121-1962N+816N^2)), \\
 c_{154} = & 20c^5D(-3+2c)(1-2N)^2(-1+N)N(1+N)(c+6N)(6N+c(-3+2N)) \\
 & \times (c+2cN+3N(-3+2N))(c^2(253+110N+48N^2) \\
 & + 3cN(-1123+912N+108N^2)+18N^2(567-1039N+450N^2)), \\
 c_{155} = & -60c^4D(c-3N)(1-2N)^2(-1+N)N(c+6N)(6N+c(-3+2N)) \\
 & \times (2c^4(-9+75N+88N^2+4N^3)+54N^3(-280+1359N-1976N^2+648N^3) \\
 & + c^3(107-103N-1724N^2-836N^3+768N^4)-9cN^2(-1268+4955N-3400N^2 \\
 & - 2732N^3+864N^4)-3c^2N(749-2202N-1394N^2+1800N^3+1560N^4)), \\
 c_{156} = & -30c^5D(-3+2c)(-1+N)N(1+N)(-1+2N)^3(c+6N)(6N+c(-3+2N)) \\
 & \times (c+2cN+3N(-3+2N))(2c(1+N)+N(-17+16N)) \\
 & \times (c(23+6N)+3N(-35+18N)), \\
 c_{157} = & -40c^4D(-3+2c)(c-3N)(-1+N)N(1+N)(-1+2N)(c+6N) \\
 & \times (6N+c(-3+2N))(c+2cN+3N(-3+2N))(c^2(-23+223N+262N^2+48N^3) \\
 & + 3cN(80-1133N+480N^2+372N^3)+9N^2(-63+1210N-1870N^2+684N^3)), \\
 c_{158} = & 30c^4D(c-3N)^2(1-2N)^2(-1+N)N(c+6N)(6N+c(-3+2N)) \\
 & \times (5c+6N(-4+3N))(609N+c^2(22+24N)-c(49+266N+288N^2)), \\
 c_{159} = & 40c^4D(3+c-9N)(c-3N)(1-2N)^2(-1+N)N(2c(-1+N) \\
 & + 3(3-4N)N)(6N+c(-3+2N))(189N^3(-9-40N+56N^2) \\
 & + 3c^2N(-1293+241N+1612N^2-468N^3)+2c^3(172+69N-115N^2+24N^3) \\
 & + 9cN^2(1183-612N-1774N^2+1104N^3)), \\
 c_{160} = & 60c^4D(c-3N)(1-2N)^2(-1+N)N(c+6N)(6N+c(-3+2N))
 \end{aligned}$$

$$\begin{aligned}
 & \times (2c^4(-145 + 247N + 556N^2 + 164N^3) + 243N^3(-315 + 1672N - 1748N^2 + 528N^3) \\
 & + 3c^3(193 - 17N - 2888N^2 - 2652N^3 + 80N^4) - 9c^2N(931 - 3480N - 4090N^2 + 128N^3 \\
 & + 824N^4) - 27cN^2(-1591 + 7453N - 1340N^2 - 3028N^3 + 1056N^4)), \\
 c_{161} = & 180c^4D(c - 3N)(1 - 2N)^2(-1 + N)N^2(c + 6N)(6N + c(-3 + 2N)) \\
 & \times (c^3(290 + 430N + 328N^2 + 296N^3) - 18cN(-407 + 1458N - 3727N^2 + 992N^3 \\
 & + 284N^4) - 3c^2(193 + 211N + 2104N^2 + 612N^3 + 560N^4) \\
 & - 27N^2(805 - 4309N + 10196N^2 - 8156N^3 + 2112N^4)), \\
 c_{163} = & -90c^4D(c - 3N)^2(-1 + N)N(-1 + 2N)(c + 6N)(6N + c(-3 + 2N)) \\
 & \times (-63N^2(-77 + 794N - 788N^2 + 168N^3) + 2c^3(-33 + 394N + 604N^2 + 168N^3) \\
 & - 6cN(217 - 3059N - 1042N^2 + 20N^3 + 936N^4) \\
 & + c^2(147 - 1898N - 6796N^2 - 4712N^3 + 1056N^4)), \\
 c_{164} = & \frac{c^4D}{480}(c - 3N)(-1 + N)N^2(-1 + 2N)(c + 6N)(6N + c(-3 + 2N))(-81N^4(-74647572 \\
 & + 60720040N - 4617815N^2 - 2106280N^3 + 103187N^4) + 27cN^3(-358514412 \\
 & + 536854368N - 451661305N^2 + 167280650N^3 - 11536283N^4 + 579982N^5) \\
 & - 18c^2N^2(-211413264 + 117831380N + 147145572N^2 - 202135535N^3 + 66630674N^4 \\
 & - 9293505N^5 + 486598N^6) + 6c^3N(-70018132 - 219125424N + 440138353N^2 \\
 & - 168239926N^3 - 22348943N^4 + 20206034N^5 - 3698398N^6 + 220316N^7) \\
 & + 4c^4(691200 + 45826132N - 22606704N^2 - 79113969N^3 + 63441646N^4 \\
 & - 20922557N^5 + 3325154N^6 - 241166N^7 + 5664N^8)), \\
 c_{165} = & \frac{1}{24}c^4D(-3 + 2c)(c - 3N)(-1 + N)N^2(-1 + 2N)(c + 6N)(6N + c(-3 + 2N)) \\
 & \times (135N^3(642420 - 313100N - 493337N^2 + 320150N^3 + 599N^4) - 9cN^2(7597620 \\
 & - 1404884N - 3834301N^2 + 823824N^3 - 11651N^4 + 848N^5) - 6c^2N(-3035232 \\
 & - 249388N + 444178N^2 + 191091N^3 + 95993N^4 - 14606N^5 + 798N^6) + 4c^3(-408492 \\
 & - 158380N + 15523N^2 - 157740N^3 - 833N^4 + 11805N^5 - 2513N^6 + 160N^7)), \\
 c_{166} = & 60c^5D(-3 + 2c)(1 - 2N)^2(-1 + N)N(1 + N)(c + 6N)(6N + c(-3 + 2N)) \\
 & \times (c + 2cN + 3N(-3 + 2N))^2(9(21 - 34N)N + c(-23 + 22N)), \\
 c_{167} = & -60c^5D(-3 + 2c)(1 - 2N)^2(-1 + N)N(1 + N)(c + 6N) \\
 & \times (6N + c(-3 + 2N))(c + 2cN + 3N(-3 + 2N))^2(c(23 + 6N) + 3N(-35 + 18N)), \\
 c_{168} = & 180c^4D(c - 3N)(1 - 2N)^2(-1 + N)N(c + 6N)(6N + c(-3 + 2N)) \\
 & \times (-2349N^4(25 - 50N + 16N^2) + 4c^4(-6 - 7N + N^2 + 2N^3) + c^3(20 \\
 & + 798N + 44N^2 - 704N^3 - 96N^4) - 3c^2N(50 + 2893N - 1902N^2 - 2804N^3 + 216N^4) \\
 & + 18cN^2(25 + 2075N - 2770N^2 - 872N^3 + 432N^4)), \\
 c_{169} = & 30c^4D(-3 + 2c)(c - 3N)(-1 + N)N(1 + N)(-1 + 2N)(c + 6N)(6N + c(-3 + 2N)) \\
 & \times (c + 2cN + 3N(-3 + 2N))(c(23 + 6N) + 3N(-35 + 18N)) \\
 & \times (2c(-2 + 5N + 6N^2) + 3N(11 - 46N + 32N^2)), \\
 c_{170} = & -2520c^4D(c - 3N)^2(1 - 2N)^2(-1 + N)N(-162N^4(-301 + 254N + 468N^2) \\
 & + 4c^5(33 - 7N - 28N^2 + 12N^3) + 27cN^3(-1248 + 1037N + 1826N^2 + 360N^3) \\
 & + c^4(258 - 2038N + 198N^2 + 1684N^3 - 720N^4) + 3c^3N(-520 + 2645N + 242N^2 \\
 & - 1840N^3 + 432N^4) + 18c^2N^2(382 - 406N - 1055N^2 - 222N^3 + 648N^4)), \\
 c_{171} = & 2520c^4D(c - 3N)^3(1 - 2N)^2(-1 + N)N(6N + c(-3 + 2N)) \\
 & \times (1566N^3 + 6c^3(1 + 2N) + c^2(-5 + 64N - 72N^2) - 3cN(-77 + 328N + 108N^2)),
 \end{aligned}$$

$$\begin{aligned}
 c_{173} &= 1260c^4D(c-3N)^4(-1+N)N(-1+2N)(6N+c(-3+2N)) \\
 &\quad \times (18c^2+c(77-256N+36N^2)-6N(10-107N+144N^2)), \\
 c_{174} &= 3780c^4D(29+2c)(c-3N)^5(-1+N)N(-1+2N)(-6N^2+c(-1+4N)), \\
 c_{175} &= 120c^4D(c-3N)(-1+N)N(1+N)(-1+2N)(2916N^5(-371-1195N+4438N^2 \\
 &\quad -3840N^3+1008N^4)+2c^6(447+683N-638N^2-800N^3+296N^4+144N^5) \\
 &\quad -486cN^4(-1647-13112N+32341N^2-20622N^3+3300N^4+168N^5) \\
 &\quad +9c^4N(1447-1210N-1448N^2-13232N^3+24192N^4-11104N^5+576N^6) \\
 &\quad +3c^5(-279-3961N-2126N^2+11772N^3-4392N^4-2752N^5+1536N^6) \\
 &\quad -162c^2N^3(580+22078N-33593N^2+682N^3+18504N^4-11688N^5+2736N^6) \\
 &\quad -27c^3N^2(1842-27517N+21888N^2+8344N^3+3584N^4-14032N^5+6144N^6)), \\
 c_{176} &= 720c^4D(c-3N)(1-2N)^2(-1+N)N^2(1+N)(2c^5(57-113N-140N^2+148N^3) \\
 &\quad +972N^4(-266+1507N-1694N^2+528N^3)-3c^4(369-2227N+552N^2+148N^3 \\
 &\quad +64N^4)+162cN^3(1913-11287N+13398N^2-5864N^3+1056N^4) \\
 &\quad -9c^3N(-2207+13530N-9818N^2-384N^3+1928N^4) \\
 &\quad -27c^2N^2(4720-28695N+31940N^2-13500N^3+2336N^4)), \\
 c_{178} &= -120c^4D(c-3N)^2(-1+N)N(-1+2N)(4c^5(-66-547N-377N^2+268N^3 \\
 &\quad +164N^4)-486N^4(1834-8569N+15776N^2-10076N^3+2112N^4) \\
 &\quad +6c^4(-132+1940N+1979N^2-1004N^3-580N^4+736N^5) \\
 &\quad -9c^3N(-2309+15418N-11528N^2-9308N^3+5696N^4+1328N^5) \\
 &\quad -54c^2N^2(3881-19339N+29449N^2-5564N^3-5228N^4+2704N^5) \\
 &\quad -81cN^3(-9661+46774N-88192N^2+59152N^3-21632N^4+4224N^5)), \\
 c_{179} &= -360c^4D(c-3N)^2(-1+N)N(-1+2N)(-162N^4(3031-10342N+24220N^2 \\
 &\quad -20832N^3+5520N^4)+4c^5(-3-269N-587N^2-88N^3+268N^4+80N^5) \\
 &\quad +54c^2N^2(-666+4433N-22569N^2+18028N^3-6532N^4+1232N^5) \\
 &\quad -27cN^3(-10916+42899N-149486N^2+151548N^3-65784N^4+13248N^5) \\
 &\quad +2c^4(51+1485N+4206N^2+1780N^3-8236N^4+1744N^5+3120N^6) \\
 &\quad +3c^3N(48-14121N+44376N^2-5456N^3+3704N^4-15248N^5+10080N^6)), \\
 c_{180} &= 180c^4D(c-3N)^3(-1+N)N(-1+2N)(1134N^3(-42+311N-212N^2+44N^3) \\
 &\quad +2c^4(-225-266N+92N^2+88N^3)+3c^3(217-682N+664N^2-24N^3+144N^4) \\
 &\quad -18c^2N(413-3242N+1326N^2-328N^3+280N^4) \\
 &\quad +27cN^2(1449-11242N+8656N^2-3736N^3+432N^4)), \\
 c_{181} &= 360c^4D(c-3N)^2(-1+N)N(-1+2N)(162N^4(-1141-3470N+6332N^2-3312N^3 \\
 &\quad +816N^4)+4c^5(-3-269N-587N^2-88N^3+268N^4+80N^5)-27cN^3(-1976-25963N \\
 &\quad +10294N^2+6900N^3-1416N^4+576N^5)+54c^2N^2(229-2714N-7685N^2+8780N^3 \\
 &\quad -4868N^4+1536N^5)+2c^4(51+1467N+4422N^2+1396N^3-8140N^4+1648N^5 \\
 &\quad +3120N^6)+3c^3N(-954-5241N+27396N^2+520N^3+4904N^4-15728N^5+10080N^6)), \\
 c_{182} &= -360c^4D(c-3N)^3(-1+N)N(-1+2N)(2c^4(66-63N-61N^2+38N^3) \\
 &\quad +108N^3(-161+759N-1000N^2+516N^3)+c^3(258-2325N+2173N^2+682N^3 \\
 &\quad -1152N^4)-9c^2N(401-2009N+1830N^2-646N^3+364N^4) \\
 &\quad +9cN^2(1721-8070N+10143N^2-7102N^3+3168N^4)), \\
 c_{183} &= -60c^4D(c-3N)^2(-1+N)N(-1+2N)(-972N^4(2828-17242N+37221N^2 \\
 &\quad -34262N^3+11368N^4)+4c^5(300-851N-799N^2+2726N^3-1616N^4+240N^5)
 \end{aligned}$$

$$\begin{aligned}
 & -162cN^3(-14610 + 99629N - 247163N^2 + 288230N^3 - 158872N^4 + 33440N^5) \\
 & -6c^4(-600 + 1793N + 2740N^2 - 28862N^3 + 47312N^4 - 26808N^5 + 4800N^6) \\
 & -27c^2N^2(20768 - 168241N + 478488N^2 - 669308N^3 + 482128N^4 \\
 & -163296N^5 + 19200N^6) + 9c^3N(2176 - 40005N + 156810N^2 - 308424N^3 \\
 & + 311816N^4 - 148560N^5 + 27840N^6)), \\
 c_{184} = & \frac{c^4 D}{1386}(c - 3N)^3(-1 + N)N(1 + N)(6N + c(-3 + 2N))(37422N^3(-607188 + 692932N \\
 & -149987N^2 - 39508N^3 + 1871N^4) - 9cN^2(-2612081772 + 3380071140N \\
 & -1506983309N^2 + 333257314N^3 - 47771527N^4 + 2526914N^5) \\
 & +12c^2N(-647089056 + 828954720N - 362423618N^2 + 66918396N^3 \\
 & -2067335N^4 - 952239N^5 + 87172N^6) + 4c^3(200737440 - 197447904N \\
 & -33958188N^2 + 119507128N^3 - 62470805N^4 + 15130604N^5 - 1786777N^6 + 83022N^7)), \\
 c_{185} = & \frac{c^4 D}{8316}(c - 3N)^2(-1 + N)N(1 + N)(224532N^5(-26588880 + 38443204N - 12620894N^2 \\
 & -6289681N^3 + 3791531N^4) + 648cN^4(17184678408 - 30615132804N + 20871541546N^2 \\
 & -5422780976N^3 - 586634977N^4 + 430249823N^5) - 27c^2N^3(29347789888 \\
 & -570089038140N + 464818497844N^2 - 198262765875N^3 + 40075198927N^4 \\
 & -1854160440N^5 + 62099116N^6) + 18c^3N^2(150087148416 - 292741884780N \\
 & +234838285558N^2 - 102080038959N^3 + 25006084826N^4 - 2325816797N^5 \\
 & -298541796N^6 + 17507132N^7) - 6c^4N(73702541412 - 133222095612N \\
 & +83496330261N^2 - 18463293380N^3 - 2322038013N^4 + 2259981346N^5 \\
 & -484741590N^6 + 16823104N^7 + 686712N^8) + 4c^5(6956292960 - 10476722844N \\
 & +1961540964N^2 + 4035022653N^3 - 2852313640N^4 + 784818761N^5 - 89044664N^6 \\
 & +1565122N^7 + 164760N^8 + 7128N^9)), \\
 c_{186} = & 360c^4 D(c - 3N)^3(-1 + N)N(-1 + 2N)(378N^3(1 - 40N - 188N^2 + 288N^3) \\
 & +4c^4(-252 + 183N + 237N^2 - 218N^3 + 40N^4) + 9cN^2(-5726 + 23163N - 2790N^2 \\
 & -34784N^3 + 19296N^4) - 2c^3(1512 - 9441N + 5101N^2 + 8612N^3 - 8816N^4 + 2040N^5) \\
 & +6c^2N(4221 - 18845N + 7614N^2 + 19674N^3 - 16964N^4 + 4320N^5)), \\
 c_{187} = & -720c^4 D(3 + c - 9N)(c - 3N)^2(-1 + N)N(-1 + 2N) \\
 & \times (2c(-2 + N) + 3(21 - 8N)N)(2c(-1 + N) + 3(3 - 4N)N)^2(6N + c(-3 + 2N)), \\
 c_{188} = & -120c^4 D(c - 3N)^2(-1 + N)N(-1 + 2N)(486N^4(-1295 + 5687N - 5326N^2 \\
 & -4272N^3 + 5600N^4) + 8c^5(120 - 224N - 61N^2 + 389N^3 - 284N^4 + 60N^5) \\
 & +81cN^3(6814 - 35827N + 50937N^2 - 1208N^3 - 31356N^4 + 10048N^5) \\
 & -12c^4(-240 + 891N + 587N^2 - 3627N^3 + 3883N^4 - 2064N^5 + 540N^6) \\
 & -18c^3N(-71 + 5988N - 24366N^2 + 29040N^3 - 9198N^4 - 2600N^5 + 1320N^6) \\
 & +27c^2N^2(-5490 + 40460N - 89523N^2 + 60140N^3 + 12916N^4 - 26112N^5 + 7680N^6)), \\
 c_{189} = & 60c^4 D(c - 3N)^2(-1 + N)N(-1 + 2N)(-972N^4(-49 + 26N + 1399N^2 - 3624N^3 \\
 & +2360N^4) + 2c^5(264 - 1097N + 521N^2 + 1514N^3 - 1580N^4 + 408N^5) - 81cN^3(2817 \\
 & -13242N - 351N^2 + 62072N^3 - 74612N^4 + 22880N^5) - 3c^4(-528 + 6839N \\
 & -16337N^2 - 1712N^3 + 30532N^4 - 24352N^5 + 5664N^6) - 27c^2N^2(-5424 + 31449N \\
 & -28979N^2 - 79268N^3 + 141412N^4 - 68240N^5 + 9024N^6) \\
 & +9c^3N(-3268 + 24055N - 35987N^2 - 36966N^3 + 103744N^4 - 63528N^5 + 11856N^6)), \\
 c_{190} = & -60c^4 D(3 + c - 9N)(c - 3N)(1 - 2N)^2(-1 + N)N(2c(-1 + N) + 3(3 - 4N)N)
 \end{aligned}$$

$$\begin{aligned}
 & \times (6N + c(-3 + 2N))(27N^3(385 - 922N + 520N^2) + 2c^3(52 + 30N - 85N^2 + 18N^3) \\
 & - 3c^2N(166 + 321N - 892N^2 + 276N^3) + 9cN^2(-251 + 967N - 1242N^2 + 528N^3)), \\
 c_{191} = & -360c^4D(c-3N)^2(-1+N)N(-1+2N)(2c^5(18+9N-17N^2-4N^3+4N^4) \\
 & + 324N^4(553-2617N+4040N^2-2648N^3+1104N^4) - 54cN^3(3373-17661N \\
 & + 29226N^2-17530N^3+4084N^4) + 9c^2N^2(5351-30960N+47373N^2-13600N^3 \\
 & - 6636N^4+144N^5) + c^4(-30-1861N+2865N^2+1004N^3-2588N^4+1104N^5) \\
 & - 3c^3N(1207-11075N+16928N^2+1786N^3-10888N^4+3576N^5)), \\
 c_{192} = & \frac{2}{231}c^4D(c-3N)^3(-1+N)N(1+N)(62370N^4(96660-151576N+41393N^2 \\
 & - 287N^3+10N^4) + 297cN^3(-17933616+21185516N-7639052N^2+4597707N^3 \\
 & - 916441N^4+44326N^5) - 9c^2N^2(-73634364-395450084N+689581619N^2 \\
 & - 415320060N^3+116512759N^4-15689206N^5+803136N^6) + 12c^3N(26768034 \\
 & - 191115792N+225419346N^2-99259402N^3+12886529N^4+1006414N^5 \\
 & - 455393N^6+32994N^7) + 4c^4(-12559320+46007100N+21686916N^2-129153896N^3 \\
 & + 126289823N^4-58085430N^5+13827169N^6-1652374N^7+78312N^8)), \\
 c_{194} = & -60c^4D(c-3N)^2(-1+N)N(-1+2N)(486N^4(-4039+5878N+6412N^2 \\
 & - 13384N^3+4128N^4) + c^5(-690+304N-4408N^2-2632N^3+2080N^4+480N^5) \\
 & - 81cN^3(-8762-8157N+30918N^2-620N^3-21624N^4+4992N^5) + 27c^2N^2(-1074 \\
 & - 3191N-54886N^2+94696N^3-48664N^4+9584N^5) + 3c^4(345+4190N-1888N^2 \\
 & + 5592N^3-14008N^4-1120N^5+4320N^6) + 9c^3N(-1271-9066N+38796N^2 \\
 & - 19268N^3+14432N^4-22384N^5+8640N^6)), \\
 c_{196} = & 1260c^4D(29+2c)(c-3N)^4(-1+N)N(-1+2N)(c^2(-3+16N+4N^2) \\
 & - 36N^2(1-8N+6N^2) + 12cN(4-21N+14N^2)), \\
 c_{197} = & -2520c^4D(29+2c)(6+5c)(c-3N)^5(1-2N)^2(-1+N)N^2, \\
 c_{198} = & -\frac{1}{231}c^4D(c-3N)^3(-1+N)N(1+N)(-74844N^4(-131220-25004N+87089N^2 \\
 & - 10444N^3+499N^4) + 594cN^3(-36502812+41875700N-21987997N^2+3127102N^3 \\
 & - 503135N^4+28062N^5) - 18c^2N^2(-760764852+1137531564N-672042139N^2 \\
 & + 153480370N^3-12194853N^4+294806N^5+27064N^6) - 3c^3N(1034113860 \\
 & - 1133703084N-428055073N^2+1026925922N^3-603493903N^4+154666562N^5 \\
 & - 19225900N^6+932936N^7) + 2c^4(111378240-26026236N-171512316N^2 \\
 & - 120145873N^3+359911574N^4-221810639N^5+61378366N^6-8038572N^7 \\
 & + 404856N^8)), \\
 c_{199} = & -2520c^5D(3+c-9N)(c-3N)^3(1-2N)^2(-1+N)N(6N+c(-3+2N)) \\
 & \times (2c(4+N)+3N(-13+16N)), \\
 c_{200} = & 120c^4D(c-3N)(1-2N)^2(-1+N)N(6N+c(-3+2N))(4c^5(130-127N-175N^2 \\
 & + 82N^3) + 1458N^4(315-400N-390N^2+528N^3) + c^4(732-8442N+8790N^2 \\
 & + 11604N^3-6144N^4) + 27c^2N^2(-349+4022N+1582N^2-2268N^3+512N^4) \\
 & + 18c^3N(11+1234N-3108N^2-2374N^3+1756N^4) \\
 & - 81cN^3(1543+3116N-6136N^2-1736N^3+4416N^4)), \\
 c_{201} = & 2520c^4D(c-3N)^3(-1+N)N(1+N)(-1+2N)(c+6N) \\
 & \times (6N+c(-3+2N))(2c^2(-9+2N)+9N(-13+10N)-3c(-5-40N+36N^2)), \\
 c_{202} = & -2520c^5D(c-3N)^2(1-2N)^2(-1+N)N(1+N)(c+6N)
 \end{aligned}$$

$$\begin{aligned}
 & \times (6N + c(-3 + 2N))(2c^2(-9 + 2N) + 9N(-13 + 10N) - 3c(-5 - 40N + 36N^2)), \\
 c_{203} = & -120c^4 D(-3 + 2c)(c - 3N)(-1 + N)N(1 + N)(-1 + 2N)(c + 6N) \\
 & \times (6N + c(-3 + 2N))(c + 2cN + 3N(-3 + 2N))^2(9(21 - 34N)N + c(-23 + 22N)), \\
 c_{204} = & -\frac{1}{924}c^4 D(c - 3N)^2(-1 + N)N(1 + N)(449064N^5(-346800 + 561876N - 305066N^2 \\
 & - 4809N^3 + 239N^4) - 14256cN^4(-13694220 + 14680948N - 3159969N^2 - 2628911N^3 \\
 & - 331059N^4 + 16711N^5) + 54c^2N^3(-1598560812 - 349252104N + 3069730897N^2 \\
 & - 3000141443N^3 + 1016895770N^4 - 48250564N^5 + 2556936N^6) \\
 & - 9c^3N^2(-2043595116 - 3524980196N + 6060217223N^2 - 3172587532N^3 + 848094755N^4 \\
 & - 85321378N^5 - 7039564N^6 + 932968N^7) - 3c^4N(881457588 + 79143996N \\
 & + 5622242571N^2 - 11829673420N^3 + 9181357733N^4 - 3683434844N^5 + 818015392N^6 \\
 & - 92510928N^7 + 4196592N^8) + 2c^5(101010240 - 264894156N + 1320376932N^2 \\
 & - 1425911005N^3 - 188223486N^4 + 1001647877N^5 - 591108222N^6 + 156677900N^7 \\
 & - 19855944N^8 + 975744N^9)), \\
 c_{205} = & -60c^4 D(c - 3N)(1 - 2N)^2(-1 + N)N(6N + c(-3 + 2N))(4c^5(181 - 124N - 280N^2 \\
 & + 88N^3) + 486N^4(1778 - 2997N - 362N^2 + 1368N^3) + c^4(930 - 12306N + 7620N^2 \\
 & + 19464N^3 - 8880N^4) + 54c^2N^2(-15 + 2922N - 1596N^2 - 2156N^3 + 1000N^4) \\
 & + 9c^3N(-121 + 4038N - 4368N^2 - 9488N^3 + 6080N^4) \\
 & - 81cN^3(3235 + 2910N - 9120N^2 - 7616N^3 + 9312N^4)), \\
 c_{206} = & 360c^4 D(c - 3N)^3(-1 + N)N(-1 + 2N)(378N^3(1 + 812N - 848N^2 + 168N^3) \\
 & + 2c^4(-252 - 553N - 60N^2 + 276N^3 + 80N^4) + 9cN^2(238 - 24549N + 25218N^2 \\
 & - 18596N^3 + 7944N^4) + 6c^2N(-588 + 6427N - 2202N^2 + 2286N^3 - 2744N^4 + 2520N^5) \\
 & + c^3(420 + 2451N + 1720N^2 - 6400N^3 + 2224N^4 + 3120N^5)), \\
 c_{207} = & 90c^4 D(c - 3N)^2(-1 + N)N(-1 + 2N)(-1134N^4(-89 + 1676N - 2120N^2 + 552N^3) \\
 & + 4c^5(-87 - 527N - 135N^2 + 344N^3 + 84N^4) - 27cN^3(1736 - 62359N + 84448N^2 \\
 & - 56156N^3 + 19824N^4) - 18c^2N^2(-1441 + 29569N - 32196N^2 + 25472N^3 - 8000N^4 \\
 & + 2736N^5) + 2c^4(237 + 3067N + 3200N^2 - 6272N^3 + 1920N^4 + 3504N^5) \\
 & + 3c^3N(-2350 + 11817N - 3652N^2 + 19176N^3 - 21232N^4 + 10128N^5)), \\
 c_{209} = & -\frac{1}{73920}c^4 D(c - 3N)^2(-1 + N)N(1 + N)(-74844N^5(87191280 - 151499604N \\
 & + 99076816N^2 - 3053239N^3 + 150361N^4) - 2268cN^4(-5251847220 + 9540938976N \\
 & - 6743912991N^2 + 1614623849N^3 - 91488776N^4 + 4246058N^5) \\
 & - 27c^2N^3(322378473600 - 604361868364N + 429225569228N^2 - 146324932805N^3 \\
 & + 19595489495N^4 - 1816562176N^5 + 69272812N^6) + 9c^3N^2(338102138160 \\
 & - 570663367708N + 185004066576N^2 + 146449169219N^3 - 128894426409N^4 \\
 & + 38009007844N^5 - 5065672580N^6 + 256284752N^7) + 12c^4N(-38233622520 \\
 & + 18612637176N + 156888130066N^2 - 276285197976N^3 + 202532062819N^4 \\
 & - 78889034550N^5 + 16932604945N^6 - 1887843948N^7 + 85219812N^8) \\
 & + 4c^5(3925687680 + 31647829104N - 151476259488N^2 + 252772338680N^3 \\
 & - 219033364088N^4 + 110406245391N^5 - 33454034437N^6 + 5993082520N^7 \\
 & - 583403652N^8 + 23728320N^9)), \\
 c_{211} = & \frac{1}{221760}c^4 D(c - 3N)^2(-1 + N)N(1 + N)(-224532N^5(80503920 - 182002324N \\
 & + 126595536N^2 - 6282759N^3 + 302441N^4) - 324cN^4(-93647595060 + 209806060656N
 \end{aligned}$$

$$\begin{aligned}
 & -157954346191N^2 + 38602856309N^3 - 2242283636N^4 + 97002538N^5) \\
 & -27c^2N^3(760610867040 - 1654445631492N + 1202422400204N^2 - 383889918975N^3 \\
 & + 39104123405N^4 - 2995368288N^5 + 91595876N^6) + 9c^3N^2(744393478320 \\
 & - 1368728592564N + 338339537608N^2 + 522142332057N^3 - 396199108387N^4 \\
 & + 110689329292N^5 - 14388388140N^6 + 717883376N^7) + 12c^4N(-76415256360 \\
 & + 11129524488N + 463017483078N^2 - 782459460928N^3 + 569222124417N^4 \\
 & - 221488661770N^5 + 47645924715N^6 - 5327534404N^7 + 241156236N^8) \\
 & + 4c^5(4434963840 + 95896703952N - 423902631744N^2 + 708943133160N^3 \\
 & - 621553546064N^4 + 317496132733N^5 - 97441426231N^6 + 17655992840N^7 \\
 & - 1735707756N^8 + 71184960N^9)), \\
 c_{212} = & -\frac{1}{5544}c^4D(c-3N)^3(-1+N)N(1+N)(1197504N^4(-14364-22996N+37256N^2 \\
 & -4361N^3+205N^4)+108cN^3(392042412-198823224N-209029297N^2+101228507N^3 \\
 & -8327054N^4+279736N^5)-9c^2N^2(3660757020-5955019668N+4312057037N^2 \\
 & -2275299554N^3+650524029N^4-79158796N^5+3775972N^6)+6c^3N(1582427124 \\
 & -3113065980N+2254779303N^2-687259790N^3-3455725N^4+55271496N^5 \\
 & -10690244N^6+637136N^7)+4c^4(-199519200+216902988N+484404120N^2 \\
 & -1095979227N^3+853473115N^4-338643053N^5+72799325N^6-8081368N^7 \\
 & +362580N^8)), \\
 c_{213} = & -\frac{1}{693}c^4D(c-3N)^3(-1+N)N(1+N)(74844N^4(-4488300+5444996N-1452661N^2 \\
 & -270074N^3+12799N^4)-54cN^3(-9343618884+14040819684N-7372647175N^2 \\
 & +1363735460N^3+10350349N^4+535006N^5)-9c^2N^2(30564243036-51492494508N \\
 & +33716391953N^2-10992015188N^3+1906862607N^4-214388440N^5+9581860N^6) \\
 & +3c^3N(21302925804-36365264676N+21949080981N^2-4872965938N^3 \\
 & -286148441N^4+346836258N^5-60380548N^6+3417160N^7)+2c^4(-2616274080 \\
 & +3753067284N-65521188N^2-2943492237N^3+2423775860N^4-930730759N^5 \\
 & +192244348N^6-20579588N^7+895440N^8)), \\
 c_{217} = & -2520c^4D(c-3N)^4(-1+N)N(1+N)(-1+2N)(c+6N)(2c^2(-9+2N) \\
 & +9N(-13+10N)-3c(-5-40N+36N^2)), \\
 c_{222} = & 180c^4D(c-3N)^2(-1+N)N(1+N)(-1+2N)(c+6N)(2c^4(-1005 \\
 & +238N+20N^2+8N^3)-81N^3(1631-1570N+68N^2+136N^3) \\
 & +27cN^2(3697+2420N-6384N^2+1648N^3+48N^4)+3c^3(543+8924N-6128N^2 \\
 & +336N^3+80N^4)+9c^2N(-2593-11032N+12488N^2-2400N^3+112N^4)), \\
 c_{224} = & -60c^4D(c-3N)^2(-1+N)N(-1+2N)(486N^4(1477+1548N-220N^2-4208N^3 \\
 & +4000N^4)+4c^5(645+1142N-473N^2-1208N^3+92N^4+240N^5) \\
 & -81cN^3(8994+4367N+22048N^2-32580N^3+10448N^4+5440N^5) \\
 & +6c^4(-435-4060N+293N^2+5364N^3-1624N^4-3952N^5+1680N^6) \\
 & -18c^3N(-670+1568N+10795N^2-5164N^3-3744N^4-1744N^5+2640N^6) \\
 & -27c^2N^2(-5492-9593N-43976N^2+57104N^3-11488N^4-22640N^5+13440N^6)), \\
 c_{225} = & \frac{1}{1386}c^4D(c-3N)(-1+N)N(1+N)(-336798N^6(4933140-11038276N \\
 & +11161651N^2-5659196N^3+1096241N^4)+5346cN^5(606646044-1310657020N \\
 & +1249237259N^2-569121554N^3+64807285N^4+23030706N^5)
 \end{aligned}$$

$$\begin{aligned}
 & +162c^2N^4(-15673645008 + 30675598004N - 24226318876N^2 + 7119528285N^3 \\
 & +2302136133N^4 - 2363991614N^5 + 506456716N^6) + 27c^3N^3(37403594280 \\
 & -58150782908N + 19427533212N^2 + 18490762297N^3 - 20200878807N^4 \\
 & +8032889410N^5 - 1247247204N^6 + 10757160N^7) + 9c^4N^2(-23664485052 \\
 & +20339565872N + 28450790965N^2 - 46804352397N^3 + 24914128446N^4 \\
 & -5854823294N^5 + 622956532N^6 - 62247480N^7 + 1579888N^8) \\
 & -6c^5N(-3747503160 - 698891988N + 12509712878N^2 - 10071672503N^3 \\
 & -593500925N^4 + 3970254092N^5 - 2050015150N^6 + 503428020N^7 - 58478792N^8 \\
 & +2682768N^9) + 8c^6(-116948880 - 179637660N + 567497940N^2 + 1719495N^3 \\
 & -567589066N^4 + 306983868N^5 + 45911870N^6 - 85516605N^7 + 27629066N^8 \\
 & -3792228N^9 + 194040N^{10}), \\
 c_{227} = & -\frac{1}{462}c^4D(c-3N)^2(-1+N)N(1+N)(37422N^5(-4672044 + 6466180N - 2200505N^2 \\
 & -199150N^3 + 9479N^4) - 1782cN^4(-172116972 + 269287380N - 146145943N^2 \\
 & +28822148N^3 - 1013081N^4 + 62788N^5) - 27c^2N^3(7619374512 - 12147629084N \\
 & +7120738810N^2 - 1921448945N^3 + 255507703N^4 - 30176576N^5 + 1141660N^6) \\
 & +18c^3N^2(3635356104 - 5183846676N + 2002631726N^2 + 210112431N^3 - 394333572N^4 \\
 & +157317619N^5 - 21091124N^6 + 1058972N^7) + 3c^4N(-3258017136 + 3258582972N \\
 & +1227685558N^2 - 2019835167N^3 + 248855333N^4 + 336647108N^5 - 144974828N^6 \\
 & +22561552N^7 - 1243552N^8) + 2c^5(264012480 + 6100848N - 758803548N^2 \\
 & +771818618N^3 - 369977641N^4 + 175652999N^5 - 86585032N^6 + 24472412N^7 \\
 & -3369904N^8 + 177408N^9)), \\
 c_{228} = & 60c^4D(c-3N)^2(-1+N)N(1+N)(-1+2N)(c+6N)(2c^4(-429 - 476N - 32N^2 \\
 & +88N^3) - 81N^3(-483 + 4206N - 4376N^2 + 1104N^3) - 3c^3(-177 - 3454N - 3796N^2 \\
 & +1176N^3 + 800N^4) + 9c^2N(-565 - 4396N - 8568N^2 + 6752N^3 + 1232N^4) \\
 & +27cN^2(81 + 4598N + 5572N^2 - 10040N^3 + 2208N^4)), \\
 c_{229} = & 2520c^4D(c-3N)^3(-1+N)N(1+N)(-1+2N)(c+6N)(-15N + 2c(2+N)) \\
 & \times (2c^2(-9 + 2N) + 9N(-13 + 10N) - 3c(-5 - 40N + 36N^2)), \\
 D = & \frac{1}{7560(-1+N)N(-1+2N)(29+2c)(6+5c)N^2(-1+2N)^2(c-6N+6N^2)^4}. \tag{E.1}
 \end{aligned}$$

F The $\mathcal{N} = 1$ superspace description

The $\mathcal{N} = 1$ superconformal algebra is described as the $\mathcal{N} = 1$ super OPE

$$\hat{T}(Z_1)\hat{T}(Z_2) = \frac{1}{z_{12}^3} \frac{c}{6} + \frac{\theta_{12}}{z_{12}^2} \frac{3}{2} \hat{T}(Z_2) + \frac{1}{z_{12}} \frac{1}{2} D \hat{T}(Z_2) + \frac{\theta_{12}}{z_{12}} \partial \hat{T}(Z_2) + \dots, \tag{F.1}$$

where $z_{12} = z_1 - z_2 - \theta_1\theta_2$, $\theta_{12} = \theta_1 - \theta_2$, $D = \partial_\theta + \theta\partial_z$ and $\partial = \partial_z$. The $\mathcal{N} = 1$ super stress energy tensor is

$$\hat{T}(Z) = \frac{1}{2}G(z) + \theta T(z), \quad Z = (z, \theta). \tag{F.2}$$

Of course the single OPE (F.1) consists of (2.8), (2.12) and (2.10). The primary superfields that we consider are given by

$$\begin{aligned}\hat{O}_2(Z) &= O_2(z) + \theta O_{\frac{5}{2}}(z), \\ \hat{O}_{\frac{7}{2}}(Z) &= O_{\frac{7}{2}}(z) + \theta O_4(z), \\ \hat{O}_4(Z) &= O_{4'}(z) + \theta O_{\frac{9}{2}}(z).\end{aligned}\tag{F.3}$$

The three OPEs (3.10), (3.11) and (3.20), together with (F.2) and (F.3), can be expressed as a single $\mathcal{N} = 1$ super OPE between $\hat{O}_2(Z)$ and itself as follows:

$$\begin{aligned}\hat{O}_2(Z_1) \hat{O}_2(Z_2) &= \frac{1}{z_{12}^4} c_{22} + \frac{\theta_{12}}{z_{12}^3} 3c_{22}^t \hat{T}(Z_2) + \frac{1}{z_{12}^2} \left[c_{22}^o \hat{O}_2 + c_{22}^t D\hat{T} \right] (Z_2) \\ &+ \frac{\theta_{12}}{z_{12}^2} \left[\frac{1}{2} c_{22}^o D\hat{O}_2 + 2c_{22}^t \partial\hat{T} \right] (Z_2) + \frac{1}{z_{12}} \left[\frac{1}{2} c_{22}^o \partial\hat{O}_2 + \frac{1}{2} c_{22}^t D\partial\hat{T} \right] (Z_2) \\ &+ \frac{\theta_{12}}{z_{12}} \left[\frac{1}{10} (3c_{22}^o + 4c_{\frac{5}{2}}^{go}) D\partial\hat{O}_2 + \frac{1}{4} (3c_{22}^t + c_{\frac{5}{2}}^{tg}) \partial^2\hat{T} - \hat{O}_{\frac{7}{2}} \right. \\ &\quad \left. - 2c_{\frac{5}{2}}^{go} \hat{T}\hat{O}_2 - 2c_{\frac{5}{2}}^{tg} \hat{T}D\hat{T} \right] (Z_2) + \dots,\end{aligned}\tag{F.4}$$

where the identity $\frac{1}{(z_1 - z_2)^n} = \frac{1}{z_{12}^n} - n \frac{\theta_{12}}{z_{12}^{n+1}}$ ($n = 1, \dots, 6$) is used. All the coefficient functions are given in section 3.

The four OPEs (3.33), (3.39), (3.47) and (3.49) can be expressed as a single $\mathcal{N} = 1$ super OPE as follows:

$$\begin{aligned}\hat{O}_2(Z_1) \hat{O}_{\frac{7}{2}}(Z_2) &= \frac{\theta_{12}}{z_{12}^4} 4c_{\frac{7}{2}}^o \hat{O}_2(Z_2) + \frac{1}{z_{12}^3} c_{\frac{7}{2}}^o D\hat{O}_2(Z_2) + \frac{\theta_{12}}{z_{12}^3} c_{\frac{7}{2}}^o \partial\hat{O}_2(Z_2) \\ &+ \frac{1}{z_{12}^2} \left[c_{\frac{7}{2}}^{o'} \hat{O}_{\frac{7}{2}} + \left(\frac{1}{5} c_{\frac{7}{2}}^o - \frac{2}{5} c_{\frac{7}{2}}^{go} \right) D\partial\hat{O}_2 + 2c_{\frac{7}{2}}^{go} \hat{T}\hat{O}_2 \right] (Z_2) \\ &+ \frac{\theta_{12}}{z_{12}^2} \left[\frac{2}{7} c_{\frac{7}{2}}^{o'} D\hat{O}_{\frac{7}{2}} + 8\hat{O}_4 + \left(\frac{1}{5} c_{\frac{7}{2}}^o - \frac{3}{10} c_{\frac{5}{2}}^{to} - \frac{1}{5} c_{\frac{5}{2}}^{go} \right) \partial^2\hat{O}_2 \right. \\ &\quad \left. + 2c_{\frac{5}{2}}^{go} \hat{T}D\hat{O}_2 + c_{\frac{5}{2}}^{to} D\hat{T}\hat{O}_2 \right] (Z_2) \\ &+ \frac{1}{z_{12}} \left[\frac{2}{7} c_{\frac{7}{2}}^{o'} \partial\hat{O}_{\frac{7}{2}} + D\hat{O}_4 + \left(\frac{1}{30} c_{\frac{7}{2}}^o - \frac{4}{35} c_{\frac{7}{2}}^{go} + \frac{1}{8} c_{\frac{7}{2}}^{go'} - \frac{1}{4} c_{\frac{7}{2}}^{to} \right) D\partial^2\hat{O}_2 \right. \\ &\quad \left. + c_{\frac{7}{2}}^{to} D\hat{T}D\hat{O}_2 + \left(\frac{4}{7} c_{\frac{7}{2}}^{go} + 2c_{\frac{7}{2}}^{go'} \right) \partial\hat{T}\hat{O}_2 + \left(\frac{4}{7} c_{\frac{7}{2}}^{go} - \frac{3}{2} c_{\frac{7}{2}}^{go'} \right) \hat{T}\partial\hat{O}_2 \right] (Z_2) \\ &+ \frac{\theta_{12}}{z_{12}} \left[3\partial\hat{O}_4 + \left(\frac{1}{30} c_{\frac{7}{2}}^o - \frac{9}{80} c_{\frac{5}{2}}^{to} - \frac{3}{40} c_{\frac{5}{2}}^{go} - \frac{1}{6} c_{\frac{5}{2}}^{to'} - \frac{1}{9} c_{\frac{5}{2}}^{go'} \right) \partial^3\hat{O}_2 \right. \\ &\quad \left. + \left(-\frac{1}{4} c_{\frac{5}{2}}^{go''} + \frac{3}{28} c_{\frac{7}{2}}^{o'} \right) D\partial\hat{O}_{\frac{7}{2}} + 2c_{\frac{5}{2}}^{go''} \hat{T}\hat{O}_{\frac{7}{2}} + \left(\frac{3}{8} c_{\frac{5}{2}}^{to} - c_{\frac{5}{2}}^{to'} \right) D\partial\hat{T}\hat{O}_2 \right. \\ &\quad \left. + \left(\frac{3}{4} c_{\frac{5}{2}}^{go} - \frac{10}{3} c_{\frac{5}{2}}^{go'} \right) \partial\hat{T}D\hat{O}_2 + \left(\frac{3}{8} c_{\frac{5}{2}}^{to} + c_{\frac{5}{2}}^{to'} \right) D\hat{T}\partial\hat{O}_2 \right. \\ &\quad \left. + \left(\frac{3}{4} c_{\frac{5}{2}}^{go} + 2c_{\frac{5}{2}}^{go'} \right) \hat{T}D\partial\hat{O}_2 \right] (Z_2) + \dots.\end{aligned}\tag{F.5}$$

All the coefficient functions in this expression are in section 3.

The four OPEs (3.51), (3.57), (3.67) and (3.70) can be expressed as a single $\mathcal{N} = 1$ super OPE as follows:

$$\begin{aligned}
 \hat{O}_2(Z_1) \hat{O}_4(Z_2) = & \frac{1}{z_{12}^4} c_{24'}^{o'} \hat{O}_2(Z_2) \\
 & + \frac{\theta_{12}}{z_{12}^3} \left[7c_{24'}^{o'} \hat{O}_{\frac{7}{2}} - \frac{2}{5} c_{\frac{5}{2}4'}^{go} D\partial\hat{O}_2 + 2c_{\frac{5}{2}4'}^{go} \hat{T}\hat{O}_2 \right] (Z_2) \\
 & + \frac{1}{z_{12}^2} \left[c_{24'}^{o''} \hat{O}_4 + c_{24'}^{o'} D\hat{O}_{\frac{7}{2}} + \hat{A}_4 - \frac{1}{5} \left(\frac{3}{2} c_{24'}^{to} + c_{24'}^{go} \right) \partial^2 \hat{O}_2 \right. \\
 & \left. + c_{24'}^{to} D\hat{T}\hat{O}_2 + 2c_{24'}^{go} \hat{T}D\hat{O}_2 \right] (Z_2) \\
 & + \frac{\theta_{12}}{z_{12}^2} \left[\frac{1}{4} c_{24'}^{o''} D\hat{O}_4 + 2c_{24'}^{o'} \partial\hat{O}_{\frac{7}{2}} + \frac{1}{4} D\hat{A}_4 + \left(-\frac{4}{35} c_{\frac{5}{2}4'}^{go} + \frac{1}{8} c_{\frac{5}{2}4'}^{go'} - \frac{1}{8} c_{\frac{5}{2}4'}^{to} \right) D\partial^2 \hat{O}_2 \right. \\
 & \left. + c_{\frac{5}{2}4'}^{to} D\hat{T}D\hat{O}_2 + 2 \left(\frac{2}{7} c_{\frac{5}{2}4'}^{go} + c_{\frac{5}{2}4'}^{go'} \right) \partial\hat{T}\hat{O}_2 + 2 \left(\frac{2}{7} c_{\frac{5}{2}4'}^{go} - \frac{3}{4} c_{\frac{5}{2}4'}^{go'} \right) \hat{T}\partial\hat{O}_2 \right] (Z_2) \\
 & + \frac{1}{z_{12}} \left[\frac{1}{4} c_{24'}^{o''} \partial\hat{O}_4 + \frac{1}{4} (c_{24'}^{o'} - c_{24'}^{go'}) D\partial\hat{O}_{\frac{7}{2}} + \frac{1}{4} \partial\hat{A}_4 \right. \\
 & \left. - \left(\frac{3}{40} c_{24'}^{to} + \frac{1}{20} c_{24'}^{go} + \frac{1}{6} c_{24'}^{to'} + \frac{1}{9} c_{24'}^{go'} \right) \partial^3 \hat{O}_2 \right. \\
 & \left. + \left(\frac{1}{4} c_{24'}^{to} - c_{24'}^{to'} \right) D\partial\hat{T}\hat{O}_2 + \left(\frac{1}{4} c_{24'}^{to} + c_{24'}^{to'} \right) D\hat{T}\partial\hat{O}_2 \right. \\
 & \left. + 2 \left(\frac{1}{4} c_{24'}^{go} - \frac{5}{3} c_{24'}^{go'} \right) \partial\hat{T}D\hat{O}_2 + 2 \left(\frac{1}{4} c_{24'}^{go} + c_{24'}^{go'} \right) \hat{T}D\partial\hat{O}_2 + 2c_{24'}^{go'} \hat{T}\hat{O}_{\frac{7}{2}} \right] (Z_2) \\
 & + \frac{\theta_{12}}{z_{12}} \left[-\hat{O}_{\frac{11}{2}} + \left(\frac{1}{12} - \frac{2}{9} c_{\frac{5}{2}4'}^{ga} \right) D\partial\hat{A}_4 \right. \\
 & \left. + \left(\frac{3}{8} c_{24'}^{o'} - \frac{3}{16} c_{\frac{5}{2}4'}^{to'} - \frac{1}{8} c_{\frac{5}{2}4'}^{go''''} - \frac{c(21+4c)(c-4cN+6N^2)}{28(6+5c)(c-6(-1+N)N)} c_{\frac{5}{2}4'}^q \right) \partial^2 \hat{O}_{\frac{7}{2}} \right. \\
 & \left. + \left(\frac{1}{12} c_{24'}^{o''} - \frac{2}{9} c_{\frac{5}{2}4'}^{go''''} - \frac{1}{3} c_{\frac{5}{2}4'}^q \right) D\partial\hat{O}_4 \right. \\
 & \left. + \left(\frac{1}{24} c_{\frac{5}{2}4'}^{go'} - \frac{3}{140} c_{\frac{5}{2}4'}^{go} - \frac{1}{12} c_{\frac{5}{2}4'}^{to} - \frac{2}{35} c_{\frac{5}{2}4'}^{go''} - \frac{1}{7} c_{\frac{5}{2}4'}^{to''} \right. \right. \\
 & \left. \left. + 2c^2 H(2304 + 5c(-654 + c(-247 + 2c))) c_{\frac{5}{2}4'}^q \right) D\partial^3 \hat{O}_2 \right. \\
 & \left. + c_{\frac{5}{2}4'}^q \hat{O}_2 \hat{O}_{\frac{7}{2}} + 2c_{\frac{5}{2}4'}^{go''''} \hat{T}D\hat{O}_{\frac{7}{2}} + c_{\frac{5}{2}4'}^{to'} D\hat{T}\hat{O}_{\frac{7}{2}} + 2c_{\frac{5}{2}4'}^{go''''} \hat{T}\hat{O}_4 + 2c_{\frac{5}{2}4'}^q \hat{T}\hat{A}_4 \right] (Z_2) \\
 & + \dots, \tag{F.6}
 \end{aligned}$$

where $H \equiv \frac{(3+c-9N)(1+N)(c+6N)(-3c+2(3+c)N)}{35(29+2c)(21+4c)(6+5c)^2(c+6(-1+N)N)^2}$ and one introduces $\hat{O}_{\frac{11}{2}}(Z) = O_{\frac{11}{2}}(z) + \theta O_6(z)$, and $\hat{A}_4(Z) = A_4(z) + \theta A_{\frac{9}{2}}(z)$. All the coefficient functions can be found in section 3.

Following [7], the equations (F.4), (F.5), and (F.6) can be summarized as follows:

$$\begin{aligned}
 \hat{O}_2 \times \hat{O}_2 &= n_2 I + C_{22}^2 \hat{O}_2 + C_{22}^{\frac{7}{2}} \hat{O}_{\frac{7}{2}}, \\
 \hat{O}_2 \times \hat{O}_{\frac{7}{2}} &= C_{2\frac{7}{2}}^2 \hat{O}_2 + C_{2\frac{7}{2}}^{\frac{7}{2}} \hat{O}_{\frac{7}{2}} + C_{2\frac{7}{2}}^4 \hat{O}_4, \\
 \hat{O}_2 \times \hat{O}_4 &= C_{24}^2 \hat{O}_2 + C_{24}^{\frac{7}{2}} \hat{O}_{\frac{7}{2}} + C_{24}^4 \hat{O}_4 + A_{24}^4 \hat{A}_4 + C_{24}^{\frac{11}{2}} \hat{O}_{\frac{11}{2}}, \tag{F.7}
 \end{aligned}$$

where the structure constants in [7] are related to our structure constants as follows

$$\hat{A}_4'(Z) = \frac{(29+2c)(6+5c)^2(c+6(-1+N)N)^2}{24c^3(-7+10c)(3+c-9N)(1+N)(c+6N)(-3c+2(3+c)N)} \hat{A}_4(Z),$$

$$\begin{aligned}
 n_2 &= c_{22}, & C_{22}^2 &= c_{22}^o, & C_{22}^{\frac{7}{2}} &= 1, \\
 C_{2\frac{7}{2}}^2 &= c_{2\frac{7}{2}}^o, & C_{2\frac{7}{2}}^{\frac{7}{2}} &= c_{2\frac{7}{2}}^{o'}, & C_{2\frac{7}{2}}^4 &= 1, \\
 C_{24}^2 &= c_{24'}^o, & C_{24}^{\frac{7}{2}} &= c_{24'}^{o'}, & C_{24}^4 &= c_{24''}^{o''}, & C_{24}^{\frac{11}{2}} &= 1, \\
 A_{24}^4 &= \frac{24c^3(-7+10c)(3+c-9N)(1+N)(c+6N)(-3c+2(3+c)N)}{(29+2c)(6+5c)^2(c+6(-1+N)N)^2}.
 \end{aligned} \tag{F.8}$$

As in [7], the following relations corresponding first few terms in (2.23) of [7] are satisfied:

$$\begin{aligned}
 C_{2\frac{7}{2}}^2 &= \frac{3(c-15)}{2(5c+6)}(C_{22}^2)^2 + \frac{12(5c+6)}{c(4c+21)}n_2, & C_{2\frac{7}{2}}^{\frac{7}{2}} &= \frac{(4c+21)}{(5c+6)}C_{22}^2, \\
 C_{24}^2 &= \frac{15(c-15)(4c+21)}{28(2c+29)(5c+6)}(C_{22}^2)^3 + \frac{30(5c+6)}{7c(2c+29)}n_2 C_{22}^2, \\
 C_{24}^{\frac{7}{2}} &= -\frac{(4c+21)(50c^2-145c+483)}{98(2c-3)(2c+29)(5c+6)}(C_{22}^2)^2 - \frac{2(4c+21)(14c-25)}{7c(2c-3)(2c+29)}n_2, \\
 C_{24}^4 &= \frac{2(25c+84)}{7(5c+6)}C_{22}^2, \\
 A_{24}^4 &= -\frac{6c(c-15)(4c+21)(10c-7)}{(2c-3)(2c+29)(5c+6)^2}(C_{22}^2)^2 - \frac{48(10c-7)}{(2c-3)(2c+29)}n_2.
 \end{aligned}$$

Note that in our case, the structure constant $c_{22}^o = C_{22}^2$ is not equal to 1.

G The OPEs between the stress energy tensor $T(z)$ and the quasi-primary (or primary) fields

To find whether a conformal field is a quasi-primary or not, the OPE between $T(z)$ and the conformal field should be computed and the vanishing of third-order pole of the OPE should be checked. We list the OPEs between the stress tensor and the quasi-primary fields (with four primary fields) appeared in the section 3 here:

$$\begin{aligned}
 T(z) \left(GO_2 - \frac{2}{5} \partial O_{\frac{5}{2}} \right) (w) &= \mathcal{O}((z-w)^{-2}), \\
 T(z) \left(TG - \frac{3}{8} \partial^2 G \right) (w) &= \frac{1}{(z-w)^4} \frac{(21+4c)}{8} G(w) + \mathcal{O}((z-w)^{-2}), \\
 T(z) \left(TT - \frac{3}{10} \partial^2 T \right) (w) &= \frac{1}{(z-w)^4} \frac{(22+5c)}{5} T(w) + \mathcal{O}((z-w)^{-2}), \\
 T(z) \left(TO_2 - \frac{3}{10} \partial^2 O_2 \right) (w) &= \frac{1}{(z-w)^4} \frac{(44+5c)}{10} O_2(w) + \mathcal{O}((z-w)^{-2}), \\
 T(z) \left(GO_{\frac{5}{2}} - \frac{1}{5} \partial^2 O_2 \right) (w) &= \frac{1}{(z-w)^4} \frac{38}{5} O_2(w) + \mathcal{O}((z-w)^{-2}), \\
 T(z) \left(G\partial G - \frac{7}{10} \partial^2 T \right) (w) &= \frac{1}{(z-w)^4} \left(-\frac{17}{5} \right) T(w) + \mathcal{O}((z-w)^{-2}), \\
 T(z) \left(\partial GO_2 - \frac{3}{4} G\partial O_2 + \frac{1}{8} \partial^2 O_{\frac{5}{2}} \right) (w) &= \frac{1}{(z-w)^4} \frac{7}{2} O_{\frac{5}{2}}(w) + \mathcal{O}((z-w)^{-2}), \\
 T(z) \left(TO_{\frac{5}{2}} - \frac{1}{4} \partial^2 O_{\frac{5}{2}} \right) (w) &= \frac{1}{(z-w)^4} \frac{(25+2c)}{4} O_{\frac{5}{2}}(w) + \mathcal{O}((z-w)^{-2}),
 \end{aligned}$$

$$\begin{aligned}
 & T(z) \left(T\partial O_2 - \partial T O_2 - \frac{1}{6}\partial^3 O_2 \right) (w) = \frac{1}{(z-w)^5} (-2c) O_2(w) \\
 & + \frac{1}{(z-w)^4} \left(-\frac{1}{4} \right) (-2c) \partial O_2(w) + \mathcal{O}((z-w)^{-2}), \\
 & T(z) \left(G\partial O_{\frac{5}{2}} - \frac{5}{3}\partial G O_{\frac{5}{2}} - \frac{1}{9}\partial^3 O_2 \right) (w) = \frac{1}{(z-w)^5} (-8) O_2(w) \\
 & + \frac{1}{(z-w)^4} \left(-\frac{1}{4} \right) (-8) \partial O_2(w) + \mathcal{O}((z-w)^{-2}), \\
 & T(z) \left(G O_{\frac{7}{2}} - \frac{1}{4}\partial O_4 \right) (w) = \mathcal{O}((z-w)^{-2}), \\
 & T(z) \left(\partial^2 G O_2 + \frac{3}{5} G \partial^2 O_2 - 2\partial G \partial O_2 - \frac{2}{35}\partial^3 O_{\frac{5}{2}} \right) (w) = \frac{1}{(z-w)^5} \frac{76}{35} O_{\frac{5}{2}}(w) \\
 & + \frac{1}{(z-w)^4} \left[\left(-\frac{1}{5} \right) \frac{76}{35} \partial O_{\frac{5}{2}} + \frac{81}{5} (G O_2 - \frac{2}{5} \partial O_{\frac{5}{2}}) \right] (w) + \mathcal{O}((z-w)^{-2}), \\
 & T(z) \left(T O_{\frac{7}{2}} - \frac{3}{16}\partial^2 O_{\frac{7}{2}} \right) (w) = \frac{1}{(z-w)^4} \frac{(161+8c)}{16} O_{\frac{7}{2}}(w) + \mathcal{O}((z-w)^{-2}), \\
 & T(z) \left(T\partial O_{\frac{5}{2}} - \frac{5}{4}\partial T O_{\frac{5}{2}} - \frac{1}{7}\partial^3 O_{\frac{5}{2}} \right) (w) = \frac{1}{(z-w)^5} \left(-\frac{5}{28} \right) (13+14c) O_{\frac{5}{2}}(w) \\
 & + \frac{1}{(z-w)^4} \left(-\frac{1}{5} \right) \left(-\frac{5}{28} \right) (13+14c) \partial O_{\frac{5}{2}}(w) + \mathcal{O}((z-w)^{-2}), \\
 & T(z) \left(G O_{4'} - \frac{2}{9}\partial O_{\frac{9}{2}} \right) (w) = \mathcal{O}((z-w)^{-2}), \\
 & T(z) \left(G A_4 - \frac{2}{9}\partial A_{\frac{9}{2}} \right) (w) = \mathcal{O}((z-w)^{-2}), \\
 & T(z) \left(G O_4 - \frac{1}{8}\partial^2 O_{\frac{7}{2}} \right) (w) = \frac{1}{(z-w)^4} \frac{119}{8} O_{\frac{7}{2}}(w) + \mathcal{O}((z-w)^{-2}), \\
 & T(z) \left(G O_{\frac{9}{2}} - \frac{1}{9}\partial^2 O_{4'} \right) (w) = \frac{1}{(z-w)^4} \frac{52}{3} O_{4'}(w) + \mathcal{O}((z-w)^{-2}), \\
 & T(z) \left(G A_{\frac{9}{2}} - \frac{1}{9}\partial^2 A_4 \right) (w) = \frac{1}{(z-w)^4} \frac{52}{3} A_4(w) + \mathcal{O}((z-w)^{-2}), \\
 & T(z) \left(T\partial^2 O_2 - \frac{5}{2}\partial T\partial O_2 + \partial^2 T O_2 - \frac{3}{28}\partial^4 O_2 \right) (w) = \frac{1}{(z-w)^6} \frac{(58+70c)}{7} O_2(w) \\
 & + \frac{1}{(z-w)^5} \left(-\frac{1}{2} \right) \frac{(58+70c)}{7} \partial O_2(w) + \frac{1}{(z-w)^4} \left[\left(\frac{1}{20} \right) \frac{(58+70c)}{7} \partial^2 O_2 + 24 \left(T O_2 - \frac{3}{10} \partial^2 O_2 \right) \right] (w) \\
 & + \mathcal{O}((z-w)^{-2}), \\
 & T(z) \left(G\partial O_{\frac{7}{2}} - \frac{7}{3}\partial G O_{\frac{7}{2}} - \frac{1}{9}\partial^2 O_4 \right) (w) = \frac{1}{(z-w)^4} \left(-\frac{25}{3} \right) O_4(w) + \mathcal{O}((z-w)^{-2}), \\
 & T(z) \left(T O_4 - \frac{1}{6}\partial^2 O_4 \right) (w) = \frac{1}{(z-w)^4} \left(\frac{24+c}{2} \right) O_4(w) + \mathcal{O}((z-w)^{-2}), \\
 & T(z) \left(T O_{4'} - \frac{1}{6}\partial^2 O_{4'} \right) (w) = \frac{1}{(z-w)^4} \left(\frac{24+c}{2} \right) O_{4'}(w) + \mathcal{O}((z-w)^{-2}), \\
 & T(z) \left(T A_4 - \frac{1}{6}\partial^2 A_4 \right) (w) = \frac{1}{(z-w)^4} \left(\frac{24+c}{2} \right) A_4(w) + \mathcal{O}((z-w)^{-2}), \\
 & T(z) \left(G\partial^2 O_{\frac{5}{2}} - 4\partial G\partial O_{\frac{5}{2}} + \frac{5}{2}\partial^2 G O_{\frac{5}{2}} - \frac{1}{14}\partial^4 O_2 \right) (w) = \frac{1}{(z-w)^6} \left(\frac{230}{7} \right) O_2(w) \\
 & + \frac{1}{(z-w)^5} \left(-\frac{1}{2} \right) \left(\frac{230}{7} \right) \partial O_2(w) + \frac{1}{(z-w)^4} \left[\left(\frac{1}{20} \right) \left(\frac{230}{7} \right) \partial^2 O_2 + \frac{75}{2} \left(G O_{\frac{5}{2}} - \frac{1}{5} \partial^2 O_2 \right) \right] (w) \\
 & + \mathcal{O}((z-w)^{-2}), \\
 & T(z) \left(O_2 O_2 - \frac{3}{10} (-2\tilde{c} + c) \partial^2 O_2 + \frac{3}{10} (\tilde{c} - c) \partial^2 T \right) (w)
 \end{aligned}$$

$$= \frac{1}{(z-w)^4} [c_{t4}^t T + c_{t4}^o O_2] (w) + \mathcal{O}((z-w)^{-2}), \quad (\text{G.1})$$

$$\begin{aligned} T(z) \left(O_2 O_{\frac{5}{2}} - \frac{3}{4} c_1' \partial T G - \frac{3}{4} c_2' G \partial O_2 - \frac{1}{120} (2c_4' - 9c_1') \partial^3 G - \frac{1}{40} (4c_3' - 9c_2') \partial^2 O_{\frac{5}{2}} - \frac{3}{7} \partial O_{\frac{7}{2}} \right) (w) \\ = \frac{1}{(z-w)^5} c_{t\frac{9}{2}}^g G(w) + \frac{1}{(z-w)^4} \left[-\frac{1}{3} c_{t\frac{9}{2}}^g \partial G + c_{t\frac{9}{2}}^o O_{\frac{5}{2}} \right] (w) + \mathcal{O}((z-w)^{-2}), \end{aligned} \quad (\text{G.2})$$

$$\begin{aligned} T(z) Q_{\frac{11}{2}}(w) &= \frac{1}{(z-w)^5} c_{t\frac{11}{2}}^o O_{\frac{5}{2}}(w) \\ &\quad \frac{1}{(z-w)^4} \left[-\frac{1}{5} c_{t\frac{11}{2}}^o \partial O_{\frac{5}{2}} + c_{t\frac{11}{2}}^{o'} O_{\frac{7}{2}} + c_{t\frac{11}{2}}^{go} \left(GO_2 - \frac{2}{5} \partial O_{\frac{5}{2}} \right) \right] (w) + \mathcal{O}((z-w)^{-2}), \\ T(z) Q_6(w) &= \frac{1}{(z-w)^6} c_{t6}^o O_2(w) + \frac{1}{(z-w)^5} \left(-\frac{1}{2} \right) c_{t6}^o \partial O_2(w) \\ &\quad + \frac{1}{(z-w)^4} \left[\frac{1}{20} c_{t6}^o \partial^2 O_2 + c_{t6}^{o'} O_4 + c_{t6}^{o''} O_{4'} + c_{t6}^{to} \left(TO_2 - \frac{3}{10} \partial^2 O_2 \right) + c_{t6}^{go} \left(GO_{\frac{5}{2}} - \frac{1}{5} \partial^2 O_2 \right) \right] (w) \\ &\quad + \mathcal{O}((z-w)^{-2}). \end{aligned} \quad (\text{G.3})$$

The two quasi-primary fields in (G.1) and (G.2) come from (3.54) and (3.59) respectively. The general forms of structure constants in (G.2) are given by

$$\begin{aligned} c_{t4}^t &= -44c^2 H' (21 + 4c)(6 + 5c)(2c(-1 + N) + 3(3 - 4N)N)(c + 2cN + 3N(-3 + 2N)), \\ c_{t4}^o &= -132cH' (21 + 4c)(6 + 5c)(c + 6(-1 + N)N)(-6N^2 + c(-1 + 4N)), \\ c_{t\frac{9}{2}}^g &= -3c^2 H' (6 + 5c)(-195 + 94c)(2c(-1 + N) + 3(3 - 4N)N)(c + 2cN + 3N(-3 + 2N)), \\ c_{t\frac{9}{2}}^o &= -15cH' (21 + 4c)(-132 + 25c)(c + 6(-1 + N)N)(-6N^2 + c(-1 + 4N)), \\ H' &\equiv \frac{1}{90(6 + 5c)(21 + 4c)(c + 6(-1 + N)N)^2}. \end{aligned}$$

The structure constants in (G.3) for general N are given by

$$\begin{aligned} c_{t\frac{11}{2}}^o &= -90c^2 H(-1791 + c(615 + 4c(302 + 55c))), \\ c_{t\frac{11}{2}}^{o'} &= \frac{23c(21 + 4c)(c - 4cN + 6N^2)}{12(6 + 5c)(c + 6(-1 + N)N)}, \quad c_{t\frac{11}{2}}^{go} = -945c^2 H(-1041 + 4c(316 + 13c)), \\ c_{t6}^o &= 28c^2 H(25137 + c(-115275 - 20302c + 2120c^2)), \\ c_{t6}^{o'} &= -\frac{c(1059 + 176c)(-6N^2 + c(-1 + 4N))}{21(6 + 5c)(c + 6(-1 + N)N)}, \quad c_{t6}^{o''} = 4, \\ c_{t6}^{to} &= -18648c^2 H(24 + c)(-3 + 2c), \quad c_{t6}^{go} = 105c^2 H(3267 + c(-26439 + 4c(-2294 + 25c))), \\ H &\equiv \frac{(3 + c - 9N)(1 + N)(c + 6N)(-3c + 2(3 + c)N)}{35(29 + 2c)(21 + 4c)(6 + 5c)^2(c + 6(-1 + N)N)^2}. \end{aligned}$$

H The OPEs between $G(z)$ and the quasi-primary (or primary) fields

To find the correct superpartner in a given superfield, the OPEs between $G(z)$ and quasi-primary (or primary) fields should be computed. We list the OPEs between $G(z)$ and the quasi-primary (or primary) fields of the section 3 here:

$$\begin{aligned} G(z) \left(GO_2 - \frac{2}{5} \partial O_{\frac{5}{2}} \right) (w) &= \frac{1}{(z-w)^3} \frac{2}{15} (6 + 5c) O_2(w) + \mathcal{O}((z-w)^{-2}), \\ G(z) \left(TG - \frac{3}{8} \partial^2 G \right) (w) &= \frac{1}{(z-w)^3} \frac{1}{6} (21 + 4c) T(w) + \mathcal{O}((z-w)^{-2}), \end{aligned}$$

$$\begin{aligned}
 G(z) \left(TT - \frac{3}{10} \partial^2 T \right) (w) &= \frac{1}{(z-w)^4} \frac{51}{20} G(w) + \frac{1}{(z-w)^3} \left(-\frac{1}{3} \right) \frac{51}{20} \partial G(w) + \mathcal{O}((z-w)^{-2}), \\
 G(z) \left(TO_2 - \frac{3}{10} \partial^2 O_2 \right) (w) &= \frac{1}{(z-w)^3} \frac{19}{10} O_{\frac{5}{2}}(w) + \mathcal{O}((z-w)^{-2}), \\
 G(z) \left(GO_{\frac{5}{2}} - \frac{1}{5} \partial^2 O_2 \right) (w) &= \frac{1}{(z-w)^3} \frac{1}{15} (69 + 10c) O_{\frac{5}{2}}(w) + \mathcal{O}((z-w)^{-2}), \\
 G(z) \left(G\partial G - \frac{7}{10} \partial^2 T \right) (w) &= \frac{1}{(z-w)^4} \left(-\frac{1}{10} \right) (3 + 20c) G(w) \\
 &+ \frac{1}{(z-w)^3} \left(-\frac{1}{3} \right) \left(-\frac{1}{10} \right) (3 + 20c) \partial G(w) + \mathcal{O}((z-w)^{-2}), \\
 G(z) \left(\partial G O_2 - \frac{3}{4} G \partial O_2 + \frac{1}{8} \partial^2 O_{\frac{5}{2}} \right) (w) &= \frac{1}{(z-w)^4} (1 + 2c) O_2(w) \\
 &+ \frac{1}{(z-w)^3} \left(-\frac{1}{4} \right) (1 + 2c) \partial O_2(w) + \mathcal{O}((z-w)^{-2}), \\
 G(z) \left(TO_{\frac{5}{2}} - \frac{1}{4} \partial^2 O_{\frac{5}{2}} \right) (w) &= \frac{1}{(z-w)^4} 8 O_2(w) + \frac{1}{(z-w)^3} \left(-\frac{1}{4} \right) 8 \partial O_2(w) + \mathcal{O}((z-w)^{-2}), \\
 G(z) \left(T \partial O_2 - \partial T O_2 - \frac{1}{6} \partial^3 O_2 \right) (w) &= \frac{1}{(z-w)^4} \left(-\frac{3}{2} \right) O_{\frac{5}{2}}(w) \\
 &+ \frac{1}{(z-w)^3} \left[\left(-\frac{1}{5} \right) \left(-\frac{3}{2} \right) \partial O_{\frac{5}{2}} - 3 \left(G O_2 - \frac{2}{5} \partial O_{\frac{5}{2}} \right) \right] (w) + \mathcal{O}((z-w)^{-2}), \\
 G(z) \left(G \partial O_{\frac{5}{2}} - \frac{5}{3} \partial G O_{\frac{5}{2}} - \frac{1}{9} \partial^3 O_2 \right) (w) &= \frac{1}{(z-w)^4} \left(-\frac{1}{3} \right) (-3 + 10c) O_{\frac{5}{2}}(w) \\
 &+ \frac{1}{(z-w)^3} \left[\left(-\frac{1}{5} \right) \left(-\frac{1}{3} \right) (-3 + 10c) \partial O_{\frac{5}{2}} - 8 \left(G O_2 - \frac{2}{5} \partial O_{\frac{5}{2}} \right) \right] (w) + \mathcal{O}((z-w)^{-2}), \\
 G(z) \left(G O_{\frac{7}{2}} - \frac{1}{4} \partial O_4 \right) (w) &= \frac{1}{(z-w)^3} \frac{1}{6} (21 + 4c) O_{\frac{7}{2}}(w) + \mathcal{O}((z-w)^{-2}), \\
 G(z) \left(\partial^2 G O_2 + \frac{3}{5} G \partial^2 O_2 - 2 \partial G \partial O_2 - \frac{2}{35} \partial^3 O_{\frac{5}{2}} \right) (w) &= \\
 &\frac{1}{(z-w)^5} \left(\frac{32 + 280c}{35} \right) O_2(w) + \frac{1}{(z-w)^4} \left(-\frac{1}{2} \right) \left(\frac{32 + 280c}{35} \right) \partial O_2(w) \\
 &+ \frac{1}{(z-w)^3} \left[\left(\frac{1}{20} \right) \left(\frac{32 + 280c}{35} \right) \partial^2 O_2 - \frac{6}{5} \left(G O_{\frac{5}{2}} - \frac{1}{5} \partial^2 O_2 \right) + 4 \left(T O_2 - \frac{3}{10} \partial^2 O_2 \right) \right] (w) \\
 &+ \mathcal{O}((z-w)^{-2}), \\
 G(z) \left(T O_{\frac{7}{2}} - \frac{3}{16} \partial^2 O_{\frac{7}{2}} \right) (w) &= \frac{1}{(z-w)^3} \frac{17}{8} O_4(w) + \mathcal{O}((z-w)^{-2}), \\
 G(z) \left(T \partial O_{\frac{5}{2}} - \frac{5}{4} \partial T O_{\frac{5}{2}} - \frac{1}{7} \partial^3 O_{\frac{5}{2}} \right) (w) &= \\
 &\frac{1}{(z-w)^5} \left(-\frac{19}{7} \right) O_2(w) + \frac{1}{(z-w)^4} \left(-\frac{1}{2} \right) \left(-\frac{19}{7} \right) \partial O_2(w) \\
 &+ \frac{1}{(z-w)^3} \left[\left(\frac{1}{20} \right) \left(-\frac{19}{7} \right) \partial^2 O_2(w) - \frac{15}{4} \left(G O_{\frac{5}{2}} - \frac{1}{5} \partial^2 O_2 \right) + 8 \left(T O_2 - \frac{3}{10} \partial^2 O_2 \right) \right] (w) \\
 &+ \mathcal{O}((z-w)^{-2}), \\
 G(z) \left(G O_{4'} - \frac{2}{9} \partial O_{\frac{9}{2}} \right) (w) &= \frac{1}{(z-w)^3} \frac{2}{9} (20 + 3c) O_{4'}(w) + \mathcal{O}((z-w)^{-2}), \\
 G(z) \left(G A_4 - \frac{2}{9} \partial A_{\frac{9}{2}} \right) (w) &= \frac{1}{(z-w)^3} \frac{2}{9} (20 + 3c) A_4(w) + \mathcal{O}((z-w)^{-2}), \\
 G(z) \left(G O_4 - \frac{1}{8} \partial^2 O_{\frac{7}{2}} \right) (w) &= \frac{1}{(z-w)^3} \frac{1}{12} (93 + 8c) O_4(w) + \mathcal{O}((z-w)^{-2}), \\
 G(z) \left(G O_{\frac{9}{2}} - \frac{1}{9} \partial^2 O_{4'} \right) (w) &= \frac{1}{(z-w)^3} \frac{1}{9} (79 + 6c) O_{\frac{9}{2}}(w) + \mathcal{O}((z-w)^{-2}),
 \end{aligned}$$

$$\begin{aligned}
 G(z) \left(GA_{\frac{9}{2}} - \frac{1}{9} \partial^2 A_4 \right) (w) &= \frac{1}{(z-w)^3} \frac{1}{9} (79+6c) A_{\frac{9}{2}}(w) + \mathcal{O}((z-w)^{-2}), \\
 G(z) \left(T \partial^2 O_2 - \frac{5}{2} \partial T \partial O_2 + \partial^2 T O_2 - \frac{3}{28} \partial^4 O_2 \right) (w) &= \frac{1}{(z-w)^5} \left(\frac{69}{14} \right) O_{\frac{5}{2}}(w) \\
 &+ \frac{1}{(z-w)^4} \left[\left(-\frac{2}{5} \right) \left(\frac{69}{14} \right) \partial O_{\frac{5}{2}} + 9 \left(GO_2 - \frac{2}{5} \partial O_{\frac{5}{2}} \right) \right] (w) \\
 &+ \frac{1}{(z-w)^3} \left[\left(\frac{1}{30} \right) \left(\frac{69}{14} \right) \partial^2 O_{\frac{5}{2}} - \frac{9}{7} \partial \left(GO_2 - \frac{2}{5} \partial O_{\frac{5}{2}} \right) + 2 \left(TO_{\frac{5}{2}} - \frac{1}{4} \partial^2 O_{\frac{5}{2}} \right) \right. \\
 &\left. + \frac{58}{7} \left(\partial GO_2 - \frac{3}{4} G \partial O_2 + \frac{1}{8} \partial^2 O_{\frac{5}{2}} \right) \right] (w) + \mathcal{O}((z-w)^{-2}), \\
 G(z) \left(G \partial O_{\frac{7}{2}} - \frac{7}{3} \partial G O_{\frac{7}{2}} - \frac{1}{9} \partial^2 O_4 \right) (w) &= \frac{1}{(z-w)^4} \left(-\frac{7}{3} \right) (3+2c) O_{\frac{7}{2}}(w) \\
 &+ \frac{1}{(z-w)^3} \left(-\frac{1}{7} \right) \left(-\frac{7}{3} \right) (3+2c) \partial O_{\frac{7}{2}}(w) + \mathcal{O}((z-w)^{-2}), \\
 G(z) \left(TO_4 - \frac{1}{6} \partial^2 O_4 \right) (w) &= \frac{1}{(z-w)^4} \left(\frac{35}{2} \right) O_{\frac{7}{2}}(w) + \frac{1}{(z-w)^3} \left(-\frac{1}{7} \right) \left(\frac{35}{2} \right) \partial O_{\frac{7}{2}}(w) \\
 &+ \mathcal{O}((z-w)^{-2}), \\
 G(z) \left(TO_{4'} - \frac{1}{6} \partial^2 O_{4'} \right) (w) &= \frac{1}{(z-w)^3} \left(\frac{13}{6} \right) O_{\frac{9}{2}}(w) + \mathcal{O}((z-w)^{-2}), \\
 G(z) \left(TA_4 - \frac{1}{6} \partial^2 A_4 \right) (w) &= \frac{1}{(z-w)^3} \left(\frac{13}{6} \right) A_{\frac{9}{2}}(w) + \mathcal{O}((z-w)^{-2}), \\
 G(z) \left(G \partial^2 O_{\frac{5}{2}} - 4 \partial G \partial O_{\frac{5}{2}} + \frac{5}{2} \partial^2 G O_{\frac{5}{2}} - \frac{1}{14} \partial^4 O_2 \right) (w) &= \frac{1}{(z-w)^5} \left(\frac{93}{7} + 20c \right) O_{\frac{5}{2}}(w) \\
 &+ \frac{1}{(z-w)^4} \left[\left(-\frac{2}{5} \right) \left(\frac{93}{7} + 20c \right) \partial O_{\frac{5}{2}} - 24 \left(GO_2 - \frac{2}{5} \partial O_{\frac{5}{2}} \right) \right] (w) \\
 &+ \frac{1}{(z-w)^3} \left[\left(\frac{1}{30} \right) \left(\frac{93}{7} + 20c \right) \partial^2 O_{\frac{5}{2}} + \frac{24}{7} \partial \left(GO_2 - \frac{2}{5} \partial O_{\frac{5}{2}} \right) + 10 \left(TO_{\frac{5}{2}} - \frac{1}{4} \partial^2 O_{\frac{5}{2}} \right) \right. \\
 &\left. - \frac{200}{7} \left(\partial GO_2 - \frac{3}{4} G \partial O_2 + \frac{1}{8} \partial^2 O_{\frac{5}{2}} \right) \right] (w) + \mathcal{O}((z-w)^{-2}), \\
 G(z) \left(O_2 O_2 - \frac{3}{10} (-2\tilde{c} + c) \partial^2 O_2 + \frac{3}{10} (\tilde{c} - c) \partial^2 T \right) (w) \\
 &= \frac{1}{(z-w)^4} c_{g4}^g G(w) + \frac{1}{(z-w)^3} \left[-\frac{1}{3} c_{g4}^g \partial G + c_{g4}^o O_{\frac{5}{2}} \right] (w) + \mathcal{O}((z-w)^{-2}), \\
 G(z) \left(O_2 O_{\frac{5}{2}} - \frac{3}{4} c_1' \partial T G - \frac{3}{4} c_2' G \partial O_2 - \frac{1}{120} (2c_4' - 9c_1') \partial^3 G - \frac{1}{40} (4c_3' - 9c_2') \partial^2 O_{\frac{5}{2}} - \frac{3}{7} \partial O_{\frac{7}{2}} \right) (w) \\
 &= \frac{1}{(z-w)^4} \left[c_{g\frac{9}{2}}^t T + c_{g\frac{9}{2}}^o O_2 \right] (w) + \frac{1}{(z-w)^3} \left[-\frac{1}{4} c_{g\frac{9}{2}}^t \partial T - \frac{1}{4} c_{g\frac{9}{2}}^o \partial O_2 \right] (w) + \mathcal{O}((z-w)^{-2}), \tag{H.1}
 \end{aligned}$$

$$\begin{aligned}
 G(z) Q_{\frac{11}{2}}(w) &= \frac{1}{(z-w)^5} c_{g\frac{11}{2}}^o O_2(w) + \frac{1}{(z-w)^4} \left(-\frac{1}{2} \right) c_{g\frac{11}{2}}^o \partial O_2(w) \\
 &+ \frac{1}{(z-w)^3} \left[\frac{1}{20} c_{g\frac{11}{2}}^o \partial^2 O_2 + c_{g\frac{11}{2}}^{o'} O_4 + c_{g\frac{11}{2}}^{o''} O_{4'} + c_{g\frac{11}{2}}^{to} (TO_2 - \frac{3}{10} \partial^2 O_2) + c_{g\frac{11}{2}}^{go} \left(GO_{\frac{5}{2}} - \frac{1}{5} \partial^2 O_2 \right) \right] (w) \\
 &+ \mathcal{O}((z-w)^{-2}), \\
 G(z) Q_6(w) &= \frac{1}{(z-w)^5} c_{g6}^o O_{\frac{5}{2}}(w) + \frac{1}{(z-w)^4} \left[-\frac{2}{5} c_{g6}^o \partial O_{\frac{5}{2}} + c_{g6}^{o'} O_{\frac{7}{2}} + c_{g6}^{go} \left(GO_2 - \frac{2}{5} \partial O_{\frac{5}{2}} \right) \right] (w) \\
 &+ \frac{1}{(z-w)^3} \left[\frac{1}{30} c_{g6}^o \partial^2 O_{\frac{5}{2}} - \frac{1}{7} c_{g6}^{o'} \partial O_{\frac{7}{2}} - \frac{1}{7} c_{g6}^{go} \partial \left(GO_2 - \frac{2}{5} \partial O_{\frac{5}{2}} \right) + c_{g6}^{g o'} \left(\partial GO_2 - \frac{3}{4} G \partial O_2 + \frac{1}{8} \partial^2 O_{\frac{5}{2}} \right) \right. \\
 &\left. + c_{g6}^{to} \left(TO_{\frac{5}{2}} - \frac{1}{4} \partial^2 O_{\frac{5}{2}} \right) + c_{g6}^{o''} O_{\frac{9}{2}} \right] (w) + \mathcal{O}((z-w)^{-2}). \tag{H.2}
 \end{aligned}$$

The general forms of structure constants in (H.1) are given by

$$\begin{aligned}
c_{g4}^g &= -3c^2 H'(21+4c)(6+5c)(2c(-1+N) + 3(3-4N)N)(c+2cN+3N(-3+2N)), \\
c_{g4}^o &= 6cH'(21+4c)(6+5c)(-108(-1+N)N^3 + 10c^3(-1+N)(1+2N) \\
&\quad - 9cN(-2+N(57-98N+40N^2)) - 3c^2(1+N(-49+20N(2+N)))), \\
c_{g\frac{9}{2}}^t &= 4c^2 H'(6+5c)(-195+94c)(2c(-1+N) + 3(3-4N)N)(c+2cN+3N(-3+2N)), \\
c_{g\frac{9}{2}}^o &= 60cH'(21+4c)(3+16c)(c+6(-1+N)N)(-6N^2+c(-1+4N)), \\
H' &\equiv \frac{1}{90(6+5c)(21+4c)(c+6(-1+N)N)^2}.
\end{aligned}$$

The general forms of structure constants in (H.2) are given by

$$\begin{aligned}
c_{g\frac{11}{2}}^o &= -24c^2 H(-2493+10c(75+2c(739+10c))), \\
c_{g\frac{11}{2}}^{o'} &= \frac{5c(21+4c)(c-4cN+6N^2)}{42(6+5c)(c+6(-1+N)N)}, \quad c_{g\frac{11}{2}}^{o''} = \frac{8}{3}, \\
c_{g\frac{11}{2}}^{to} &= -84c^2 H(1953+100(-21+c)c), \quad c_{g\frac{11}{2}}^{go} = -630c^2 H(-3+2c)(-18+23c), \\
c_{g6}^o &= 140c^2 H(3078+c(-2709+c(-15777-3794c+40c^2))), \\
c_{g6}^{o'} &= \frac{c(201+46c)(c-4cN+6N^2)}{3(6+5c)(c+6(-1+N)N)}, \quad c_{g6}^{go} = -420c^2 H(-675+2c(-1731-724c+8c^2)), \\
c_{g6}^{go'} &= 8c^2 H(90207+c(-307725+2c(-44221+500c))), \\
c_{g6}^{to} &= 28c^2 H(7101+c(-31785+4c(-1829+25c))), \quad c_{g6}^{o''} = -\frac{8}{3}, \\
H &\equiv \frac{(3+c-9N)(1+N)(c+6N)(-3c+2(3+c)N)}{35(29+2c)(21+4c)(6+5c)^2(c+6(-1+N)N)^2}.
\end{aligned}$$

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