

On k-spanned projective surfaces

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This note can be considered as an appendix to [BS], since here we give an improvement of [BS], th.2.4.

First we recall a few notations. We work over the complex number field. Let $T \subset \mathbb{P}^N$ be a scheme of dimension 0; T is called curvilinear if it is contained in a smooth curve, or equivalently if it has embedding dimension at most 1. Let X be a complete variety embedded in a projective space by a linear subspace W of $H^0(S, L)$, $L \in \text{Pic}(X)$. (X, W) (or X if there is no danger of misunderstanding) is called k-spanned if for all curvilinear subschemes T of X with $\text{length}(T) = k+1$, the restriction map from W to $H^0(T, LT)$ is surjective. L is called k-spanned if $(X, H^0(X, L))$ is k-spanned.

Here we prove the following result.

Theorem *Let (S, W) be a k-spanned smooth surface with $k \geq 3$. Then $\dim(W) \geq k+5$.*

Proof. Set $P := P(W)$, hence $S \subset P$. Assume $w := \dim(W) \leq k+4$. Take a general hyperplane H of P and set $C := H \cap S$. Then C is a smooth, k-spanned curve in H . It is easy to check that the projection from a point of a smooth m-spanned curve, $m \geq 2$, is a smooth $(m-1)$ -spanned curve in the appropriate projective space. After $(k-2)$ general projections, we find a smooth 2-spanned curve Z in a projective space U , $\dim(U) \leq 4$. If $\dim(U) < 4$, this is a contradiction. Assume $\dim(U) = 4$, hence $w = k+4$. Let d, g be respectively the degree and genus of Z . Note that S has sectional genus g . By Castelnuovo's formula for the number of trisecant lines to a smooth curve in \mathbb{P}^4 ([LB], p.182) we get $d \leq 8$, $g \leq 5$, and that Z is linearly normal. But then C and S are linearly normal. In [BS], 5.1, 5.2, 5.3, there is a complete classification of all linearly normal surfaces with $k \geq 2$ and sectional genus $g \leq 5$. For the surfaces listed in [BS], §5, with $k \geq 3$, we have always $h^0(S, L) \geq k+5$, proving the theorem.

This note was born in the warm atmosphere of Max-Planck-Institut (Bonn).

References

- [BS] M. Beltrametti, A.J. Sommese: *On k-spannedness for projective surfaces*, preprint MPI/88 - 14.
- [LB] P. Le Barz: *Formule multi-secantes pour les courbes gauches quelconques*, in *Enumerative geometry and classical algebraic geometry*, p. 165-197, Progress in Math. **24**, Birkhäuser.