ON INDECOMPOSABLES IN PREPROJECTIVE COMPONENTS

Dieter Vossieck

Fakultät für Mathematik, Universität

D-4800 Bielefeld

We want to give a simple proof of the following result of Bongartz [1]. Note that this result implies, in particular, that each omnipresent (= sincere) indecomposable in a preprojective component actually is faithful. We simply write $(,)$ instead of $\text{Hom}_A(,)$.

Let A be a finite-dimensional k -algebra, U an indecomposable A -module contained in a preprojective Auslander-Reiten component C , and V an indecomposable A -module with minimal projective presentation.

$$(+)\quad P_1 \xrightarrow{P} P_0 \longrightarrow V \longrightarrow 0.$$

Then $(p, U) : (P_0, U) \longrightarrow (P_1, U)$ is injective or surjective.

Proof. The Auslander-Reiten translate τV is defined by means of the exact sequence

$$(-)\quad 0 \longrightarrow \tau V \longrightarrow vP_1 \xrightarrow{vP} vP_0,$$

where $v = D(, A)$. Applying $(, U)$ and $D(U,)$ to $(+)$ and $(-)$ respectively, and using the canonical isomorphisms $D(U, vP_i) \xrightarrow{\sim} (P_i, U)$, we get the following commutative diagram with exact rows:

$$\begin{array}{ccccccc} 0 & \longrightarrow & (V, U) & \longrightarrow & (P_0, U) & \xrightarrow{(p, U)} & (P_1, U) \\ & & & & \uparrow \wr & & \uparrow \wr \\ & & & & D(U, vP_0) & \xrightarrow{D(U, vp)} & D(U, vP_1) \longrightarrow D(U, \tau V) \longrightarrow 0. \end{array}$$

In case (p, U) is neither injective nor surjective, we have both $(V, U) \neq 0$ and $(U, \tau V) \neq 0$. Since C is a preprojective component, $(V, U) \neq 0$ and $U \in C$ imply $V \in C$. We obtain a non-trivial cycle $U \longrightarrow \dots \longrightarrow \tau V \longrightarrow * \longrightarrow V \longrightarrow \dots \longrightarrow U$ in C , in contrast to C being a preprojective component.

Reference

- [1] K. BONGARTZ: On omnipresent modules in preprojective components.
Comm. Algebra 11 (1983), 2125-2128.

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