(4) Conditions for integrability of certain equations J.Kurzweil

We consider systems of total differential equations of the form

$$(1) \quad \frac{\partial y_i}{\partial z_j} = H_{ij}(y, z), \quad i = 1, 2, \dots, m, \quad j = 1, 2, \dots, n,$$

$$y = (y_1, \dots, y_m), \quad z = (z_1, \dots, z_n),$$

or, briefly, $dy = Hdz$, and give a new proof of the following result due to P. Hartman [1,2,3]:

Assume that H_{ij} are continuous. For a differential 1 - form ξ and 2 - form ω (both with continuous coefficients) write $\omega = d\xi$ if the Stokes formula $\int_{\partial B} \xi = \int_B \omega$ holds for all rectangles B . For any such ξ there exists at most one $d\xi$. For a continuous matrix $A(y, z) = (A_{ij}(y, z))$ put $\xi_i = \sum_j A_{ij}(dy_j - \sum_k H_{jk} dz_k)$, i.e. $\xi = A(dy - Hdz)$.

THEOREM. (1) is integrable if and only if there exists an A such that ξ_i have continuous derivatives for $i = 1, 2, \dots, m$ and $\xi_1 \wedge \xi_2 \wedge \dots \wedge \xi_m \wedge d\xi_i = 0$ for $i = 1, 2, \dots, m$.

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- [1] Hartman, P. Ordinary Differential Equations, J. Wiley and Sons, New York, 1964.
- [2] Hartman, P. On exterior derivatives and solutions of ordinary differential equations, Trans. A.M.S. 91 (1959) 277-292.
- [3] Hartman, P. On uniqueness and differentiability of solutions of ordinary differential equations, Proc. Symposium on Nonlinear Problems, Madison (Wis) 1963, 255-259.

(5) Formalisme Lagrangien

C. Godbillon

Les équations de Lagrange se transformant de façon contravariante. Elles sont donc associées à un champ de vecteurs sur l'espace tangent $T(M)$ à la variété de configuration M . La