Section 13 Examples of Maximal Valuation Rings

The main theorem 9.1 states that the FGC rings are exactly the rings which are finite direct products of maximal valuation rings, almost maximal Bezout domains, and torch rings. The next three sections present examples of the indecomposable FGC rings of these three types.

We begin by considering the maximal valuation domains. Fields are maximal valuation domains. Besides fields, perhaps the best known examples of maximal valuation domains are the p-adic integers, for p a prime integer. The divisibility group of the p-adic integers is order isomorphic to Z with the standard total ordering, and the residue field of the p-adic integers is isomorphic to the field Z/pZ. If R is the long power series ring relative to Z/pZ and Z, then it is not hard to see that R is isomorphic to the p-adic integers.

More generally long power series rings are maximal valuation domains by 11.5; and given any totally ordered group and any field, there is a long power series ring with that divisibility group and that residue field by 11.4(3). For examples of maximal (i.e., maximally complete) valuation domains other than long power series rings, the reader is refered to the paper by I. Kaplansky [13] or the text by O. Schilling [26].

Examples of maximal valuation rings which are not domains, include R/I for R a maximal valuation domain and I an ideal of R, I not a prime ideal of R. For example Z/p^nZ for p a prime integer and $n \in N - \{1\}$. Other examples include quotients of long power series rings (of which Z/p^nZ is a special case).