SQ-Universal 1-Relator Groups

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A countable group G is SQ-universal if for each countable group H , there is a quotient Q of G in which H can be embedded.

A group is a 1-relator group if it has a presentation by generators and relations involving only one relation.

Theorem: Let G be a countable 1-relator group. Then either G is SQ-universal or else G is one of the following: (i) cyclic
(ii) metabelian and isomorphic to $\langle a, b; a^{-1}b^ma = b^n \rangle$
where either $|m|=1$ or $|n|=1$.

Corollary: The Baumslag-Solitar group $\langle a, b; a^{-1}b^2a = b^3 \rangle$ is SQ-universal.

A group is deficient if it has a presentation in which the number of generators exceeds the number of relations by at least two. Clearly a deficient group with zero relations is SQ-universal, for it is free. By the preceding theorem a deficient group with one relation is SQ-universal.

Conjecture (Peter Neumann): Every finitely generated deficient group is SQ-universal.