Compositional Verification of Reactive Systems Specified by Graph Transformation^{*}

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Abstract. A loose semantics for graph transformation rules which has been developed recently is used in this paper for the compositional verification of specifications. The main conceptual tool here is the notion of *view*, that is, an incomplete specification describing only a certain aspect of the overall system. A view anticipates the (potential) behavior of the complete system by its loose semantics. This ensures that properties of the view are inherited by the complete system.

Based on this result one may verify temporal properties by decomposing a specification into several views, analyzing them separately, and deriving the desired property from properties shown for the views.

1 Introduction

One of the most challenging problems in specifying reactive systems is the complexity of the development and verification of large specifications. An important approach to solve this problem is the separation of different aspects or *views*. They may be used to split a team of developers into subgroups specifying only that aspect of a system which is later responsible for a certain task or is seen and used by a certain type of user. In the software engineering field, this view-oriented approach is known by the notion of viewpoints (cf. e.g., [FKN⁺92]).

Reactive systems in the sense of [MP92] are distinguished by the complex interaction between the system and its environment (as opposed to classical programs which basically transform input to output). They include concurrent, object-oriented, or distributed systems like operating systems, process control systems, seat reservation systems, etc., typically consisting of several components which can be seen as reactive systems themselves.

Probably, the most natural representation of such a system is a *graph*, where nodes represent components (objects, processes, ...) and edges communication links. If the system's topology is dynamic (since e.g., components are created and deleted) some sort of *graph transformation* has to be performed. Among the

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various possibilities of specifying the transformation of graphs, I prefer a rule-based approach which exploits the visual character of the graph representation for describing transformations. In particular, the “algebraic, Double Pushout (DPO) approach” is applied [EPS73,CMR⁺97], which owes its name to the basic construction of a direct derivation step: This is modeled indeed by two gluing diagrams (i.e., pushouts) in the category of graphs and total graph morphisms.

In [EHTE97] a specification technique based on graph transformations has been presented which supports a view-based development approach. Since each view specifies only a part of the operations, it may be that a view operation is executed in parallel with operations of other views. Thus, a view operation specifies only what *at least* has to happen on a system’s state, but it allows also additional effects which may be caused by the environment. In this sense, the semantics of a view is a loose one, in contrast to the semantics of the complete specification. A loose operational semantics for graph transformation rules, called *graph transitions* as opposed to direct derivations, has been developed in [HEWC97b] and used in [EHTE97] as the semantics of views.

Here, this loose semantics is exploited for compositional reasoning about reactive systems. The hope is that a meaningful decomposition of a specification into views reduces the complexity of the verification and, which seems equally important, that it allows to concentrate on the verification of the most important (e.g., safety-critical) aspects.

The general idea of compositionality through loose semantics can be traced back to several sources. In the area of algebraic specification a class of algebras forms the loose semantics of an incomplete specification, as opposed to a single (initial) algebra associated to a complete specification. Extending a specification (e.g., by adding equations) reduces its class of models [EM85]. Similar to the present approach is [MP92], where a loose interpretation of assignments in a shared variable languages (allowing for unspecified change of values) is used for compositional proofs of temporal properties.

The approach here is to verify temporal properties of a big specification by decomposing it into views and analyzing them separately. Then, the properties of the big specification are derived from properties of the views. The verification of properties of views is not addressed in this paper. The main technical result shows the correctness of this compositional verification approach, that is, temporal properties of the views remain valid in the complete specification. The technique is applied to a small case study specifying an algorithm for mutual exclusion and deadlock detection in a distributed system with dynamic topology.

2 Specifying a Distributed Algorithm with Graph Transformation Rules

The following specification of a distributed mutual exclusion (MUTEX) algorithm shall illustrate the expressive power of rule-based graph transformation and serves as running example throughout the paper. The ideas are standard in distributed computing and have been communicated to me through [Sch94].

Along with this specification the basic notions of the algebraic DPO approach to graph transformation [EPS73,CMR+97] are introduced.

Distributed MUTEX algorithms are used in (distributed) operating systems and data base systems. Their task is to ensure that a resource is never used by more than one process at a time. However, each request of a process for a resource must eventually be granted. Here the main problem is to prevent processes from running into a deadlock.

Graph transformation rules are used for specifying a MUTEX algorithm with deadlock detection for a variable number of processes and resources. The specification is presented in three views, the *system view* (SYS) specifying the creation and deletion of processes and resources, the *token ring view* (TR) ensuring mutual exclusion, and the *distributed deadlock detection view* (DDD) detecting and announcing deadlocks. All views share a common graph signature specifying the structure of system states. They use disjoint sets of graph transformation rules for describing the evolution of the system.

A *graph signature* is an algebraic signature $GS = \langle S, OP \rangle$ having only unary operation symbols $op : s \rightarrow s'$.¹ GS -Algebras and $-$ homomorphisms are called *graphs* and *graph morphisms*, respectively.² The category of graphs and graph morphisms for a signature GS is denoted by $Gr(GS)$. In the MUTEX specification *type graphs* (in the sense of e.g., [HCEL96]) are used as a more intuitive, graphical notation for signatures.

Fig. 1. Type graph of *MUTEX* (left) and deadlock state (right)

Example 1. The type graph of the *MUTEX* specification is shown in the left of Figure 1. It may be read like an entity/relationship schema specifying the node and edge types which may occur in graphs modeling system states. *Processes* are drawn as black nodes and *resources* as light boxes. An edge from a process to a resource models a *request*. A solid edge in the opposite direction shows that the resource is currently *held_by* the process. A dashed edge from a resource to a process asks the process to *release* the resource.

The token ring is a cyclic list of processes, where an edge between two processes points to the *next* process. For each resource there is a *token*, represented

¹ Some basic notions of universal algebra are assumed, like *signature* Sig , Sig -algebra A and Sig -homomorphism $h : A \rightarrow B$, *subsignature* $Sig' \subseteq Sig$, and Sig' -reduct $(A)_{Sig'}$ and $(h)_{Sig'}$, see e.g., [EM85].

² Graphs of this kind have been mainly used in the algebraic single-pushout approach under the name of *graph structures* [Lw93]. They include, beside ordinary graphs, also more general kinds of structures like hyper-graphs or higher-order graphs.

by an edge with a white flag, which is passed from process to process along the ring. If a process wants to use a resource, it waits for the corresponding token. Mutual exclusion is ensured because there is only one token for each resource in the system. The distributed deadlock detection uses *blocked* messages in order to detect cyclic dependencies. They are represented by edges with a black flag from a resource to a process.

The graph signature $GS_{MUTEX} = \langle S, OP \rangle$ has as sorts the nodes and edges of the type graph, i.e., the node sorts P and R and the edge sorts *request*, *held_by*, *release*, *next*, *token*, and *blocked*. The operations of the signature are the obvious *source* and *target* assignments from edge sorts to node sorts, like *source* : *request* $\rightarrow P$ and *target* : *request* $\rightarrow R$. A graph over this signature is shown in the right of Figure 1. It represents a system state with two processes and two resources, where each process requests one resource while holding the other one, i.e., both processes are deadlocked.

Graph transformation rules (or productions) according to the DPO approach are specified by spans of injective graph morphisms $sp = (L \xleftarrow{l} K \xrightarrow{r} R)$, called *production spans*. The *left-hand side* L and the *right-hand side* R represent pre- and postconditions, i.e., items which have to be present before and after the application. The *interface graph* K specifies the objects which are required and preserved, that is, the “intersection” of L and R .

Example 2. The rules of the MUTEX specification are given in Figure 2. In the upper part, the SYS-rules are shown modeling the operations *new* and *kill*(p) for inserting and removing processes to and from the ring, and the operations *mount* and *unmount*(r) for creating a resource with a token, and deleting it. The interface graph K is omitted. It can be reconstructed from the graphical layout of productions: Items that appear in the same position in the left- and right-hand side are usually preserved and belong to the interface.

The complete representation of the production *new* is given by the top span of the left diagram in Figure 3 which shows an application modeling the insertion of a new process in the ring. Given the match $m : L \rightarrow G$ which maps the processes 1 and 2 to the process in G and the edge in between to the loop, the application consists of two steps: The objects of G matched by $L - l(K)$ are removed which leads to the graph D without loop. Then, the objects of $R - r(K)$ are added to D leading to the derived graph H . Notice that the application deletes and creates exactly what is specified by the production: There is an *implicit frame condition* ensuring that everything that is not rewritten explicitly by the production is left unchanged.

The gluing of L and D over K is again the given graph G , i.e., the left-hand square (1) of d forms a so-called *pushout complement*. Only in this case the application is permitted. Similarly, the derived graph H is the gluing of D and R over K , which creates the right-hand side pushout square (2). The resulting *double-pushout (DPO) diagram* d represents the transformation of G into H , denoted $G \xRightarrow{d} H$. Since this diagram is symmetric, it can also be read from

Fig. 2. The MUTEX-productions of the SYS-, TR- and DDD-view (from top to bottom).

Fig. 3. A sample direct DPO derivation using *new* (left), and a generic double-pushout (resp., double-pullback) diagram *d* (right).

right to left, as *inverse transformation*. The corresponding *inverse production* *span* is $sp^{-1} = (R \xleftarrow{r} K \xrightarrow{l} L)$.

The existence of the pushout complement (1), and hence of a direct derivation³ $G \xRightarrow{d} H$, is characterized by the *gluing conditions*: The *dangling condition* ensures that the structure D obtained by removing from G all objects to be deleted is indeed a graph, that is, no edges are left “dangling” without source or target node. The *identification condition* states that objects from the left-hand side may only be identified by the match if they also belong to the interface (and are thus preserved).

Example 3. The TR-rules are shown in the middle of Figure 2, where $pass(p, r)$ describes that a process having the token may pass it to the next process in the ring, provided that it does not have a request on the corresponding resource. This *negative application condition* is visualized by the crossed-out *request* edge from the process to the resource, see [HHT96] for a formalization of this concept. If a process wants to use a resource, it may generate a request. This is modeled by the rule $req(p, r)$, which is only applicable if the process does not have any requests yet, and if the particular resource is not used already by this process. If a process receives a token and there *is* a request for the resource, the process will chose the rule $take(p, r)$ replacing the token and the request by a *held_by* edge from the resource to the process. When it has finished its task, the process may release its resource and give the token to the next process using $rel(p, r)$ and $give(p, r)$. This will happen only when there are no pending requests, which is modeled by a negative application condition at $rel(p, r)$.

The algorithm for distributed deadlock detection specified by the DDD-rules in the lower part is a variant of [CMH83]: In a graph representing a state, a deadlock is represented as a cycle of *request* and *held_by* edges. In order to detect such cycles in a distributed way, a process holding a resource requested by another process will use the rule $blocked(p, r)$ to send a *blocked*-message to that process. If this process itself blocks another one, he will use the $waiting(p, r)$ rule to pass this message on. The inequation in this rule, which is also a negative application condition in the sense of [HHT96], ensures that resource r is not the original one. If the process does not hold any resource, it deletes the message with $ignore(p, r)$.

Thanks to the mutual exclusion, each resource is held by only one processes. Hence, if the message arrives at a process who holds the original resource, this is the original sender of the message. The deadlock thus detected is broken by the $unlock(p, r)$ rule which replaces the *held_by* edge and the *blocked* message with a *release* edge, asking the TR-view to give the token to the next process.

3 Temporal Specification of Graph Transition Systems

Graph transition systems are labeled transition systems whose states are graphs. In [HEWC97a] they are defined for a *graph grammar* (i.e., a set of productions

³ The pushout (2) always exists since the category of graphs over GS is cocomplete.

with a start graph) using coalgebraic techniques. Following [GR97] in this section graph transition systems are defined for a *signature*, independently of their generation by productions. Like in [HEWC97a] a temporal logic is developed for specifying graph transition systems axiomatically.

The common base for graph transition systems, temporal formulas, and graph transition specifications (in the next section) are *graph transition signatures*:

Definition 4 (graph transition signature). A *graph transition signature* $GTSig = \langle GS, C, P, Q \rangle$ consists of a graph signature GS , a *global context graph* C over GS , and two families of sets $P = (P_w)_{w \in S^*}$ and $Q = (Q_w)_{w \in S^*}$ of *production names* and *state formula symbols*. For $p \in P_v$ and $q \in Q_w$ I write $p : v$ and $q : w$.

Given an S -indexed family of sets $A = (A_s)_{s \in S}$, the set of *transition expressions over A* is defined by $\mathcal{TE}_P(A) = \{p(a) | p : w, a \in A_w\}$. Similarly, the set of *state formulas over A* is defined by $\mathcal{SF}_Q(A) = \{q(a) | q : w, a \in A_w\}$.

Example 5. The graph transition signature of the MUTEX algorithm is $GTSig_{MUTEX} = \langle GS_{MUTEX}, CMUTEX, P_{SYS} \cup P_{TR} \cup P_{DDD}, Q_{MUTEX} \rangle$ where

- GS_{MUTEX} is the graph signature introduced in Example 1
- $CMUTEX = \langle (C_s)_{s \in S}, (op^C)_{op \in OP} \rangle$ is a “complete graph structure” of size $\mathbb{N}$, that is, $C_s = \mathbb{N}$ for all node sorts s and $C_{s'} = \mathbb{N} \times \mathbb{N} \times \mathbb{N}$ for all edge sorts s' . The *source* and *target* mappings from node to edge sorts are given, respectively, by the first and third projection. Hence $CMUTEX$ has for each node type all natural numbers as nodes, and for each edge type the set of natural numbers as edges between every pair of nodes.
- $P_{SYS} = \{new, kill : P, mount, unmount : R\}$,
 $P_{TR} = \{pass : PR, req : PR, take : PR, rel : PR, give : PR\}$, and
 $P_{DDD} = \{blocked : PR, waiting : PR, ignore : PR, unlock : PR\}$.
- $Q_{MUTEX} = \{no_request : PR, no_request : P, not_held_by : RP, \dots\}$

The global context graph C provides a common name space for the states, formally represented as injective graph morphisms $g : G \rightarrow C$. The set of all states is denoted by S_C . Transitions are labeled by sets of transition expressions representing parallel transitions. For evaluating temporal formulas, a valuation $\mathcal{V}$ of state formulas over C is assumed.

Definition 6 (graph transition system). A *graph transition system* over a signature $GTSig = \langle GS, C, P, Q \rangle$ is a triple $GTSys = \langle \mathcal{V}, \rightsquigarrow, \mathcal{R} \rangle$ where

- $\mathcal{V} : \mathcal{SF}_Q(C) \rightarrow \mathcal{P}(S_C)$ is a *valuation* assigning to each state formula $q(c)$ over C a class $\mathcal{V}(q(c))$ of states (at which $q(c)$ is assumed to be true)
- $\rightsquigarrow \subseteq S \times \mathcal{P}_f(\mathcal{TE}_P(C)) \times S$ is a labeled transition relation⁴, where $\langle g, L, h \rangle \in \rightsquigarrow$ is denoted by $g \xrightarrow{L} h$.
- $\mathcal{R}$ is a class of *runs*, i.e., infinite sequences $\rho : g_0 g_1 g_2 \dots$ of states over C such that for all $i \in \mathbb{N}$ there exists a transition $g_i \xrightarrow{L_i} g_{i+1}$.

⁴ For a set A , $\mathcal{P}_f(A)$ denotes the set of finite subsets of A .

Graph transition systems can be specified axiomatically (via their runs) by temporal logic formulas. Assume in the following definitions a graph transition signature $GTSig = \langle GS, C, P, Q \rangle$.

Definition 7 (temporal formulas). Given a family of sets of variables $X = (X_s)_{s \in S}$, the syntax of *temporal formulas over X* has the form

$$\Phi ::= q(x) \mid \neg\Phi \mid \Phi_1 \wedge \Phi_2 \mid \exists y. \Phi \mid \Diamond\Phi$$

where $q(x)$ is a state formula over X , and $y \in X$ is a variable. A *temporal formula with variables* is a pair $X : \Phi$ where Φ is a temporal formula over X . The set of all such formulas is $\mathcal{TF}_{GTSig}$.

Here, $\Diamond$ is the usual *sometime* operator of temporal logic. The *always* operator is defined by: $\Box\Phi$ iff $\neg\Diamond\neg\Phi$. The usual definitions are applied for the propositional operators $T, F, \vee$, and $\implies$, and for the universal quantification $\forall$.

Satisfaction of temporal formulas is defined for a *temporal model* $M = \langle GTSys, ass \rangle$ over $GTSig$ consisting of a graph transition system $GTSys = \langle \mathcal{V}, \rightsquigarrow, \mathcal{R} \rangle$ over $GTSig$, and an *assignment* $ass : X \rightarrow C$.

Definition 8 (satisfaction). Let $M = \langle GTSys, ass \rangle$ be a temporal model. The satisfaction of temporal formulas $X : \Phi \in \mathcal{TF}_{GTSig}$ by runs $\rho \in \mathcal{R}$ is defined inductively as follows:⁵

- $\rho \models_M q(x)$ iff $\rho(0) \in \mathcal{V}(q(ass(x)))$
- $\rho \models_M \neg\Phi$ iff $\rho \not\models_M \Phi$,
- $\rho \models_M \Phi_1 \wedge \Phi_2$ iff $\rho \models_M \Phi_1$ and $\rho \models_M \Phi_2$,
- $\rho \models_M \exists y. \Phi$ iff there is an assignment ass' s.t. $ass'(x) = ass(x)$ for all variables $x \neq y$ and $\rho \models_{\langle GTSys, ass' \rangle} \Phi$
- $\rho \models_M \Diamond\Phi$ iff there is $k \in \mathbb{N}$ such that $\rho_k \models_M \Phi$.

The model M satisfies $X : \Phi$, written $\models_M X : \Phi$, if $\rho \models_M \Phi$ for all runs $\rho \in \mathcal{R}$. A transition system $GTSys$ satisfies $X : \Phi$, written $\models_{GTSys} X : \Phi$, if $\models_M X : \Phi$ for all models $M = \langle GTSys, ass \rangle$.

Example 9. The temporal specification of the MUTEX algorithm consists of the *safety property* *MUTEX-safe*: $\Box[held_by(r, p_1) \wedge held_by(r, p_2) \implies p_1 = p_2]$ which states that a resource must not be held by two different processes, and the *liveness property* *MUTEX-live*: $\Box[requests(p, r) \implies \Diamond held_by(r, p)]$ saying that each request will eventually be served. The set of state formulas is extended by equations and edge predicates, like $edge_sort : node_sort_1 node_sort_2$, with the obvious interpretations.

The MUTEX liveness property is decomposed according to the different tasks performed by the TR- and the DDD-view. The latter has to announce all permanent deadlocks by a *release* edge, that is, *DDD-live* reads:

$$\Box[deadlocked(p_1) \implies \exists p_2, r. path(p_1, p_2) \wedge \Diamond[release(r, p_2) \vee \neg deadlocked(p_1)]]$$

⁵ $\rho(i) = g_i$ denotes the i th state and $\rho_i = g_i g_{i+1} \dots$ the i th suffix of ρ .

Here $path(p_1, p_2)$ is true in a state if the processes p_1 and p_2 are connected by a path of *request* and *held.by* edges, and $deadlocked(p)$ is equivalent to $path(p, p)$. Notice that the *release* edge may point to any process in the deadlock cycle.

The liveness property of the TR-view is that each request will eventually be served, provided that deadlocks are always detected and the number of processes is bounded, i.e., $TR\text{-live} : \Box bounded(n) \wedge DDD\text{-live} \implies MUTEX\text{-live}$. Here $bounded(n)$ is true in a state if the number of P nodes does not exceed n .

4 A Loose Semantics for Graph Productions

A graph transition system (or rather its transition relation) can be specified constructively by a set of graph productions (plus a set of initial states and some additional constraints). One can ask if this specification is *correct* w.r.t. a certain temporal formula by checking the validity of this formula in the transition system. If the specification is incomplete like the DDD-view which describes only a particular aspect of the MUTEX algorithm, the closed behavior of this view given by the classical DPO interpretation is rather poor. Starting for example with the deadlock state in Figure 1 on the right, the DDD-productions could (at most) replace two *held.by* edges by *release* edges, and stop. Properties verified for this transition system are obviously not very interesting and of little use for the verification of the MUTEX algorithm.

One way out is, of course, to verify the DDD-view in the context of the SYS- and TR-view, but this would contradict the aim of compositional verification. The solution I propose in this paper is a loose semantics of graph productions which anticipates the effects of applying them in a bigger context. So-called *graph transitions* will ensure that an application preserves, deletes, and adds at least as much as it is specified by the productions, but it permits also additional effects which may be caused, in the example, by concurrent application of the SYS- and TR-productions. Thus the implicit frame condition is dropped. Instead, *explicit frame conditions* are introduced which protect only particular parts of the graphs from unspecified changes.

Definition 10 (graph transition). A *sequential graph transition* is defined by replacing the double-pushout diagram of direct derivations with a *double-pullback (DPB) diagram*, that is, a diagram d like in the right of Figure 3 where (1) and (2) are pullbacks. This diagram represents a transition from G to H with production span $sp = (L \leftarrow K \rightarrow R)$ and bottom span $t = (G \leftarrow D \rightarrow H)$, denoted by $G \xrightarrow{d} H$.

The *parallel production span* $Par(Sp)$ of a finite set Sp of production spans $sp = (L_{sp} \leftarrow K_{sp} \rightarrow R_{sp})$ is constructed as component-wise disjoint union (coproduct)

$$Par(Sp) = \sum_{sp \in Sp} L_{sp} \xleftarrow{l} \sum_{sp \in Sp} K_{sp} \xrightarrow{r} \sum_{sp \in Sp} R_{sp}$$

A graph transition using a parallel production span is called *parallel graph transition*.

Each pushout in $\mathcal{Gr}(GS)$ where two opposite morphisms are injective is also a pullback. Hence, graph transitions generalize direct derivations defined by double-pushouts. From an operational point of view a span $G \xleftarrow{l^*} D \xrightarrow{r^*} H$ represents a transition where $G - l^*(D)$ is deleted, $l^*(D) \subseteq G$ is preserved as $r^*(D) \subseteq H$, and $H - r^*(D)$ is newly created. Then, referring to the right diagram of Figure 3:

- Commutativity of (1) and (2) ensures that the image of K in G is preserved in D and H , i.e., $m(l(K)) \subseteq l^*(D)$ and $m^*(r(K)) \subseteq r^*(D)$.
- Pullback property of (1) ensures that at least every image of $L - l(K)$ in G is deleted, i.e., $m(L - l(K)) \cap l^*(D) = \emptyset$.
- Pullback property of (2) ensures that at least every image of $R - r(K)$ in H is newly created, i.e., $m^*(R - r(K)) \cap r^*(D) = \emptyset$.

Example 11. A sample graph transition is shown in the left of Figure 4. It applies the TR-production *take(p, r)* replacing a request and the corresponding token by a *held.by* edge. Meanwhile, another process is inserted in the ring, which is not specified by the production but permitted by the loose semantics. In fact, none of the two squares is a pushout: The given graph G adds to the gluing of L and D a *next* loop which is “spontaneously deleted”, and in the derived graph H , an additional process with two *next* edges has been “spontaneously created”. This effect could be obtained by applying production *new* in parallel.

Fig. 4. A sample graph transition (left) and a graph transition not satisfying the dangling condition (right).

It is worth stressing that graph transitions may not only have additional effects but are also more likely to exist than DPO derivations. In general, the match m of a DPB diagram may satisfy neither the identification nor the dangling condition of the corresponding production, and so does the comatch m^* . Consider for example the transition in the right of Figure 4, using the SYS-production *kill(p)*, where the match does not satisfy the dangling condition: Process 2 is removed from the ring while still holding the resource. Here deleting the dangling *held.by* edge is an unspecified effect. Symmetrically, it would be possible to attach edges to newly created processes (for example in the inverse transition).

Motivated by these observations *safe* transitions are introduced:

Definition 12 (safe transitions). A graph transition $G \xrightarrow{d} H$ like in the right of Figure 3 is *safe* if m satisfies the gluing conditions w.r.t. $sp = (L \leftarrow K \rightarrow R)$, and m^* satisfies the gluing conditions w.r.t. $sp^{-1} = (R \leftarrow K \rightarrow L)$.

In the following all transitions are assumed to be safe.

Hence, a safe graph transition exists if and only if there is also a direct DPO derivation with the same match.

In order to ensure that, e.g., processes and resources are only created and deleted by SYS-productions, one may declare certain sorts as *input* and/or *output sorts*. Only elements of such sorts may be spontaneously created and deleted. Sorts that are neither input nor output are called *private* since they are protected from the influence of the environment.

Definition 13 (explicit frame condition). An *explicit frame condition* over GS is a pair of sets $FC = \langle IS, OS \rangle$ of *input* and *output sorts* $IS, OS \subseteq S$. Denote by $\overline{IS}$ ($\overline{OS}$) the biggest subsignature of GS not containing input (output) sorts. Then, a graph transition $G \xrightarrow{d} H$ *respects* IS (OS) if the reduct of the left-hand side (right-hand side) pullback square of d to $\overline{IS}$ ($\overline{OS}$) is a pushout.⁶

If all sorts are private (that is, $FC = \langle \emptyset, \emptyset \rangle$), transitions are reduced to direct derivations. In this way, the classical DPO interpretation of productions becomes a special case of the loose one.

As constructive specification of a graph transition system, *production definitions* interpret production names by spans with application conditions. Together with frame conditions, initial graphs, and fairness conditions for runs, this forms a graph transition specification.

Definition 14 (graph transition specification). A *graph transition specification* over a signature $GTSig = \langle GS, C, P, Q \rangle$ is a five-tuple $GTSpec = \langle \mathcal{V}, \mathcal{PD}, FC, I, F \rangle$ where $\mathcal{V} : \mathcal{SF}_Q(C) \rightarrow \mathcal{P}(S_C)$ is a valuation of state formulas, and

- $\mathcal{PD}$ is a set of *production definitions* of the form $p(a) : sp \text{ if } AC$, where $sp = (L \leftarrow K \rightarrow R)$ is a production span, $p(a) \in \mathcal{TE}_P(L)$ is a transition expression over L , and $AC \subseteq \mathcal{SF}_Q(L)$ is a set of state formulas over L representing application conditions.
- $FC = \langle IS, OS \rangle$ is a frame condition
- $I \subseteq S_C$ is a set of *initial states*
- $F = \langle weak, strong \rangle$ is a *fairness condition*⁷ where $weak, strong \subseteq \mathcal{TE}_P(C)$

⁶ Here, the reduct is extended from graphs and graph morphisms to diagrams in $Gr(GS)$. This is possible because it forms a functor, called *forgetful functor* in [EM85]. It follows from the component-wise construction of pullbacks and pushouts in $Gr(GS)$ that their reduct is a pullback or pushout again. Hence, it preserves direct derivations as well as transitions.

⁷ In [MP92] weak and strong fairness are called *justice* and *fairness*, respectively.

Example 15. The graph transition specification $MUTEX = \langle \mathcal{V}_{MUTEX}, \mathcal{PD}_{SYS} \cup \mathcal{PD}_{TR} \cup \mathcal{PD}_{DDD}, FC_{MUTEX}, IM_{MUTEX}, FM_{MUTEX} \rangle$ is given by

- the obvious valuation $\mathcal{V}_{MUTEX}$ of state formula symbols, e.g., $g : G \rightarrow C \in \mathcal{V}_{MUTEX}(no_request(p, r))$ iff there is no *request* edge from p to r in $g(G)$.
- $\mathcal{PD}_{SYS}$, $\mathcal{PD}_{TR}$, and $\mathcal{PD}_{DDD}$ are given, respectively, by the productions in the upper, middle and lower part of Figure 2. The second TR-production, e.g., leads to the definition $req(p, r) : sp_{req} \text{ if } \{no_request(p), not_held_by(r, p)\}$.
- $FC_{MUTEX} = \langle \emptyset, \emptyset \rangle$ that is, the specification is considered as complete.
- IM_{MUTEX} is the set of all states $g : G_0 \rightarrow C$ where G_0 is the graph in the left of Figure 3, which is put in all possible naming contexts.
- $FM_{MUTEX} = \langle weak, strong \rangle$ where $weak = \mathcal{TE}_{P_{TR} \cup P_{DDD}}(C)$ and $strong = \mathcal{TE}_{\{rel:PR, unlock:PR\}}(C)$

A transition expression $p(c) \in \mathcal{TE}_P(C)$ is *enabled* by a state g if there is a transition $g \xrightarrow{L} h$ with $p(c) \in L$. Then, a run $\rho : g_0 g_1 g_2 \dots$ satisfies the fairness condition $F = \langle weak, strong \rangle$ if

Weak Fairness: For all $p(c) \in weak$ and $k \in \mathbb{N}$: if for an $i \geq k$, g_i enables $p(c)$, then there exists $n \geq k$ such that $g_n \xrightarrow{L} g_{n+1}$ with $p(c) \in L$.

Strong Fairness: For all $p(c) \in strong$ and $k \in \mathbb{N}$: if there exist infinitely many $i \geq k$ where g_i enables $p(c)$, then there exist infinitely many $n \geq k$ such that $g_n \xrightarrow{L} g_{n+1}$ with $p(c) \in L$.

Hence, the weak fairness condition of FM_{MUTEX} above says that a TR- or DDD-transition which is continuously enabled must eventually take place. The strong fairness condition implies that transitions using *rel* : *PR* or *unlock* : *PR* which are enabled infinitely many times, must be taken infinitely often.

Each graph transition specification generates a transition systems:

Definition 16 (generated transition system, correctness). The transition specification $GTSpec = \langle \mathcal{V}, \mathcal{PD}, FC, I, F \rangle$ over $GTSig$ generates the transition system $TS(GTSpec) = \langle \mathcal{V}, \rightsquigarrow, \mathcal{R} \rangle$ where

- $g \xrightarrow{L} h$ holds for two states $g : G \rightarrow C$ and $h : H \rightarrow C$ iff there exists a parallel graph transition $G \xrightarrow{d} H$ respecting FC with
 - production span $\mathcal{Par}(Sp(L))$ where

$$Sp(L) = \{sp | p(c) \in L \wedge p(a) : sp \text{ if } AC \in \mathcal{PD}\}$$

is the set of all production spans for production names in L

- bottom span $t = (G \longleftarrow D \longrightarrow H)$ given by the pullback of g and h modeling the “intersection” D of G and H in C , such that $c = g \circ m \circ in_{sp}(a)$ ⁸ for all $p(c) \in L$ with $p(a) : sp \text{ if } AC$, i.e., the formal parameter a is mapped to the actual parameter c , and the application conditions are satisfied, that is, $g \in \mathcal{V}(q(g \circ m \circ in_{sp}(b)))$ for all $q(b) \in AC$.

⁸ Here, m is the match of the DPB diagram d , and in_{sp} the embedding of the left-hand side of sp in the parallel production span.

- each run $\rho : g_0 g_1 g_2 \dots$ in $\mathcal{R}$ starts in an initial state ($g_0 \in I$) and satisfies the fairness conditions F .

The transition specification $GTSpec$ is *correct w.r.t.* $X : \Phi$ iff $\models_{TS(GTSpec)} X : \Phi$.

5 Compositional Verification of System Properties

This section formalizes the concept of a view on a specification. The idea is to ensure that the behavior of the overall specification is permitted by the view's loose semantics.

Definition 17 (view on a specification). Let $GTSig = \langle GS, C, P, Q \rangle$ and $GTSig_0 = \langle GS, C, P_0, Q_0 \rangle$ be graph transition signatures such that $P_0 \subseteq P$ and $Q_0 \subseteq Q$. Then, a graph transition specification $GTSpec_0 = \langle \mathcal{V}_0, \mathcal{PD}_0, FC_0, I_0, F_0 \rangle$ over $GTSig_0$ is a *view of* $GTSpec = \langle \mathcal{V}, \mathcal{PD}, FC, I, F \rangle$ over $GTSig$, written $GTSpec_0 \subseteq GTSpec$, iff $\mathcal{V}_0 = \mathcal{V}|_{\mathcal{SF}_{Q_0}(C)}$, $\mathcal{PD}_0 \subseteq \mathcal{PD}$, $FC \subseteq FC_0$, $I \subseteq I_0$ and $F_0 \subseteq F$, such that for each *hidden production name* $p \in P \setminus P_0$ with definition $p(a) : (L \xleftarrow{l} K \xrightarrow{r} R)$ if AC , the reduct of l and r to non-input and non-output sorts, $(l)_{\overline{IS}_0}$ and $(r)_{\overline{OS}_0}$, respectively, yields isomorphisms.

Both transition specifications have the same graph signature GS and context graph C , that is, their generated transition systems have the same set of states. All input (output) sorts in $GTSpec$ are also input (output) sorts in the view. Hence, the frame conditions in $GTSpec$ are stronger, which means that $GTSpec$ is the more complete specification and effects that are covered by the loose semantics of $GTSpec_0$ are explicitly specified in $GTSpec$. On the other hand, each production of $GTSpec$ which is hidden from the view must not have effects on its private sorts since for those the view is assumed to be complete.

A view of $GTSpec$ can be induced by choosing a suitable subset of production names P_0 together with a frame condition FC_0 . This choice is *valid* if the five-tuple $GTSpec_0 = \langle \mathcal{V}|_{\mathcal{SF}_{Q_0}(C)}, \mathcal{PD}_0, FC_0, I, F_0 \rangle$ where $\mathcal{PD}_0 = \{p(a) : sp \text{ if } AC \in \mathcal{PD} | p \in P_0\}$ and $F_0 = \langle weak \cap \mathcal{TE}_{P_0}(C), strong \cap \mathcal{TE}_{P_0}(C) \rangle$ for $F = \langle weak, strong \rangle$ is indeed a view of $GTSpec$.

Example 18. The DDD-view of the *MUTEX* specification is induced by the DDD-productions with the private sort *blocked*. The TR-view is given by all TR-productions and the frame condition $IS = \{P, R, next, token, blocked, release\}$ and $OS = IS \setminus \{release\}$.

Semantically, $GTSpec_0 \subseteq GTSpec$ implies that every run in $GTSpec$ is also a run in $GTSpec_0$, or vice versa, the extension of a specification leads to a restricted behavior.

Proposition 19 (restriction of behavior). Let $GTSpec_0$ and $GTSpec$ be transition specifications such that $GTSpec_0 \subseteq GTSpec$, $TS(GTSpec) = \langle \mathcal{V}, \rightsquigarrow, \mathcal{R} \rangle$, and $TS(GTSpec_0) = \langle \mathcal{V}_0, \rightsquigarrow_0, \mathcal{R}_0 \rangle$. Then, $\mathcal{R} \subseteq \mathcal{R}_0$.

Proof. A sequence of states ρ is a run in $GTSpec$ iff each pair of consecutive states g, h is related by $g \xrightarrow{L} h$ (i), ρ starts in an initial state (ii), and it satisfies the fairness conditions (iii). In this case, ρ is also a run in $GTSpec_0$: For (i), $g \xrightarrow{L} h$ implies $g \xrightarrow{L'}_0 h$ where $L' = \{p(c) \in L \mid p \in P_0\}$, because $\mathcal{PD}_0 \subseteq \mathcal{PD}$. If (the parallel graph transition generating) $g \xrightarrow{L} h$ respects FC , then it also respects the weaker condition FC_0 . In this case, also $g \xrightarrow{L'}_0 h$ respects FC_0 since the hidden productions in $P \setminus P_0$ do not create anything of non-input sort or delete anything of non-output sort, which concludes the proof of (i).

By assumption, $I \subseteq I_0$, that is if $\rho(0) \in I$ then $\rho(0) \in I_0$ implying (ii). For (iii), $F_0 \subseteq F$ that is, the fairness condition is weakened. A state g enables a transition expression $p(c)$ in $TS(GTSpec_0)$ if and only if it enables $p(c)$ in $TS(GTSpec)$. This follows from the fact that, on the one hand, $g \xrightarrow{L} h$ implies $g \xrightarrow{L'}_0 h$, as shown above. On the other hand, $g \xrightarrow{L'}_0 h$ implies $g \xrightarrow{L} k$, for some state $k \in S_C$ because, by default, all transitions are safe, and strengthening the frame conditions does not disable safe transitions. $\square$

This result is based on the notion of (safe) graph transition which anticipates the additional effects of the rules in the bigger specification. The main theoretical result of this paper follows from the definition of correctness and the proposition above.

Theorem 20 (correctness of compositional verification). *Let $GTSpec_0$ and $GTSpec$ be graph transition specifications such that $GTSpec_0 \subseteq GTSpec$, and $X : \Phi \in \mathcal{TF}_{GT\text{Sig}_0}$ be a temporal formula such that $GTSpec_0$ is correct w.r.t. $X : \Phi$. Then, also $GTSpec$ is correct w.r.t. $X : \Phi$.*

This allows to verify the properties of the MUTEX specification in a compositional way, by analyzing its views separately and deriving the MUTEX properties from the properties the views:

Example 21. For safety properties, the decomposition of the specification depends on the property to be shown. In order to prove *MUTEX-safe*, the stronger *consistency condition* $\square \text{MUTEX-cc}$ is established, where *MUTEX-cc* is satisfied in a state $g : G \rightarrow C$ iff for each resource $r \in g(G)$ there is a unique *token* edge t with $\text{source}(t) = r$, or a unique *held_by* edge h with $\text{source}(h) = r$, or a unique *release* edge e with $\text{source}(e) = r$. This property is verified for the view induced by the production names $\{\text{mount}, \text{unmount} : R, \text{pass} : PR, \text{take} : PR, \text{rel} : PR, \text{give} : PR, \text{unlock} : PR\}$ with private sorts $\{R, \text{held_by}, \text{token}, \text{release}\}$, i.e., all other sorts are input and output sorts. This is obviously a view of *MUTEX* since all the productions not mentioned do not affect the private sorts. Moreover, it is correct w.r.t. *MUTEX-cc*. In fact, the initial graphs satisfy the constraint and, since the relevant sorts are private and the condition is not violated by the productions of the view, the constraint is preserved by all transitions. By Theorem 20 one concludes that the complete specification *MUTEX* is correct w.r.t. *MUTEX-cc* and hence w.r.t. *MUTEX-safe*.

Another basic consistency condition is the integrity of the ring structure. Here, *ring-cc* shall be satisfied by a state g if its *next* edges form a cycle containing all processes of the state. For proving $\Box$ *ring-cc* let's pick up the view given by $P_0 = \{new, kill : P\}$ with private sorts $\{P, next\}$. Again, the chosen productions are the only ones affecting the private sorts, and it is similarly easy to see that the view is correct. Hence, also *MUTEX* is correct w.r.t. $\Box$ *ring-cc*.

It is worth mentioning here that most of the requirements checked so far can be verified automatically: Given the production names and frame conditions, it is easy to test if they induce a valid view, and for verifying that the productions preserve certain consistency conditions, a procedure has been developed in [HW95].

The more difficult task is presented by the verification of liveness properties. Here, *MUTEX-live* has been decomposed into the local properties *DDD-live* and *TR-live*. Since the focus is on the composition of properties rather than on their verification for views, I do not present a proof of the correctness of *DDD* and *TR*. Notice, however, that the consistency properties above have to be assumed.

Like safety properties, also the liveness properties *DDD-live* and *TR-live* carry over to the global system by Theorem 20. Combining them one obtains $\Box bounded(n) \implies MUTEX-live$, that is, the boundedness of the system is left as an assumption for the schedule of SYS-productions.

6 Conclusion

In this paper I introduced graph transitions as a loose semantics of productions. By formalizing the notion of view I provided conceptual tools for compositional verification, and proved their correctness. An example showed the usefulness of the approach for specifying reactive systems and verifying their properties.

This notion of view is not very flexible yet since it does not allow the renaming of types or the extension of productions. It is not difficult, however, to generalize the concepts and results to some sort of specification morphisms, like the view relations in [EHTE97] or GTS morphisms in [HCEL96]. These papers also consider the problem of composing specifications.

A topic of ongoing work is the verification of views w.r.t. temporal properties. First results have been presented in [GHK] using graphical constraints [HW95] in order to define the evaluation of state formulas.

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