Network coding with diversity and outdated channel state information

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Abstract: Physical-layer network coding (PNC) has the potential to significantly improve the throughput of wireless networks where the channels can be modeled as additive white Gaussian noise (AWGN) channel. As extending to multiple channels, this technique requires both amplitude and phase compensation at each transmitter and will lead to inefficient systems yielding no diversity even with perfect channel state information (CSI). In order to avoid these limitations, we apply network coding with diversity (NCD) to achieve a form of selection diversity and extend NCD to cooperative multiple access channels in this paper. However, in practical wireless communication systems, the CSI could become outdated due to the difference between the CSI used in the relay selection and data transmission phases. Hence, the selected relay may not be the best one during data transmission phase due to the dynamic change in the wireless channels. Therefore, we first explore the relation between the present and past CSIs. Exploiting this relationship, the NCD scheme with outdated CSI is investigated based on the past CSI. To evaluate the performance of this scheme, an information-theoretic metric, namely the outage capacity, is studied under this condition.

Key words: network coding with diversity (NCD); cooperative multiple access channels; channel state information (CSI)

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1. Introduction

Network coding as a potentially powerful tool in designing communication networks has been widely used in wireless communications [1]. This new approach is exploited in intermediate nodes to combine and code packets from different transmitters which exchange the information through a relay [2]. By using physical layer network coding (PNC), the relay only combines the received packets and then broadcasts the network-coded information. The existing network coding with diversity (NCD) provides a significant throughput gain for distributed relay selection strategy while introducing a form of multi-user diversity in the network [3].

Most of the existing networks assume that each transceiver has the perfect channel state information (CSI). However, in practical scenarios the CSI could become outdated due to the difference between the CSIs during the relay selection and data transmission phases. Due to the variations of the wireless channels, the relay selec-

tion is carried out based on outdated CSI, which does not guarantee that the selected relay is the best one during the data transmission phase. This relay selection based on outdated CSI significantly influences the overall performance of the network. Hence, in this paper we investigate the performance of such a system.

The contributions of this paper are as follows. First, the effects of outdated CSI are provided that the selected relay may not be the best one during data transmission phase. The same distributed relay selection strategy will be considered as in [3]. Second, we develop a relationship between the present and the past CSIs based on the correlation coefficient [4]. By exploiting this relationship, we study the performance of NCD scheme with the available outdated CSI and demonstrate the effects of the outdated CSI on the overall system performance. Third, we will apply a simplified approximation for high signal-to-noise (SNR) region. Using order statistics [5], we set a normal situation that the relay we have chosen already is the k -th worst relay. From the existing expression of outage capacity for NCD scheme with perfect CSI, we can get it with outdated CSI. Finally, in our case, the relay is the best one in the past period. An expression of outage capacity for this relay will be obtained under such a condition.

This paper is organized as follows. The network coding with diversity protocol and the distributed relay se-

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lection strategy are described in Section 2. The effects of outdated CSI are discussed in Section 3 whereas the outage capacities with the perfect and the outdated CSIs are provided in Section 4. Finally, Section 5 concludes the paper.

2. Network coding with diversity protocol description and distributed relay selection strategy

2.1. Protocol description

Let us consider a half-duplex two-hop relaying system with $N+2$ nodes, where two source nodes exchange information with the help of N relaying nodes. It is assumed that there is no direct link between the two source nodes and all channel coefficients in each link are independent and identically distributed.

Fig. 1 A diagram for the information exchanging scenario and the transmission strategy for the proposed network coding with diversity protocol

The two source nodes S_1 and S_2 exchange their information in two time slots as shown in Fig. 1. During the first time slot, both sources broadcast their information to the whole network at the same time. The signals received at N relays experience different types of attenuations, such as additive white Gaussian noise (AWGN), large scale path loss, and Rayleigh fading. These N relays assist the communication between S_1 and S_2 . In addition, these relays are assumed to be located randomly and independently between the two sources. Hence, the channel coefficients between each relay and the two sources are independent. Both sources are assumed to have the transmission power constraint of P . Let us define the forward channel between S_1 and the relay R_n as h_{1R_n} , and the backward channel between the relay R_n and S_1 as h_{R_n1} . Similarly, the forward channel between S_2 and the relay R_n is defined as h_{2R_n} , and the backward channel between the relay R_n and S_2 is h_{R_n2} . The received signal at the relay R_n can be represented as

$$y_{R_n} = \sqrt{P}h_{1R_n}S_1 + \sqrt{P}h_{2R_n}S_2 + w_{R_n}, \quad (1)$$

where P is the source transmission power, s_i is the transmitted signal from the source S_i with unit power ($i \in \{1, 2\}$), and w_{R_n} represents the AWGN with power P_w .

With the help of training symbols, each relay can obtain their own forward channel state information. Because of the symmetry of time division duplex systems considered here, the forward and the backward channels are symmetric, $h_{iR_n} = h_{R_n,i}$. It is assumed that the transceivers have the CSI. Therefore, each relay node could have the access to local CSI. As a result, a distributed relay selection strategy is carried out to maintain the quality of the transmission process between both the sources. Let us consider that the node R_k has been chosen as the best relay, and the channels between the sources and the best relay are denoted by h_{1R_k} and h_{2R_k} , respectively.

During the second time slot, the best relay R_k broadcasts the compressed mixture

$$\frac{\sqrt{P}h_{1R_k}S_1 + \sqrt{P}h_{2R_k}S_2 + w_R}{\sqrt{P|h_{1R_k}|^2 + P|h_{2R_k}|^2 + P_w}}\sqrt{P}$$

to the two sources based on the amplify-and-forward protocol. Since the transmitted information s_i is already available at the source S_i , this part could be easily removed at each source from the received signal. Hence, the rest of the signal can be written as follows:

$$y_i = \frac{\sqrt{P}h_{iR_k}}{\sqrt{P|h_{1R_k}|^2 + P|h_{2R_k}|^2 + P_w}}(\sqrt{P}h_{jR_k}S_j + w_R) + w_i, \quad (2)$$

where w_i is the additive noise at the source S_i . Based on this signal model, the mutual information that the source S_i could receive from the source S_j is expressed as

$$I_{ij, \text{NCD}} = \log \left[1 + \frac{\rho^2 |h_{1R_k}|^2 |h_{2R_k}|^2}{2\rho |h_{iR_k}|^2 + \rho |h_{jR_k}|^2 + 1} \right], \quad \forall i \neq j \text{ \& } i, j \in [1, 2], \quad (3)$$

where $\rho = P/P_w$ is the SNR. To compare the performance of the proposed protocol, the direct transmission (DR) scheme is used and the mutual information of this scheme can be written as follows:

$$I_{\text{DR}} = \log [1 + \rho |h|^2]. \quad (4)$$

2.2. Distributed relay selection strategy

In the medium access layer, a distributed cooperative routing strategy is employed to select the best relay node. Specifically, each relay node uses the carrier sensing with deterministically mapped back-off time [6]. Here, the carrier sensing technique is the basis of the collision avoidance mechanism, and the back-off time is a period of time by which each relay node delays its transmission. The back-off time is inversely proportional to the quality of its local CSI [7]. In this case, the routing with the best quality is applied for data transmission. Each relay node continues monitoring the condition of the channel and starts its data transmission. Hence, the relay will refrain from forwarding data if another relay is in data transmission phase. Hence, the selection of back-off time at different relay nodes ensures that the relay node with the best quality of local CSI will be the one responsible for data transmission [6].

We focus on how to determine the evaluation criterion for the link quality, which is used to implement the relay selection strategy as discussed. From (4), it can be seen that the two sources have different preferences. Fortunately, the preferences do not conflict with each other. The relay who has the channel $I_{12,NCD}$ also has the channel yielding a large value for $I_{21,NCD}$, if not the maximum one. The results of simulations in [3] could present this claim. The best relay is chosen by the following criterion:

$$\frac{\rho^2 |h_{1R_k}|^2 |h_{2R_k}|^2}{2\rho |h_{1R_k}|^2 + \rho |h_{2R_k}|^2 + 1},$$

which maximizes $I_{12,NCD}$. After choosing it, $I_{21,NCD}$ can also be calculated. This is suboptimal for the second source. However, such losses due to this suboptimal relay selection could be neglected as discussed in [3].

3. Effects of outdated CSI

In this section, we study the effects of outdated CSI on the proposed NCD scheme. In order to explain how the outdated CSI influences the system, we divide the whole information exchanging process into two phases: the relay selection phase and the data transmission phase. Due to the channel variations, the actual channel during the data transmission phase could be different from the CSI used in the relay selection phase. Recall NCD scheme and the distributed relay selection strategy; the proposed NCD selects the best relay R_k among the N relay nodes, and then the two source nodes transmit their information to each other through the best relay.

The procedure of the whole information exchanging process is as follows.

3.1. Relay selection phase

The two sources broadcast the training symbols to all the relays to determine the quality of $S_1 - R_k$ and $R_k - S_2$ links in advance. All relays decode the training symbol and estimate their own CSI. Then the relays feed the CSI back to the sources (the relays send back the CSI of the $R_k - S_2$ link to S_1 , and the CSI of the $S_1 - R_k$ link to S_2). After receiving the CSI, the sources S_1 and S_2 select the best relay R_k using the criterion

$$\frac{\rho^2 |h_{1R_k}|^2 |h_{2R_k}|^2}{2\rho |h_{1R_k}|^2 + \rho |h_{2R_k}|^2 + 1},$$

which maximizes $I_{12,NCD}$ to participate in the data transmission phase.

3.2. Data transmission phase

This phase can be divided into two stages: the source transmission stage and the relay transmission stage. To exchange information, both two sources first transmit the data to the selected relay R_k . The relay R_k then broadcasts the information corresponding to each sources.

In practical scenarios, the CSI used for relay selection may be different from the channel experienced in data transmission stage due to the variations of the wireless channels. That means the relay selection is being performed based on the outdated CSI, not with actual CSI in data transmission stage. Similarly, there is a delay between the relay selection phase and the source transmission stage, so the CSI could also be outdated. Hence, the selected relay R_k may not be the best during the actual data transmission phase.

4. The information-theoretic metric: outage capacity

Now we will utilize one information theoretic metric to evaluate the performance of the NCD. Outage capacity or outage probability is the quantity that describes the robustness of a communication system and the data rate that can be supported with a certain error probability.

Definition 1 $\alpha\%$ outage capacity is the data rate that can be supported with $\alpha\%$, i.e.,

$$P(I < R) \leq \alpha\%.$$

The outage capacity can be calculated from the cumulative distribution function (CDF) of the mutual information I_{NCD} . Assuming that the relay selection strategy is not applied, a random relay R_n is used to exchange the information between both the sources. From (3), the mutual information obtained by such a relay can be expressed as

$$I_n = \log \left[1 + \rho \frac{\rho |h_{1R_n}|^2 |h_{2R_n}|^2}{2\rho |h_{1R_n}|^2 + \rho |h_{2R_n}|^2 + 1} \right] = \log \left[1 + \rho \frac{x_n y_n}{2x_n + y_n + 1/\rho} \right], \quad (5)$$

where $x_n = |h_{1R_n}|^2$ and $y_n = |h_{2R_n}|^2$. Due to the symmetric system, we only focus on the mutual information $I_{12,\text{NCD}}$. It should be mentioned that $|h_{iR_n}|^2$ is exponentially distributed with the parameter $d_{iR_n}^\alpha$. Thus, x_n will be exponentially distributed with the parameter $\lambda_{x_n} = d_{1R_n}^\alpha$ and so will y_n with $\lambda_{y_n} = d_{2R_n}^\alpha$, where α is the path loss exponent factor. In order to simplify the analytical development as in [8], we assume that the distance between relays is far less than the distance d_{iR_n} .

That means $d_{iR_n} = d_{iR_k} = d_i$, $\forall n \neq k$. Therefore, the N mutual information will be identical and independently distributed, and they can be ordered as

$$I_{(1)} \leq I_{(2)} \leq \dots \leq I_{(N)}. \quad (6)$$

And the relay with the largest mutual information will be chosen according to the relay selection strategy, which means $I_{(N)} = I_{\text{NCD}}$. Hence, we should first find the probability density function (PDF) of I_n , and then apply the order statistics to derive the cumulative density function (CDF) of $I_{(N)}$. Let us define

$$z_n = \frac{x_n y_n}{2x_n + y_n + 1/\rho},$$

and find the distribution function of z_n , which is shown in the following lemma.

Lemma 1 [3] Provided that x and y are exponentially distributed with λ_x and λ_y , respectively, the CDF of z can be obtained as

$$P(z < Z) = 1 - \exp[-(\lambda_x + 2\lambda_y)Z] \times \sqrt{4\lambda_x \lambda_y Z \left(2Z + \frac{1}{\rho}\right)} K_1 \left(\sqrt{4\lambda_x \lambda_y Z \left(2Z + \frac{1}{\rho}\right)} \right), \quad (7)$$

where $K_1(x)$ is the modified Bessel function of the

second kind.

Then, the CDF of the mutual information I_n can be expressed as

$$P(I_n < R) = P\left(\frac{x_n y_n}{2x_n + y_n + 1/\rho} < \frac{2^R - 1}{\rho}\right). \quad (8)$$

After some algebraic manipulations, we could simplify (8) as

$$P(I_n < R) = 1 - \exp[-(\lambda_{x_n} + 2\lambda_{y_n})\gamma] \sqrt{\theta_n} K_1(\sqrt{\theta_n}), \quad (9)$$

where

$$\theta_n = 4\lambda_{x_n} \lambda_{y_n} \gamma (2\gamma + 1), \quad \text{and} \quad \gamma = (2^R - 1)/\rho.$$

Using the order statistics, the outage capacity of NCD can be shown as [5]

$$P(I_{\text{NCD}} < R) = P(I_{(N)} < R) = [P(I_n < R)]^N = \left[1 - \exp[-(\lambda_{x_n} + 2\lambda_{y_n})\gamma] \sqrt{\theta_n} K_1(\sqrt{\theta_n}) \right]^N. \quad (10)$$

At the high SNR region, the outage capacity of NCD can be shown as [3]

$$P(I_{\text{NCD}} < R) \approx \frac{[(d_1^\alpha + 2d_2^\alpha)(2^R - 1)]^N}{\rho^N}. \quad (11)$$

From the expression of the outage capacity, it is clear that the NCD scheme can achieve the diversity gain N due to its ability to explore the diversity of wireless multipath fading channel. In this case, it is possible to find the best relay among multiple relays to connect both the sources, which yields multi-user/relay diversity.

Compared with the direct transmission scheme, the NCD scheme can achieve the outage capacity as [3]

$$P(I_{\text{DR}} < R) \approx d_{12}^\alpha \frac{2^R - 1}{\rho}, \quad (12)$$

where d_{12} is the direct distance between the two sources. From (12), we can see that the DR scheme can only achieve the diversity gain 1. Therefore, at high SNR regions, it can be concluded that the NCD scheme has larger outage capacity than the DR scheme, which means that the NCD scheme is much sturdier than the DR scheme.

Now we will analyze the performance of the proposed NCD scheme when the CSIs are outdated. In order to explore the effects of outdated CSI on overall system performance, let us define the correlation coefficient between $\hat{h}_{iR_n}$ and h_{iR_n} ($i \in \{1, 2\}$) on the link of the n -th relay as ϕ_{iR_n} . It is assumed that the correlation coefficients for all channels are identical (i.e., $\phi_{iR_n} = \phi_{in}$ for $n \in \{1, 2, \dots, N\}$). Under Jakes' scattering model, the

correlation coefficient is defined as $\varphi_{\text{in}} = J_0(2\pi f_{d_{\text{in}}} T_{s_{\text{in}}})$, where $J_0(\cdot)$ denotes the zero-th-order Bessel function of the first kind, $f_{d_{\text{in}}}$ stands for the maximum Doppler frequency, and $T_{s_{\text{in}}}$ is the time delay between relay selection phase and data transmission phase [4].

Applying the Bayes' theorem, the PDF of h_{iR_n} conditioned on $\hat{h}_{iR_n}$ could be written as

$$f_{h_{iR_n}|\hat{h}_{iR_n}}(h_{iR_n}|\hat{h}_{iR_n}) = \frac{f_{h_{iR_n}, \hat{h}_{iR_n}}(h_{iR_n}, \hat{h}_{iR_n})}{f_{\hat{h}_{iR_n}}(\hat{h}_{iR_n})} = \frac{\exp[-(h_{iR_n} - \phi_{\text{in}} \hat{h}_{iR_n})^H R_i^{-1} (h_{iR_n} - \phi_{\text{in}} \hat{h}_{iR_n})]}{\pi^M \det(\mathbf{G}_{\text{in}})}, \quad (13)$$

where $\mathbf{G}_{\text{in}} = (1 - \varphi_{\text{in}}^2) \mathbf{I}_M$ is the covariance matrix. Note that $\mathbf{G}_{\text{in}}$ models the degree of CSI uncertainty, and when $h_{iR_n} = \hat{h}_{iR_n}$, $\mathbf{G}_{\text{in}} = \mathbf{0}$.

Recalling the quotation that $x_n = |h_{1R_n}|^2$ and $y_n = |h_{2R_n}|^2$, we can obtain $\hat{x}_n = |\hat{h}_{1R_n}|^2$ and $\hat{y}_n = |\hat{h}_{2R_n}|^2$. Since x_n is conditioned on $\hat{x}_n$, and y_n is conditioned on $\hat{y}_n$ following a non-central Chi-square distribution with two degrees of freedom, from (13), we can obtain their PDF as

$$f_{x_n|\hat{x}_n}(x_n|\hat{x}_n) = \frac{\lambda_{x_n}}{(1 - \varphi_{\text{in}}^2)} \times \exp\left[\frac{-\lambda_{x_n}(x_n + \hat{x}_n \varphi_{\text{in}}^2)}{1 - \varphi_{\text{in}}^2}\right] I_0\left(\frac{2\lambda_{x_n} \varphi_{\text{in}} \sqrt{x_n \hat{x}_n}}{1 - \varphi_{\text{in}}^2}\right), \quad (14)$$

$$f_{y_n|\hat{y}_n}(y_n|\hat{y}_n) = \frac{\lambda_{y_n}}{(1 - \varphi_{\text{in}}^2)} \times \exp\left[\frac{-\lambda_{y_n}(y_n + \hat{y}_n \varphi_{\text{in}}^2)}{1 - \varphi_{\text{in}}^2}\right] I_0\left(\frac{2\lambda_{y_n} \varphi_{\text{in}} \sqrt{y_n \hat{y}_n}}{1 - \varphi_{\text{in}}^2}\right), \quad (15)$$

where $I_0(\cdot)$ denotes the zero-order modified Bessel function of the first kind.

From (6), it is noted that at high SNR region the equation can be written as

$$I_n = \log\left[1 + \rho \frac{x_n y_n}{2x_n + y_n}\right]. \quad (16)$$

Now let us consider the practical scenario. Assume that the relay we have chosen is the k -th worst relay, which means that $I_{(1)} \leq I_{(2)} \leq \dots \leq I_{(k)} \leq \dots \leq I_{(N)}$. Ac-

cording to the theorem of order statistics in [5], the PDF of $\hat{x}_n$ can be expressed as

$$f_{X_{(k)}}(\hat{x}) = \frac{N!}{(k-1)!(N-k)!} f_X(\hat{x}) [F_X(\hat{x})]^{k-1} [1 - F_X(\hat{x})]^{N-k}, \quad (17)$$

where

$$f_X(\hat{x}) = \lambda_x \exp(-\lambda_x \hat{x}), \\ F_X(\hat{x}) = 1 - \exp(-\lambda_x \hat{x}).$$

Because x is conditioned on $\hat{x}$, the PDF of x can be obtained from the marginal distribution density function of $f(x, \hat{x})$ as

$$f_{X_{(k)}}(x) = \int_0^\infty f_{X_{(k)}}(x, \hat{x}) d\hat{x}. \quad (18)$$

According to the conditional distribution function, $f_{X_{(k)}}(x, \hat{x}) = f_{X_{(k)}}(x|\hat{x}) f_{X_{(k)}}(\hat{x})$, $f_{X_{(k)}}(x)$ can be rewritten as

$$f_{X_{(k)}}(x) = \int_0^\infty f_{X_{(k)}}(x|\hat{x}) f_{X_{(k)}}(\hat{x}) d\hat{x}. \quad (19)$$

Following the approach in [9], we can obtain the PDF of x as

$$f_{X_{(k)}}(x) = k \binom{N}{k} \sum_{m=0}^{k-1} \lambda_x (-1)^m \binom{k-1}{m} \times \frac{1}{(N-k+m)(1-\varphi_i^2)+1} \times \exp\left[-\frac{\lambda_x(N-k+m+1)x}{(N-k+m)(1-\varphi_i^2)+1}\right]. \quad (20)$$

In this case, the PDF of y can also be derived using the same approach:

$$f_{Y_{(k)}}(y) = k \binom{N}{k} \sum_{m=0}^{k-1} \lambda_y (-1)^m \binom{k-1}{m} \times \frac{1}{(N-k+m)(1-\varphi_i^2)+1} \times \exp\left[-\frac{\lambda_y(N-k+m+1)y}{(N-k+m)(1-\varphi_i^2)+1}\right]. \quad (21)$$

Lemma 2 The CDF of $z = xy/(2x + y)$ is written as

$$P(z < Z) = \iint_{\frac{xy}{2x+y} < Z} f_{X_{(k)}}(x) f_{Y_{(k)}}(y) dx dy = \int_0^{2Z} f_{Y_{(k)}}(y) \int_0^\infty f_{X_{(k)}}(x) dx dy + \int_{2Z}^\infty f_{Y_{(k)}}(y) \int_{\frac{yZ}{y-2Z}}^\infty f_{X_{(k)}}(x) dx dy. \quad (22)$$

Combining (20) and (21), the first part at the left side of (22) can be derived as

$$\begin{aligned} \int_0^{2z} f_{Y_{(k)}}(y) \int_0^\infty f_{X_{(k)}}(x) dx dy = \\ k \binom{N}{k} \sum_{m=0}^{k-1} \lambda_x (-1)^m \binom{k-1}{m} k \binom{N}{k} \sum_{n=0}^{k-1} \lambda_y (-1)^n \binom{k-1}{n} \times \\ \frac{1}{(N-k+m)(1-\phi_i^2)+1} \times \frac{1}{(N-k+n)(1-\phi_i^2)+1} \times \\ \frac{1}{\lambda_x A} \times \frac{1}{\lambda_y B} [1 - \exp(-2\lambda_y Bz)], \end{aligned} \quad (23)$$

where

$$A = \frac{N-k+m+1}{(N-k+m)(1-\phi_i^2)+1},$$

and

$$B = \frac{N-k+n+1}{(N-k+n)(1-\phi_i^2)+1}.$$

For the second part of (23), we can simply it as

$$\begin{aligned} \int_{2z}^\infty f_{Y_{(k)}}(y) \int_0^\infty f_{X_{(k)}}(x) dx dy = \\ \frac{k \binom{N}{k} \sum_{m=0}^{k-1} \lambda_x (-1)^m \binom{k-1}{m}}{(N-k+m)(1-\phi_i^2)+1} \times \int_{2z}^\infty f_{Y_{(k)}}(y) \frac{1}{\lambda_x A} \left[1 - \exp\left(-\lambda_x A \frac{yz}{y-2z}\right) \right] dy \stackrel{t=y-2z}{=} \\ k \binom{N}{k} \sum_{m=0}^{k-1} \lambda_x (-1)^m \binom{k-1}{m} \times k \binom{N}{k} \sum_{n=0}^{k-1} \lambda_y (-1)^n \binom{k-1}{n} \times \frac{1}{(N-k+m)(1-\phi_i^2)+1} \times \frac{1}{(N-k+n)(1-\phi_i^2)+1} \times \\ \frac{1}{\lambda_x A} \frac{1}{\lambda_y B} \left[\exp(-2\lambda_y Bz) - \lambda_y B \exp[-(\lambda_x A + 2\lambda_y B)z] \int_0^\infty \exp\left(-\lambda_y Bt - \lambda_x \frac{2z^2}{t}\right) dt \right]. \end{aligned} \quad (24)$$

From Eq. (3.324.1) in [10], (24) can be rewritten as

$$\begin{aligned} \int_{2z}^\infty f_{Y_{(k)}}(y) \int_0^\infty f_{X_{(k)}}(x) dx dy = k \binom{N}{k} \sum_{m=0}^{k-1} \lambda_x (-1)^m \binom{k-1}{m} \times k \binom{N}{k} \sum_{n=0}^{k-1} \lambda_y (-1)^n \binom{k-1}{n} \times \\ \frac{1}{(N-k+m)(1-\phi_i^2)+1} \times \frac{1}{(N-k+n)(1-\phi_i^2)+1} \times \\ \frac{1}{\lambda_x A} \frac{1}{\lambda_y B} \left[\exp(-2\lambda_y Bz) - \exp[-(\lambda_x A + 2\lambda_y B)z] \sqrt{8\lambda_x \lambda_y ABz^2} K_1\left(\sqrt{8\lambda_x \lambda_y ABz^2}\right) \right]. \end{aligned} \quad (25)$$

Combining (23) and (25), after some algebraic manipulations, we can obtain the CDF of z as

$$\begin{aligned} P(z < Z) = \\ k \binom{N}{k} \sum_{m=0}^{k-1} \lambda_x (-1)^m \binom{k-1}{m} \times k \binom{N}{k} \sum_{n=0}^{k-1} \lambda_y (-1)^n \binom{k-1}{n} \times \frac{1}{(N-k+m)(1-\phi_i^2)+1} \times \frac{1}{(N-k+n)(1-\phi_i^2)+1} \times \\ \frac{1}{\lambda_x A} \frac{1}{\lambda_y B} \left[1 - \exp[-(\lambda_x A + 2\lambda_y B)Z] \sqrt{8\lambda_x \lambda_y ABZ^2} K_1\left(\sqrt{8\lambda_x \lambda_y ABZ^2}\right) \right]. \end{aligned} \quad (26)$$

The CDF of the mutual information $I_{(k)}$ of the k -th worst relay is obtained as

$$P(I_{(k)} < R) = P\left(\frac{xy}{2x+y} < \frac{2^R - 1}{\rho}\right). \quad (27)$$

Introducing (26) into (27), we obtain

$$\begin{aligned} P(I_{(k)} < R) = k \binom{N}{k} \sum_{m=0}^{k-1} \lambda_x (-1)^m \binom{k-1}{m} \times \\ k \binom{N}{k} \sum_{n=0}^{k-1} \lambda_y (-1)^n \binom{k-1}{n} \times \end{aligned}$$

$$\frac{1}{(N-k+m)(1-\phi_i^2)+1} \times \frac{1}{(N-k+n)(1-\phi_i^2)+1} \times \frac{1}{\lambda_x A} \frac{1}{\lambda_y B} \left[1 - \exp[-(\lambda_x A + 2\lambda_y B)\gamma] \sqrt{\theta} K_1(\sqrt{\theta}) \right], \quad (28)$$

where $\theta = 8\lambda_x \lambda_y AB\gamma^2$ and $\gamma = (2^R - 1)/\rho$. We can have $\theta \rightarrow 0$ for $\rho \rightarrow \infty$ and a fixed value of the data rate R .

Using the following approximations:

$$K_1(x) \approx \frac{1}{x} \quad \text{and} \quad e^{-x} \approx 1 - x, \quad \text{as } x \rightarrow 0$$

the CDF of the outage capacity of the k -th worst relay can be approximated as

$$P(I_{(k)} < R) \approx \frac{k \binom{N}{k} \sum_{m=0}^{k-1} (-1)^m \binom{k-1}{m} \times k \binom{N}{k} \sum_{n=0}^{k-1} (-1)^n \binom{k-1}{n} \times \frac{1}{(N-k+m)(1-\varphi_i^2)+1} \times \frac{1}{(N-k+n)(1-\varphi_i^2)+1} \times \frac{1}{AB} \frac{(d_1^\alpha A + 2d_2^\alpha B)(2^R - 1)}{\rho}. \quad (29)$$

In our case, the selected k -th worst relay is the best relay, so that we can have $k = N$, and (28) could be simplified as

$$P(I_{(k)} < R) \approx N \binom{N}{N} \sum_{m=0}^{N-1} (-1)^m \binom{N-1}{m} \times N \binom{N}{N} \sum_{n=0}^{N-1} (-1)^n \binom{N-1}{n} \times \frac{1}{m(1-\varphi_i^2)+1} \times \frac{1}{n(1-\varphi_i^2)+1} \times \frac{1}{AB} \frac{(d_1^\alpha A + 2d_2^\alpha B)(2^R - 1)}{\rho}. \quad (30)$$

To make a comparison, the DR scheme will also be introduced into the circumstance of outdated CSI. Hence, we can have $z = |h|^2$, and $\hat{z} = |\hat{h}|^2$ which is exponentially distributed with the parameter $\lambda_z = d_{12}^\alpha$. Also, due to the outdated CSI, z is conditioned on $\hat{z}$. Under Jakes' model, the correlation coefficient between z and $\hat{z}$ is defined as φ , which has the same definition with φ_{in} . We can obtain the PDF using the same method as x and y , as follows:

$$f(z) = \int_0^\infty f(z|\hat{z})f(\hat{z})d\hat{z} = \int_0^\infty \frac{\lambda_z}{(1-\varphi^2)} \exp\left(\frac{-\lambda_z(z+\hat{z}\varphi^2)}{1-\varphi^2} - \lambda_z\hat{z}\right) I_0\left(\frac{2\lambda_z\varphi\sqrt{z\hat{z}}}{1-\varphi^2}\right) \lambda_z d\hat{z} = \frac{\lambda_z^2}{(1-\varphi^2)} \int_0^\infty \exp\left(\frac{-\lambda_z(z+\hat{z})}{1-\varphi^2}\right) I_0\left(\frac{2\lambda_z\varphi\sqrt{z\hat{z}}}{1-\varphi^2}\right) d\hat{z} = \lambda_z \exp(-\lambda_z z). \quad (31)$$

Also, the outage capacity of DR is obtained as

$$P(I_{DR} < R) = P\left(z < \frac{2^R - 1}{\rho}\right) \approx d_{12}^\alpha \frac{2^R - 1}{\rho}. \quad (32)$$

From (32), we can see that the DR scheme still only achieve the diversity gain 1. Therefore, it is clear that the NCD scheme can still achieve higher diversity gain than DR scheme with the outdated CSI. Also, provided that SNR is large enough, it can be concluded that the NCD scheme has larger outage capacity than the DR

scheme, which means the NCD scheme outperforms the DR scheme in terms of diversity gain.

5. Conclusion

As discussed in [3], NCD has demonstrated that it is much better than DR scheme with perfect CSI. In this paper, we evaluated the performance of the system when the CSIs are outdated. The information-theoretic metrics, outage capacity, has been derived to evaluate the NCD scheme with perfect and outdated CSIs. The results show that the NCD scheme outperforms the DR scheme.

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