Reliability analysis of circular concrete-filled steel tube with material and geometrical nonlinearity

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Abstract: Most of the previous research on concrete-filled steel tube is restricted to a deterministic approach. To gain clear insight into the random properties of circular concrete-filled steel tube, reliability analysis is carried out in the present study. To obtain the structural nonlinear response and ultimate resistance capacity, material and geometrical nonlinear analysis of circular concrete-filled steel tube is performed with a three-dimensional degenerated beam element. Then we investigate the reliability of concrete-filled steel tube using the first-order reliability method combined with nonlinear finite element analysis. The influences of such parameters as material strength, slenderness, initial geometrical imperfection, etc. on the reliability of circular concrete-filled steel tube column are studied. It can be concluded that inevitable random fluctuation of those parameters has significant influence on structural reliability, and that stochastic or reliability methods can provide a more rational and subjective evaluation on the safety of CFT structures than a deterministic approach.

Key words: concrete-filled steel tube; reliability; material nonlinearity; geometrical nonlinearity

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1. Introduction

Concrete-filled steel tube (CFT), which combines the advantages of steel and concrete, has been developed as an excellent structural element for building columns, bridge piers, and arch ribs throughout worldwide engineering practices. Since the steel tube can serve as a form for casting of core concrete, CFT structures possess economical merits in construction. Moreover, compared with hollow steel tube, core concrete can prevent buckling of steel tube, which may improve the compressive stability enormously. The most attractive advantage of concrete-filled steel tube structures is that the element exhibits an excellent compressive resistance capacity, ductility, and energy dissipation ability owing to the confining effect provided by steel tube.

A systematical and comprehensive collection of the research work about CFT structures can be found in Ref. [1]. In the previous research, the deterministic method plays a key role in the establishment of the design theory of CFT structures. However, it should be noted that those parameters, which influence the me-

chanical behaviors of CFT structures, always possess a random nature. Therefore, it may be logical to involve those uncertainties in the analysis of CFT structures. In the present study, the authors intend to investigate the reliability of CFT structures with the combination of the first-order reliability method (FORM) and nonlinear finite element analysis.

Previous studies [2-4] have indicated that FORM is sufficiently accurate for engineering application. With linearization of the failure surface at the design point, which is the point closest to the origin in the standard normal space, FORM gives the first-order estimate of the probability of failure, $P_f = \Phi(-\beta)$, where β , the reliability index, is the distance between the origin and design points in the standard normal space, and Φ is the cumulative distribution function of standard normal distribution.

The reliability of nonlinear structures has received an increasing interest in the past twenty years, and a great amount of important research has been continuously contributed to this subject. The reliability of geometrical nonlinear structures has been studied in Refs. [5-6], in which FORM and SORM (second-order reliability method) are both used to calculate the reliability index, and the response gradient of the limit state function to random variables is directly derived from the equilibrium equation. Impollonia and Sofi [7] studied the reliability of geometrical nonlinear structures by adopting the response surface method (RSM). For reliability analysis of concrete structures, to avoid the underdesign

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or overdesign of RC structures, material nonlinearity should not be neglected [8]. Teigen et al. [9] presented a method based on the perturbation technique for reliability analysis of reinforced concrete planar frame. Biondini et al. [10] studied the reliability of material and geometrical nonlinear reinforced concrete and prestressed concrete structures by Monte Carlo method, and analyzed the reliability of an existing prestressed concrete arch bridge. Through combining fiber element analysis and Monte Carlo method, Lee and Mosalam [11] also studied the reliability of RC structures. With FORM and RSM, Val et al. [12] studied the reliability of nonlinear reinforced concrete structures. Soares et al. [13] also investigated the reliability of nonlinear reinforced concrete structures by FORM and RSM. In Refs. [12-13], the ultimate capacity of reinforced concrete structure with material and geometrical nonlinearity are mainly concerned. To determine the ultimate limit state, Val et al. [12] assumed that if the nonlinear iteration did not converge, the ultimate limit state was reached. A similar way was adopted by Soares et al. [13], and the singularity of the tangent stiffness matrix was used to characterize the ultimate limit state.

In the present study, a nonlinear analysis program for concrete structures with degenerated beam elements developed by Xiang et al. [14] is adopted to investigate the nonlinear behaviors of CFT structure. For confined concrete, the stress-strain relationship proposed by Xu et al. [15] is used. To investigate the influence of geometrical nonlinearity, updated Lagrangian formulation will be employed. When strong material nonlinearity is involved in finite element analysis, the failure surface may no longer be smooth, which may lead to the convergence difficulty in the FORM algorithm. To improve the convergence dilemma, the reliability scheme is built by coupling nonlinear finite element analysis with RSM in the present study.

2. Nonlinear finite element analysis of circular concrete-filled steel tube

2.1. Formulation of degenerated beam element

The position of any point in the element shown in Fig. 1 can be given by

$$\mathbf{X} = \sum_{i=1}^2 N_i(r) (\mathbf{X}_i + y' \mathbf{V}_2 + z' \mathbf{V}_3), \quad (1)$$

in which $\mathbf{X}$ is the coordinates of any point in the element; $\mathbf{X}_i$ is the coordinates of node i ($i=1,2$); $\mathbf{V}_1$, $\mathbf{V}_2$ and $\mathbf{V}_3$ form the local orthogonal coordinate system illustrated in Fig. 1; y' and z' are the local coordinates

in $\mathbf{V}_2$ and $\mathbf{V}_3$ directions, respectively; the interpolation function $N_i(r)$ is

$$N_i(r) = \frac{1}{2}(1 + r_i r), \quad (2)$$

where r_i is the isoparametric coordinate of node i .

Fig. 1 The beam element

The local orthogonal coordinate system can be constructed in the following way

$$\mathbf{V}_3 = \mathbf{V}_1 \times \mathbf{V}_2, \quad (3)$$

in which $\mathbf{V}_1$ is expressed as

$$\mathbf{V}_1 = ((x_2 - x_1)/L \ (y_2 - y_1)/L \ (z_2 - z_1)/L)^T, \quad (4)$$

where L is the length of the element. With the given $\mathbf{V}_2$, $\mathbf{V}_3$ can be finally determined through Eq. (3).

The displacement field can be expressed as

$$\mathbf{U} = \sum_{i=1}^2 N_i(r) [\mathbf{U}_i + \mathbf{U}_\alpha(y', z')], \quad (5)$$

where $\mathbf{U}$ is the displacement vector at any point in the element, $\mathbf{U}_i$ is the nodal displacement vector at node i , and $\mathbf{U}_\alpha(y', z')$ is the relative nodal displacement produced by normal rotations. With a hypothesis of the plane section, $\mathbf{U}_\alpha(y', z')$ can be expressed as

$$\begin{pmatrix} u_i(\alpha) \\ v_i(\alpha) \\ w_i(\alpha) \end{pmatrix} = \begin{bmatrix} \mathbf{V}_1 & \mathbf{V}_2 & \mathbf{V}_3 \\ \alpha'_{i1} & \alpha'_{i2} & \alpha'_{i3} \\ 0 & y' & z' \end{bmatrix}, \quad (6)$$

where α'_{i1} , α'_{i2} , and α'_{i3} are the normal rotations about local axes of $\mathbf{V}_1$, $\mathbf{V}_2$, and $\mathbf{V}_3$.

With displacement transformation relation, Eq. (6) can be written in terms of the normal rotations about global axes as

$$\begin{pmatrix} u_i(\alpha) \\ v_i(\alpha) \\ w_i(\alpha) \end{pmatrix} = \begin{bmatrix} 0 & y'n_2 + z'n_3 & -y'm_2 - z'm_3 \\ -y'n_2 - z'n_3 & 0 & y'l_2 + z'l_3 \\ y'm_2 + z'm_3 & -y'l_2 - z'l_3 & 0 \end{bmatrix} \begin{pmatrix} \alpha_{i1} \\ \alpha_{i2} \\ \alpha_{i3} \end{pmatrix}, \quad (7)$$

in which

$$(l_2 \quad m_2 \quad n_2)^T \text{ and } (l_3 \quad m_3 \quad n_3)^T$$

represent the direction cosine of V_2 and V_3 in the global coordinate system, respectively. Then, Eq. (5) can be re-written as

$$U = \sum_{i=1}^2 N_i(r) (U_i + \Phi a_i), \quad (8)$$

where

$$\Phi = \begin{bmatrix} 0 & y'n_2 + z'n_3 & -y'm_2 - z'm_3 \\ -y'n_2 - z'n_3 & 0 & y'l_2 + z'l_3 \\ y'm_2 + z'm_3 & -y'l_2 - z'l_3 & 0 \end{bmatrix}, \quad (9)$$

and a_i is the normal rotation vector about the global axes.

2.2. Constitutive model of concrete and steel

In the degenerated beam element, the cross-section will be subdivided into a series of pieces, and the axial stress of every piece will be computed with the one-dimensional constitutive model [14]. In this section, the uniaxial constitutive model of confined concrete and steel will be discussed.

2.2.1. Confined concrete

The experimental studies have revealed that the confining effect of steel tube can increase the compressive strength and ductility of concrete. To describe a complete curve of one-dimensional stress-strain relation of concrete confined by steel tube, several characteristic points must be identified. For the ascending branch, the compressive strength of confined concrete f_{cc} , the corresponding strain ε_{cc} , and the initial elastic modulus of concrete E_0 should be determined. The compressive strength of confined concrete can be written as

$$f_{cc} = f_c + \alpha m f_{rp}, \quad (10)$$

where f_c is the compressive strength of unconfined concrete, f_{rp} is the average radial pressure acted on the core concrete, and m is an empirical constant which is assumed to be 4.2 in the present study. It has been found

that the average radial pressure f_{rp} depends on the width-to-thickness ratio (D/t) [16]. An empirical formula of f_{rp} presented by Xu et al. [15] is adopted

$$\frac{f_{rp}}{f_y} = 0.3111 \left(\frac{D}{t} - 2 \right)^{-1.027}. \quad (11)$$

Since the confining stress of steel tube to core concrete will decrease gradually with increasing load eccentricity [16-17], a reduction parameter α is introduced into Eq. (10), which is given as [15]

$$\alpha = \begin{cases} \left(1 - \frac{2e}{D} \right) \exp \left(-\frac{2e}{D} \right) & \text{for } \frac{2e}{D} < 1, \\ 0 & \text{for } \frac{2e}{D} \geq 1, \end{cases} \quad (12)$$

where e represents the load eccentricity, and can be expressed as M/N (M and N are the moment and axial force acted on the given section, respectively).

ε_{cc} , the strain of the confined concrete corresponding to maximum stress f_{cc} , is defined as

$$\varepsilon_{cc} = \frac{f_{cc}}{f_c} \varepsilon_0, \quad (13)$$

where ε_0 is the strain corresponding to maximum compressive stress f_c of unconfined concrete.

The confining effect does not exist in the initial elastic stage of loading due to the fact that the Poisson ratio of concrete is lower than that of steel in this period. In this paper, it is assumed that the initial elastic modulus of confined concrete is equal to that of unconfined common concrete.

With the three parameters f_{cc} , ε_{cc} , and E_c , the stress-strain relation of confined concrete in the ascending branch can be described by Saenz equation [18]:

$$\sigma = \frac{E_0 \varepsilon}{1 + \left(\frac{E_0}{E_{cs}} - 2 \right) \left(\frac{\varepsilon}{\varepsilon_{cc}} \right) + \left(\frac{\varepsilon}{\varepsilon_{cc}} \right)^2}, \quad (14)$$

where E_{cs} is the secant modulus of concrete at the peak stress point, which is given as f_{cc}/ε_{cc} .

For the descending branch of the stress and strain relationship, an exponential degrading function is adopted [15]:

$$\sigma = f_{cu} + (f_{cc} - f_{cu}) \exp \left(-k \frac{\varepsilon - \varepsilon_{cc}}{\varepsilon_{cc}} \right), \quad (15)$$

in which f_{cu} is the ultimate residual compressive stress, and k is a parameter to describe the descending rate which is assumed to be 0.3. f_{cu} is determined through the following equation:

$$f_{cu} = \begin{cases} f_{cc} \left(\frac{0.1 f_y}{f_c} \right)^{0.1} & \text{for } \frac{D}{t} \leq 40, \\ f_{cc} \left(\frac{0.1 f_y}{f_c} \right)^{0.1} \left[\lambda + (1-\lambda) \exp \left(-\frac{\frac{D}{t} - 40}{40} \right) \right] & \text{for } \frac{D}{t} \geq 40, \end{cases} \quad (16)$$

where λ is an empirical parameter, which is assumed to be 0.6 in this paper.

For tension of confined concrete, the tensile strength is neglected in the present study.

2.2.2. Steel

Under compressive load, owing to volume expansion of core concrete, steel tube will be subjected to hoop tension stress, and biaxial stress state occurs. With Von Mises criterion, the yield envelope of steel tube under biaxial stresses can be written as

$$\sigma_v^2 - \sigma_v \sigma_h + \sigma_h^2 = f_y^2, \quad (17)$$

in which σ_v is the axial stress of steel tube. With the assumption that hoop stress σ_h is $0.1 f_y$ [19], the vertical yield stress of steel tube can be solved from Eq. (17):

$$f_{yv}^c \approx 0.95 f_y, \quad f_{yv}^t \approx 1.05 f_y, \quad (18)$$

where f_{yv}^c and f_{yv}^t represent the axial compressive and tensile yield stress of steel tube, respectively.

2.3. Geometrical nonlinearity

To describe the geometrical nonlinear effect, updated Lagrangian formulation (UL) is adopted. For a large displacement motion, the virtual work equation can be written as

$$\int_{V_0} S_{ij} \delta E_{ij} dV_0 = \int_{V_0} p_{0i} \delta u_i dV_0 + \int_{A_0} q_{0i} \delta u_i dA_0, \quad (19)$$

where S_{ij} and E_{ij} are the Kirchhoff stress tensor and Green strain tensor, p_{0i} and q_{0i} is the body force and surface trace, u_i is the displacement tensor, and δ stands for variation.

Since UL formulation is referred to the last configuration, the Green strain should be given in an incremental form, which is defined as

$$\begin{aligned} \Delta E_{ij} &= \Delta E_{ij}^L + \Delta E_{ij}^N \\ &= \frac{1}{2} \left(\frac{\partial \Delta u_i}{\partial x_j} + \frac{\partial \Delta u_j}{\partial x_i} \right) + \frac{1}{2} \frac{\partial \Delta u_k}{\partial x_i} \frac{\partial \Delta u_k}{\partial x_j}, \end{aligned} \quad (20)$$

where ΔE_{ij}^L and ΔE_{ij}^N represent the linear and nonlinear strain, respectively. Substituting Eq. (8) into Eq. (20), the matrix form of the linear and nonlinear strain-displacement relation can be obtained

$$\Delta E_L = [\mathbf{B}]_L \Delta \mathbf{U}, \quad (21)$$

$$\Delta E_N = [\mathbf{B}]_N \Delta \mathbf{U}, \quad (22)$$

where $[\mathbf{B}]_L$ and $[\mathbf{B}]_N$ are the linear and nonlinear strain matrix, respectively. Using the virtual work equation given in Eq. (19), the geometrical stiffness matrix can be finally obtained, and the details about derivation procedure can be referred to Ref. [20].

2.4. Arc-length method

In the present study, the arc-length method is employed in nonlinear FEM analysis. As a displacement control algorithm, the arc-length method is more numerically stable in nonlinear finite element analysis than the Newton-Raphson approach, which may be efficient for capturing ultimate resistance capacity when softening behavior occurs.

Fig. 2 Arc-length method

As shown in Fig. 2, the constraint equation of iteration can be expressed as [21]

$$\Delta \mathbf{p}_i^T \Delta \mathbf{p}_i + A_0 \Delta \lambda_i^2 = \Delta \mathbf{p}_{i+1}^T \Delta \mathbf{p}_{i+1} + A_0 \Delta \lambda_{i+1}^2 = \Delta l^2, \quad (23)$$

where Δp_i is the incremental displacement after the i th iteration, $\Delta \lambda_i$ is the incremental load factor after the i th iteration, A_0 is a scalar parameter, and Δl is the arc length.

After the i th iteration, the out-of-balance force can be expressed as

$$\mathbf{e}_i = \mathbf{E}_i - \mathbf{F}_i, \quad (24)$$

in which $\mathbf{E}_i$ is the external load, which can be written as $(\lambda_0 + \Delta \lambda_i) \mathbf{q}$, and $\mathbf{F}_i$ is the internal nodal force. $\mathbf{e}_i$ can be decomposed into two components

$$\mathbf{g}_i = \mathbf{e}_i^T \mathbf{q} / \mathbf{q}^T \mathbf{q}, \quad \mathbf{h}_i = \mathbf{e}_i - \mathbf{g}_i \mathbf{q}. \quad (25)$$

In the $(i+1)$ th iteration, to eliminate the out-of-balance force, $\mathbf{e}_i$ is applied on the structure. Furthermore, to satisfy Eq. (23), an extra force $\tilde{x}_i \mathbf{q}$ is also applied. Then, the incremental displacement after the $(i+1)$ th iteration is

$$\Delta \mathbf{p}_{i+1} = \Delta \mathbf{p}_i + \mathbf{K}^{-1} (\mathbf{e}_i + \tilde{x}_i \mathbf{q}) = \Delta \mathbf{p}_i + x_i \delta_q + \eta_i \delta_{hi}, \quad (26)$$

where $\mathbf{K}$ is the tangential stiffness matrix, $\delta_q = \mathbf{K}^{-1} \mathbf{q}$, $\delta_{hi} = \mathbf{K}^{-1} \mathbf{h}_i$, and η_i is a scalar factor. Then, the incremental load factor $\Delta \lambda_{i+1}$ can be written as

$$\Delta \lambda_{i+1} = \Delta \lambda_i - \mathbf{g}_i + x_i. \quad (27)$$

Substituting Eqs. (26) and (27) into Eq. (23), then

$$ax_i^2 + 2bx_i + c = 0, \quad (28)$$

where

$$a = A_0 + \delta_q^T \delta_q, \quad (29)$$

$$b = A_0 (\Delta \lambda_i - \mathbf{g}_i) + \delta_q^T (\Delta \mathbf{p}_i + \eta_i \delta_{hi}), \quad (30)$$

$$c = A_0 (\Delta \lambda_i - \mathbf{g}_i)^2 + (\Delta \mathbf{p}_i + \eta_i \delta_{hi})^T (\Delta \mathbf{p}_i + \eta_i \delta_{hi}) - \Delta l^2. \quad (31)$$

From Eq. (28) x_i can be solved. Generally, two real roots can be solved from Eq. (28). A scalar product $\Delta \mathbf{p}_{i+1}^T \Delta \mathbf{p}^{\text{pr}}$ is calculated, and the root that gives the larger product is chosen ($\Delta \mathbf{p}^{\text{pr}}$ is the incremental displacement of the previous converged solution). Occasionally, when no real roots can be solved from Eq. (28), the pseudo line search technique is suggested by Lam and Morley [21].

With nonlinear analysis results obtained by the arc-length method, two criteria are employed to determine the failure state of CFT in the present study. First, if the negative load increment is acquired by the arc-length

method, the peak point of the load-displacement curve is captured, and the failure state is viewed to be achieved. On the contrary, if no softening behavior is observed, the yield of steel and crushing of concrete are chosen as criteria to judge the failure.

3. Reliability analysis

3.1. The first-order reliability method (FORM)

Approaches such as the Monte Carlo method, importance sampling method, FORM, and the second-order reliability method (SORM), are intensively used in structural reliability analysis. Generally, the failure probability can be written as

$$P_f = \int_{g(\mathbf{x}) \leq 0} f_X(\mathbf{x}) d\mathbf{x}, \quad (32)$$

where $\mathbf{X}$ is the vector of random variables, $f_X(\mathbf{x})$ denotes the joint probability density function of $\mathbf{X}$, and $g(\mathbf{x})$ is the limit state function.

For reliability analysis by FORM, the basic random variables must be transformed into the uncorrelated standard normal space by $\mathbf{Y} = \mathbf{Y}(\mathbf{X})$. Then, Eq. (32) is rewritten as

$$P_f = \int_{G(\mathbf{Y}) \leq 0} \varphi(\mathbf{y}) d\mathbf{y}, \quad (33)$$

where $\varphi(\mathbf{y})$ is the standard normal density of $\mathbf{Y}$ and $G(\mathbf{Y})$ is the limit state function in an uncorrelated standard normal space. Then, on the limit-state surface $G(\mathbf{Y})=0$, the point that has the minimum distance to the origin will be found. This point is named as the design point, which has the highest likelihood of being in the failure domain and makes the dominant contribution to the failure integral. The corresponding first-order estimate of failure probability is $P_f = \Phi(-\beta)$, where β , the reliability index, is the distance between the origin and design point. Therefore, the key task for FORM is to find the design point, and an effective iterative algorithm proposed in Refs. [5,22] is adopted in this paper.

3.2. Response surface method

To find the design point, the gradient of the limit state function to random variables should be calculated. For static reliability analysis of linear or geometrical nonlinear structures, Der Kiureghian and Ke [22], and Liu and Der Kiureghian [5] derived the analytical formula of the implicit gradient of the limit state function with respect to random variables. Since the material and geometrical nonlinear finite element analysis of CFT structures will be carried, the response gradient of the limit state

function with respect to random variables may be difficult to obtain directly, and the response surface method (RSM) is an effective way to meet this requirement.

The basic idea behind RSM is to approximate the real limit state function by an equivalent polynomial function. Compared with the traditional FORM algorithm, the search for the design point with RSM may be more effective and more tractable [23]. Moreover, such problems as numerical noise in the simulation process, non-smooth failure surface caused by multi-criterion, and model coupling in multidisciplinary analysis, also motivate the use of the response surfaces method.

The form of the response surface is usually chosen to be of a second-order polynomial. In this paper, a second-order polynomial without cross terms is adopted:

$$g(x) = a + \sum_{i=1}^n b_i x_i + \sum_{i=1}^n c_i x_i^2, \quad (34)$$

where n is the number of random variables X ; and a_0 , b_i , and c_{ii} are $2n+1$ unknown coefficients to be determined. Two kinds of experimental design, saturated design, and central composite design (CCD), are usually used to determine the unknown coefficients [24-26]. In the saturated design, the required number of sampling points is the same as that of unknown coefficients, and the unknown coefficients are obtained through solving a set of linear equations. CCD requires more sampling points, and regression analysis should be introduced to determine the unknown coefficients. Compared with saturated design, CCD are more accurate but less efficient. To reduce the computational cost, the saturated design method is adopted in this paper.

4. Example study

4.1. Reliability of ultimate resistance capacity of CFT column

The reliability of resistance of a CFT column with $D \times t = 216 \text{ mm} \times 4 \text{ mm}$ is investigated. The slenderness of the column is 18.52. The following material properties are employed in the deterministic analysis: compressive strength of unconfined concrete $f_c = 36.30 \text{ MPa}$, elastic modulus of concrete $E_0 = 32.50 \text{ GPa}$, strain corresponding to peak stress of unconfined concrete $\varepsilon_0 = 0.0022$, yield strength of steel $f_y = 343.55 \text{ MPa}$, and elastic modulus of steel $E_s = 20.00 \text{ GPa}$. The results of the resistance of column with deterministic analysis are plotted in Fig. 3.

In reliability analysis, the compressive strength of unconfined concrete f_c , yield strength of steel f_y , axial load P , and eccentricity e are chosen as random vari-

ables. f_c and f_y are assumed to be lognormal random variables with the following stochastic parameters:

$$\mu_{f_c} = 36.30 \text{ MPa},$$

$$\sigma_{f_c} = 4.21 \text{ MPa},$$

$$\mu_{f_y} = 343.35 \text{ MPa},$$

$$\sigma_{f_y} = 27.47 \text{ MPa}.$$

The axial load P is assumed as a normal random variable with mean of $0.57P_u$ (P_u is the ultimate resistance capacity obtained from deterministic analysis shown in Fig. 3) and $\text{COV}=0.05$. Eccentricity e is assumed to be normally distributed. Fig. 4 illustrates the influences of eccentricity e on the reliability index of ultimate resistance capacity.

Fig. 3 Ultimate resistance capacity with deterministic analysis

Fig. 4 Reliability of ultimate resistance capacity

It can be observed from Fig. 4 that although safety factors of ultimate resistance capacity obtained from deterministic analysis are equal, the results of the reliability index are quite different. With the increase in mean value of eccentricity, the reliability index of ultimate resistance capacity increases. Therefore, it can be deduced that it is almost impossible to define an identical safety factor for different load cases, and reliability analysis

method should be suggested for calibration of design code of CFT structures. Fig. 5 shows the results of sensitivity $\partial\beta/\partial y$ when standard deviation of eccentricity σ_e varied from $0.02D$ to $0.06D$. It can be viewed that with the increase in mean of eccentricity, the reliability index becomes more sensitive to the fluctuation of f_y and

its sensitivity to f_c remains almost unchanged. This phenomenon may be due to the fact that with the increase in eccentricity, bending moment will dominate the failure of column gradually, and as a result, the influence of uncertainty of f_y on structure reliability enhances. More over, it is should be noted that when the mean value of e

(a) $\sigma_e=0.02D$ (b) $\sigma_e=0.03D$ (c) $\sigma_e=0.04D$ (d) $\sigma_e=0.05D$ (e) $\sigma_e=0.06D$

Fig. 5 Results of sensitivity

is comparably small (i.e., concentrically loaded or small eccentrically loaded column), e is overwhelmingly sensitive to the reliability index among all variables.

As shown in Fig. 5, when μ_e equals zero and σ_e is greater than $0.02D$, uncertainty of e plays a dominant role for reliability of ultimate resistance capacity, which means that randomness of eccentricity may strongly harm the structural safety in the concentrically loading case. This phenomenon should be given enough attention in analysis and design for concentrically loaded CFT column.

4.2. Reliability of ultimate resistance capacity of imperfect slender CFT column

It has been widely accepted that the initial geometrical imperfection should be considered in design and analysis of slender compressive structures, especially when stability and buckling problems are concerned. To deal with the imperfection, it can be simply and conservatively assumed that the initial geometrical imperfection possess the same shape of the classical buckling mode. However, it should be worthwhile to note that the imperfection has a stochastic property in nature, and it may be rational to treat this phenomenon as a random field.

A slender concentrically loaded CFT column with simple support is studied. The geometrical configuration of the column is $L \times D \times t = 5000 \text{ mm} \times 216 \text{ mm} \times 4 \text{ mm}$. The material properties given in Example 1 are also adopted. In this example, axial load P is assumed to be a deterministic variable with value of 500 kN. The imperfection is assumed as a normal random field with mean value of zero, and the standard deviation σ_I is assumed to be $L/400$, $L/300$, and $L/200$, respectively. The autocorrelation coefficient function of the random field is defined as

$$\rho = \exp\left(-\frac{|\Delta x|}{\alpha L}\right), \quad (35)$$

where Δx is the distance between two points, L is the length of the column, and α is a dimensionless measure of the correlation length.

Generally, for use in such reliability algorithms as FORM, the random field should be represented in terms of random variables. To meet this end, several methods have been proposed in previous studies. In this paper, the domain of the random field is discretized into a mesh of random field elements, and the random field will be described through interpolation of the random element value represented by a set of random variables. The size of the random field element is an important consideration in random field discretization. Experience has shown that a

size of random field element equal to one-half to one-quarter of the correlation length is appropriate [5,22]. In the present example, the column is discretized into 9 random field elements with equal length.

Fig. 6 shows the reliability results of ultimate resistance when α varies from 0.1 to 0.6. From Fig. 6, it can be found that with the increase in standard deviation of initial geometrical imperfection, the reliability of slender CFT column decreases enormously. Moreover, it can also be viewed that with the increase in α , i. e., the random field approaches the random variable gradually, the reliability index decreases gradually.

Fig. 7 presents the geometrical imperfection at the design point when $\alpha=0.01$. It can be observed that the geometrical imperfection found at the design point is similar to the density function subject to the normal distribution, which is distinctively different from the imperfection with sine curve assumed in deterministic analysis.

To give a further discussion about this phenomenon, Fig. 8 gives the comparison of ultimate resistance capacity when imperfection is sine, triangular, and the present

Fig. 6 Influence of imperfection on reliability index

Fig. 7 Geometrical imperfection at design point

Fig. 8 Ultimate resistance capacity

form with the same peak value of $L/200$.

Although ultimate resistance capacity obtained from the present imperfection is about 1.12 times that computed with sine imperfection mode, the present imperfection has a higher likelihood of being in the failure domain, which should be paid more attention to imperfection control during construction process.

5. Conclusions

In this paper, a systematical approach for reliability analysis of CFT structures through combining FORM and nonlinear FEM is presented. Geometrical and material nonlinearity are involved in nonlinear analysis, and a modified constitutive model for core concrete is adopted to simulate the confining effect of CFT structures. Through combining FORM and RSM, the reliability of ultimate resistance capacity of CFT columns is evaluated.

Through a numerical example study, the influence of such parameters as material strength, eccentricity, slenderness, geometrical imperfection, etc., on the reliability of CFT columns is investigated. It can be concluded that inevitable random fluctuation of those parameters has significant influences on structural reliability, and that stochastic or reliability methods can give a more rational and subjective evaluation on the safety of CFT structures than a deterministic approach. Although the research of this paper focuses mainly on evaluating the CFT columns, the reliability of other CFT structures can also be obtained by following the same procedure.

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