Chaotic movement of vortices in superconductors of high- T_c superconducting maglev system at a high speed

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Abstract: Vortex motion in superconductors of high- T_c superconducting maglev system is studied by a computational simulation. The vortex system is treated in a similar fashion, as a system where defects are arranged in periodic arrays and driven by an alternating current (AC) field. Computational simulation reveals that due to the high nonlinearity of the vortex-defect interaction, the system shows typical chaotic characters. Effects of the vortex-vortex coupling, the driving frequency, and the vortex viscosity on the vortex motion have been studied to reveal the interaction between the spatial and temporal chaos. It is found that the mean-field approach is a good approximation to describe the vortex motion in one dimensional vortex system under an AC driving field. The vortex motion under the damping mode is a kind of self-organized motion. Lastly, the spatial chaos can dominate the chaotic behavior of the system.

Key words: superconductors; vortex motion; spatial chaos; temporal chaos

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1. Introduction

High temperature superconductor (HTS) bulk materials have many potential applications such as Maglev transportation [1]. However, issues of the vortex motion in HTS bulks in high-frequency-related applications, for example high speed HTS maglev trains, have not been well addressed to date.

In order to study such a topic, vortex pinning in systems where defects are arranged in periodic arrays and driven by an AC field are established as a “model” system for clarifying the optimal pinning arrangements and commensurability effects in high- T_c superconductors since the properties of the pinning sites, such as the size, periodicity of the array, and array geometry, can be highly controlled. Typical examples of periodic pinning systems include (1) one-dimensional periodic systems constructed by multilayers where the vortices interact with a corrugated substrate [2], and (2) two-dimensional periodic arrays constructed by periodically arranged microholes [3-6] or magnetic dots [7-9] of various geometries. With these “model” systems, interactions between vortex lattice and pinning array, such as commensurability effects, can be conveniently investigated. Usually,

at commensurate fields the vortices can form a highly ordered lattice while at incommensurate fields the vortex lattice can be disordered. This will result in very different pinning effects. Experimentally, it has been observed that different types of ordered vortex crystals can be stabilized at the different matching fields; and beyond the first matching field all the pinning sites are occupied with a single vortex and that the additional vortices sit in the interstitial regions [6]. In actuality, more complicated phenomena may be observed in these systems because besides the matching effect, the interaction between the defect and vortex is highly nonlinear, which introduces intrinsic disorder or chaotic nature into the system.

In this work, we investigate the spatial arrangement of vortices under the background of a periodic pinning array driven by an AC field in a simple one-dimensional system through a computational simulation. We demonstrate that arrangement of the vortices in a periodic pinning array shows a glassy state, depending on the vortex-defect interaction state and vortex-vortex interaction. Therefore, two types of disordered vortex states may exist in the system: one arises from the intrinsically chaotic nature of the nonlinear system, existing when the pinning disorder is low and pinning strength is weak; the other is related to the pinning disorder, which dominates when both the pinning disorder and pinning strength are strong.

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2. Interaction of vortices with periodic pinning array

When a bias current is applied to the vortex lattice system of a type-II superconductor with periodic pinning, a Lorentz force acts on the vortices, some of which are confined inside potential wells. This is the parabolic model proposed by Pla and Nori [10]. Therefore, the pinning center will act as parabolic traps for the vortices. Generally, considering the finite depth of the potential well and the nonlinear interaction between the flux and the pinning, the potential acting on the i th vortex, V_{pi} , can be described below in the one-dimensional case:

$$V_{pi} = \sum_j V \{1 - \cos[(x_i^v - x_j^p) / \xi_p]\}, \quad (1)$$

where x_i^v , x_j^p , and ξ_p represent the position of the vortex, the position of the pinning site, and the range of the pinning force, respectively. In addition, the vortex-vortex potential felt by a particular vortex i is approximately [10]:

$$V_{pi} = \begin{cases} \sum_{j \neq i} A_v (|x_i^v - x_j^v| - \xi_v)^2 & \text{for } |x_i^v - x_j^v| \leq \xi_v, \\ C & \text{for } |x_i^v - x_j^v| > \xi_v, \end{cases} \quad (2)$$

where C is a constant, ξ_v is the vortex-vortex interaction range, and A_v is the vortex-vortex interaction constant. If the driving force F_L is applied in the positive x direction, the total force that acts on the vortex i is

$$F_i = -\nabla_i V_{pi} - \nabla_i V_{vi} - \eta v_i + F_L, \quad (3)$$

where v_i is the velocity of the vortex i , and η is the friction coefficient. By denoting $2\pi\phi_n = (x_i^v - x_j^p) / \xi_p$, we can obtain the dynamic equation [11-12] for ϕ_n as

$$\frac{d^2\phi_n}{dt^2} + \eta \frac{d\phi_n}{dt} = A(\phi_{n+1} - \phi_n) - A'(\phi_n - \phi_{n-1}) - BP(n)\sin(2\pi\phi_n) + F_L, \quad (4)$$

where $n=1,2,3,\dots,N$, numbering the pinning site. The quantities and parameters involved in the equation are all dimensionless and defined as: ϕ_n is the displacement of the n th vortex from its balance position; η is the friction coefficient to the vortex motion; A and A' are the vortex-vortex interaction constant; B is the vortex-defect interaction constant; F_L is the Lorentz force; $P(n)$ satisfying $0 \leq P(n) \leq 1$, is the distribution function of the pinning center: when $P(n)=1$, it represents a periodic pinning potential; when $P(n)=\text{random function}$, it represents a disordered distribution of the pinning centers in superconductors. Most of the real systems can be well described by these two cases.

Different from what we have studied previously [13], the Lorentz force F_L here is no longer a constant, but a time-dependent term, $F_L = F_{L0} + \Delta \sin(\omega t)$, where F_{L0} is the conventional DC (direct current) Lorentz force, Δ is the amplitude of the AC driving force, and ω is the driving frequency which is related with the driving speed of the maglev train as defined in Ref. [14], $\omega = 2\pi v_0 / S_0$, with v_0 the velocity of the maglev speed and S_0 the length of permanent magnetic guideway segment. Therefore, the dynamic equation for ϕ_n at speed v_0 is given as:

$$\frac{d^2\phi_n}{dt^2} + \eta \frac{d\phi_n}{dt} = A(\phi_{n+1} - \phi_n) - A'(\phi_n - \phi_{n-1}) - BP(n)\sin(2\pi\phi_n) + F_{L0} + \Delta \sin(\omega t). \quad (5)$$

If there is no vortex-vortex interaction, i.e., $A=0$ and $A'=0$, Eq. (5) is reduced to an equation of a free damped pendulum. It is the case of a single vortex, and the motion is periodic. If the interaction of a vortex with other vortices is treated as a harmonic driven force, Eq. (5) is reduced to the form of a Josephson junction driven by an AC current. In this case, bifurcation and chaos are expected. This may be the cause of the exponential-law decay of the power spectra observed in the experiments [11-12,15]. Generally, Eq. (5) is a system consisting of many pendulums which couple with each other randomly. Thus it reflects the multi-body dynamics of a vortex. According to Bak et al. [16], this system should have a power-law distribution of lifetime, leading to the power spectrum of $1/f$, where f is frequency. It is a kind of self-organized criticality. Based on this understanding, Zhao et al. [17] provides a simulation on the noise spectrum of the vortex motion, which explains the experimental observation [16], indicating that Eq. (5) contains fundamental components necessary to describe the vortex motion in the system where defects are arranged in periodic arrays.

3. Results and discussion

The simulation presented below is done by the mean-field approach, which is a good approximation to describe the vortex motion in one dimensional vortex system. With the mean-field idea, the background of the motion of other vortices is described as a function $A(y,t)$ which can be expanded as

$$A(y,t) = \sum_{l=1}^L \sum_{m=1}^M \bar{A}_{m,l} \exp(i\omega_m t) \exp(ik_l y), \quad (6)$$

where k_l is the wave vector.

For a given pinning centre, y can be regarded as a constant, and Eq. (2) can be further simplified as

$$A(y_n, t) = \sum_{m=1}^M A_{m,n} \exp(i\omega_m t). \quad (7)$$

Therefore, Eq. (5) can be written as

$$\frac{d^2\phi_n}{dt^2} + \eta \frac{d\phi_n}{dt} = F_L - B \sin(2\pi\phi_n) + \sum_{m=1}^M A_{m,n} \cos(\omega_m t), \quad (8)$$

where $A_{m,n}$ is the vortex-vortex coupling strength between the n th and $(n-1)$ th vortices with the m th motion mode. It should be pointed out that the model is a highly simplified one. Usually, a two-dimensional model is used to describe the vortex system since a collection of vortices forms triangular lattices with or without disorders, amorphous, or liquid depending on temperature, magnetic field, pinning potentials, etc. To simplify the calculation, a one-dimensional model is also proposed to investigate the vortex motion or flux flow related phenomena [18-20]. As pointed out previously [21], a one-dimensional model is a good choice for simplifying the study on the chaotic behaviors of the vortex motion. The investigation on the present system may reveal some features unique to the vortex-defect interaction.

It is noticeable that Eq. (5) is mathematically equivalent with a model of the Josephson junction array [22], which has been intensively investigated from contexts of the resonance and synchronization including chaotic behaviors [23-25], although the present periodically pinned vortices and Josephson junction array are two different physical systems. The mathematical equivalency of the two systems provides a useful connection to reveal their universal features, thus, deepening the understandings of the intrinsic mechanism behind the different physical phenomena.

First we have examined the effect of the coupling strength A on the vortex motion. Results of $\phi(t)$ and its power spectrum have been calculated for the conditions of $F_L=0.01$, $B=0.45$, $\omega=0.5$, and with A changing from 0 to 0.6. The typical results are shown in Figs. 1 and 2.

The results show that the chaotic motion of the vortex exists as indicated by the broad peaks in the power spectra. These peaks are superposed on the power-law background. This is also the feature of the power spectra for the vortex noise experiments [11-12,15]. This feature is also consistent with those obtained in the previous studies on this system [13,21]. Therefore, the physics behind Eq. (8) is similar to that of Eq. (5). This gives us a good basis to discuss the influence of the coupling strength, A , on the vortex motion. As shown in Fig. 1 where A is significantly smaller than B , the vortex motion is unstable, showing a divergent movement. When the value of A is getting closer to that of B , the vortex motion becomes stable and convergent (see Fig. 2). When A be-

comes significantly larger than B , the vortex motion becomes divergent again (here the results are not shown). This is similar to what we observed in Refs. [13,21] which showed the mismatch between A and B leading to a dominant effect on the occurrence of the attractor crisis. The similarity of the effect of A on the attractor crisis in the present system with those shown previously [13,21] indicates again that the physics behind these system is the same, and the approximation of Eq. (8) for Eq. (5) still keeps the basic nature of the physics in the system.

As stated previously, the frequency of the background vortex motion may cover a wide range and with many different modes. Therefore, it is interesting to know the effect of the frequency change on the vortex motion, and then to estimate the effect of the single-mode approximation made for Eq. (5). Figs. 3 and 4 show the results of $\phi(t)$ and its power spectra in the conditions of $F_L=0.01$, $B=0.4$, $A=0.41$, $\eta=0$, and $0.3 \leq \omega \leq 0.9$. We identify that different values of ω in the range studied have no significant effect on the feature of the chaotic motion, especially on the power spectrum.

Fig. 1 Time evolution of the oscillation for the n th vortex (a) and the related power spectrum for the n th vortex (b) in the conditions of $F_L=0.01$, $B=0.45$, $A=0$, $\eta=0$, and $\omega=0.5$

(a)

(b)

Fig. 2 Time evolution of the oscillation for the n th vortex (a) and related power spectrum for the n th vortex (b) in the conditions of $F_L=0.01$, $B=0.45$, $A=0.4$, $\eta=0$, and $\omega=0.5$

However, like the effect of A , when the value of ω deviates from B , the motion becomes divergent, independent of whether or not the value of ω is significantly larger or smaller than B and A . This suggests that a match among B , A , and ω is important to sustain a stable and convergent vortex motion in the conditions considered here.

The influence of the viscosity on the vortex motion is not taken into account in the calculation. This may be one of the reasons that flux jumping is often observed in the calculation. Here we examine such a friction on the vortex motion. In the following calculation, we set $F_L=0$, $B=0.5$, $A=2.0$, $\omega=2.0$, and η between 0 and 0.04. When $\eta=0$, the vortex motion is divergent. This is a quite interesting phenomenon that the vortex motion is divergent at $\eta=0$ although there is no driving force existing ($F_L=0$). The real reason may be due to the intrinsic instability of such a system like Eq. (5). As analyzed by Zhao et al. [26], $\eta=0$ is an unstable fixed point for the system like Eq. (5). It means that the system will be driven away from its equilibrium position even by a small fluctuation, as observed in our numerical simula-

(a)

(b)

Fig. 3 Time evolution of the oscillation for the n th vortex (a) and related power spectrum for the n th vortex (b) in the conditions of $F_L=0.01$, $B=0.4$, $A=0.41$, $\eta=0$, and $\omega=0.3$

tion since computation fluctuation intrinsically exists. It is rare to observe such a phenomenon experimentally since it is hard to find out a real type-II superconducting system without any friction because edges of samples always play a role as a surface pinning force.

When $\eta \neq 0$, the motion soon becomes convergent. A typical result is shown in Fig. 5. With an increase in the value of η , the vortex motion rapidly converges to a chaotic and stable motion, i.e., a kind of self-organized motion. Because $\eta \neq 0$ means the existence of dissipation in the system, the decay of the amplitude of the oscillation is reasonable. The vortex motion does not decay to a static state or a periodic state driven by the “external force” [26] when the system is in the spatially chaotic state; however, it always reaches a stable and chaotic state, whereas it may reach a chaotic state or a periodic state when the system is not in the spatially chaotic state. This interesting observation indicates that the spatial chaos can dominate the chaotic behavior of the system. This may be a good explanation for a more complicated vortex motion in vortex-glass systems observed in high-

(a)

(b)

Fig. 4 Time evolution of the oscillation for the n th vortex (a) and related power spectrum for the n th vortex (b) in the conditions of $F_L=0.01$, $B=0.4$, $A=0.41$, $\eta=0$, and $\omega=0.6$

T_c superconductors [11-12,15], since spatial chaos dominates the system. In addition, a unique peak appears in the power spectrum which corresponds to the frequency of the “external force” (Here the “external force” is actually the mean-field effect of vortex-vortex interaction, and not a real external driving force). Therefore, the chaotic state possesses the “fingerprint” of the background of the vortices. This suggests that only collective modes belonging to the whole vortex system can be reflected by the motion of each individual vortex in its power spectrum.

The intensity of the characteristic peak in the power spectrum becomes stronger with the increase of the value of η ; the vortex motion is closer to a quasi-periodic one. This is to say, the viscosity to the vortex motion tends to “wash out” the nature of each individual vortex and to “emphasize” the feature of the driving force.

4. Conclusions

We have studied the vortex motion in the background

(a)

(b)

Fig. 5 Time evolution of the oscillation for the n th vortex (a) and related power spectrum for the n th vortex (b) in the conditions of $F_L=0$, $B=0.5$, $A=2$, $\eta=0.01$, and $\omega=2$

of many vortices in high- T_c superconductors under a high velocity. Effects of the vortex-vortex coupling, the driving frequency, and the vortex viscosity on the vortex motion have been studied to reveal the interaction between the spatial and temporal chaos.

A number of conclusions can be drawn from this study as follows:

The mean-field approach is a good approximation to describe the vortex motion. The vortex motion under the damping mode is a kind of self-organized motion. All of the transient states and the extrinsic effects are “washed out” during the damping process. The final state is largely determined by the stable spatial attractors of the nonlinear system with the fingerprints from the driving forces.

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