circumscribed about a circle meet at one point)?

Proof: Consider the circle as a plane section going through the center of a hyperboloid of one sheet. There are two families of straight lines on the hyperboloid, say A and B, such that through every point on the hyperboloid passes exactly one line of each family, and every line in family A meets every line in family B. Let $1, 2, \dots, 6$ be the six points where the sides of the hexagon are tangent to the circle. Consider the lines $l_1, \dots, l_6$ that lie on the hyperboloid and pass through $1, \dots, 6$, respectively. Lines l_1, l_3, l_5 are in family A and l_2, l_4, l_6 are in family B. Since the neighbor lines are in different families, they intersect to form a 3-dimensional hexagon; its top view is our original hexagon. (The vertices of this 3-dimensional hexagon are denoted by 12, 23, 34, 45, 56, 61 in the sequel.)

Consider now lines l_1 and l_4 . They belong to different families, so they intersect each other and lie in some plane p_{14} . Planes p_{25} and p_{36} are defined in a similar way. Now let us look at the intersection line of planes p_{14} and p_{25} . Points 12 and 45 lie on both planes, so the intersection of these two

planes is the diagonal 12–45. Since we have three planes, there are three intersection lines (12–45, 23–56, 34–61), and the point Q where the three planes meet is the point where these three lines meet. The top view of each of these lines is a main diagonal of our original plane hexagon, hence the top view of Q is the point where the main diagonals meet.

There is a version of this proof which works for all fields k of characteristic $\neq 2$ ("circle" must be replaced by "conic"; in characteristic 2 the Brianchon theorem makes no sense, since in that case all the tangent lines to a conic meet at one point and hence the diagonals of a circumscribed hexagon are not defined). Suppose that we have a conic C in a projective plane P^2 over k . We may assume that k is algebraically closed, that P^2 lies in the 3-dimensional projective space P^3 with homogeneous coordinates $(x:y:z:w)$, that P^2 is given by the equation $x + w = 0$, and that the conic C is the intersection of the "hyperboloid" H given by the equation $xw - yz = 0$ with P^2 . Writing every point $(x:y:z:w) \in P^3$ as a matrix with the rows (xy) and (zw) , we see that points of P^3 correspond to

FIGURE 10

non-zero 2×2 matrices X , considered up to a scalar factor. In terms of matrices the equation of H is $\det X = 0$; the equation of the plane P^2 is $\text{Tr } X = 0$; and the equation of their intersection C is $X^2 = 0$. There are two families of lines on H , say A and B, and every tangent plane to H intersects H in the union of an A-line and a B-line. For every point on C , represented by a (nilpotent) matrix X_0 , the equation of the plane tangent to H at this point is $\text{Tr } XX_0 = 0$. Since $\text{Tr } X_0 = 0$, scalar matrices satisfy this equation. Thus all planes tangent to H at points of C pass through the point $E \in P^3$ represented by scalar matrices. Now we can repeat the above proof: given a hexagon S in P^2 whose sides are tangent to C , construct a 3-dimensional hexagon S' , using in turn A- and B-lines, such that S is the projection of S' from E onto P^2 . (In the real case considered above, E was the point at infinity in the direction of the axis of the hyperboloid. Note that the assumption $\text{char } k \neq 2$ implies that the point E is not on P^2 .) The main diagonals of S' meet at one point, since the three planes p_{14} , p_{25} , and p_{36} through the opposite sides of S' meet at one point. It follows that the main diagonals of S also meet at one point.

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Soliloquy

John B. Thoo

To use a calculator, or not to use a calculator, that is the question:
Whether 'tis nobler in the mind to suffer
The slings and arrows of outrageous paper and pencil computations,
Or to take arms against a sea of troubles,
And by opposing, end them. To long multiply, to long divide—
No more, and by using a calculator to say we end
The heart-ache and the thousand natural shocks
That paper and pencil computations are heir to, 'tis a consummation
Devoutly to be wish'd.

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