Anaesthesia Equipment

A mass balance model for the Mapleson D anaesthesia breathing system

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A mathematical model is described which calculates the alveolar concentration of CO_2 (F_{ACO_2}) in a patient breathing through a Mapleson D anaesthesia system. The model is derived using a series of mass balances for CO_2 in the alveolar space, dead space, breathing system limb volume and reservoir. The variables included in the model are tidal volume (V_T), respiratory rate, fresh gas flow rate ($\dot{V}_f$), dead space volume, I:E ratio, and expiratory limb volume (V_I), time constant of lung expiration, and carbon dioxide production rate. The model predictions are compared with measurements made using a mechanical lung simulator in both spontaneous and controlled ventilation. Both the model and the experimental data predict that at high fresh gas flow rates and low respiratory rates, F_{ACO_2} is independent of $\dot{V}_f$; at low fresh gas flow rates and high respiratory rates, F_{ACO_2} is independent of respiratory rate. The model and the data show that the V_T influences F_{ACO_2} independent of minute ventilation alone, during both partial re-breathing and non-rebreathing operation. Therefore, describing the operation in terms of minute ventilation is ambiguous. It is also shown that V_I influences F_{ACO_2} such that, for any combination of patient and breathing-system variables, there is a V_I that minimizes the $\dot{V}_f$ required to maintain F_{ACO_2} . In addition, expiratory resistance can increase the fresh gas flow rate required to maintain a given F_{ACO_2} . The respiratory patterns

observed with spontaneous and controlled ventilation are responsible for the difference in $\dot{V}_f$ required with each mode of ventilation.

On décrit un modèle mathématique qui calcule la concentration alvéolaire du CO_2 (F_{ACO_2}) d'un patient qui ventile avec un système d'anesthésie du type Mapleson D. Le modèle utilise une série d'équilibrage statique pour le CO_2 dans l'espace alvéolaire, l'espace mort, le volume des tubulures et le réservoir. Les variables considérées dans le modèle sont le volume courant (V_T), la fréquence respiratoire, le débit des gaz frais ($\dot{V}_f$), le volume de l'espace mort, le rapport I/E, et le volume du circuit expiratoire (V_I), la constante de temps de l'expiration, et la production du CO_2 . Les prédictions du modèle sont comparées avec des mesures faites sur un simulateur de ventilation mécanique en ventilation spontanée et en ventilation contrôlée. Le modèle mathématique autant que les mesures expérimentales prédisent qu'à hauts débits de gaz frais et à basses fréquences respiratoires, la F_{ACO_2} est indépendante du $\dot{V}_f$; à bas débits de gaz frais et à hautes fréquences respiratoires, la F_{ACO_2} est indépendante de la fréquence respiratoire. Le modèle et les mesures montrent que le volume courant influence la F_{ACO_2} indépendamment de la ventilation minute seule, pendant la réinspiration partielle ou sans réinspiration des gaz expirés. Dès lors, décrire l'utilisation du système en termes de ventilation minute est ambigu. On montre aussi que V_I influence F_{ACO_2} de telle manière que pour quelle que soit la combinaison entre patient et variables du circuit respiratoire, il y a un V_I qui diminue le $\dot{V}_f$ requis pour maintenir la F_{ACO_2} . En plus, la résistance expiratoire peut augmenter le débit de gaz frais requis pour maintenir une F_{ACO_2} donnée. Les caractéristiques respiratoires observées en ventilation spontanée et contrôlée sont responsables des différents $\dot{V}_f$ requis dans chaque mode ventilatoire.

Key words

EQUIPMENT: anaesthesia circuits, non-rebreathing, Mapleson D.

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Despite their many desirable features, semi-open breathing systems, of which the Mapleson D is the best known, are not widely used for adult anaesthesia in many countries. Commonly cited advantages include a low resistance

to breathing, the absence of one-way valves, light weight, and the ability to change inspired anaesthetic concentration rapidly. Most of the objections to these breathing systems are based on the relatively high fresh gas flow

Abbreviations

E/I:	Expiratory to inspiratory time ratio.
f:	Frequency, respiratory rate.
$f_{\min}$:	Minimum respiratory rate achievable on an isopleth.
F_{ACO_2} :	Fractional concentration of CO_2 in alveolar gas.
F_b :	Concentration of reservoir gas.
F_D :	Concentration of gas occupying the dead space at end inspiration.
F_0 :	Concentration of fresh gas, equal to 0.
P_{ACO_2} :	Partial pressure of CO_2 in alveolar gas, directly proportional to F_a .
t:	time
T_{ee} :	Time taken for the alveolar gas that will remain in the limb to pass the Y piece (zones I and II).
T_{ei} :	Time taken during expiration for all the alveolar gas in V_{II} to pass the Y piece (zone III).
T_{eb} :	Time from start expiration for all of V_{Eb} to flow past the Y piece.
T_I :	Time of inspiration.
T_{pe} :	Time period of expiration.
T_{Ib} :	Time from start inspiration for all of V_{Ib} to flow past the Y piece.
τ :	Time constant of lung expiration.
V_b :	Reservoir (bellows/bag) stroke volume.
V_b :	Total reservoir volume less V_b .
V_{CO_2} :	Volume of CO_2 produced by metabolic processes per breath.
V_D :	Physiologic dead space volume.
V_{Eb} :	Volume of alveolar gas directly expired to the reservoir.
$V_{Ex}(t)$:	Volume of expired gas as a function of time.
V_{Ib} :	Volume of gas inspired from reservoir to the alveoli.
V_{II} :	Volume of gas inspired from limb to the alveoli.
V_I :	Limb volume (volume between fresh gas inflow and reservoir).
V_r :	Volume of rebreathed alveolar gas.
V_T :	Tidal volume.
$\dot{V}_A$:	Alveolar ventilation.
$\dot{V}_{\text{CO}_2}$:	Metabolic CO_2 production rate.
$\dot{V}_{Ex}(t)$:	Expiratory flow rate as a function of time.
$\dot{V}_f$:	Fresh gas flow rate.
$\dot{V}_{f,\min}$:	Minimum fresh gas flow rate achievable on an isopleth.
$\dot{V}_{Is}$:	Flow rate of gas delivered from the breathing-system.

rate required to prevent rebreathing of CO_2 . The high fresh gas flow rate wastes anaesthetic gases, heat, and humidity. If fresh gas flow rate is lowered to cause partial rebreathing, then P_{ACO_2} will depend on both minute ventilation and fresh gas flow rate in a manner that is not easily predicted. Furthermore, the relationship between P_{ACO_2} , fresh gas flow rate, and minute volume varies for different circuit configurations, and between spontaneous versus controlled ventilation. Clinical guidelines for fresh gas flow rate selection are empirical and vary widely,^{1,2} adding to the confusion.

Our goals were to improve upon the quantitative understanding of the mechanisms by which the Mapleson D breathing system operates and to investigate the effect of several system variables on CO_2 elimination. We hoped that the process of working towards these goals would provide some explanation for the wide variety of clinical recommendations for the use of the Mapleson D cited in anaesthesia practice and in the literature. We addressed this problem by mathematically modeling the mass balance of CO_2 in the lungs and breathing system to establish the relationship between the system variables (such as tidal volume, respiratory rate, and fresh gas flow rate) and the alveolar ventilation. The model yielded general solutions that describe the operation of the breathing system over a wide range of conditions and includes the effects of dead space, breathing system limb volume and breathing pattern. Model predictions were then validated by comparing them to experimental results obtained using a mechanical lung simulator.

Operation of the Mapleson D

The Mapleson D system can be operated with three different modes of ventilation, each requiring a slightly different mechanical configuration. Spontaneous ventilation is usually implemented with a hanging reservoir bag and an adjustable pressure limiting valve (APL) set at its minimal opening pressure (1–2 cm H_2O). Manually assisted ventilation is performed using the same equipment, but with the APL valve set to a higher opening pressure. Controlled automatic ventilation is implemented via a standing-bellows ventilator with an integral volume-limiting exhaust valve. For convenience, the ventilator bellows hose is typically connected to the bag port of the Mapleson D manifold and the APL valve is fully closed, thus approximately doubling the expiratory limb volume.

In both spontaneous and controlled automatic ventilation, gas is exhausted only during expiration. Assisted ventilation with a partially closed APL valve, however, is pressure limited, and a variable amount of gas is exhausted during inspiration. This analysis is limited to the first two modes of ventilation, in which the time when the exhaust valve opens is easily predicted.

In any configuration or mode of ventilation, two extremes of operation have been described. At high fresh gas flow rates relative to minute volume, there is no re-breathing of expired alveolar gas; P_{ACO_2} depends only on minute volume and is fresh gas flow independent. At low fresh gas flow rates relative to minute volume, there is partial re-breathing of expired alveolar gas; P_{ACO_2} depends only on the fresh gas flow rate and is minute volume independent.^{1,3-5}

Model overview

The details of the model are shown in the appendix; the following is a conceptual overview:

The model calculates the alveolar ventilation by relating the results of the mass balances for CO_2 in the alveolar space, the dead space, the reservoir, and expiratory limb of the breathing system. It assumes "plug flow," with no mixing across boundaries between adjacent plugs, but with instantaneous and complete mixing within compartments (alveolar space or reservoir) and when gas streams meet (fresh gas inflow and expired gas). The model further assumes constant inspiratory flows and exponentially decaying expiratory flows,⁶ consisting of plugs of dead space followed by alveolar gas. Because only volume-limited modes of ventilation are considered, the flow patterns and the distribution of gases are assumed to be identical for spontaneous ventilation and controlled automatic ventilation.⁷ Therefore, the same model is applied to both modes of ventilation.

The independent system variables included in the model are tidal volume (V_T), respiratory rate (f), fresh gas flow rate ($\dot{V}_f$), physiological dead space volume (V_D , the volume of gas on the patient side of the Y piece that does not contribute to the elimination of CO_2), I:E ratio, and breathing system limb volume (V_l , the undistended volume of tubing between the reservoir (bag or bellows) and the fresh gas input), time constant of lung expiration (τ), and metabolic carbon dioxide production rate ($\dot{V}CO_2$). Together, these determine the value of the dependent variables: fractional concentration of CO_2 in alveolar gas (F_{ACO_2}) and alveolar ventilation ($\dot{V}_A$). Note that F_{ACO_2} is directly proportional to P_{ACO_2} .

We define alveolar ventilation as the volume of fresh gas (CO_2 -free gas) that reaches the alveoli and is available for exchange per unit time.^{1,8-10} Rebreathing is defined as the re-inspiration of alveolar gas above the unavoidable amount left in the dead space at end expiration. By these definitions, *rebreathed alveolar gas* (V_r), *similar to dead space gas*, does not contribute to the alveolar ventilation:

$$\dot{V}_A = (V_T - V_D - V_r) \cdot f \quad (1)$$

We find this physiological definition more descriptive than the mechanical definition sometimes used, in which al-

veolar ventilation is the minute ventilation less the dead space ventilation, regardless of rebreathing. By the former definition, breathing gas at a fixed minute volume but with an inspired P_{ACO_2} that is equal to one third the alveolar P_{ACO_2} will provide two-thirds the alveolar ventilation that would be achieved breathing CO_2 -free gas. Using the latter definition, the alveolar ventilation would be the same in both cases.

The following mass balance allows the measurement of alveolar ventilation. Under steady-state conditions, the amount of CO_2 entering the lung must equal the amount leaving the lung:

$$\dot{V}_{CO_2} = F_{ACO_2} \cdot \dot{V}_A$$

and therefore,

$$\dot{V}_A = \frac{\dot{V}_{CO_2}}{F_{ACO_2}} = (V_T - V_D - V_r) \cdot f \quad (2)$$

Note that for a given $\dot{V}_{CO_2}$, F_{ACO_2} and $\dot{V}_A$ are inversely proportional.

The model determines the steady-state relationship between alveolar ventilation and the system variables by determining F_{ACO_2} . Rearranging Equation 2:

$$F_{ACO_2} \cdot (V_T - V_D) \cdot f = \dot{V}_{CO_2} + f \cdot V_r \cdot F_{ACO_2} \quad (3)$$

where the left side of the equation gives the amount of CO_2 leaving the lung, and the right side, that entering the lung.

The model is divided into four steady-state zones of operation, which identify the sources of all the rebreathed alveolar gas, V_r . For example, gas inspired to the alveoli may originate in the reservoir and/or in the limb; either source of gas may include some previously expired alveolar gas. For each zone of operation, an algebraic relationship is determined between the system variables and V_r . Equation 3 then relates the system variables to the alveolar concentration. These relationships are solved computationally to simulate the operation of the breathing system under a wide range of f , $\dot{V}_f$, F_{ACO_2} , V_T , V_D , and V_l . The computations were programmed in C and executed on a Sun Sparc Station 1.

Since one of the goals of this exercise is to provide an explanation for the variety of clinical recommendations for the use of the Mapleson D, the results of the model are presented in terms of the fresh gas flow rate required to keep a constant alveolar CO_2 concentration.

EXPERIMENTAL APPARATUS

The mathematical model predictions were compared to a mechanical model consisting of a Mapleson D breathing system connected to a Bio-Tek Instruments VT-2 lung simulator (Figure 1). The compliance was set to 0.05 l/cm H_2O^{-1} and the airway resistor used was R_{P20} (following Bio-Tek's recommendation for the simulation of

FIGURE 1 Experimental apparatus. Mapleson D breathing system is shown configured for controlled ventilation with patient model and measurement devices. Extra dead space was added to bring the total to 150 ml.

a normal adult). Carbon dioxide production was simulated by adding $240 \pm 7.5 \text{ ml} \cdot \text{min}^{-1}$ of 100% CO_2 gas into the alveolar chamber via a pressure-compensated rotameter. A fan installed in the alveolar chamber enhanced mixing. Concentration measurements were taken from the alveolar chamber using an Ohmeda 5200 CO_2 monitor, accurate to $\pm 0.2\%$ CO_2 , with a sample flow rate of $300 \pm 100 \text{ ml} \cdot \text{min}^{-1}$. Since this monitor effectively removed part of the added CO_2 before it could reach the breathing system, CO_2 production was calculated as $(\dot{V}_{\text{CO}_2} = 240 - 300 \cdot F_{\text{ACO}_2}) \text{ ml} \cdot \text{min}^{-1}$. The total volumetric lung simulator dead space was set to 150 ml.

The respiratory rate, tidal volume, and inspiratory flow were controlled by a personal computer (Epson Equity III+), which was interfaced to a specially modified Ohmeda 7800 ventilator. Tidal volume was measured with a Fleisch pneumotachograph placed within the dead space. The pneumotachograph was calibrated at the beginning of each data set. The computer integrated the expiratory flow signal and provided feedback control of the delivered tidal volume by continually correcting the set tidal volume of the ventilator. The fresh gas consisted of medical grade air, controlled by a pressure-compensated rotameter and measured with a pressure and temperature-compensated laminar flow element ($\pm 4\%$). The fresh gas flow rate and the carbon dioxide concentration measurements were also acquired by the computer via an A/D board (Data Translation DT2801A).

For simulation of spontaneous respiration, a second Bio-Tek lung simulator was placed adjacent to the original in the manner described by Katz *et al.*¹¹ The second lung was powered by the ventilator, which inflated the original lung simulator via a mechanical linkage while allowing passive expiration.

The physiological difference between O_2 uptake and CO_2 production was simulated by the CO_2 monitor's sample flow rate of $300 \text{ ml} \cdot \text{min}^{-1}$. Since the CO_2 input was set at $240 \text{ ml} \cdot \text{min}^{-1}$, a respiratory quotient of 0.8 was obtained.

Data sets were taken with tidal volumes of 1050, 450, and 350 ml with $V_I = 425 \text{ ml}$, $I:E = 1:2$, $F_{\text{ACO}_2} = 6\%$ CO_2 , and $\dot{V}_{\text{CO}_2} = 222 \text{ ml} \cdot \text{min}^{-1}$ under controlled and spontaneous ventilation. Additional data sets were taken with limb volumes of 60, 180 and 425 ml, with $V_T = 700 \text{ ml}$, $I:E = 1:2$, $F_{\text{ACO}_2} = 6\%$ CO_2 , and $\dot{V}_{\text{CO}_2} = 222 \text{ ml} \cdot \text{min}^{-1}$ under spontaneous ventilation alone. Each data set was taken at respiratory rates between 5 and 30 bpm.

Each data point was obtained by linear interpolation of the measured alveolar concentration between either two fresh gas flow rates (at constant respiratory rate) or two respiratory rates (at constant fresh gas flow rate), which bracketed the desired alveolar concentration. The system variables were allowed to stabilize for at least four minutes between any adjustment.

Despite the fixed volumetric dead space of 150 ml,

the physiological dead space is known to vary with V_T ^{1,12,13} and thus V_D was measured for each V_T used. This was done by operating the breathing system at high-enough fresh gas flow rates to ensure zero inspired CO_2 ($15 \text{ L} \cdot \text{min}^{-1}$) and using the following rearrangement of Equation 3:

$$V_D = V_T - \frac{\dot{V}_{CO_2}}{f \cdot F_{ACO_2}} \quad (4)$$

These measured physiological dead space volumes were used in subsequent comparisons of computational and mechanical model results.

The expiratory resistance was quantified for the configuration used with controlled ventilation (bellows assembly with internal exhaust valve) and for the configuration used with spontaneous ventilation (APL valve set open). The pressure drop due to steady flow ($15 \text{ L} \cdot \text{min}^{-1}$) was measured from the Y-piece to the exhaust of each configuration. The measured pressure drop was $3.6 \text{ cm H}_2\text{O}$ for the bellows assembly and $0.6 \text{ cm H}_2\text{O}$ for the APL valve.

The time constant of lung expiration (τ) used in the computations was determined experimentally with the pneumotachograph. It was noted that τ was consistently larger for controlled ventilation than for spontaneous ventilation (presumably because the resistance to flow through the bellows assembly used in controlled ventilation is greater than the resistance to flow through the APL valve used with spontaneous ventilation). A single τ was obtained for each mode of ventilation by curve-fitting expiratory flow data with various combinations of V_T and f . For controlled ventilation $\tau = 0.47 \text{ sec}$ ($R^2 = 0.989$) and for spontaneous ventilation $\tau = 0.24 \text{ sec}$ ($R^2 = 0.912$).

Results

Computer model predictions

In agreement with the observations of previous investigators, the model predicts that the Mapleson D has two operating extremes (Figure 2).^{1,3-5} With a fixed tidal volume, at high fresh gas flow rates and low respiratory rates, P_{ACO_2} (F_{ACO_2}) depends only on respiratory rate and is fresh gas flow rate-independent; at low fresh gas flow rates and high respiratory rates, P_{ACO_2} depends only on the fresh gas flow rate and is respiratory rate-independent. Both the minimum respiratory rate achievable on an isopleth ($f_{\min}$) and the minimum fresh gas flow rate achievable on an isopleth ($\dot{V}_{f,\min}$) decrease with increasing alveolar concentration. The model also predicts the manner in which the transition is made from one extreme to the other along each isopleth.

The individual effects of tidal volume, physiological dead space volume, and limb volume on the fresh gas

FIGURE 2 Model predictions of isopleths for various alveolar concentrations of CO_2 (F_{ACO_2}). Alveolar concentration ranges from 3 to 7% CO_2 , with all $V_T = 700 \text{ ml}$, $V_I = 425 \text{ ml}$, $V_D = 150 \text{ ml}$, $I:E = 1:2$, $\tau = 0.47 \text{ seconds}$, and $\dot{V}_{CO_2} = 22 \text{ ml} \cdot \text{min}^{-1}$. Three regions are observed: one in which respiratory rate controls concentration, one in which fresh gas flow rate controls the alveolar concentration, and a third transition zone between the first two, in which both fresh gas flow rate and respiratory rate influence alveolar concentration.

flow rate and respiratory rate required to keep a constant level of F_{ACO_2} (6%) are shown (Figure 3). At a single level of F_{ACO_2} , as V_T increases, both $\dot{V}_{f,\min}$ and $f_{\min}$ decrease (Figure 3a). Further increases in V_T above some threshold, however, do not additionally lower $\dot{V}_{f,\min}$. Decreases in V_D behave similarly to increases in V_T (Figure 3b). In general, for a given f , as V_I is increased, $\dot{V}_f$ decreases (Figure 3c). For example, at an f of 15 bpm, $\dot{V}_f$ decreases from $7 \text{ L} \cdot \text{min}^{-1}$ to $4 \text{ L} \cdot \text{min}^{-1}$ as V_I is increased from 50 to 450 ml. However, if V_I becomes large enough, $\dot{V}_f$ will jump to some higher value and will become independent of further increases in V_I . For example, at an f of 15 bpm, $\dot{V}_f$ jumps to $4.5 \text{ L} \cdot \text{min}^{-1}$ when V_I is 500 ml or larger.

In general, small tidal volumes, large physiological dead space volumes, and small limb volumes produce isopleths whose transitions between the two operating extremes are less sharp, drawn out over a greater range of respiratory rates.

Mechanical model findings

Data obtained with the mechanical lung simulator are shown in Figures 4 and 5. The effect of V_T on $\dot{V}_{f,\min}$ for both spontaneous and controlled ventilation, is compared with the model predictions (Figure 4). The physiological dead space volumes measured in the simulator and then used in the model are 369, 192, 173 ml for the tidal volumes of 1050, 450, and 350 ml, respectively.

The experimental data follow the two operating extremes predicted by the model. For each V_T , $f_{\min}$ matches

FIGURE 3 Model predictions showing the individual effect of: (a) tidal volume (350 to 1050 ml), (b) dead space volume (50 to 450 ml), and (c) the limb volume (50 to 1000 ml) on the fresh gas flow rate and respiratory rate required to maintain a normal alveolar concentration of CO_2 . Unless otherwise noted the conditions were $V_T = 700$ ml, $V_I = 425$ ml, $V_D = 150$ ml, $I:E = 1:2$, $\dot{V}_{CO_2} = 222$ ml $\cdot \min^{-1}$, $\tau = 0.47$ seconds, and $F_{ACO_2} = 6\%$ CO_2 .

FIGURE 4 Results of mechanical model testing. Data for spontaneous ventilation (open symbols) and controlled ventilation (closed symbols) are compared to the model predictions for spontaneous ventilation (dashed lines) and controlled ventilation (solid lines). Three tidal volumes are shown, 1050 ml (squares), 450 ml (triangles), 350 ml (circles) and their respective, experimentally determined, physiological dead space volumes: 369, 192, 173 ml. The time constant of expiration used in the model was $\tau = 0.47$ seconds for controlled ventilation isopleths and $\tau = 0.24$ seconds for spontaneous ventilation. The conditions were: $F_{ACO_2} = 6\%$ CO_2 , $I:E = 1:2$, $V_I = 425$ ml, and $\dot{V}_{CO_2} = 222$ ml $\cdot \min^{-1}$.

FIGURE 5 Results of mechanical testing comparing limb volumes of 60 ml (squares), 180 ml (triangles), and 425 ml (circles). The conditions were: $F_{ACO_2} = 6\%$ CO_2 , $I:E = 1:2$, $V_T = 700$ ml, $\tau = 0.24$ sec, and $\dot{V}_{CO_2} = 222$ ml $\cdot \min^{-1}$ under spontaneous ventilation. V_D was determined experimentally to be 266 ml.

the model predictions well. However, the model consistently under-predicts $\dot{V}_{f,min}$ (Table I). This discrepancy is much more pronounced with smaller tidal volumes. In addition, the model also predicts that $\dot{V}_{f,min}$ is higher with controlled than with spontaneous ventilation.

The $\dot{V}_{f,min}$ for each isopleth shown in Figure 4 are tabulated (Table Ia). Percentage differences between the

TABLE I Comparison of the $\dot{V}_{f,min}$ predicted by the model and exhibited by the data. (a) $\dot{V}_{f,min}$ s for the model predictions and data of Figure 4. (b) Percentage difference of the $\dot{V}_{f,min}$ s between the model and the data for spontaneous and controlled ventilation. (c) Percentage difference of the $\dot{V}_{f,min}$ s between controlled and spontaneous ventilation for both the model and the data.

(a)

Type	Ventilation mode	V_T (ml)	$\dot{V}_{f,min}$ ($L \cdot \text{min}^{-1}$)
Data	Spontaneous	350	7.57
Data	Spontaneous	450	6.10
Data	Spontaneous	1050	4.25
Data	Controlled	350	9.88
Data	Controlled	450	7.01
Data	Controlled	1050	4.73
Model	Spontaneous	350	6.04
Model	Spontaneous	450	5.17
Model	Spontaneous	1050	3.86
Model	Controlled	350	7.87
Model	Controlled	450	6.06
Model	Controlled	1050	3.95

(b)

Ventilation mode	V_T (ml)	% Difference between data and model
Spontaneous	350	25.3
Spontaneous	450	18.1
Spontaneous	1050	10.3
Controlled	350	25.5
Controlled	450	15.7
Controlled	1050	19.7

(c)

Type	V_T (ml)	% Difference spontaneous and controlled
Data	350	23.4
Data	450	13.0
Data	1050	10.0
Model	350	23.3
Model	450	14.8
Model	1050	2.4

$\dot{V}_{f,min}$ predicted by the model and exhibited by the data are shown for both spontaneous and controlled ventilation (Table Ib). Percentage differences between the $\dot{V}_{f,min}$ for controlled and spontaneous ventilation are tabulated for both the model and the data (Table Ic).

The data taken at different limb volumes show that V_l does influence $\dot{V}_{f,min}$, however, not as dramatically as the model predicts (Figure 5). Of the three limb volumes tested, the median sized limb volume (180 ml) has the lowest $\dot{V}_{f,min}$. This finding is both predicted by the model and confirmed by the experimental data.

Discussion

This modeling effort was motivated by a desire to develop a quantitative understanding of the mechanisms by which the Mapleson D breathing system operates, to describe the individual effects of respiratory rate, fresh gas flow rate, tidal volume, and limb volume on CO_2 elimination, and to provide some explanation for the variety of clinical recommendations for its use.

Comparison with other models

From the time Mapleson classified rebreathing systems, mathematical characterizations have been sought to relate fresh gas flows, minute ventilation and alveolar P_{ACO_2} . Mapleson noted that a fresh gas flow greater than peak inspiratory flow would prevent rebreathing in the D system.¹⁴ Later analyses established algebraic relationships between some system variables to determine alveolar concentrations of CO_2 under rebreathing conditions,¹⁵ and these sometimes contain a semi-empirical term related to the distribution of alveolar gas within the breathing system.^{1,3,9} Such analyses describe the fresh gas requirement in terms of fresh gas efficiency, which is essentially the alveolar ventilation normalized by the fresh gas flow rate.

Other analyses have been entirely computational.^{16,17} Finite element analysis can demonstrate the effect of any system variable, such as limb volume, sample sites, and complex breathing patterns. It can also be used to quantify the concentration of CO_2 anywhere in the breathing system. Although these computational solutions can be accurate, they are not intuitive.

The "semi-analytic" solutions for each zone of operation shown in the appendix incorporate more system variables than previous algebraic models (limb volume, respiratory pattern, time constant of expiration). Compared to the purely computational approach, this model provides a more intuitive feel for how different variables dominate the control of F_{ACO_2} . For example, the solution shown in the appendix for zone III shows that the $\dot{V}_f$ required to maintain F_{ACO_2} depends upon the ratio V_D/V_T and not on V_l .

Validation of the model

The overall behaviour of the model is generally confirmed by the data (Figure 4), with some important differences. The fresh gas flow rates predicted by the model generally underestimate those observed in the data (Table I). Fur-

thermore, the transitions between operating extremes observed in the data occur over wider ranges of respiratory rates than in the model.

At the high fresh gas flow rate, non-rebreathing operating extreme, the model predicts the same $f_{\min}$ that the data exhibits. Although the volumetric dead space volume was 150 ml, the physiological dead space, the portion of V_T that does not contribute to gas exchange, increases with V_T .^{1,12,13} Our V_D measurements confirm this. The experimentally determined dead space volumes were used in the model, causing the model predictions of $f_{\min}$ to match closely that of the data.

At the high respiratory rate, rebreathing operating extreme, the model consistently under-predicted the fresh gas flow rates observed in the data (Figure 4). One possible explanation for this discrepancy is deviation from the plug flow assumption. Consider, for example, the superposition of the expired alveolar gas and the fresh gas added to it during expiration as they flow away from the patient (Figure 6). Because the fresh gas flow rate is constant and the expired alveolar gas flow rate decays exponentially, a sigmoidal concentration gradient is formed in the limb. (This concentration profile was created by dividing the time of expiration into small time increments and calculating the volume and concentration of each corresponding segment from the relative velocities and the known concentrations of each contributing gas.) This combined alveolar and fresh gas forms a concentration profile that will be referred to as the "expiratory wave." The distal edge of the expiratory wave has a concentration close to alveolar and the proximal edge has a concentration of zero (Figure 6a). A portion of the expiratory wave (to the right of the market arrow) will be inspired to the alveoli on the next breath. With the assumption of "plug flow," increasing $\dot{V}_f$ will shift the expiratory wave to the left, decreasing the amount of CO_2 rebreathed on the next inspiration. In reality, axial mixing and dispersion in the limb will cause the concentration gradient to appear flatter (Figure 6b), causing the amount of reinspired CO_2 to increase. Thus, the plug flow assumption causes the model to under-predict the amount of CO_2 inspired and hence under-predict the fresh gas requirement.

The data show that the transitions between operating extremes are not as sharp as the model predicts. Again, this may be the result of axial mixing and dispersion, causing some alveolar gas to be inspired under conditions not predicted by the model.

The model also consistently under-predicts the fresh gas flow rate by an amount that generally increases with decreasing tidal volume. Since expiration is exponential, the lower tidal volumes will have lower velocities and, likely, less mixing due to turbulence. Thus, these argu-

FIGURE 6 "Expiratory wave" in the limb at end expiration. During expiration, the fresh gas flow, with constant flow rate, and alveolar gas, with exponentially decaying flow rate, result in a sigmoidal concentration gradient of CO_2 in the limb. (a) With the plug flow assumption, the distal end of the expiratory wave has a concentration close to F_a and the proximal end contains no alveolar gas. The gas to the right of the arrow will be inspired on the next breath. (b) If dispersion is allowed, the concentration gradient flattens and more alveolar gas will be reinspired.

ments concerning the dispersion of the expiratory wave can not account for this trend.

Tidal volume and respiratory rate act independently

The operating extremes demonstrated in this study have been described in the past as occurring at certain minute volume to fresh gas flow rate ratios.^{1,3-5} In fact, most of the analyses performed on the Mapleson D and other rebreathing systems have described the fresh gas requirements in terms of minute ventilation.^{2,7,10,15,18-20} Our results, however, clearly show that at the high minute ventilation and low fresh gas flow rate operating extreme, f does not effect the fresh gas requirement while V_T does (Figures 3a and 4). Thus, characterizations of the Mapleson D as a function of minute volume alone are inadequate.

For example, the notion that, under rebreathing conditions, alveolar concentration in the Mapleson D is controlled solely by fresh gas flow rate may lead to confusion. In the rebreathing extreme of operation, if minute ventilation were increased by increasing V_T , $\dot{V}_{f,\min}$ would be decreased (Figure 3a). In the rebreathing extreme of operation, the use of a $\dot{V}_f$ that is higher than necessary will result in a decrease in F_{ACO_2} (Figure 2). Thus, if $\dot{V}_f$ is not lowered, F_{ACO_2} will decrease. This helps explain

FIGURE 7 The impact of limb volume on the storage of fresh gas and alveolar gas. (a) A small limb volume stores little fresh gas in the limb, stores alveolar gas in the reservoir, and alveolar gas is inspired from the reservoir. (b) The limb volume that just barely allows alveolar gas to enter the reservoir stores more fresh gas in the limb, and almost no alveolar gas is inspired from the reservoir. (c) A large limb volume that does not allow alveolar gas to enter the reservoir stores fresh gas and alveolar gas in the limb.

some of the difficulty associated with using the Mapleson D with partial rebreathing and also helps to explain the unpredictability associated with maintaining P_{ACO_2} during spontaneous ventilation, where V_T is not controlled.

The independent influence of tidal volume and respiratory rate suggests recommendations for minimizing $\dot{V}_f$ during controlled ventilation. The data show that at constant minute ventilation, a V_T of 1050 ml and f of 10 bpm requires half the $\dot{V}_f$ as would operation at V_T of 350 ml and f of 30 bpm (Figure 4). Thus, for the same minute ventilation, a high V_T and low f combination

requires a lower $\dot{V}_f$ than a high f and low V_T combination. The tidal volume and respiratory rate must be considered separately.

The limb volume influences the fresh gas requirement

Although the limb volumes available to anaesthetists are limited to several discrete sizes, an analysis of the limb volume can be insightful. The influence of the limb volume on the $\dot{V}_f$ required to maintain F_{ACO_2} is shown by the model (Figure 3c) and the experimental data (Figure 5). In both of these analyses, over most respiratory rates, the limb volume with the lowest $\dot{V}_f$ is neither the largest nor the smallest evaluated.

To explain these phenomena, consider how the limb and the reservoir store fresh gas and alveolar gas. At end expiration, depending on the relative volumes of the limb and expiratory wave, the limb will contain all or the most proximal portion of the expiratory wave. As described earlier, the fresh gas in the expiratory wave is unevenly distributed and is concentrated in the proximal end (Figure 6a). This fresh gas is useful for alveolar ventilation. In contrast, the gases in the reservoir are mixed. Gas inspired to the alveoli, but not supplied by the fresh gas flow, must come from the limb, and if necessary, from the reservoir.

If the limb volume is very small, at end expiration it will contain no alveolar gas; if V_T exceeds $V_I + V_D$, alveolar gas will have entered directly into the reservoir (Figure 7a). Gas from the reservoir, which contains alveolar gas, will be required to contribute to the next V_T . As the limb volume is increased, with all other variables fixed, the storage capacity of the limb for fresh gas increases. Also, less alveolar gas will enter the reservoir directly during expiration, and the reservoir will be required to contribute less gas to the next V_T . Thus, while V_I is small, increasing V_I decreases the amount of rebreathed alveolar gas.

Eventually, as the limb volume is increased further, alveolar gas does not enter the reservoir and is entirely stored in the limb (Figure 7b). This condition is approximated by the relation:

$$V_I \geq V_T - V_D - \dot{V}_f \cdot T_I \quad (5)$$

where T_I is the time of inspiration. At limb volumes below this condition, the amount of rebreathed alveolar gas (V_r) decreases with increasing V_I . At limb volumes above this condition, a slight increase in V_I is observed, the amount of which is independent of V_I . Thus, Equation 5 determines the optimal V_I which minimizes $\dot{V}_{f,min}$.

When V_I is small enough to allow alveolar gas into the reservoir, the most distal gas inspired to the alveoli is reservoir gas that contains diluted alveolar gas. When V_I is larger and does not allow gas into the reservoir

(Equation 5), the most distal gas inspired to the alveoli contains mostly undiluted expired alveolar gas (Figure 7c). Thus, with all other variables fixed, increasing V_I above the level determined by Equation 5 slightly increases the amount of rebreathed alveolar gas.

The use of a V_I that is too small can increase the V_T , and consequently, can increase the $\dot{V}_f$ required to maintain F_{ACO_2} (Figures 3c and 5). The increase in $\dot{V}_f$ resulting from the use of too large a V_I , over the optimal V_I , is measurable, but not clinically significant. Therefore, for reasons of fresh gas economy and because V_T and other variables are constantly changing, it is recommended that the limb volume be larger than necessary rather than smaller.

The limb volumes provided by manufacturers are typically about 425 ml. Given the typical adult's V_T , V_D , $\dot{V}_f$ and inspiratory times, this V_I may not be optimal. The resulting $\dot{V}_f$ required to maintain F_{ACO_2} may be higher than if V_I were larger. Depending on the ventilatory variables chosen, the optimal V_I can range from 425 to 900 ml. Connecting a remote ventilator via a corrugated tube will roughly double V_I , and probably increase the $\dot{V}_f$ required to maintain F_{ACO_2} slightly over the $\dot{V}_f$ required to maintain F_{ACO_2} by the optimal V_I . The use of adult-sized limb volumes with paediatric patients would result in a similar small increase in the required $\dot{V}_f$.

Clearly, the limb volume influences the fresh gas flow rate required to maintain P_{ACO_2} (F_{ACO_2}) in the rebreathing extreme of operation. Many of the previous studies regarding the appropriate use of the Mapleson D did not report the limb volumes used.^{1-4,10,18,19,21-24} Thus, one cause of the many conflicting recommendations may result from the use of different limb volumes in those studies.

The fresh gas requirement increases with expiratory resistance

The model predictions shown (Figure 4) were generated under identical conditions, except that the experimentally determined time constant, τ , used with controlled ventilation was higher than that for spontaneous ventilation. As a result, the model predicted a higher fresh gas requirement with controlled ventilation. The experimental data support this result.

The time constant of expiration of a lung is proportional to the resistance to flow the gas must overcome. A passively expiring lung should have a single, fixed τ that is determined by its own compliance and resistance to flow. However, when a breathing system is attached, the added resistance to flow can increase τ . An increase in τ will cause the lung to empty more slowly, effectively lowering the concentration gradient in the expiratory wave

(Figure 6a), and increase the amount of rebreathed alveolar gas.

The measured pressure drop across the expiratory pathway due to a steady flow of $15 \text{ L} \cdot \text{min}^{-1}$ was much greater with a bellows assembly (3.6 cm H_2O) than with an APL valve (0.6 cm H_2O). Therefore, the resistance, and hence, the expiratory time constant observed in controlled ventilation should be larger than that for spontaneous ventilation. This was verified by the expiratory flow data and computed time constants.

Therefore, increases in expiratory resistance to flow can increase τ and subsequently, the fresh gas required to maintain P_{ACO_2} .

The fresh gas requirement observed with each mode of ventilation is determined by the respiratory pattern

Many clinical studies of the Mapleson D in the literature recommend fresh gas flow rates two to three times minute ventilation for spontaneous ventilation^{14,25,26} while others recommend fresh gas flow rates on the order of minute ventilation with controlled ventilation.^{2,10,18} These recommendations are commonly cited in anaesthesia practice.

The data in this study were taken at a constant I:E ratio (1:2), for both modes of ventilation. It is shown that controlled ventilation actually requires more fresh gas flow than spontaneous ventilation, for any respiratory rate and tidal volume combination (Figure 4), in part, because of the time constants that result from the configuration of each mode of ventilation. Thus, the recommendations for higher fresh gas flow rates during spontaneous ventilation cannot be a result of the mode of ventilation itself. More likely, they are a result of the respiratory patterns typically observed with these models of ventilation.

Many researchers have noted a link between respiratory pattern and alveolar ventilation in the Mapleson D.^{1,5,13,27,28} This study shows that, at a given minute ventilation, slow deep breathing requires a lower fresh gas flow rate than rapid shallow breathing (Figure 4). Although data were not taken to elucidate specifically the effect of I:E ratio, others have shown that a lower I:E ratio results in a lower alveolar ventilation^{1,28} (or in the terms of this study, a higher required fresh gas flow rate).

Since spontaneously breathing patients tend to breathe more rapidly, with lower V_T , and at lower I:E ratios than patients under controlled ventilation, they tend to require a higher fresh gas flow rate. Opposing this trend can be the reduced expiratory time constants generated by the equipment typically used with spontaneous ventilation. The balance of these three effects on the respiratory pattern of the typical patient has led clinicians to recommend higher fresh gas flows during spontaneous ventilation. These recommendations may not always be ap-

appropriate, for example, for the patient breathing spontaneously but with an atypical pattern of respiratory rate and tidal volume, or for a patient made to breathe spontaneously through a bellows assembly.

Summary

We have modeled the flow pattern of gases and the mass balance for CO₂ within the Mapleson D system and applied it to both controlled and spontaneous ventilation. Our experimental data show that the model describes some aspects of the breathing system well, and provides new insight into the mechanisms of its operation. Some of its deficits can be explained by the mixing that occurs in the breathing system.

The data and model revealed some clinically useful results:

- 1 In contrast to previous descriptions, the tidal volume does influence the alveolar ventilation during both re-breathing and non-rebreathing operation. Similar to previous descriptions, the respiratory rate is not influential during rebreathing operation and the fresh gas flow rate is not influential during non-rebreathing operation.
- 2 The fresh gas required to maintain P_{ACO₂} during re-breathing can be minimized by using the combination of a large tidal volume and a low respiratory rate. Clearly the fresh gas requirement of the Mapleson D breathing system cannot be expressed in terms of minute ventilation alone.
- 3 The limb volume influences the fresh gas required to maintain P_{ACO₂}. The most appropriate limb volume is in the order of the tidal volume minus dead space volume minus the fresh gas added during inspiration. A limb volume that is too small can greatly increase the demand for fresh gas, but too large a limb volume can only increase the demand by a small amount. The current limb volumes used clinically may be somewhat small for adults and thus result in an increase in the fresh gas flow rate required to maintain alveolar concentration. Doubling the limb volume for convenience, as when a ventilator is used, does not increase the fresh gas requirement substantially.
- 4 Increased expiratory resistance can increase the fresh gas flow rate required to maintain a given P_{ACO₂}. This may have implications for the management of patients with asthma or obstructive pulmonary diseases.
- 5 The V_T and f combination, I:E ratio, and time constant of expiration differ between spontaneous and controlled ventilation, and result in the familiar need for higher fresh gas flows during spontaneous ventilation in the typical patient.

There are many system and patient variables that influence the fresh gas requirement of the Mapleson D circuit. A spectrum of conflicting recommendations has

come from clinical studies, possibly because rigorous attention was not paid to some of the variables that we have shown to be important. Tidal volume and respiratory rate have traditionally been combined into minute volume and the influence of the limb volume is not usually considered. The mode of ventilation has been seen as a criterion for choosing the fresh gas flow rate, rather than the respiratory pattern observed in that mode of ventilation. All of these simplifications have led to ambiguity in the proper use of the Mapleson D breathing system. We hope that these studies provide a basis for the better understanding and rational use of these breathing systems.

Appendix

Expiratory flow, $\dot{V}_{\text{Ex}}$, decays exponentially from the peak flow rate with an appropriate time constant, (τ):

$$\dot{V}_{\text{Ex}}(t) = \frac{V_T}{\tau} e^{-t/\tau}$$

$$V_{\text{Ex}}(T) = \int_0^T \frac{V_T}{\tau} e^{-t/\tau} dt$$

This relation will be used to quantify expired volumes of gas that flow past the Y piece during finite time intervals.

The time duration of inspiration (in seconds) is defined as:

$$T_I = \frac{60}{f(1 + E/I)} \quad (\text{A1})$$

Where f has units of bpm.

The reservoir (bellows/bag) excursion, neglecting gas compression in the breathing system, is defined as:

$$V_b = V_T - \dot{V}_f T_I \quad (\text{A2})$$

The flow rate of gas delivered from the breathing system during inspiration, $\dot{V}_{\text{Is}}$, is given by:

$$\dot{V}_{\text{Is}} = \frac{V_T}{T_I} - \dot{V}_f \quad (\text{A3})$$

The model calculates the volume of CO₂ in all the gas contributing to inspiration and from that computes the alveolar concentration of CO₂, F_{ACO₂}. Rebreathing and CO₂ production are the only sources of CO₂ to the alveoli. It must be determined whether the limb or the reservoir contain alveolar gas at end expiration, and independently, the volume of gas inspired from these components to the alveoli.

Gas that enters the alveoli must come from either the limb, the reservoir, or the fresh gas added during inspiration:

$$V_T - V_D = V_1 + V_{lb} + \dot{V}_f \cdot T_{lb} \quad (A4)$$

The volume of gas from the reservoir inspired to the alveoli, V_{lb} , depends on the volume of fresh gas added during inspiration that reaches the alveoli. This volume of fresh gas is proportional to the time it takes from the start of inspiration for all of V_{lb} to flow past the Y piece, T_{lb} .

$$T_{lb} = \frac{V_1 + V_{lb}}{\dot{V}_{ls}} = \frac{(V_1 + V_{lb}) \cdot T_1}{V_b} \quad (A5)$$

By combining Equations A2–A5:

$$V_{lb} = V_T - V_D - V_1 - \dot{V}_f \cdot T_1 \cdot (1 - V_D/V_T) \quad (A6)$$

Gas that enters the reservoir during expiration can only come from the limb, the dead space, the alveoli or the fresh gas flow. The volume of alveolar gas directly expired to the reservoir, V_{Eb} , is determined through:

$$V_b = V_1 + V_D + V_{Eb} + \dot{V}_f \cdot T_{eb}$$

Substituting in Equation A2 and rearranging:

$$V_{Eb} = V_T - V_D - V_1 - \dot{V}_f \cdot (T_1 + T_{eb}) \quad (A7)$$

T_{Eb} , the time taken from start expiration for all of V_{Eb} to flow past the Y piece is found through:

$$V_{Eb} + V_D = \int_0^{T_{eb}} \frac{V_T}{\tau} e^{-t/\tau} dt$$

Carrying out the integration and substituting in Equations A2 and A7:

$$\dot{V}_f \cdot (T_1 + T_{eb}) + V_1 = V_T e^{-T_{eb}/\tau} \quad (A8)$$

Thus, V_{Eb} and T_{Eb} need to be determined computationally. Note that if $V_{Eb} < 0$ then no alveolar gas enters the reservoir and Equation A7 becomes the exact form of Equation 5 in the text.

By subtracting Equation A7 from Equation A6:

$$V_{lb} - V_{Eb} = \dot{V}_f \left(\frac{V_D}{V_T} T_1 + T_{eb} \right) > 0$$

$$\therefore V_{lb} > V_{Eb}$$

Therefore, the amount of gas inspired from the reservoir to the alveoli is always greater than the alveolar gas directly expired to the reservoir.

Four scenarios are possible:

- Zone I – alveolar gas is expired into the reservoir as well as the limb, and alveolar gas is re-inspired from the reservoir as well as the limb.
- Zone II – alveolar gas is expired into the limb but not the reservoir, and although gas is inspired from both the reservoir and the limb, only the gas re-inspired from the limb contains alveolar gas.

- Zone III – alveolar gas is expired to the limb only and alveolar gas is re-inspired from the limb only.
- Zone IV – no alveolar gas is re-inspired.

Zone I – alveolar gas is expired into the reservoir as well as the limb, and alveolar gas is re-inspired from the reservoir as well as the limb.

$$V_{Eb} > 0 \quad V_{lb} > 0$$

The concentration in the reservoir must be quantified. At end inspiration the limb is filled with reservoir gas and the dead space is filled with diluted reservoir gas. The gas from the limb that returns to the reservoir has not changed in concentration, but the gas that returns from the dead space has a concentration to be determined, F_D . A mass balance on gas returning to the bellows yields:

$$F_b(V_b + V_b) = F_{ACO_2} V_{Eb} + F_b V_1 + F_D V_D + F_b V_b + F_0 \dot{V}_f T_{eb} \quad (A9)$$

Where V_b is the balance of reservoir gas that does not exit during inspiration. $F_0 = 0$ since fresh gas contains no alveolar gas.

The concentration of the gas in the dead space at end inspiration is related to the reservoir concentration via:

$$F_D = F_b \frac{\dot{V}_{ls}}{\dot{V}_{ls} + \dot{V}_f} = F_b \left[1 - \frac{\dot{V}_f \cdot T_1}{V_T} \right] = F_b \frac{V_b}{V_T} \quad (A10)$$

The concentration of the gas in the reservoir at end expiration is related to the alveolar concentration by combining Equations A2, A6, A9, and A10:

$$F_b = \frac{V_{Eb} F_{ACO_2}}{V_b - V_1 - \frac{V_b V_D}{V_T}} = \frac{V_{Eb} F_{ACO_2}}{V_{lb}} \quad (A11)$$

The amount of CO_2 introduced to the alveoli from re-breathing is given by:

$$V_r \cdot F_{ACO_2} = (V_1 - \dot{V}_f \cdot T_{ee}) \cdot F_{ACO_2} + V_{lb} \cdot F_b \quad (A12)$$

The second term on the right side of the equality is the volume of CO_2 inspired from the reservoir. The first term is the volume of CO_2 rebreathed from the limb. Note that at end expiration the limb is completely filled with alveolar gas, except for the fresh gas added during the time interval it takes for the alveolar gas that will remain in the limb to pass the Y piece, T_{ee} :

$$V_1 = \dot{V}_f \cdot T_{ee} + \int_{T_{pe}-T_{ee}}^{T_{pe}} \frac{V_T}{\tau} e^{-t/\tau} dt \quad (A13)$$

where the time period of expiration, T_{pe} is given by:

$$T_{pe} = \frac{60}{f} - T_1$$

and by assuming that the expiratory flow is zero at end expiration:

$$\frac{V_T}{\tau} e^{-T_{pe}/\tau} \approx 0$$

T_{ee} can be solved numerically via:

$$V_l = \dot{V}_f \cdot T_{ee} + V_T e^{-(T_{pe}-T_{ee})/\tau}$$

By substituting Equation A12 into Equation 3 and dividing by f :

$$F_{ACO_2} (V_T - V_D) = V_{CO_2} + F_{ACO_2} (V_l - \dot{V}_f T_{ee}) + F_b V_{lb} \quad (A14)$$

The left side of this mass balance is the amount of CO_2 leaving the lung and the right side is the CO_2 added through metabolic production and rebreathing.

Note that V_{CO_2} is the volume of CO_2 produced per breath:

$$V_{CO_2} = \dot{V}_{CO_2}/f$$

Therefore:

$$F_{ACO_2} = \frac{V_{CO_2}}{\dot{V}_f \cdot (T_l + T_{ee} + T_{eb})} \quad (A15)$$

and

$$\dot{V}_f = \frac{V_{CO_2}}{F_{ACO_2} \cdot (T_l + T_{ee} + T_{eb})} \quad (I) \quad (A16)$$

Zone II - alveolar gas is expired into the limb but not the reservoir, and although gas is inspired from both the reservoir and the limb, only the gas reinspired from the limb contains alveolar gas.

$$V_{Eb} < 0 \quad V_{lb} > 0$$

$$F_b = 0$$

$$V_r = V_l - \dot{V}_f \cdot T_{ee} = V_T e^{-(T_{pe}-T_{ee})/\tau} \quad (A17)$$

The value of T_{ee} is the same as in zone I.

$$F_{ACO_2} (V_T - V_D) = V_{CO_2} + F_{ACO_2} (V_l - \dot{V}_f T_{ee}) \quad (A18)$$

Therefore:

$$F_{ACO_2} = \frac{V_{CO_2}}{V_T - V_D - V_l - \dot{V}_f \cdot T_{ee}} \quad (A19)$$

and

$$\dot{V}_f = \frac{V_{CO_2} - F_{ACO_2} (V_T - V_D - V_l)}{T_{ee} F_{ACO_2}} \quad (II) \quad (A20)$$

Zone III - alveolar gas is expired to the limb only and alveolar gas is reinspired from the limb only.

$$V_{Eb} < 0 \quad V_{lb} < 0$$

In contrast to zones I and II, all of the gas from the limb is not inspired to the alveoli. The volume of gas

inspired from the limb to the alveoli, V_{lb} , is the tidal volume minus the dead space and fresh gas delivered to the alveoli during inspiration (some fresh gas is left in the dead space at end inspiration)

$$V_{lb} = V_T - V_D - \dot{V}_f \cdot T_l \cdot \left[1 - \frac{V_D}{V_T} \right] \quad (A21)$$

This gas is composed of expired alveolar gas and fresh gas added during expiration, which is delivered at a constant flow rate. The fresh gas contribution to V_{lb} can be determined by finding the time duration taken at end expiration for all the alveolar gas in V_{lb} to pass the Y piece, T_{el} . By subtracting that fresh gas contribution to V_{lb} , the exact volume of alveolar gas reinspired is known:

$$V_r = V_T - V_D - \dot{V}_f \cdot T_l \cdot \left[1 - \frac{V_D}{V_T} \right] - \dot{V}_f \cdot T_{el} \quad (A22)$$

This rebreathed volume is also the last portion of alveolar gas expired to the limb under the time interval t_{el} . Therefore:

$$V_r = \int_{T_{pe}-T_{el}}^{T_{pe}} \frac{V_T}{\tau} e^{-t/\tau} dt$$

$$V_r = V_T e^{-(T_{pe}-T_{el})/\tau} \quad (A23)$$

By combining Equations A22 and A23, T_{el} can be determined computationally via:

$$V_T - V_D - \dot{V}_f \cdot T_l \cdot \left[1 - \frac{V_D}{V_T} \right] - \dot{V}_f \cdot T_{el} = V_T e^{-(T_{pe}-T_{el})/\tau} \quad (A24)$$

again assuming:

$$\frac{V_T}{\tau} e^{-T_{pe}/\tau} \approx 0$$

Adding the rebreathed volume of alveolar gas into the mass balance for CO_2 in the lungs yields:

$$F_{ACO_2} (V_T - V_D) = V_{CO_2} + F_{ACO_2} \left(V_T - V_D - \dot{V}_f \cdot T_l \cdot \left[1 - \frac{V_D}{V_T} \right] - \dot{V}_f \cdot T_{el} \right) \quad (A25)$$

Therefore,

$$F_{ACO_2} = \frac{V_{CO_2}}{\dot{V}_f [T_l (1 - V_D/V_T) + T_{el}]} \quad (A26)$$

and

$$\dot{V}_f = \frac{V_{CO_2}}{F_{ACO_2} [T_l (1 - V_D/V_T) + T_{el}]} \quad (III) \quad (A27)$$

Zone IV – no alveolar gas is reinspired. The expiratory velocity is assumed to decay exponentially, and as a consequence, expiratory flow will never reach zero. In the Mapleson D, this implies that there must always be some rebreathing. In practice, the breathing system will operate without rebreathing. Therefore, the model defines zone IV as meeting the same criteria as zone III ($V_{Eb} < 0$ and $V_{Ib} < 0$) and in addition, when a very small change in respiratory rate (0.25 bpm) produces an extremely large change in fresh gas requirement ($10 \text{ L} \cdot \text{min}^{-1}$). Alveolar concentration is assumed:

$$F_{\text{ACO}_2} = \frac{V_{\text{CO}_2}}{V_T - V_D}$$

F_{ACO_2} is independent of fresh gas flow.

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