HOW REBREATHING ANAESTHETIC SYSTEMS CONTROL P_{aCO_2} : STUDIES WITH A MECHANICAL AND A MATHEMATICAL MODEL

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REBREATHING ANAESTHETIC SYSTEMS without soda lime have been clinically feasible since Mapleson published his classic study of "semi-closed" systems in 1958.¹ Two systems now show promise because they are simple and familiar. These are the modified Mapleson "D" t-tube system suggested by Bain and Spoerel^{2,3} and the partial rebreathing circle system without soda lime suggested by Benson, *et al.*⁴ General acceptance of such systems, however, cannot be expected until their dynamics of carbon dioxide removal are understood. Mapleson predicted in his theoretical study that fresh gas flow, rather than alveolar minute volume, would determine alveolar carbon dioxide levels in a rebreathing system.¹ However, Bain and Spoerel's attempt to verify Mapleson's prediction³ met with disappointment. Apparently other factors besides fresh gas flow are involved. This study attempts to define and measure those factors.

METHODS

1. *Mathematical Model.* The equation derived by de Silva⁵ is used as a basis for a mathematical model:

$$P_{ACO_2} = \frac{\dot{V}_{CO_2}(P_B - P_{H_2O})}{\dot{V}_A} + \frac{\dot{V}_{CO_2}(P_B - P_{H_2O})}{\dot{V}_F} \times \frac{\dot{V}_E - \dot{V}_F}{\dot{V}_E} \quad (1)$$

where P_{ACO_2} = alveolar partial pressure of carbon dioxide, torr
 $\dot{V}_{CO_2}$ = carbon dioxide excretion, ml/min
 $\dot{V}_A$ = alveolar minute volume, ml/min
 $\dot{V}_E$ = expired minute volume, ml/min

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$\dot{V}_F$ = fresh gas inflow rate, ml/min

$P_B - P_{H_2O}$ = barometric less water vapor pressure, torr.

The first term on the right-hand side of the equation,

$$\frac{\dot{V}_{CO_2}(P_B - P_{H_2O})}{\dot{V}_A}$$

is the standard expression of the relationship between carbon dioxide excretion and alveolar ventilation at a given alveolar carbon dioxide concentration, with the term $(P_B - P_{H_2O})$ inserted to convert fractional carbon dioxide concentration to partial pressure. The second term,

$$\frac{\dot{V}_{CO_2}(P_B - P_{H_2O})}{\dot{V}_F}$$

is an expression for the P_{CO_2} of the mixed exhaled gas within the system which is available for rebreathing. The last term,

$$\frac{\dot{V}_E - \dot{V}_F}{\dot{V}_E}$$

expresses the degree of rebreathing which occurs at given values of $\dot{V}_E$ and $\dot{V}_F$. (For a formal derivation of this equation, see de Silva's paper.⁵)

Equation (1) was modified as follows:

Assuming

$$\dot{V}_A = \dot{V}_E(1 - V_D/V_T) \quad (2)$$

where V_D/V_T = physiologic dead space/tidal volume ratio, and

$$P_{ACO_2} = P_{aCO_2} \quad (3)$$

where P_{aCO_2} = arterial partial pressure of carbon dioxide, torr; then, substituting (2) and (3) into (1),

$$P_{aCO_2} = \frac{\dot{V}_{CO_2}(P_B - P_{H_2O})}{\dot{V}_E(1 - V_D/V_T)} + \frac{\dot{V}_{CO_2}(P_B - P_{H_2O})}{\dot{V}_F} \times \frac{\dot{V}_E - \dot{V}_F}{\dot{V}_E} \quad (4)$$

Equation (4) was written into a programme for use with a Hewlett-Packard HP-97 calculator, whereby P_{aCO_2} could be calculated for given values of $\dot{V}_{CO_2}$, $\dot{V}_D/\dot{V}_T$, $\dot{V}_E$ and $\dot{V}_F$. The term $(P_B - P_{H_2O})$ was assigned the value 713.

2. Mechanical Model. Figure 1 illustrates the mechanical model. An artificial "lung" was constructed of two anaesthesia breathing bags joined with a Y-connector. Carbon dioxide gas in metered quantities (a controllable " $\dot{V}_{CO_2}$ ") was flowed into the tail of one bag (the "alveolus"). A controllable "physiological dead space" was provided by the second bag, the volume of which could be changed by altering the position of a clamp. Values of $\dot{V}_D/\dot{V}_T$ were only roughly estimated.

The breathing system included an Air-Shields anaesthesia ventilator by which known minute volume was delivered. $\dot{V}_E$ was measured with a Wright ventilometer. $\dot{V}_F$ was provided as oxygen through the flowmeter of an Ohio "2000" series anaesthesia machine. To determine " P_{aCO_2} ", carbon dioxide concentration was measured with an infrared analyzer at the neck of the "alveolus" breathing bag. The analyzer was calibrated with known concentrations of carbon dioxide. Fractional concentrations were converted to partial pressure by multiplying with 713. The model system was completed by insertion of a commercially obtained Bain Breathing Circuit, as illustrated in Figure 1, or an anaesthetic circle absorber system of the Eger-Ethans type A (fresh gas inflow between inspiratory valve and patient),⁶ with soda lime excluded.

" P_{aCO_2} " was measured at various settings of $\dot{V}_D/\dot{V}_T$, $\dot{V}_{CO_2}$, $\dot{V}_E$ and $\dot{V}_F$. At least 15 minutes were allowed at each setting before P_{aCO_2} was measured. It was assumed that the model weighed 70 kg, and values for $\dot{V}_{CO_2}$, $\dot{V}_E$ and $\dot{V}_F$

were selected accordingly. For ease of comparison with the mathematical model and clinical data from the literature, results are expressed in terms of ml/kg.

RESULTS

Figure 2 demonstrates the relationship between P_{aCO_2} and $\dot{V}_F$ found with the mechanical and mathematical models. The circles represent actual measurements of P_{aCO_2} at various fresh gas flows ($\dot{V}_F$) with the mechanical model. $\dot{V}_{CO_2}$ was held constant at 2.25 ml/kg, $\dot{V}_D/\dot{V}_T$ at 0.5, and $\dot{V}_E$ at 140 ml/kg. These values of $\dot{V}_{CO_2}$ and $\dot{V}_D/\dot{V}_T$ are typical for anaesthetized, artificially ventilated humans.^{7,8} Data from both the Bain Breathing Circuit (open circles) and circle system (closed circles) are included.

The solid curve in Figure 2 represents calculated P_{aCO_2} values produced by the mathematical model at various assigned values of $\dot{V}_F$, when $\dot{V}_{CO_2} = 2.25$ ml/kg, $\dot{V}_D/\dot{V}_T = 0.5$, and $\dot{V}_E = 140$ ml/kg. Figure 2 demonstrates that the mathematical model, and the mechanical model with both breathing systems, performed identically.

To test whether this identity of performance holds when values of $\dot{V}_{CO_2}$, $\dot{V}_D/\dot{V}_T$, and $\dot{V}_E$ are varied, $\dot{V}_F$ was held constant at 70 ml/kg while each of those parameters was varied individually. Figure 3 compares results from the model using the Bain Circuit (circles) with those from the equation (lines). Again, nearly identical results were

FIGURE 1 Mechanical model. For details see text.

FIGURE 2 Relationship of P_{aCO_2} to $\dot{V}_F$ in re-breathing systems when $\dot{V}_{CO_2} = 2.25$ ml/kg, $\dot{V}_D/\dot{V}_T = 0.5$, $\dot{V}_E = 140$ ml/kg. Circles: Data from mechanical model with Bain circuit (open circles) and circle system without soda lime (closed circles). Solid curve: Results with mathematical model.

FIGURE 3 Effect of changes in (A) $\dot{V}_{CO_2}$, (B) $\dot{V}_D/\dot{V}_T$, (C) $\dot{V}_E$. For all curves, $\dot{V}_F = 70$ ml/kg. Except as specified otherwise, $\dot{V}_{CO_2} = 2.25$ ml/kg, $\dot{V}_D/\dot{V}_T = 0.5$, $\dot{V}_E = 140$ ml/kg. Circles: Data from mechanical model with Bain circuit. Solid curves: Results with mathematical model.

obtained from the mathematical and mechanical models.

Figure 4 compares the results of the mathematical model (solid line) with the clinical results obtained by Bain and Spoerel (dashed line) from anaesthetized, artificially ventilated patients.³ A close similarity is obvious.

DISCUSSION

For comparison with the models in this study, the clinical data of Bain and Spoerel³ were used. In addition, almost identical clinical data were obtained by Benson, *et al.*⁴ and by Akkineni, *et al.*,⁹ using the circle system without soda lime.

FIGURE 4 Comparison of model studies with clinical data. Solid line: From mathematical model when $\dot{V}_{CO_2} = 2.25$ ml/kg, $\dot{V}_D/\dot{V}_T = 0.5$, $\dot{V}_E = 140$ ml/kg. Dashed line: From clinical data of Bain and Spoerel.³

Slightly higher P_{aCO_2} values for comparable fresh gas flows were reported by Henville and Adams¹⁰ with a Mapleson D system, while lower P_{aCO_2} values were obtained by de Silva⁵ with a circle system. Chu, *et al.*,¹¹ using the Bain Breathing Circuit, obtained an average P_{aCO_2} within 3 torr of that which the mathematical model would predict from their values of $\dot{V}_F$ and $\dot{V}_E$.

With identical results from the mathematical equation and mechanical model and a reasonable consistency in duplicating clinical data, it is safe to conclude that the equation proposed by de Silva, as modified, is an accurate mathematical representation of rebreathing anaesthetic systems without soda lime. Certain important inferences may be drawn from this study.

It has often been suggested that P_{aCO_2} is independent of changes in ventilation (provided ventilation exceeds normal) with rebreathing systems.^{1,2,3,9,12} Figure 3C demonstrates that this is not precisely correct. It is only when $\dot{V}_E$ exceeds 160 ml/kg, which is nearly double the normal figure, that P_{aCO_2} becomes relatively insensitive to changes in $\dot{V}_E$. Thus, if precision control of P_{aCO_2} is desired, as in neuroanaesthesia, then $\dot{V}_E$ must be measured and controlled in rebreathing systems, just as in nonrebreathing systems. The fact that Bain and Spoerel did not precisely specify a value for $\dot{V}_E$ in their clinical studies may account for the slight discrepancy seen between their data and those from the models in this study. Bain and Spoerel recommended tidal volumes of at least 10 ml/kg, with respiratory rates of 12 to 16 per minute. Thus, $\dot{V}_E$ could have been from 120 to over 160 ml/kg in their patients, whereas in this study $\dot{V}_E$ was fixed at 140 ml/kg for the data presented in Figures 2 and 4. It is also probable that the scatter in the data reported by Bain and Spoerel would have been less had a fixed value of $\dot{V}_E$ been used.

It is not surprising that $\dot{V}_{CO_2}$ and $\dot{V}_D/\dot{V}_T$ would affect the P_{aCO_2} produced by a rebreathing system, as they play a role in all ventilatory systems. It is well known that these variables are not exactly predictable.¹³ Therefore a certain imprecision in estimating P_{aCO_2} with rebreathing systems is an inevitable fact of clinical life. But with the other factors ($\dot{V}_F$ and $\dot{V}_E$) controlled, the imprecision should be no greater than with other methods of artificial ventilation.

The variable $\dot{V}_D/\dot{V}_T$ provides a key to the understanding of how rebreathing systems work. Bain and Spoerel, following the prediction that fresh gas flow controls P_{aCO_2} , proposed that the equation

$$Pa_{CO_2} = \frac{\dot{V}_{CO_2}}{\dot{V}_F} (P_B - PH_2O) \quad (5)$$

should be valid for a rebreathing system.³ Yet when they compared theoretical calculations from this equation with their own clinical results, a large discrepancy was seen, for which the authors had "no satisfactory explanation." It can now be shown that physiological dead space, or wasted ventilation,* can provide the explanation.

Figure 5 demonstrates the effect of wasted ventilation. Using the mathematical model (equation 4), the relationship between $\dot{V}_F$ and Pa_{CO_2} was calculated at two values of V_D/V_T when $\dot{V}_{CO_2} = 2.25$ ml/kg and $\dot{V}_E = 140$ ml/kg. The solid curve ($V_D/V_T = 0.5$) is a duplicate of that in Figure 4 which was used for comparison with the clinical data of Bain and Spoerel.³ The dashed curve ($V_D/V_T = 0$) describes what would occur if there were no wasted ventilation. Interestingly, the dashed curve is also produced by equation (5) when the same value for $\dot{V}_{CO_2}$ (2.25 ml/kg) is used. Furthermore, equation (4) can be reduced algebraically to equation (5) when $V_D/V_T = 0$. Therefore, the discrepancy between what Bain and Spoerel predicted with equation (5) and what actually occurs with a rebreathing system may be accounted for by the presence of wasted ventilation.

It appears that wasted ventilation makes $\dot{V}_F$ inefficient in its removal of carbon dioxide. For example, Figure 5 demonstrates that when $V_D/V_T = 0$, a $\dot{V}_F$ of only 65 ml/kg is required to produce a Pa_{CO_2} of 25 torr. Yet when $V_D/V_T = 0.5$ almost twice as much $\dot{V}_F$ (120 ml/kg) is required; obviously only half the fresh gas reaches actively exchanging alveoli when $V_D/V_T = 0.5$. The other half reaches underperfused or unperfused alveoli and is "wasted". What is less obvious, but just as important to the understanding of rebreathing systems, is that the proportion of $\dot{V}_F$ wasted with a given V_D/V_T is not always the same. Figure 5 shows that the two curves tend to converge as $\dot{V}_F$ decreases, suggesting that the effect of wasted ventilation is progressively diminished as the value for $\dot{V}_F$ is reduced. Thus, for example when

FIGURE 5 Effect of dead space on performance of rebreathing systems. For both curves $\dot{V}_{CO_2} = 2.25$ ml/kg, $\dot{V}_E = 140$ ml/kg.

$V_D/V_T = 0$, a $\dot{V}_F$ of 30 ml/kg produces a Pa_{CO_2} of 50 torr. Yet when $V_D/V_T = 0.5$, a $\dot{V}_F$ of only 42 ml/kg produces the same Pa_{CO_2} , although one would expect the required $\dot{V}_F$ to be 60 ml/kg, if half the $\dot{V}_F$ were wasted. Obviously, some factor is at work at low values of $\dot{V}_F$ which directs a portion of the "wasted" $\dot{V}_F$ back into effectively exchanging alveoli.

This factor is rebreathing. Some of the fresh gas ($\dot{V}_F$) which is initially breathed into unperfused alveoli may be re-breathed into effectively exchanging alveoli. The extent to which this phenomenon occurs will be determined by the extent to which rebreathing occurs. The fraction (from equation 4)

$$\frac{\dot{V}_E - \dot{V}_F}{\dot{V}_E}$$

defines the extent of rebreathing. Thus when $\dot{V}_E$ is held constant a decrease in $\dot{V}_F$ will bring about a proportionate increase in rebreathing. This explains why the two curves in Figure 5 tend to converge at low values of $\dot{V}_F$.

Therefore, the dynamics of carbon dioxide control in a rebreathing system may be characterized as an interaction of wasted ventilation and rebreathing on the effect of fresh gas flow. Wasted ventilation tends to diminish and rebreathing tends to enhance the efficiency of fresh gas flow in the removal of carbon dioxide from a rebreathing system.

SUMMARY

Results from a proposed equation for rebreathing systems,

*The term "wasted ventilation," for the purposes of this discussion, may be considered identical to physiological dead space, as described by West.¹⁴ Briefly, it is "made up of contributions from alveoli which have ventilation-perfusion ratios exceeding that of ideal alveoli, and the volume of gas contained in the anatomic dead space."

$$Pa_{CO_2} = \frac{\dot{V}_{CO_2}(P_B - P_{H_2O})}{\dot{V}_E(1 - V_D/V_T)} + \frac{\dot{V}_{CO_2}(P_B - P_{H_2O})}{\dot{V}_F} \times \frac{\dot{V}_E - \dot{V}_F}{\dot{V}_E}$$

$\dot{V}_E$ = Ventilation-minute
 V_D/V_T = Rapport espace mort/volume courant
 $\dot{V}_F$ = Apport de gaz frais-minute
 $P_B - P_{H_2O}$ = Pression barométrique - Pression de vapeur d'eau.

were compared with results from a mechanical model "lung" ventilated either with a Bain Breathing Circuit, or a circle system (Eger-Ethans type A) without soda lime. When values for carbon dioxide excretion ($\dot{V}_{CO_2}$), dead space-tidal volume ratio (V_D/V_T), minute volume ventilation ($\dot{V}_E$), and fresh gas flow ($\dot{V}_F$) were varied over a wide range, the model and the equation yielded identical values of Pa_{CO_2} .

When $\dot{V}_{CO_2} = 2.25$ ml/kg, $V_D/V_T = 0.5$, and $\dot{V}_E = 140$ ml/kg, then fresh gas flows ($\dot{V}_F$) with both the equation and the model produced values of Pa_{CO_2} which were very close to those found by Bain and Spoerel in anaesthetized, artificially ventilated humans. It is concluded that the equation is an accurate mathematical representation of how rebreathing anaesthetic systems control Pa_{CO_2} .

It is expected that the equation will be useful in the clinical application of rebreathing anaesthetic systems, allowing the selection of minute volumes and fresh gas flows which will yield predictable Pa_{CO_2} values.

RÉSUMÉ

Les auteurs proposent un modèle mathématique représentatif du contrôle de la PCO_2 dans les circuits anesthésiques avec réinspiration. L'usage d'un modèle mécanique de poumon, ventilé au moyen d'un circuit de Bain ou d'un circuit sans réinspiration (Eger-Ethans) a permis de vérifier la validité de l'équation.

La formule proposée est la suivante:

$$Pa_{CO_2} = \frac{\dot{V}_{CO_2}(P_B - P_{H_2O})}{\dot{V}_E(1 - V_D/V_T)} + \frac{\dot{V}_{CO_2}(P_B - P_{H_2O})}{\dot{V}_F} \times \frac{\dot{V}_E - \dot{V}_F}{\dot{V}_E}$$

Pa_{CO_2} = Pression partielle de la PCO_2 artérielle

$\dot{V}_{CO_2}$ = Excrétion-minute de CO_2

Cette équation apparaît d'utilité clinique car elle permet la sélection des volumes de ventilation-minute et des débits de gaz frais nécessaires pour maintenir la PCO_2 à des niveaux prévisibles.

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