SECTION 5 OF "SOME APPLICATIONS OF LOEWNER'S ORDERING ON SYMMETRIC MATRICES"

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Soon after the above titled paper was published (this Annals, Vol. 19, No. 2 (1967), 245-259), the author found that there was a big mistake, that is, Lemmas 5.1 and 5.2 do not hold, so the whole of Section 5 should be canceled.

In fact, for any vector with $aa'=1$, we have

$$\begin{aligned}\frac{a'\Sigma xx'\Sigma a}{x'\Sigma x} &= \frac{a'\Sigma^{1/2}yy'\Sigma^{1/2}a}{y'y} & (\Sigma^{1/2}x=y) \\ &= \frac{y'\Sigma^{1/2}aa'\Sigma^{1/2}y}{y'y} \\ &\leq \max ch(\Sigma^{1/2}aa'\Sigma^{1/2}) = ch(a'\Sigma a) = a'\Sigma a \leq \lambda_1\end{aligned}$$

where ch denotes a characteristic root and λ_1 is the largest characteristic root of Σ . The equality in the first inequality is attained by $y=\Sigma^{1/2}a$ or $x=a$ for fixed a , which depends on a , and the equality in the second inequality can not be attained generally except when a is equal to the characteristic vector corresponding to λ_1 .