Addendum to the article *Some Unsolved Problems in Number Theory* by K Ramachandra in *Resonance* Vol.2, No.5, May 1997, pp.77–80

1. S S Pillai – The omission of the name S S Pillai (Siva Sankaranarayana Pillai) in connection with Waring's problem is very serious. In a series of papers, Pillai proved that if $k \geq 6$ and further if $(3^k + 1)/(2^k - 1) \leq [1.5^k] + 1$ then Waring's Conjecture is correct for that k . Around the same time (but a little later) L E Dickson proved this with $k \geq 7$ and $(3^k + 1)/(2^k - 1) \leq [1.5^k] + 1$. The inequality $(3^k + 1)/(2^k - 1) \leq [1.5^k] + 1$ was proved for all integers exceeding a certain constant C (same as in para on Waring's problem) by K Mahler. The history of this discovery is very well explained in G H Hardy and E M Wright, *Introduction to the Theory of Numbers* (see notes at the end of the chapter XXI). For another treasure house of information regarding priority of Pillai's work see K Chandrasekharan, S S Pillai (obituary), *J. Indian Math. Soc.*, Vol.15, pp.1–10, 1951. Regarding Pillai's achievements I mention the following: when I was in the Institute for Advanced Studies, Princeton, USA, during 1970–71, I noticed in the Institute Library a book by G H Hardy where he places Pillai as the greatest Indian mathematician after Srinivasa Ramanujan. Waring's Conjecture was proved for $k=5$ by Chen-Jing-Run (around 1970) and for $k=4$ by R Balasubramanian, J-M Deshouillers and F Dress in 1989. Cases $k=2$ and 3 were disposed off (by simpler methods) by Lagrange and Wieferich respectively. About Pillai I have the following comment: Once I was talking to a responsible Indian specialist dealing with History of Mathematics. I was very surprised when I came to know that he had not heard of Pillai at all. I can account for it as follows. Pillai was very unassuming; he was a member of the Indian Mathematical Society alright; but he was not a fellow of any of the academies and he had no publicity whatsoever amongst mathematicians who had not looked at the book by G H Hardy and E M Wright mentioned earlier.

2. Last line in the text on page 78 should read $\left| \sum_{1 \leq n \leq x} \mu(an+b) \right| \leq x^h$.

3. A comment on *The Circle Method* in the box on page 78.

The function $f(z)$ is analytic in $|z| < 1$ and it does not exist anywhere in $|z| \geq 1$. (So the terminology *poles of $f(z)$* is not correct). We have to make r a suitable function of n but still less than 1. Then decompose this circle into small bits in a particular way and obtain asymptotics of each bit. The cumulative effect of adding all these asymptotics will give the Hardy–Ramanujan formula for partitions. Actually Ramanujan in his first letter (this letter was

written from Madras Port Trust) to Hardy mentions (see equation 1.14 of *Twelve Lectures*), that the integer $q(n)$ defined by

$$\left(\sum_{n=-\infty}^{\infty} (-x)^{n^2} \right)^{-1} = \sum_{n=0}^{\infty} q(n) x^n$$

(note that LHS is the product $\prod_{h=1}^{\infty} \{(1-x^h)(1-x^{2h-1})\}^{-1}$) is the integer nearest to

$$\frac{1}{2} \frac{d}{dn} \left(\frac{\sinh(\pi \sqrt{n})}{\pi \sqrt{n}} \right).$$

When questioned about this he wrote in a letter that it is "not the integer nearest to but this main term plus ...". (Compare this main term with the first term of the Hardy–Ramanujan–Rademacher formula for $p(n)$).

K Ramachandra

Errata

Resonance, Vol.2, No.3, March 1997

Page 88: Line 8, first paragraph in the text

...which began a thousand years...

should read ...which began a million years...

Resonance, Vol.2, No.5, May 1997

Page 76: Line 6 in the text

...in these m trials.

should read ...in these n trials.

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