Commutation Properties of the Nonleptonic Weak Hamiltonian and K Decays.

Erratum.

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Summary.—Errors in a previous paper about the application of the soft-pion method to $K \rightarrow 3\pi$ decays are corrected. The conclusions about the $\Delta T = \frac{3}{2}$ contribution are changed; this improves the prediction for the $K^+ \rightarrow \pi^+\pi^+\pi^-$ spectrum.

Our previous paper ⁽¹⁾ contained errors affecting the $\Delta T = \frac{3}{2}$ contributions. In eqs. (2) and (3) the expressions for $\langle \pi^-\pi^+(\pi^+) | H | K^+ \rangle$ and $\langle \pi^0\pi^0(\pi^+) | H | K^+ \rangle$ have the wrong sign. Equations (5) and (6) then become

$$\begin{aligned}
 (5) \quad & \left\{ \begin{aligned} A_0(+ + -) &= \frac{c}{3\sqrt{2}} A(+ -), \\ A_0(+ 0 0) &= \frac{c}{6} [|A(0 0)| + \sqrt{2}\eta \cos\delta |A(+ 0)|], \\ A_0(+ - 0) &= \frac{c}{6} [|A(+ -)| - 2\eta \cos\delta |A(+ 0)|], \\ A_0(0 0 0) &= \frac{c}{\sqrt{12}} A(0 0); \end{aligned} \right. \\
 (6) \quad & \left\{ \begin{aligned} a(+ + -) &\simeq \frac{1}{8} [1 - 6\eta \cos\delta |A(+ 0)| / |A(+ -)|], \\ a(+ 0 0) &\simeq -\frac{1}{4} [1 + \frac{3}{2} \sqrt{2}\eta \cos\delta |A(+ 0)| / |A(0 0)|], \\ a(+ - 0) &\simeq -\frac{1}{4} [1 + 3\eta \cos\delta |A(+ 0)| / |A(+ -)|], \\ a(0 0 0) &= 0. \end{aligned} \right.
 \end{aligned}$$

⁽¹⁾ D. K. ELIAS and J. C. TAYLOR: *Nuovo Cimento*, **44**, 518 (1966).

If we assume the $\Delta T = \frac{1}{2}$ or $\frac{3}{2}$ rule⁽²⁾, eq. (11) of ref. (1), eq. (5) becomes⁽³⁾

$$(5') \quad \left\{ \begin{array}{l} A_0(+ + -) = \frac{c}{3\sqrt{2}} A(+ - -), \\ A_0(+ 0 0) = -\frac{c}{6\sqrt{2}} A(+ - -), \\ A_0(+ - 0) = -\frac{c}{3\sqrt{2}} A(0 0 0), \\ A_0(0 0 0) = \frac{c}{\sqrt{12}} A(0 0 0). \end{array} \right.$$

Equation (5') gives the following predictions, independent of the unknown phase δ :

$$(7) \quad \left\{ \begin{array}{l} |A_0(+ + -)| = 0.33, \\ |A_0(+ 0 0)| = 0.17, \\ |A_0(+ - 0)| = 0.23, \\ |A_0(0 0 0)| = 0.28. \end{array} \right.$$

These values each fall significantly below the experimental values in eq. (8) of ref. (1). If the theoretical Goldberger-Treiman value $c = Gg_V/(mg_A)$ were replaced by the experimental value $c = f_\pi^{-1}$ the agreement would be worse.

Turning now to the slope parameters, eq. (6) gives

$$(9) \quad \left\{ \begin{array}{l} a(+ + -) = 0.125 - 0.03\eta \cos \delta, \\ a(+ 0 0) = -0.25 - 0.03\eta \cos \delta, \\ a(+ - 0) = -0.25 - 0.03\eta \cos \delta, \\ a(0 0 0) = 0. \end{array} \right.$$

Thus $a(+ + -)$ is rather sensitive to the $\Delta T = \frac{3}{2}$ contribution, which could alter it by up to 25 %. This stems from the large numerical coefficient 6 in the first of eqs. (6). This $\Delta T = \frac{3}{2}$ correction *might* exceed the errors inherent

(2) That is, assuming a current $\times$ current weak Hamiltonian with $\Delta T = \frac{1}{2}$ or $\frac{3}{2}$. If the $K^+ \rightarrow \pi^+ \pi^0$ decay is electromagnetic, then there is no reason to suppose that the soft-pion method works for the $\Delta T = \frac{3}{2}$ corrections.

(3) These equations have been given by S. K. BOSE and S. N. BISWAS: *Phys. Rev. Lett.*, **16**, 330 (1966). One of us (J.C.T.) is grateful to Dr. BOSE for a conversation which ought to have made us discover the error in ref. (1) earlier.

in the method (though the central amplitudes A_0 in (7) are in error by about 15%). The experimental values for the a 's (including a new result ⁽⁴⁾ for $a(+ + -)$) are respectively

$$(10) \quad 0.103 \pm 0.010, \quad -0.25 \pm 0.02, \quad -0.24 \pm 0.02.$$

A value for $\eta \cos \delta$ of about ± 0.4 brings each of (9) into within 1 standard deviation of (10). This positive value of $\eta \cos \delta$ is consistent with the conclusion from $K \rightarrow 2\pi$ decays, eq. (12) of ref. (1). The chief point of this erratum is that the correct sign in the first equation (9) makes it consistent with eq. (12).

Note added in proof.

Using recent data on the $K_1^0 \rightarrow \pi^+\pi^-$, $\pi^0\pi^0$ branching ratio (A. H. ROSENFELD, *et al.*: *Data on Particles and Resonant States*, *Rev. Mod. Phys.*, January 1967), the $\Delta T = \frac{1}{2}$ or $\frac{3}{2}$ rule, eq. (11) of ref. (11), gives $\eta \cos \delta = 0.7 \pm 0.3$, which is stronger evidence for a positive value of $\eta \cos \delta$ than the older data in eq. (12) of ref. (1). We are indebted to Dr. P. B. JONES for pointing this out to us.

⁽⁴⁾ S. Y. FUNG, *et al.*: *Study of a Sample of K^+ Tau Decays from Rest*, reported at the *XIII International Conference on High-Energy Physics at Berkeley* (1966). We are grateful to Dr. D. G. SUTHERLAND for calling our attention to this report.

RIASSUNTO (*)

Si correggono gli errori di un articolo precedente sull'applicazione del metodo del pione molle ai decadimenti $K \rightarrow 3\pi$. Le conclusioni sul contributo $\Delta T = \frac{3}{2}$ cambiano; ciò migliora la predizione sullo spettro $K^+ \rightarrow \pi^+\pi^+\pi^-$.

(*) *Traduzione a cura della Redazione.*

Коммутационные свойства нелептонного слабого гамильтониана и K-распады. — Опечатка.

Резюме (*). — Исправляются ошибки в предыдущей статье, связанные с применением метода мягких пионов к распадам $K \rightarrow 3\pi$. Изменяются заключения относительно вклада $\Delta T = \frac{3}{2}$; это улучшает предсказания для спектра $K^+ \rightarrow \pi^+\pi^+\pi^-$.

(*) *Переведено редакцией.*