Path Integrals in Stochastic Electrodynamics.

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In eq. (15) of the paper a factor $\delta(p - p_0)$ is lacking. Below that equation, there is a misprint in the expression of Δx for $T \gg \tau$, which should read

$$\Delta x \simeq (\hbar\tau/\pi m) \log(T/\tau).$$

Note also that Δx is defined in the paper as half the variance. For very short times, of order τ , the basic approximations of the calculation (*e.g.* eq. (9)) do not hold. Therefore, the value of Δx for $T \ll \tau$, quoted in the paper, is not meaningful.

In eq. (14) and in the definitions of x_c , p_c , γ the parameter τ should be replaced by $\tau/2$ everywhere. For instance, γ should read

$$\gamma \equiv [1 - \exp[-\tau\omega_0^2 T/2]]^2.$$

Also, the term $\tau\omega_0(x - x_c)(p - p_c)$ should be removed, because other small terms of the same order of magnitude have been neglected. Besides, it is necessary to specify the range of validity of the different expressions. In fact, there are three characteristic times involved: $\tau \ll \omega_0^{-1} \ll (\tau\omega_0^2)^{-1}$. Equation (14), corrected as above, is only valid for times, T , of order $(\tau\omega_0^2)^{-1}$ or larger. For shorter times the correct expression is rather cumbersome except for times of order ω_0^{-1} or shorter (but $T \gg \tau$). For times of order ω_0^{-1} the following approximate expression holds:

$$(16) \quad \varrho(x, p, T|x_0, p_0, 0) = (1/D) \exp[-(x - x_c)^2/A - (p - p_c)^2/B - (x - x_c)(p - p_c)/C],$$

where $x_c(T)$ and $p_c(T)$ are the classical paths as in the paper, and

$$A \equiv (2\hbar\tau/\pi m) \log(1/\tau\omega_0),$$

$$B \equiv m^2\omega_0^2 A \sin^2(\omega_0 T)[1 + \cos^2(\omega_0 T)]^{-1},$$

$$C \equiv (m\omega_0 A/2) \operatorname{tg}(\omega_0 T),$$

$$D \equiv \pi m\omega_0 A \sin(\omega_0 T).$$

For times $T \ll \omega_0^{-1}$ but $T \gg \tau$, the probability distribution of the oscillator is the same as the one of the free particle, eq. (15) (which, in fact, almost coincides with eq. (16) for these times; note that $\log(\tau\omega_0) \simeq \log(\tau/T)$ in this case). This should be expected because, for short times, the relevant Fourier components of the random force are the ones of high frequency. But, in view of the spectrum of our random force (eq. (13)), these components are so strong that every charged particle behaves as free. On the other hand, for larger time intervals, the effect of the high-frequency components of the random-force averages out to a small value and the deterministic forces are quite relevant.

Our results do not agree with the ones reported by MARSHALL⁽¹⁾, obtained by a different method. However, Marshall's calculations are valid only for times $T \gg (\tau\omega_0^2)^{-1}$ because some of the approximations involved are only correct at lowest order in the small quantity $\tau\omega_0$. For these times Marshall's calculations agree with ours.

In eq. (3), the first y should read x .

(1) T. W. MARSHALL: *Proc. Roy. Soc.*, **276 A**, 475 (1963).