Particle Fluctuations in Bose Statistics.

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(ricevuto il 29 Ottobre 1968)

In a recent article by DUNNING-DAVIES ⁽¹⁾ it was pointed out that the some of the current formulae of quantum statistics—as derived from the grand canonical ensemble—may fail in some particular cases.

This brought to my mind an observation which probably has been made by many people, *viz.* that the well-known expression ⁽²⁾

$$(1) \quad \frac{\overline{(n_s - \bar{n}_s)^2}}{(\bar{n}_s)^2} = \frac{1}{\bar{n}_s} \pm 1$$

for the fluctuation of the occupation number in Bose-Einstein (+) and Fermi-Dirac statistics (−) is not valid for the lowest quantum state ($s = 0$).

That is, for Fermi statistics the relation (1) is all right, since there the fluctuation goes to zero with temperature as it must. For Bose statistics however, the formula above would indicate a fluctuation of absolute order $n^{\frac{1}{2}}$ persisting right down to the absolute zero. This is of course not true, and it may be one of the very few cases where the grand canonical ensemble does not give the correct result.

Here one actually has to take the constancy of the total particle number (N) more accurately into account. When this is done one finds that, at the lowest temperatures, the relative fluctuation in the occupation number of the ground state is given by

$$(2) \quad \frac{\overline{(n_0 - \bar{n}_0)^2}}{(\bar{n}_0)^2} = \frac{N}{\bar{n}_0} - 1$$

instead of formula (1) with the + sign. And this fluctuation goes to zero with the temperature, as it should.

⁽¹⁾ J. DUNNING-DAVIES: *Nuovo Cimento*, **57** B, 315 (1968).

⁽²⁾ Any text, *e.g.* D. TER HAAR: *Elements of Thermostatistics* (New York, 1965), p. 235.